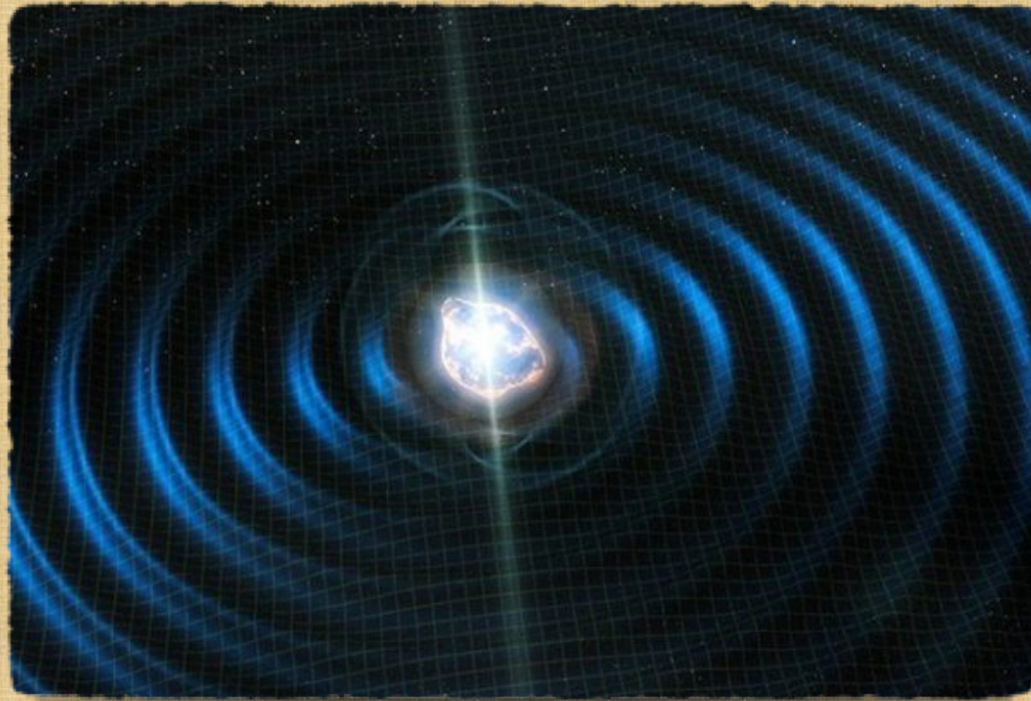


Neutron star physics with gravitational waves

Cristiano Palomba – INFN Roma
(on behalf of the LIGO-Virgo-KAGRA Collaborations)



Gravitational Waves

Spacetime tells matter how to move; matter tells spacetime how to curve. (J. A. Wheeler)

Space-time curvature

Matter-energy

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu}$$

A. EINSTEIN

- GWs are perturbative solutions of the GR equations in vacuum

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1$$



$$\left(-\frac{1}{c^2} \frac{\partial^2}{\partial t^2} + \nabla^2 \right) h_{\mu\nu} = 0$$

Wave equation



TT-gauge

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & h_{yy} & h_{yz} \\ 0 & 0 & h_{yz} & -h_{yy} \end{pmatrix}$$

Two independent degrees of freedom (often called + and x)

Source (rough) classification

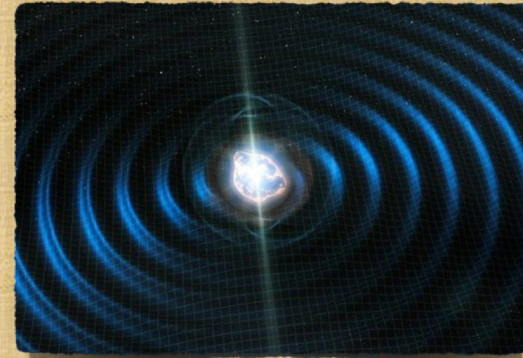
Coalescing compact binaries
(black holes, neutron stars)



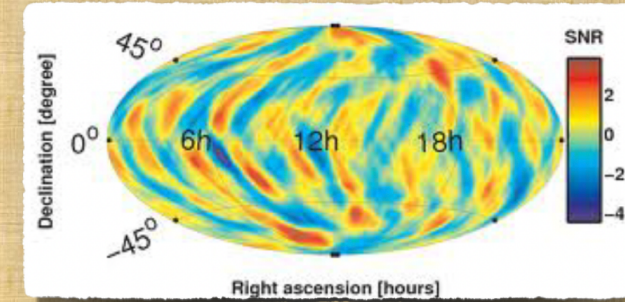
Supernova explosions



CW (e.g. spinning neutron stars)



Stochastic background
(Astrophysical, Cosmological)



Transient signals (from ms to minutes)

Persistent signals (from hours to "infinite" duration)

$$\text{GW strain tensor } h_{ij}(t, \vec{x}) = \frac{2G}{rc^4} \ddot{Q}_{ij} \left(t - \frac{r}{c} \right)$$

$$Q_{ij} = \int \rho(t, \vec{x}) \left(x_i x_j - \frac{1}{3} r^2 \delta_{ij} \right) d^3x$$

Quadrupole mass moment

Compact binaries is the only kind of source detected so far, but many others are expected to exist!

50+ years of hard experimental work

Since the pioneering work of Joseph Weber in the '60, the search for GWs has never stopped, with an increasing effort of manpower and ingenuity



60': Joe Weber pioneering work



90': Cryogenic Bars



Edoardo Amaldi & Guido Pizzella



2000' - : Large Interferometers



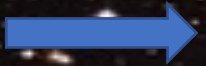
Rai Weiss,
Kip Thorne



Alain Brillet,
Adalberto Giazzotto

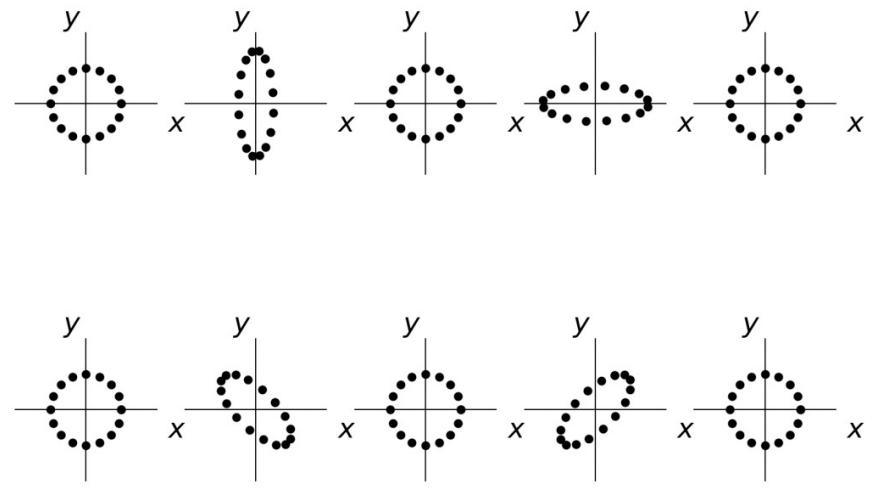
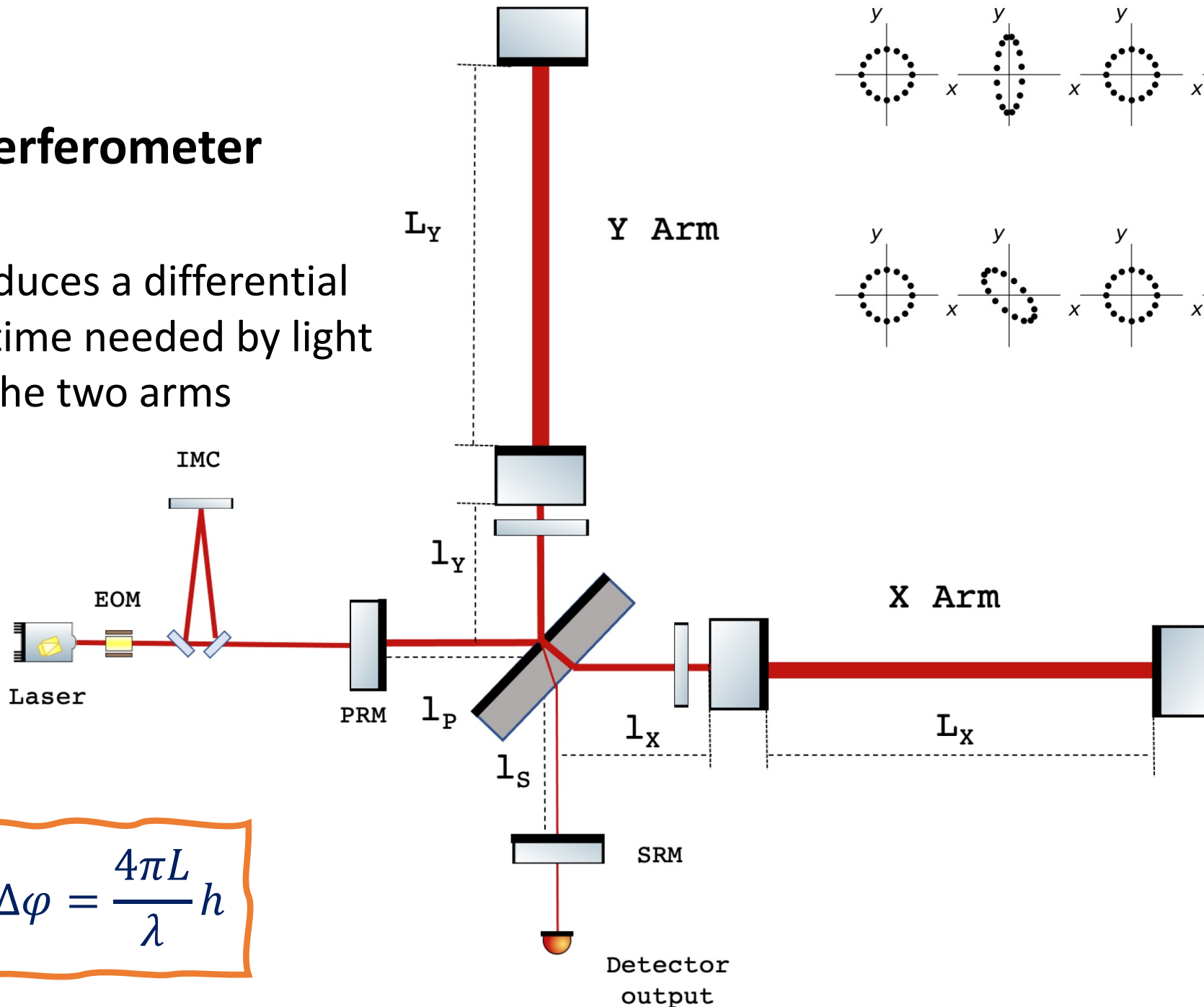


First detection: Sept. 14th 2015



Basic concept:
Michelson interferometer

A passing GW induces a differential variation of the time needed by light to travel across the two arms

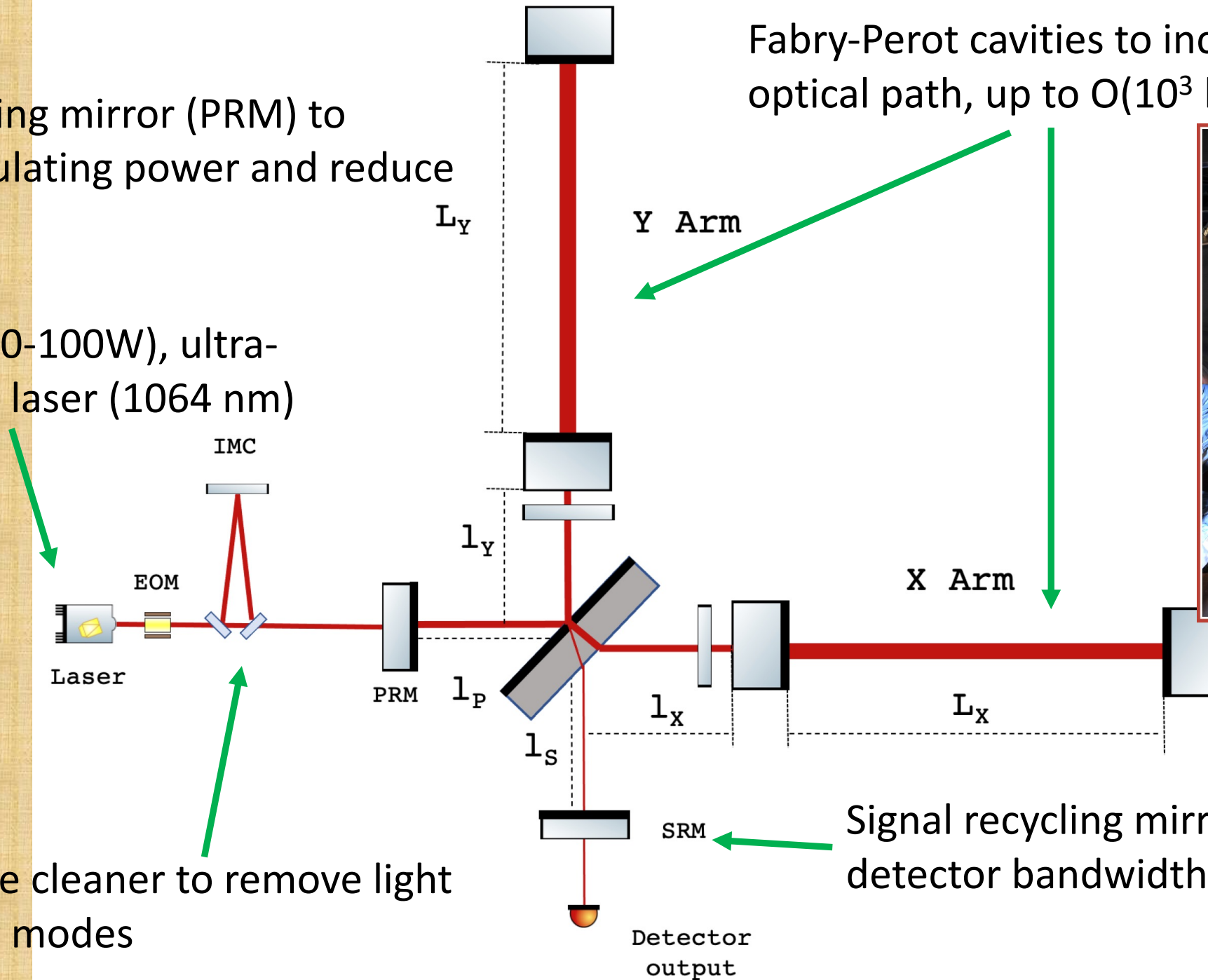


$$\Delta\tau = \frac{2L}{c} h \rightarrow \Delta\phi = \frac{4\pi L}{\lambda} h$$

Power recycling mirror (PRM) to increase circulating power and reduce shot noise

High power (10-100W), ultra-stable Nd:YAG laser (1064 nm)

Input mode cleaner to remove light high-order modes

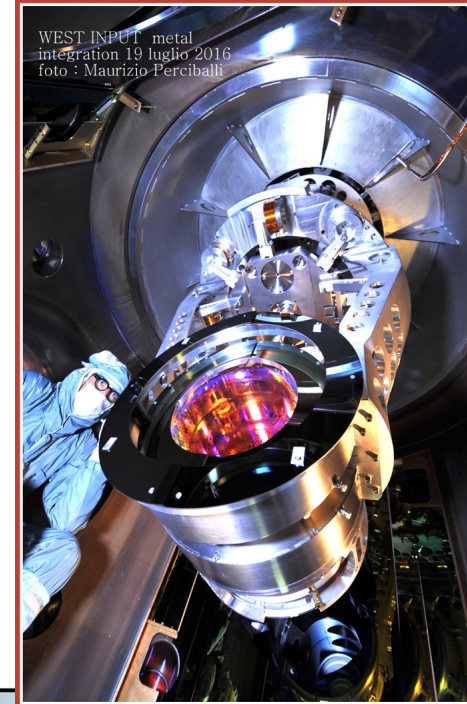


Fabry-Perot cavities to increase the light optical path, up to $O(10^3 \text{ km})$

Y Arm

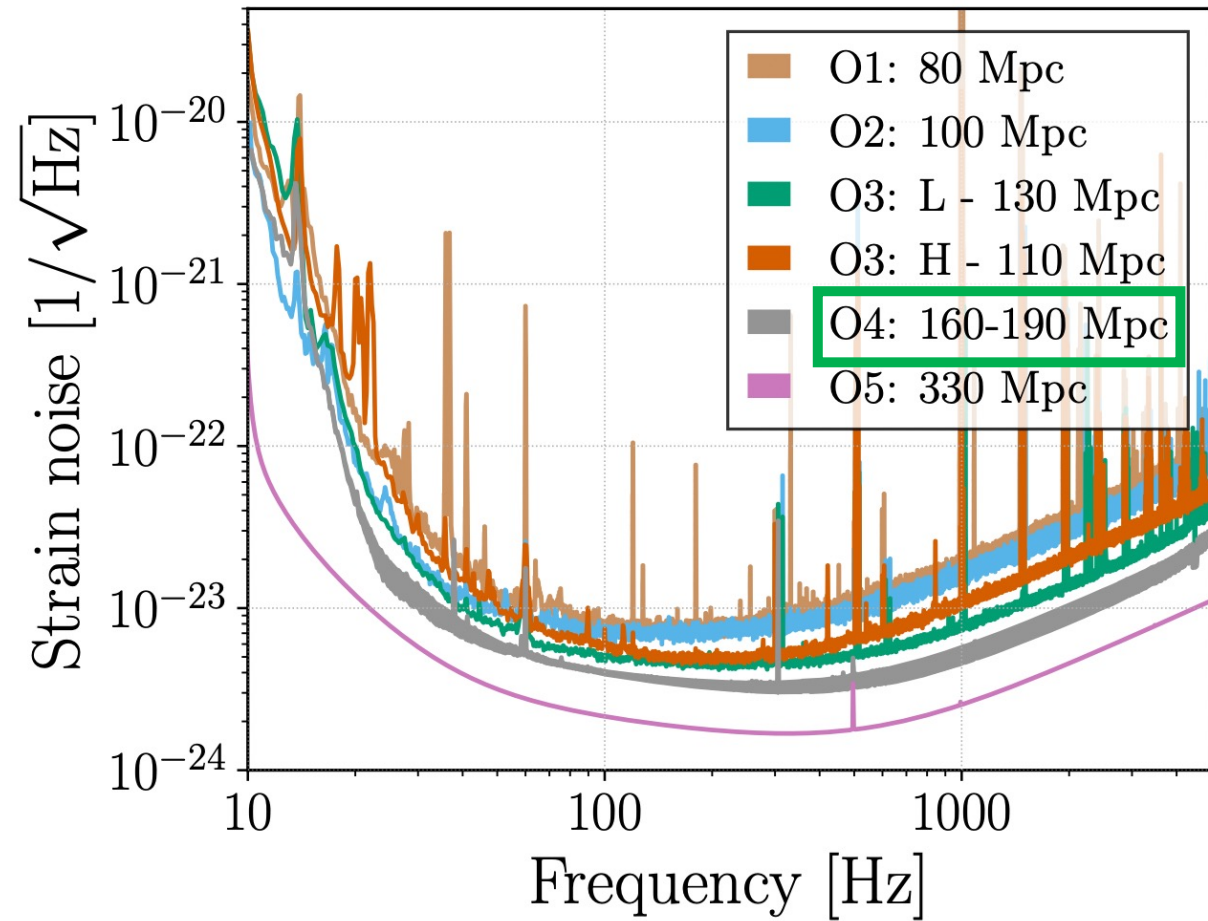
X Arm

Signal recycling mirror to increase detector bandwidth

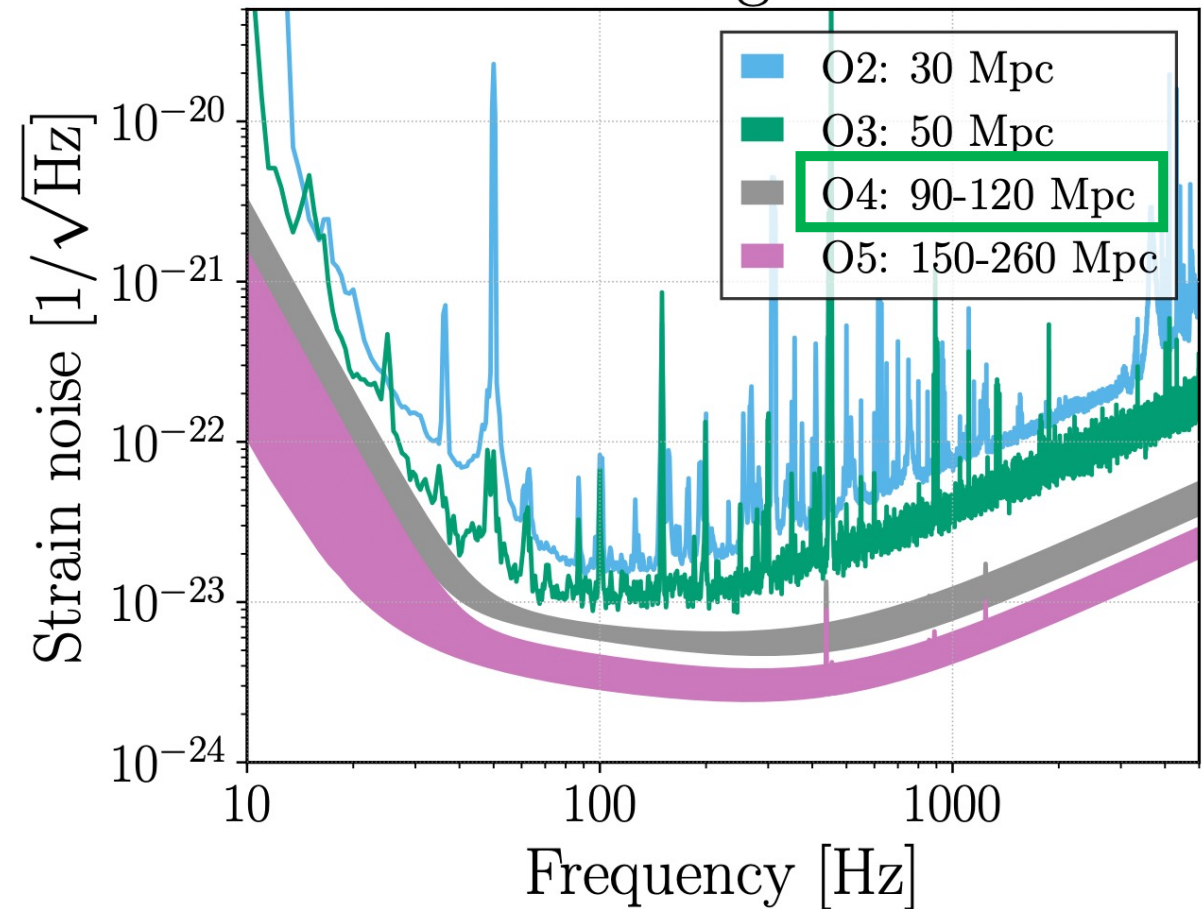


Detector noise curves

LIGO

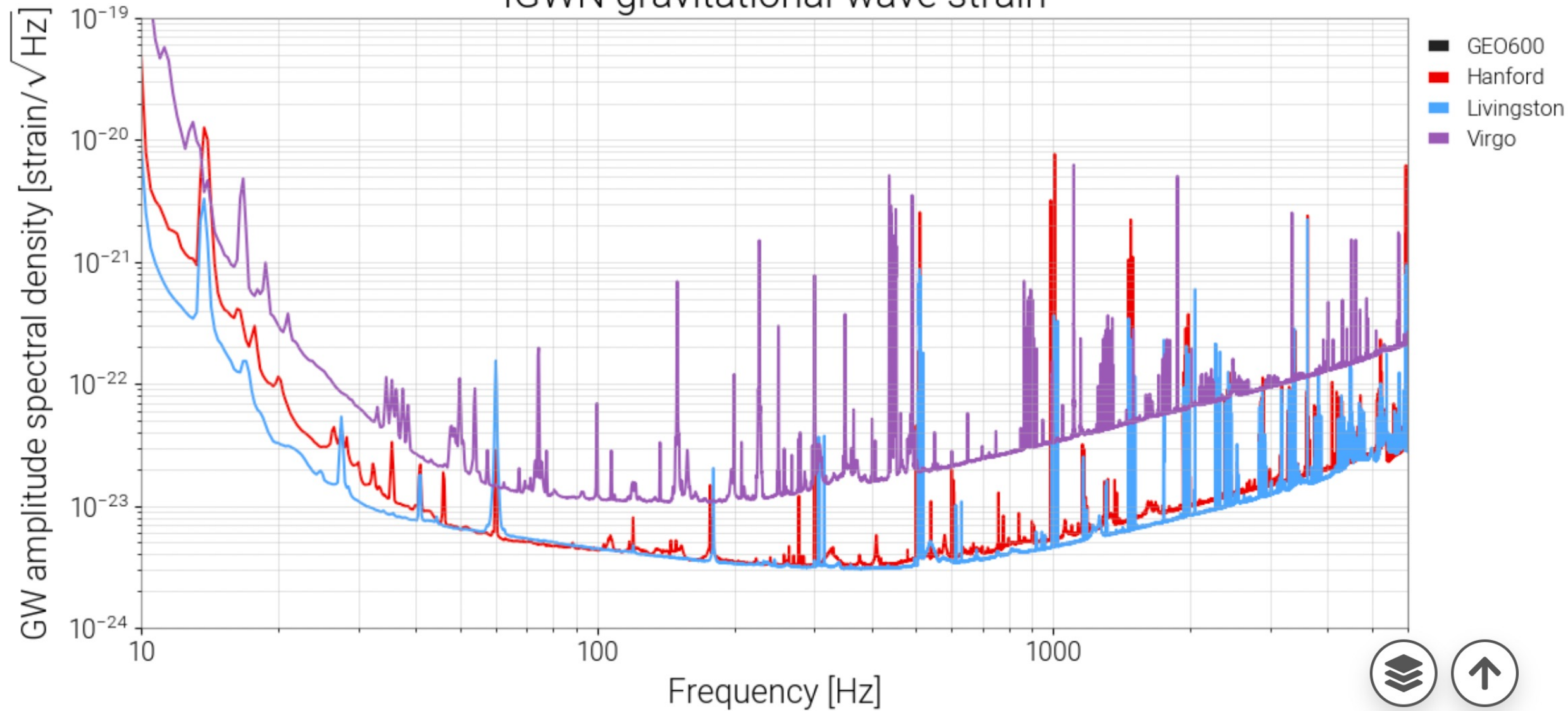


Virgo

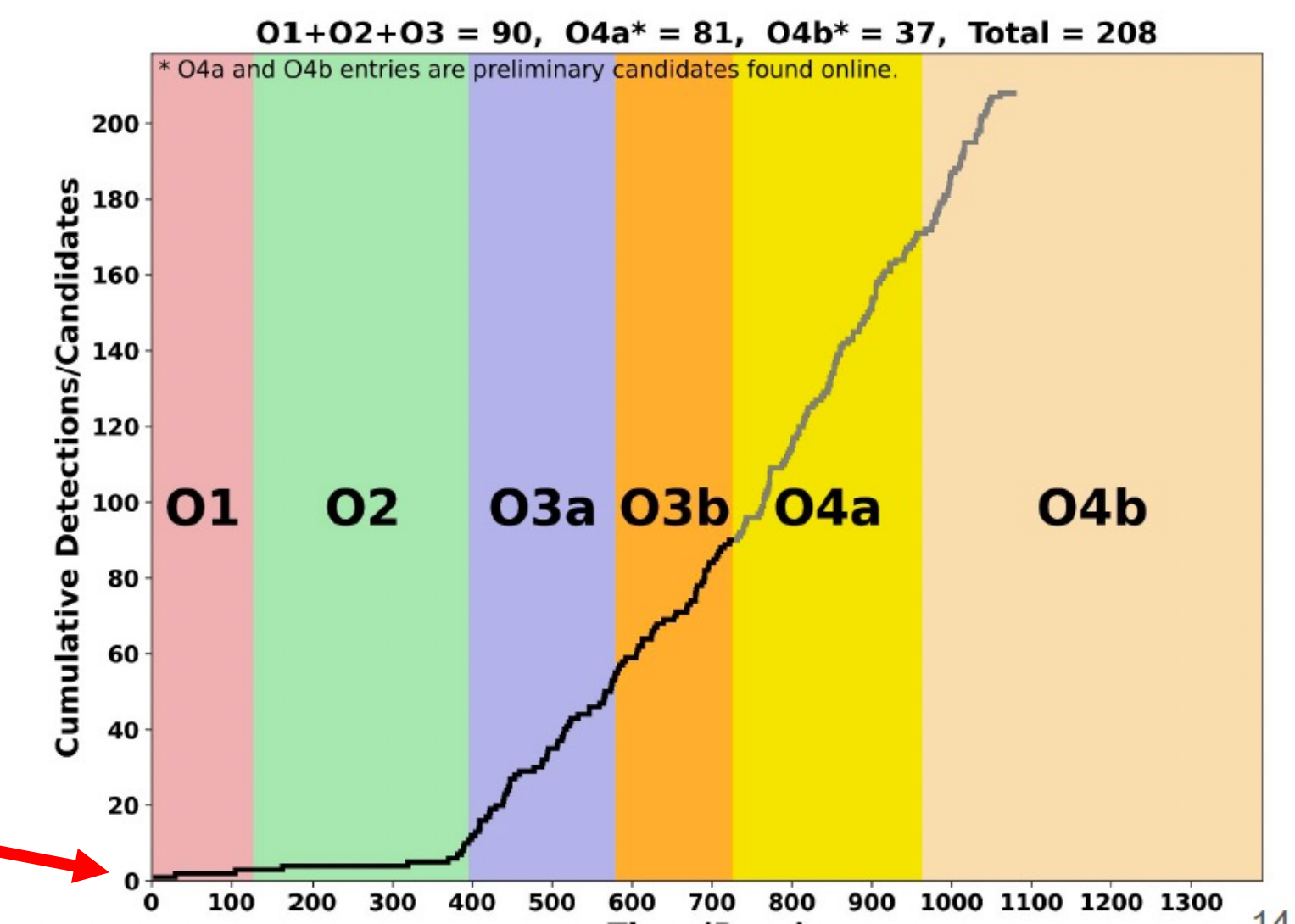


Sensitivity of Sept. 23th (yesterday)

IGWN gravitational-wave strain



Number of detections steadily increasing over time (as of yesterday: 228)



First detection:
Sept 14th 2015



Mostly BHBH systems, plus a few BHNS and NSNS

Neutron Stars (NS)

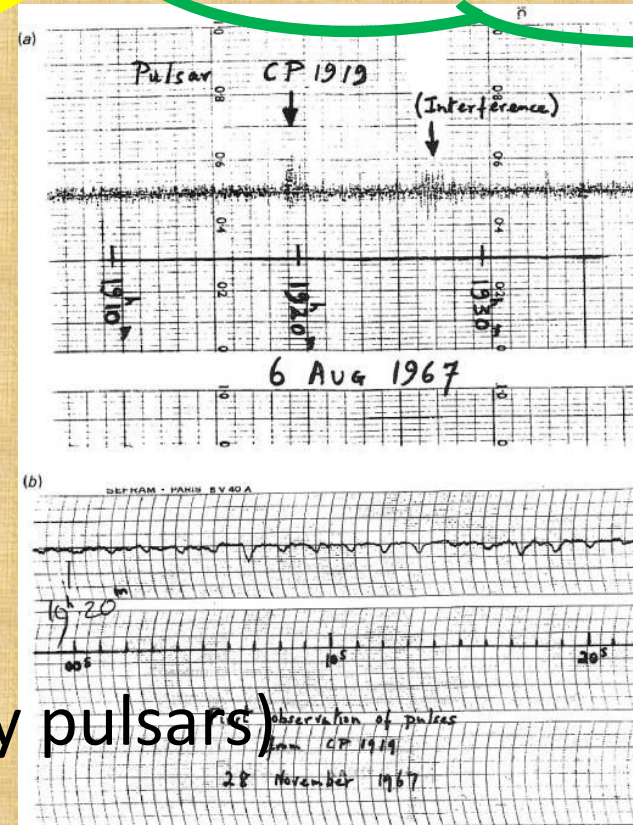
NS proposed by Baade & Zwicky in 1934 (two years after the neutron discovery)

In 1969 it was interpreted by Pacini and Gold as a pulsar: a rotating NS with an intense magnetic field

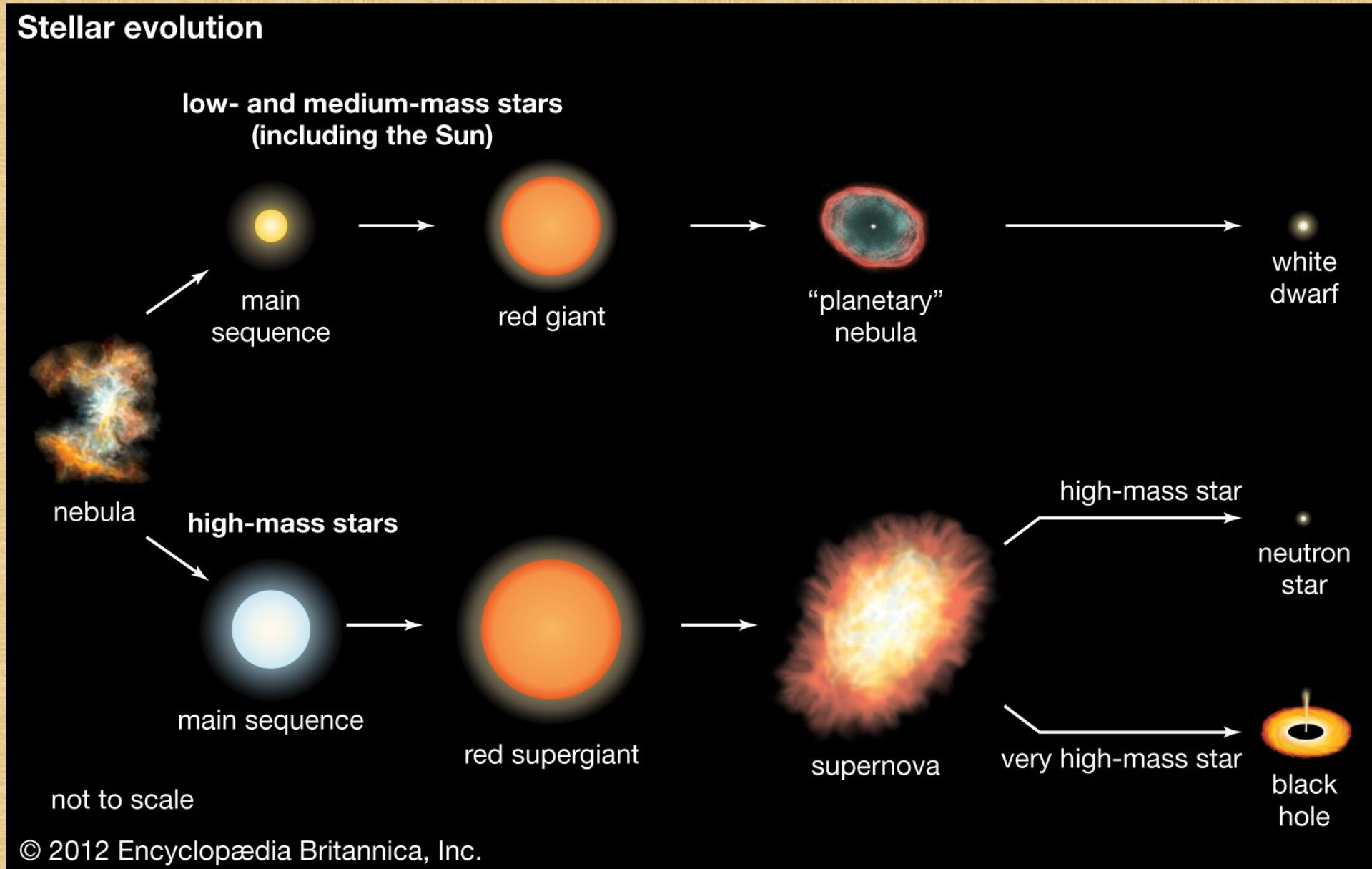
Today, about 3600 NS are known (mostly pulsars)

10^8 - 10^9 are expected to exist in our galaxy

First NS detected – through pulsed EM emission - by Jocelyn Bell in 1967

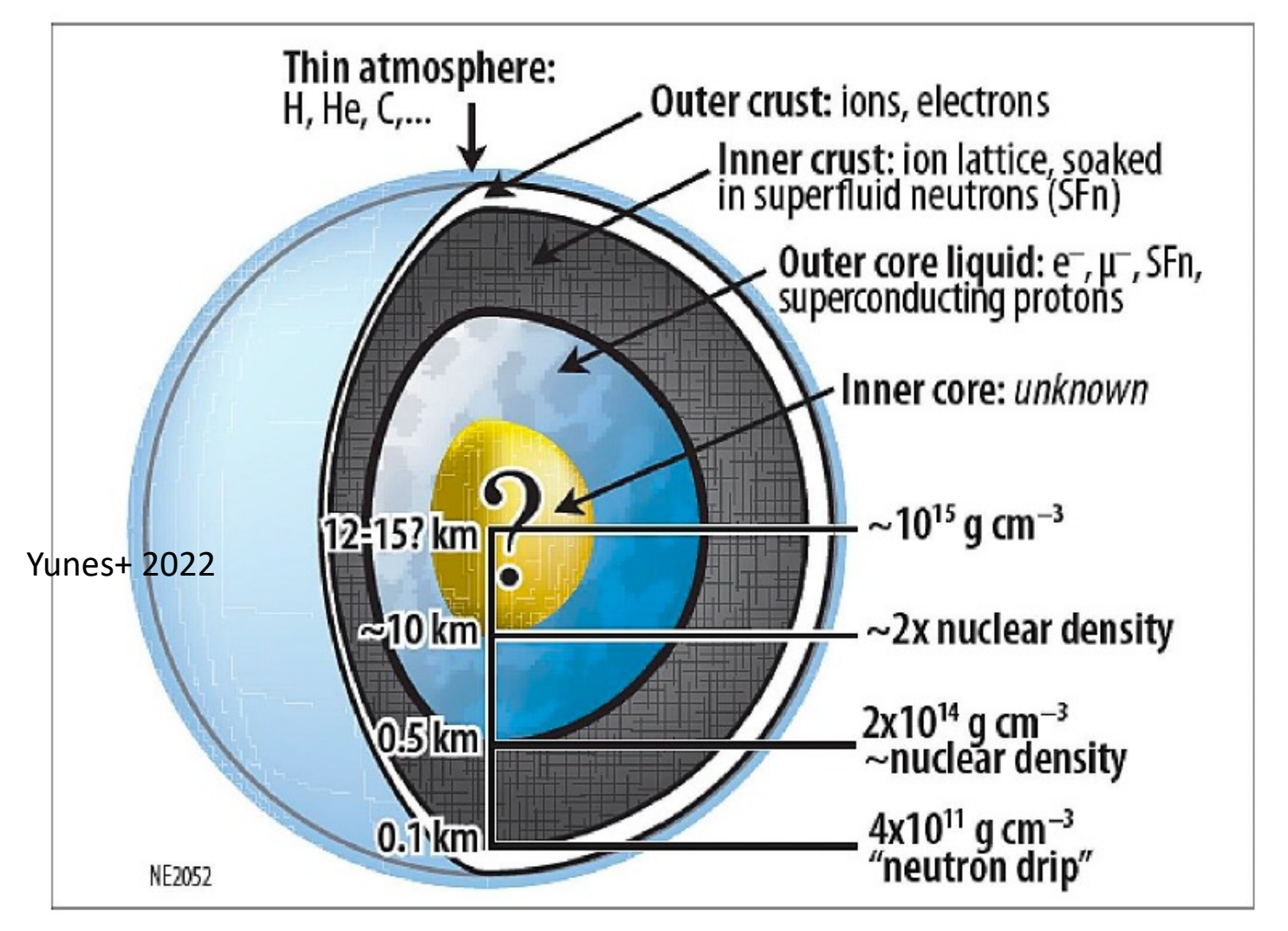


End product of the collapse of progenitor stars with mass in the range $8-20 M_{\text{sun}}$



Typical masses of about 1-2 solar masses and radius of 10-15 km

Neutron stars reach densities many times nuclear saturation density and largely exceed anything can be created in a laboratory on the Earth



In particular, core structure is largely unknown

NS Equation of State (EoS) relates pressure and mass density

Up to about 2x saturation density, the EoS can be guessed extrapolating results of nuclear physics experiments and from ab-initio calculations

The EoS is connected to astrophysical observables: mass, radius, moment of inertia, tidal deformability

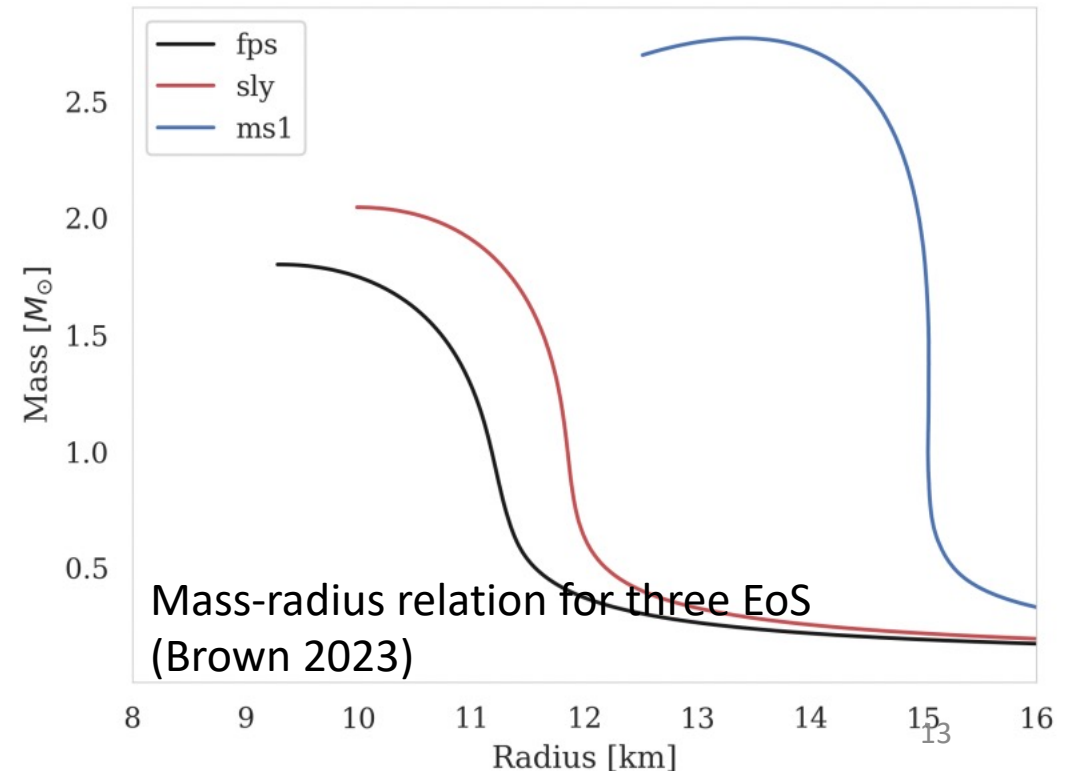
$$\frac{dm}{dr} = 4\pi\rho r^2,$$

$$\frac{d\nu}{dr} = \frac{m(r) + 4\pi r^3 p}{r(r - 2m(r))},$$

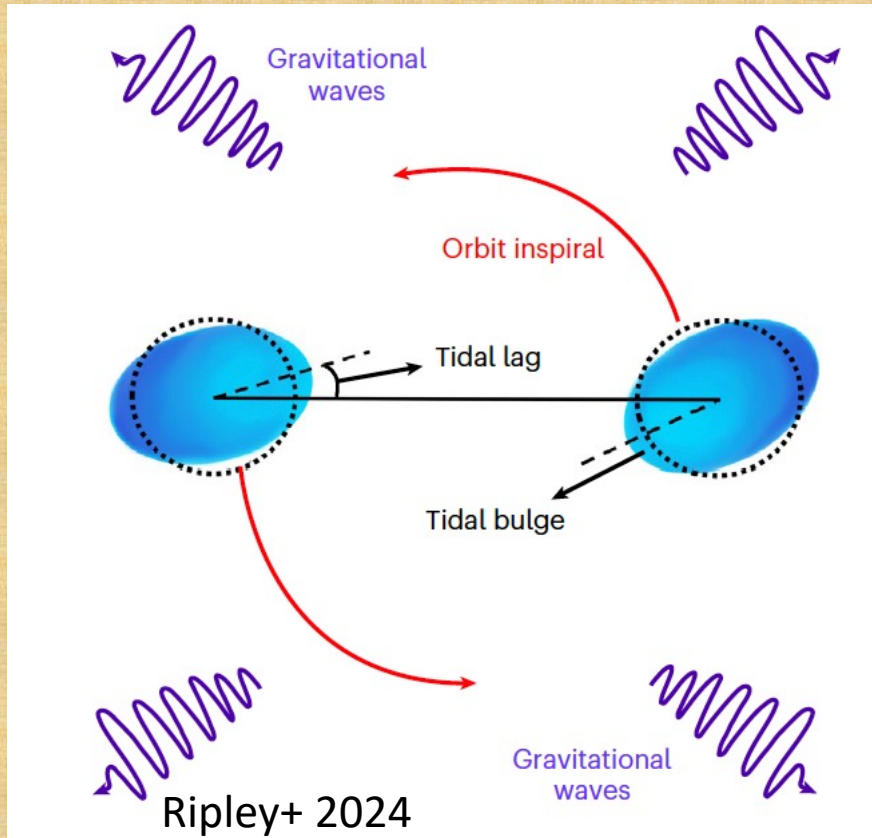
$$\frac{dp}{dr} = -(p + \rho) \frac{m(r) + 4\pi r^3 p}{r(r - 2m(r))}.$$

+ EoS \rightarrow
($p=p(\rho)$)

Tolman-Oppenheimer-Volkov (TOV) Equation



Tidal deformability measures the strength of a NS's response to an external tidal field



(leading order) tidal deformability

$$Q_{ij} = -\lambda \xi_{ij}$$

[Hinderer 2008]

Quadrupole moment

External (quadrupole) tidal field

Analogous to the tides on the Earth due to the Moon!

Tidal deformability depends on the EoS and affects the GW signal (in particular, the phase) emitted during the last stages of the coalescence of a NSNS binary system

We can use GWs to study the structure of NSs

To measure macroscopic parameters from GW data, **Bayesian inference** is typically used [see e.g. LVC, PRL121 161101 (2018)]

The source is described by a set of parameters $\vec{\vartheta}$

The goal is to compute the posterior PDF of the parameters, $p(\vec{\vartheta}|d)$, given GW data d

This can be done using Bayes' Theorem:

Likelihood to obtain data d given the presence of a signal with parameters $\vec{\vartheta}$

$$p(\vec{\vartheta}|d) \propto p(\vec{\vartheta}) \cdot p(d|\vec{\vartheta})$$

Parameter prior PDF

The likelihood requires a signal model (waveform template)

Evaluation of the posterior PDF is based on sampling techniques (MCMC, Nested sampling,...)

Approach 1.

Take a parametrized EoS model (depending on a number of free parameters)

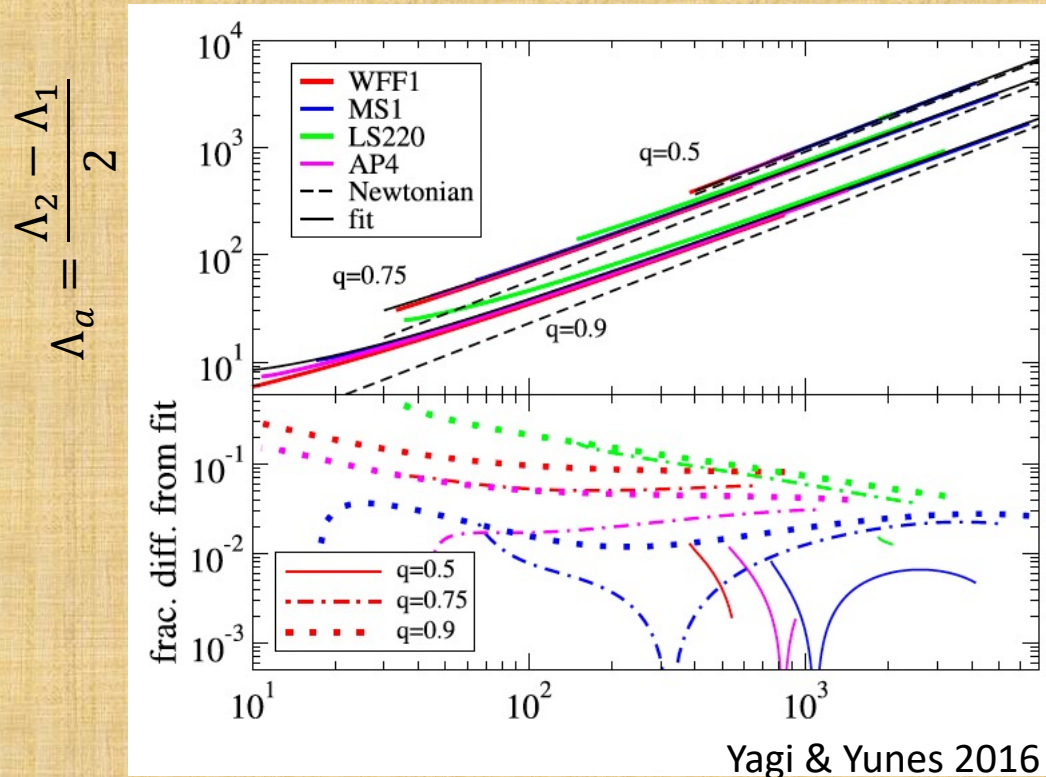
- ➔ For each set of the parameters, sampled from their prior distributions, they and measured NS masses are mapped into the tidal deformability, by solving the TOV equations
- ➔ Compute GW signal (waveform template)
- ➔ Compare to GW data
- ➔ Posterior distribution of observables ($M(r)$ or $p(\rho), \dots$)

Prior constraint can be incorporated, e.g. causality ($v_s < c$), masses observational consistency, masses must be supported by the EoS,...

Hopefully, reasonable models produce similar posteriors

Approach 2. Use universal relations (weakly dependent on the EoS)

➔ *Binary-Love* relations connects combination of the two star tidal deformabilities, Λ_1, Λ_2 , and mass ratio $q = m_2/m_1$: $\Lambda_a = \Lambda_a(q, \Lambda_s)$



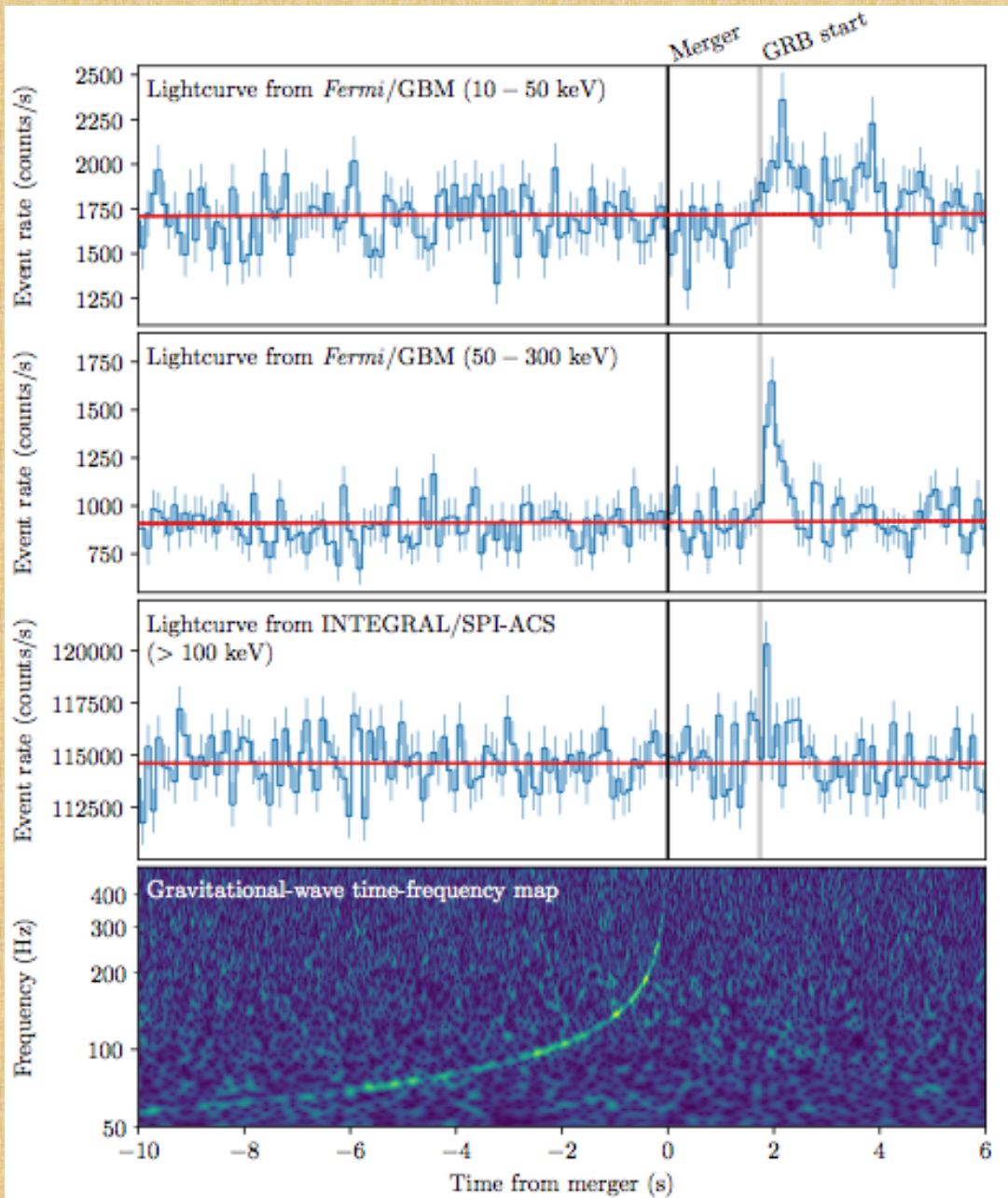
$$\Lambda_s = \frac{\Lambda_2 + \Lambda_1}{2}$$

➔ Sample over $\Lambda_s \rightarrow$ get $\Lambda_a \rightarrow$ compute $\Lambda_1, \Lambda_2 \rightarrow$ generate waveform template

➔ Use $\Lambda - C$ universal relation to get compactness $C = Gm/c^2 R$ [Maselli+ 2013]

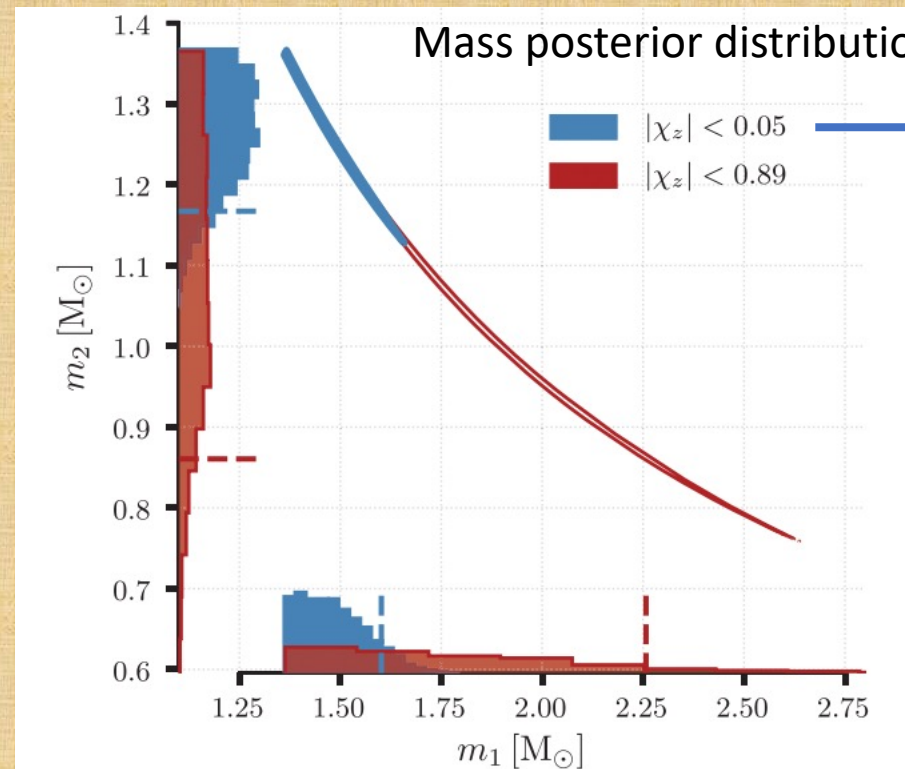
➔ Combine with GW measurement of masses to get stars' radii

GW170817: the birth of multi-messenger astronomy



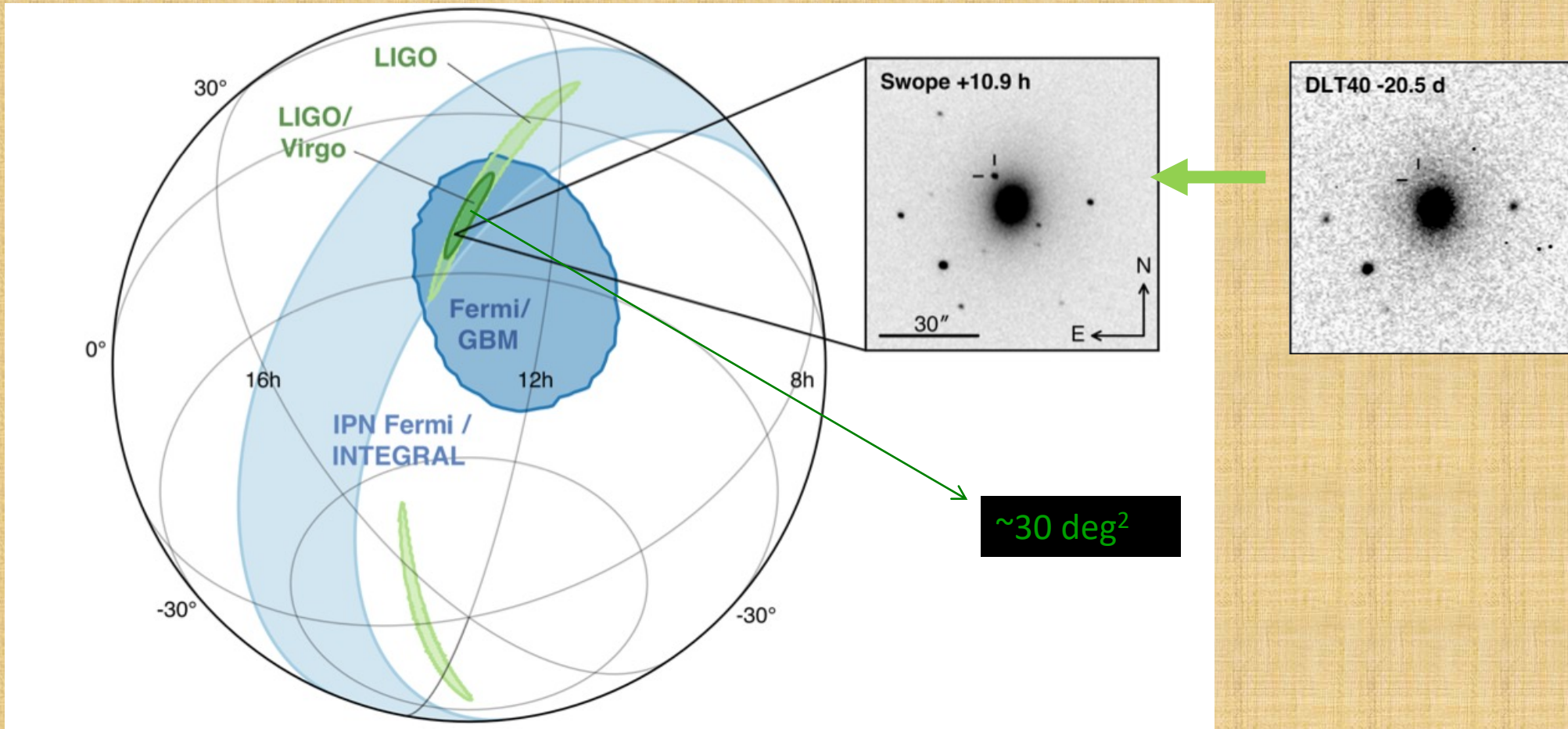
□ The *Fermi* Gamma-ray Burst Monitor independently detected a **gamma-ray burst** (GRB170817A) with a time-delay of ~ 1.7 s with respect to the merger time.

□ Confirmed by INTEGRAL



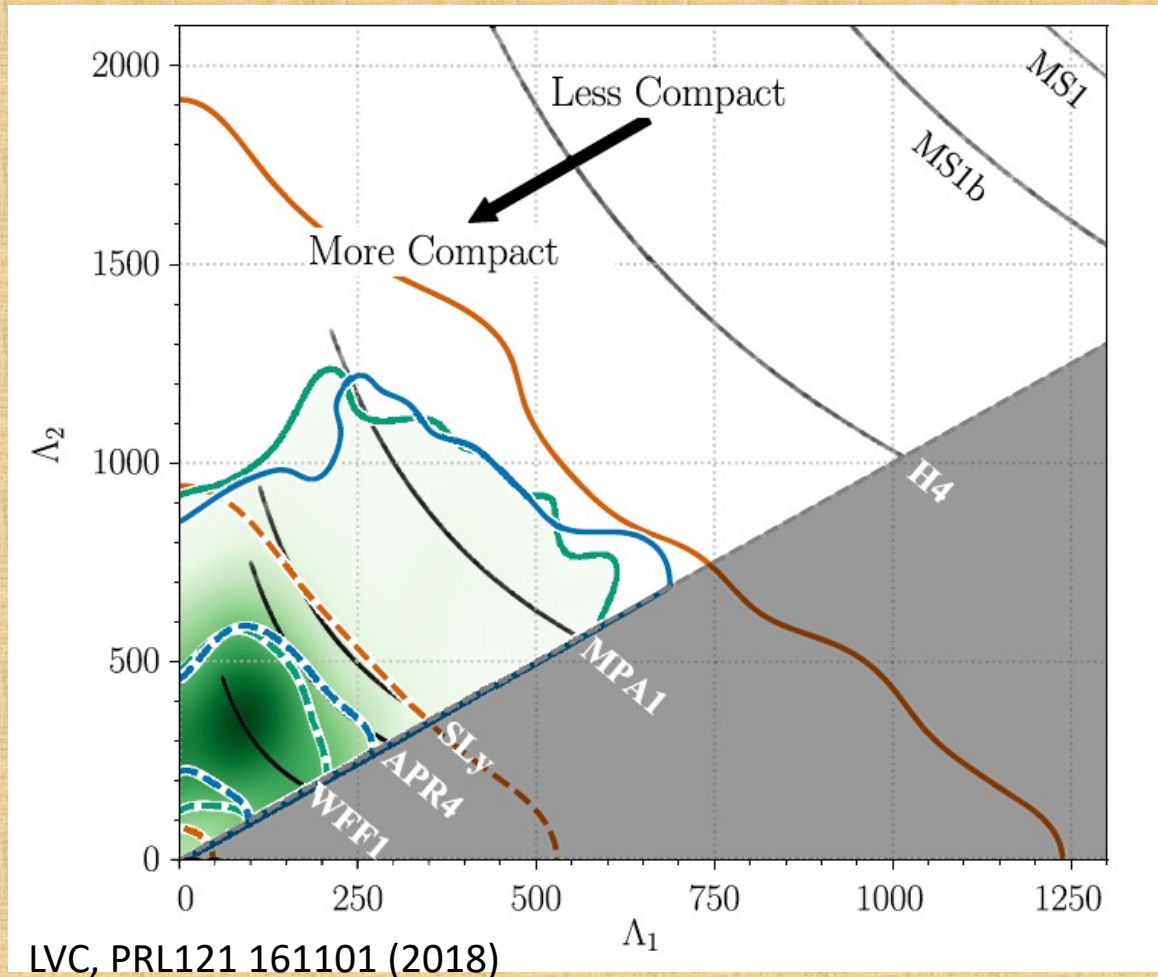
Clear NSNS binary system

- Highly accurate position from GW data (the power of triangulation!)
- Identification of the host galaxy **NGC 4993** in the **Swope optical discovery** image at 10.9 h after the merger $\rightarrow D = 41.0 \pm 3.1 \text{ Mpc}$



Constraints on NS EoS from GW170817

Assume a common but unknown EoS for the two NSs



Assuming $1.4 M_{\odot}$ NS:

$$\Lambda_{1.4} = 190^{+390}_{-120} \text{ at 90\% level}$$

“Soft” EoS (like WFF1 or APR4) are favored w.r.t. to “stiff” EoS

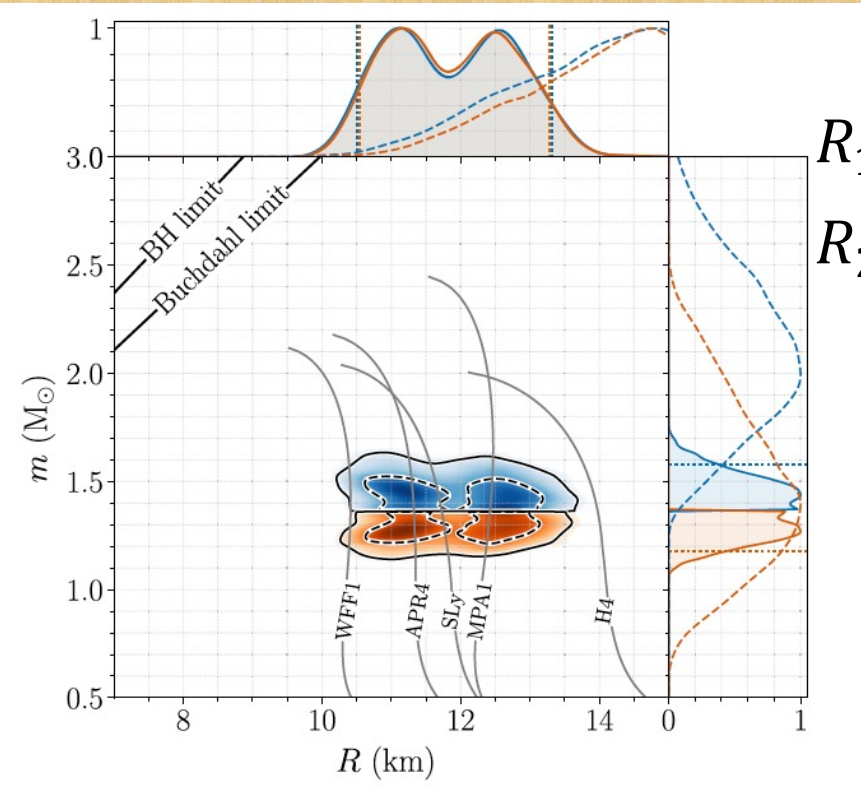
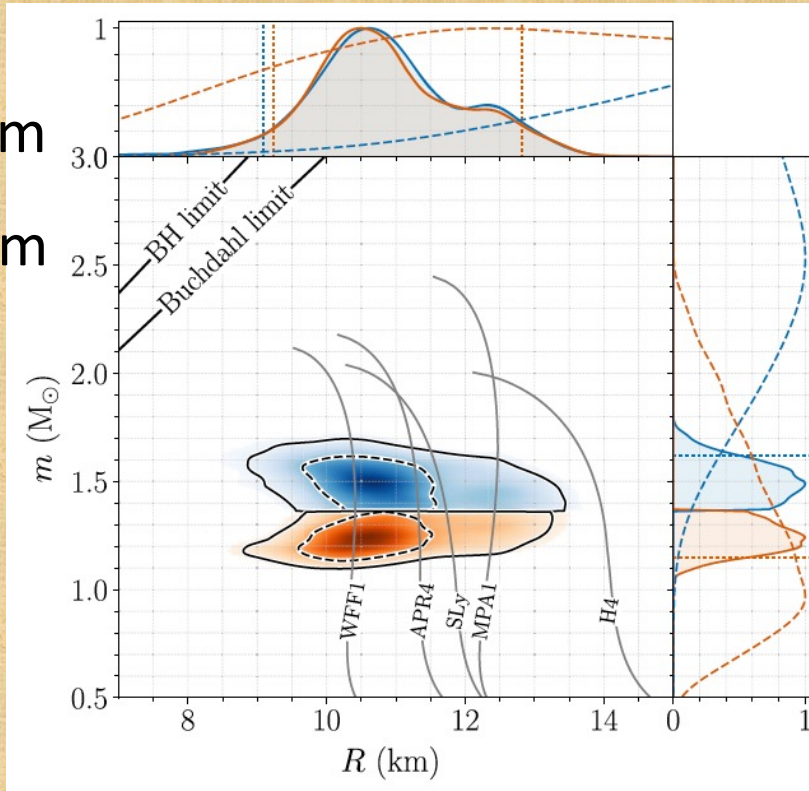
Mass-radius posterior [LVC, PRL121 161101 (2018)]

EoS-insensitive relations

Parametrized EoS

$$R_1 = 10.8^{+2.0}_{-1.8} \text{ km}$$

$$R_2 = 10.7^{+2.1}_{-1.5} \text{ km}$$



$$R_1 = 11.9^{+1.4}_{-1.4} \text{ km}$$

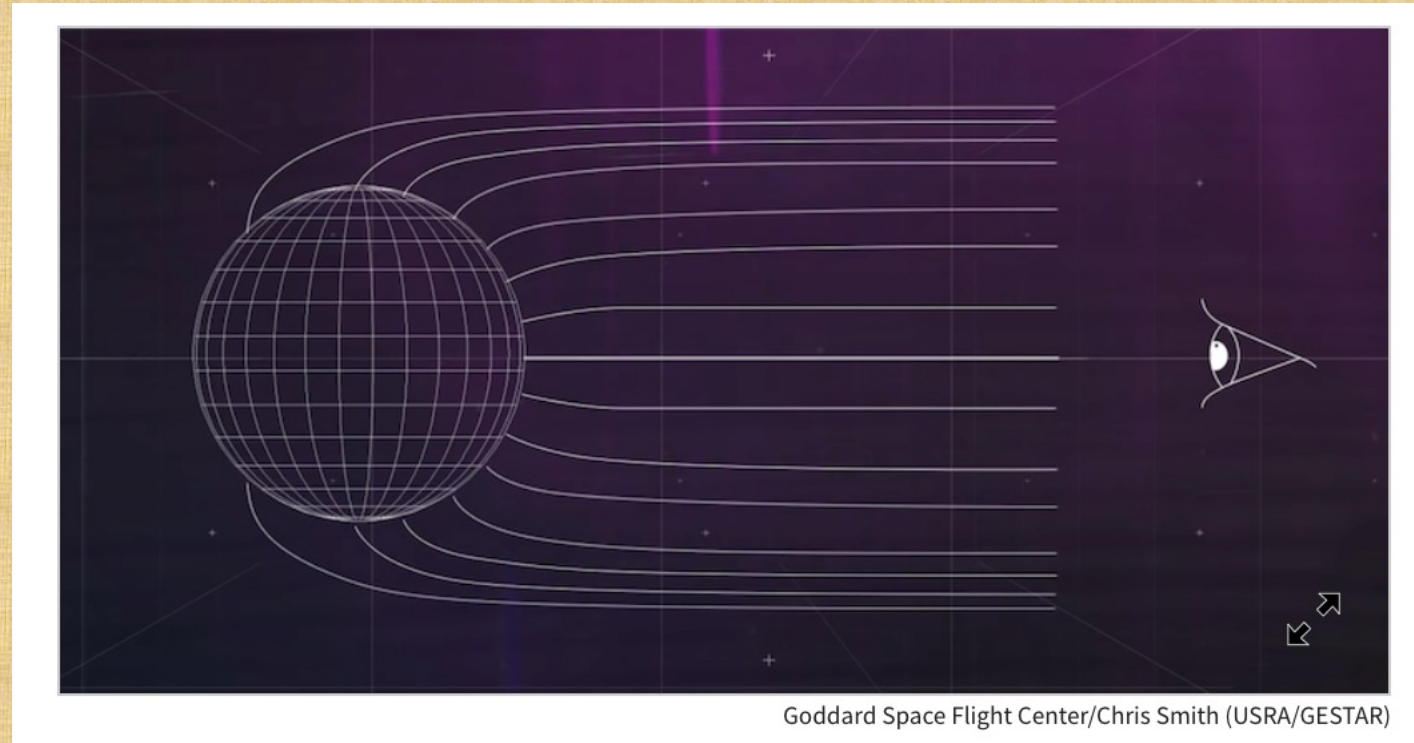
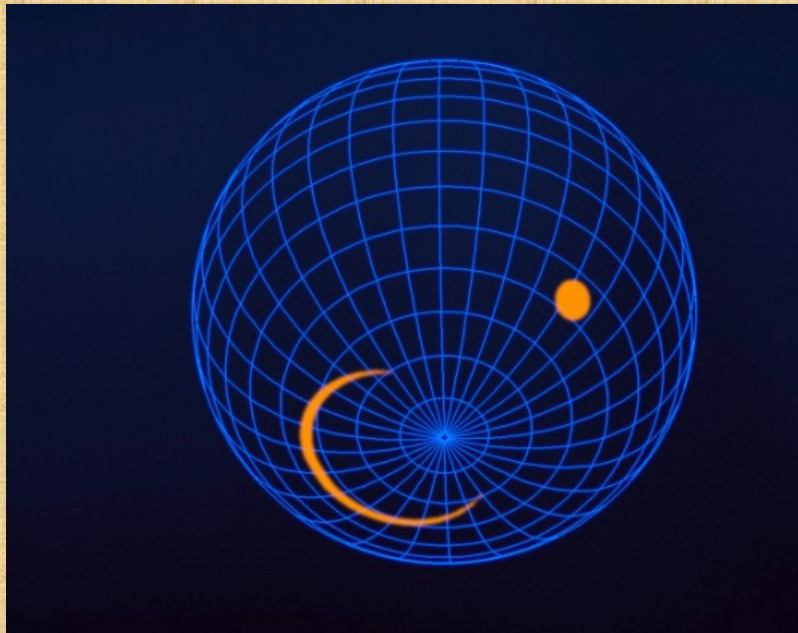
$$R_2 = 11.9^{+1.4}_{-1.4} \text{ km}$$

Consistent posteriors among the two approaches

Mass and radius measurements can come also from EM observations

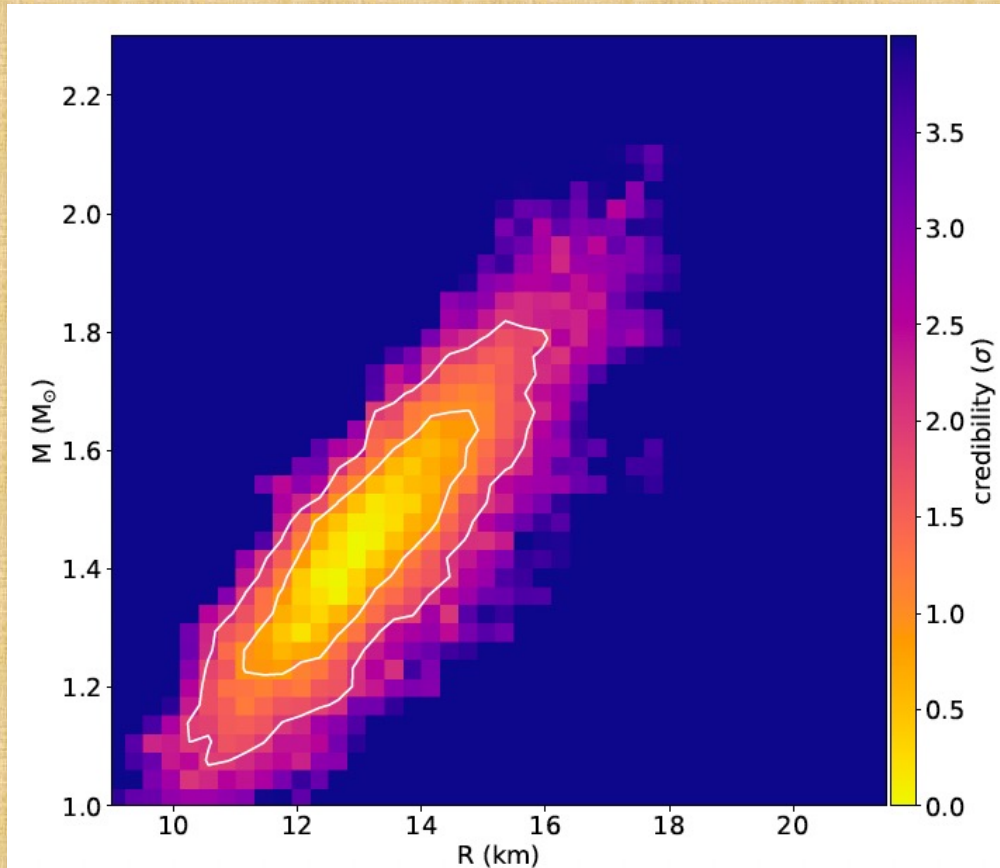
E.g. NICER (on board of the International Space Station) has been able to measure mass and radius for two pulsars: J0030+0451 and J0740+6620

It uses very precise (100 ns) observations of the X-rays photons emitted by 'hot spots' on the star surface

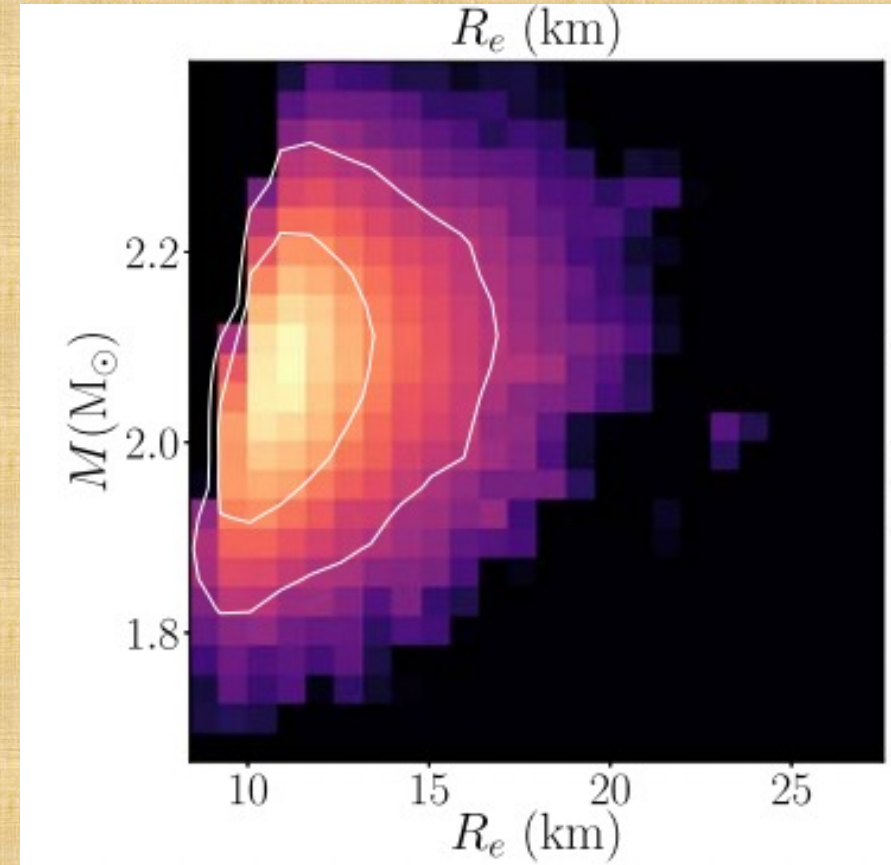


NICER M-R posterior PDF

J0030+0451 [Miller+ 2019]

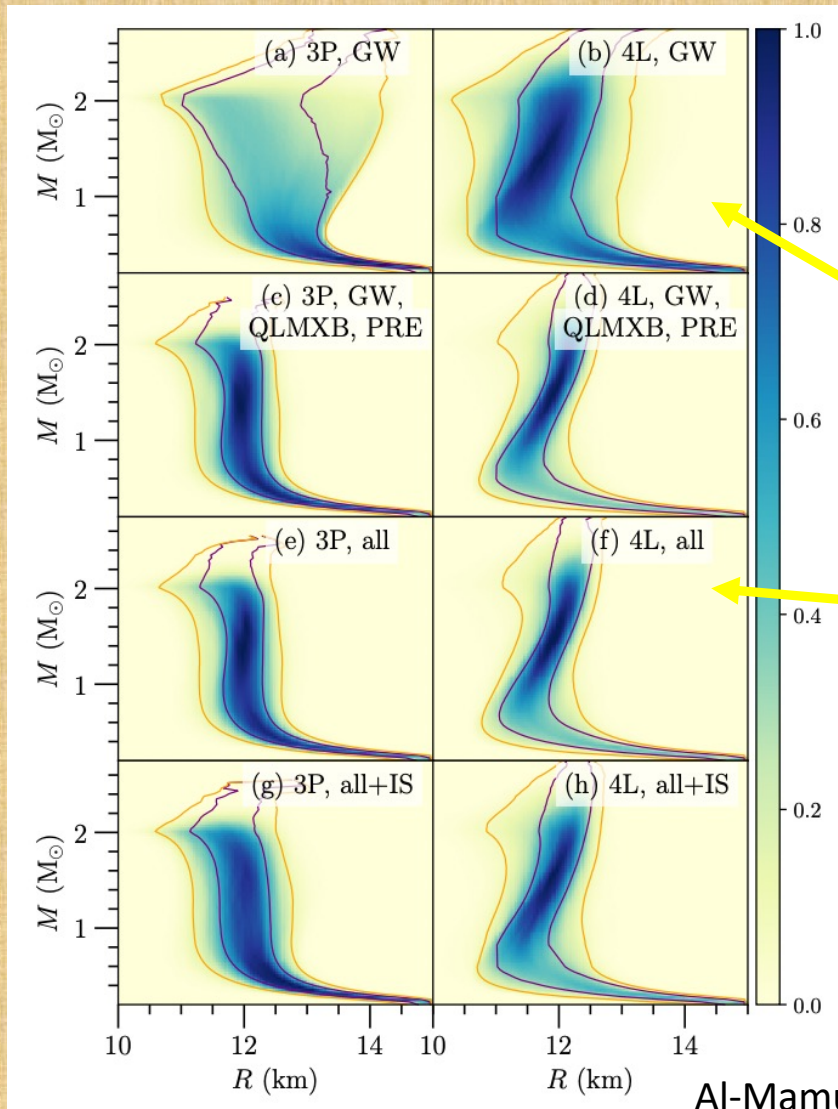


J0740+6620 [Miller+ 2021]



Combining GW and EM observations

It can allow to put more stringent constraints on the NS parameters



Only GW obs.

All obs.

Combination of :
EoS model,
GW,
isolated NSs,
QLMXB (quiescent low-mass X-ray
binaries),
PRE (NSs with photospheric radius
expansion X-ray bursts)

A multi-messenger approach can be fruitful

GW170817 has been a lucky event: with current (uncertain) NSNS coalescence rate, an event at $D=40$ Mpc would have been detected from $\sim 1/500$ to $1/3$ yrs

A few others NSNS and NSBH coalescences have been observed in runs O2-O3-O4

Event	m_1	m_2	distance	SNR
GW170817	[1.36, 1.60]	[1.16, 1.36]	41.0 ± 3.1	32
GW190425	[1.60, 1.87]	[1.40, 1.69]	159^{+69}_{-72}	12.9
GW190814	[21.9, 24.7]	[2.5, 2.7]	230^{+40}_{-50}	25.3
GW 191219	[28.3, 33.3]	[1.11, 1.24]	550^{+240}_{-160}	9.1
GW200105	[7.4, 10.8]	[1.67, 2.24]	270^{+120}_{-110}	13.7
GW200115	[3.4, 7.9]	[1.16, 2.29]	290^{+150}_{-100}	11.3
GW200210	[6.2, 6.9]	[2.41, 3.3]	940^{+430}_{-340}	8.4
GW230529	[2.4, 4.4]	[1.2, 2]	201^{+101}_{-96}	11.6

Matter effects have not been measured in none of them

In order to improve constraints on the EoS we would need:

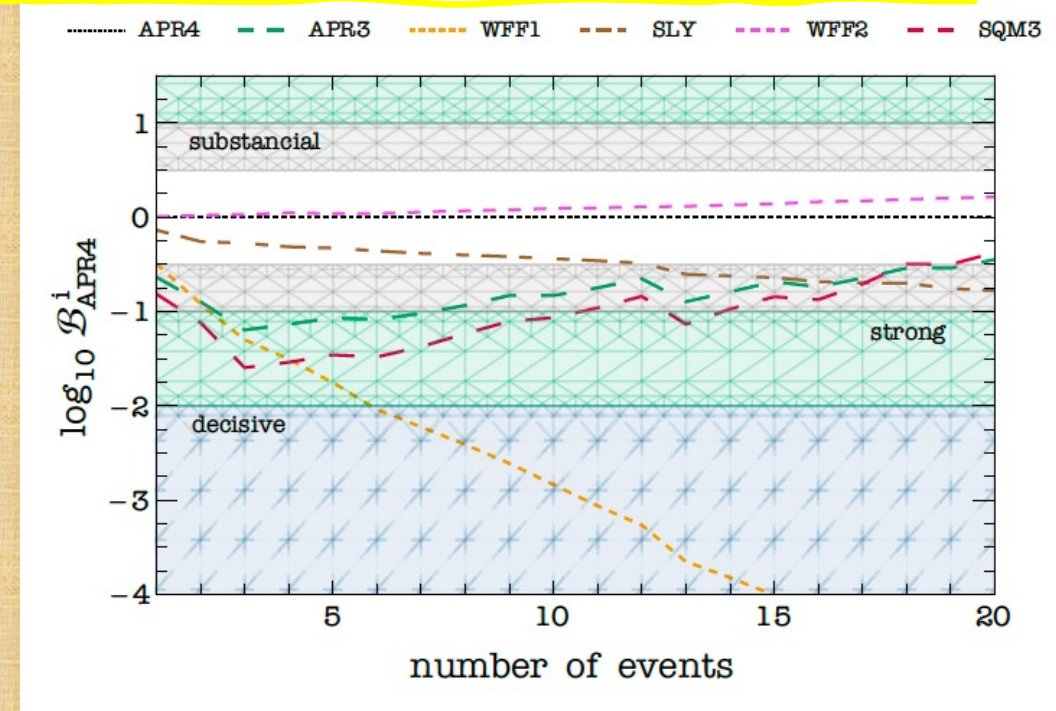
- many (near) events
- or more sensitive detectors

With current detectors, even at design sensitivity, it is not possible to discriminate among EoS with similar softness but different particle content.

[Carson+ 2020, Pacilio+ 2022]

Most “similar” EoS cannot be excluded even with tens of events by 2G detectors

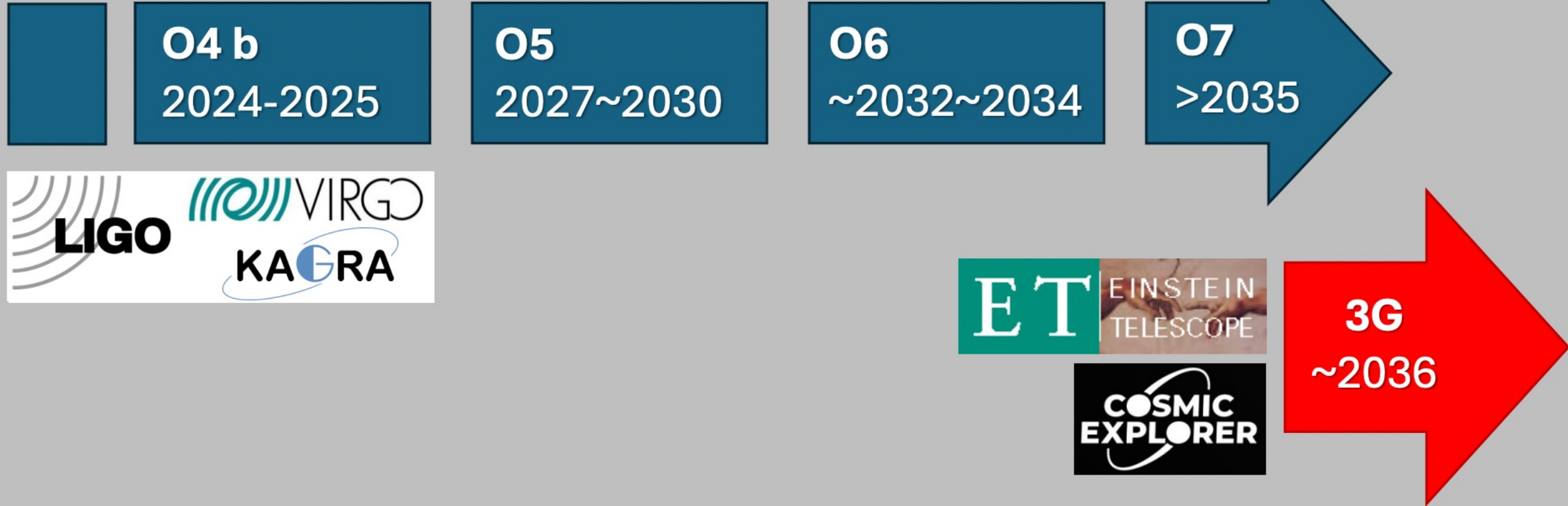
Phase transitions to non-nucleonic matter in the NS core cannot be detected too

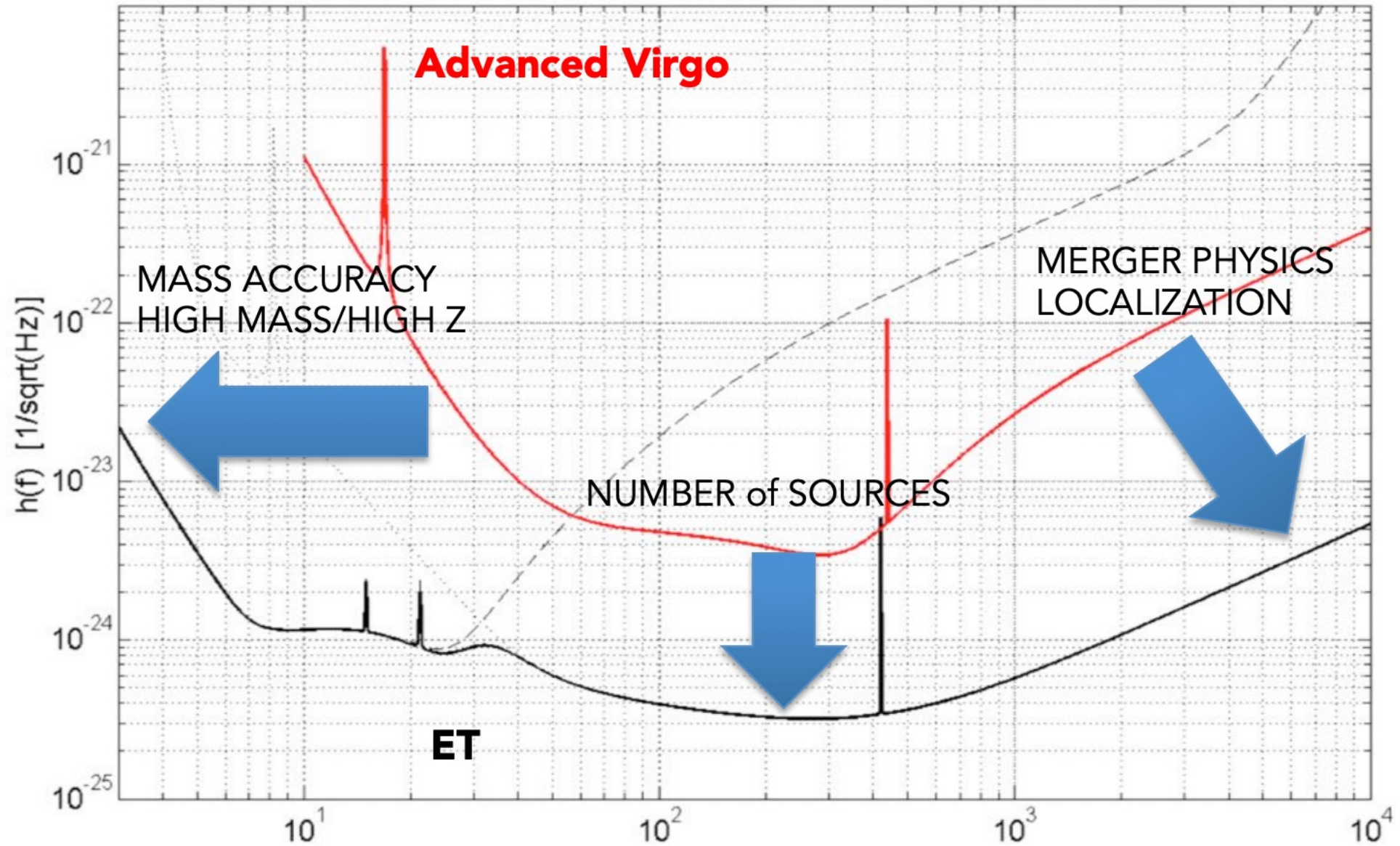


AP4 injected, normalized
Bayes factor for other EoS

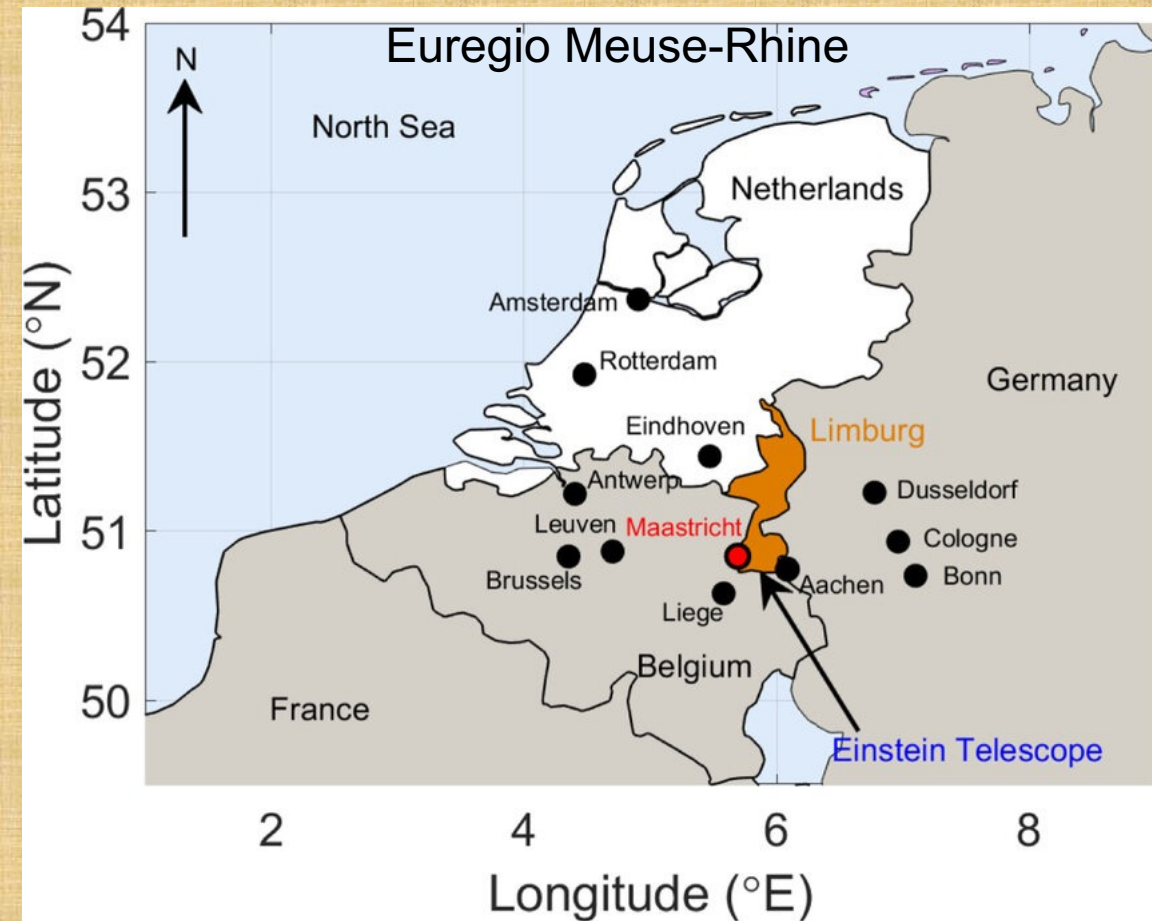
2G/2G+ expected timeline

Observation Runs





Two official candidate sites

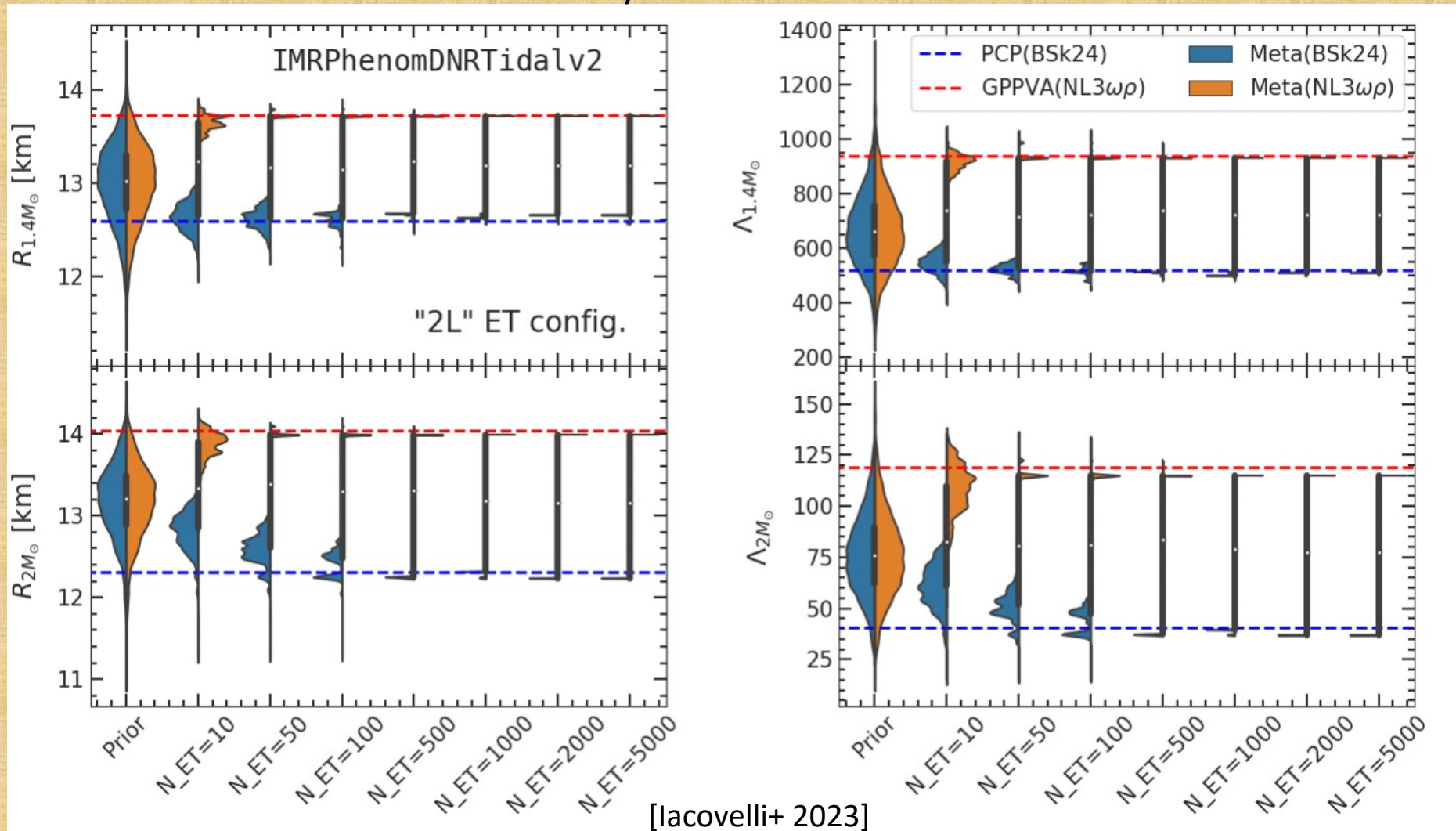


A third (still not official) candidate site in Saxony (Germany)

Site selection should be around half of 2026

3G detectors, like ET or CE, will detect $O(10^5)$ NSNS coalescences per year!

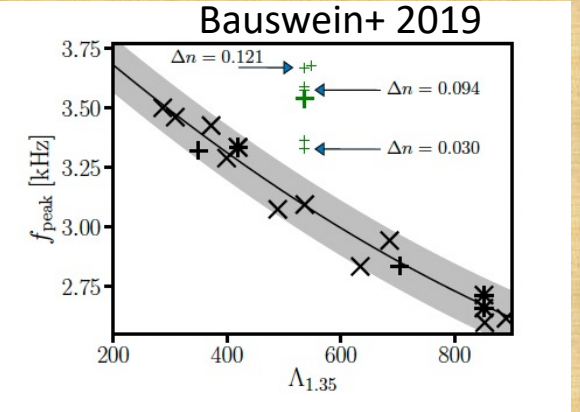
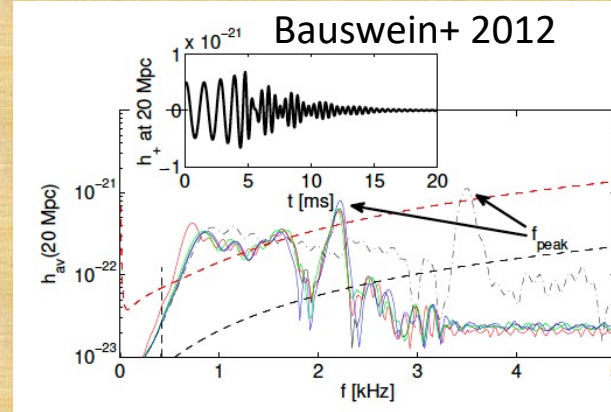
Measure of radius and tidal deformability for two different EoS vs number of detections with ET



Radius could be measured with a precision of few tens of meters from GW observations alone!

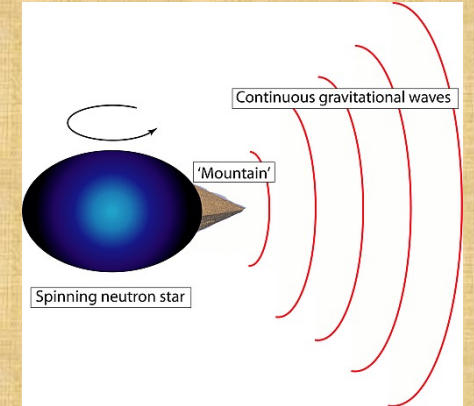
Other ways exists to get information on NS internal structure

NSNS post-merger signals

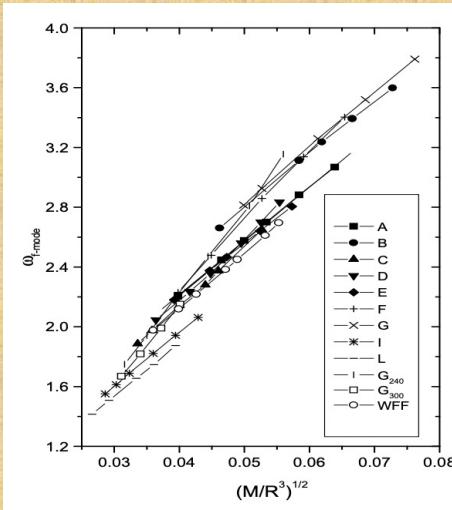
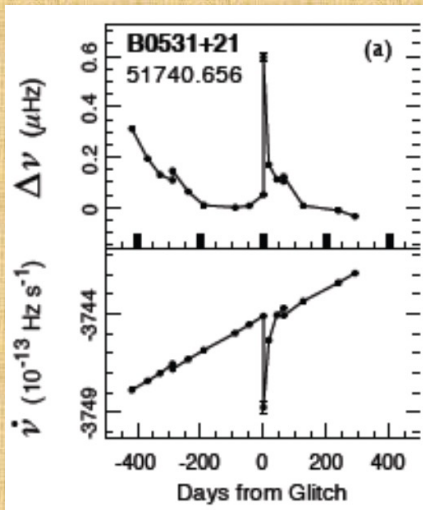


Persistent signals from spinning NS

$$h_0 \simeq 10^{-25} \left(\frac{10 \text{ kpc}}{d} \right) \left(\frac{\epsilon}{10^{-6}} \right) \left(\frac{I_3}{10^{45} \text{ g cm}^2} \right) \left(\frac{\nu}{500 \text{ Hz}} \right)^2$$



Pulsar glitches



Conclusions

Gravitational waves are a powerful tool also to understand properties of neutron stars and of high density nuclear matter

Some relevant results are being obtained with current detectors, but their potentialities will be fully deployed with third generation detectors, like the Einstein Telescope

A synergic approach with nuclear physics (both theory and experiments) and telescopes will allow to maximize the science return

Stay hungry, nuclear pasta is waiting for us!



Image credit: Card, the Universe and Everything Wiki



Backup slides

Element Origins

1 H																	2 He	
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne	
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar	
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr	
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe	
55 Cs	56 Ba			72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra																	
		57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu		
		89 Ac	90 Th	91 Pa	92 U													

Merging Neutron Stars
Dying Low Mass Stars

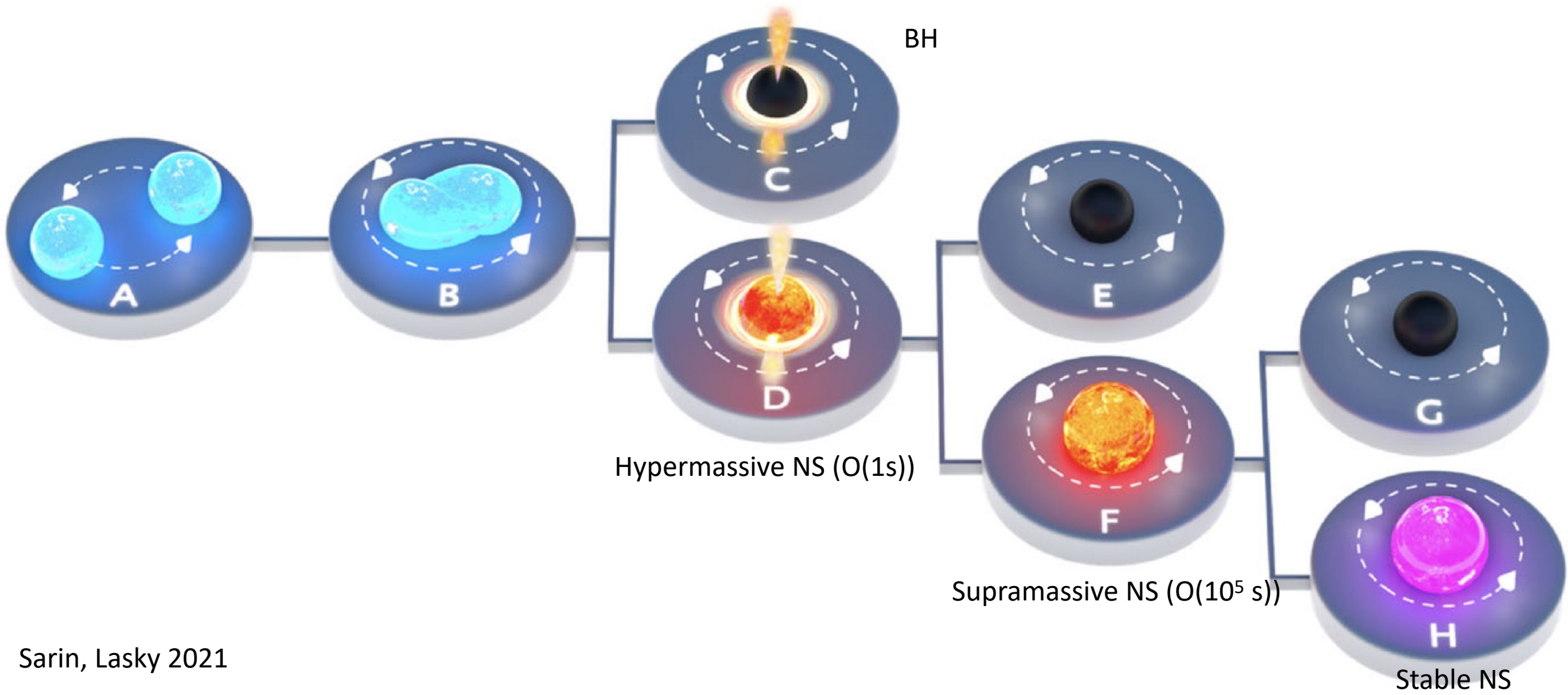
Exploding Massive Stars
Exploding White Dwarfs

Big Bang
Cosmic Ray Fission

Based on graphic created by Jennifer Johnson



GM
The Average Human Body contains **0.2 mg of Gold**



Sarin, Lasky 2021

A passing GW changes the time needed by light to travel across interferometer arms

$$\text{Space-time interval: } ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -c^2 dt^2 + (1 + h) dx^2$$

$$\int dt = \frac{1}{c} \int_0^L \sqrt{1 + h} dx \approx \frac{1}{c} \int_0^L \left(1 + \frac{1}{2} h\right) dx \quad \text{Time to cross a detector arm}$$

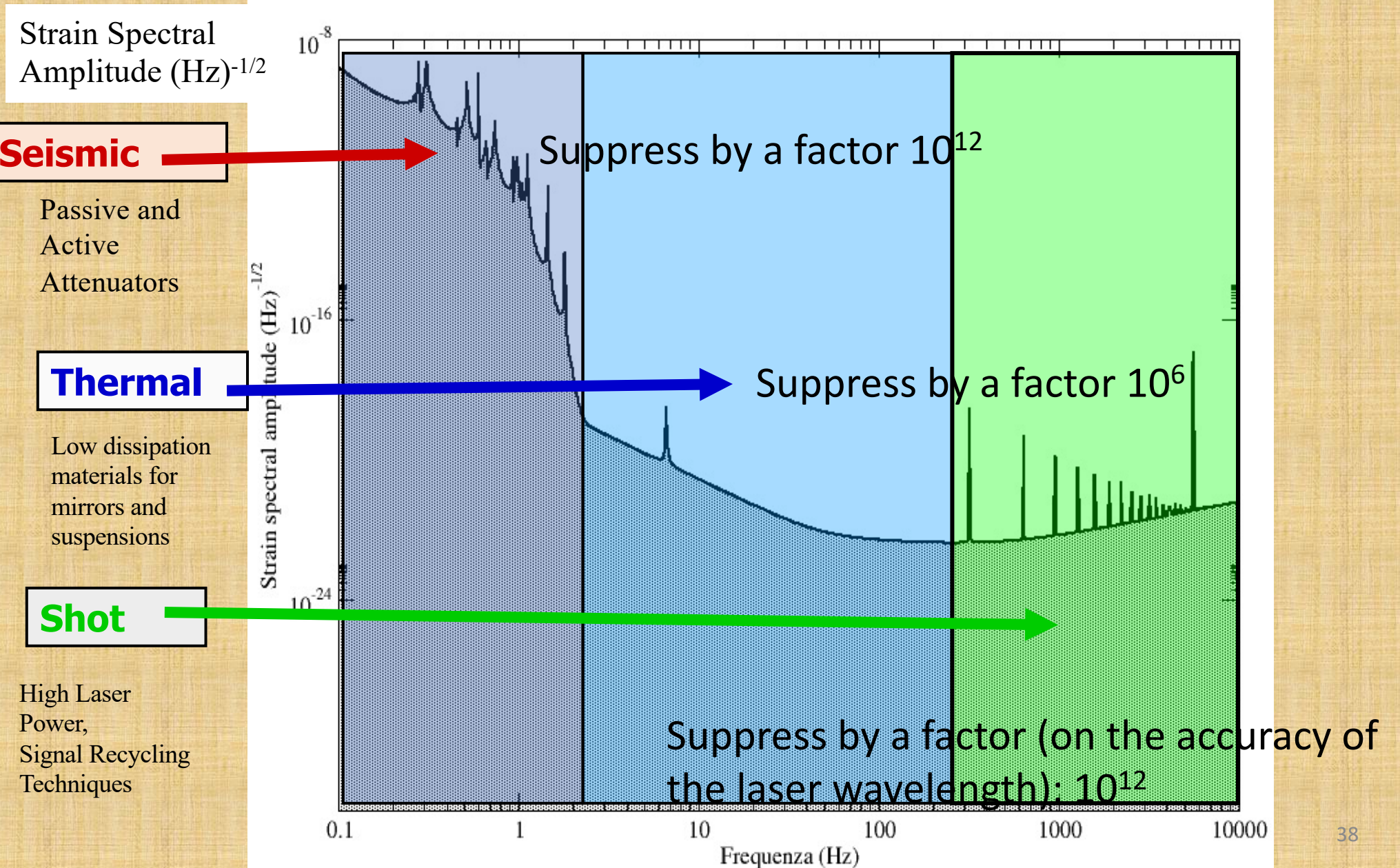
The total time difference among the light travelling across the two arms (and the resulting phase difference) are

$$\Delta\tau = \frac{2L}{c} h \rightarrow \Delta\varphi = \frac{4\pi L}{\lambda} h$$

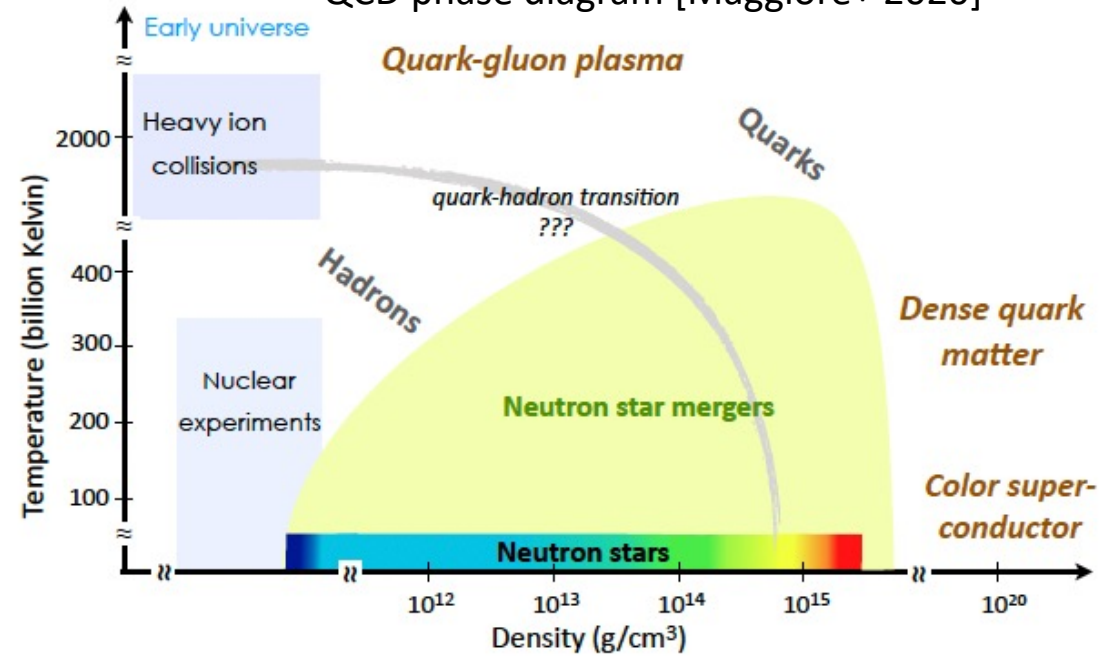
Principio di funzionamento dei rivelatori interferometrici

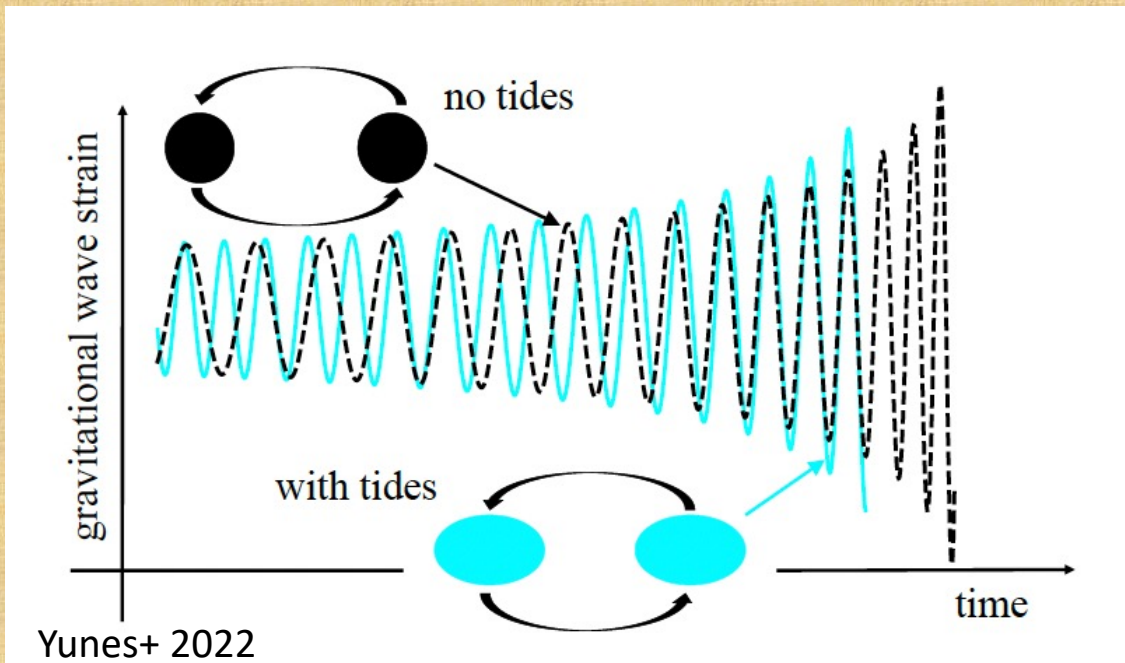


Detector fundamental noises



QCD phase diagram [Maggiore+ 2020]





The tidal deformation requires energy, which is taken from the binary gravitational binding energy



The binary inspirals faster!

The GW phase is modified



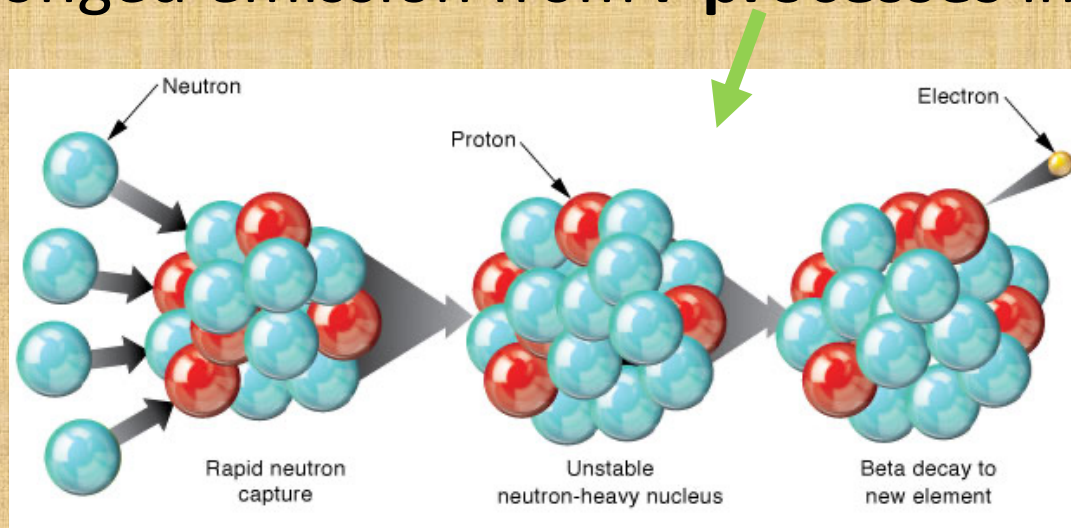
By measuring the GW signal phase we can, in principle, extract information on the star's tidal deformability.

Other effects depend on the EoS but are more difficult to observe

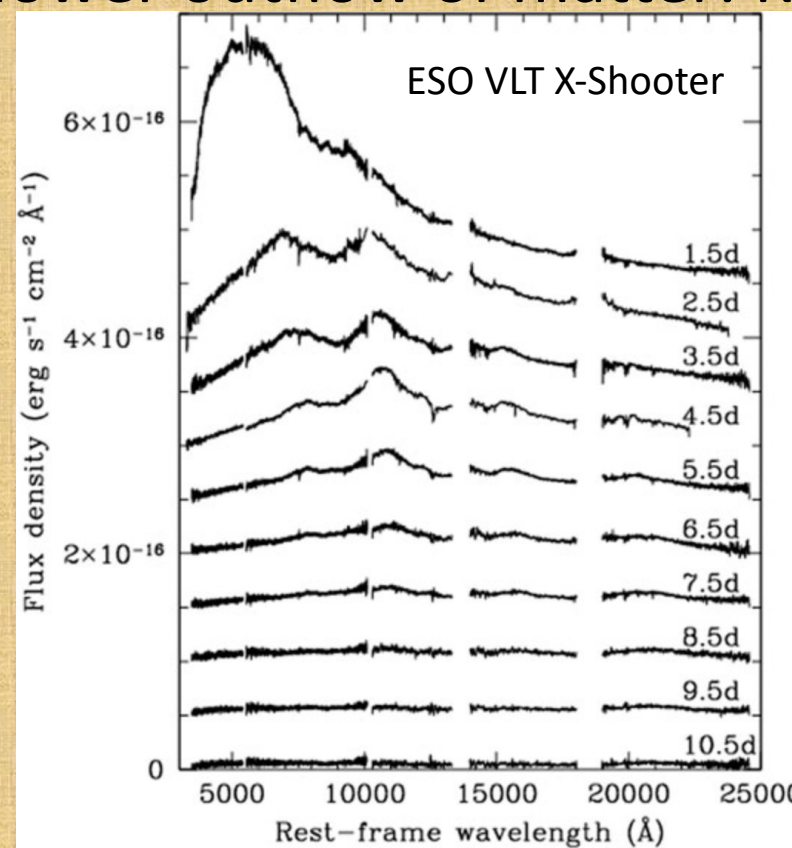
EM counterpart of GW170817 was observed across a large portion of the EM spectrum (from gamma to radio)

Observations have been shown to be consistent with the standard predictions:

- GWs
- Short GRB (Gamma Ray Burst), produced by an highly relativistic blast wave
- Prolonged emission from **r-processes** in a slower outflow of matter: Kilonova



The kilonova has produced ~ 10 Earth masses of gold and platinum!



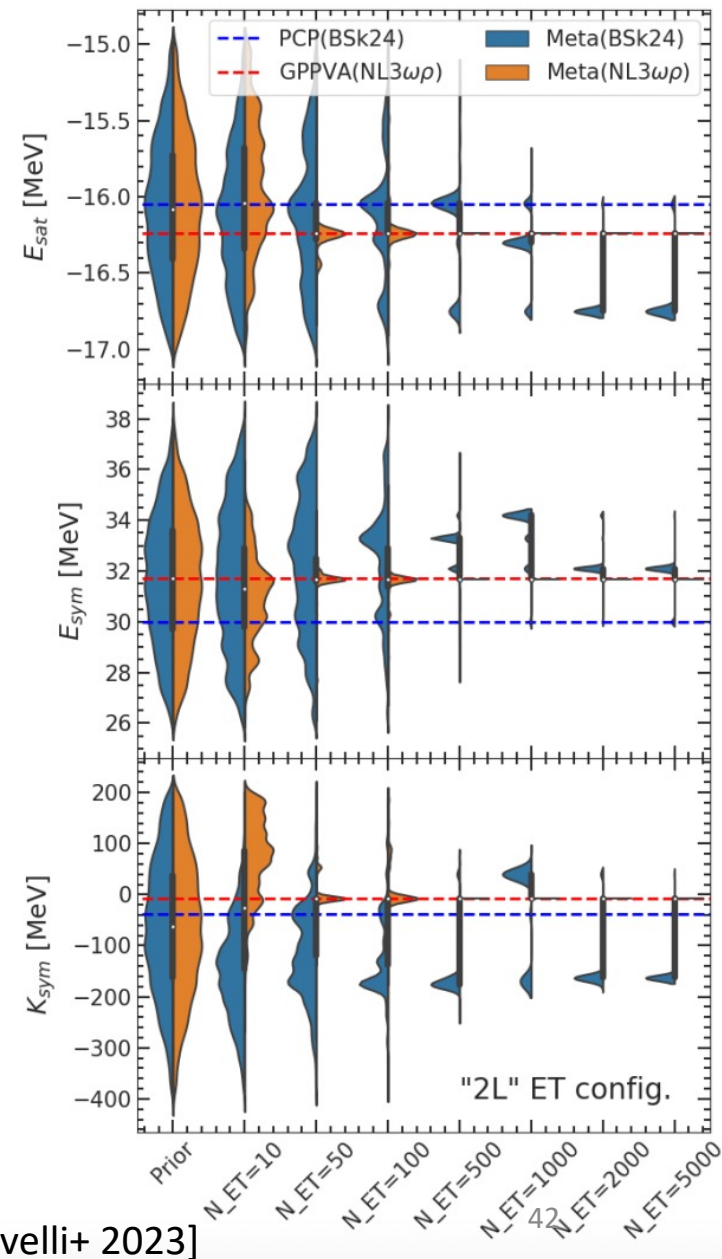
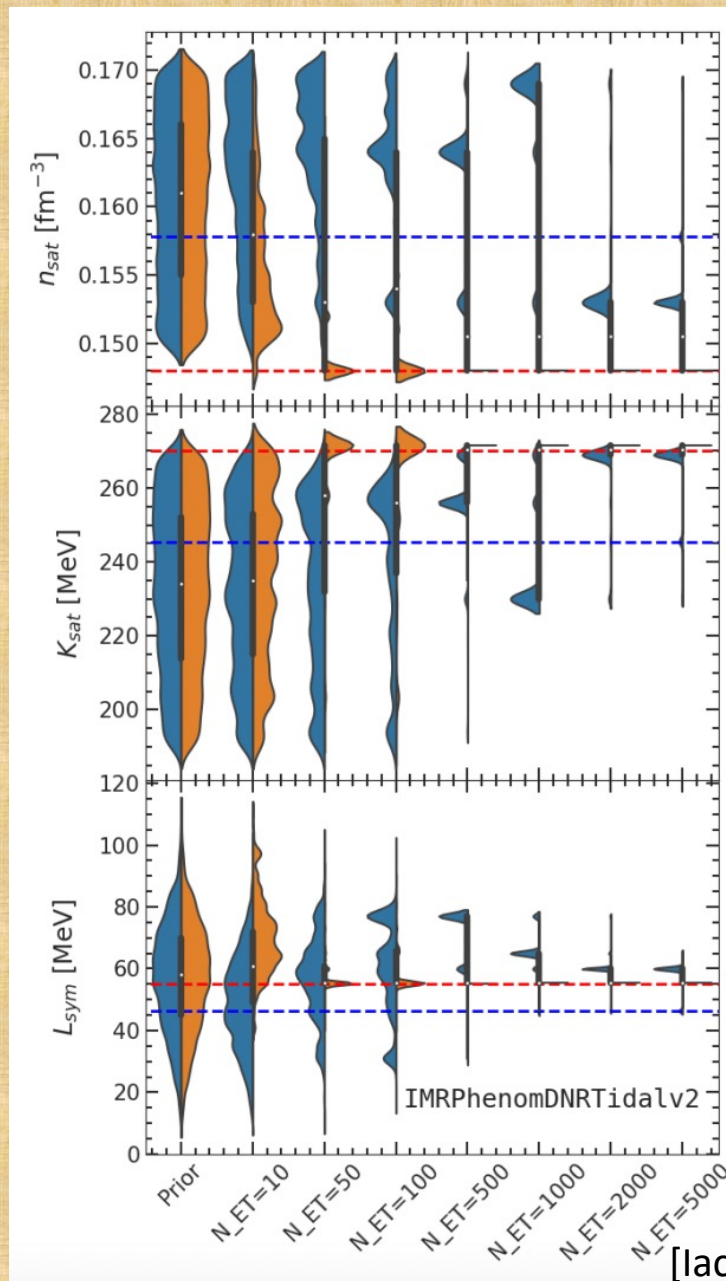
Pian+ 2017
Smarrt+ 2017

Reconstruction of nuclear physics parameters

n_{sat} : saturation density
 E_{sat} : binding energy at saturation
 K_{sat} : incompressibility
 E_{sym} : symmetry energy
 L_{sym} : symmetry energy slope
 K_{sym} : symmetry incompressibility

Nuclear parameters are reconstructed with high accuracy and precision for some EoS. For others, a degeneracy exists, such that different combinations of parameters lead to \sim the same EoS.

In these cases, **additional information from nuclear physics (theory and experiments) and EM observations is needed** to determine nuclear matter properties with high accuracy



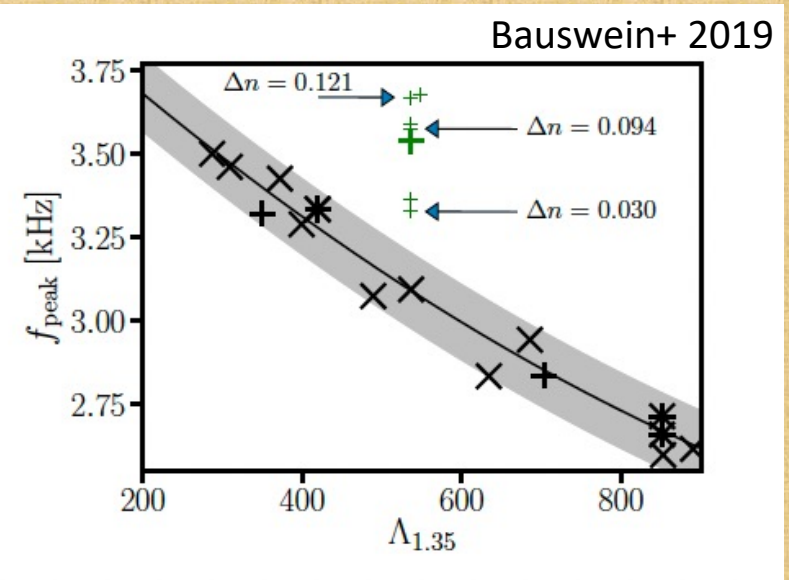
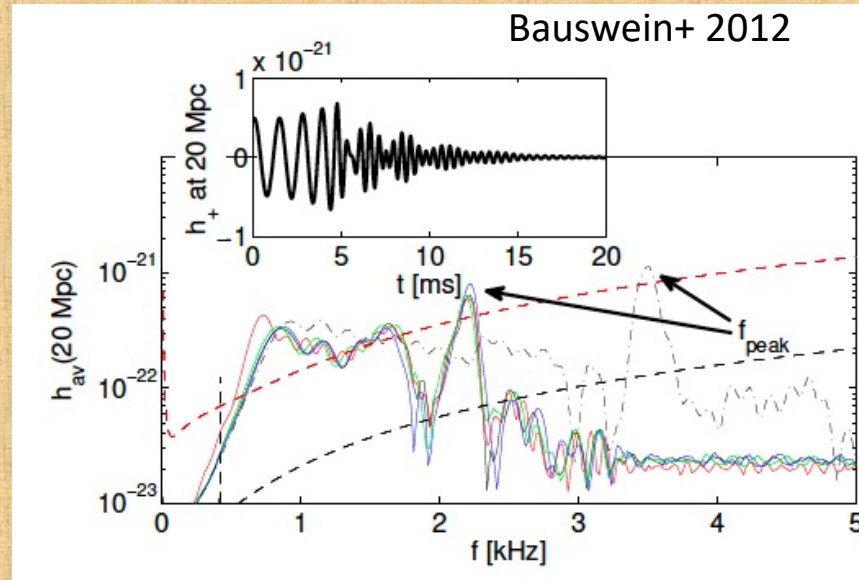
[Iacovelli+ 2023]

Other ways: NSNS post-merger GWs

High-frequency (2-4 kHz) GWs are emitted after the NSNS merger

The signal encodes information on hot high density matter

E.g., a spectral peak is expected, which frequency is related to the tidal deformability



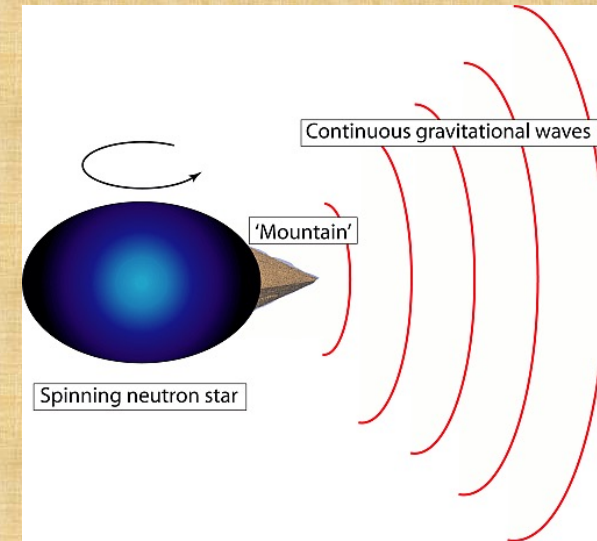
A phase transition in the NS inner core would determine a shift of the peak

Other ways: CW from spinning neutron stars

Persistent GW signals, from NS asymmetric w.r.t. the rotation axis

Very weak signals, but can be integrated over long times

$$h_0 \simeq 10^{-25} \left(\frac{10 \text{ kpc}}{d} \right) \left(\frac{\epsilon}{10^{-6}} \right) \left(\frac{I_3}{10^{45} \text{ g cm}^2} \right) \left(\frac{\nu}{500 \text{ Hz}} \right)^2$$



From a detection we estimate amplitude and frequency. If the distance is measured (from GW parallax or from an EM counterpart)

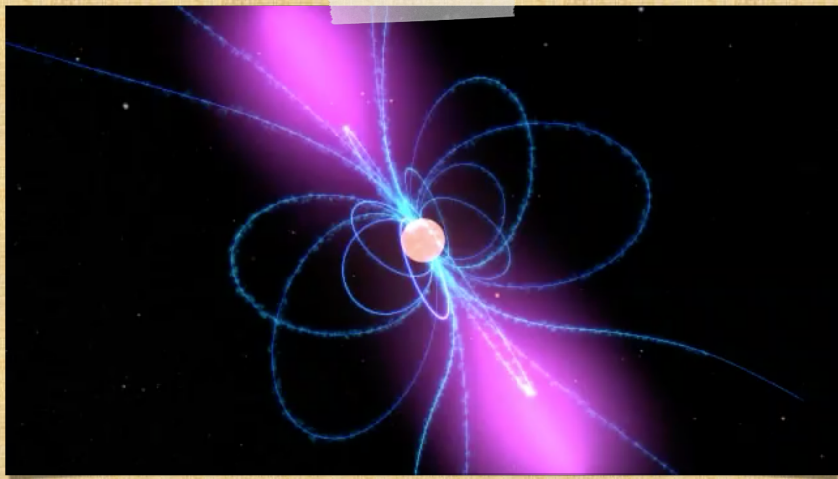
$$Q_{22} = \sqrt{\frac{15}{8\pi}} I_{zz} \epsilon$$

Quadrupole moment

Exclude EoS which do not allow given quadrupole moments

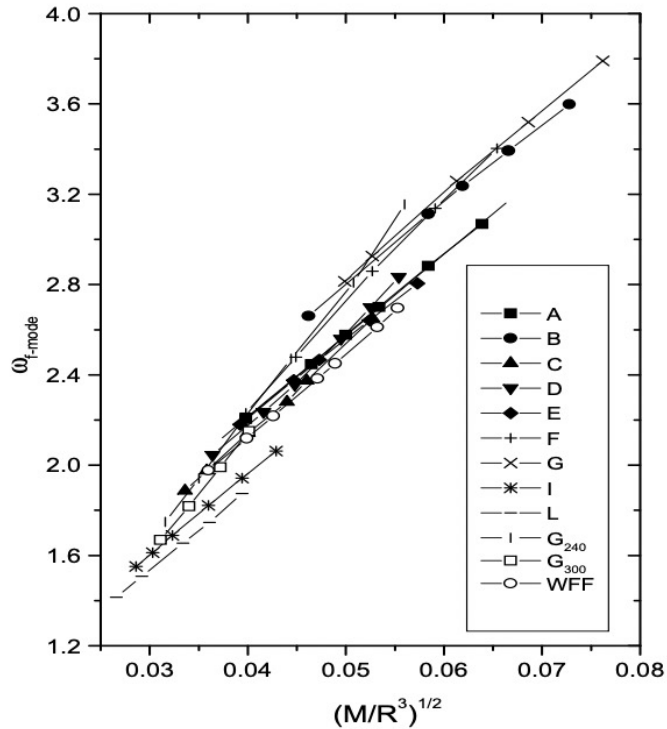
If the evolution is dominated by the emission of GWs \longrightarrow measure I_{zz}

An independent measure of mass or radius is needed to pinpoint the EoS



Other ways: pulsar glitches

Many pulsars sometimes show a sudden spin-up, likely due to the interaction among inner superfluid and crust



NS oscillation modes can be excited.

In principle, from the mode frequency and damping time the mass and radius of the NS can be estimated

