Recent progress in many-body theory for nuclei and matter

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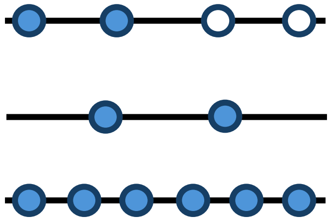
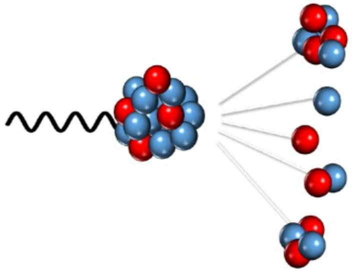
Johannes Gutenberg-Universität Mainz

Electroweak Physics InterseCtions workshop
(EPIC 2024), Sardinia, Italy

Outline

Open-shell nuclei with coupled-cluster





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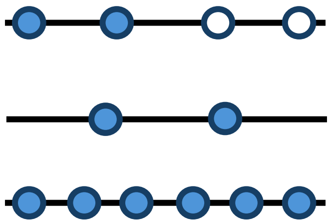
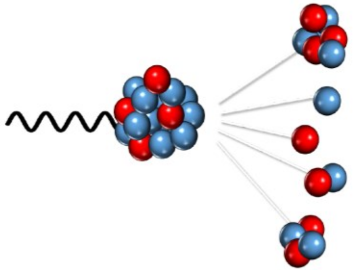
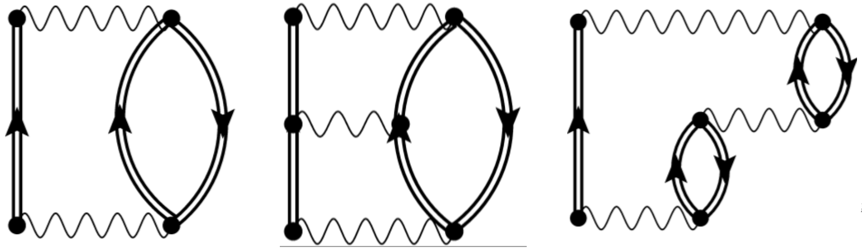
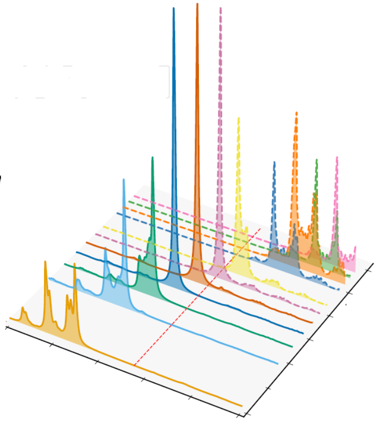


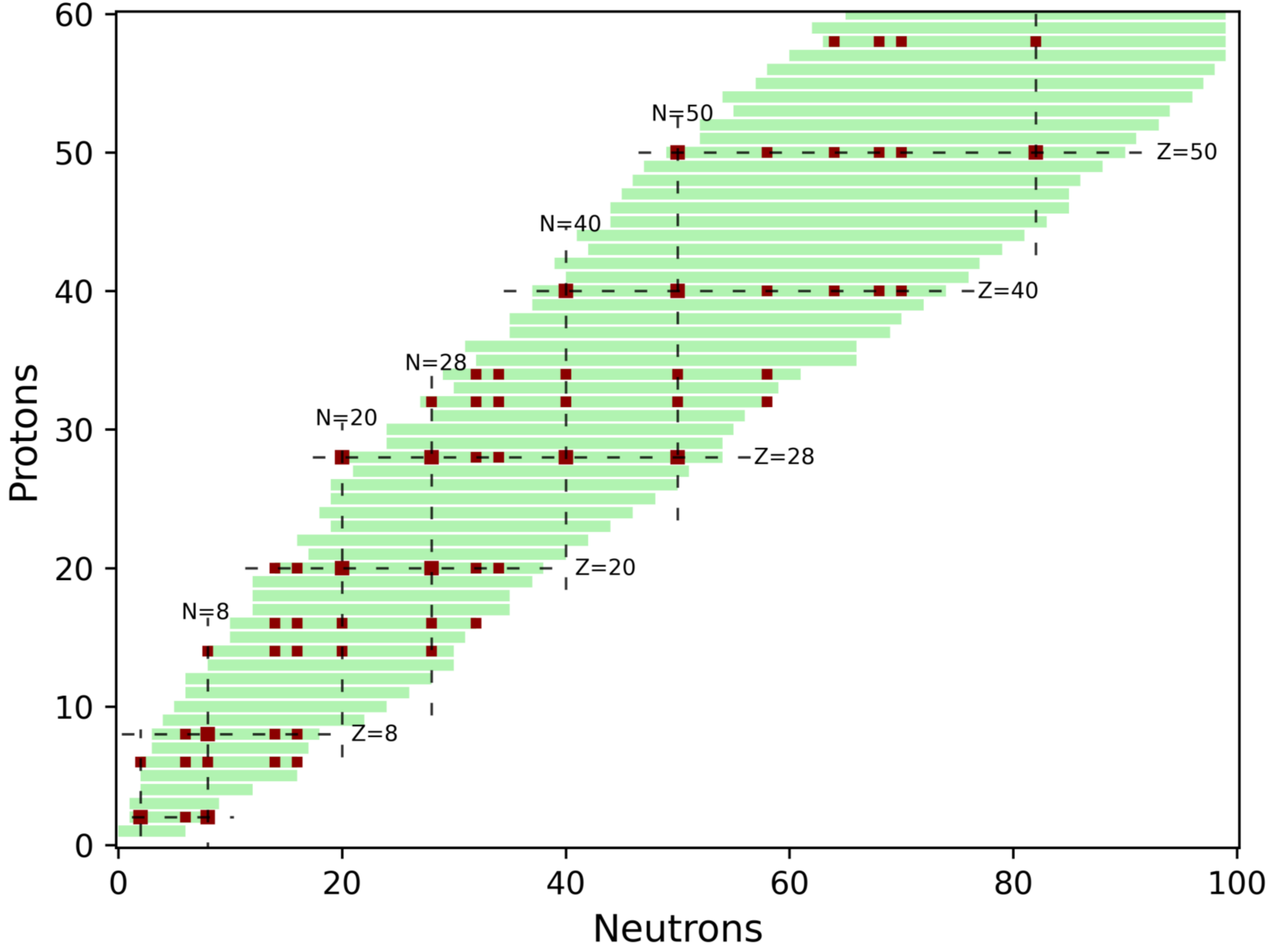


Infinite nuclear matter with **Green's functions** and coupled-cluster





Part 1:
Open-shell nuclei with coupled-cluster



Protons
Neutrons
N=8
N=20
N=28
N=40
N=50
Z=8
Z=20
Z=28
Z=40
Z=50

Coupled-cluster

Coupled-cluster (CC) ground-state ansatz

$$|\Psi_0\rangle = e^T|\Phi_0\rangle$$

Hagen et al., Rep. Prog. Phys. **77**, 096302 (2014)

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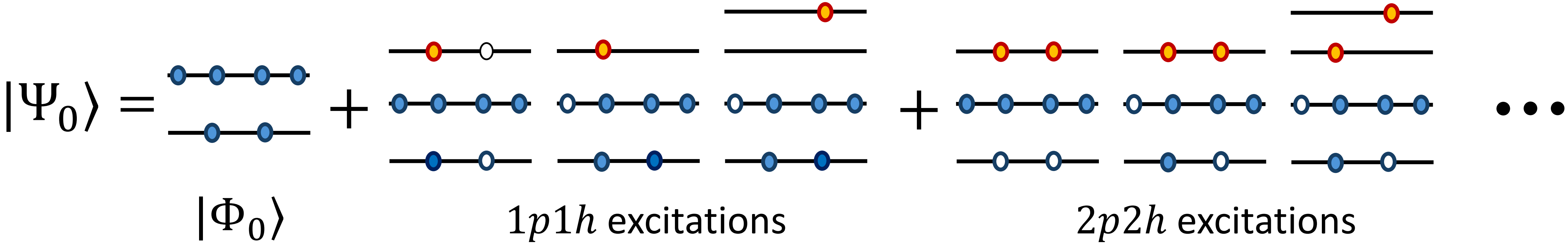

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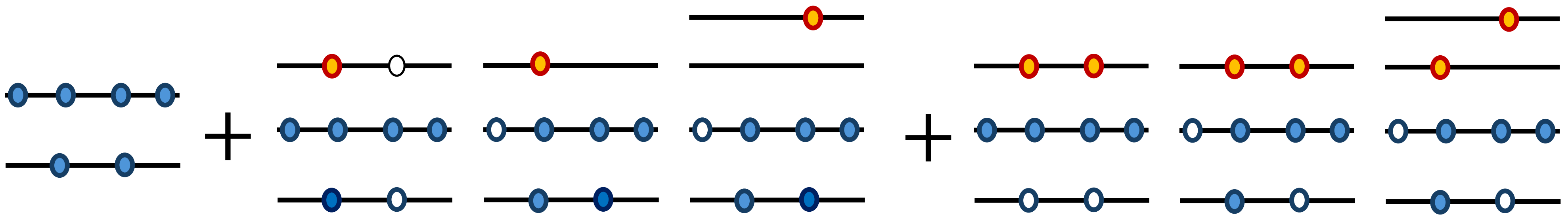
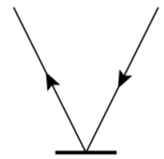
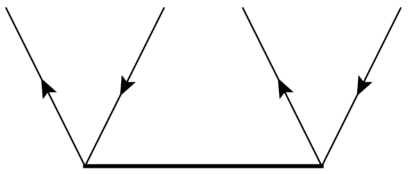
$|\Psi_0\rangle =$  $\cdots$

$|\Phi_0\rangle$ $\quad$ $1p1h$ excitations $\quad$ $2p2h$ excitations

CCSD: truncate at doubles (*2p2h)* level

CCSD(T): approximate triples (*3p3h)*

$T_1 =$ 

$T_2 =$ 

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Challenges for open-shell nuclei

Closed-shell nuclei ⇒ $|\Phi_0\rangle$: spherical reference state

Challenges for open-shell nuclei

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Open-shell nuclei exhibit pairing and deformation

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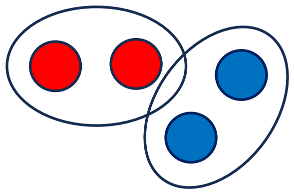
⟹ Use symmetry-breaking reference

Bogoliubov CC



Tichai et al., Physics Letters B **851**, 138571 (2024)

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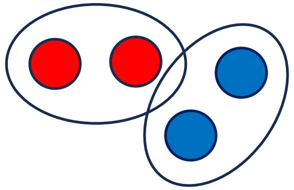
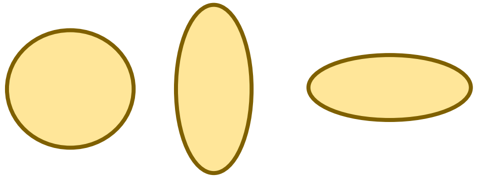


Deformed CC



See Gaute Hagen's talk

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Hagen et al. Phys. Rev. C **105**, 064311 (2022)
Sun et al., arXiv:2404.00058

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⟹ Use symmetry-breaking reference

✗ … but computationally expensive

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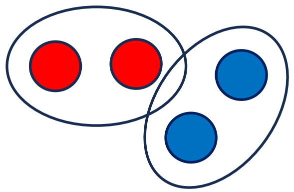

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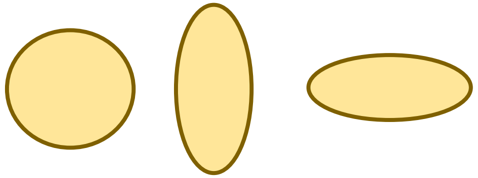

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⟹ Equation-of-motion CC

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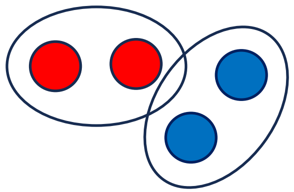

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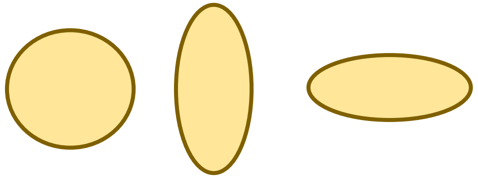

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Closed-shell nuclei → $|\Phi_0\rangle$: spherical reference state

Open-shell nuclei exhibit pairing and deformation

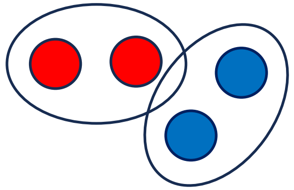
→ Use symmetry-breaking reference

✗ … but computationally expensive

→ Equation-of-motion CC

✓ Efficient ✓ Response functions

Bogoliubov CC



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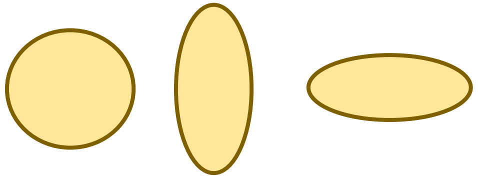

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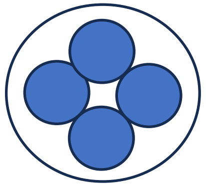
Equation-of-motion approach

Closed-shell



$$|\Psi_0\rangle = e^{T}|\Phi_0\rangle$$

Equation-of-motion approach

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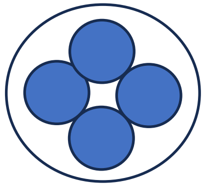

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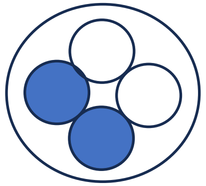
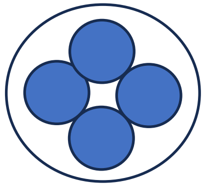


$$|\Psi_f^{(A-2)}\rangle$$

Two-particle-removed (2PR) nucleus

Equation-of-motion approach

Closed-shell



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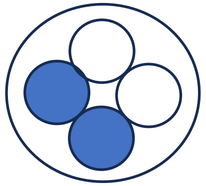
Open-shell



$|\Psi_f^{(A-2)}\rangle$

Two-particle-removed (2PR) nucleus

$$H|\Psi_f^{(A-2)}\rangle = E_f|\Psi_f^{(A-2)}\rangle$$

Equation-of-motion approach

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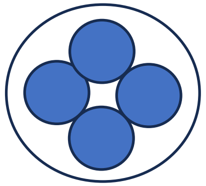

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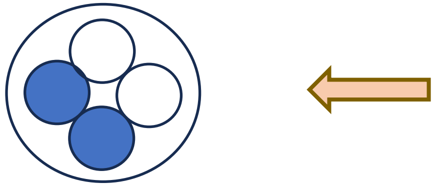

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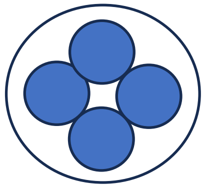

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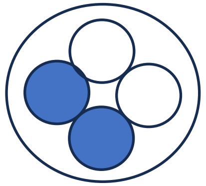

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Two-particle-removed (2PR) nucleus

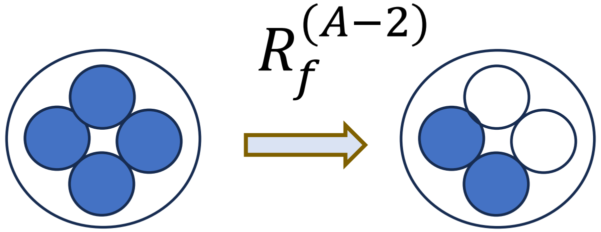
$$H|\Psi_f^{(A-2)}\rangle = E_f|\Psi_f^{(A-2)}\rangle$$

Equation-of-motion ansatz

$$|\Psi_f^{(A-2)}\rangle = R_f^{(A-2)}|\Psi_0\rangle$$

Excitation operator

Two-particle-removed CC



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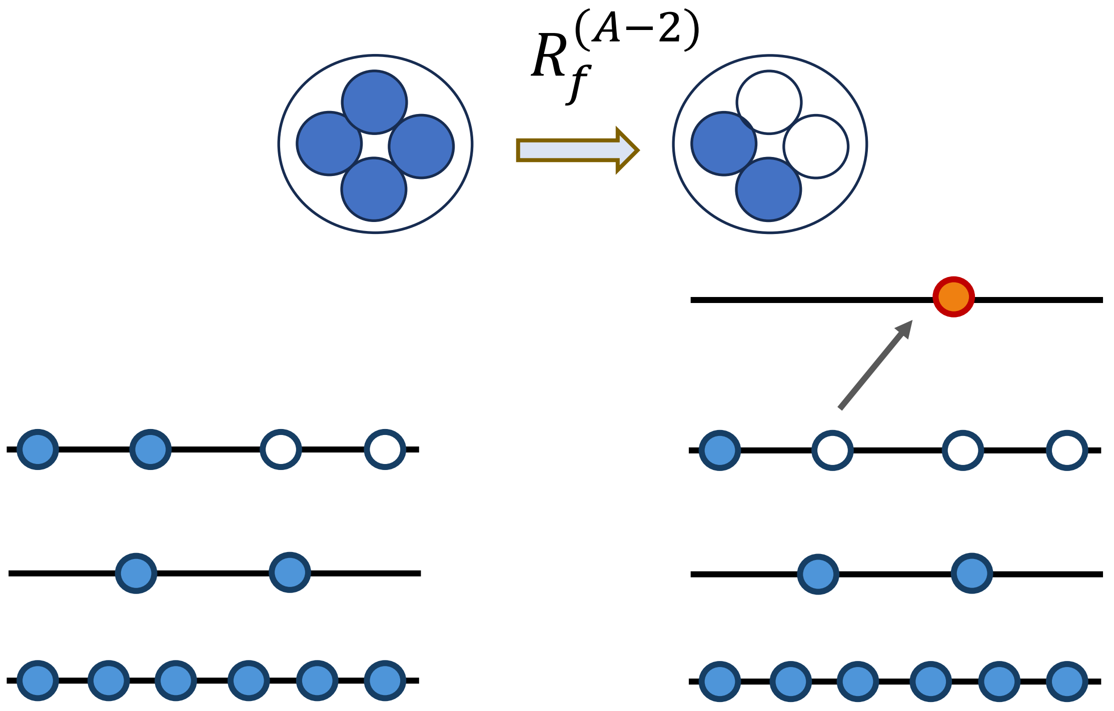
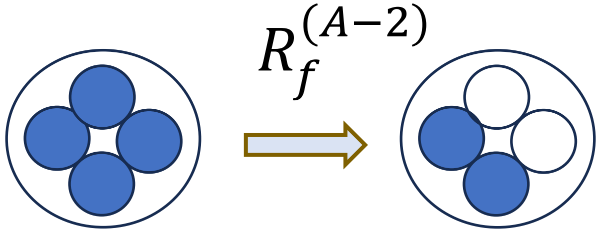
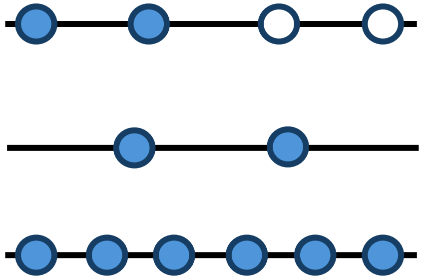


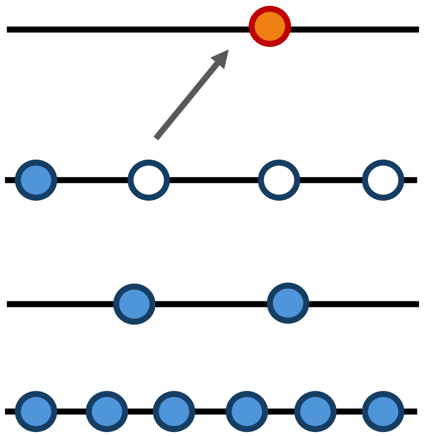
Two-hole   One-particle three-hole

$$R_f^{(A-2)} = \frac{1}{2}\sum_{ij} r_{ij} c_j c_i + \frac{1}{6}\sum_{ijka} r_{ijk}^{a}\, c_a^\dagger c_k c_j c_i$$

Two-particle-removed CC







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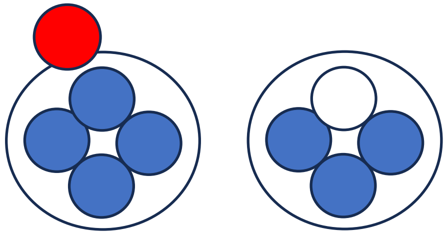
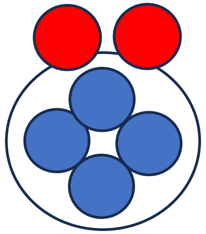


One-particle-attached/ removed (1PA/1PR)



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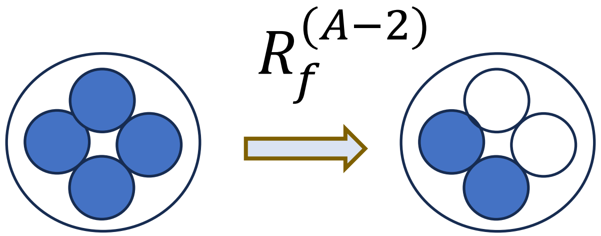

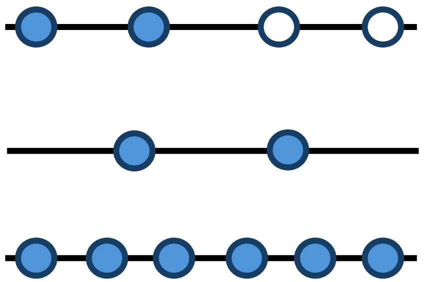

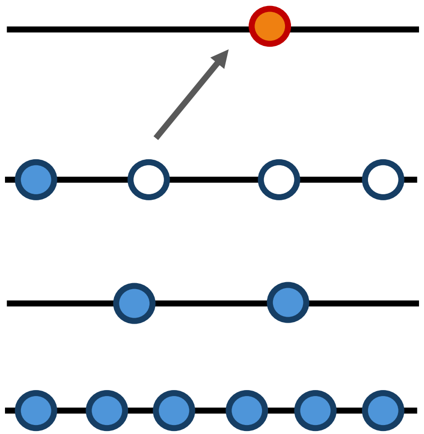

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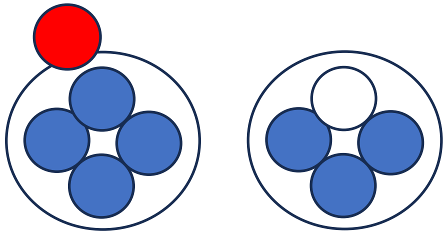

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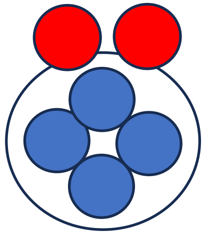
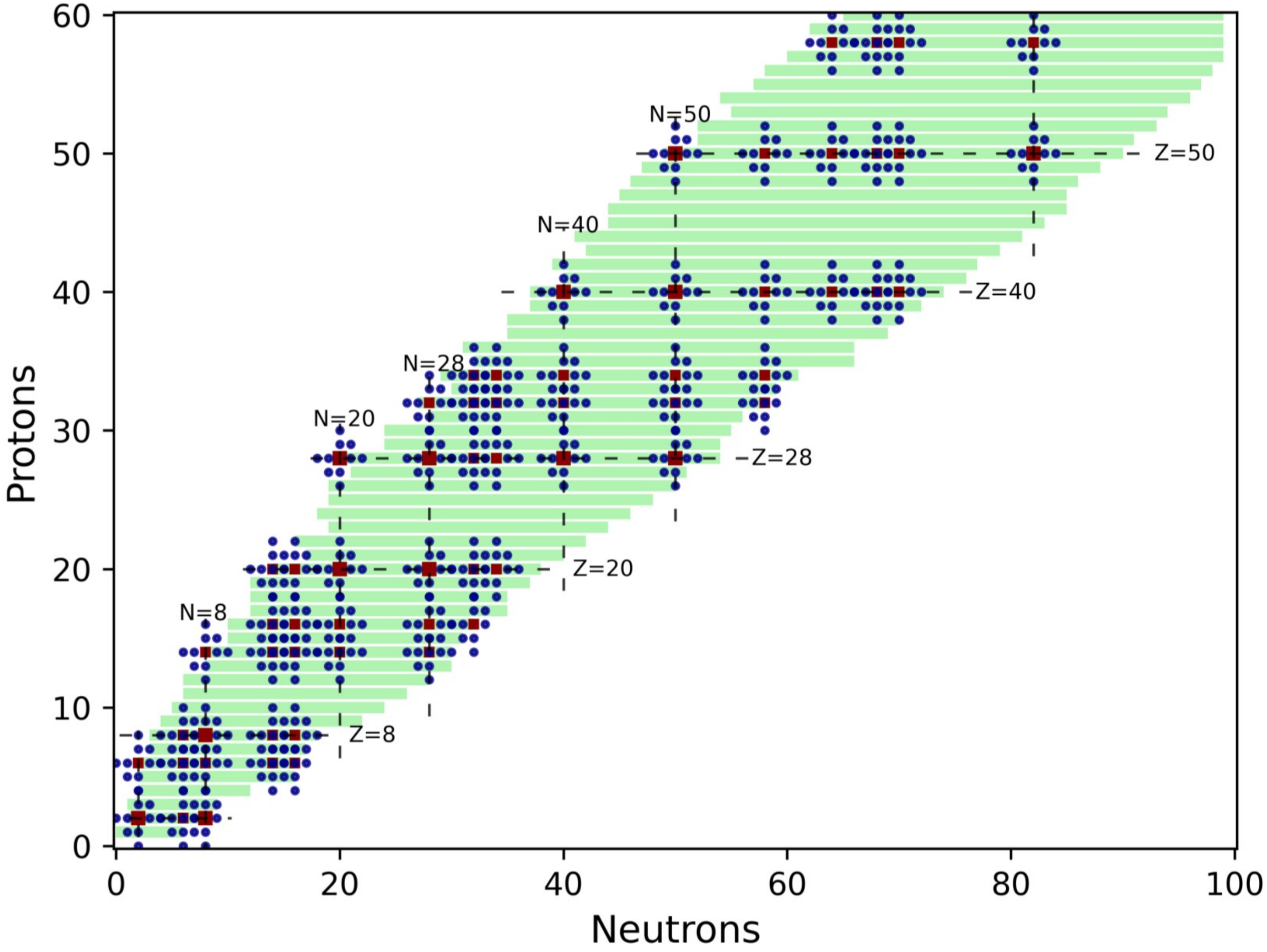


Two-particle-attached (2PA)

See **Francesca Bonaiti**'s talk

Two-particle-removed CC



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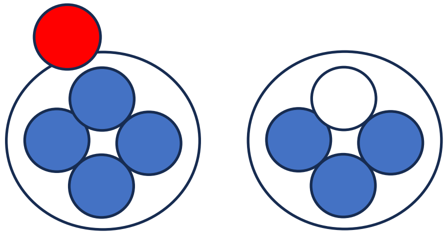

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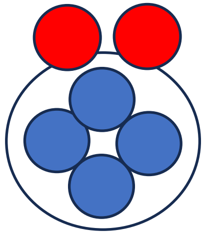

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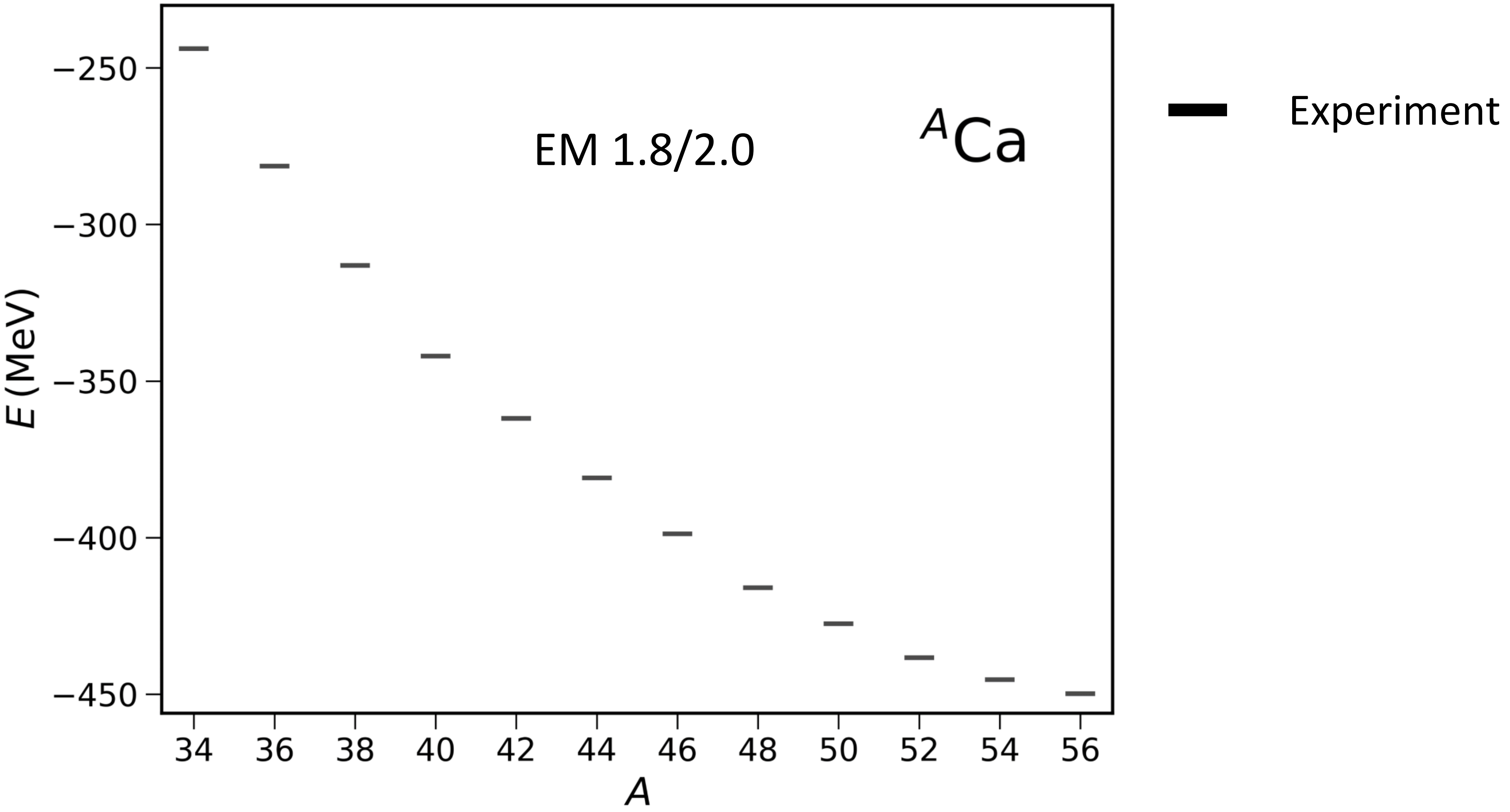
Binding energies in Ca chain



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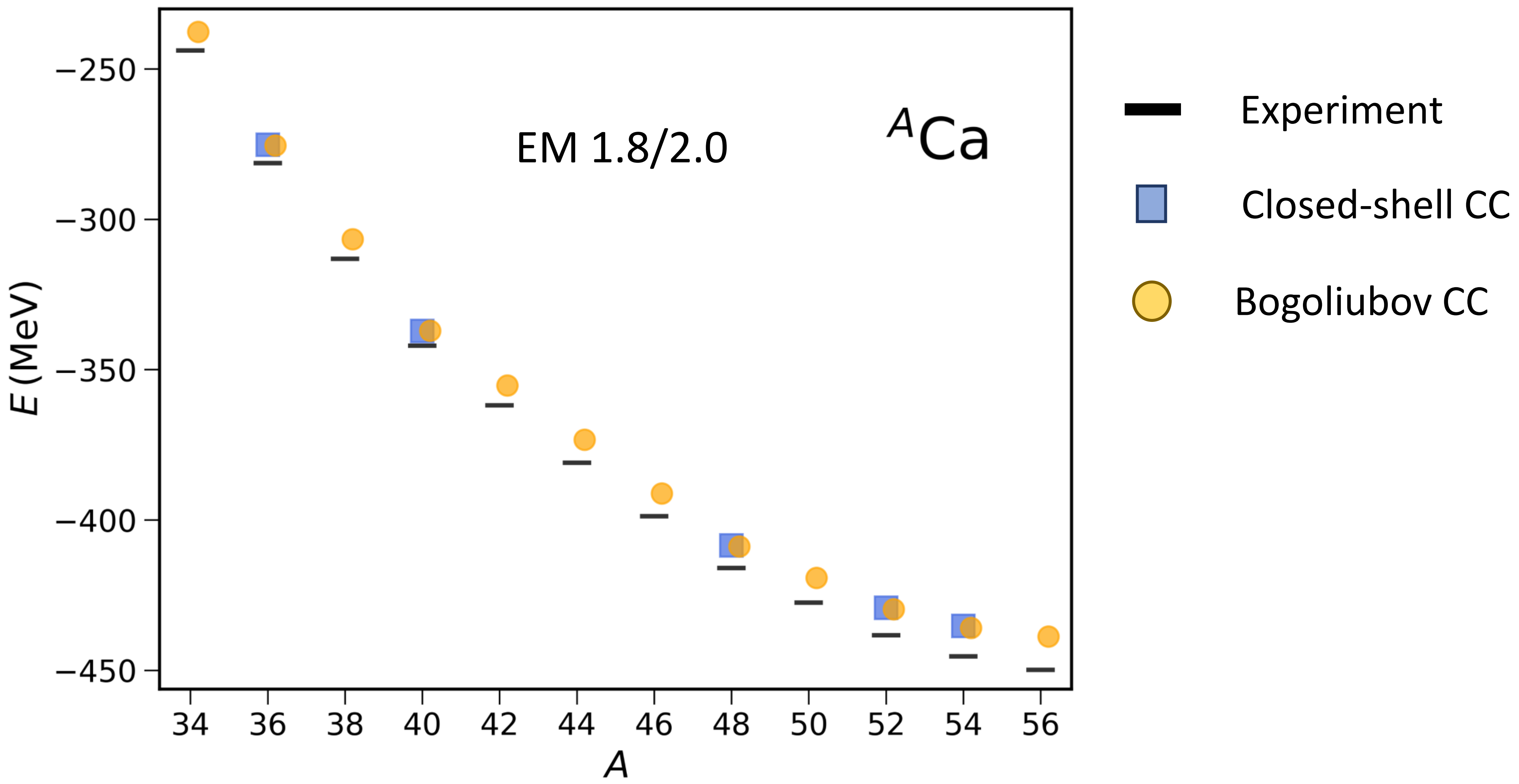

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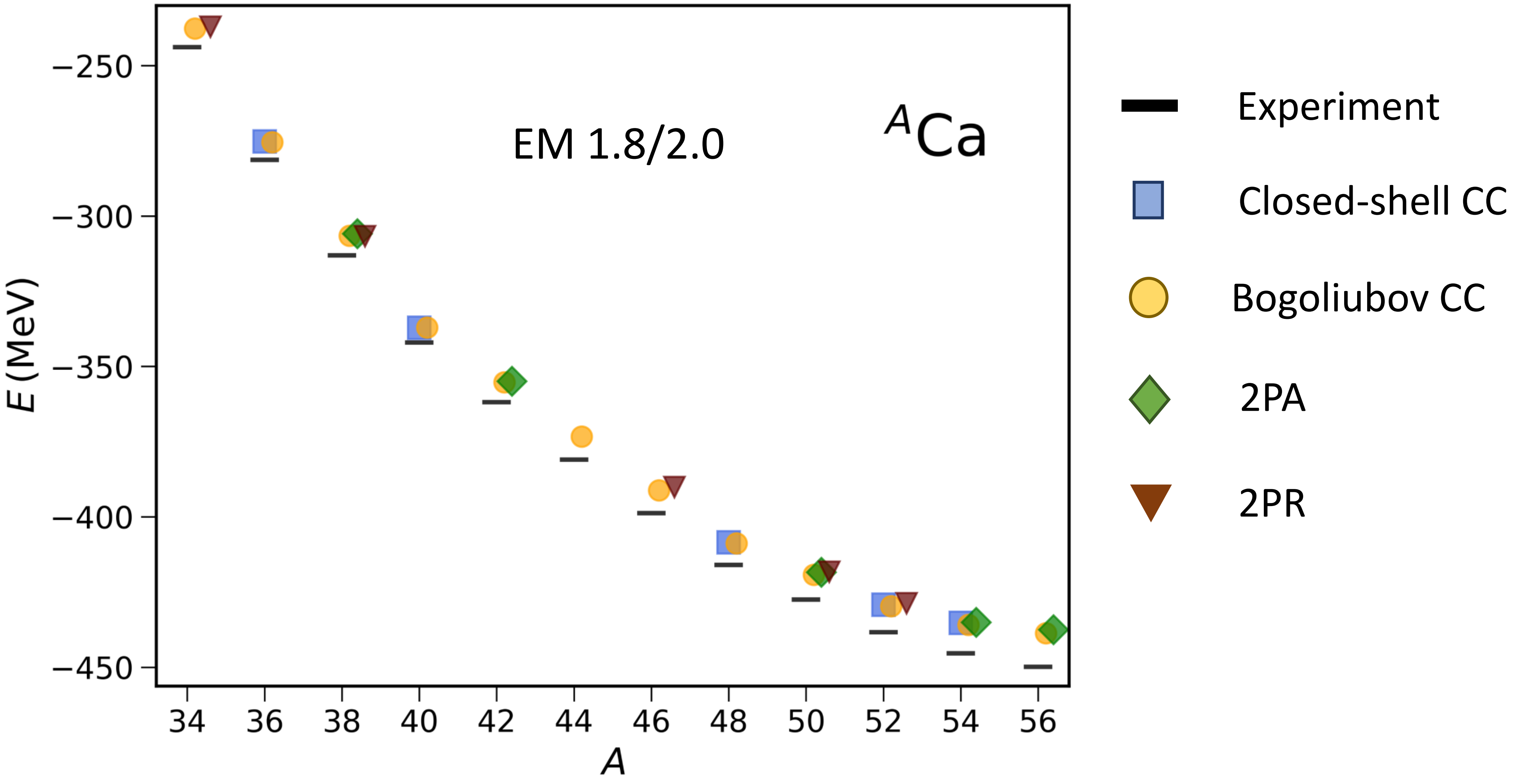

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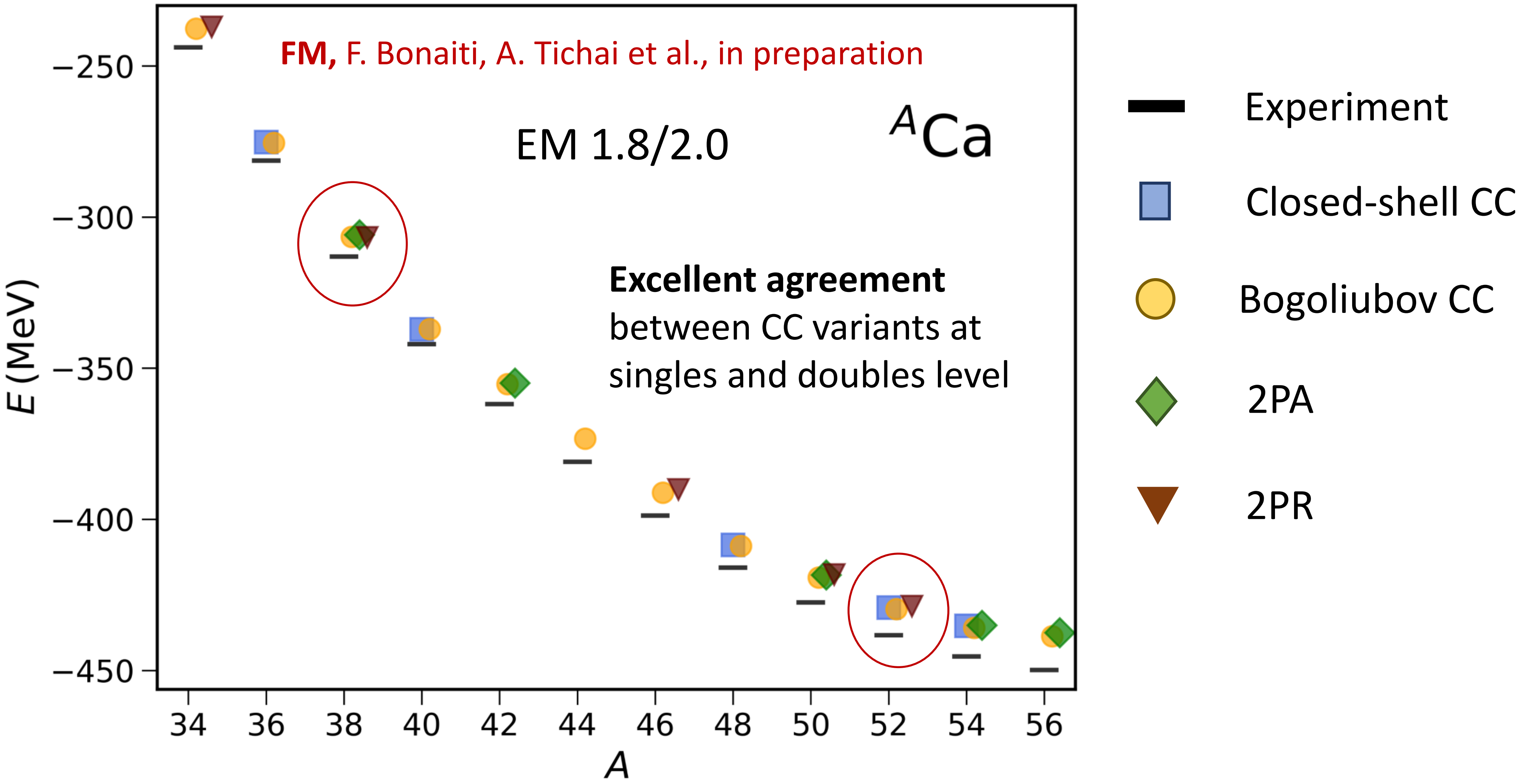

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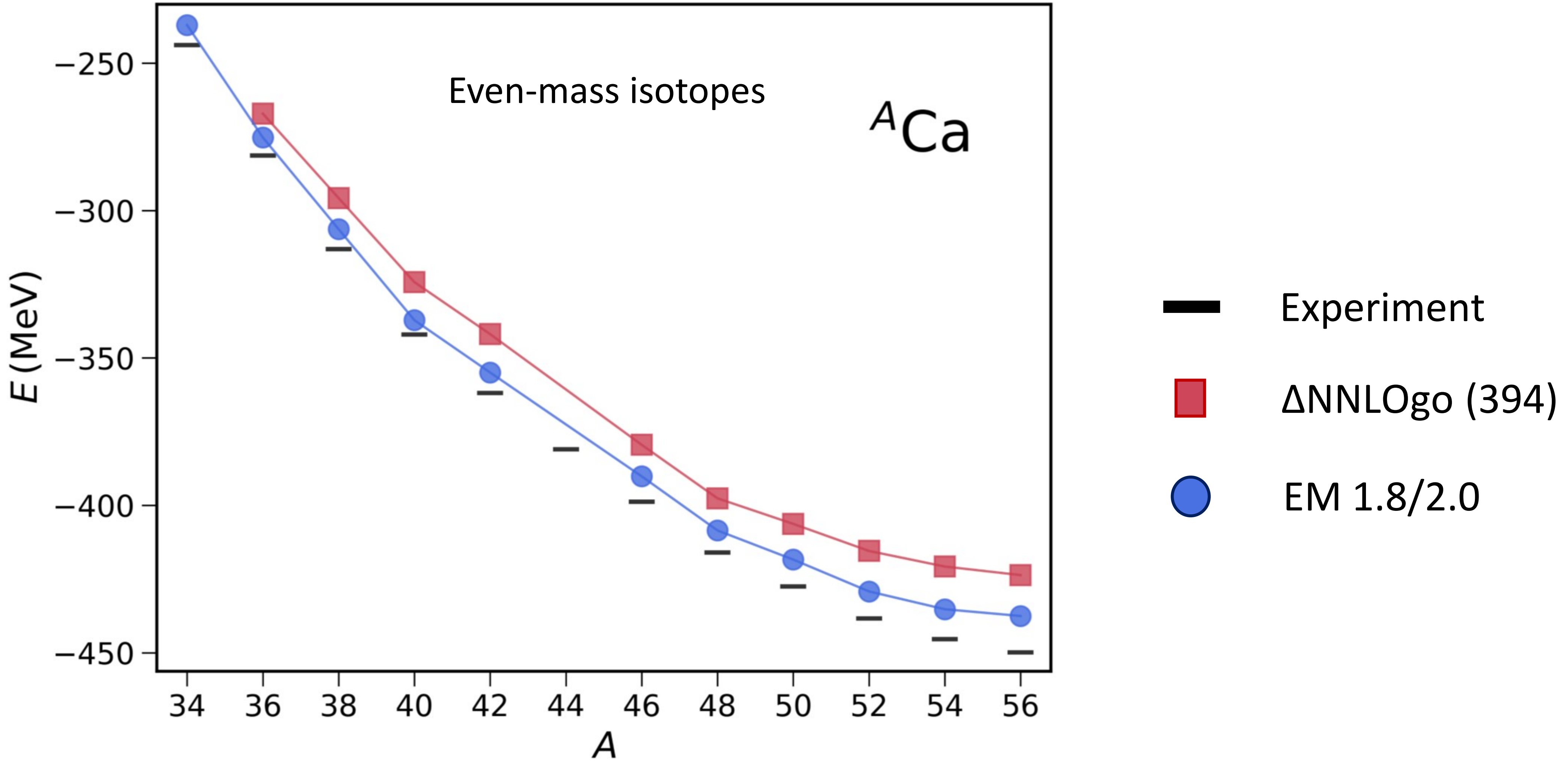

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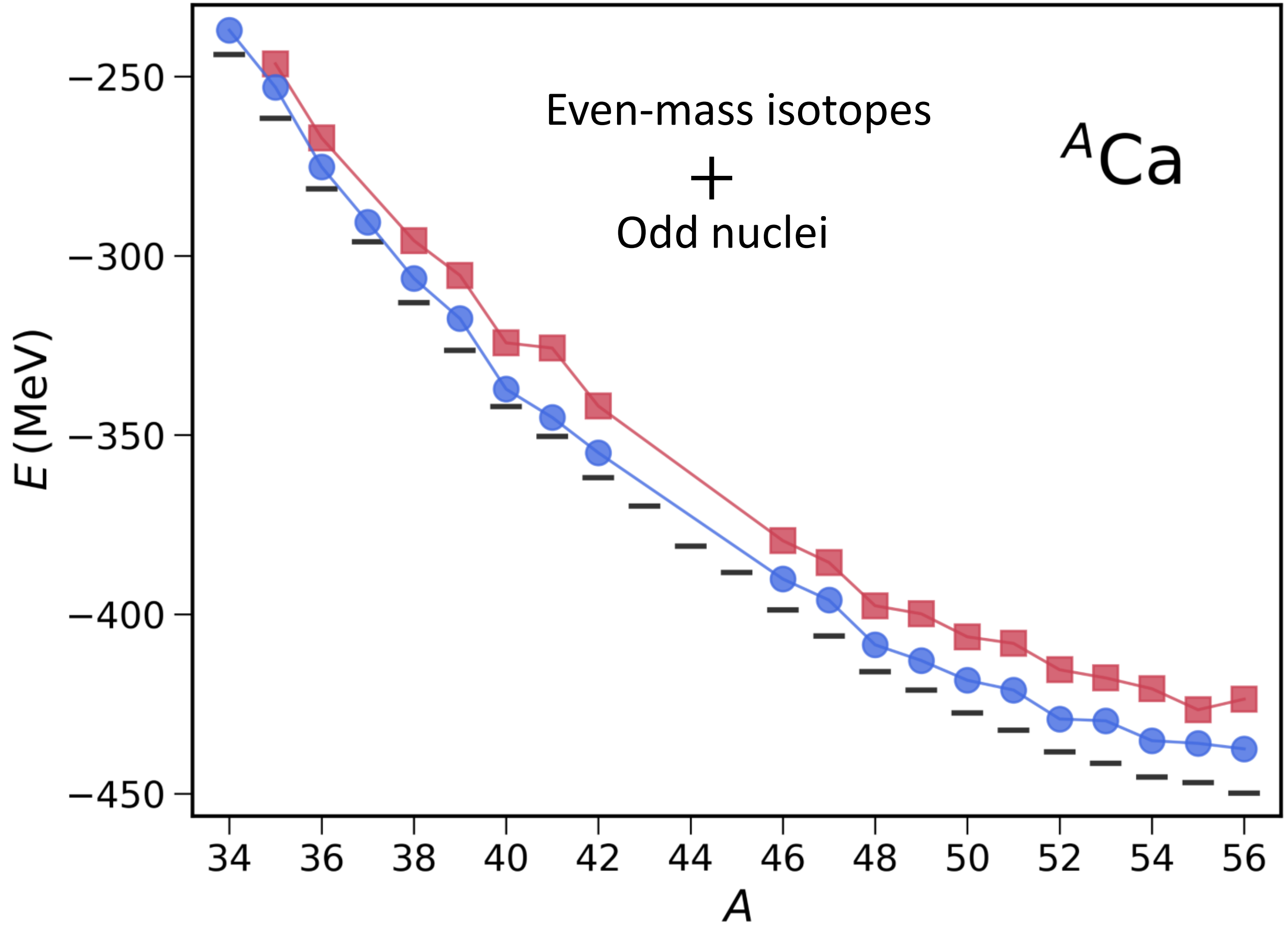
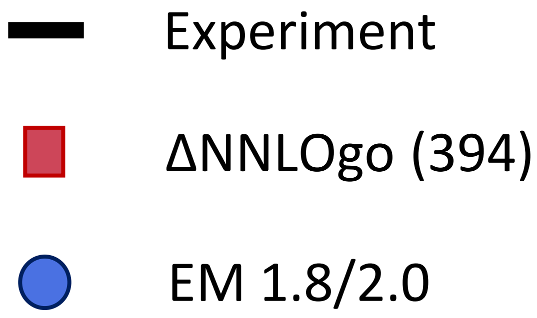


Comprehensive picture from **particle-attached/removed** coupled-cluster



Eletric dipole polarizability

Response function

$$R(\omega) = \sum_f \left|\langle \Psi_f | \Theta | \Psi_0 \rangle\right|^2 \delta(E_f - E_0 - \omega)$$

Θ: excitation operator

Electric dipole polarizability

$$\alpha_D = 2\alpha \int d\omega \frac{R(\omega)}{\omega}$$

See talks by **Francesca Bonaiti** and **Weiguang Jiang**

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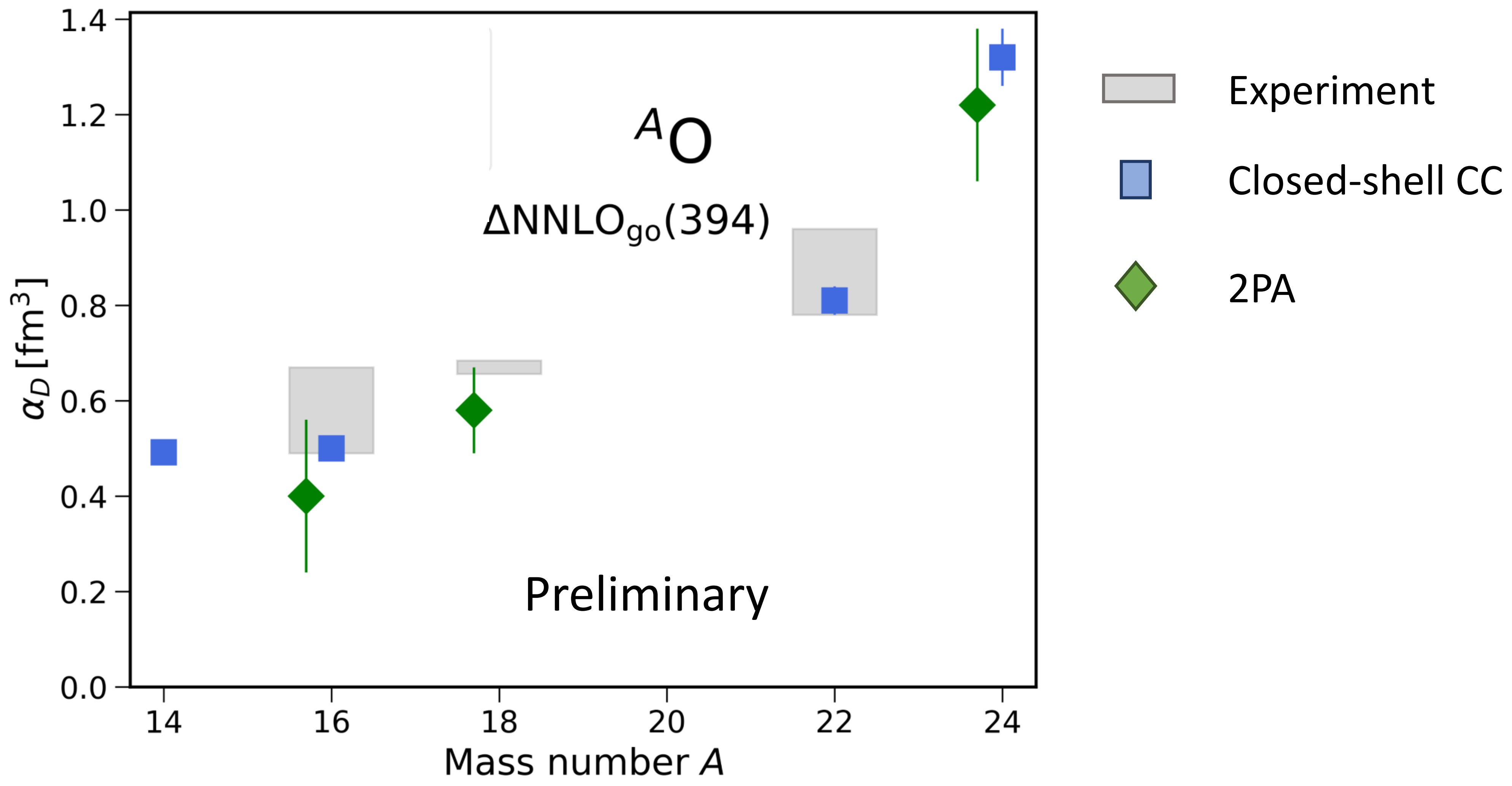
See talks by **Francesca Bonaiti** and **Weiguang Jiang**

Strong linear **correlation** between α_D and the **slope** of the symmetry energy *L* → Constraints on neutron matter equation of state from dipole response of nuclei

Eletric dipole polarizability in O isotopes



Bonaiti et al., arXiv: 2405.05608 (2024)

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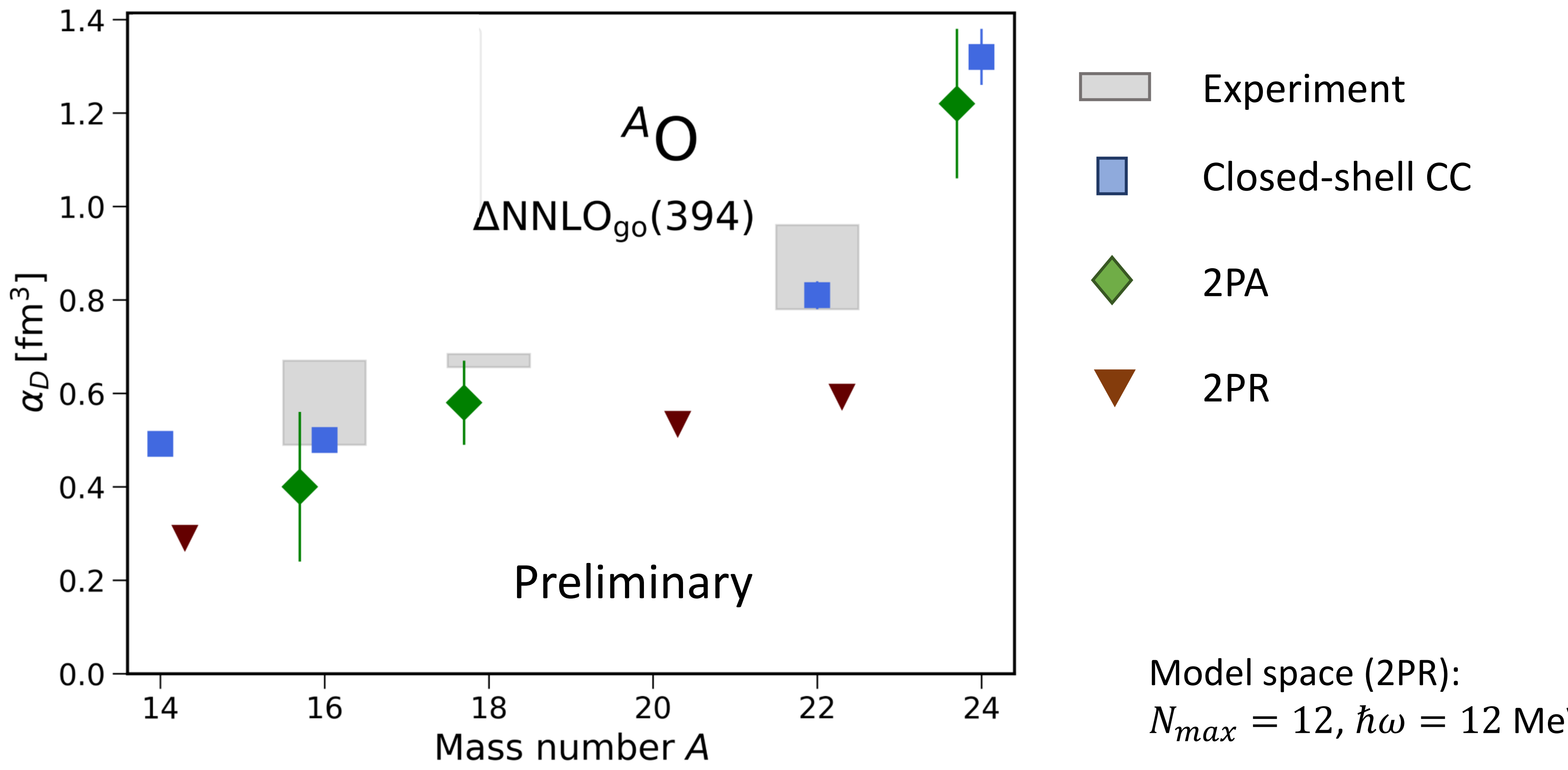
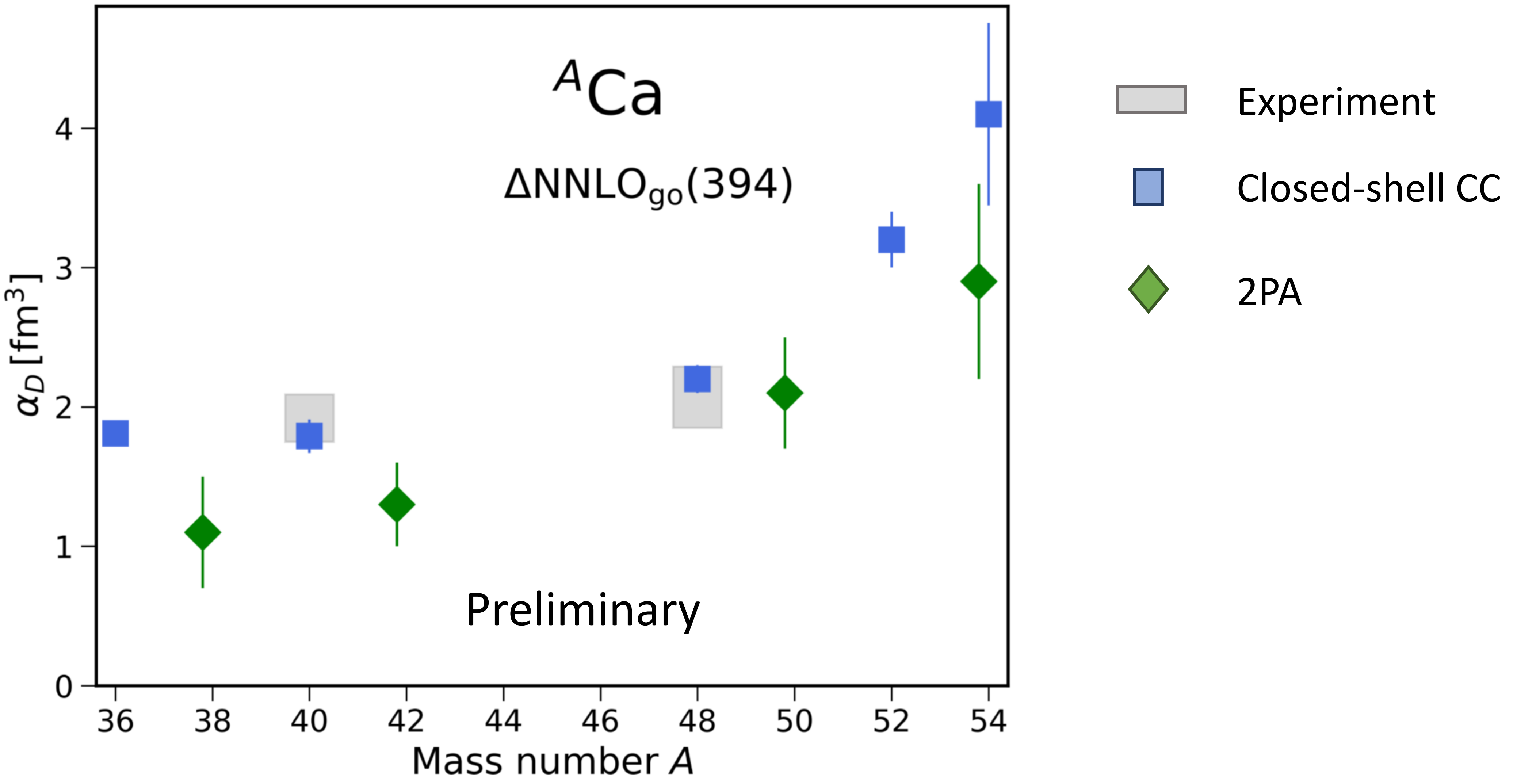


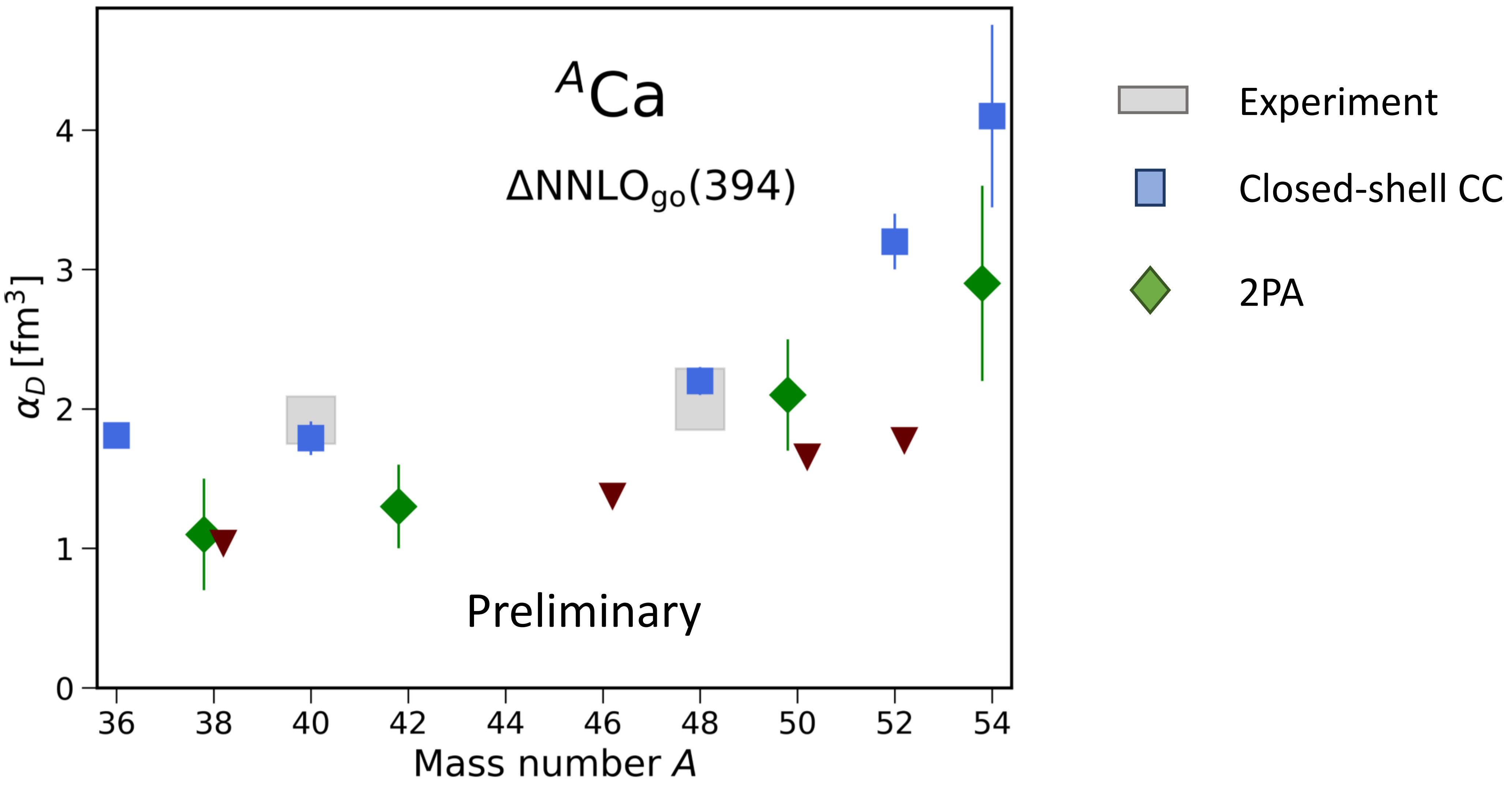
Model space (2PR):
$N_{max} = 12$, $\hbar\omega = 12$ MeV

Bonaiti et al., arXiv: 2405.05608 (2024)

Eletric dipole polarizability in Ca isotopes



Eletric dipole polarizability in Ca isotopes



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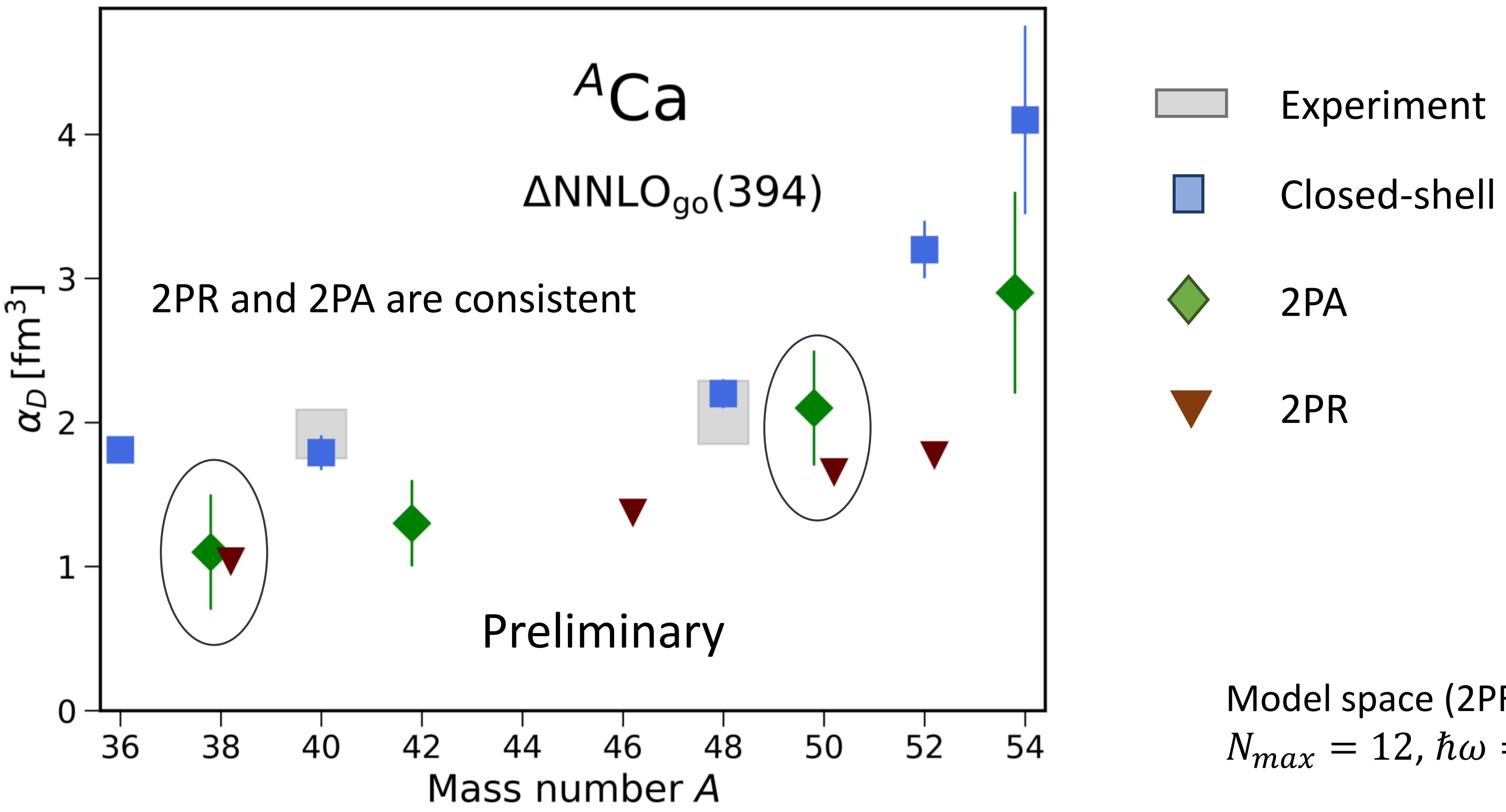

Model space (2PR):
$N_{max} = 12$, $\hbar\omega = 16$ MeV

Part 2:
Green’s functions for infinite nuclear matter

Infinite nuclear matter

Infinite nuclear matter is a homogeneous system of nucleons that interact through the strong interaction only

Equation of state (EOS): energy per particle $e = E/A$ as a function of the density ρ and isospin asymmetry $\beta = (\rho_n - \rho_p)/\rho$

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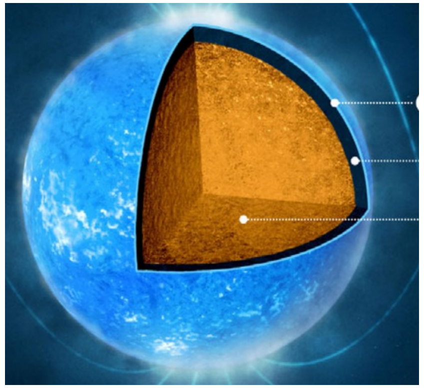

Watts et al., Rev. Mod. Phys. **88**, 021001 (2016)

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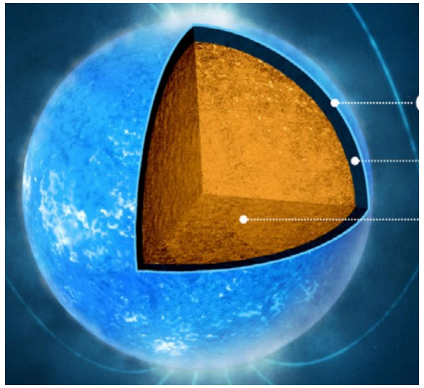
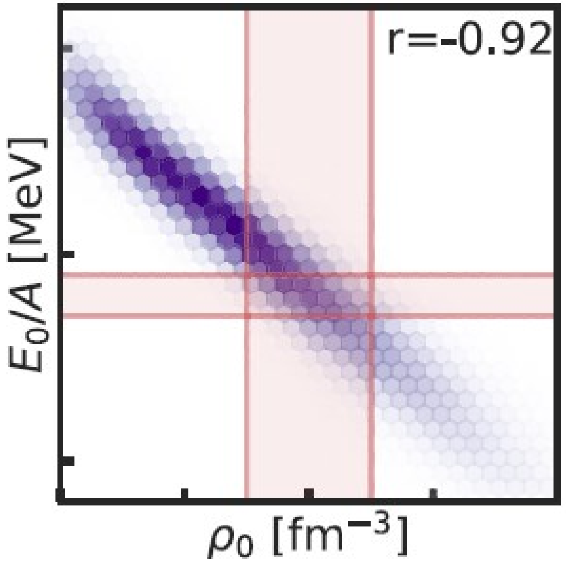
- Astrophysical implications
- Constraint for nuclear interactions



Watts et al., Rev. Mod. Phys. **88**, 021001 (2016)



Saturation density ρ_0

vs.

Saturation energy E_0/A

Jiang et al., Phys. Rev. C **109**, L061302 (2024)

Self-consistent Green's functions

Gorkov theory = Green's functions for **superfluid** systems

Somà et al., Phys. Rev. C **84**, 064317 (2011)
Barbieri et al., Phys. Rev. C **105**, 044330 (2022)

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Normal self-energy — Anomalous self-energy — Amplitudes — Excitation energies

$$\begin{pmatrix} T - \mu\mathbb{1} + \Sigma^{11}(\omega) & \Sigma^{12(\infty)} \\ (\Sigma^{12(\infty)})^\dagger & -(T - \mu\mathbb{1}) + \Sigma^{22}(\omega) \end{pmatrix} \Bigg|_{\omega=\omega_q} \begin{pmatrix} \mathcal{U}^q \\ \mathcal{V}^q \end{pmatrix} = \hbar\omega_q \begin{pmatrix} \mathcal{U}^q \\ \mathcal{V}^q \end{pmatrix}$$

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Propagator

$$g^{11}(\mathbf{k},\omega) = \sum_q \frac{|\mathcal{V}^q_{\mathbf{k}}|^2}{\hbar\omega-\hbar\omega_q-i\eta} + \frac{|\mathcal{U}^q_{\mathbf{k}}|^2}{\hbar\omega+\hbar\omega_q+i\eta}$$

→ Total energy

→ One-particle quantities

Algebraic diagrammatic construction

$$\Sigma^{11}(\omega) = \Sigma^{11(\infty)} + \tilde{\Sigma}(\omega)$$

Static self-energy ⇑ $\Sigma^{11(\infty)}$ Dynamical self-energy ⇑ $\tilde{\Sigma}(\omega)$

Self-consistent GF $\Sigma^{\star} = \Sigma^{\star}[g(\omega)]$

Raimondi et al., Phys Rev C **97**, 054308 (2018)
Barbieri et al., Phys. Rev. C **105**, 044330 (2022)

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$$\Sigma^{11}(\omega) = \Sigma^{11(\infty)} + \tilde{\Sigma}(\omega)$$

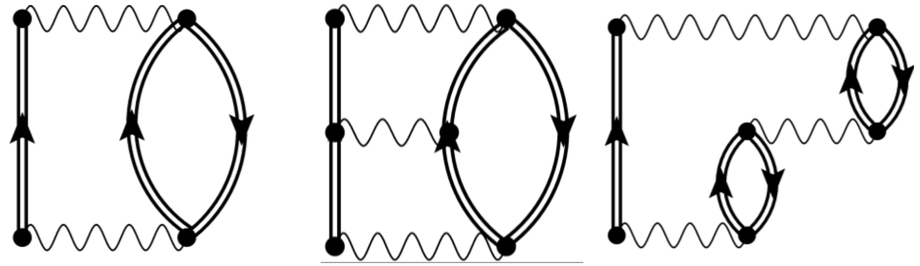
Static self-energy   Dynamical self-energy

Self-consistent GF $\Sigma^{\star} = \Sigma^{\star}[g(\omega)]$

Algebraic diagrammatic construction (ADC)

ADC(3) includes third-order perturbation theory, ladders, rings…

ADC(2)   ADC(3)



Ladder   Ring

Raimondi et al., Phys Rev C **97**, 054308 (2018)
Barbieri et al., Phys. Rev. C **105**, 044330 (2022)

ADC + coupled-cluster amplitudes

Extension of ADC(3) ⟹ **ADC(3)-D**

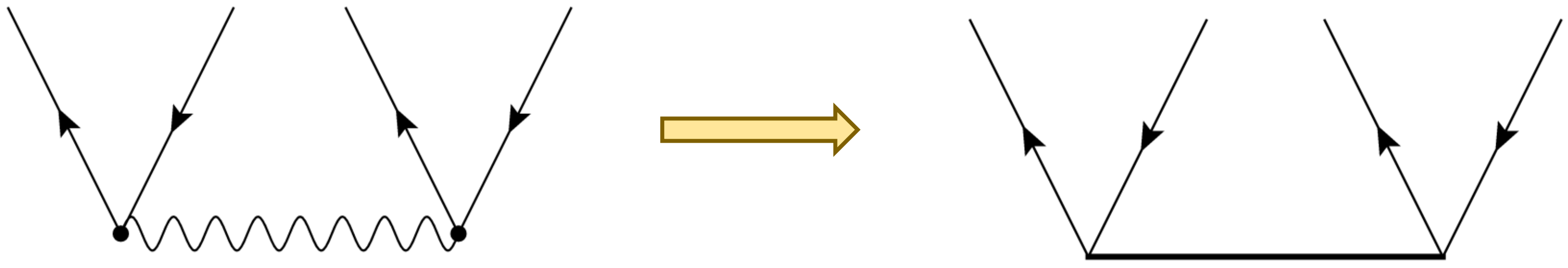
ADC + coupled-cluster amplitudes

Extension of ADC(3) → **ADC(3)-D**

In $\tilde{\Sigma}(\omega)$, replace $t^{(0)}$ with converged $\boldsymbol{T_2}$ **amplitudes**

$$(t^{(0)})_{ij}^{ab} = \frac{\langle ab|v|ij\rangle_A}{\epsilon_i + \epsilon_j - \epsilon_a - \epsilon_b}$$

$t^{(0)}$ → $t^{(CCD)}$



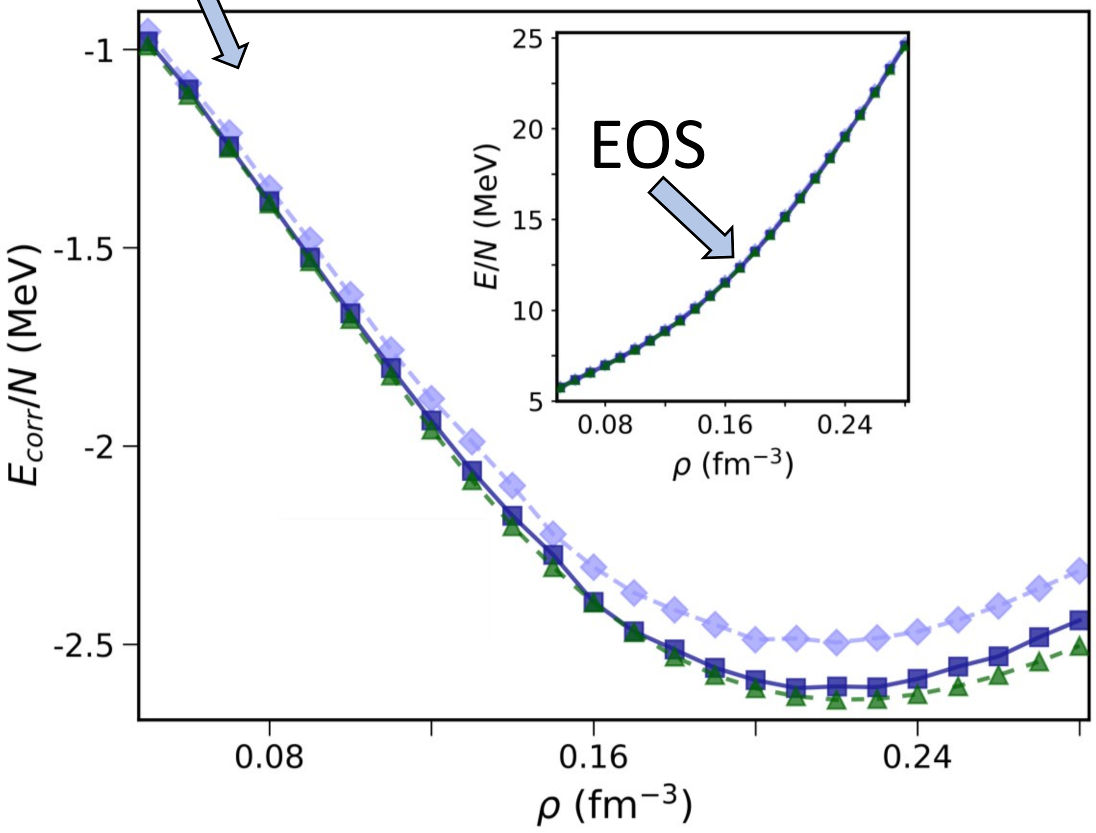
Barbieri et al., Lect. Notes Phys. **936**, 571 (2017)
FM et al., arXiv:2407.17098

Equation of state

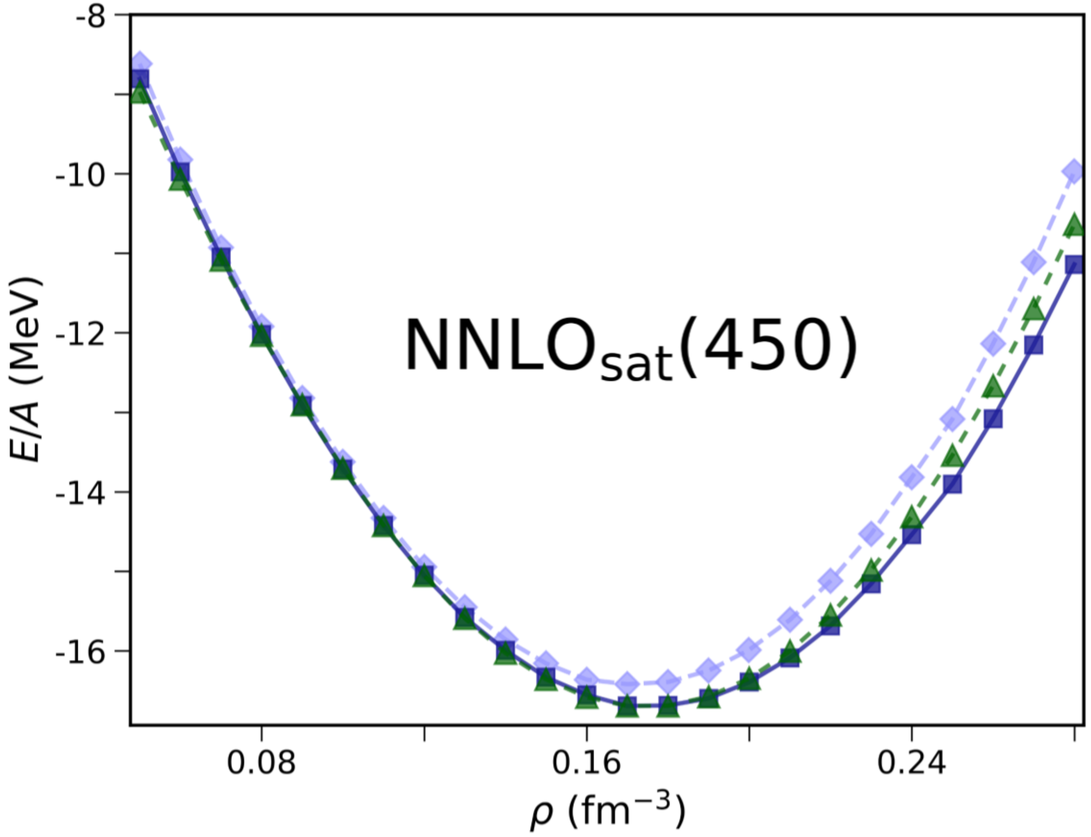
Correlation energy
$E_{corr} = E - E_{HF}$

ADC(3)
ADC(3)-D
CCD(T)

Pure neutron matter

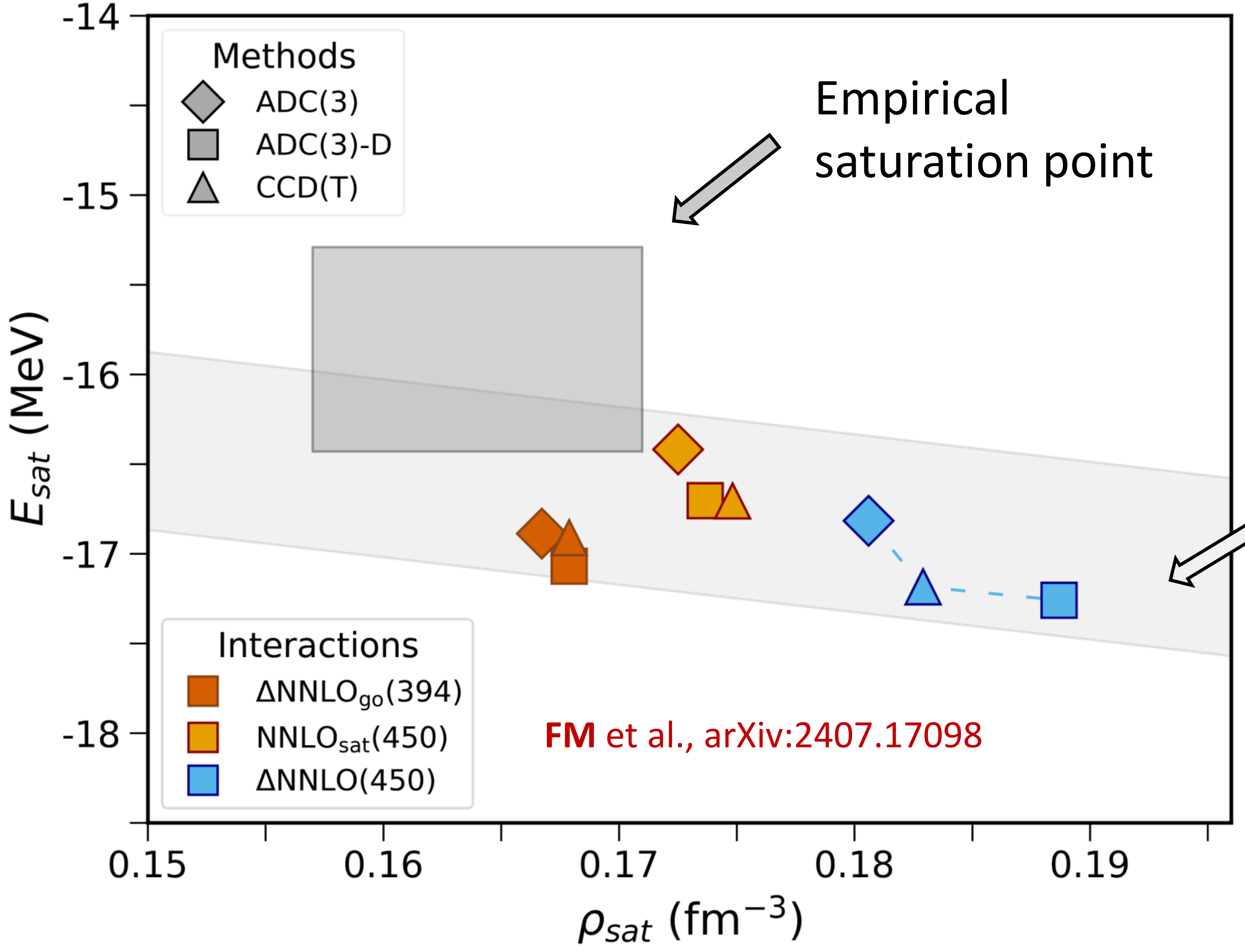


Symmetric nuclear matter



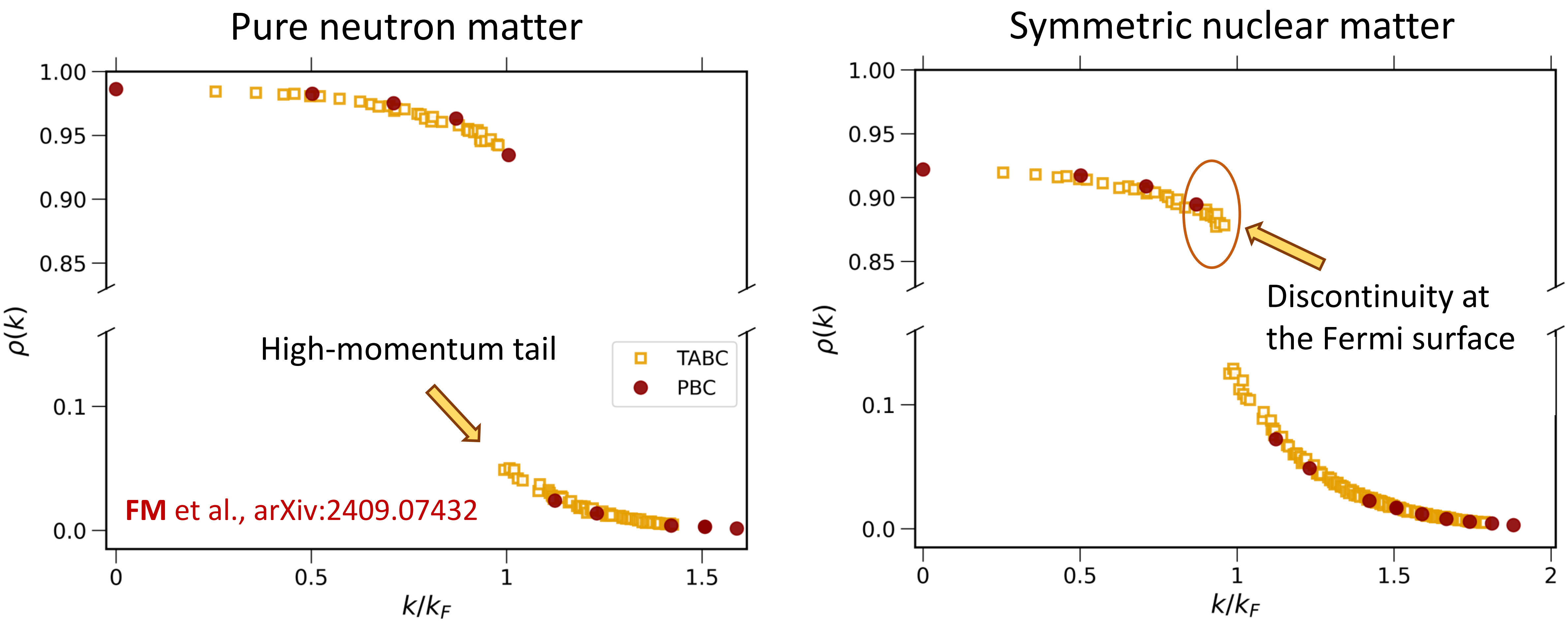
FM et al., arXiv:2407.17098

Saturation point of symmetric nuclear matter



Linear correlation between saturation density and energy

Momentum distributions

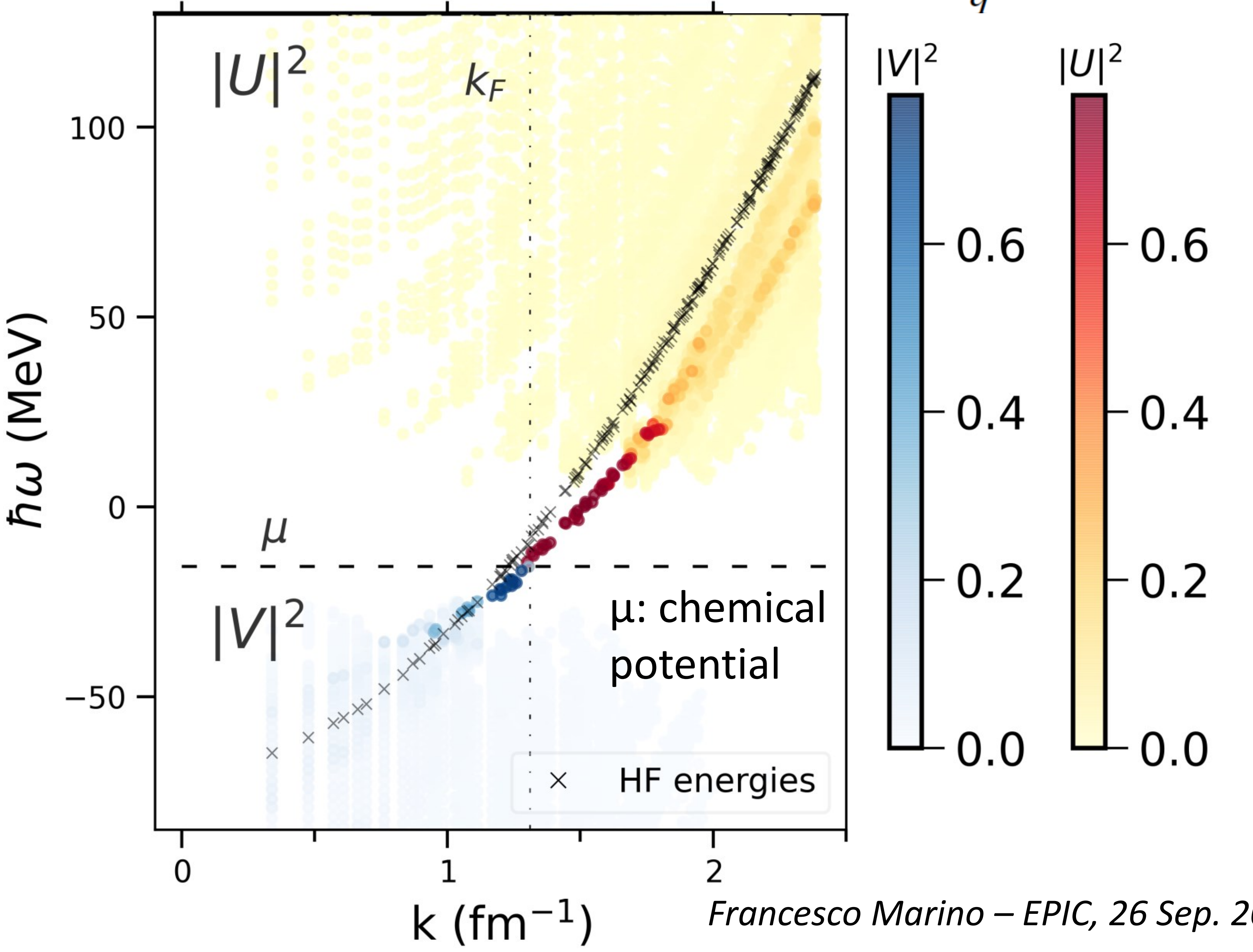


ΔNNLO$_{go}$(450) interaction
$\rho = 0.16\ \mathrm{fm}^{-3}$

Spectral functions

FM et al., arXiv:2409.07432

$$S(\mathbf{k},\omega) = \sum_q \left[\left|\mathcal{V}_{\mathbf{k}}^q\right|^2 \delta(\hbar\omega + \hbar\omega_q) + \left|\mathcal{U}_{\mathbf{k}}^q\right|^2 \delta(\hbar\omega - \hbar\omega_q) \right]$$



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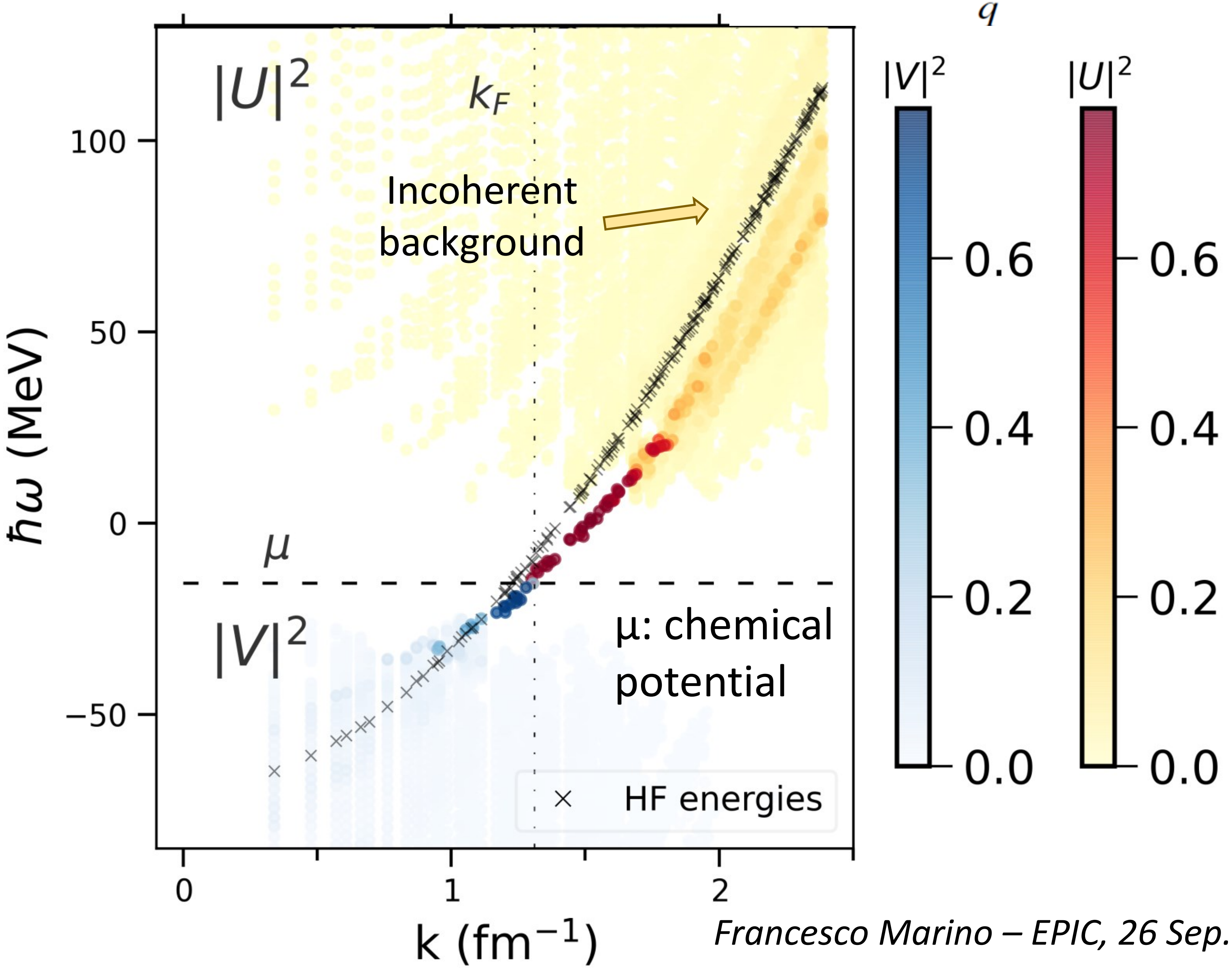

Fragmentation of the single-particle strength

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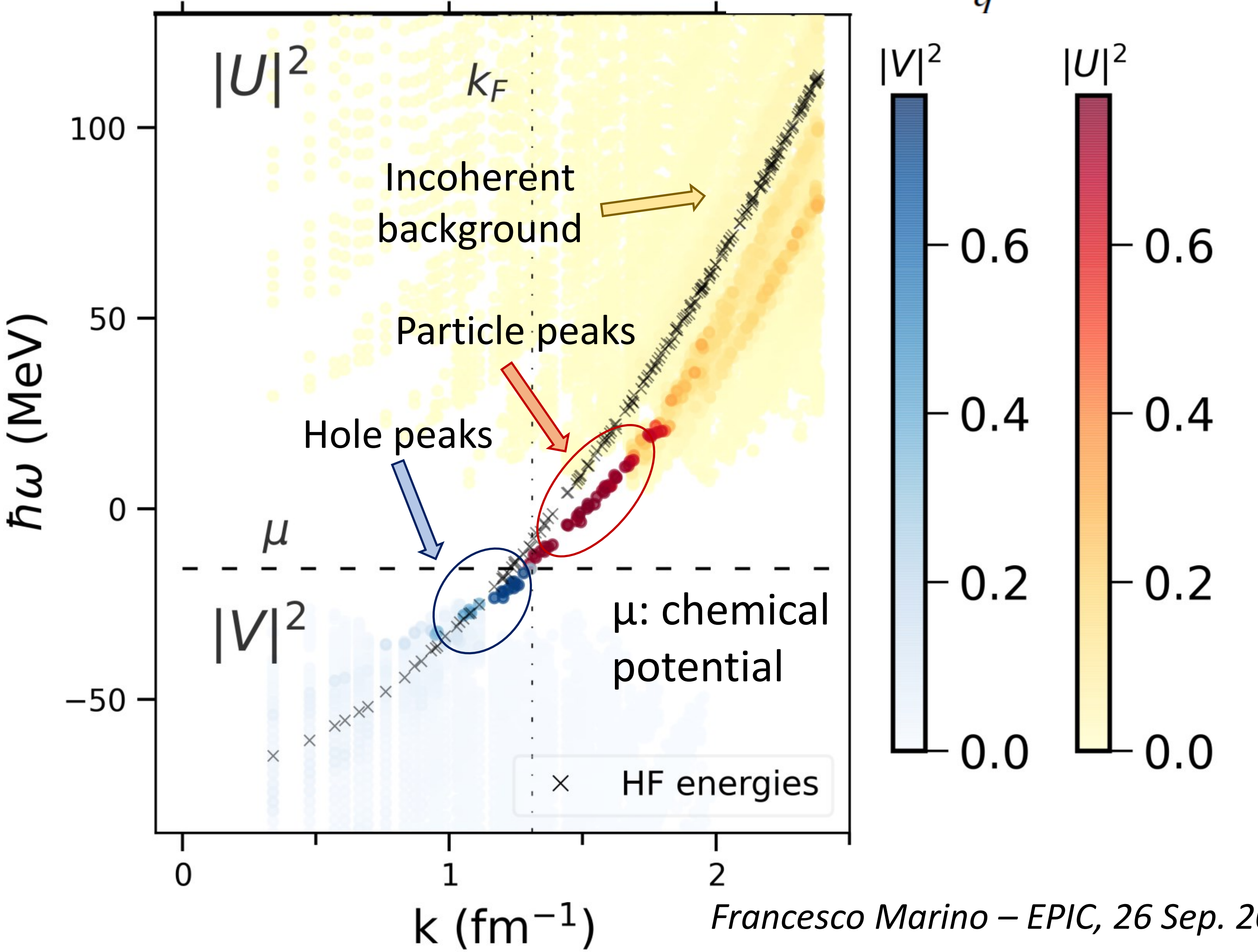

Fragmentation of the single-particle strength

Quasi-particle excitations at the Fermi surface

Symmetric nuclear matter
ΔNNLO$_{go}$(450) interaction
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Conclusions and perspectives

- Coupled-cluster has been used to study the ground state and electric dipole polarizability of **open-shell** nuclei

- Ongoing: dipole polarizability of open-shell nuclei; binding energies and radii with higher-order CC truncations

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- Coupled-cluster has been used to study the ground state and electric dipole polarizability of **open-shell** nuclei

- Ongoing: dipole polarizability of open-shell nuclei; binding energies and radii with higher-order CC truncations

- The **ADC(3) Green's functions** method is a powerful ab initio tool to study the EOS and single-particle properties of **nuclear matter**

- Future developments: low-density neutron matter, quasi-particle properties …

Questions

How can we improve theoretical predictions of the electric dipole polarizability?

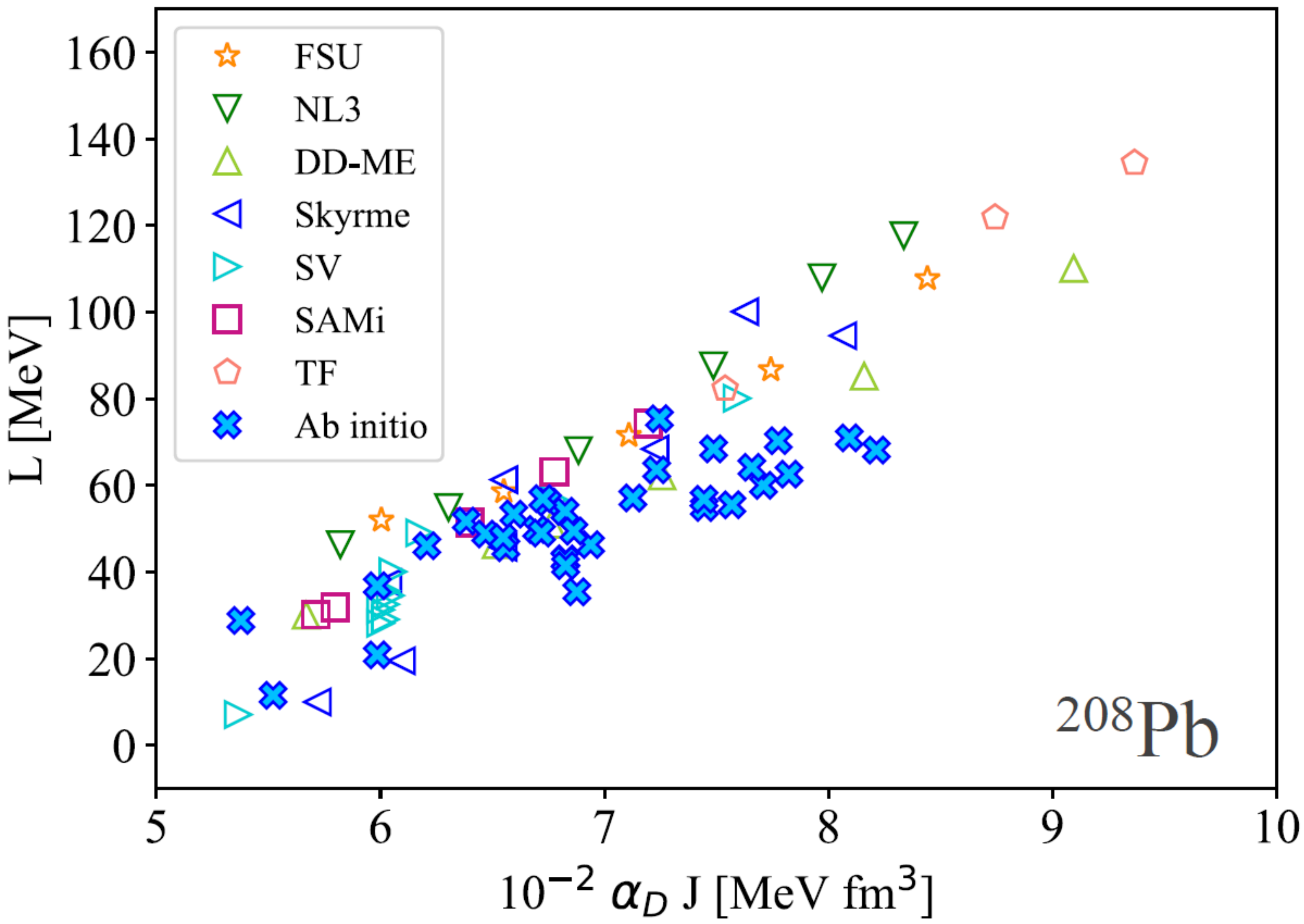
How can we connect nuclear matter quantities to finite nuclei?

Thank you for your attention!

Collaborators

Darmstadt:	Alex Tichai
Mainz:	Sonia Bacca, Francesca Bonaiti, Weiguang Jiang
Milano:	Carlo Barbieri, Gianluca Colò
Oak Ridge:	Gaute Hagen, Gustav Jansen
St. Louis:	Sam Novario

Eletric dipole polarizability

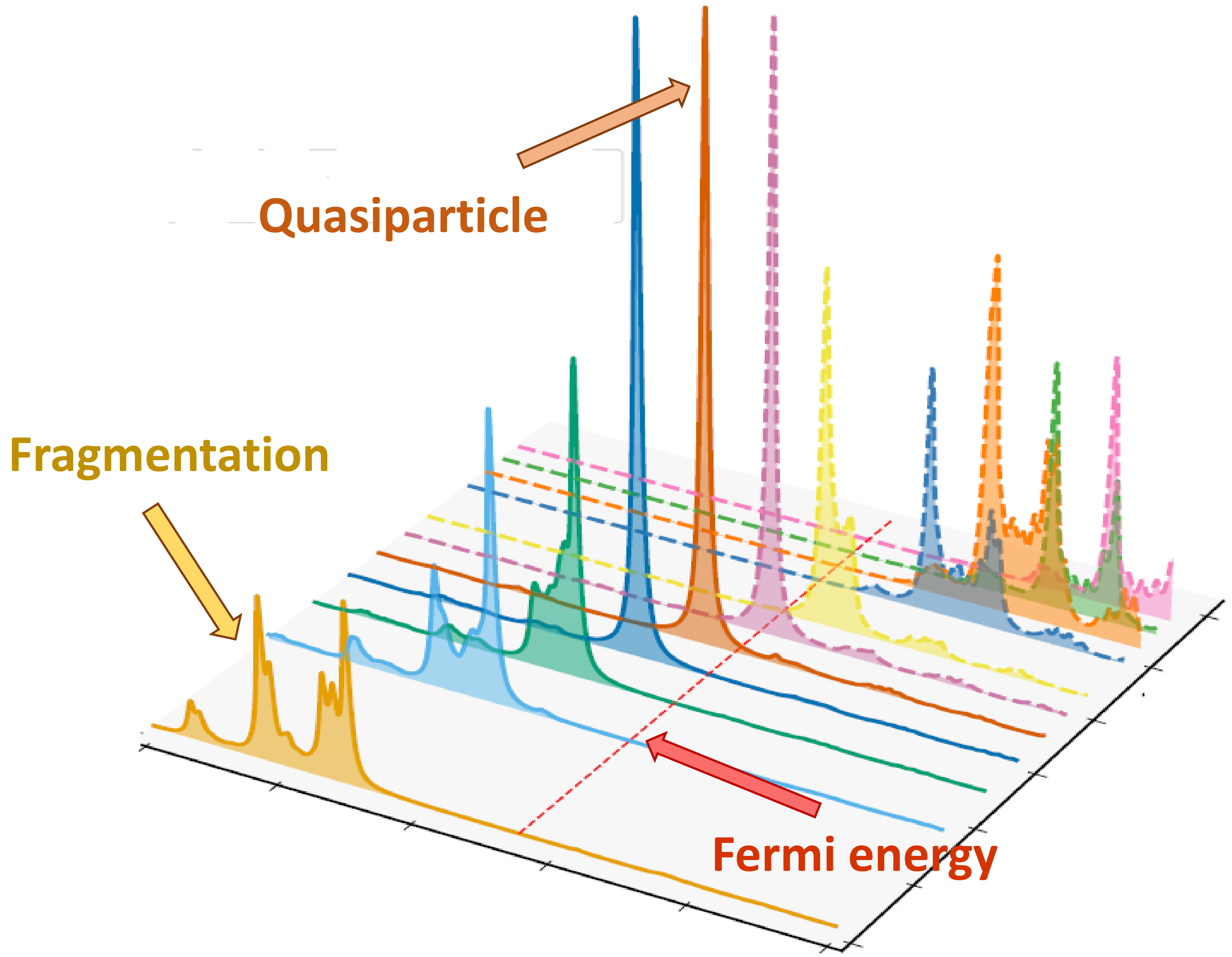


DFT data from:
Roca-Maza, Phys. Rev. C **88**, 024316 (2013)

Ab initio data from:
Hu, Nat. Phys. **18**, 1196-1200 (2022)

Courtesy: Francesca Bonaiti

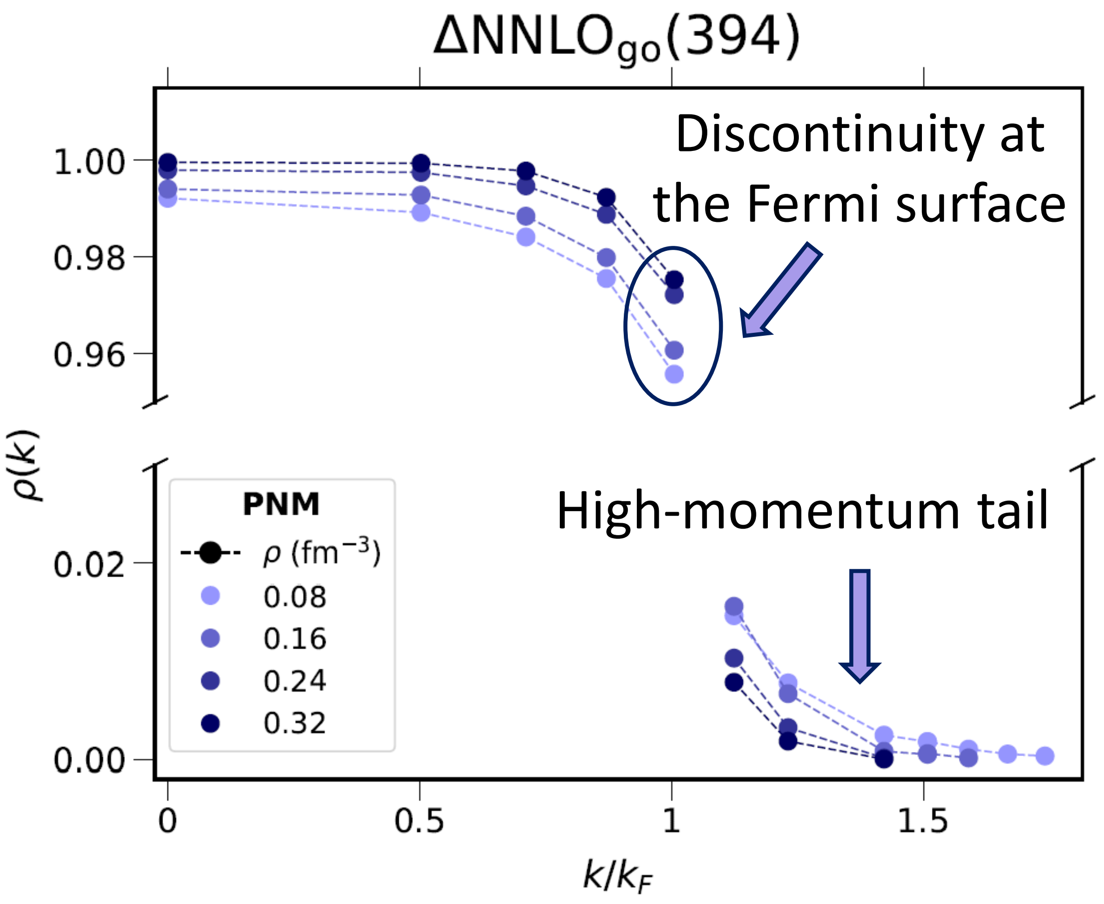
Spectral functions – neutron matter

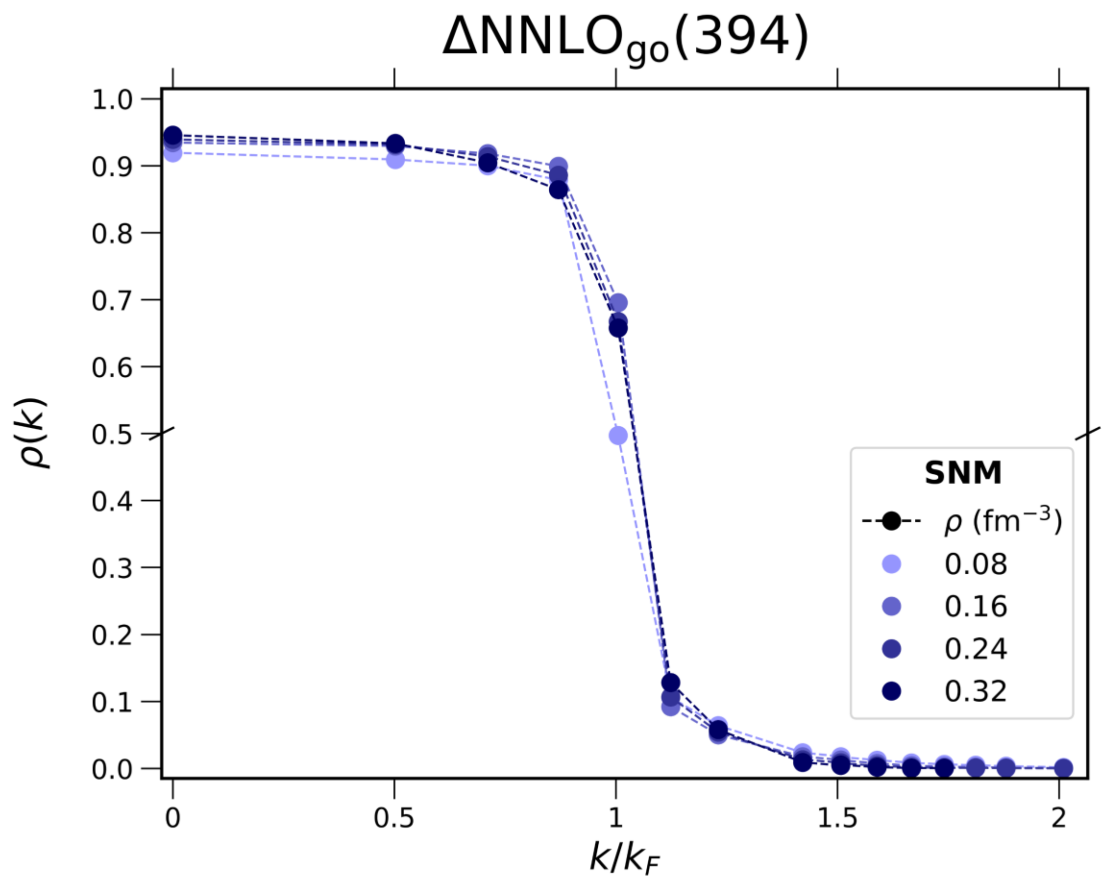


Spectral function

$$A(\boldsymbol{k},\omega) = \sum_j |Z_j|^2 \delta(\omega - \epsilon_j)$$

Momentum distributions





Preliminary!