Ab Initio Nuclear Theory for Precision Electroweak Physics

Electroweak **P**hysics **I**nterse**C**tions – **EPIC 2024**

Calaserena Resort, Geremeas, Sardinia

September 22-27, 2024

Petr Navratil

TRIUMF

2024-09-24

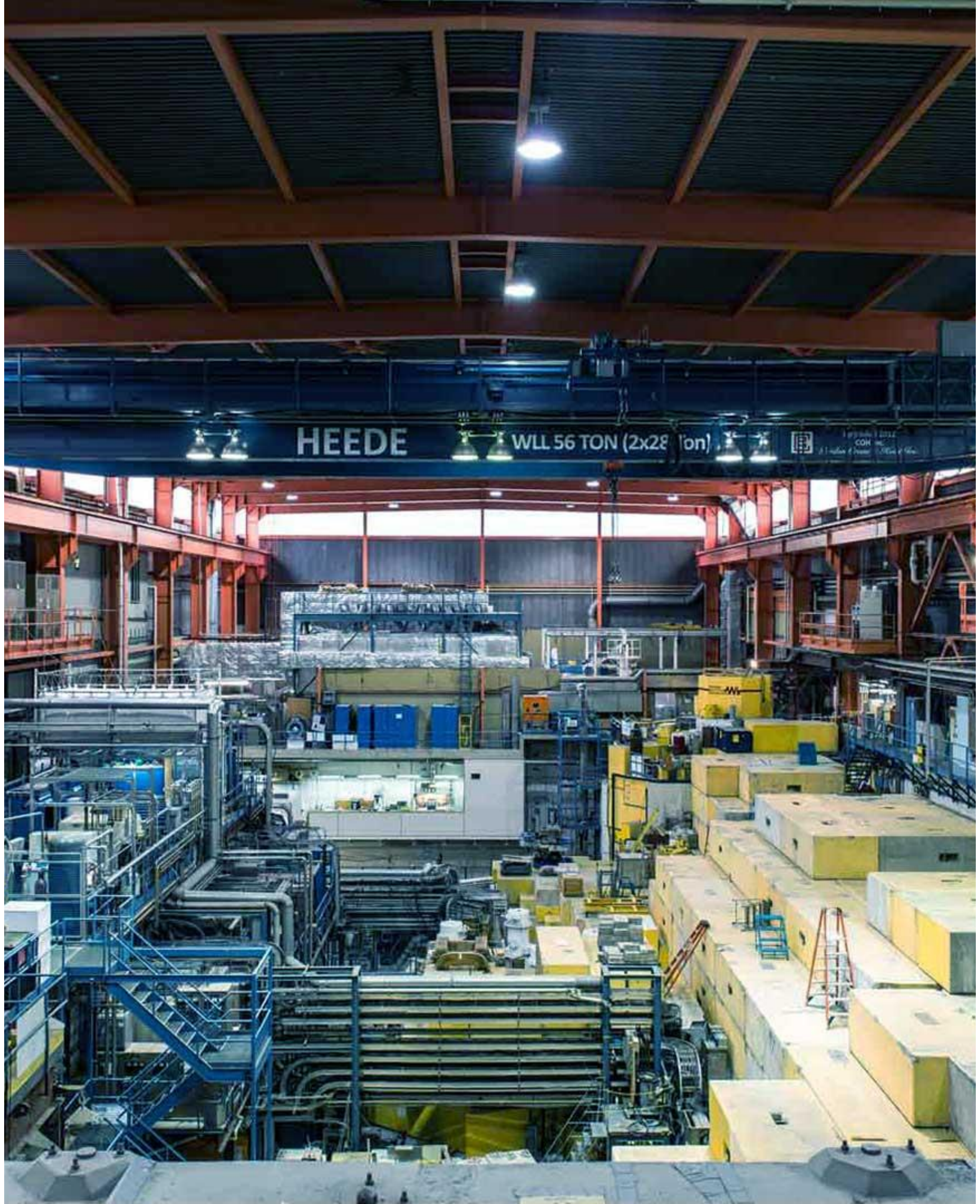
Outline

- Introduction – *Ab initio* nuclear theory – no-core shell model (NCSM)
- *Ab initio* calculations of parity-violating moments
- ^{6}He β-decay
- Super-allowed Fermi transitions - electroweak radiative correction δ_{NS}
  - isospin-symmetry breaking correction δ_{C}
- ^{7}Li(p,$e^{+}e^{-}$)^{8}Be internal pair creation and the X17 anomaly
- Conclusions & topics for discussion

- Backup slides – muon capture on light nuclei, ^{16}N beta decay

TRIUMF

Ab initio nuclear theory - no-core shell model (NCSM)



Discovery, accelerated

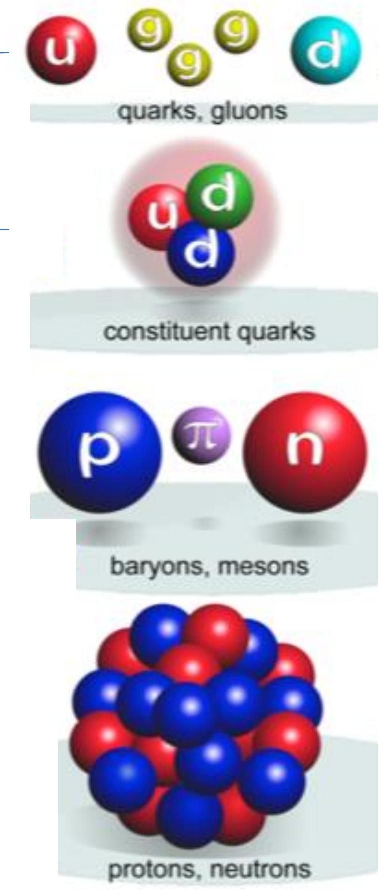
First principles or *ab initio* nuclear theory

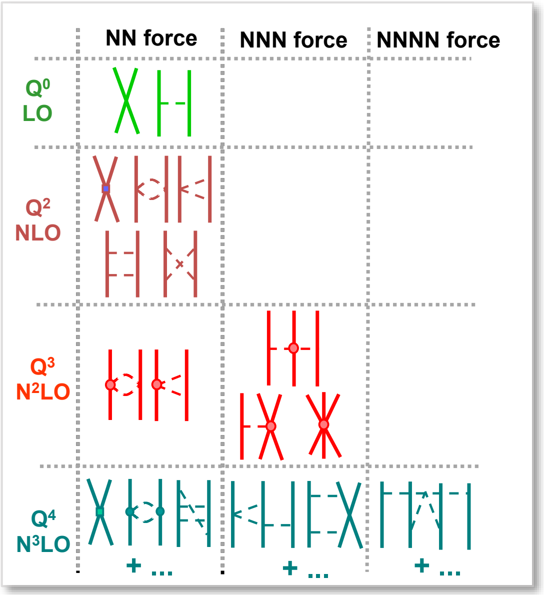
Quantum Chromodynamics (QCD)

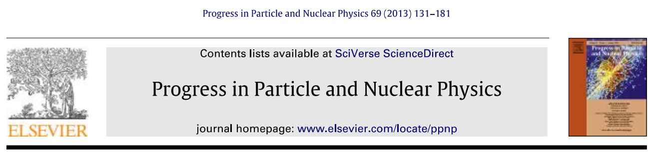
Chiral Effective Field Theory (parameters fitted to NN data)

Current *ab initio* nuclear theory

$$H\Psi^{(A)} = E\Psi^{(A)}$$



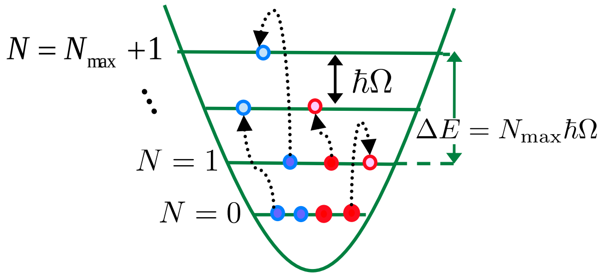



Progress in Particle and Nuclear Physics 69 (2013) 131–181
Contents lists available at SciVerse ScienceDirect
Progress in Particle and Nuclear Physics
journal homepage: www.elsevier.com/locate/ppnp
Review
Ab initio no core shell model
Bruce R. Barrett [a], Petr Navrátil [b], James P. Vary [c,*]

Conceptually simplest *ab initio* method: No-Core Shell Model (NCSM)

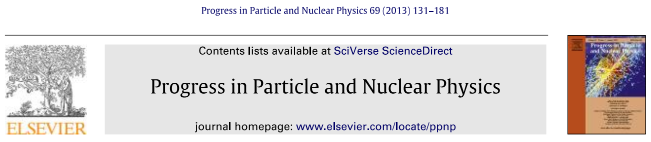
NCSM

- Basis expansion method
  - Harmonic oscillator (HO) basis truncated in a particular way (N_{max})
  - Why HO basis?
    - Lowest filled HO shells match magic numbers of light nuclei (2, 8, 20 – ^{4}He, ^{16}O, ^{40}Ca)
    - Equivalent description in relative(Jacobi)-coordinate and Slater determinant basis
- Short- and medium range correlations
- Bound-states, narrow resonances



$$\Psi^A = \sum_{N=0}^{N_{max}} \sum_i c_{Ni}\, \Phi^{HO}_{Ni}(\vec{\eta}_1, \vec{\eta}_2, \ldots, \vec{\eta}_{A-1})$$

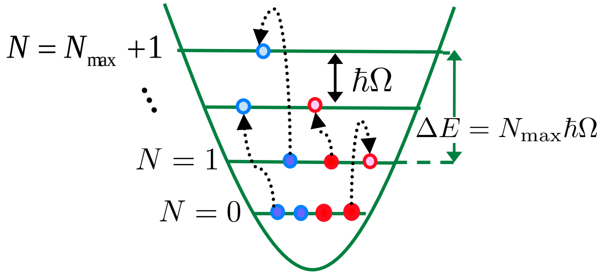
$$\Psi^A_{SD} = \sum_{N=0}^{N_{max}} \sum_j c^{SD}_{Nj}\, \Phi^{HO}_{SD\,Nj}(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_A) = \Psi^A\, \varphi_{000}(\vec{R}_{CM})$$

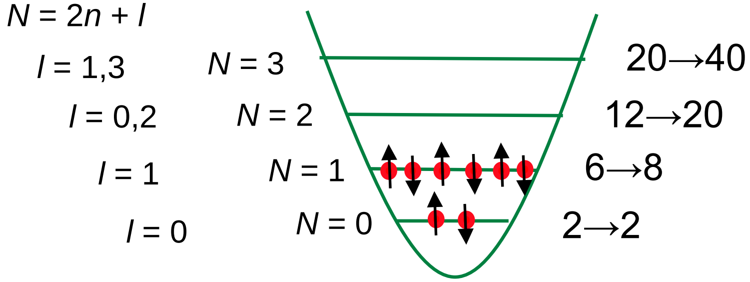

Progress in Particle and Nuclear Physics 69 (2013) 131–181
Contents lists available at SciVerse ScienceDirect
Progress in Particle and Nuclear Physics
journal homepage: www.elsevier.com/locate/ppnp
Review
Ab initio no core shell model
Bruce R. Barrett[a], Petr Navrátil[b], James P. Vary[c,*]

Conceptually simplest *ab initio* method: No-Core Shell Model (NCSM)


NCSM

- Basis expansion method
  - Harmonic oscillator (HO) basis truncated in a particular way (N_{max})
  - Why HO basis?
    - Lowest filled HO shells match magic numbers of light nuclei (2, 8, 20 – ^{4}He, ^{16}O, ^{40}Ca)
    - Equivalent description in relative(Jacobi)-coordinate and Slater determinant basis
- Short- and medium range correlations
- Bound-states, narrow resonances





$$E = (2n + l + \tfrac{3}{2})\hbar\Omega$$

$$\Psi^A = \sum_{N=0}^{N_{max}} \sum_i c_{Ni} \Phi^{HO}_{Ni}(\vec{\eta}_1, \vec{\eta}_2, \ldots, \vec{\eta}_{A-1})$$

$$\Psi^A_{SD} = \sum_{N=0}^{N_{max}} \sum_j c^{SD}_{Nj} \Phi^{HO}_{SD\,Nj}(\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_A) = \Psi^A \varphi_{000}(\vec{R}_{CM})$$

PHYSICAL REVIEW C 101, 014318 (2020)

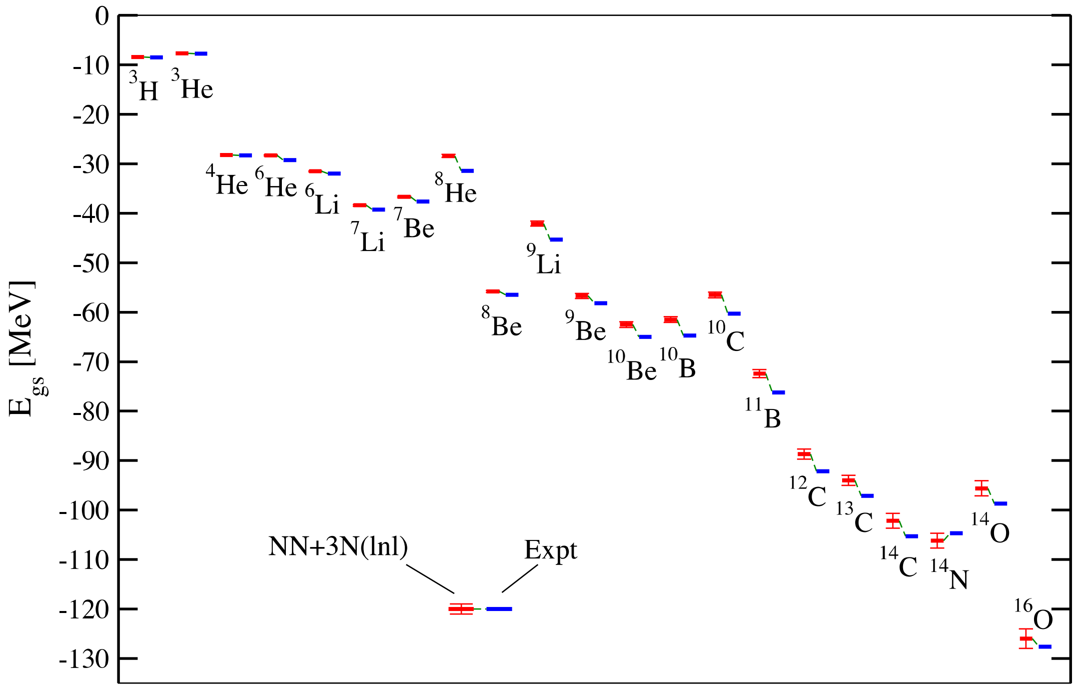
Novel chiral Hamiltonian and observables in light and medium-mass nuclei

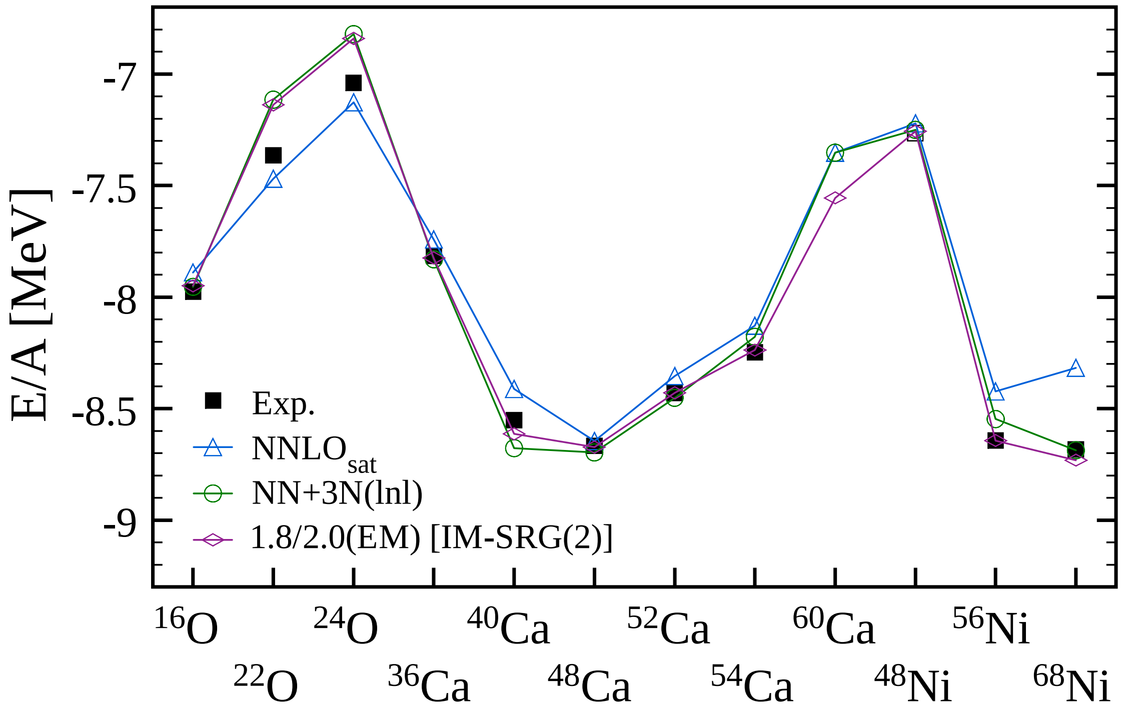
V. Somà,[1,*] P. Navrátil,[2,†] F. Raimondi,[3,4,‡] C. Barbieri,[4,§] and T. Duguet[1,5,‖]

Binding energies of atomic nuclei with NN+3N forces from chiral Effective Field Theory

- Quite reasonable description of binding energies across the nuclear charts becomes feasible
  - **The Hamiltonian fully determined in *A*=2 and *A*=3,4 systems**
    - Nucleon–nucleon scattering, deuteron properties, ^{3}H and ^{4}He binding energy, ^{3}H half life
  - Light nuclei – NCSM
  - Medium mass nuclei – Self-Consistent Green's Function method

NN N^{3}LO (Entem-Machleidt 2003)
3N N^{2}LO w local/non-local regulator





1.8/2.0 (EM) results: J. Simonis, S. R. Stroberg, K. Hebeler, J. D. Holt, and A. Schwenk, Phys. Rev. C 96, 014303 (2017).

PHYSICAL REVIEW C **101**, 014318 (2020)

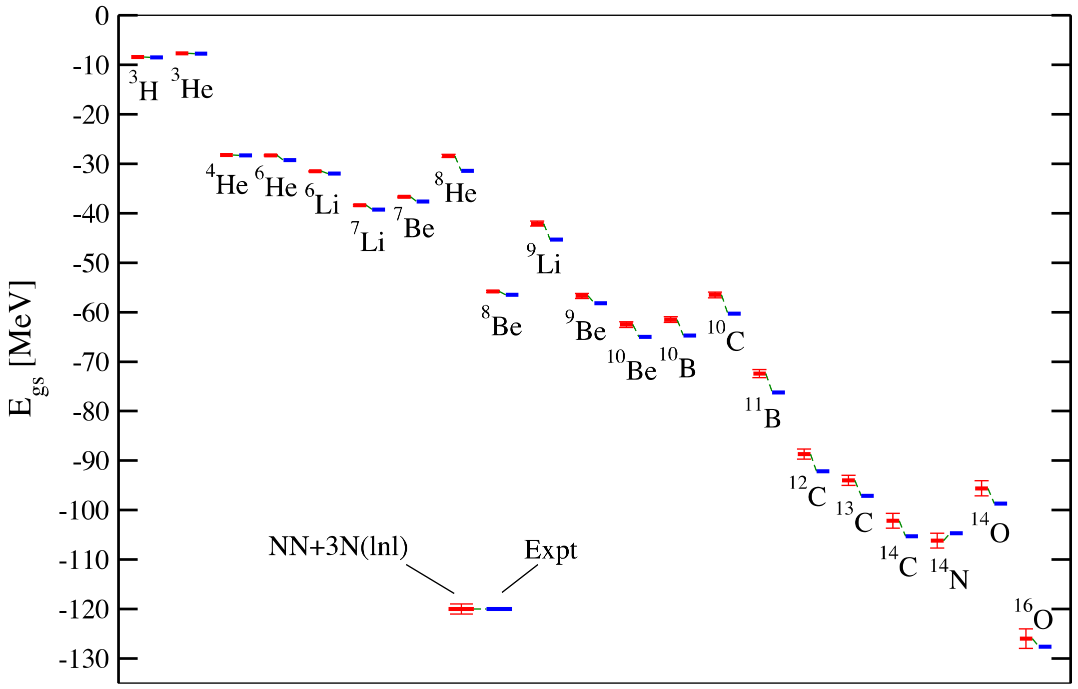
Novel chiral Hamiltonian and observables in light and medium-mass nuclei

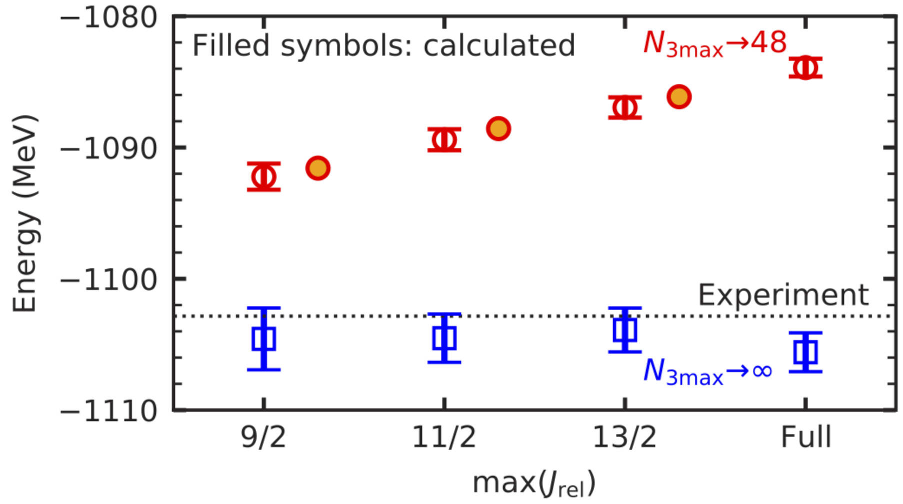
V. Somà,[1,*] P. Navrátil,[2,†] F. Raimondi,[3,4,‡] C. Barbieri,[4,§] and T. Duguet[1,5,∥]

Binding energies of atomic nuclei with NN+3N forces from chiral Effective Field Theory

- Quite reasonable description of binding energies across the nuclear charts becomes feasible
  - **The Hamiltonian fully determined in *A*=2 and *A*=3,4 systems**
    - Nucleon–nucleon scattering, deuteron properties, ^{3}H and ^{4}He binding energy, ^{3}H half life
  - Light nuclei – NCSM
  - Medium mass nuclei – Self-Consistent Green's Function method

NN N^{3}LO (Entem-Machleidt 2003)
3N N^{2}LO w local/non-local regulator





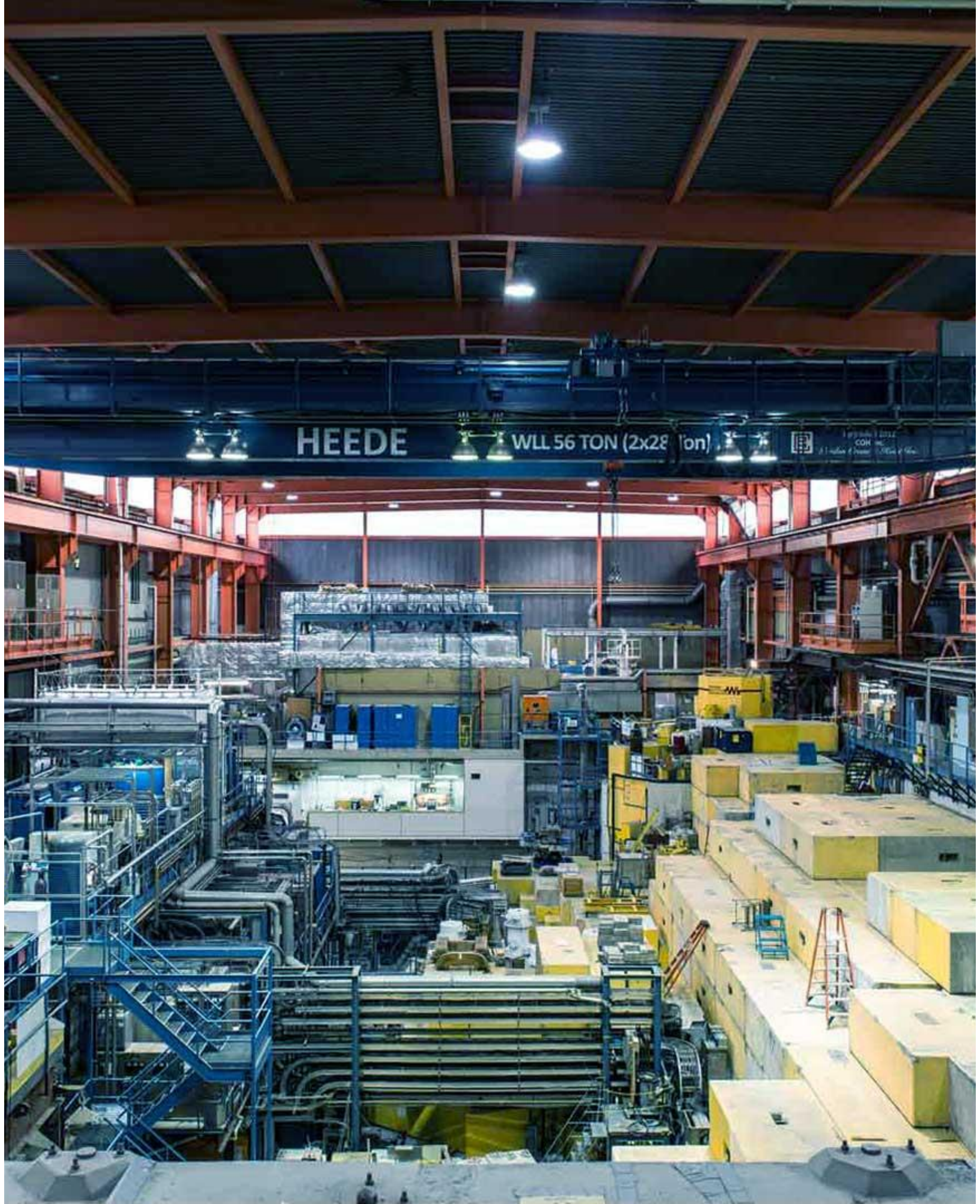
PHYSICAL REVIEW C **105**, 014302 (2022)

Converged *ab initio* calculations of heavy nuclei

T. Miyagi,[1,*] S. R. Stroberg,[2,†] P. Navrátil,[1,‡] K. Hebeler,[3,4,5,§] and J. D. Holt[1,6,∥]

TRIUMF

Ab initio calculations of parity-violating moments



Discovery, accelerated

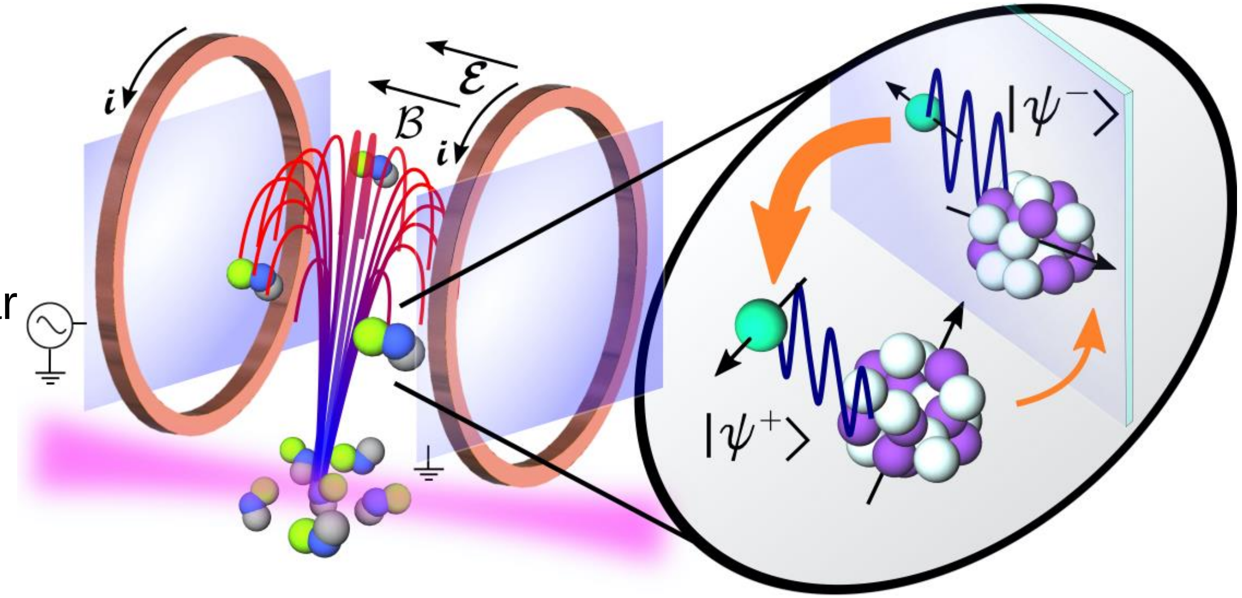
Why investigate the anapole moment and the EDM?

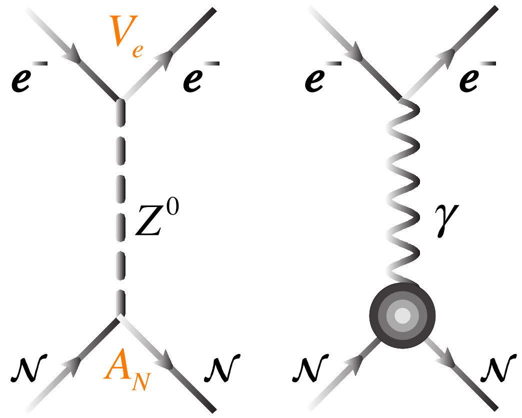
- Parity violation in atomic and molecular systems sensitive to a variety of "new physics"
  - Probes electron-quark electroweak interaction
  - Best limits on the Z' boson parity violating interaction with electrons and nucleons

- The EDM is a promising probe for CP violation beyond the standard model as well as CP violating QCD $\bar{\theta}$ parameter
  - Nuclear structure can enhance the EDM
  - Nuclear EDMs can be measured in storage rings (CERN feasibility study: arXiv:1912.07881)

Nuclear spin dependent parity violating effects in light polyatomic molecules

- Experiments proposed for ^{9}BeNC, ^{25}MgNC
- To extract the underlying physics, atomic, molecular and **nuclear** structure effects must be understood
  - *Ab initio* calculations
- Spin dependent PV
  - Z-boson exchange between nucleon axial-vector and electron-vector currents (b)
  - Electromagnetic interaction of atomic electrons with the nuclear anapole moment (c)





(b) (c)

Parity violating nucleon-nucleon interaction

- Meson exchange approach
- Chiral EFT
- Unknown parameters (LECs)
  - DDH (1980) – estimates based on the quark model
  - Experiments give conflicting limits on the weak couplings

ANNALS OF PHYSICS 124, 449–495 (1980)

Unified Treatment of the Parity Violating Nuclear Force

BERTRAND DESPLANQUES*

Institut de Physique Nucleaire, Division de Physique Théorique, 91406 Orsay Cedex—France

JOHN F. DONOGHUE†

Center for Theoretical Physics, Laboratory for Nuclear Science and Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

AND

BARRY R. HOLSTEIN

Physics Division, National Science Foundation, Washington, D. C. 20550

$$V_{12}^{\mathrm{p.v.}} = \frac{f_\pi g_{\pi NN}}{2^{1/2}}\, i\left(\frac{\vec{\tau}_1 \times \vec{\tau}_2}{2}\right)^z (\vec{\sigma}_1 + \vec{\sigma}_2)\cdot\left[\frac{\vec{p}_1 - \vec{p}_2}{2M}, f_\pi(r)\right]$$

$$- g_\rho\left(h_\rho^{\,0}\vec{\tau}_1\cdot\vec{\tau}_2 + h_\rho^{\,1}\left(\frac{\vec{\tau}_1 + \vec{\tau}_2}{2}\right)^z + h_\rho^{\,2}\,\frac{(3\tau_1^{\,z}\tau_2^{\,z} - \vec{\tau}_1\cdot\vec{\tau}_2)}{2(6)^{1/2}}\right)$$

$$\times\left((\vec{\sigma}_1 - \vec{\sigma}_2)\cdot\left\{\frac{\vec{p}_1 - \vec{p}_2}{2M}, f_\rho(r)\right\} + i(1+\chi_v)\,\vec{\sigma}_1\times\vec{\sigma}_2\cdot\left[\frac{\vec{p}_1 - \vec{p}_2}{2M}, f_\rho(r)\right]\right.$$

$$- g_\omega\left(h_\omega^{\,0} + h_\omega^{\,1}\left(\frac{\vec{\tau}_1 + \vec{\tau}_2}{2}\right)^z\right)$$

$$\times\left((\vec{\sigma}_1 - \vec{\sigma}_2)\cdot\left\{\frac{\vec{p}_1 - \vec{p}_2}{2M}, f_\omega(r)\right\} + i(1+\chi_S)\,\vec{\sigma}_1\times\vec{\sigma}_2\cdot\left[\frac{\vec{p}_1 - \vec{p}_2}{2M}, f_\omega(r)\right]\right.$$

$$- (g_\omega h_\omega^{\,1} - g_\rho h_\rho^{\,1})\left(\frac{\vec{\tau}_1 - \vec{\tau}_2}{2}\right)^z (\vec{\sigma}_1 + \vec{\sigma}_2)\cdot\left\{\frac{\vec{p}_1 - \vec{p}_2}{2M}, f_\rho(r)\right\}$$

$$- g_\rho h_\rho^{\prime 1} i\left(\frac{\vec{\tau}_1 \times \vec{\tau}_2}{2}\right)^z (\vec{\sigma}_1 + \vec{\sigma}_2)\cdot\left[\frac{\vec{p}_1 - \vec{p}_2}{2M}, f_\rho(r)\right],$$

$$f_\pi(r) = \frac{e^{-m_\pi r}}{4\pi r},$$

$$f_\rho(r) = f_\omega(r) = \frac{e^{-m_\rho r}}{4\pi r}.$$

frontiers in Physics

REVIEW
published: 21 July 2020
doi: 10.3389/fphy.2020.00218

Parity- and Time-Reversal-Violating Nuclear Forces

Jordy de Vries [1,2], Evgeny Epelbaum [3], Luca Girlanda [4,5], Alex Gnech [6], Emanuele Mereghetti [7] and Michele Viviani [8]*

Parity violating nucleon-nucleon interaction

- Meson exchange approach
- Chiral EFT
- Unknown parameters (LECs)
  - DDH (1980) – estimates based on the quark model
  - Experiments give conflicting limits on the weak couplings

$$\mathscr{H}^{\text{p.v.}}_{MNN} = (2)^{-1/2} f_\pi \bar{N}(\vec{\tau} \times \vec{\phi}^\pi)^3 N + \bar{N}\left[h_\rho^0 \vec{\tau}\cdot\vec{\phi}_\mu^{\,\rho} + h_\rho^1 \phi_\mu^{\rho 3} + h_\rho^2 \frac{(3\tau^3\phi_\mu^{\rho 3} - \vec{\tau}\cdot\vec{\phi}_\mu^{\,\rho})}{2(6)^{1/2}}\right]\gamma^\mu\gamma_5 N + \bar{N}[h_\omega^0\phi_\mu^\omega + h_\omega^1\tau^3\phi_\mu^\omega]\gamma^\mu\gamma_5 N - h_\rho'^1 \bar{N}(\vec{\tau}\times\vec{\phi}_\mu^{\,\rho})^3 \frac{\sigma^{\mu\nu}k_\nu}{2M}\gamma_5 N.$$

$$\mathscr{H}^{\text{p.c.}}_{MNN} = i g_{\pi NN}\bar{N}\gamma_5\vec{\tau}\cdot\vec{\phi}^\pi N + g_\rho \bar{N}\left(\gamma_\mu + \frac{i\chi_V}{2M}\sigma_{\mu\nu}k^\nu\right)\vec{\tau}\cdot\vec{\phi}^{\mu\rho}N + g_\omega\bar{N}\left(\gamma^\mu + \frac{i\chi_S}{2M}\sigma^{\mu\nu}k_\nu\right)\phi_\mu^\omega N$$

ANNALS OF PHYSICS 124, 449–495 (1980)

Unified Treatment of the Parity Violating Nuclear Force

BERTRAND DESPLANQUES*

Institut de Physique Nucleaire, Division de Physique Théorique, 91406 Orsay Cedex—France

JOHN F. DONOGHUE†

Center for Theoretical Physics, Laboratory for Nuclear Science and Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

AND

BARRY R. HOLSTEIN

Physics Division, National Science Foundation, Washington, D. C. 20550

$$\begin{aligned}
V_{12}^{\text{p.v.}} = {} & \frac{f_\pi g_{\pi NN}}{2^{1/2}}\, i \left(\frac{\vec{\tau}_1\times\vec{\tau}_2}{2}\right)^z (\vec{\sigma}_1+\vec{\sigma}_2)\cdot\left[\frac{\vec{p}_1-\vec{p}_2}{2M}, f_\pi(r)\right] \\
& - g_\rho\left(h_\rho^0\vec{\tau}_1\cdot\vec{\tau}_2 + h_\rho^1\left(\frac{\vec{\tau}_1+\vec{\tau}_2}{2}\right)^z + h_\rho^2\frac{(3\tau_1^z\tau_2^z - \vec{\tau}_1\cdot\vec{\tau}_2)}{2(6)^{1/2}}\right) \\
& \times\left((\vec{\sigma}_1-\vec{\sigma}_2)\cdot\left\{\frac{\vec{p}_1-\vec{p}_2}{2M}, f_\rho(r)\right\} + i(1+\chi_V)\,\vec{\sigma}_1\times\vec{\sigma}_2\cdot\left[\frac{\vec{p}_1-\vec{p}_2}{2M}, f_\rho(r)\right]\right) \\
& - g_\omega\left(h_\omega^0 + h_\omega^1\left(\frac{\vec{\tau}_1+\vec{\tau}_2}{2}\right)^z\right) \\
& \times\left((\vec{\sigma}_1-\vec{\sigma}_2)\cdot\left\{\frac{\vec{p}_1-\vec{p}_2}{2M}, f_\omega(r)\right\} + i(1+\chi_S)\,\vec{\sigma}_1\times\vec{\sigma}_2\cdot\left[\frac{\vec{p}_1-\vec{p}_2}{2M}, f_\omega(r)\right]\right) \\
& - (g_\omega h_\omega^1 - g_\rho h_\rho^1)\left(\frac{\vec{\tau}_1-\vec{\tau}_2}{2}\right)^z(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\left\{\frac{\vec{p}_1-\vec{p}_2}{2M}, f_\rho(r)\right\} \\
& - g_\rho h_\rho'^1\, i\left(\frac{\vec{\tau}_1\times\vec{\tau}_2}{2}\right)^z(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\left[\frac{\vec{p}_1-\vec{p}_2}{2M}, f_\rho(r)\right],
\end{aligned}$$

$$f_\pi(r) = \frac{e^{-m_\pi r}}{4\pi r},$$

$$f_\rho(r) = f_\omega(r) = \frac{e^{-m_\rho r}}{4\pi r}.$$

Parity violating nucleon-nucleon interaction and the nuclear anapole moment

- Parity violating (non-conserving) V_{NN}^{PNC} interaction
  - Conserves total angular momentum *I*
  - Mixes opposite parities
  - Has isoscalar, isovector and isotensor components
  - Admixes unnatural parity states in the ground state

$$
\begin{aligned}
|\psi_{\rm gs}\ I\rangle &= |\psi_{\rm gs}\ I^{\pi}\rangle + \sum_j |\psi_j\ I^{-\pi}\rangle \\
&\quad \times \frac{1}{E_{\rm gs} - E_j} \langle \psi_j\ I^{-\pi} | V_{\rm NN}^{\rm PNC} | \psi_{\rm gs}\ I^{\pi}\rangle
\end{aligned}
$$

$$
\begin{aligned}
V_{12}^{\rm p.v.} &= \frac{f_\pi g_{\pi NN}}{2^{1/2}}\, i \left(\frac{\vec{\tau}_1 \times \vec{\tau}_2}{2}\right)^z (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \left[\frac{\vec{p}_1 - \vec{p}_2}{2M}, f_\pi(r)\right] \\
&- g_\rho \left(h_\rho^{\,0} \vec{\tau}_1 \cdot \vec{\tau}_2 + h_\rho^{\,1} \left(\frac{\vec{\tau}_1 + \vec{\tau}_2}{2}\right)^z + h_\rho^{\,2} \frac{(3\tau_1^{\,z}\tau_2^{\,z} - \vec{\tau}_1 \cdot \vec{\tau}_2)}{2(6)^{1/2}}\right) \\
&\times \left((\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \left\{\frac{\vec{p}_1 - \vec{p}_2}{2M}, f_\rho(r)\right\} + i(1 + \chi_v)\, \vec{\sigma}_1 \times \vec{\sigma}_2 \cdot \left[\frac{\vec{p}_1 - \vec{p}_2}{2M}, f_\rho(r)\right]\right. \\
&- g_\omega \left(h_\omega^{\,0} + h_\omega^{\,1} \left(\frac{\vec{\tau}_1 + \vec{\tau}_2}{2}\right)^z\right) \\
&\times \left((\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \left\{\frac{\vec{p}_1 - \vec{p}_2}{2M}, f_\omega(r)\right\} + i(1 + \chi_S)\, \vec{\sigma}_1 \times \vec{\sigma}_2 \cdot \left[\frac{\vec{p}_1 - \vec{p}_2}{2M}, f_\omega(r)\right]\right. \\
&- (g_\omega h_\omega^{\,1} - g_\rho h_\rho^{\,1}) \left(\frac{\vec{\tau}_1 - \vec{\tau}_2}{2}\right)^z (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \left\{\frac{\vec{p}_1 - \vec{p}_2}{2M}, f_\rho(r)\right\} \\
&- g_\rho h_\rho^{\prime 1} i \left(\frac{\vec{\tau}_1 \times \vec{\tau}_2}{2}\right)^z (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \left[\frac{\vec{p}_1 - \vec{p}_2}{2M}, f_\rho(r)\right],
\end{aligned}
$$

$$
f_\pi(r) = \frac{e^{-m_\pi r}}{4\pi r},
$$

$$
f_\rho(r) = f_\omega(r) = \frac{e^{-m_\rho r}}{4\pi r}.
$$

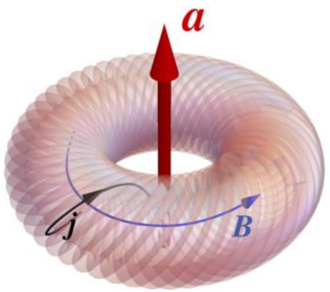
Parity violating nucleon-nucleon interaction and the nuclear anapole moment

- Parity violating (non-conserving) V_{NN}^{PNC} interaction
  - Conserves total angular momentum *I*
  - Mixes opposite parities
  - Has isoscalar, isovector and isotensor components
  - Admixes unnatural parity states in the ground state

$$|\psi_{\rm gs}\ I\rangle = |\psi_{\rm gs}\ I^{\pi}\rangle + \sum_j |\psi_j\ I^{-\pi}\rangle$$
$$\times \frac{1}{E_{\rm gs}-E_j}\langle\psi_j\ I^{-\pi}|V_{\rm NN}^{\rm PNC}|\psi_{\rm gs}\ I^{\pi}\rangle$$

- Anapole moment operator dominated by spin contribution

$$\boldsymbol{a} = -\pi\int d^3r\, r^2\, \boldsymbol{j}(\boldsymbol{r})$$



$$\hat{\boldsymbol{a}}_s = \frac{\pi e}{m}\sum_{i=1}^{A}\mu_i(\boldsymbol{r}_i\times\boldsymbol{\sigma}_i)$$

$$\mu_i{=}\mu_p(1/2{+}t_{z,i}) + \mu_n(1/2{-}t_{z,i})$$

$$a_s = \langle\psi_{\rm gs}\ I\ I_z{=}I|\hat{a}^{(1)}_{s,0}|\psi_{\rm gs}\ I\ I_z{=}I\rangle$$

- Here is what we want to calculate:

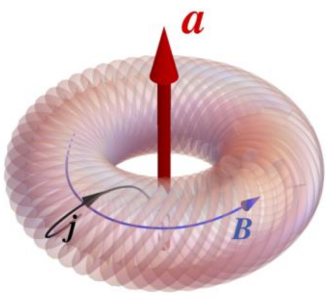
$$\kappa_A = \frac{\sqrt{2}e}{G_F}a_s$$

$$\kappa_A = -i4\pi\frac{e^2}{G_F}\frac{\hbar}{mc}\frac{(II10|II)}{\sqrt{2I+1}}\sum_j\langle\psi_{\rm gs}\ I^{\pi}||\sqrt{4\pi/3}\sum_{i=1}^{A}\mu_i r_i[Y_1(\hat{r}_i)\sigma_i]^{(1)}||\psi_j\ I^{-\pi}\rangle\frac{1}{E_{\rm gs}-E_j}\langle\psi_j\ I^{-\pi}|V_{\rm NN}^{\rm PNC}|\psi_{\rm gs}\ I^{\pi}\rangle$$

NCSM applications to parity-violating moments:
How to calculate the sum of intermediate unnatural parity states?

$$a_s = \langle \psi_{\rm gs}\ I\ I_z{=}I | \hat{a}^{(1)}_{s,0} | \psi_{\rm gs}\ I\ I_z{=}I \rangle$$

$$|\psi_{\rm gs}\ I\rangle = |\psi_{\rm gs}\ I^{\pi}\rangle + \sum_j |\psi_j\ I^{-\pi}\rangle \frac{1}{E_{\rm gs} - E_j} \langle \psi_j\ I^{-\pi} | V^{\rm PNC}_{\rm NN} | \psi_{\rm gs}\ I^{\pi}\rangle$$



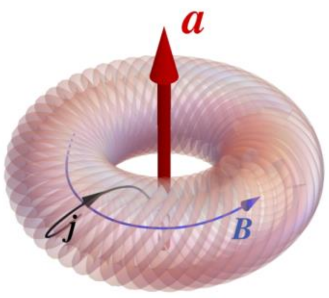
- Solving Schroedinger equation with inhomogeneous term

$$(E_{\rm gs} - H)|\psi_{\rm gs}\ I\rangle = V^{\rm PNC}_{\rm NN} |\psi_{\rm gs}\ I^{\pi}\rangle$$

- To invert this equation, we apply the Lanczos algorithm

NCSM applications to parity-violating moments:
How to calculate the sum of intermediate unnatural parity states?

$$a_s = \langle \psi_{\rm gs}\ I\ I_z{=}I | \hat{a}^{(1)}_{s,0} | \psi_{\rm gs}\ I\ I_z{=}I \rangle$$



$$|\psi_{\rm gs}\ I\rangle = |\psi_{\rm gs}\ I^{\pi}\rangle + \sum_j |\psi_j\ I^{-\pi}\rangle \frac{1}{E_{\rm gs} - E_j} \langle \psi_j\ I^{-\pi} | V^{\rm PNC}_{\rm NN} | \psi_{\rm gs}\ I^{\pi}\rangle$$

- Solving Schroedinger equation with inhomogeneous term

$$(E_{\rm gs} - H)|\psi_{\rm gs}\ I\rangle = V^{\rm PNC}_{\rm NN} |\psi_{\rm gs}\ I^{\pi}\rangle$$

- To invert this equation, we apply the Lanczos algorithm
  - Bring matrix to tri-diagonal form ($\mathbf{v}_1$, $\mathbf{v}_2$ … orthonormal, H Hermitian)

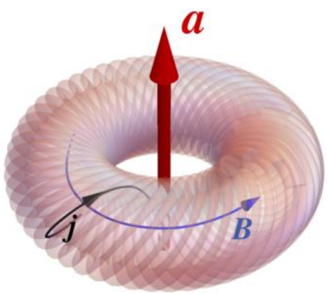
$$\begin{aligned}
H\mathbf{v}_1 &= a_1\mathbf{v}_1 + b_1\mathbf{v}_2 \\
H\mathbf{v}_2 &= b_1\mathbf{v}_1 + a_2\mathbf{v}_2 + b_2\mathbf{v}_3 \\
H\mathbf{v}_3 &= \qquad\quad b_2\mathbf{v}_2 + a_3\mathbf{v}_3 + b_3\mathbf{v}_4 \\
H\mathbf{v}_4 &= \qquad\qquad\qquad b_3\mathbf{v}_3 + a_4\mathbf{v}_4 + b_4\mathbf{v}_5
\end{aligned}$$

  - n^{th} iteration computes $2n^{th}$ moment
  - Eigenvalues converge to extreme (largest in magnitude) values
  - ~ 150-200 iterations needed for 10 eigenvalues (even for 10^9 states)

NCSM applications to parity-violating moments:
How to calculate the sum of intermediate unnatural parity states?

$$a_s = \langle \psi_{\rm gs}\ I\ I_z{=}I | \hat{a}^{(1)}_{s,0} | \psi_{\rm gs}\ I\ I_z{=}I \rangle$$

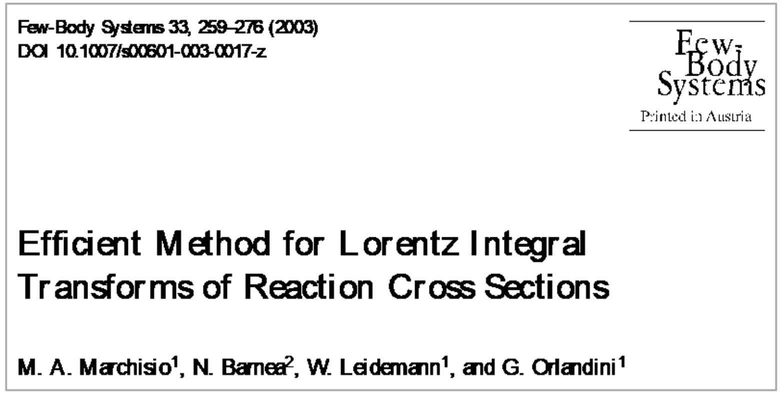
$$|\psi_{\rm gs}\ I\rangle = |\psi_{\rm gs}\ I^{\pi}\rangle + \sum_j |\psi_j\ I^{-\pi}\rangle \frac{1}{E_{\rm gs} - E_j} \langle \psi_j\ I^{-\pi} | V^{\rm PNC}_{\rm NN} | \psi_{\rm gs}\ I^{\pi}\rangle$$



- Solving Schroedinger equation with inhomogeneous term

$$(E_{\rm gs} - H)|\psi_{\rm gs}\ I\rangle = V^{\rm PNC}_{\rm NN} |\psi_{\rm gs}\ I^{\pi}\rangle$$

- To invert this equation, we apply the Lanczos algorithm



Few-Body Systems 33, 259–276 (2003)
DOI 10.1007/s00601-003-0017-z

Few-Body Systems
Printed in Austria

Efficient Method for Lorentz Integral Transforms of Reaction Cross Sections

M. A. Marchisio[1], N. Barnea[2], W. Leidemann[1], and G. Orlandini[1]

$$|\mathbf{v}_1\rangle = V^{\rm PNC}_{\rm NN} |\psi_{\rm gs}\ I^{\pi}\rangle$$

$$|\psi_{\rm gs}\ I\rangle \approx \sum_k g_k(E_0) |\mathbf{v}_k\rangle$$

$$\hat{g}_1(\omega) = \cfrac{1}{\omega - \alpha_1 - \cfrac{\beta_1^2}{\omega - \alpha_2 - \cfrac{\beta_2^2}{\omega - \alpha_3 - \beta_3^2}}}$$

$$\ddots$$

Lanczos continued fraction method

Parity and time-reversal violating nucleon-nucleon interaction

PHYSICAL REVIEW C **70**, 055501 (2004)

***P*- and *T*-odd two-nucleon interaction and the deuteron electric dipole moment**

C.-P. Liu* and R. G. E. Timmermans[†]

Introduced through Hamiltonian H_{PVTV} :

$$
\begin{aligned}
H_{PVTV}(\boldsymbol{r}) = \frac{1}{2m_n} \boldsymbol{\sigma}_- \cdot \boldsymbol{\nabla}\left(-\bar{G}_\omega^0 y_\omega(r)\right) \\
+\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2\, \boldsymbol{\sigma}_- \cdot \boldsymbol{\nabla}\left(\bar{G}_\pi^0 y_\pi(r) - \bar{G}_\rho^0 y_\rho(r)\right) \\
+\frac{\tau_+^z}{2} \boldsymbol{\sigma}_- \cdot \boldsymbol{\nabla}\left(\bar{G}_\pi^1 y_\pi(r) - \bar{G}_\rho^1 y_\rho(r) - \bar{G}_\omega^1 y_\omega(r)\right) \\
+\frac{\tau_-^z}{2} \boldsymbol{\sigma}_+ \cdot \boldsymbol{\nabla}\left(\bar{G}_\pi^1 y_\pi(r) + \bar{G}_\rho^1 y_\rho(r) - \bar{G}_\omega^1 y_\omega(r)\right) \\
+(3\tau_1^z\tau_2^z - \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)\boldsymbol{\sigma}_- \cdot \boldsymbol{\nabla}\left(\bar{G}_\pi^2 y_\pi(r) - \bar{G}_\rho^2 y_\rho(r)\right)
\end{aligned}
$$

- Based on one meson exchange model
- $y_x(r) = e^{-m_x r}/(4\pi r)$

$$\sigma_\pm = \sigma_1 \pm \sigma_2$$

$$\tau_\pm^z = \tau_1^z \pm \tau_2^z$$

Parity and time-reversal violating nucleon-nucleon interaction

Introduced through Hamiltonian H_{PVTV} :

PHYSICAL REVIEW C **70**, 055501 (2004)

***P*- and *T*-odd two-nucleon interaction and the deuteron electric dipole moment**

C.-P. Liu* and R. G. E. Timmermans†

$$
\begin{aligned}
H_{PVTV}(\boldsymbol{r}) = \frac{1}{2m_n} & \boldsymbol{\sigma}_- \cdot \nabla \left(-\bar{G}_\omega^0 y_\omega(r)\right) \\
& + \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \, \boldsymbol{\sigma}_- \cdot \nabla \left(\bar{G}_\pi^0 y_\pi(r) - \bar{G}_\rho^0 y_\rho(r)\right) \\
& + \frac{\tau_+^z}{2} \boldsymbol{\sigma}_- \cdot \nabla \left(\bar{G}_\pi^1 y_\pi(r) - \bar{G}_\rho^1 y_\rho(r) - \bar{G}_\omega^1 y_\omega(r)\right) \\
& + \frac{\tau_-^z}{2} \boldsymbol{\sigma}_+ \cdot \nabla \left(\bar{G}_\pi^1 y_\pi(r) + \bar{G}_\rho^1 y_\rho(r) - \bar{G}_\omega^1 y_\omega(r)\right) \\
& + (3\tau_1^z \tau_2^z - \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2) \boldsymbol{\sigma}_- \cdot \nabla \left(\bar{G}_\pi^2 y_\pi(r) - \bar{G}_\rho^2 y_\rho(r)\right)
\end{aligned}
$$

- Based on one meson exchange model
- $y_x(r) = e^{-m_x r}/(4\pi r)$
- Coupling constants

$$\sigma_\pm = \sigma_1 \pm \sigma_2$$

$$\tau_\pm^z = \tau_1^z \pm \tau_2^z$$

Parity and time-reversal violating nucleon-nucleon interaction and nuclear EDM or Schiff moment

H_{PVTV} introduces parity admixture in the ground state (perturbation theory):

$$|0\rangle \longrightarrow |0\rangle + |\tilde{0}\rangle$$

$$|\tilde{0}\rangle = \sum_{n\neq 0} \frac{1}{E_0 - E_n} |n\rangle\langle n| H_{PVTV} |0\rangle$$

Nuclear EDM is dominated by polarization contribution:

$$\boldsymbol{S} = \frac{e}{10} \sum_{i=1}^{Z} \left(r_i^2 \boldsymbol{r}_i - \frac{5}{3} \langle r^2 \rangle_{\mathrm{ch}} \boldsymbol{r}_i \right)$$

$$D^{(pol)} = \langle 0 | \hat{D}_z | \tilde{0} \rangle + c.c.$$

$$\hat{D}_z = \frac{e}{2} \sum_{i=1}^{A} (1 + \tau_i^z)\, z_i$$

Parity and time-reversal violating nucleon-nucleon interaction and nuclear EDM or Schiff moment

H_{PVTV} introduces parity admixture in the ground state (perturbation theory):

$$|0\rangle \longrightarrow |0\rangle + |\tilde{0}\rangle$$

$$|\tilde{0}\rangle = \sum_{n\neq 0} \frac{1}{E_0 - E_n} |n\rangle\langle n| H_{PVTV} |0\rangle$$

Low lying states of opposite parity can lead to enhancement!

Nuclear EDM is dominated by polarization contribution:

$$\boldsymbol{S} = \frac{e}{10}\sum_{i=1}^{Z}\left(r_i^2\boldsymbol{r}_i - \frac{5}{3}\langle r^2\rangle_{\mathrm{ch}}\boldsymbol{r}_i\right)$$

$$D^{(pol)} = \langle 0|\hat{D}_z|\tilde{0}\rangle + c.c.$$

$$\hat{D}_z = \frac{e}{2}\sum_{i=1}^{A}(1+\tau_i^z)\, z_i$$

PHYSICAL REVIEW C **104**, 025502 (2021)

***Ab initio* calculations of electric dipole moments of light nuclei**

Paul Froese*
TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada
and Department of Physics and Astronomy, University of British Columbia, Vancouver, British Columbia V6T 1Z1, Canada

Petr Navrátil†
TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada

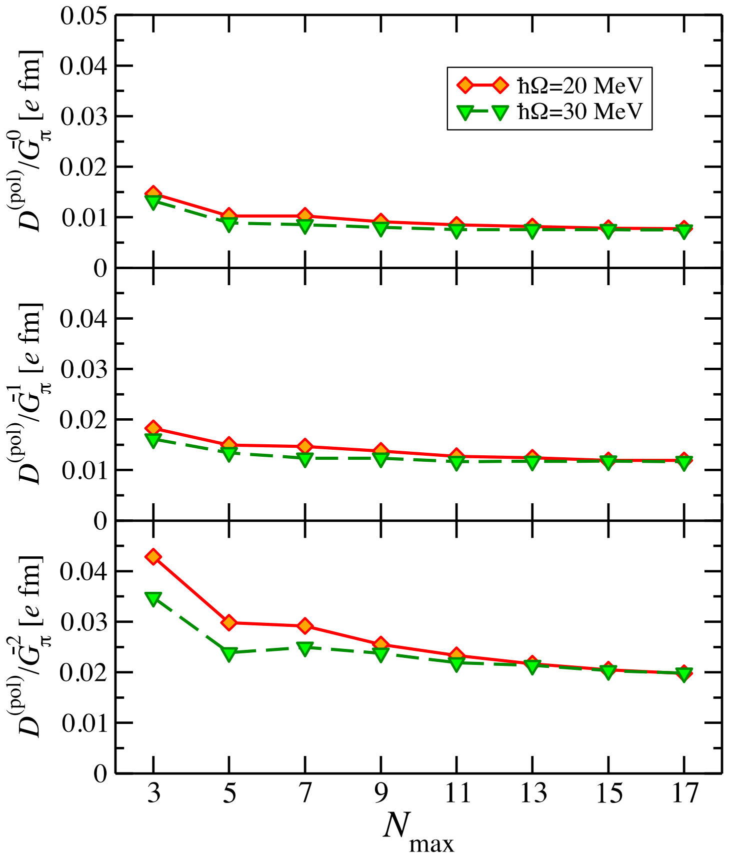
^{3}He EDM Benchmark Calculation

Discrepancy between calculations?

	PLB 665:165-172 (2008) (NN EFT)	PRC 87:015501 (2013)	PRC 91:054005 (2015)	Our calculation (NN EFT)
$\bar{G}_\pi^0$	0.015	(x 1/2)	(x 1/2)	0.0073 (x 1/2)
$\bar{G}_\pi^1$	0.023	(x 1/2)	(x 1/2)	0.011 (x 1/2)
$\bar{G}_\pi^2$	0.037	(x 1/5)	(x 1/2)	0.019 (x 1/2)
$\bar{G}_\rho^0$	-0.0012	(x 1/2)	(x 1/2)	-0.00062 (x 1/2)
$\bar{G}_\rho^1$	0.0013	(x 1/2)	(x 1/2)	0.00063 (x 1/2)
$\bar{G}_\rho^2$	-0.0028	(x 1/5)	(x 1/2)	-0.0014 (x 1/2)
$\bar{G}_\omega^0$	0.0009	(x 1/2)	(x 1/2)	0.00042 (x 1/2)
$\bar{G}_\omega^1$	-0.0017	(x 1/2)	(x 1/2)	-0.00086 (x 1/2)

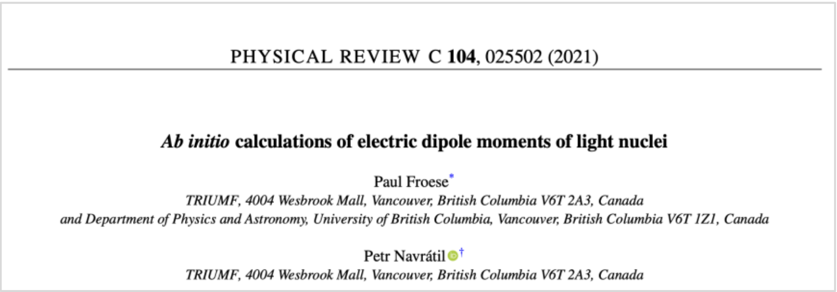
N_{max} convergence for ^{3}He

N^{3}LO NN



Our results confirm those of Yamanaka and Hiyama, PRC 91:054005 (2015)

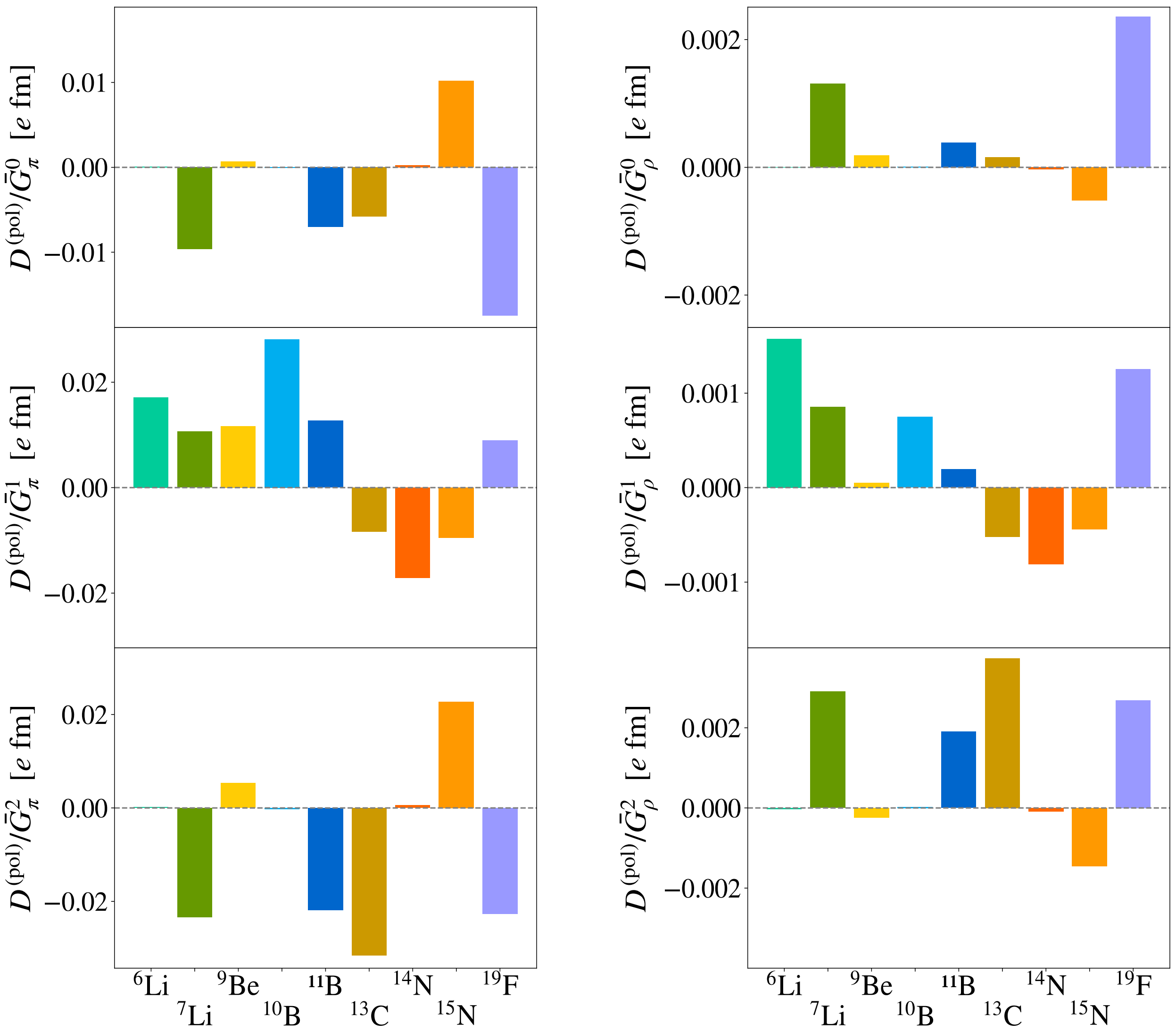
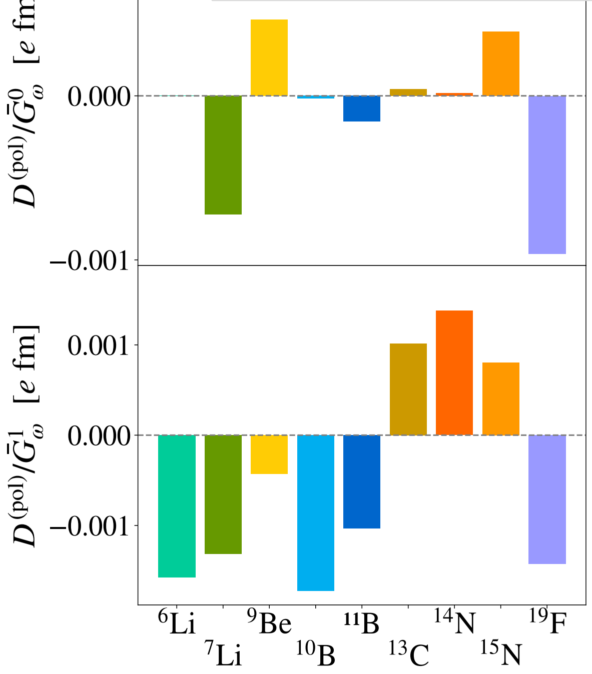
NCSM applications to parity-violating moments:
EDMs of light stable nuclei


PHYSICAL REVIEW C **104**, 025502 (2021)

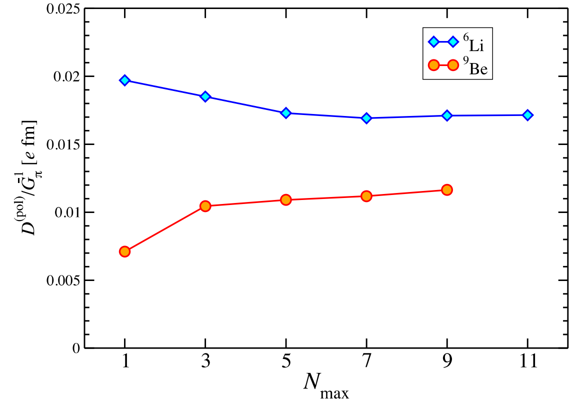
Ab initio **calculations of electric dipole moments of light nuclei**

Paul Froese*
TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada
and Department of Physics and Astronomy, University of British Columbia, Vancouver, British Columbia V6T 1Z1, Canada

Petr Navrátil†
TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada

Examples of N_{max} convergence



PHYSICAL REVIEW A **102**, 052828 (2020)
Editors' Suggestion

Nuclear spin-dependent parity-violating effects in light polyatomic molecules

Yongliang Hao,[1] Petr Navrátil,[2] Eric B. Norrgard,[3] Miroslav Iliaš,[4] Ephraim Eliav,[5] Rob G. E. Timmermans,[1] Victor V. Flambaum,[6] and Anastasia Borschevsky[1,*]

Nuclear spin-dependent parity-violating effects from NCSM

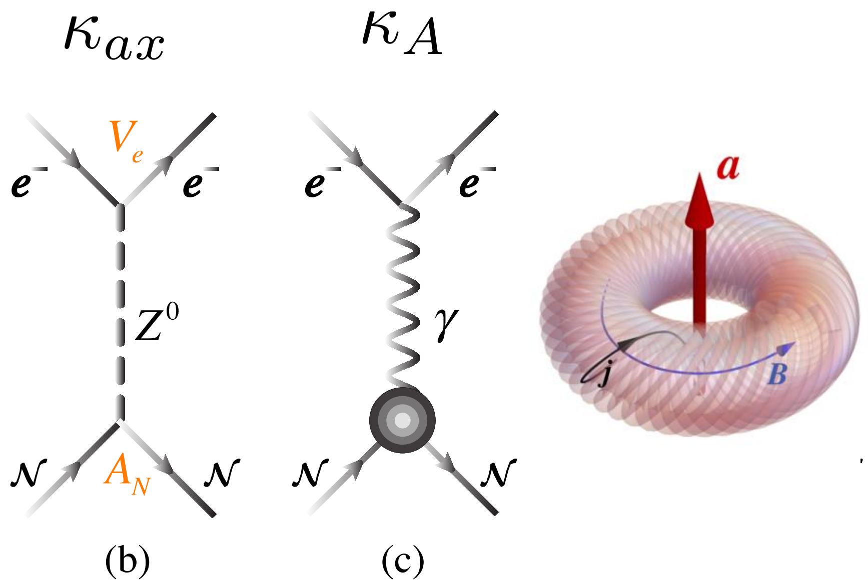
- Contributions from nucleon axial-vector and the anapole moment

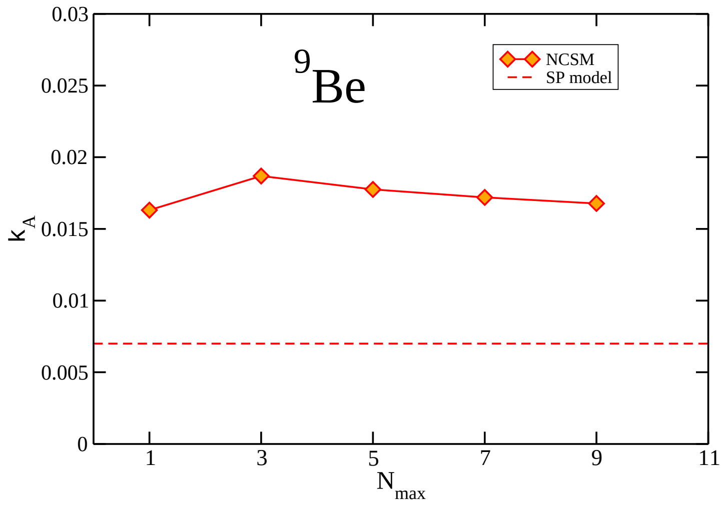
	^{9}Be	^{13}C	^{14}N	^{15}N	^{25}Mg
I^π	$3/2^-$	$1/2^-$	1^+	$1/2^-$	$5/2^+$
$\mu^{\text{exp.}}$	−1.177[a]	0.702[b]	0.404[c]	−0.283[d]	−0.855[e]
		NCSM calculations			
μ	−1.05	0.44	0.37	−0.25	−0.50
κ_{A}	0.016	−0.028	0.036	0.088	0.035
$\langle s_{p,z}\rangle$	0.009	−0.049	−0.183	−0.148	0.06
$\langle s_{n,z}\rangle$	0.360	−0.141	−0.1815	0.004	0.30
κ_{ax}	0.035	−0.009	0.0002	0.015	0.024
κ	0.050	−0.037	0.037	0.103	0.057

$$\kappa_{ax} \simeq -2C_{2p}\langle s_{p,z}\rangle - 2C_{2n}\langle s_{n,z}\rangle \simeq -0.1\langle s_{p,z}\rangle + 0.1\langle s_{n,z}\rangle$$

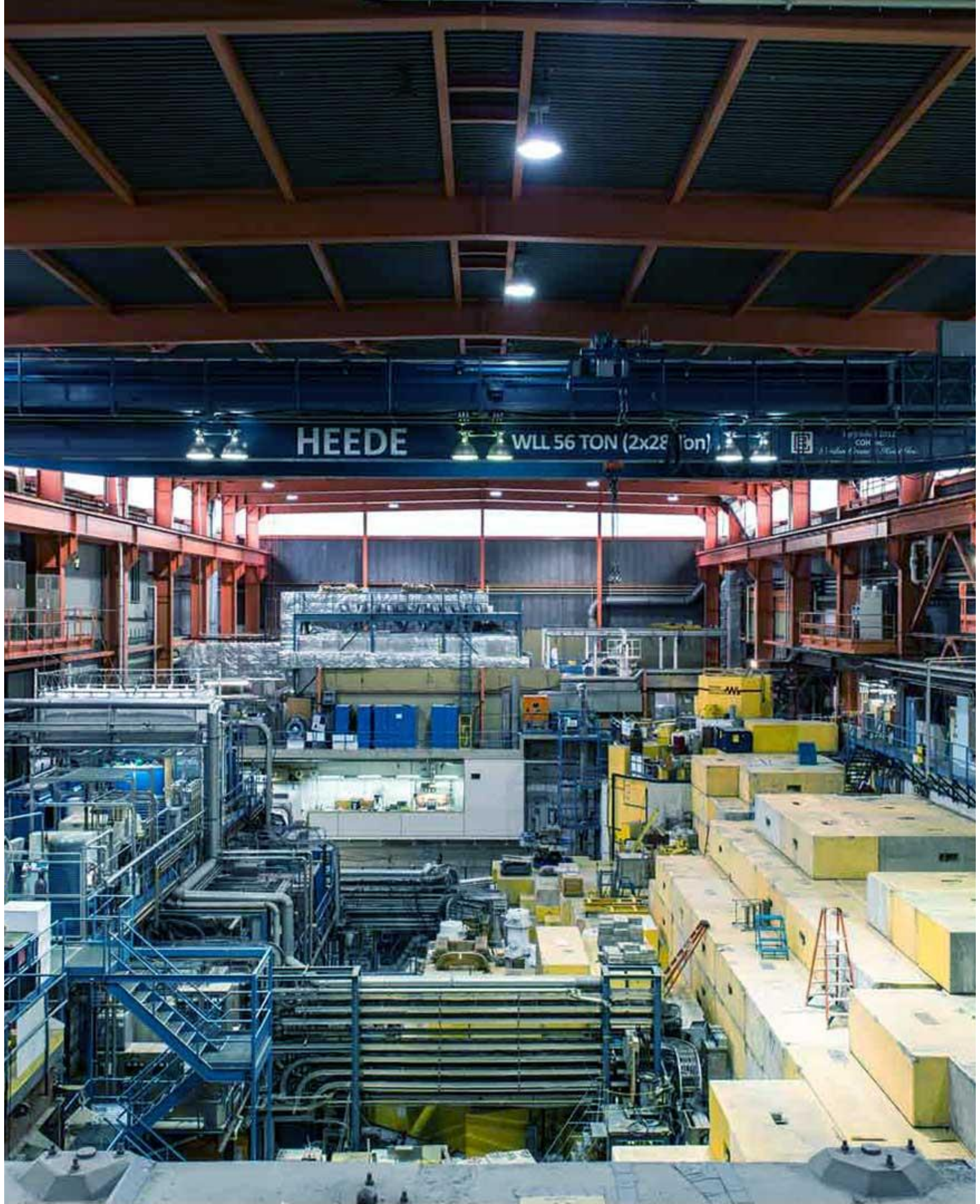
$$\langle s_{\nu,z}\rangle \equiv \langle \psi_{\text{gs}}\ I^\pi I_z{=}I|\hat{s}_{\nu,z}|\psi_{\text{gs}}\ I^\pi I_z{=}I\rangle$$

$$C_{2\text{p}} = -C_{2\text{n}} = g_A(1-4\sin^2\theta_W)/2 \simeq 0.05$$





^{6}He β-decay



Discovery, accelerated

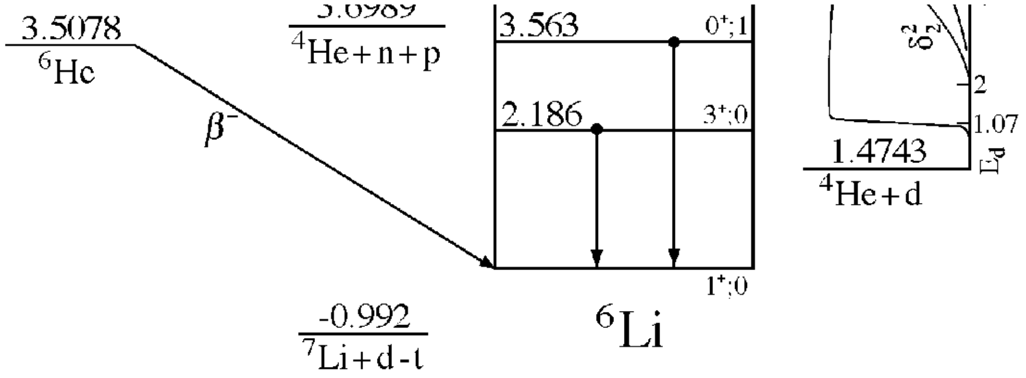
Precise measurements of β decays to search for Physics Beyond the Standard Model

- Precision measurements of β-decay observables offer the possibility to search for deviations from the Standard Model
  - β-decay observables are sensitive to interference of currents of SM particles and hypothetical BSM physics
  - Discovering such small deviations from the SM predictions demands also high-precision theoretical calculations
    - ⇒ Nuclear structure calculations with quantified uncertainties

^{6}He β-decay

- Decay rate proportional to

$$d\omega \propto 1 + a_{\beta\nu}\vec{\beta}\cdot\hat{\nu} + b_{\mathrm{F}}\frac{m_e}{E} \qquad \vec{\beta} = \frac{\vec{k}}{E} \qquad \vec{\nu} = \nu\hat{\nu}$$



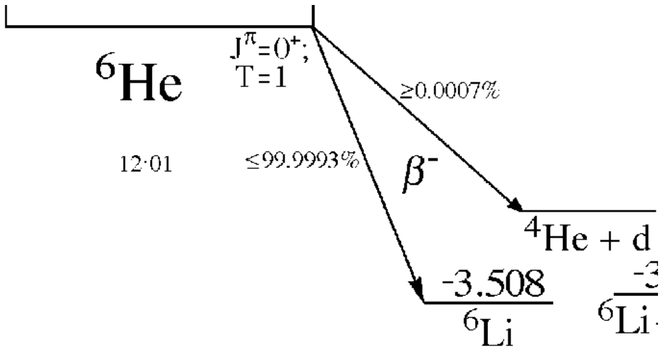
$a_{\beta\nu}$ angular correlation coefficient between the emitted electron and the antineutrino

b_{F} Fierz interference term that can be extracted from electron energy spectrum measurements

- The *V-A* structure of the weak interaction in the Standard Model implies for a Gamow-Teller transition

$$a_{\beta\nu} = -\frac{1}{3}$$

$$b_{\mathrm{F}} = 0$$



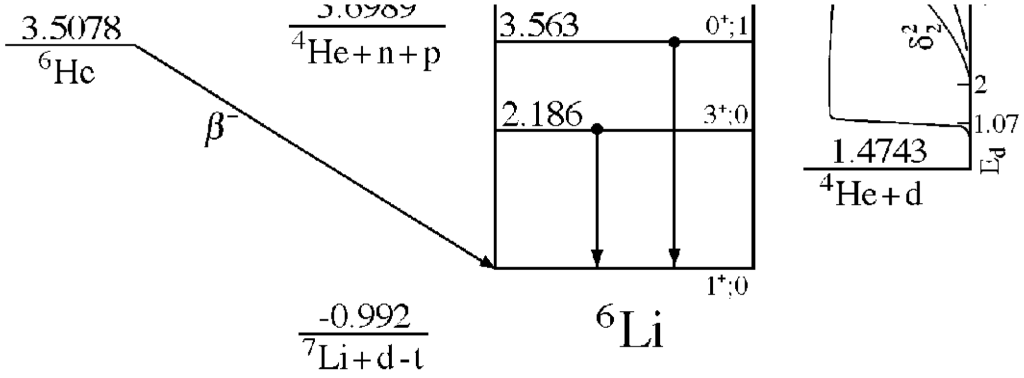
Precise measurements of β decays to search for Physics Beyond the Standard Model

- In the presence of Beyond the Standard Model interactions

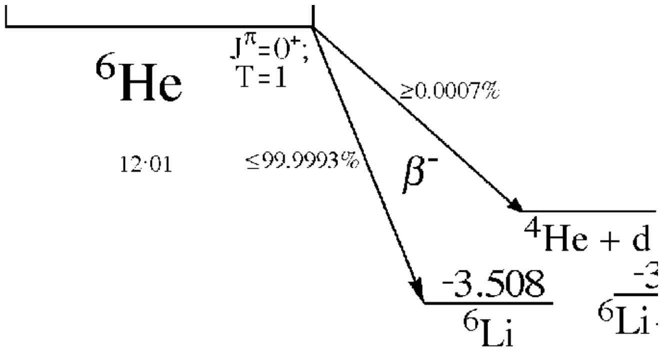
$$a_{\beta\nu}^{\mathrm{BSM}} = -\frac{1}{3}\left(1 - \frac{|C_T|^2 + \left|C_T'\right|^2}{2|C_A|^2}\right)$$

$$b_{\mathrm{Fierz}}^{\mathrm{BSM}} = \frac{C_T + C_T'}{C_A}$$

  - with tensor and pseudo-tensor contributions



- However, deviations also within the Standard Model caused by the finite momentum transfer, higher-order transition operators, and nuclear structure effects
  - Detailed, accurate, and precise calculations required



Precise measurements of β decays to search for Physics Beyond the Standard Model

- Higher-order Standard Model recoil and shape corrections

$$a_{\beta\nu}^{1^+\beta^-} = -\frac{1}{3}\left(1 + \tilde{\delta}_a^{1^+\beta^-}\right)$$

$$b_{\mathrm{F}}^{1^+\beta^-} = \delta_b^{1^+\beta^-}$$

$$\delta_1^{1^+\beta^-} \equiv \frac{2}{3}\mathfrak{Re}\left[-E_0\frac{\langle\|\hat{C}_1^A/q\|\rangle}{\langle\|\hat{L}_1^A\|\rangle} + \sqrt{2}\,(E_0 - 2E)\,\frac{\langle\|\hat{M}_1^V/q\|\rangle}{\langle\|\hat{L}_1^A\|\rangle}\right] - \frac{4}{7}ER\alpha Z_f - \frac{233}{630}\left(\alpha Z_f\right)^2,$$

$$\tilde{\delta}_a^{1^+\beta^-} \equiv \frac{4}{3}\mathfrak{Re}\left[2E_0\frac{\langle\|\hat{C}_1^A/q\|\rangle}{\langle\|\hat{L}_1^A\|\rangle} + \sqrt{2}\,(E_0 - 2E)\,\frac{\langle\|\hat{M}_1^V/q\|\rangle}{\langle\|\hat{L}_1^A\|\rangle}\right] + \frac{4}{7}ER\alpha Z_f - \frac{2}{5}E_0R\alpha Z_f,$$

$$\delta_b^{1^+\beta^-} \equiv \frac{2}{3}m_e\mathfrak{Re}\left[\frac{\langle\|\hat{C}_1^A/q\|\rangle}{\langle\|\hat{L}_1^A\|\rangle} + \sqrt{2}\frac{\langle\|\hat{M}_1^V/q\|\rangle}{\langle\|\hat{L}_1^A\|\rangle}\right],$$

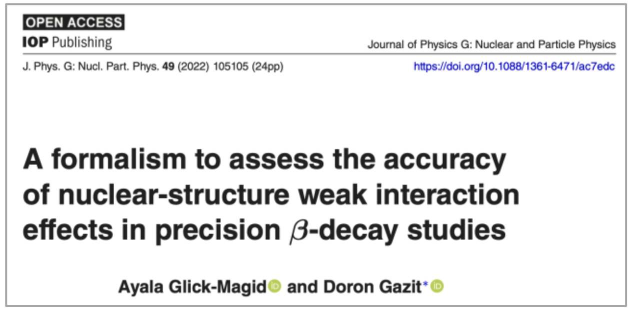
$\vec{q} = \vec{k} + \vec{\nu}$ momentum transfer

$\hat{C}_1^A$ axial charge

$\hat{M}_1^V$ vector magnetic or weak magnetism

$\hat{L}_1^A \propto 1$ Gamow-Teller leading order

$\hat{C}_1^A$ $\hat{M}_1^V$ NLO recoil corrections, order q/m_N


OPEN ACCESS

IOP Publishing — Journal of Physics G: Nuclear and Particle Physics

J. Phys. G: Nucl. Part. Phys. 49 (2022) 105105 (24pp) https://doi.org/10.1088/1361-6471/ac7edc

A formalism to assess the accuracy of nuclear-structure weak interaction effects in precision β-decay studies

Ayala Glick-Magid and Doron Gazit*

Precise measurements of β decays to search for Physics Beyond the Standard Model

- Higher-order Standard Model recoil and shape corrections

$$\frac{\hat{C}^A_{JM_J}}{q} = \sum_{j=1}^{A} \frac{i}{m_N}\Big[g_A \hat{\Omega}'_{JM_J}(q\vec{r}_j) - \frac{1}{2}\frac{\tilde{g}_P}{2m_N}(E_0+\Delta E_c)\,\hat{\Sigma}''_{JM_J}(q\vec{r}_j)\Big]\tau_j^+,$$

$$\hat{L}^A_{JM_J} = \sum_{j=1}^{A} i\left(g_A + \frac{\tilde{g}_P}{(2m_N)^2}q^2\right)\hat{\Sigma}''_{JM_J}(q\vec{r}_j)\,\tau_j^+,$$

$$\frac{\hat{M}^V_{JM_J}}{q} = \sum_{j=1}^{A} \frac{-i}{m_N}\Big[g_V\,\hat{\Delta}_{JM_J}(q\vec{r}_j) - \frac{1}{2}\mu\hat{\Sigma}'_{JM_J}(q\vec{r}_j)\Big]\tau_j^+$$

$$\hat{\Sigma}''_{JM_J}(q\vec{r}_j) = \left[\frac{1}{q}\vec{\nabla}_{\vec{r}_j} M_{JM_J}(q\vec{r}_j)\right]\cdot\vec{\sigma}(j),$$

$$\hat{\Omega}'_{JM_J}(q\vec{r}_j) = M_{JM_J}(q\vec{r}_j)\,\vec{\sigma}(j)\cdot\vec{\nabla}_{\vec{r}_j} + \frac{1}{2}\hat{\Sigma}''_{JM_J}(q\vec{r}_j),$$

$$\hat{\Delta}_{JM_J}(q\vec{r}_j) = \vec{M}_{JJM_J}(q\vec{r}_j)\cdot\frac{1}{q}\vec{\nabla}_{\vec{r}_j},$$

$$\hat{\Sigma}'_{JM_J}(q\vec{r}_j) = -i\left[\frac{1}{q}\vec{\nabla}_{\vec{r}_j}\times\vec{M}_{JJM_J}(q\vec{r}_j)\right]\cdot\vec{\sigma}(j),$$

$$M_{JM_J}(q\vec{r}_j) = j_J(qr_j)Y_{JM_J}(\hat{r}_j)$$

$$\vec{M}_{JLM_J}(q\vec{r}_j) = j_L(qr_j)\vec{Y}_{JLM_J}(\hat{r}_j)$$

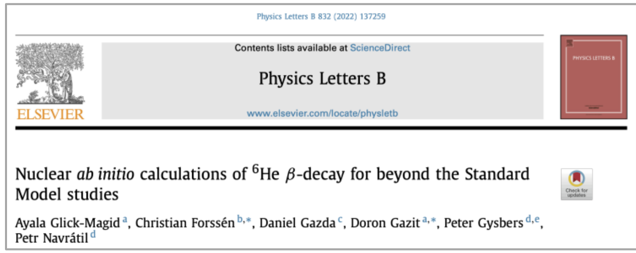
Hadronic vector, axial vector and pseudo-scalar charges

$g_V = 1$ $\quad g_A = -1.2756(13)$ $\quad \tilde{g}_P = -\frac{(2m_N)^2}{m_\pi^2 - q^2} g_A$

$\mu \approx 4.706$ is the nucleon isovector magnetic moment

$$\Delta E_c \equiv \langle {}^6\text{Li}\ 1^+_{gs}|V_c|{}^6\text{Li}\ 1^+_{gs}\rangle - \langle {}^6\text{He}\ 0^+_{gs}|V_c|{}^6\text{He}\ 0^+_{gs}\rangle$$

Ultimately, we need to calculate ^{6}He(0^+ 1) → ^{6}Li(1^+ 0) matrix elements of these “one-body” operators


Physics Letters B 832 (2022) 137259
Contents lists available at ScienceDirect
Physics Letters B
www.elsevier.com/locate/physletb

Nuclear *ab initio* calculations of ^{6}He β-decay for beyond the Standard Model studies

Ayala Glick-Magid[a], Christian Forssén[b,*], Daniel Gazda[c], Doron Gazit[a,*], Peter Gysbers[d,e], Petr Navrátil[d]

Apply *ab initio* No-Core Shell Model to calculate the ^{6}Li and ^{6}He wave functions and the operator matrix elements

NCSM

- Matrix elements of the relevant operators

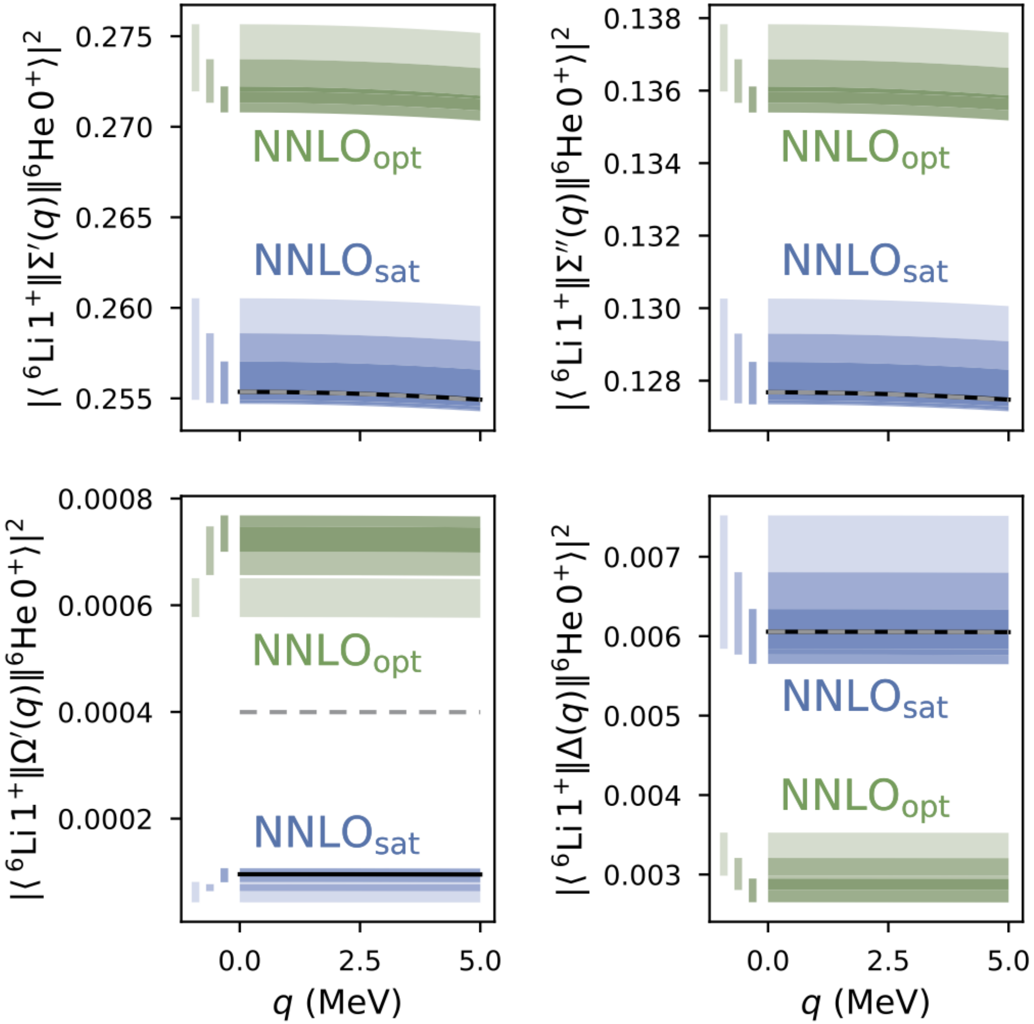
$$\hat{\Sigma}''_{JM_J}(q\vec{r}_j) = \left[\frac{1}{q}\vec{\nabla}_{\vec{r}_j} M_{JM_J}(q\vec{r}_j)\right]\cdot\vec{\sigma}(j),$$

$$\hat{\Omega}'_{JM_J}(q\vec{r}_j) = M_{JM_J}(q\vec{r}_j)\,\vec{\sigma}(j)\cdot\vec{\nabla}_{\vec{r}_j} + \frac{1}{2}\hat{\Sigma}''_{JM_J}(q\vec{r}_j),$$

$$\hat{\Delta}_{JM_J}(q\vec{r}_j) = \vec{M}_{JJM_J}(q\vec{r}_j)\cdot\frac{1}{q}\vec{\nabla}_{\vec{r}_j},$$

$$\hat{\Sigma}'_{JM_J}(q\vec{r}_j) = -i\left[\frac{1}{q}\vec{\nabla}_{\vec{r}_j}\times\vec{M}_{JJM_J}(q\vec{r}_j)\right]\cdot\vec{\sigma}(j),$$

- Convergence investigation
  - Variation of HO frequency
    - $h\boldsymbol{\Omega}$ = 16 - 24 MeV
  - Variation of basis size
    - N_{max}= 0 - 14 for $NNLO_{opt}$
    - N_{max}= 0 - 12 for $NNLO_{sat}$



Overall results for $^{6}He(0^{+}\ 1) \rightarrow {}^{6}Li(1^{+}\ 0) + e^{-} + \overline{\nu}$

- We find up to 1% correction for the β spectrum and up to 2% correction for the angular correlation
- Propagating nuclear structure and χEFT uncertainties results in an overall uncertainty of 10^{-4}
  - Comparable to the precision of current experiments

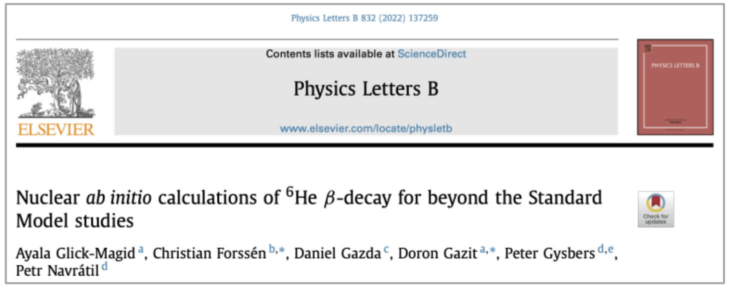
$$b_{\mathrm{F}}^{1^{+}\beta^{-}} = \delta_{b}^{1^{+}\beta^{-}} = -1.52\,(18)\cdot 10^{-3}$$

$$\left\langle \tilde{\delta}_{a}^{1^{+}\beta^{-}} \right\rangle = -2.54\,(68)\cdot 10^{-3}$$

Non-zero Fierz interference term due to nuclear structure corrections

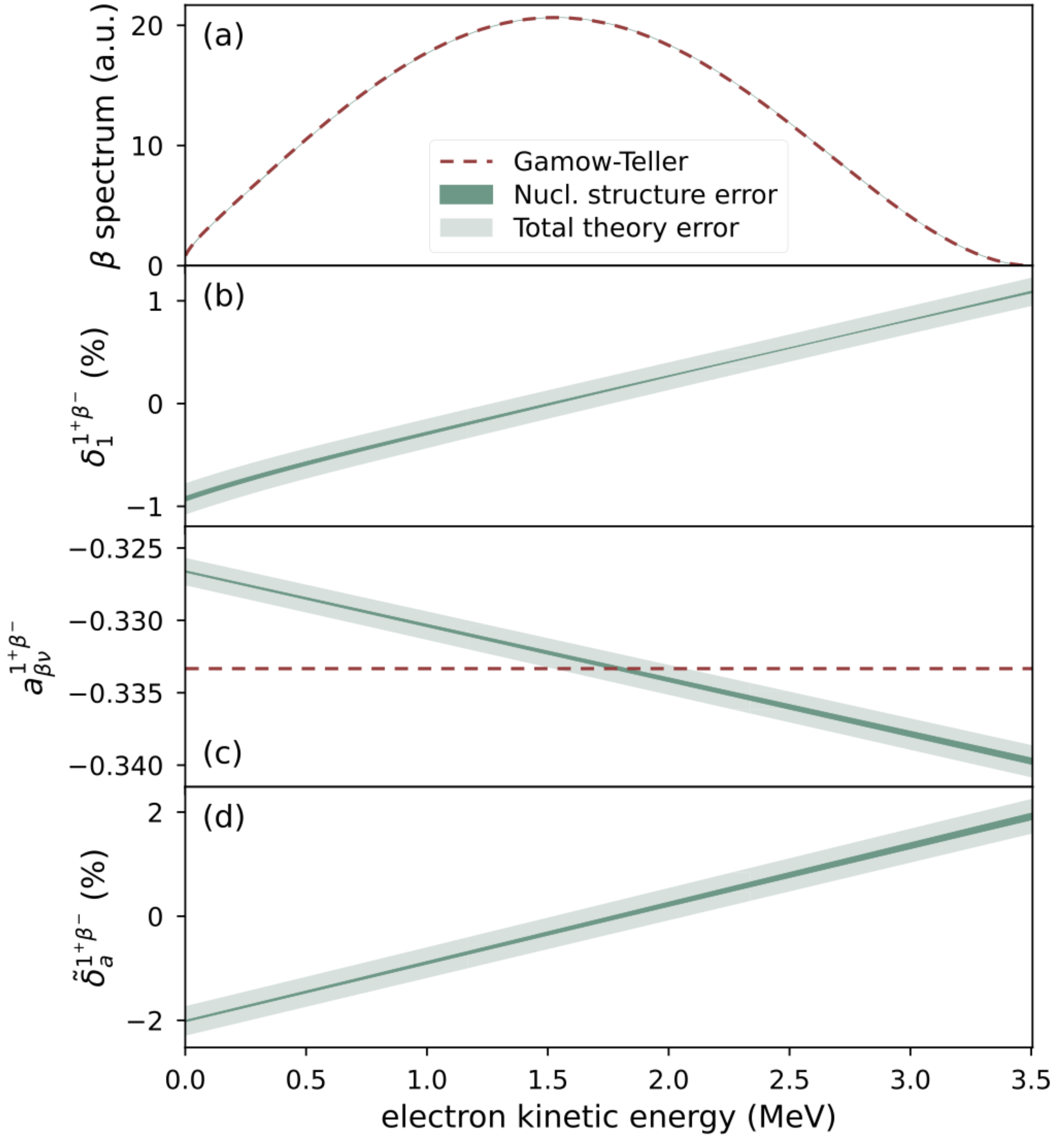
Note that new physics at TeV scale implies

$$b_{\mathrm{Fierz}}^{\mathrm{BSM}} = \frac{C_T + C_T'}{C_A} \sim 10^{-3}$$


Physics Letters B 832 (2022) 137259
Contents lists available at ScienceDirect
Physics Letters B
www.elsevier.com/locate/physletb
ELSEVIER

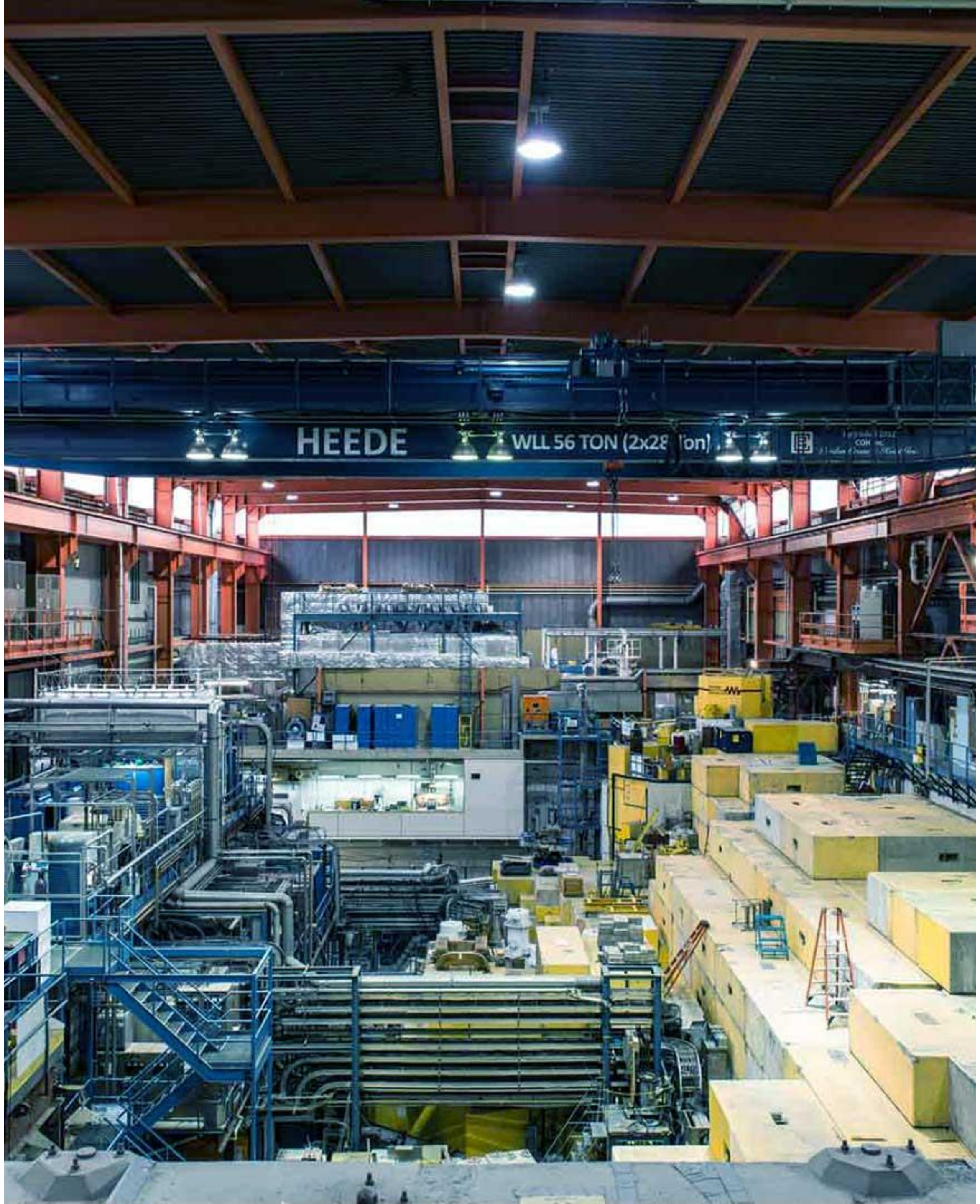
Nuclear *ab initio* calculations of ^{6}He β-decay for beyond the Standard Model studies

Ayala Glick-Magid [a], Christian Forssén [b,*], Daniel Gazda [c], Doron Gazit [a,*], Peter Gysbers [d,e], Petr Navrátil [d]



TRIUMF

Super-allowed Fermi transitions - electroweak radiative correction δ_{NS}



Discovery, accelerated

Synergy of precision experiments and *ab initio* nuclear theory to test CKM unitarity

Structure corrections for the extraction of the V_{ud} matrix element from the $^{10}C \rightarrow {}^{10}B$ Fermi transition

See poster by Michael Gennari

- CKM unitarity sensitive probe of BSM physics
  - V_{ud} element from super-allowed Fermi transitions

$$|V_{ud}|^2 = \frac{\hbar^7}{G_F^2 m_e^5 c^4} \frac{\pi^3 \ln(2)}{\mathcal{F}t} \qquad \mathcal{F}t = \frac{K}{G_V^2 |M_{F0}|^2 (1+\Delta_R^V)}$$

$$\mathcal{F}t(1+\Delta_R^V) = ft(1+\delta_R')(1-\delta_C+\delta_{NS})$$

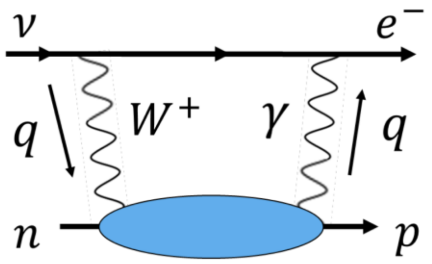
  - δ_{NS} parametrizes correction to free γW box
  - *Ab initio* no-core shell model (NCSM)
    - A very good convergence – consistent with what used in latest evaluation with a substantially reduced theoretical uncertainties

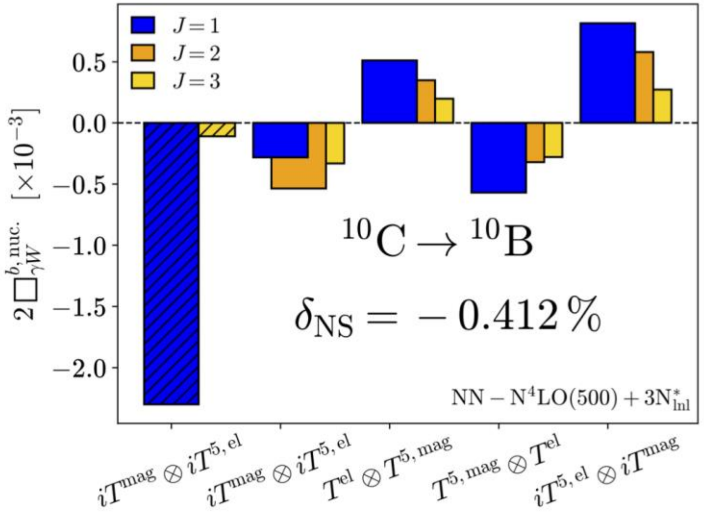
$$\delta_{NS} = 2[\Box_{\gamma W}^{VA,\mathrm{nuc.}} - \Box_{\gamma W}^{VA,\mathrm{free\ n}}]$$

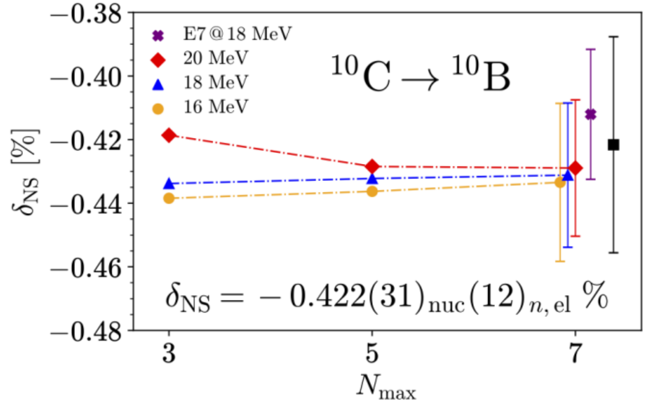
An *ab initio* strategy for taming the nuclear-structure dependence of V_{ud} extractions: the $^{10}C \rightarrow {}^{10}B$ superallowed transition

arXiv: 2405.19281

Michael Gennari[1,2], Mehdi Drissi[1], Mikhail Gorchtein[3,4], Petr Navrátil[1,2], and Chien-Yeah Seng[5,6]



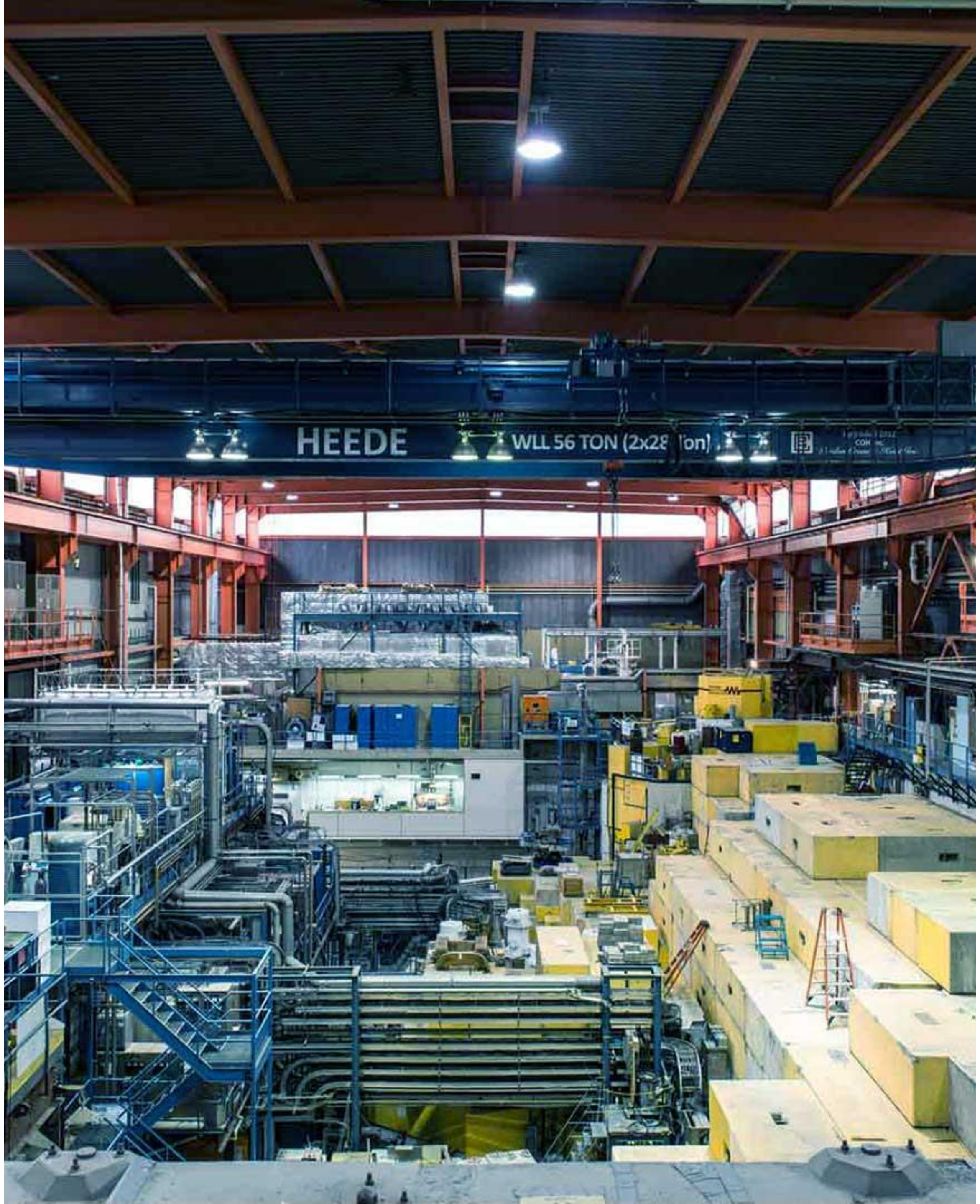




NCSM applicable also to $^{14}O \rightarrow {}^{14}N$ and possibly $^{18}Ne \rightarrow {}^{18}F$, $^{22}Mg \rightarrow {}^{22}Na$

TRIUMF

Isospin-symmetry breaking correction δ_C



Discovery, accelerated

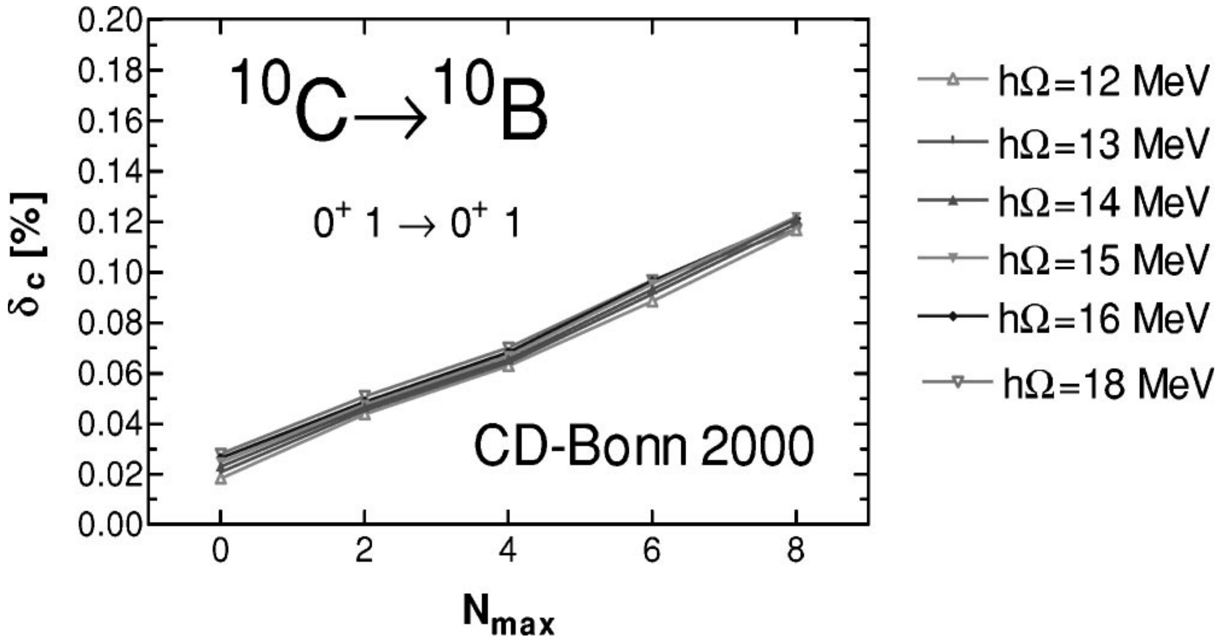
The pathway to δ_C

- δ_C in *ab initio* NCSM over 20 years ago

PHYSICAL REVIEW C **66**, 024314 (2002)

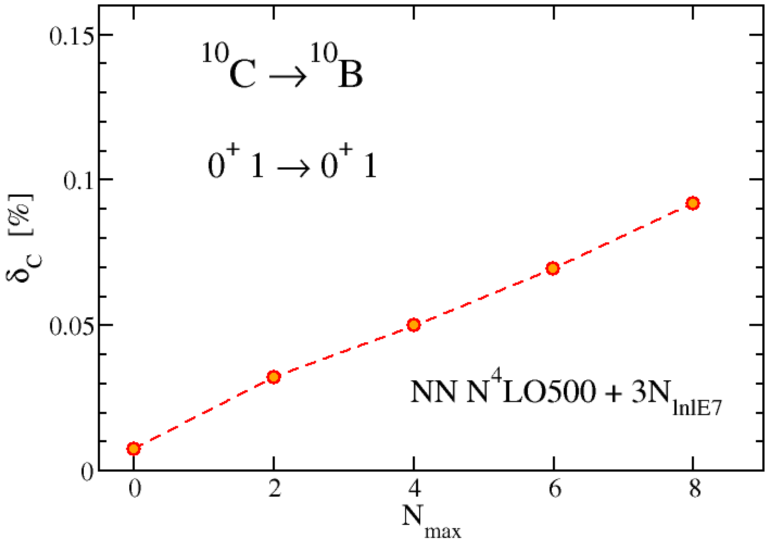
***Ab initio* shell model for $A=10$ nuclei**

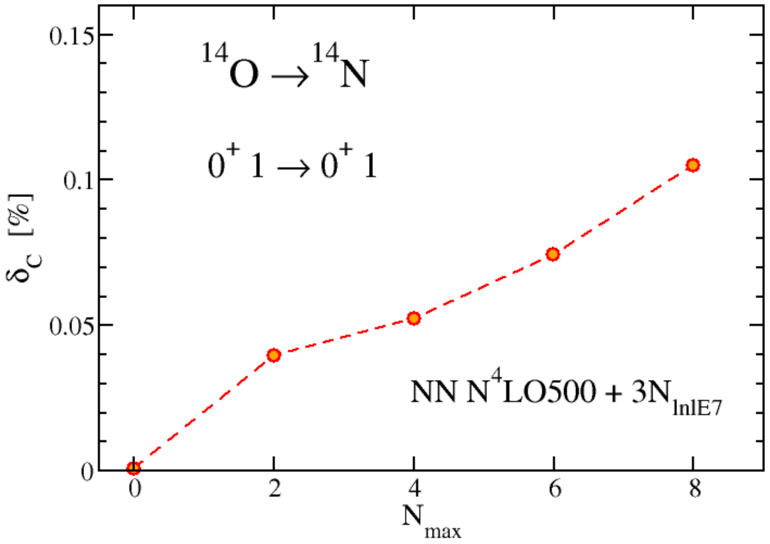
E. Caurier,[1] P. Navrátil,[2] W. E. Ormand,[2] and J. P. Vary[3]
[1]*Institut de Recherches Subatomiques (IN2P3-CNRS-Université Louis Pasteur), Batiment 27/1, 67037 Strasbourg Cedex 2, France*
[2]*Lawrence Livermore National Laboratory, L-414, P.O. Box 808, Livermore, California 94551*
[3]*Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011*
(Received 10 May 2002; published 13 August 2002)



The pathway to δ_C

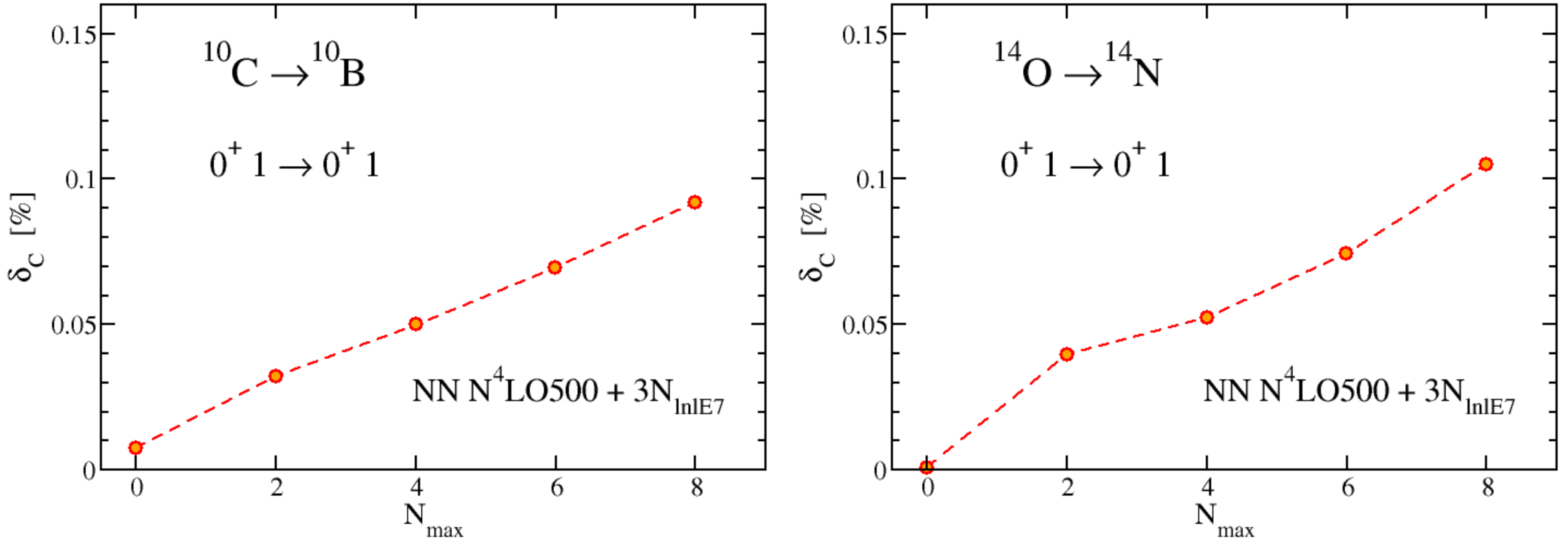
- δ_C in *ab initio* NCSM now





The pathway to δ_C

- δ_C in *ab initio* NCSM now



Isospin-symmetry breaking interaction admixes continuum intruder states in the ground state

- Poorly described in the HO expansion
- Need to include continuum effects explicitly

→ No-Core Shell Model with Continuum

Combine NCSM with resonating group method (RGM)

Ab Initio Calculations of Structure, Scattering, Reactions
Unified approach to bound & continuum states

No-Core Shell Model with Continuum (NCSMC)

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| {}^{(A)}\, , \lambda \right\rangle + \sum_{\nu} \int d\vec{r}\; \gamma_{\nu}(\vec{r})\; \hat{A}_{\nu} \left| \underset{(A-a)}{} \overset{\vec{r}}{\longleftarrow} \underset{(a)}{}\, , \nu \right\rangle$$


IOP Publishing | Royal Swedish Academy of Sciences Physica Scripta

Phys. Scr. **91** (2016) 053002 (38pp) doi:10.1088/0031-8949/91/5/053002

Invited Comment

Unified *ab initio* approaches to nuclear structure and reactions

Petr Navrátil[1], Sofia Quaglioni[2], Guillaume Hupin[3,4], Carolina Romero-Redondo[2] and Angelo Calci[1]

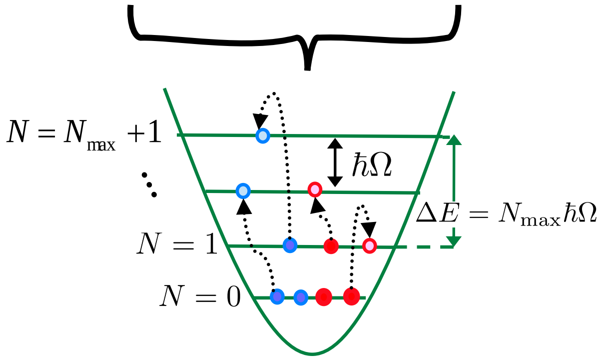
S. Baroni, P. Navratil, and S. Quaglioni,
PRL **110**, 022505 (2013); PRC **87**, 034326 (2013).

Ab Initio Calculations of Structure, Scattering, Reactions

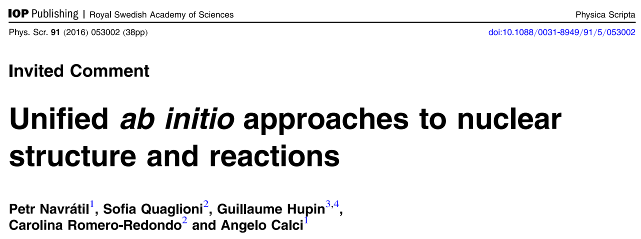
Unified approach to bound & continuum states

No-Core Shell Model with Continuum (NCSMC)

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| (A), \lambda \right\rangle + \sum_{\nu} \int d\vec{r}\, g_{\nu}(\vec{r})\, \hat{A}_{\nu} \left| \vec{r}, (A-a), (a), \nu \right\rangle$$



Static solutions for aggregate system, describe all nucleons close together


IOP Publishing | Royal Swedish Academy of Sciences Physica Scripta

Phys. Scr. **91** (2016) 053002 (38pp) doi:10.1088/0031-8949/91/5/053002

Invited Comment

Unified *ab initio* approaches to nuclear structure and reactions

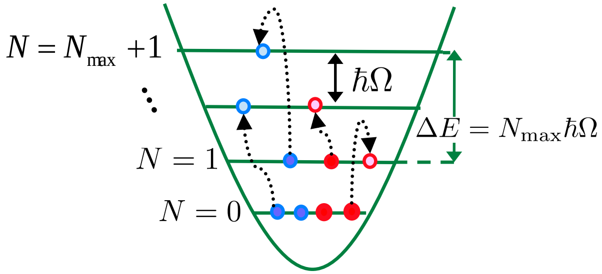
Petr Navrátil[1], Sofia Quaglioni[2], Guillaume Hupin[3,4], Carolina Romero-Redondo[2] and Angelo Calci[1]

S. Baroni, P. Navratil, and S. Quaglioni,
PRL **110**, 022505 (2013); PRC **87**, 034326 (2013).

Ab Initio Calculations of Structure, Scattering, Reactions
Unified approach to bound & continuum states

No-Core Shell Model with Continuum (NCSMC)

$$\Psi^{(A)} = \sum_{\lambda} c_\lambda \left|{}^{(A)}, \lambda\right\rangle + \sum_{\nu} \int d\vec{r}\, \gamma_\nu(\vec{r})\, \hat{A}_\nu \left| {}_{(A-a)} \vec{r}\ {}_{(a)}, \nu \right\rangle$$



Static solutions for aggregate system, describe all nucleons close together

Continuous microscopic cluster states, describe long-range projectile-target

IOP Publishing | Royal Swedish Academy of Sciences Physica Scripta
Phys. Scr. 91 (2016) 053002 (38pp) doi:10.1088/0031-8949/91/5/053002

Invited Comment

Unified *ab initio* approaches to nuclear structure and reactions

Petr Navrátil[1], Sofia Quaglioni[2], Guillaume Hupin[3,4], Carolina Romero-Redondo[2] and Angelo Calci[1]

S. Baroni, P. Navratil, and S. Quaglioni,
PRL **110**, 022505 (2013); PRC **87**, 034326 (2013).

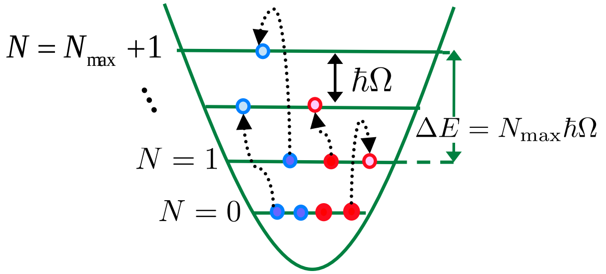
Ab Initio Calculations of Structure, Scattering, Reactions

Unified approach to bound & continuum states

No-Core Shell Model with Continuum (NCSMC)

Unknowns

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| (A), \lambda \right\rangle + \sum_{\nu} \int d\vec{r}\, \gamma_{\nu}(\vec{r})\, \hat{A}_{\nu} \left| (A-a)\ \vec{r}\ (a), \nu \right\rangle$$



Static solutions for aggregate system, describe all nucleons close together

Continuous microscopic cluster states, describe long-range projectile-target

IOP Publishing | Royal Swedish Academy of Sciences — Physica Scripta

Phys. Scr. 91 (2016) 053002 (38pp) — doi:10.1088/0031-8949/91/5/053002

Invited Comment

Unified *ab initio* approaches to nuclear structure and reactions

Petr Navrátil[1], Sofia Quaglioni[2], Guillaume Hupin[3,4], Carolina Romero-Redondo[2] and Angelo Calci[1]

S. Baroni, P. Navratil, and S. Quaglioni, PRL **110**, 022505 (2013); PRC **87**, 034326 (2013).

PHYSICAL REVIEW C **105**, 054316 (2022)

***Ab initio* calculation of the β decay from ^{11}Be to a ^{10}Be + p resonance**

M. C. Atkinson,[1] P. Navrátil,[1] G. Hupin,[2] K. Kravvaris,[3] and S. Quaglioni[3]

δ_C in NCSMC

- Compute Fermi matrix element in NCSMC

$$M_F = \left\langle \Psi^{J^\pi T_f M_{T_f}} \middle| T_+ \middle| \Psi^{J^\pi T_i M_{T_i}} \right\rangle \longrightarrow |M_F|^2 = |M_{F0}|^2(1-\delta_C)$$

- Total isospin operator $T_+ = T_+^{(1)} + T_+^{(2)}$ for partitioned system

$$M_F \sim \langle A\lambda_f J_f T_f M_{T_f} | T_+ | A\lambda J_i T_i M_{T_i} \rangle + \langle A\lambda J_f T_f M_{T_f} | T_+ \mathcal{A}_{\nu i} | \Phi_{\nu r}^{J_i T_i M_{T_i}} \rangle$$
$$+ \langle \Phi_{\nu r}^{J_f T_f M_{T_f}} | \mathcal{A}_{\nu f} T_+ | A\lambda_i J_i T_i M_{T_i} \rangle + \langle \Phi_{\nu r}^{J_f T_f M_{T_f}} | \mathcal{A}_{\nu f} T_+ \mathcal{A}_{\nu i} | \Phi_{\nu r}^{J_i T_i M_{T_i}} \rangle$$

NCSM matrix element

NCSM-Cluster matrix elements

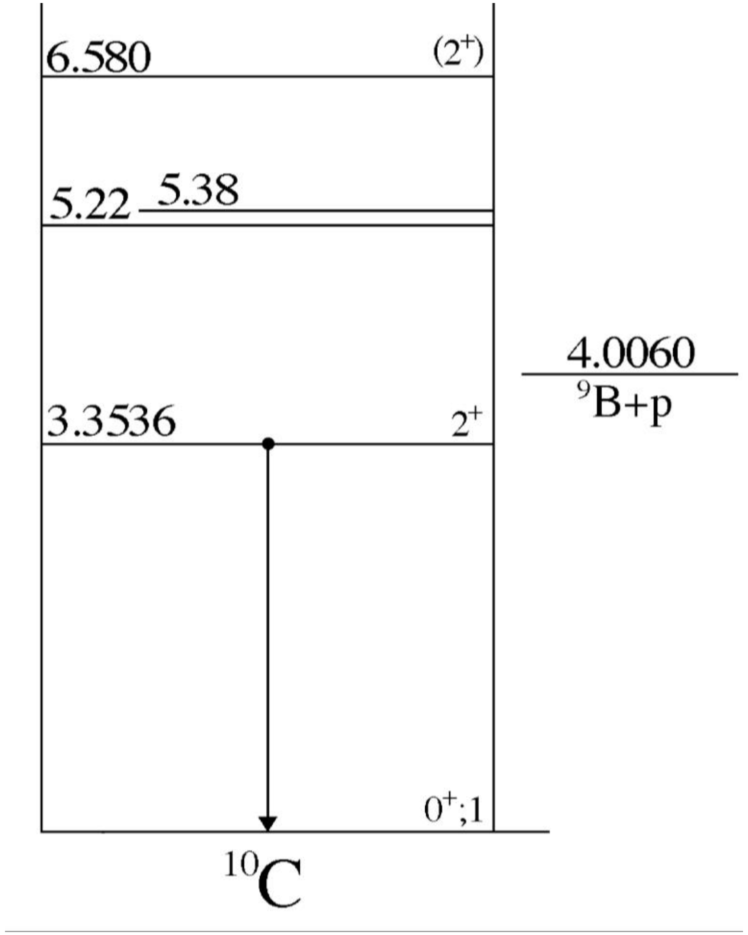
Continuum (cluster) matrix element

^{10}C structure from chiral EFT NN(N^{4}LO)+3N(N^{2}LO,lnl) interaction ($N_{max} = 9$)

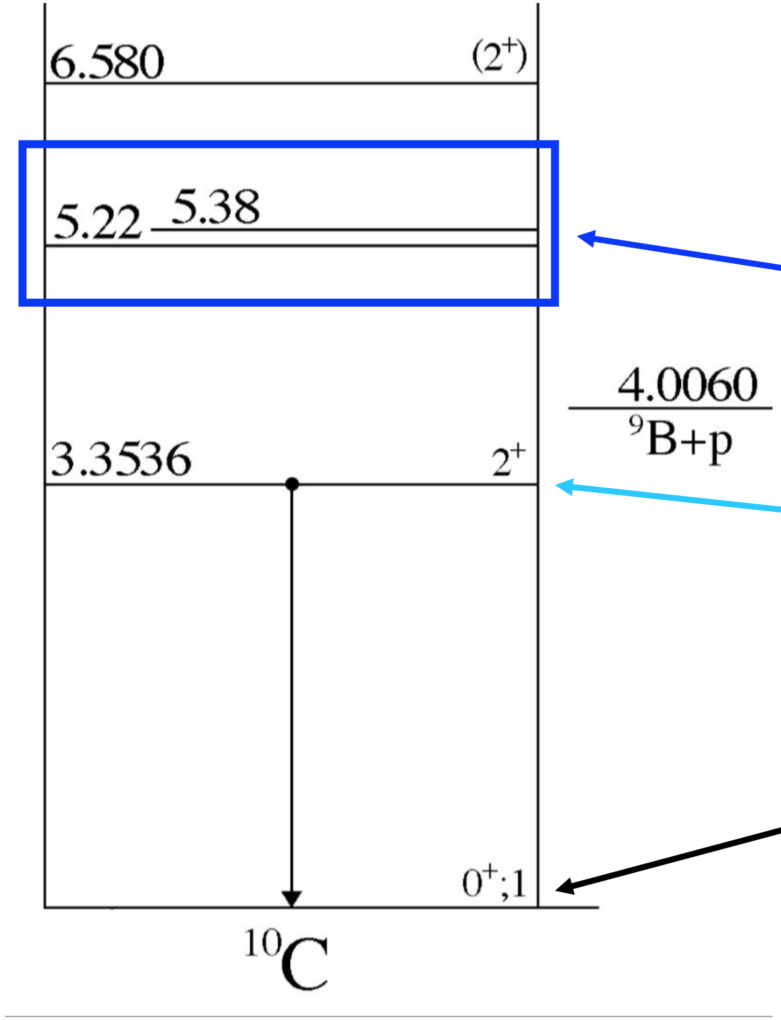
$$|{}^{10}\mathrm{C}\rangle = \sum_{\alpha} c_\alpha |{}^{10}\mathrm{C}, \alpha\rangle_{\mathrm{NCSM}} + \sum_{\nu} \int dr\, \gamma_\nu^{J^\pi T}(r) \mathcal{A}_\nu |{}^9\mathrm{B} + \mathrm{p}, \nu\rangle$$

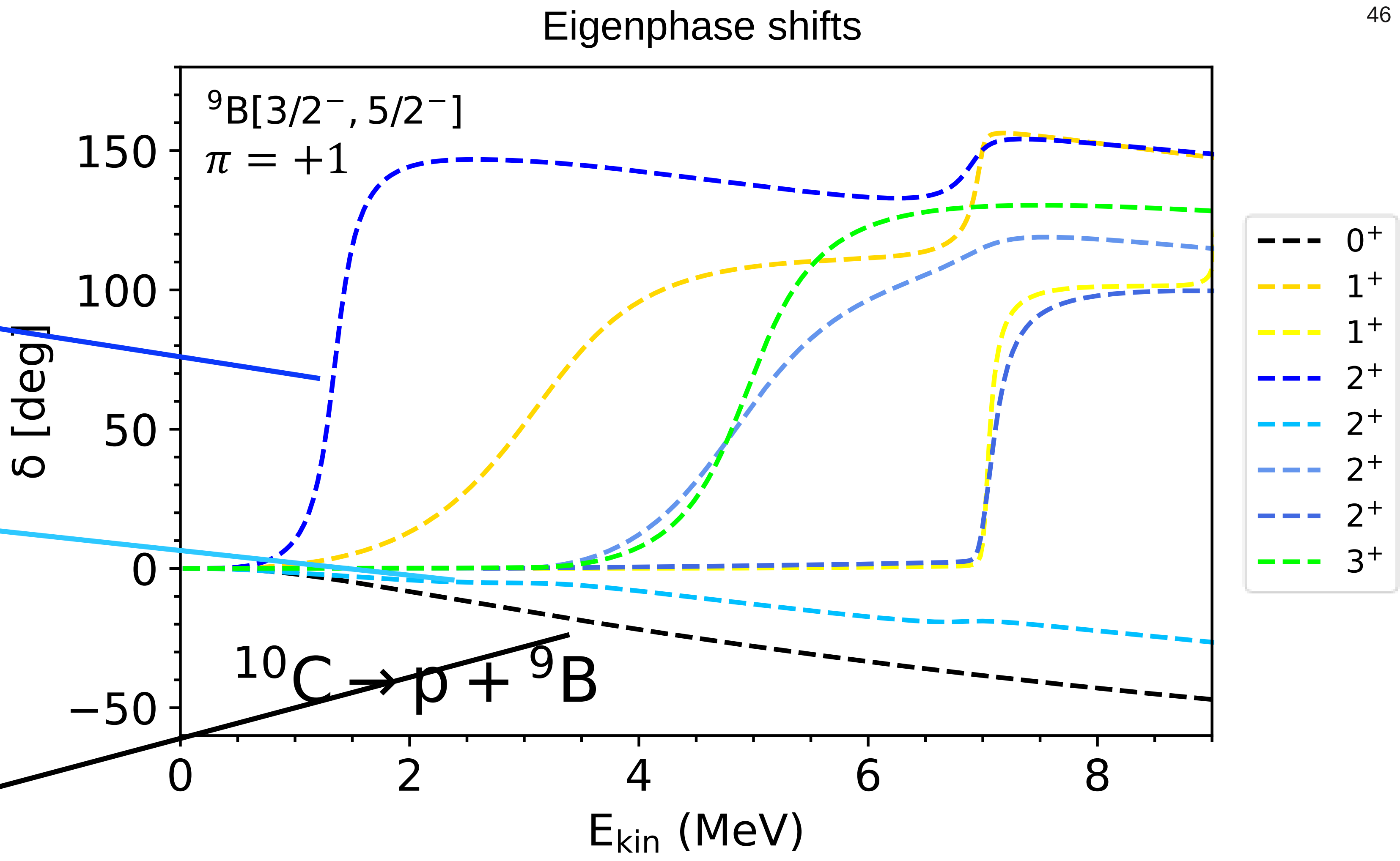
- Treat as mass partition of proton plus ^{9}B
- Use $3/2^-$ and $5/2^-$ states of ^{9}B
- Known bound states captured by NCSMC

State	E_{NCSM} (MeV)	E (MeV)	E_{exp} (MeV)
0^+	-3.09	-3.46	-4.006
2^+	$+0.40$	-0.03	-0.652



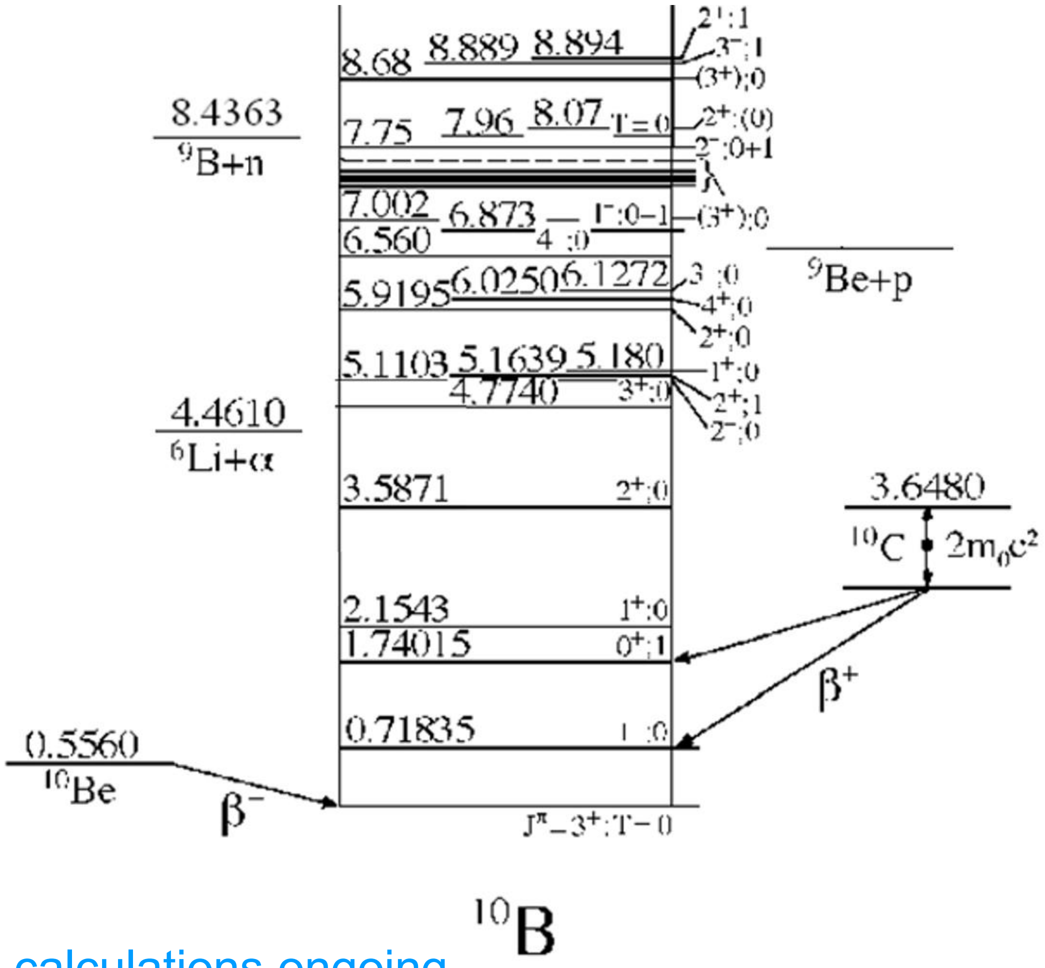
^{10}C structure from chiral EFT NN(N^{4}LO)+3N(N^{2}LO,lnl) interaction ($N_{max} = 9$)





^{10}B structure from chiral EFT NN(N^{4}LO)+3N(N^{2}LO,lnl) interaction ($N_{max} = 9$)

$$|^{10}\mathrm{B}\rangle = \sum_\alpha c_\alpha \, |^{10}\mathrm{B}, \alpha\rangle_{\mathrm{NCSM}} + \sum_\nu \int dr \, \gamma_\nu(r) \mathcal{A}_\nu \, |^{9}\mathrm{Be} + p, \nu\rangle + \sum_\mu \int dr \, \gamma_\mu(r) \mathcal{A}_\mu \, |^{9}\mathrm{B} + n, \mu\rangle$$



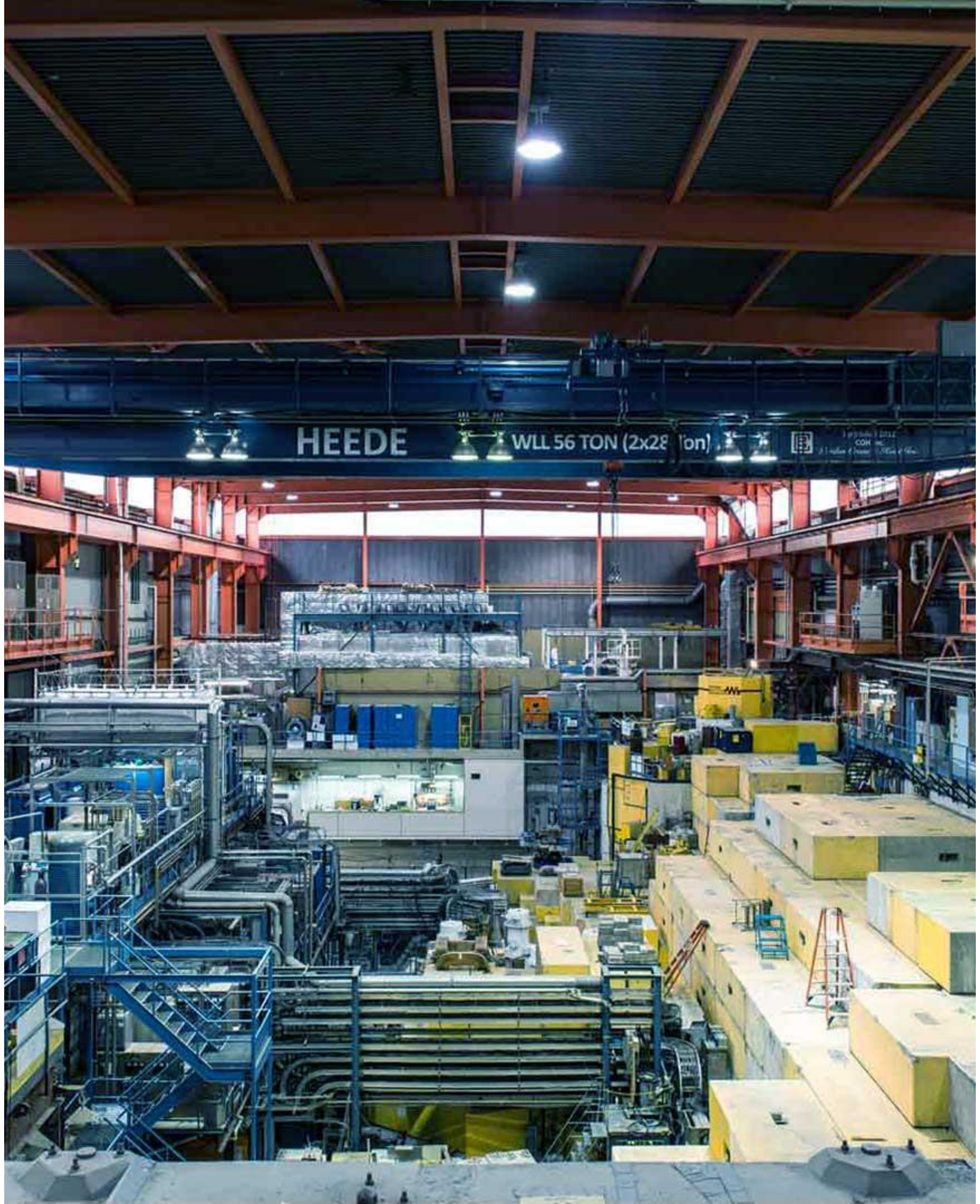
- Use $3/2^-$ and $5/2^-$ states of ^{9}B and ^{9}Be
- Eight of twelve bound states predicted

State	E (MeV)	E_{exp} (MeV)
3^+	−5.75	−6.5859
1^+	−5.33	−5.8676
0^+	−4.30	−4.8458
1^+	−4.26	−4.4316
2^+	−2.69	−2.9988
2^+	−0.93	−1.4220
2^+	−0.70	−0.6664
4^+	−0.19	−0.5609

δ_C calculations ongoing …

TRIUMF

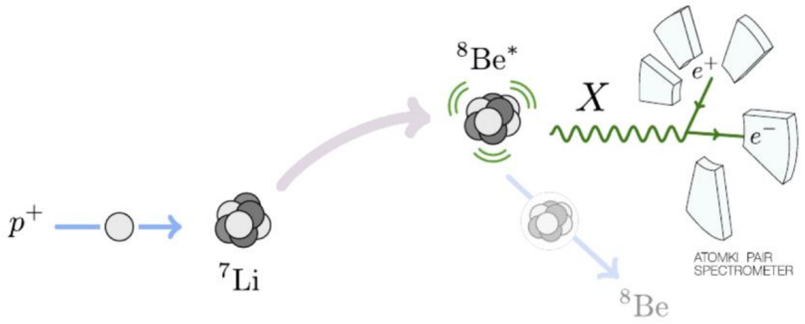
^{7}Li(p,e^{+}e^{-})^{8}Be internal pair creation and the X17 anomaly



Discovery, accelerated

X17 Anomaly

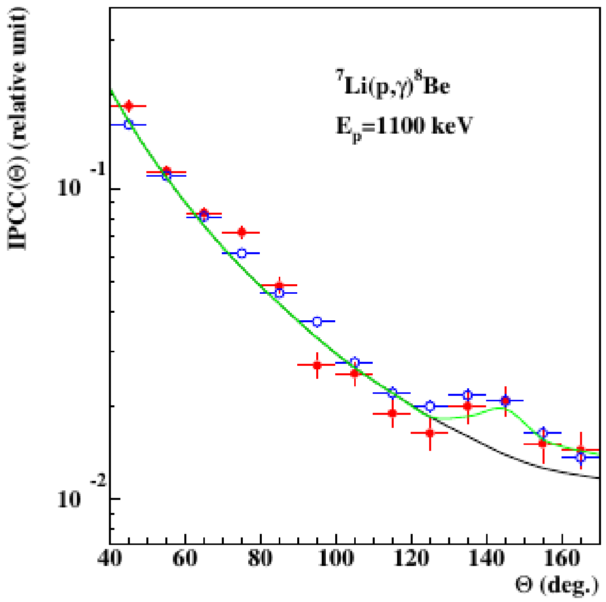
Phys. Rev. Lett. **116**, 042501 (2016) – ^{7}Li+p -> ^{8}Be
Phys. Rev. C **104**, 044003 (2021) – ^{3}H+p -> ^{4}He
Phys. Rev. C **106**, L061601 (2022) – ^{11}B+p ->^{12}C

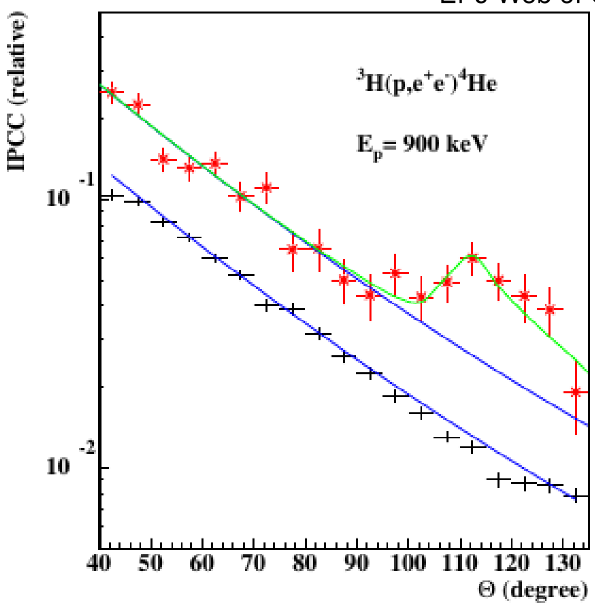


Feng PRD **95**, 035017 (2017)

"An anomaly in the internal pair creation on the M1 transition depopulating the 18.15 MeV isoscalar 1^{+} state on $^{8}\mathrm{Be}$ was observed. This could be explained by the creation and subsequent decay of a new boson .. mass 17.01(16) MeV"

Firak, Krasznahorkay, et al
EPJ Web of Conferences **232 04005 (2020)**





Angle between e^{-} and e^{+}

IPCC:
Internal Pair Creation
Angular Correlation

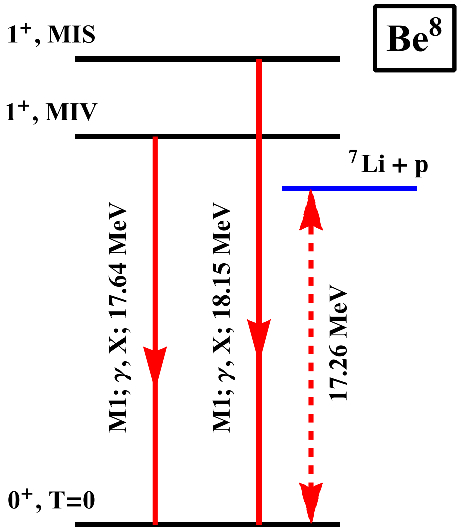


Fig. from PLB 813, 136061 (2021)

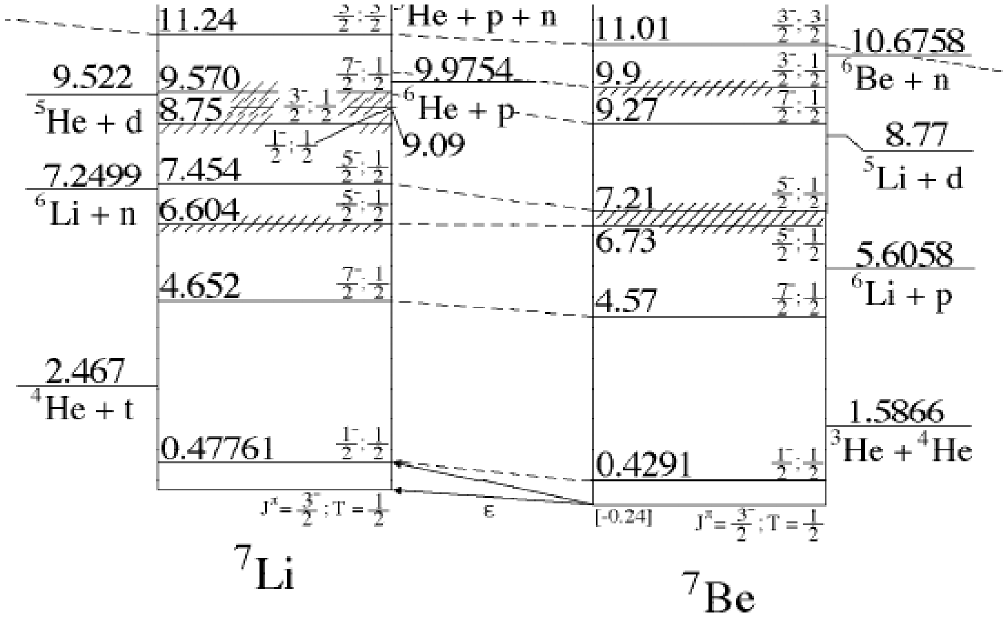
Can *ab initio* nuclear theory help interpret the anomaly?

NCSMC calculations of ^{8}Be structure and ^{7}Li+p scattering and capture

- Wave function ansatz

$$\Psi^{(8)}_{\text{NCSMC}} = \sum_{\lambda} c_{\lambda} \left|^{8}\text{Be}, \lambda\right\rangle + \sum_{\nu} \int dr \gamma_{\nu}(r) \hat{A}_{\nu} \left|^{7}\text{Li} + p, \nu\right\rangle + \sum_{\mu} \int dr \gamma_{\mu}(r) \hat{A}_{\mu} \left|^{7}\text{Be} + n, \mu\right\rangle$$

- $3/2^-$, $1/2^-$, $7/2^-$, $5/2^-$, $5/2^-$ ^{7}Li and ^{7}Be states in cluster basis
- 15 positive and 15 negative parity states in ^{8}Be composite state basis



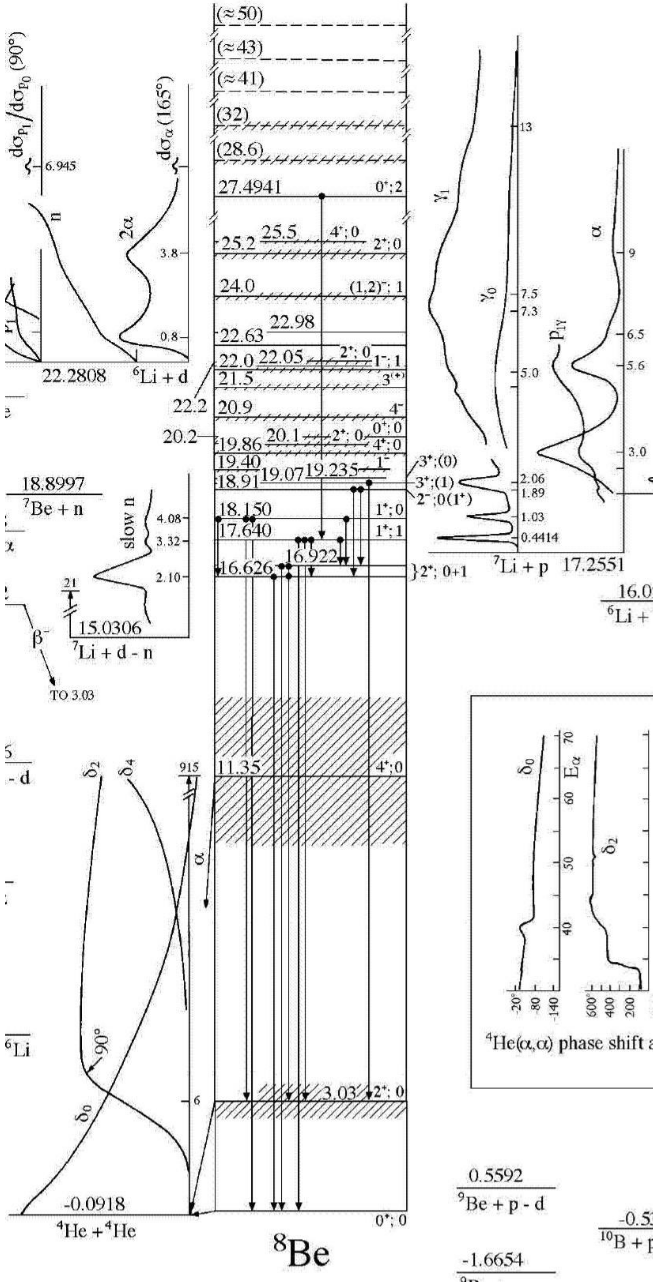
TUNL Nuclear Data Evaluation Project

NCSMC calculations of ^{8}Be structure and ^{7}Li+p scattering and capture

- Wave function ansatz

$$\Psi^{(8)}_{\text{NCSMC}} = \sum_\lambda c_\lambda \left|{}^8\text{Be}, \lambda\right\rangle + \sum_\nu \int \mathrm{d}r \gamma_\nu(r) \hat{A}_\nu \left|{}^7\text{Li} + p, \nu\right\rangle + \sum_\mu \int \mathrm{d}r \gamma_\mu(r) \hat{A}_\mu \left|{}^7\text{Be} + n, \mu\right\rangle$$

- $3/2^-$, $1/2^-$, $7/2^-$, $5/2^-$, $5/2^-$ ^{7}Li and ^{7}Be states in cluster basis
- 15 positive and 15 negative parity states in ^{8}Be composite state basis



^{8}Be structure – calculated positive-parity eigenphase shifts

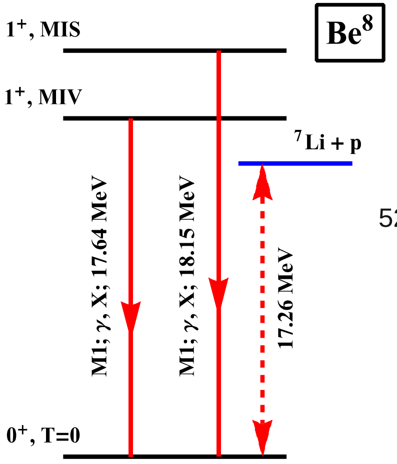
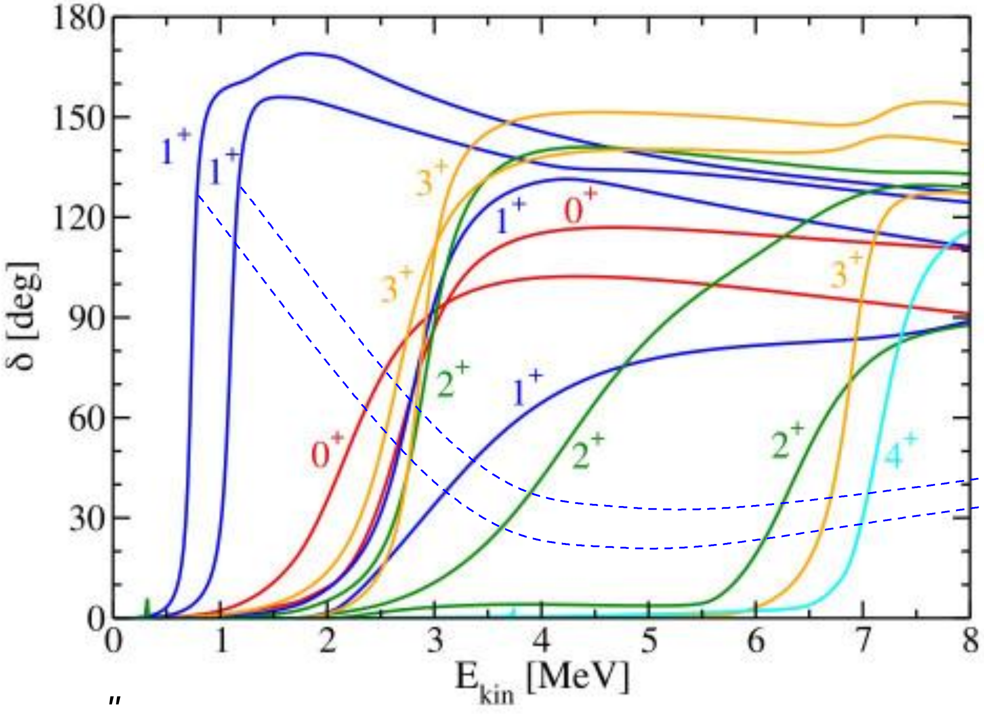
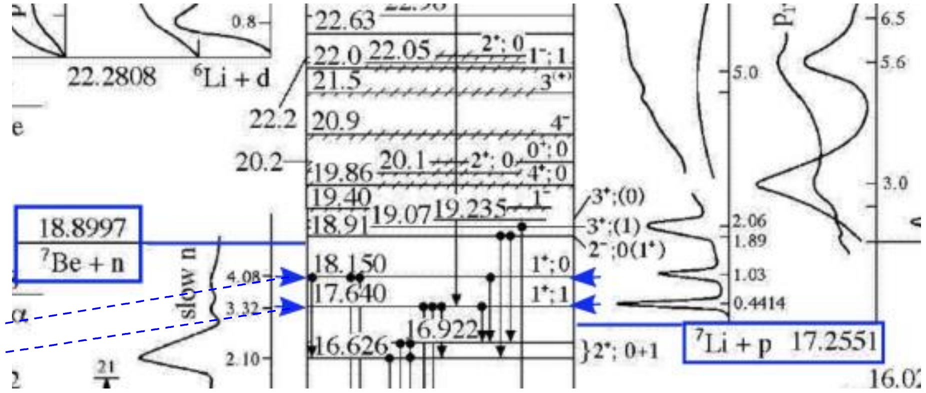


Fig. from PLB 813, 136061 (2021)



"
$p + {}^{7}$Li threshold



Additional resonances are seen compared to TUNL data

PHYSICAL REVIEW C **110**, 015503 (2024)

Editors' Suggestion

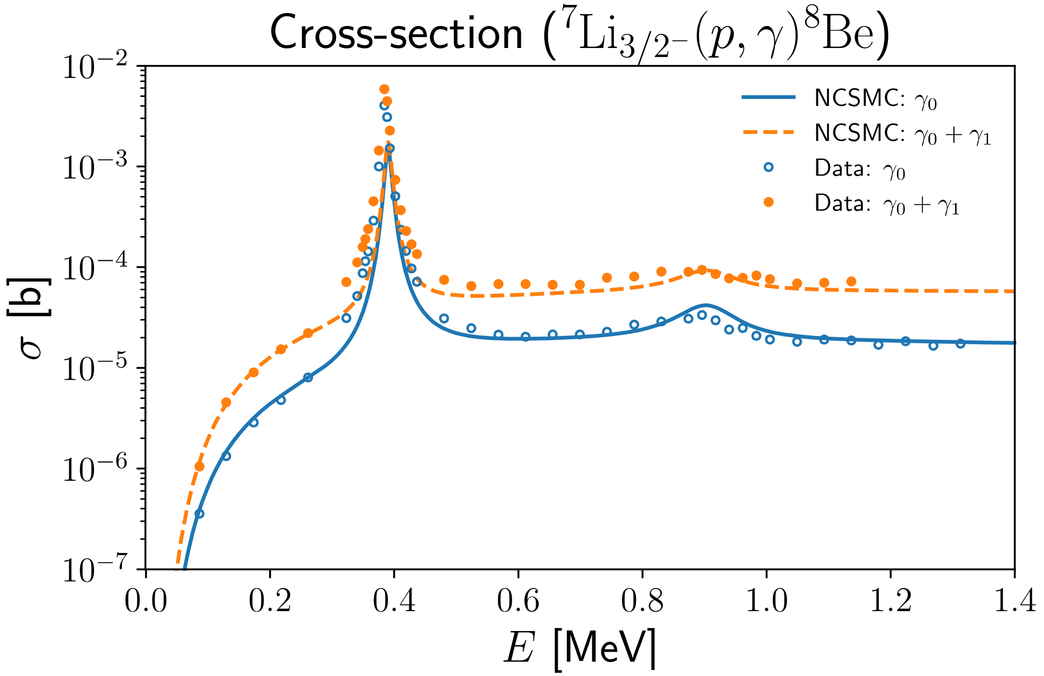
***Ab initio* investigation of the ^{7}Li(p, $e^{+}e^{-}$)^{8}Be process and the X17 boson**

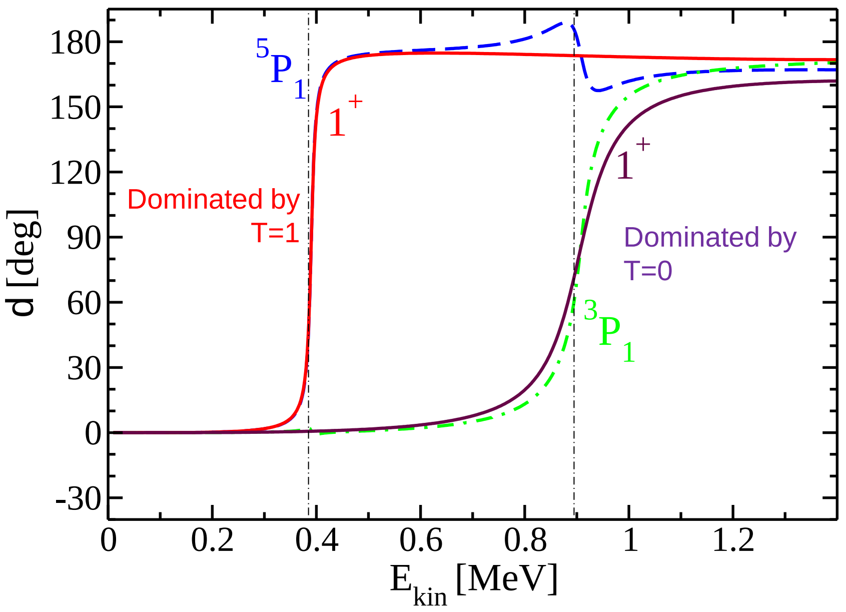
P. Gysbers,[1,2,3] P. Navrátil,[1,4] K. Kravvaris,[5] G. Hupin,[6] and S. Quaglioni[5]

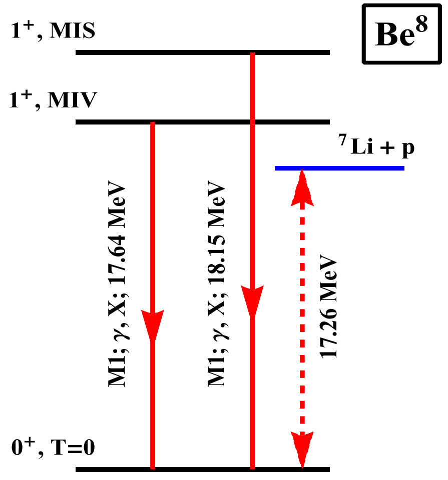
Ab initio calculations of $^7Li(p,\gamma)^8Be$ radiative capture, $^7Li(p,e^+e^-)^8Be$ pair production & X17 boson

- Motivated by ATOMKI experiments (Firak, Krasznahorkay *et al.*, EPJ Web of Conferences **232**, 04005 (2020))
- No-core shell model with continuum (NCSMC) with wave function ansatz

$$\Psi^{(8)}_{\text{NCSMC}} = \sum_\lambda c_\lambda \left|{}^8\text{Be}, \lambda\right\rangle + \sum_\nu \int \mathrm{d}r \gamma_\nu(r) \hat{A}_\nu \left|{}^7\text{Li} + p, \nu\right\rangle + \sum_\mu \int \mathrm{d}r \gamma_\mu(r) \hat{A}_\mu \left|{}^7\text{Be} + n, \mu\right\rangle$$





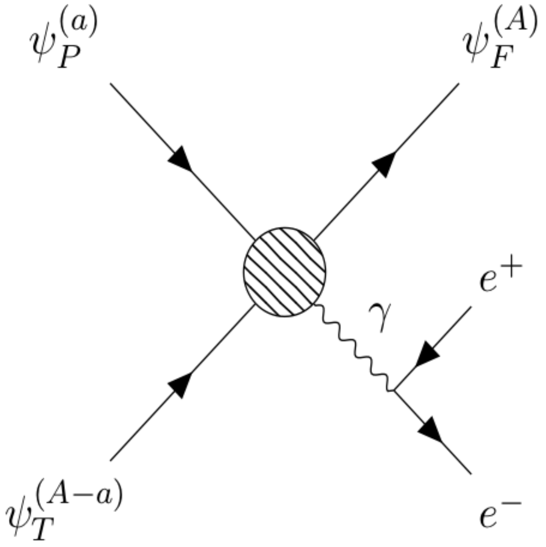


Data: Zahnow *et al.*
Z.Phys.A **351** 229-236 (1995)

γ_0: decay to ground state (0^+)
γ_1: decay to first excited (2^+)

Ab initio calculations of ^{7}Li(p,γ)^{8}Be radiative capture, ^{7}Li(p,e$^+$e$^-$)^{8}Be pair production & X17 boson

Internal electron-positron pair conversion correlation



Calculating properly the pair production cross section
with the interference of different multipoles improves description.

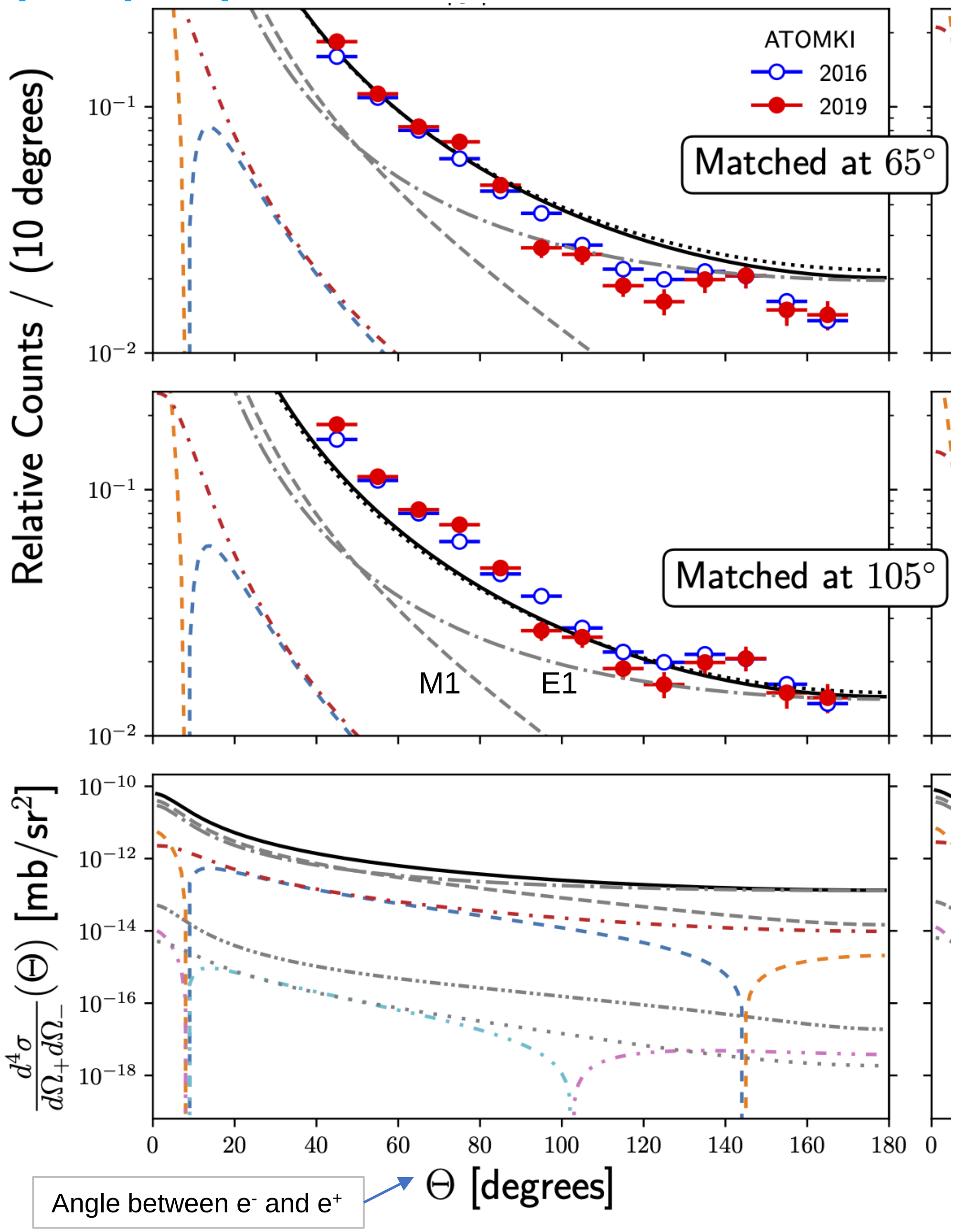
Still not a perfect agreement with ATOMKI data

PHYSICAL REVIEW C **110**, 015503 (2024)

Editors' Suggestion

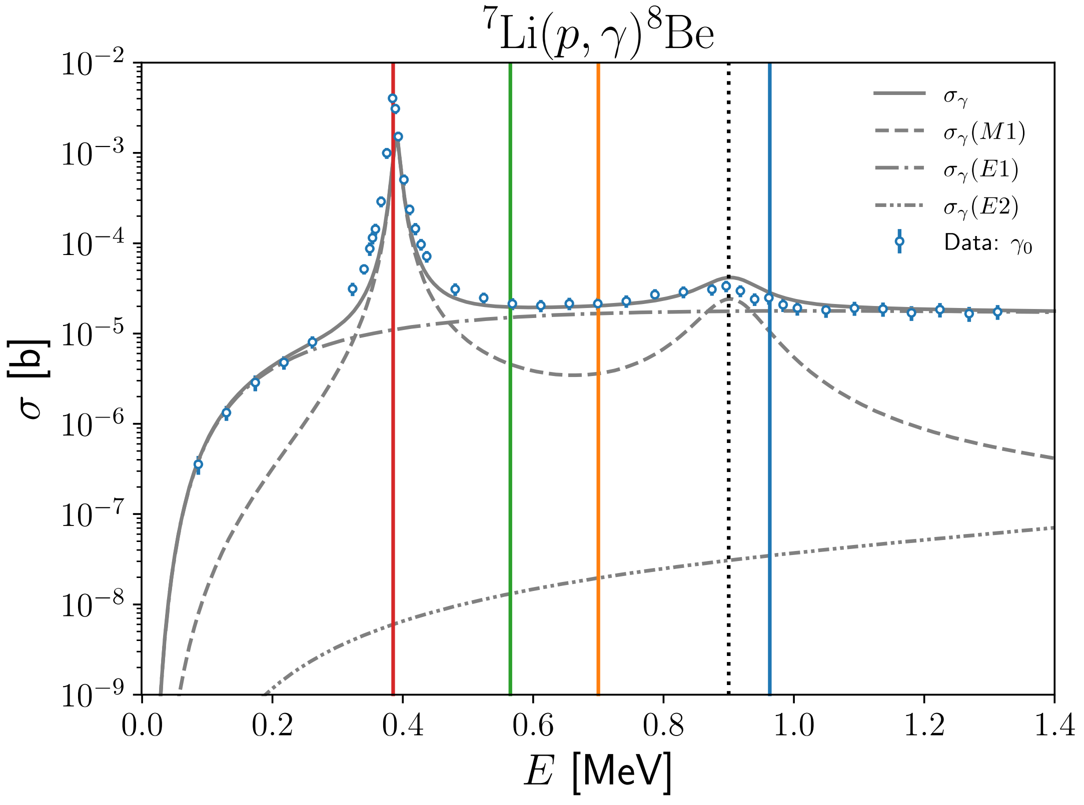
***Ab initio* investigation of the ^{7}Li(p, e^+e^-)^{8}Be process and the X17 boson**

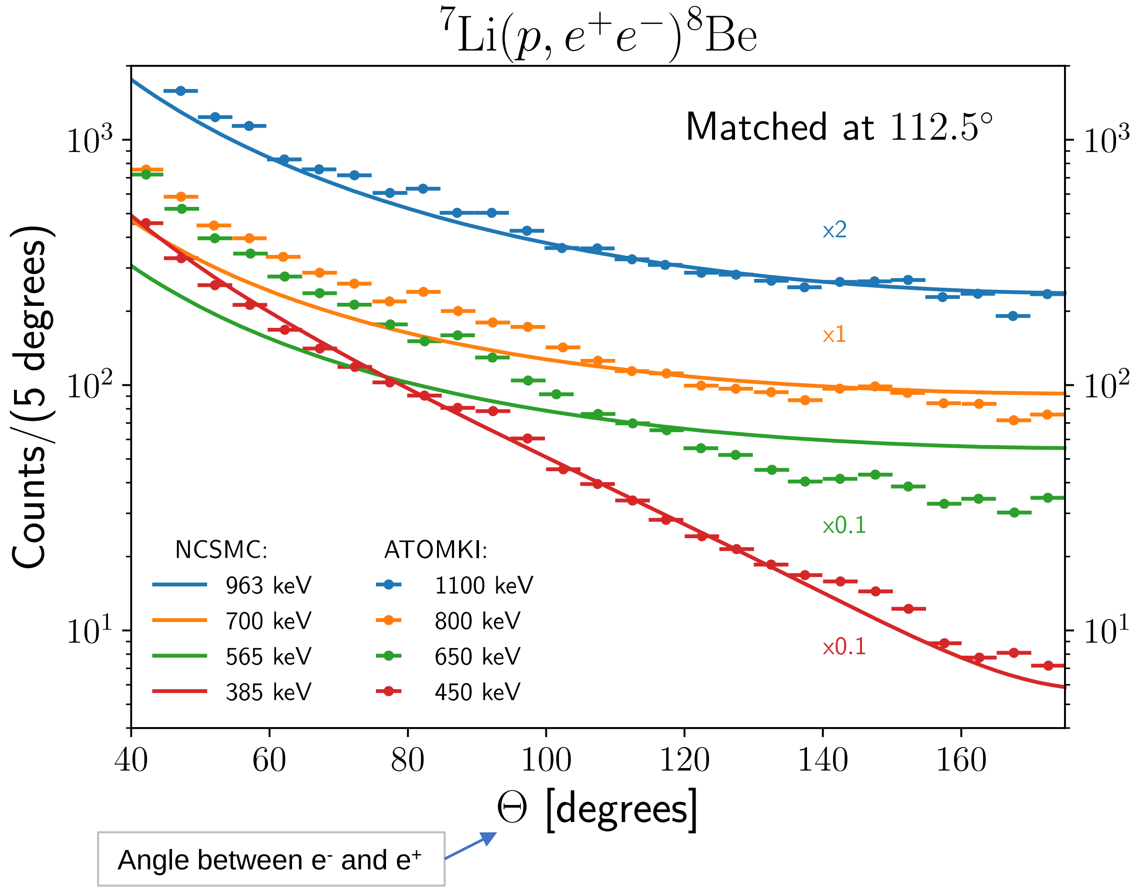
P. Gysbers,[1,2,3] P. Navrátil,[1,4] K. Kravvaris,[5] G. Hupin,[6] and S. Quaglioni[5]



New ATOMKI measurements in-between & at resonance energies

N. J. Sas et al., "Observation of the X17 anomaly in the ^{7}Li(p,e$^+$ e$^-$)^{8}Be direct proton-capture reaction," arXiv:2205.07744



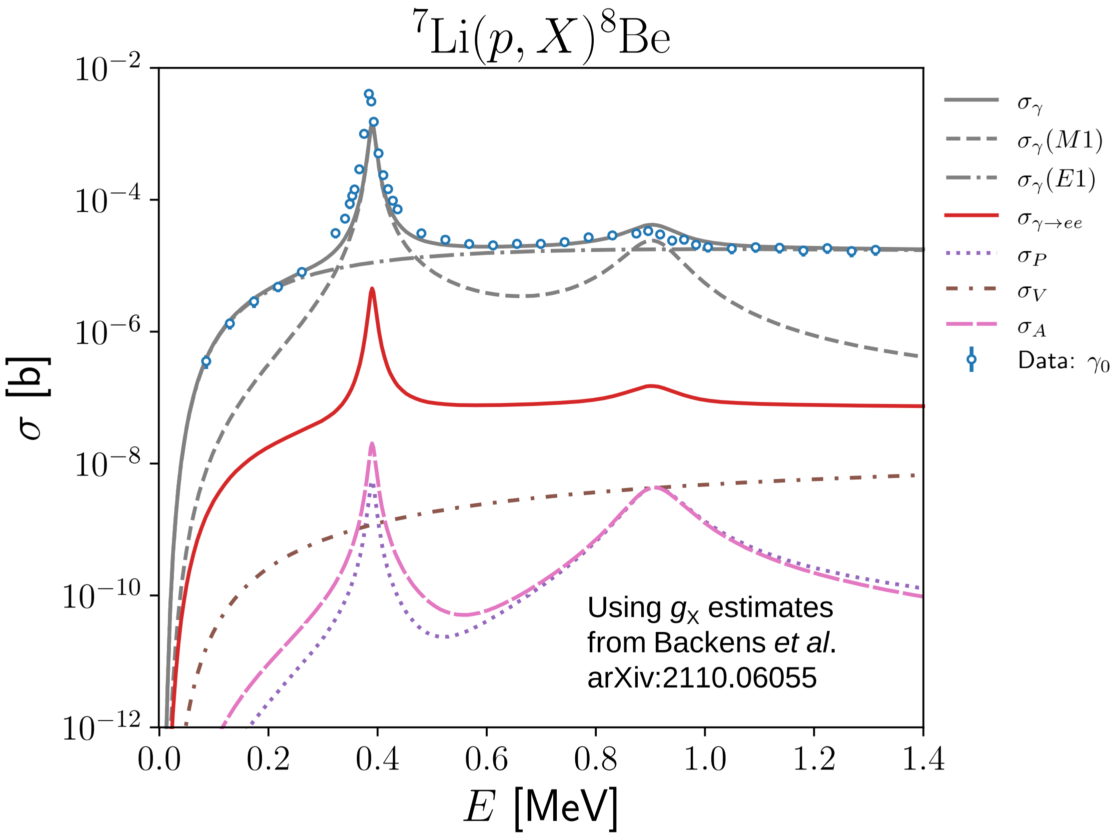


NCSMC calculations match well resonance data.
Disagree in-between resonances – flat E1 distribution.
Proton slow-down in the thick target?

Ab initio calculations of ^{7}Li(p,γ)^{8}Be radiative capture, ^{7}Li(p,e$^+$e$^-$)^{8}Be pair production & X17 boson

Modeling hypothetical X17 boson

Integrated cross sections



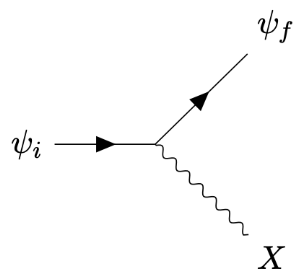
Gamma capture data: Zahnow *et al.*
Z.Phys.A **351** 229-236 (1995)

X17 Candidate Bosons

$(m_X \simeq 17$ MeV, $\Delta E \geq 17.2251$ MeV $[^7\mathrm{Li} + p]$,
$k_X = \sqrt{\Delta E^2 - m_X^2}$, $k_\gamma = \Delta E)$

Operators for $1^\pm \to 0^+$ decay (in the long-wavelength approximation)

- Pseudo-scalar (0^-):$\langle X_P \rangle \sim \epsilon_P \left\langle \hat{S} \right\rangle k_X$
- Axial-vector (1^+): $\langle X_A \rangle \sim \epsilon_A \left\langle \hat{S} \right\rangle \sqrt{2 + \frac{m_X^2}{\Delta E^2}}$
- Vector (1^-): $\langle X_V \rangle \sim \epsilon_V \langle E1 \rangle \frac{k_X}{k_\gamma}$
- For comparison: γ (E1 (1^-), M1 (1^+), E2 (2^+), etc)
  $\langle E1 \rangle \sim \langle r Y_1 \rangle k_\gamma$
  $\langle M1 \rangle \sim \left(g_l \left\langle \hat{L} \right\rangle + g_s \left\langle \hat{S} \right\rangle\right) k_\gamma$



PHYSICAL REVIEW C **110**, 015503 (2024)

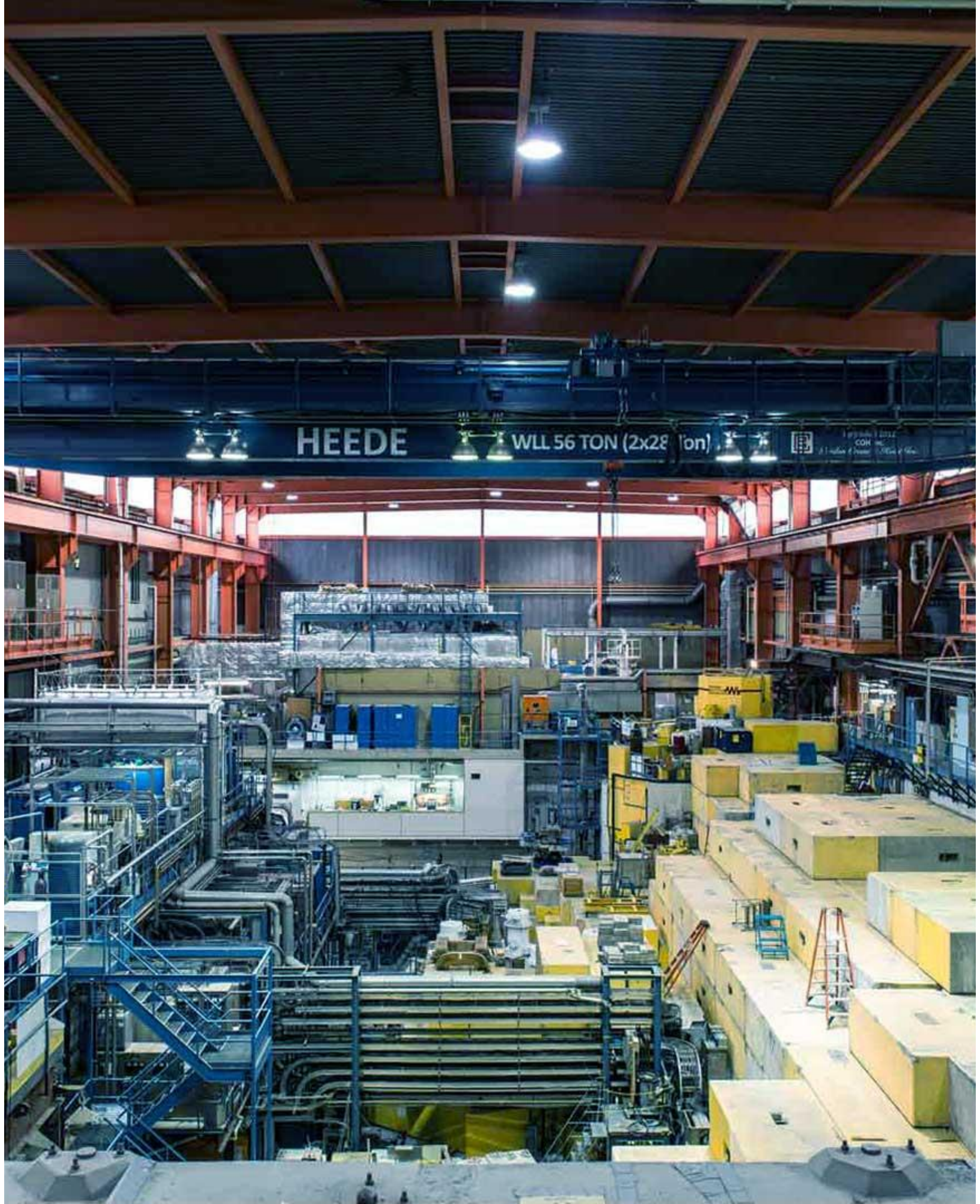
Editors' Suggestion

Ab initio investigation of the ^{7}Li$(p, e^+e^-)^8$Be process and the X17 boson

P. Gysbers,[1,2,3] P. Navrátil,[1,4] K. Kravvaris,[5] G. Hupin,[6] and S. Quaglioni[5]

TRIUMF

Conclusions & topics for discussion



Discovery, accelerated

Conclusions & topics for discussion

- *Ab initio* nuclear theory
  - Makes connections between the low-energy QCD and many-nucleon systems

- No-core shell model is an *ab initio* extension of the original nuclear shell model
  - Applicable to nuclear structure, reactions including those relevant for astrophysics, electroweak processes, tests of fundamental symmetries

- **Open questions**
  - How to accurately and precisely evaluate the isospin-symmetry breaking correction δ_C?
  - How to evaluate the radiative nuclear structure correction δ_{NS} beyond light nuclei?
  - What is the importance of sub-leading chiral 3N contributions for electro-weak processes in nuclei?
  - What is the particle physics interpretation of the X17 anomaly?

TRIUMF

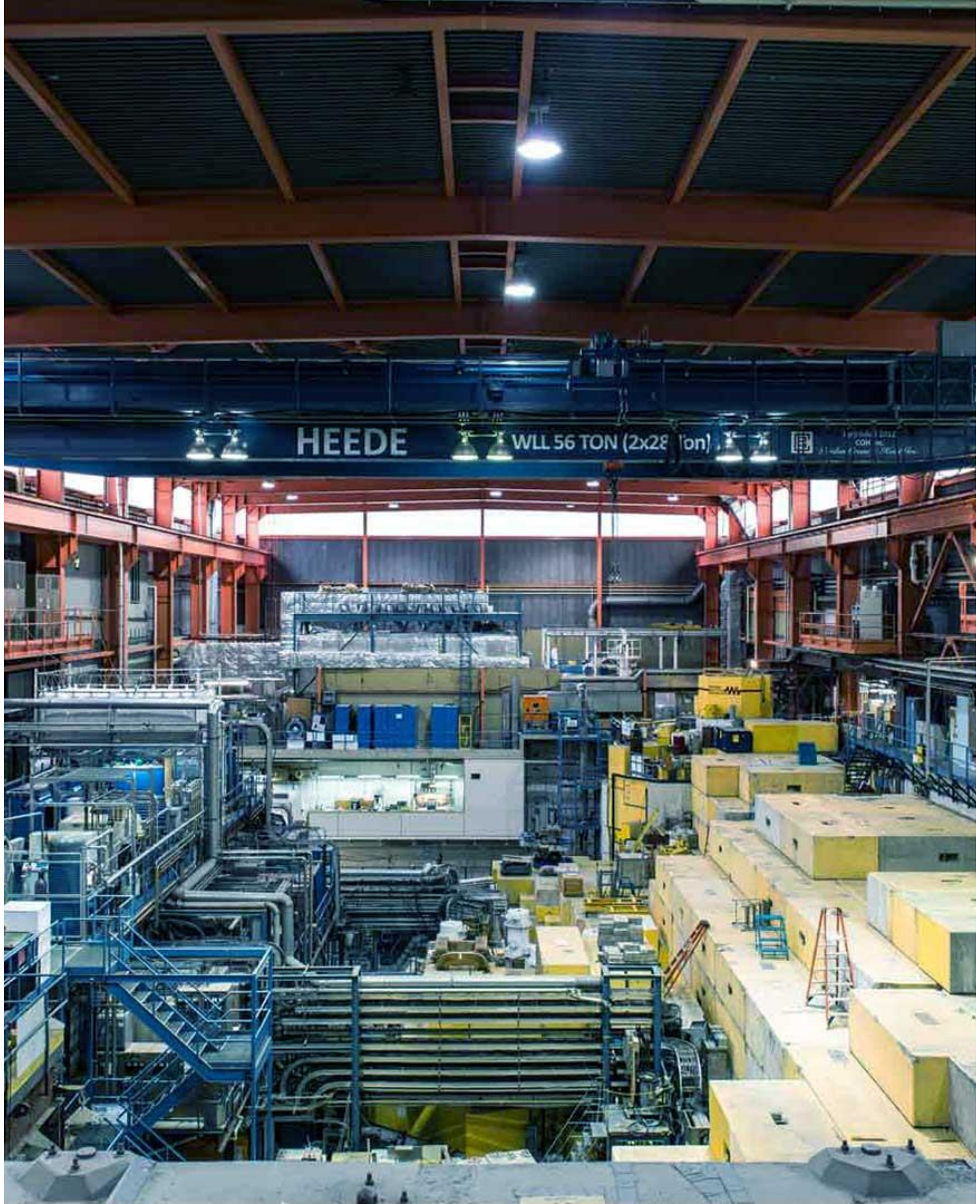
Thanks to my collaborators

Peter Gysbers (MSU), Michael Gennari (UVic/TRIUMF), Paul Froese (UBC), Lotta Jokiemi (TRIUMF), Mehdi Drissi (TRIUMF), Ayala Glick-Magid (INT), Doron Gazit (Hebrew U), C. Forssen (Chalmers UT), Daniel Gazda (NPI Rez), Chien-Yeah Seng (INT), Misha Gorshteyn (U Mainz), Sofia Quaglioni (LLNL), Guillaume Hupin (IJCLab), Kostas Kravvaris (LLNL), Mack Atkinson (LLNL)

Discovery, accelerated

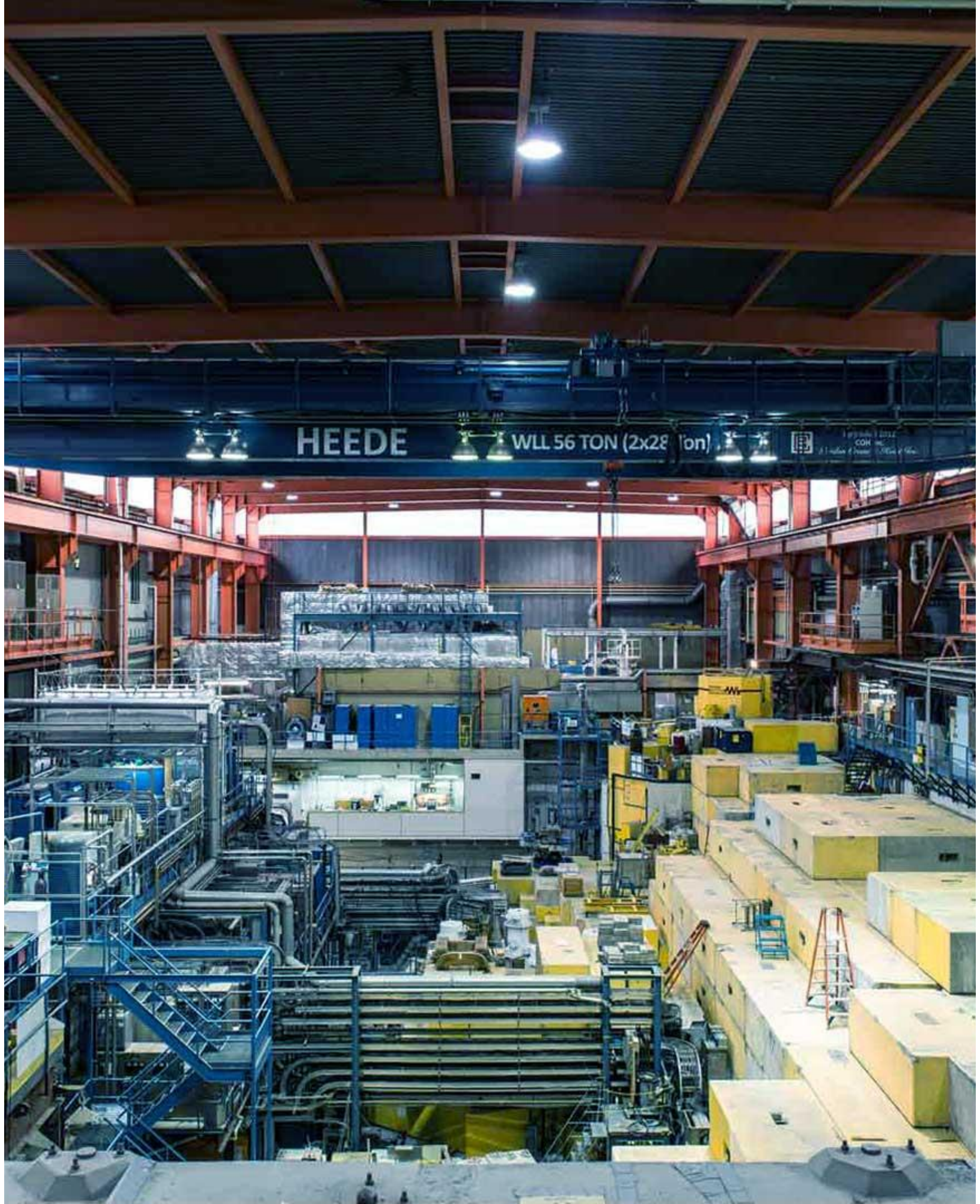
TRIUMF

Backup slides



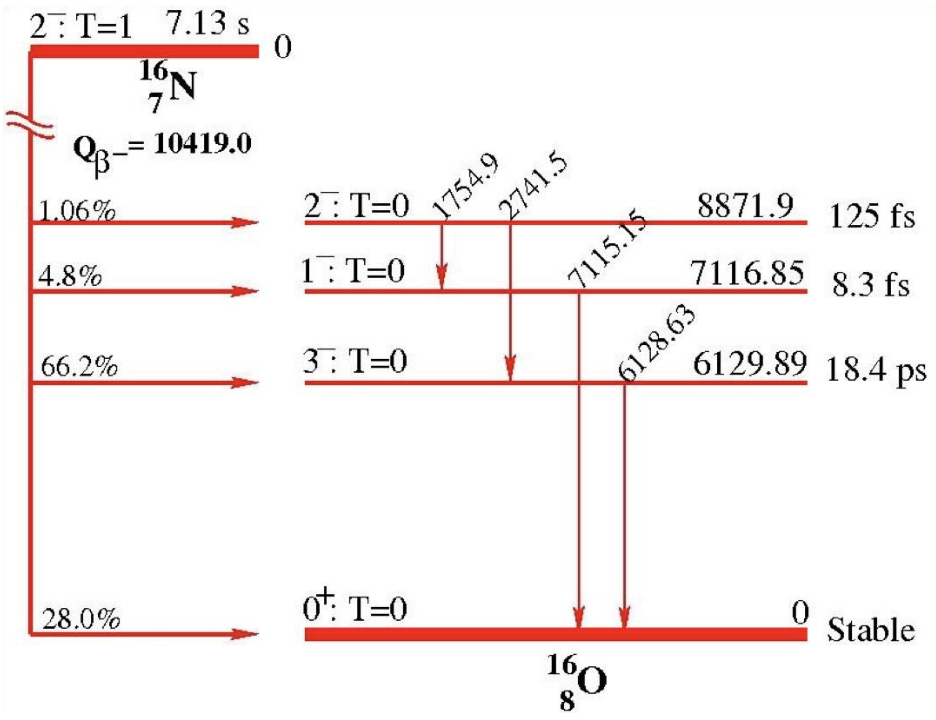
Discovery, accelerated

^{16}N β-decay



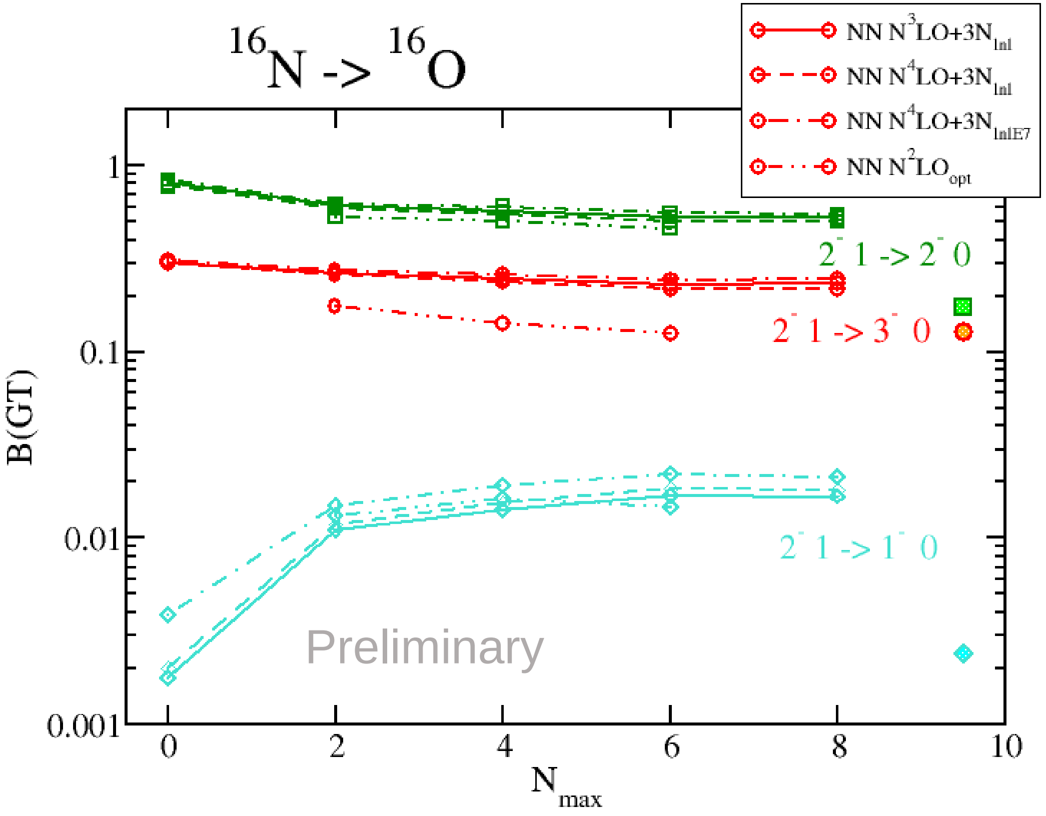
Unique first-forbidden beta decay $^{16}N(2^-) \rightarrow {}^{16}O(0^+)$

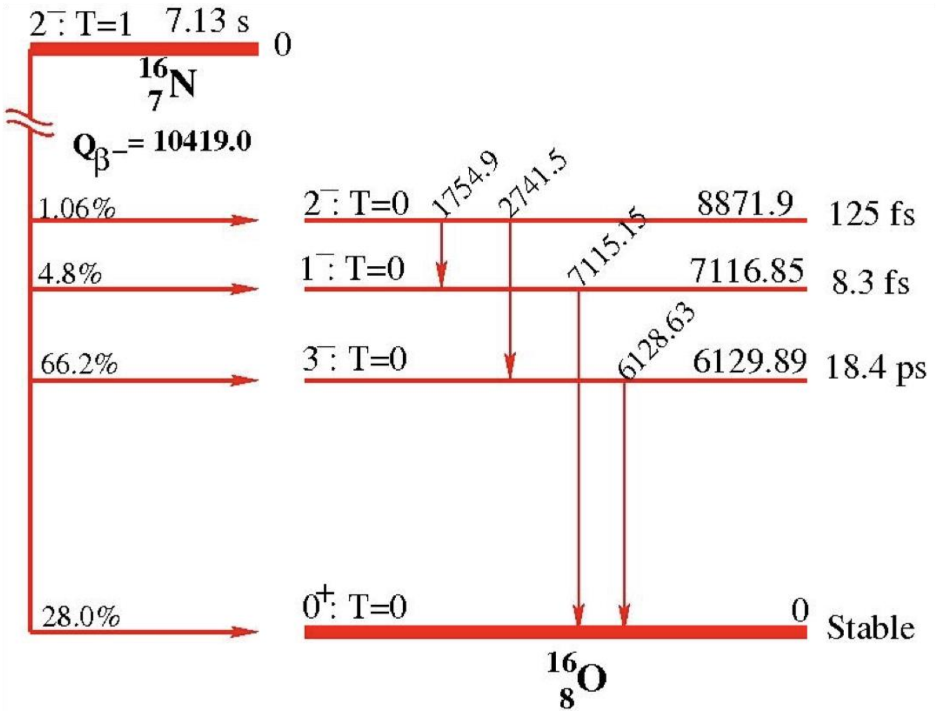
- The unique first-forbidden transition, $J^{\Delta\pi} = 2^-$, is of great interest for BSM searches
  - Energy spectrum of emitted electrons sensitive to the symmetries of the weak interaction, gives constraints both in the case of right and left couplings of the new beyond standard model currents
  - Ayala Glick-Magid *et al.*, PLB 767 (2017) 285
- Ongoing experiment at SARAF, Israel



$^{16}N(2^-)$ Gamow-Teller transitions to the negative parity excited states of ^{16}O

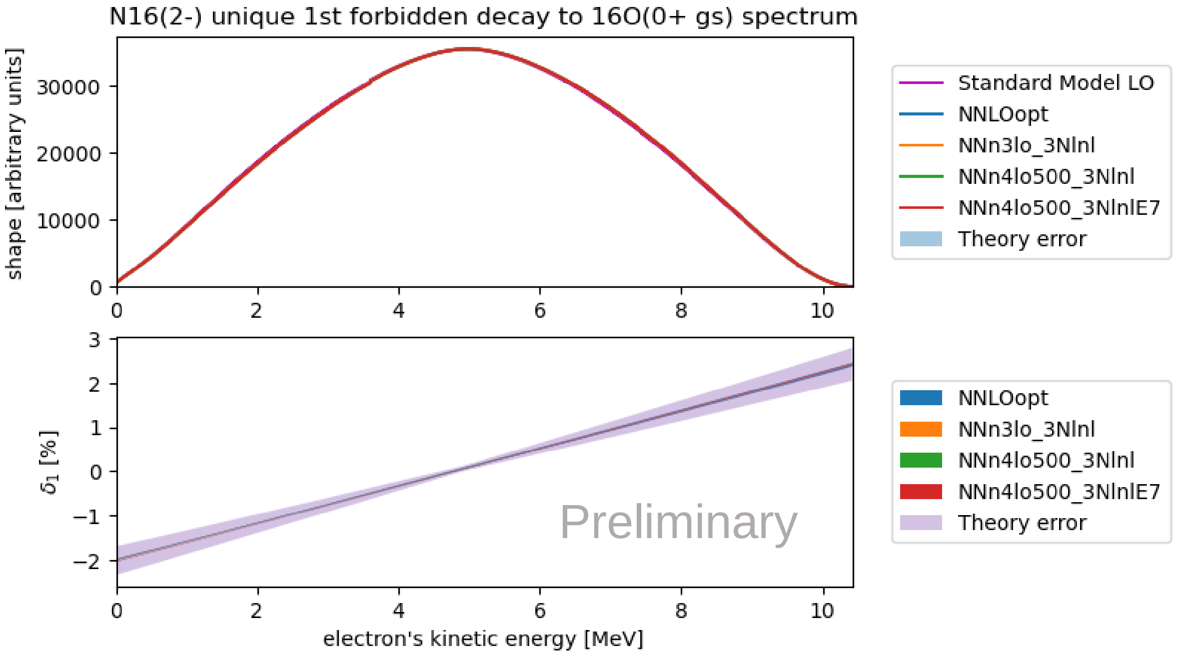
- Tests of NCSM wave functions
  - B(GT)s overestimated – operator SRG, 2BC need to be included, continuum
  - Correct hierarchy of transitions

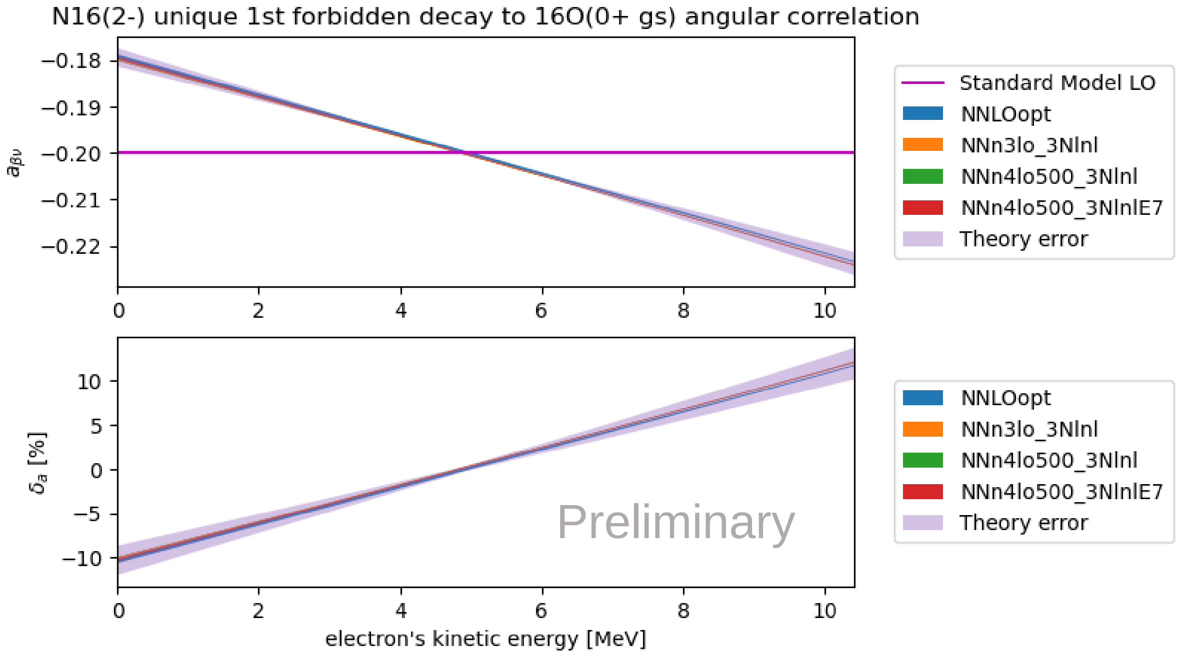




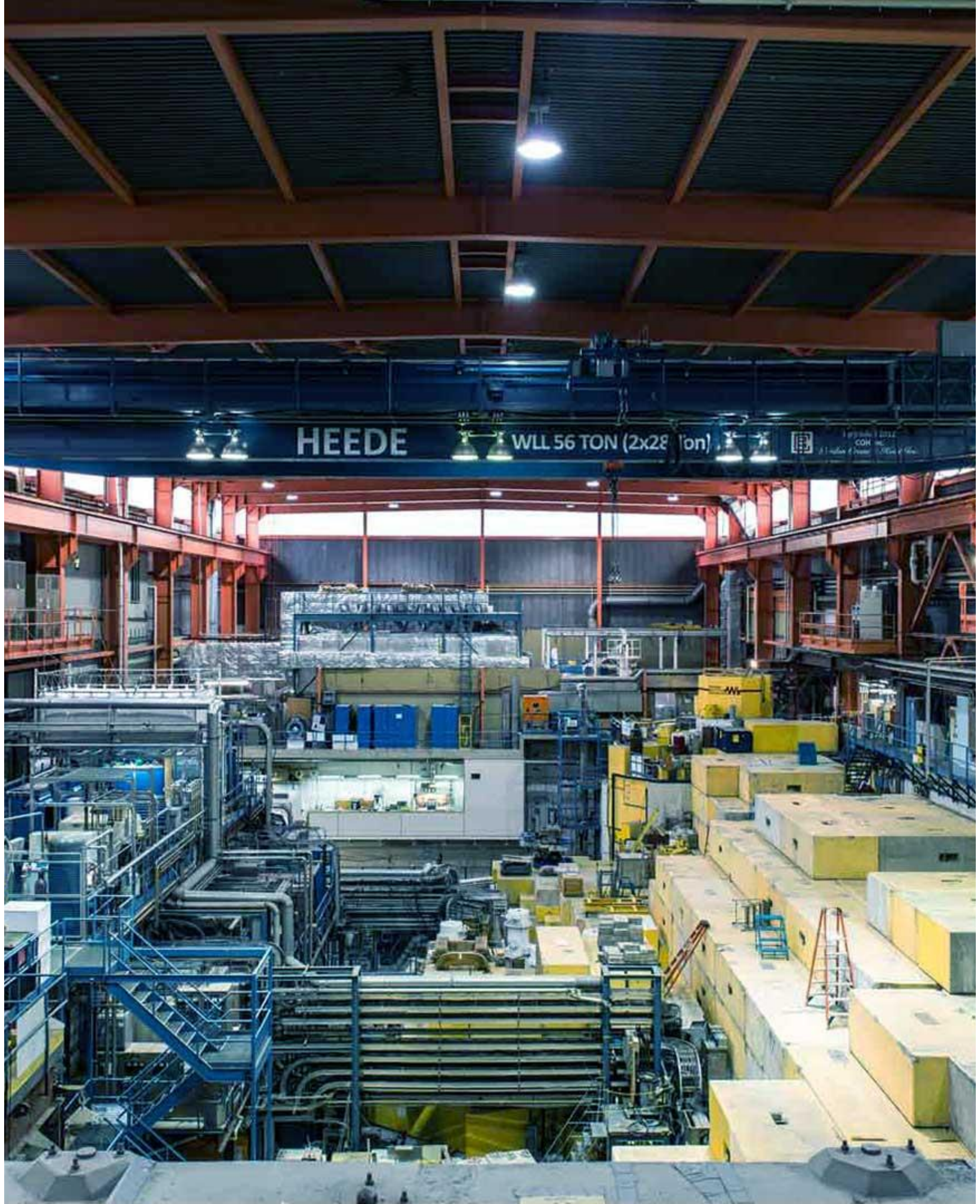
Unique first-forbidden beta decay $^{16}N(2^-) \rightarrow {}^{16}O(0^+)$

- Preliminary results for electron energy spectrum and angular correlations





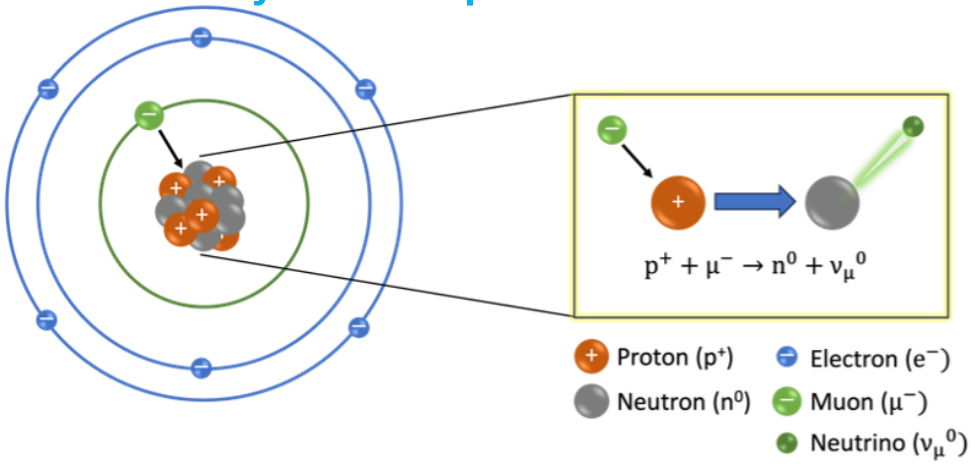
Ab initio calculations of muon capture on light nuclei



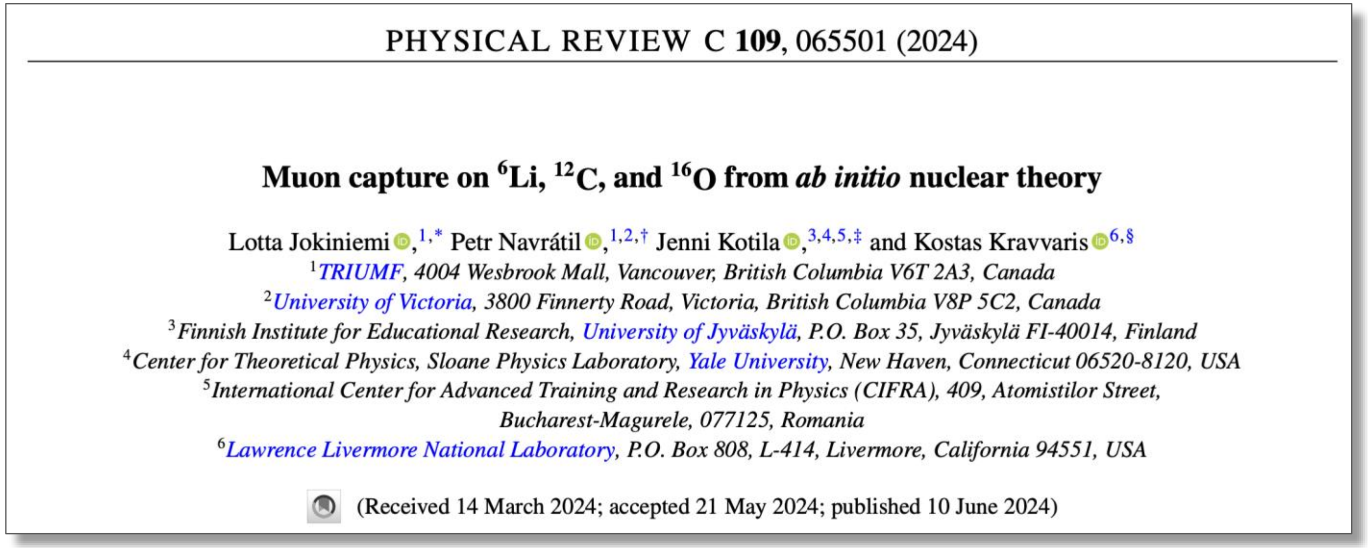
Discovery, accelerated

Muon capture on ^{6}Li, ^{12}C, ^{16}N from *ab initio* nuclear theory

Ordinary muon capture on a nucleus



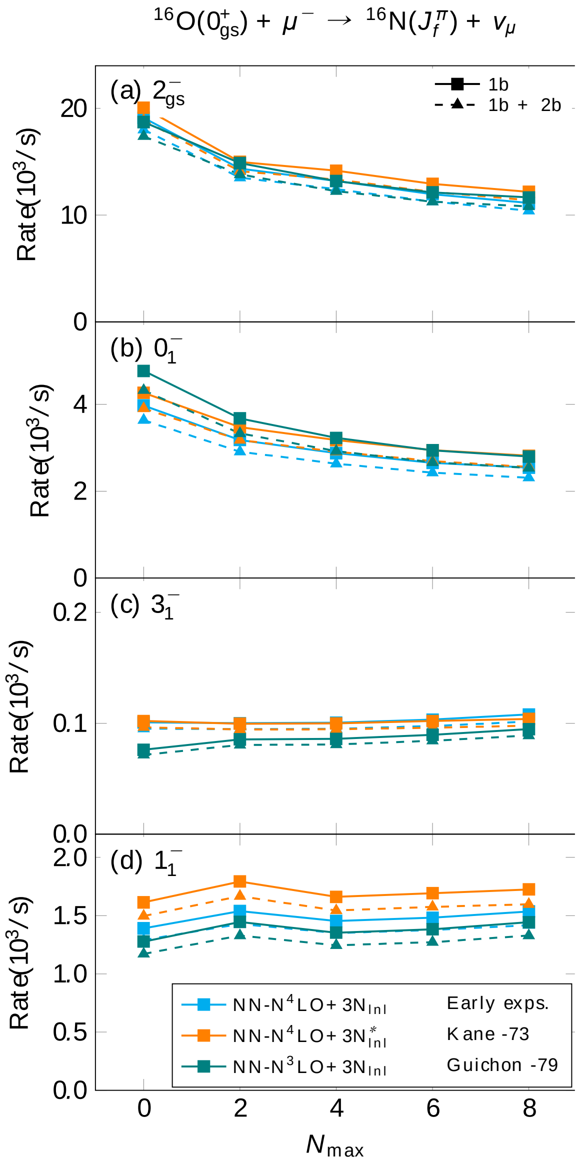
- ❑ Momentum exchange $q = m_\mu + E_i - E_f \approx 100$ MeV
- ❑ Involves vector, axial-vector, magnetic and pseudoscalar nuclear-weak currents
  - → **Can be used as a probe of $0\nu\beta\beta$ decay**


PHYSICAL REVIEW C 109, 065501 (2024)

Muon capture on ^{6}Li, ^{12}C, and ^{16}O from *ab initio* nuclear theory

Lotta Jokiniemi[1,*] Petr Navrátil[1,2,†] Jenni Kotila[3,4,5,‡] and Kostas Kravvaris[6,§]
[1]*TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada*
[2]*University of Victoria, 3800 Finnerty Road, Victoria, British Columbia V8P 5C2, Canada*
[3]*Finnish Institute for Educational Research, University of Jyväskylä, P.O. Box 35, Jyväskylä FI-40014, Finland*
[4]*Center for Theoretical Physics, Sloane Physics Laboratory, Yale University, New Haven, Connecticut 06520-8120, USA*
[5]*International Center for Advanced Training and Research in Physics (CIFRA), 409, Atomistilor Street, Bucharest-Magurele, 077125, Romania*
[6]*Lawrence Livermore National Laboratory, P.O. Box 808, L-414, Livermore, California 94551, USA*

(Received 14 March 2024; accepted 21 May 2024; published 10 June 2024)



- ❑ ***Ab initio no-core* shell-model calculations in good agreement with experiments**

See talk by Lotta Jokiniemi on Saturday