Electroweak constraints for nucleonic matter

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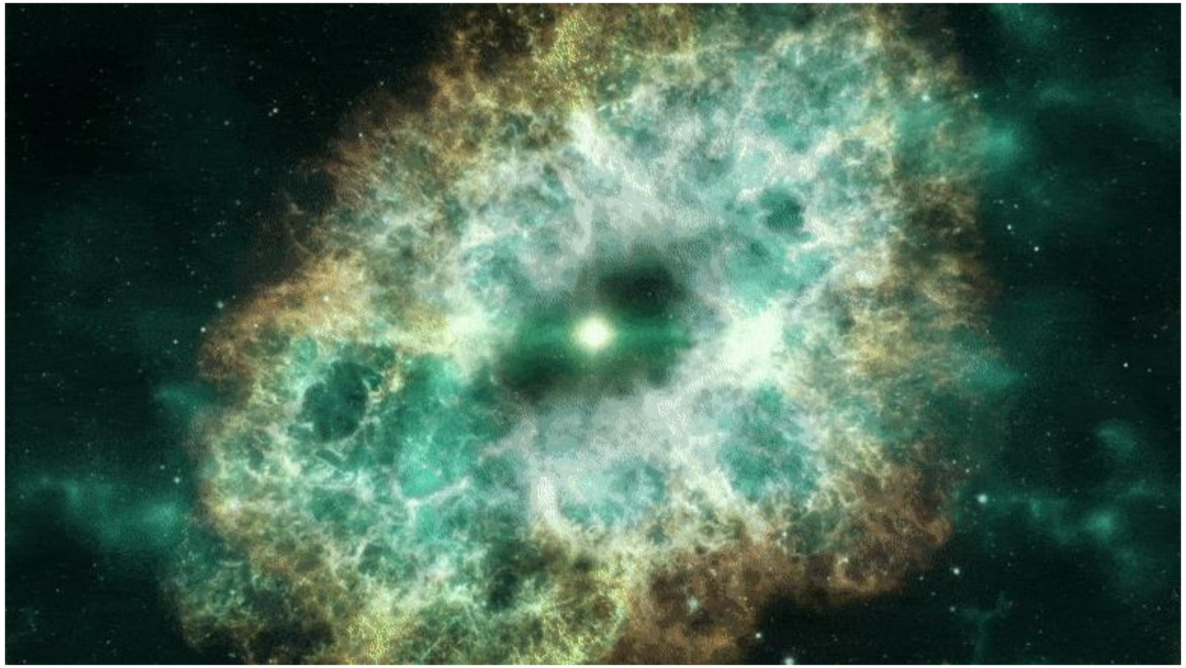
Outline

- Neutrino interactions with finite nuclei and matter

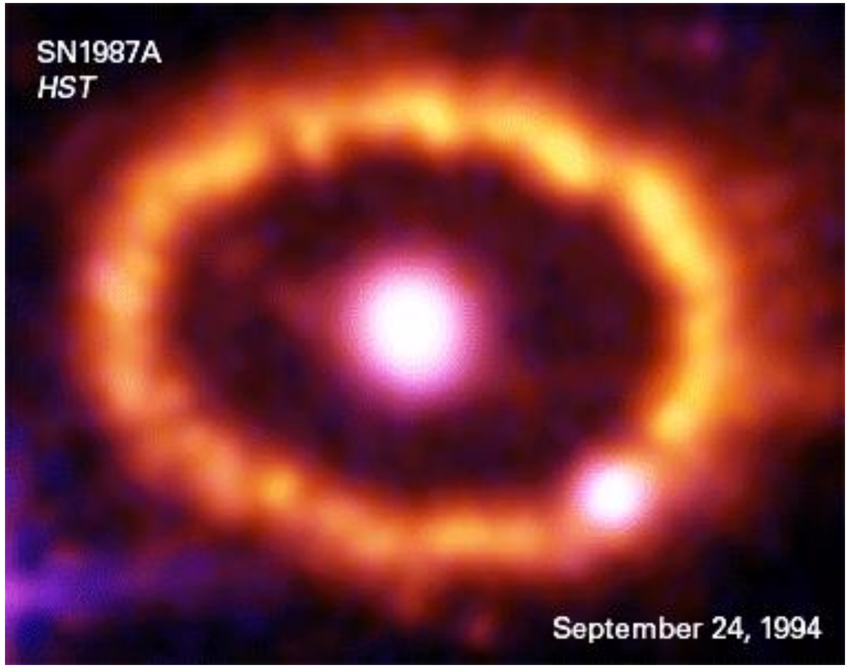
- Data-driven AI analysis of dipole strength functions

Motivation – neutrino, nucleon and supernova

supernova explosion animation



Hubble space telescope (X-rays)



Neutrino plays an important role in astrophysics, particularly in the dynamics of core-collapse supernovas.

During a supernova explosion, the collapsing core of a massive star releases an immense flux of neutrinos (10^{58}).

It is crucial to understand how neutrinos interact with nucleonic matter.

SN1987A is the first supernova (core-collapse supernova) that modern astronomers were able to study in detail. Also, it is the first time neutrinos are known to be emitted from a supernova.

Motivation - neutrinos interact with nucleonic matter

Differential cross section:

$$\frac{d^2\sigma}{dE'd\Omega'} = \frac{1}{16\pi^2}\frac{G^2}{2}L_{\mu\nu}R^{\mu\nu}$$

Leptonic tensor:

$$L_{\mu\nu} = 8\left[k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu}k\cdot k' \pm i\epsilon_{\mu\nu\alpha\beta}k'^\alpha k^\beta\right]$$

Hadronic Tensor:

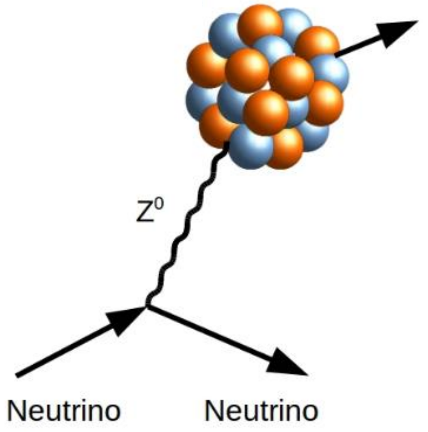
$$R^{\mu\nu} = \sum\!\!\!\!\!\!\!\int_f \langle 0|J^{\mu\dagger}(q)|f\rangle\langle f|J^\nu(q)|0\rangle\,\delta(E_0+\omega-E_f)$$

response function

weak current operator

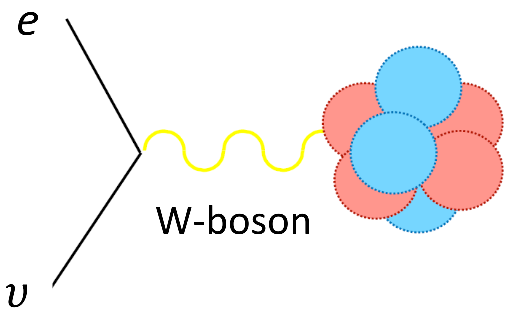
nucleonic system (g.s.)

scattering of neutrinos off nucleonic system



$$\nu + X \rightarrow \nu + X^*$$

$$\nu + {}_zX \rightarrow e^- + {}_{z+1}X^*$$



^{16}O electroweak response

See G. Hagen's talk

charge current neutrino scattering

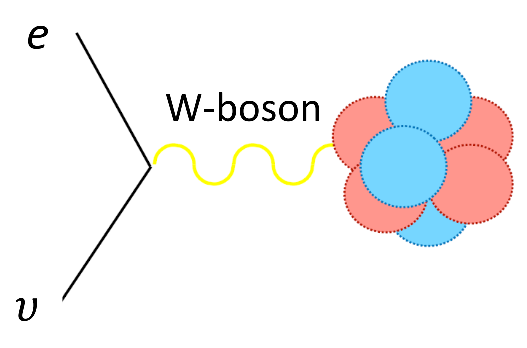
charge-changing weak current

spacelike component

$$\mathbf{j}_{\alpha}^{5(\pm)} = -G_A(Q^2)\left(\boldsymbol{\sigma}_j - \frac{\mathbf{q}\,\boldsymbol{\sigma}_j\cdot\mathbf{q}}{m_\pi^2+q^2}\right)e^{i\mathbf{q}\cdot\mathbf{r}_j}\,\frac{\tau_{j,\pm}}{2}$$

timelike (axial charge) component

$$j_0^5 = -G_A(Q^2)\left(\boldsymbol{\sigma}_j\cdot\frac{\bar{\mathbf{p}}_j}{m} - \frac{\omega\,\boldsymbol{\sigma}_j\cdot\mathbf{q}}{m_\pi^2+q^2}\right)e^{i\mathbf{q}\cdot\mathbf{r}_j}\,\frac{\tau_{j,\pm}}{2}$$



$$\nu + {}_{Z}X \rightarrow e^- + {}_{Z+1}X^*$$

Acharya, Sobcyzk, Bacca, Hagen, Jiang (2024)

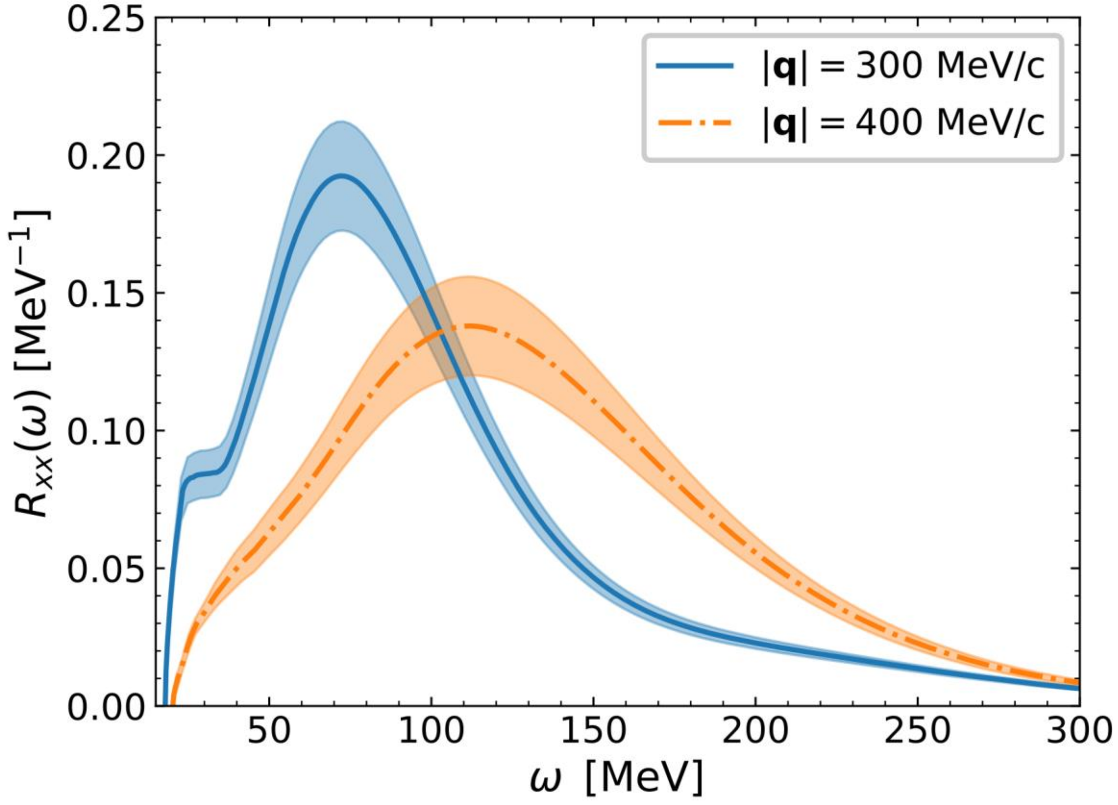
EW responses for finite nuclei with LIT-CC

See G. Hagen's talk

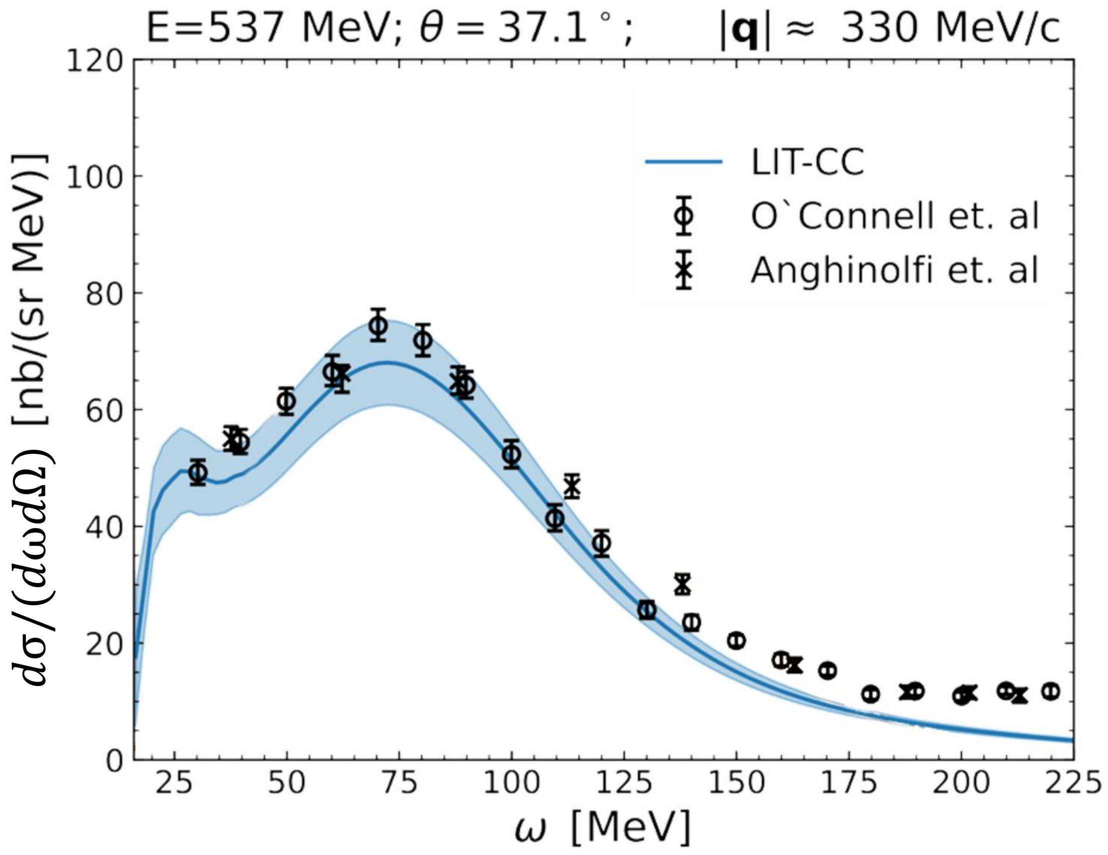
Acharya, Sobcyzk, Bacca, Hagen, Jiang (2024)

$^{16}O(\nu_e,e)X$

One of the 5 response functions

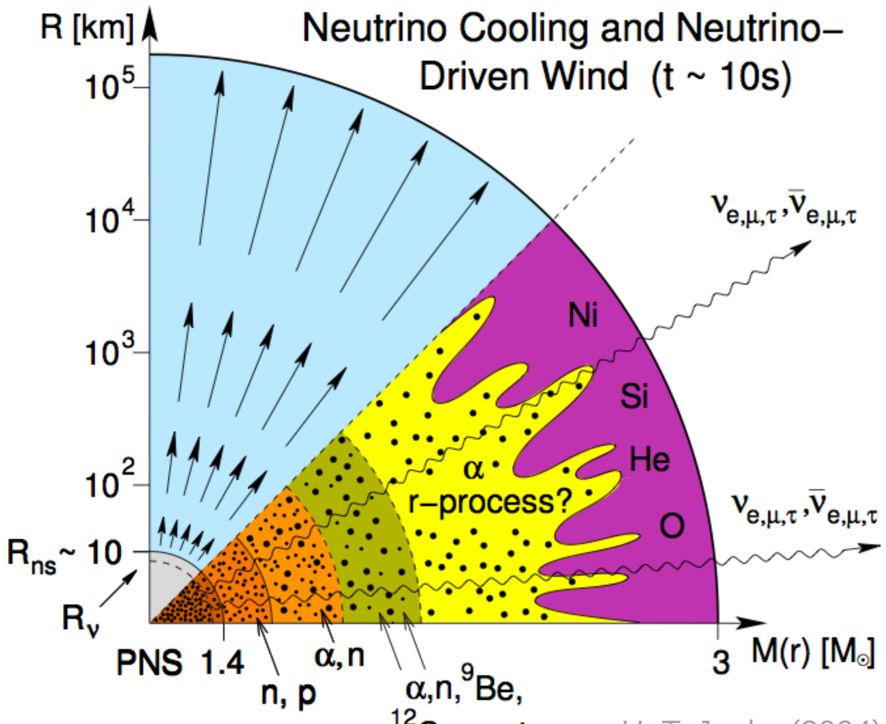


$^{16}O(e,e')X$



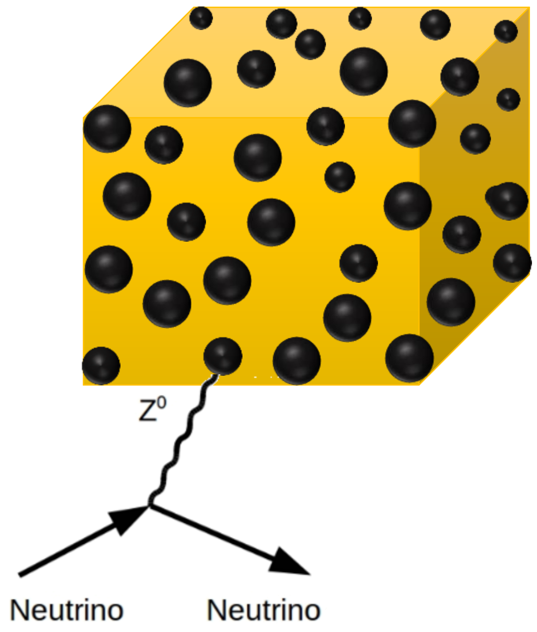
Spin response of neutron matter

A proto-neutron star (PNS) is the early stage of a neutron star, originating from a core-collapse supernova



H. T. Janka (2004)

neutral current neutrino scattering



$$\nu + X \to \nu + X^*$$

long-wavelength limit q → 0

$$\boldsymbol{J}_z \to \hat{O}_{\boldsymbol{q}}^{\boldsymbol{\sigma}} = \sum_i \hat{O}_{\boldsymbol{q}}^{\boldsymbol{\sigma}}(i) = \sum_i e^{i\boldsymbol{q}\cdot\boldsymbol{r}_i}\boldsymbol{\sigma}_i$$

spin response

neutron matter

Method – Integral transform

Response:

$$R^{\mu\nu} = \sum\!\!\!\!\!\!\!\int_f \langle 0|J^{\mu\dagger}(q)|f\rangle\langle f|J^{\nu}(q)|0\rangle\,\delta(E_0+\omega-E_f)$$

See Immo C. Reis's poster

^{16}O: Lorentz kernel
neutron matter: Gaussian kernel

$$I(\nu;\lambda) = \int d\omega K(\nu,\omega;\lambda)R(\omega)$$

Integral transform

Lorentz kernel

Gaussian kernel

$$K(\nu,\omega;\lambda) = \sum_k^{\infty} c_k(\nu;\lambda)T_k(\omega)$$

Expansion in a complete basis of orthogonal polynomials

Moments m_k
this we can calculate

$$I(\nu;\lambda) = \sum_k^{\infty} c_k(\nu;\lambda)m_k$$

$$m_k = \int d\omega T_k(\omega)R(\omega) = \frac{\langle\Phi_0|\hat{\Theta}T_k(\hat{H})\hat{\Theta}|\Phi_0\rangle}{\langle\Phi_0|\hat{\Theta}^2|\Phi_0\rangle}$$

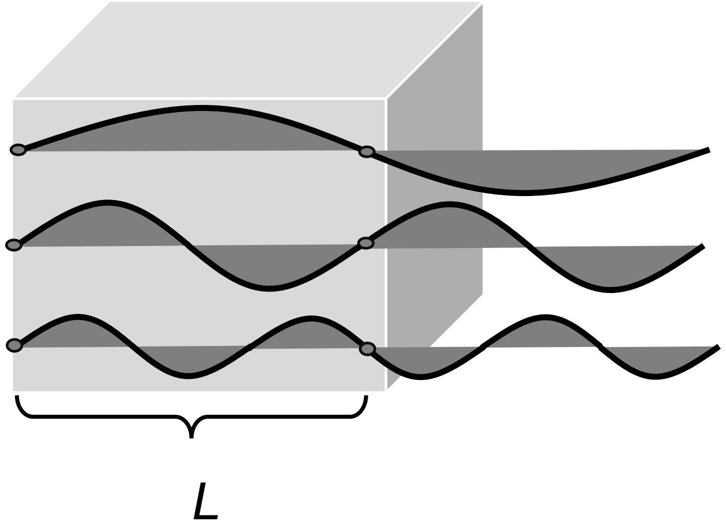
Method - CC nuclear matter

See Francesco Marino’s talk

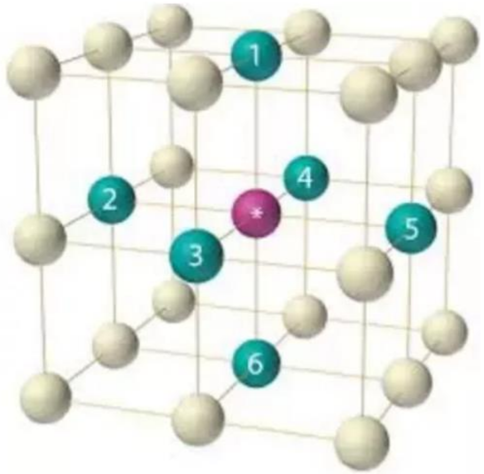
The spin response of the neutron matter computed under the coupled cluster framework

$$H_N \, e^T |\Phi_0\rangle = E \, e^T |\Phi_0\rangle$$

Nuclear matter with transitional invariance
The basis of the system is discrete momentum state on a cubic lattice (kx, ky, kz)



Compute the g.s. and response function under twist-averaged boundary conditions (TABC)

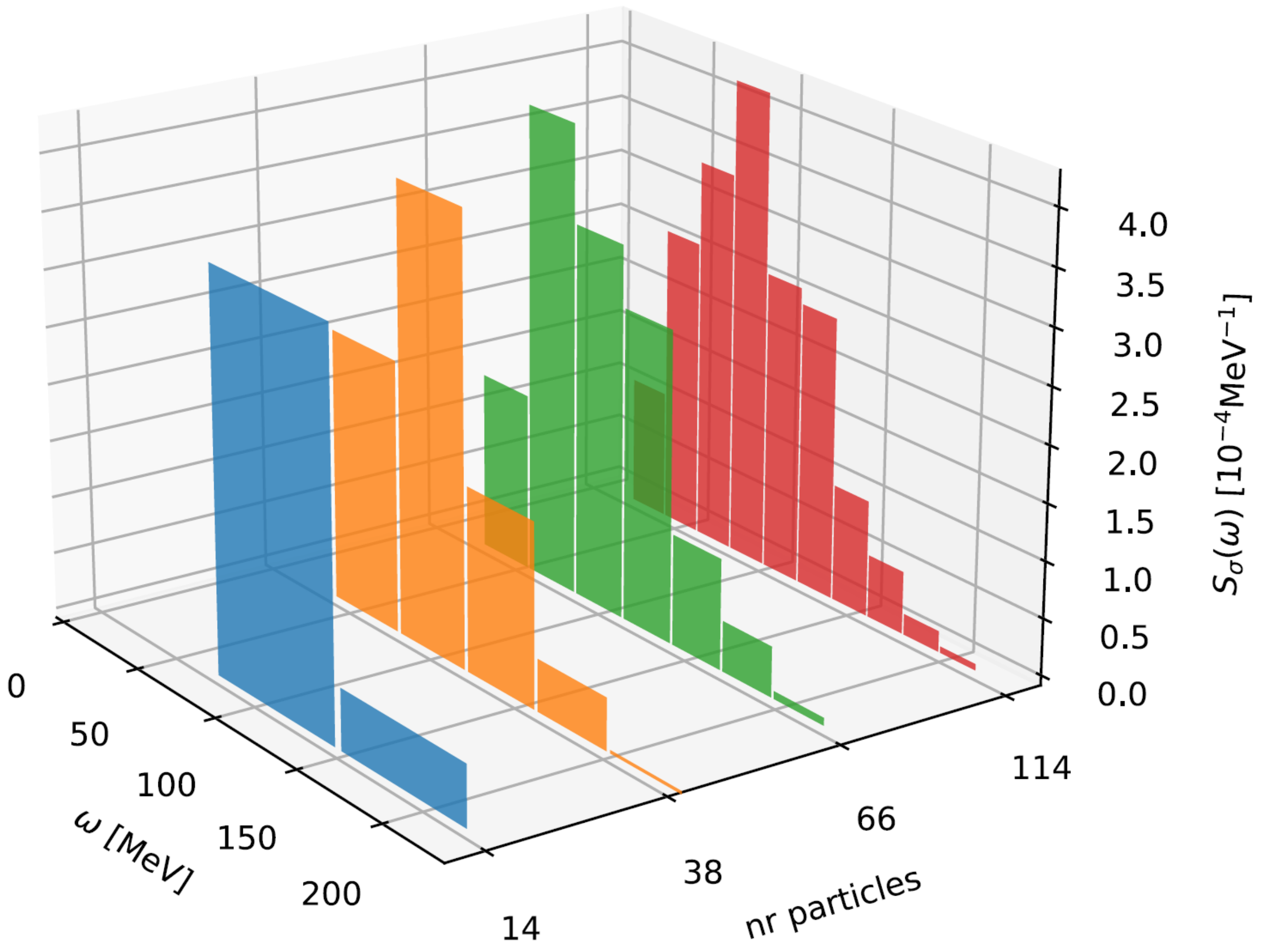


Chiral interaction DNNLO_GO

Finite-size effect

Magic number: N = 2, 14, 38, 54, 66, 114, 132, . . . for pure neutron matter

Spin response of neutron matter

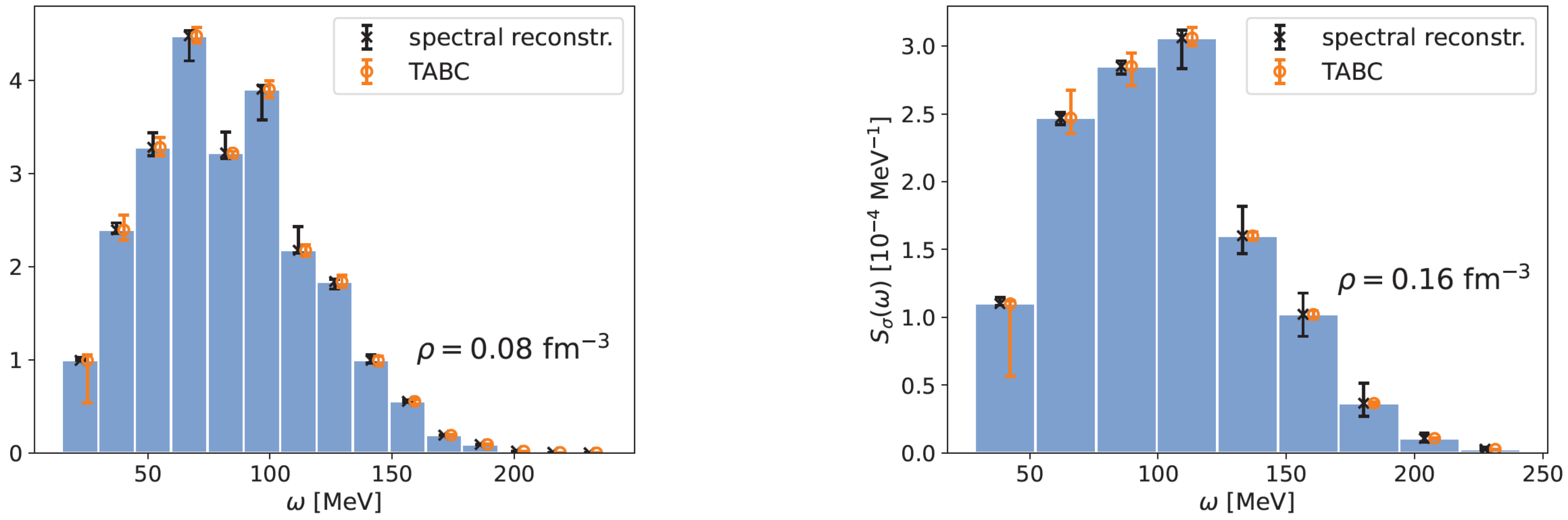


arXiv:2407.20986

Finite-size effect

Spin response of neutron matter

Mode of the response function shifted towards higher ω when density of the neutron matter increases



arXiv:2407.20986

Interaction sensitive (tensor part)

Δ LO(450 MeV) Hamiltonian

method	$\mathcal{H}_{kin}$	$\mathcal{H}_{contact}$	$\mathcal{H}_{\pi}$	$\mathcal{H}_{\pi} - \mathcal{H}_{T}$	E_{tot}	m_1	m_0
CIMC	37.489(31)	-31.379(97)	11.300(59)	13.161(46)	17.400(15)	7.467(55)	0.0662
CC	37.0939	-31.3374	11.6571	13.107	17.4141	5.809	0.0434

Similar binding energy

deviated spin response
(energy-weighted sum rule)

Δ LO(500 MeV) Hamiltonian

method	$\mathcal{H}_{kin}$	$\mathcal{H}_{contact}$	$\mathcal{H}_{\pi}$	$\mathcal{H}_{\pi} - \mathcal{H}_{T}$	E_{tot}	m_1	m_0
CIMC	38.307(50)	-32.338(88)	11.471(63)	14.359(47)	17.433(26)	11.553(85)	0.1015
CC	37.8515	-32.41071	11.91662	14.36435	17.3574	10.02966	0.070

CIMC: Configuration interaction Montecarlo, Alessandro Roggero, etc.
CC: Coupled cluster

Outline

- Neutrino interactions with finite nuclei and matter
- Data-driven AI analysis of dipole strength functions

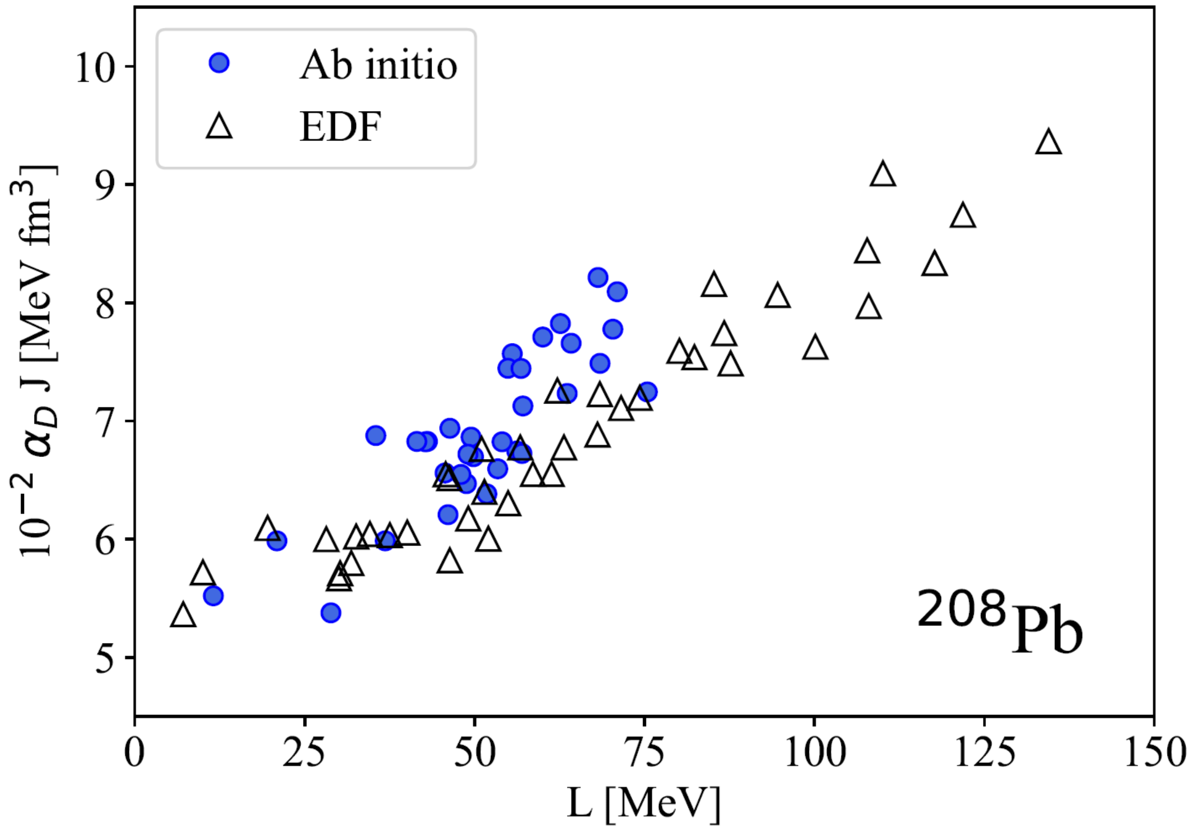
Motivation – Electric dipole polarizability (α_D)

See Francesca Bonaiti's talk

α_D is a fundamental observable that characterizes the response of a nucleus to an external electric field.

Provides key insights into the behavior of nuclear matter under varying conditions.

- neutron skin thickness
- symmetry energy, slope



Motivation – Electric dipole polarizability (α_D)

There is a need for accurate predictions of α_D to constrain the nuclear EoS and deepen our understanding of neutron-rich matter.

Limited experimental α_D data

Expensive theoretical calculations

Experiment:

$$f_{E1}(E) = \frac{\sigma_\gamma(E)}{3(\pi\hbar c)^2 E}$$

$$\alpha_D = \frac{3(\hbar c)^3}{2} \int \frac{f_{E1}(E)}{E} dE$$

Theory:

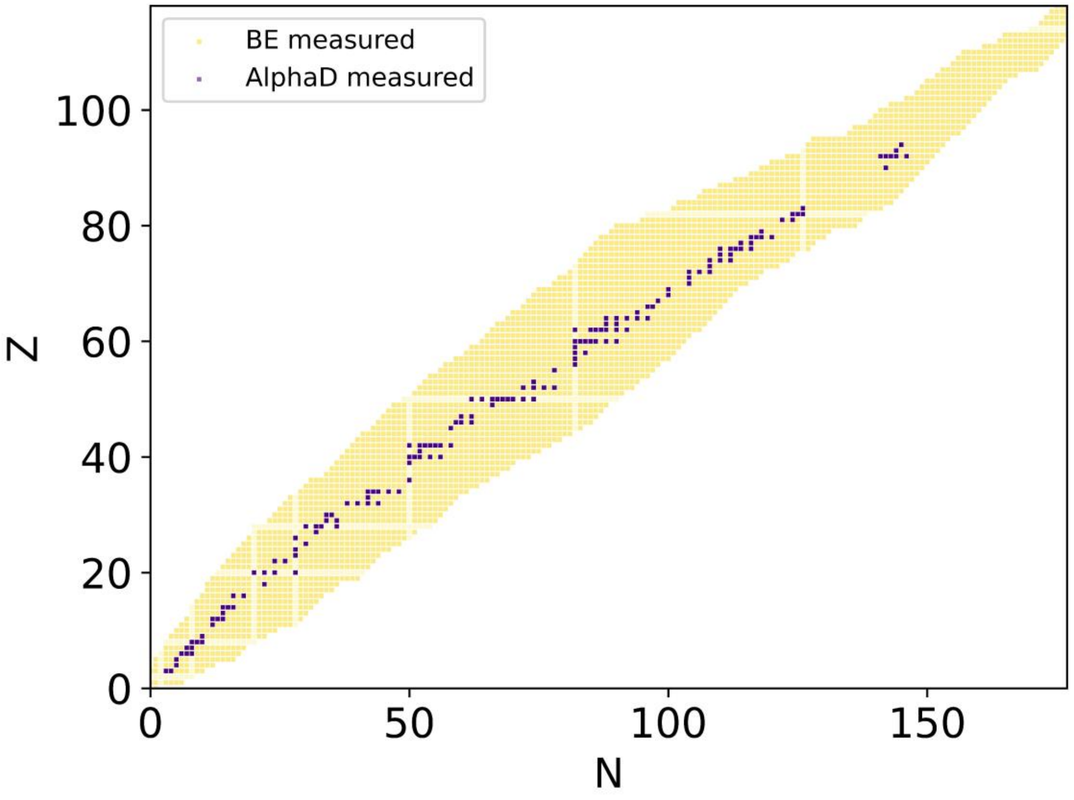
$$R = \sum\!\!\!\!\!\!\int_f |\langle f|\Theta|i\rangle|^2 \, \delta(E_0 + \omega - E_f)$$

$$\alpha_D = 2\alpha \int d\omega \frac{R(\omega)}{\omega} = 2\alpha \sum\!\!\!\!\!\!\int_f \frac{|\langle f|\Theta|i\rangle|^2}{E_f - E_i}$$

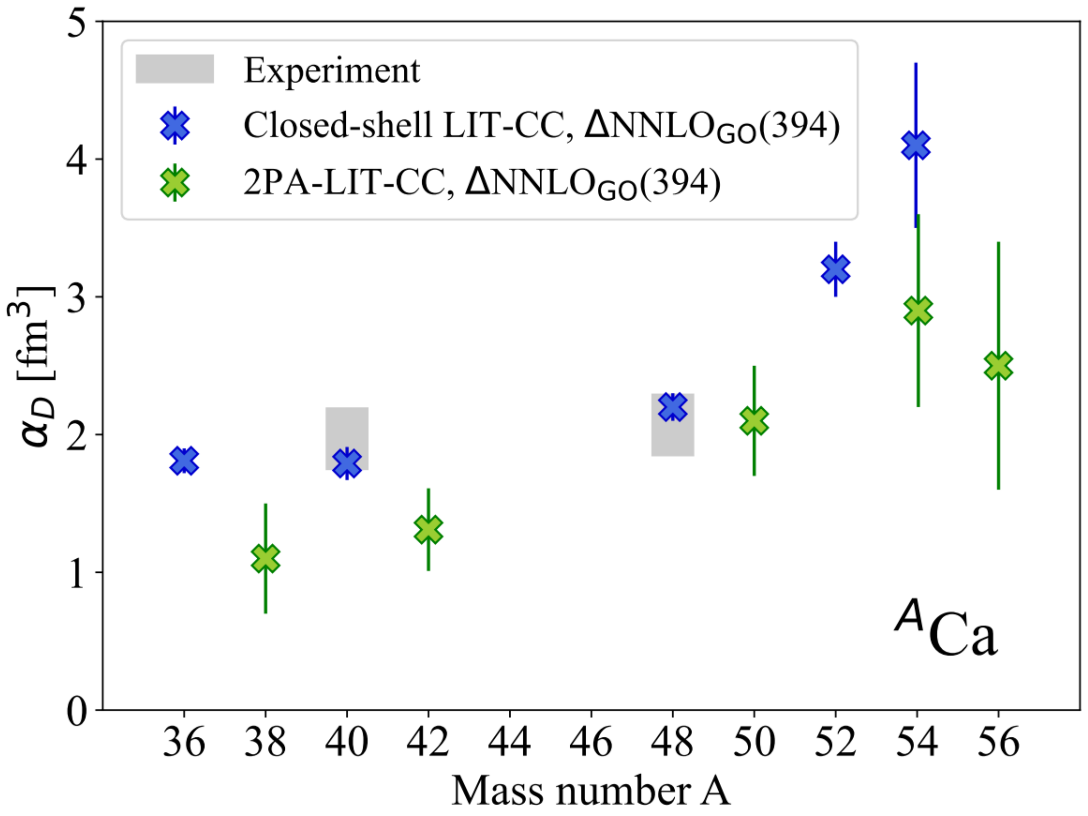
Motivation – Electric dipole polarizability (α_D)

There is a need for accurate predictions of α_D to constrain the nuclear EoS and deepen our understanding of neutron-rich matter.

Limited experimental α_D data



Expensive theoretical calculations



What about machine learning models (AI emulators)?

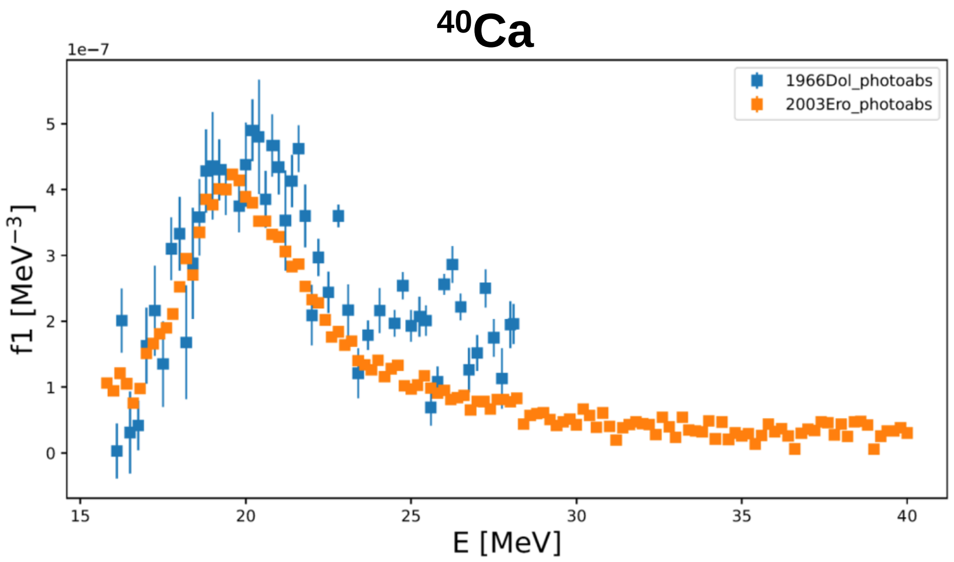
Method – data preparation

See Tim Egert's poster

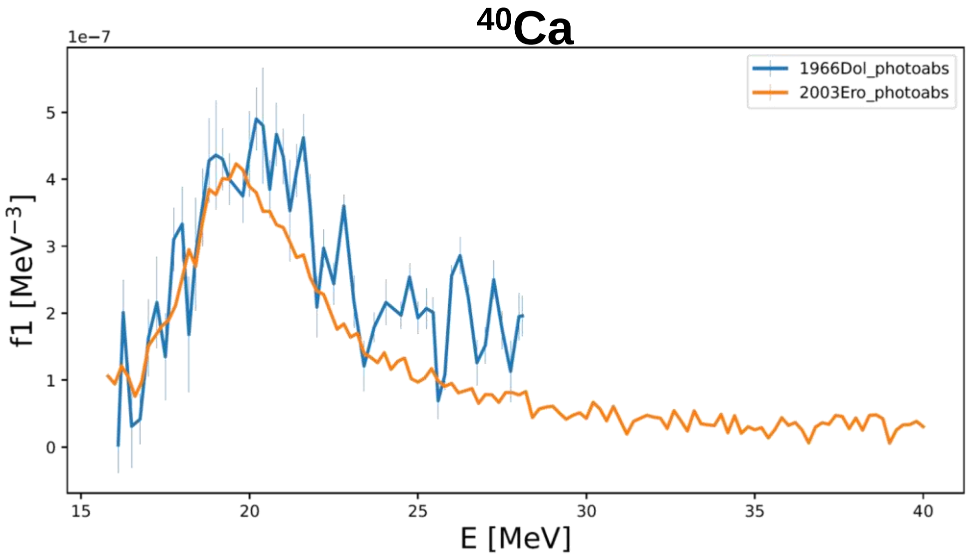
Data-driven α_D emulator based on machine learning technique

Capture the pattern of cross-section instead of α_D

$$\alpha_D \propto \int \frac{\sigma_\gamma(E)}{E^2} dE$$



Data augmentation (by interpolation)



Expt. data points (used): 30,635

data points after (used): 162,968

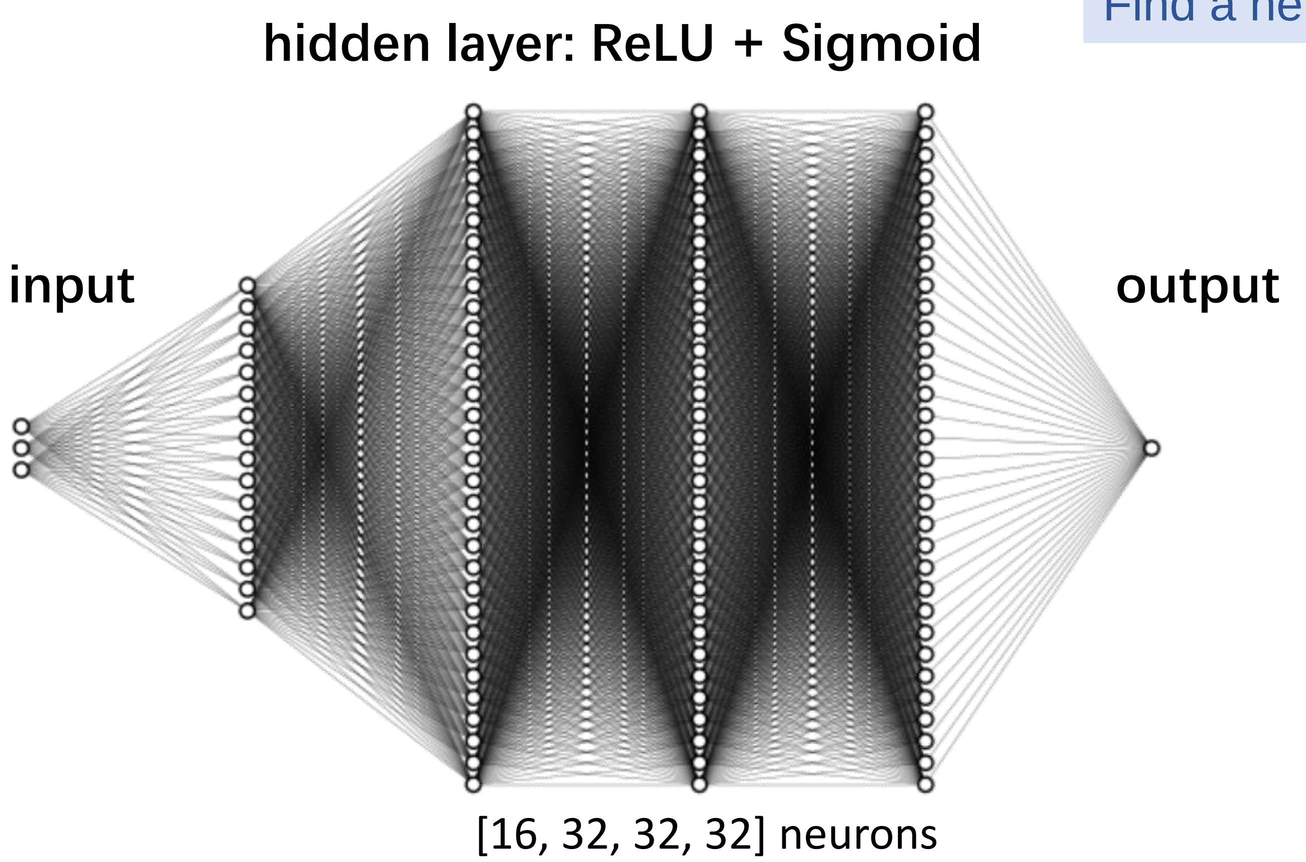
experimental data extracted from the well-established EXFOR database

Method – Neural Network architecture

See Tim's poster

Our goal: given *A, Z,* E_γ , predict σ_γ or f_{E1}

Find a network that is able to capture the relation between them



Schematic representation of a selected fully connected feed-forward neural network used in this work

Layer (type)	Output Shape	Param #
dense (Dense)	(None, 16)	64
dense_1 (Dense)	(None, 32)	544
dense_2 (Dense)	(None, 32)	1056
dense_3 (Dense)	(None, 32)	1056
dense_4 (Dense)	(None, 1)	33

Total params: 2753 (10.75 KB)
Trainable params: 2753 (10.75 KB)
Non-trainable params: 0 (0.00 Byte)

number of all weights:2640

Method – model complexity vs overfitting

balance between model complexity and overfitting

training set: 90%
validation set: 10%

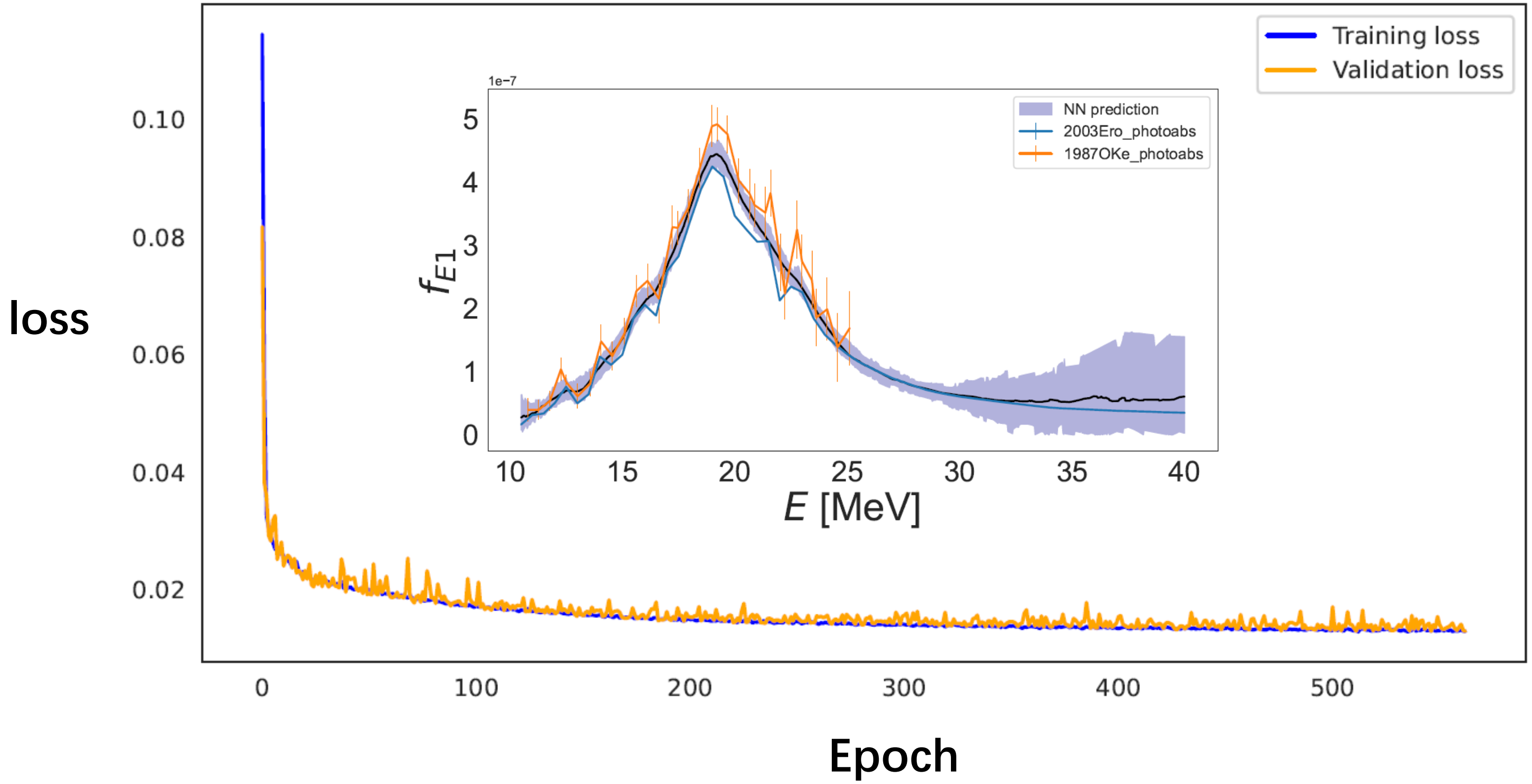
1. Is the network giving satisfactory predictions for the training data?

2. Is the network overfitting?

input data size: 162,968 data points

trainable parameters: 2,753

There is still potential to further deepen or widen the network before encountering significant overfitting.

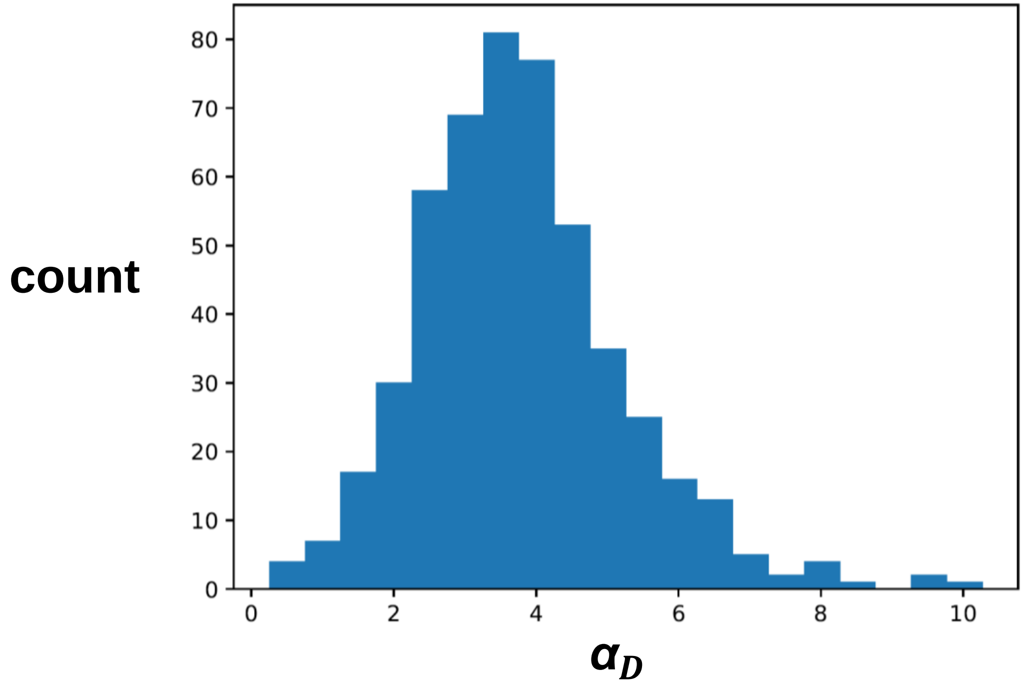


Method – Uncertainty Analysis

To fully understand the limitations and reliability of our machine learning model, it is essential to discuss and quantify the uncertainties embedded in the neural network prediction

Model uncertainty: parameter uncertainty, architecture sensitivity …

Data uncertainty: quality and consistency of the input data …

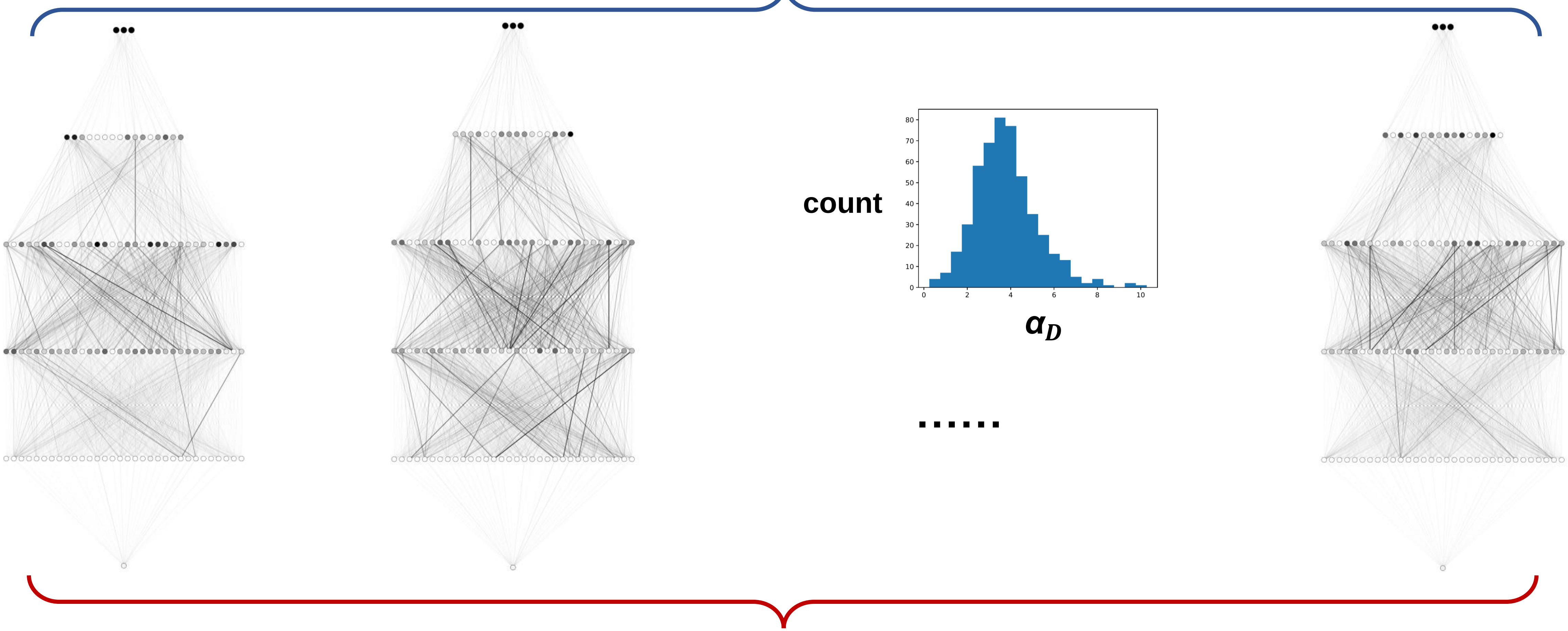


To quantify the main uncertainties we employed **the ensemble learning methods** that train multiple neural network models (500) with different initializations and different subsets of data.

Method – ensemble learning methods

weights (lines) and biases (dots), represented in grayscale and normalized to the range [0,1]

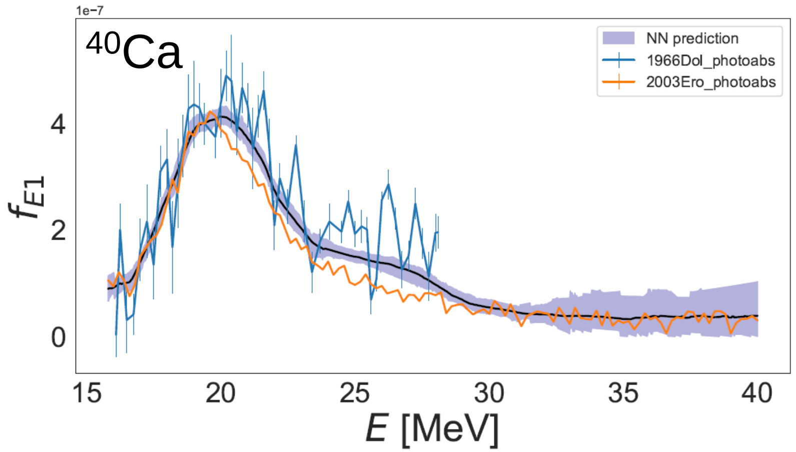
Input: A, Z, E_γ

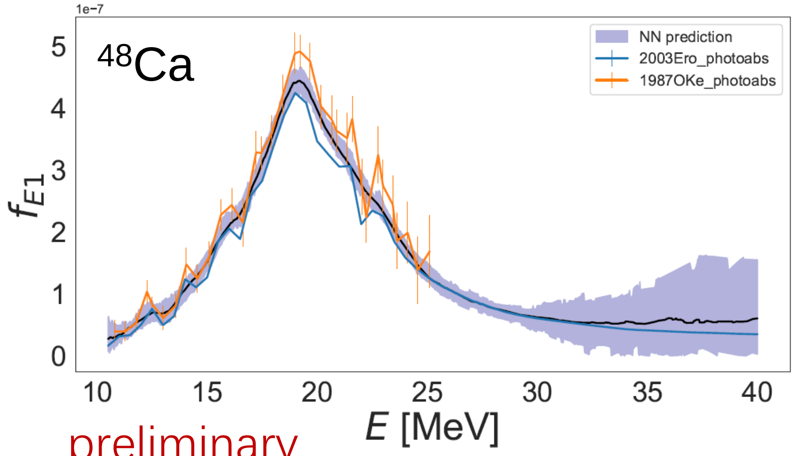


cross-section / α_D (mean value), error bar (68% credible interval)

Results – NN vs experiments

Training nuclei

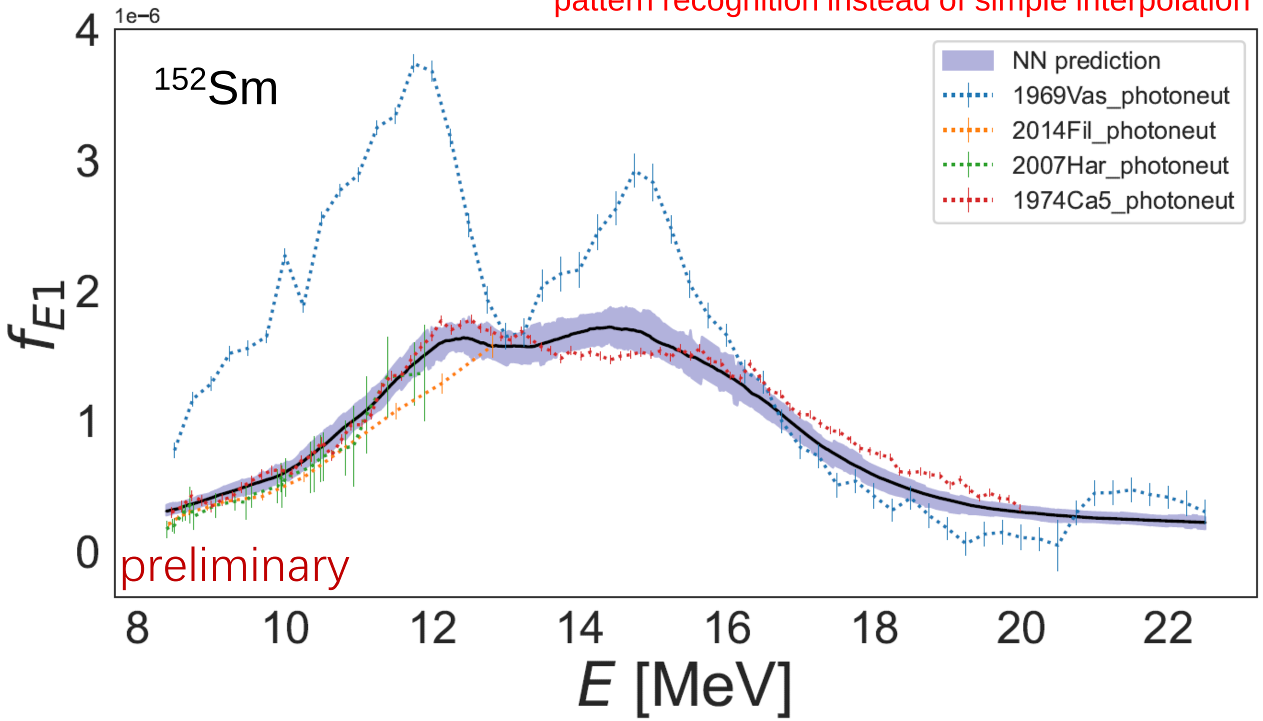




preliminary

Testing nuclei (prediction)

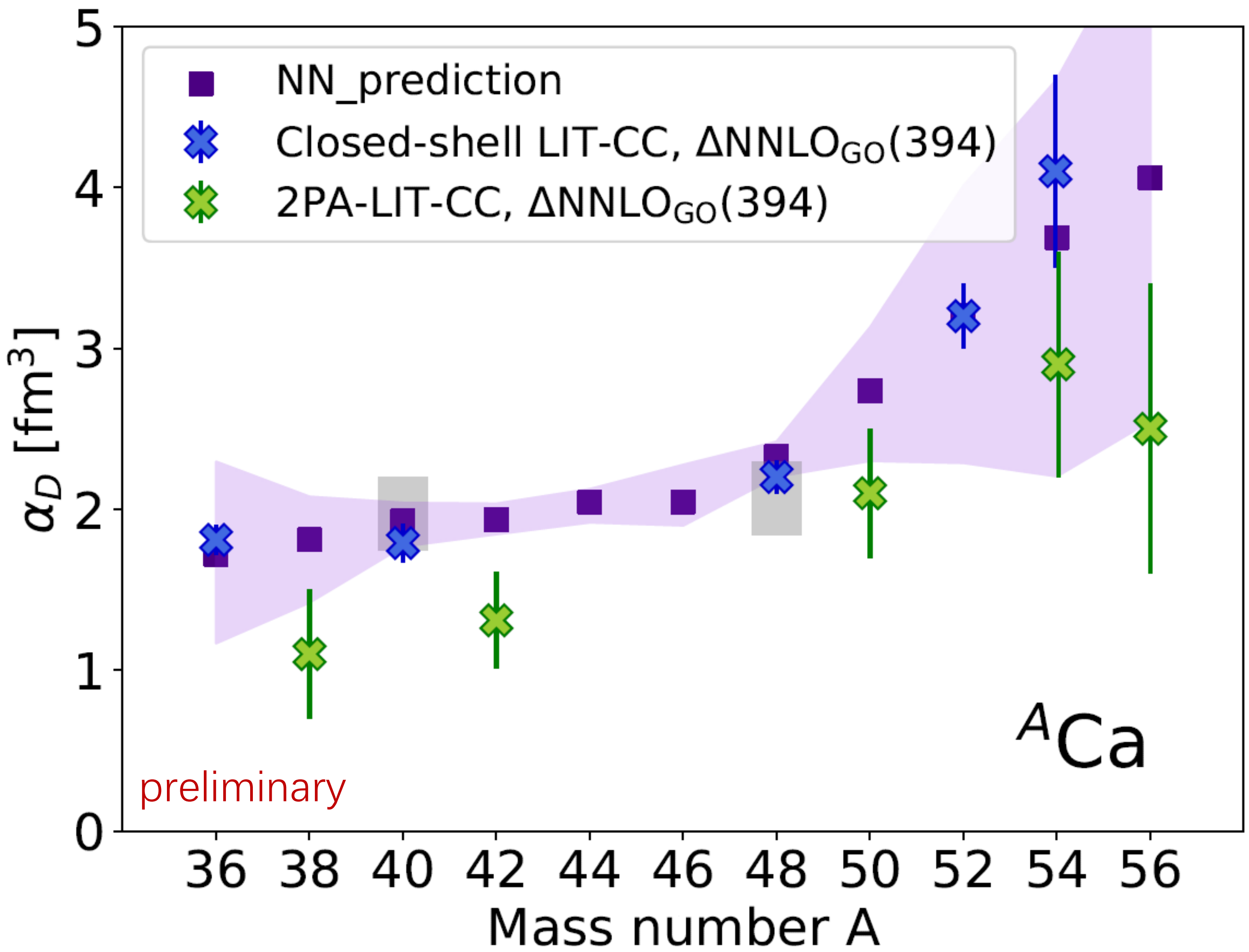
pattern recognition instead of simple interpolation



10 out of 160 nuclei serve as the testing set

Results – NN vs theoretical calculations

Neural network predictions compared with different coupled cluster approaches



Questions

- Is there other calculations for the response function that we can benchmark with?

- New experiment?

Collaborators

Mainz: Sonia Bacca, Francesca Bonaiti, Tim Egert,
Joanna Ewa Sobczyk, Francesco Marino

Oak Ridge: Bijaya Acharya, Gaute Hagen

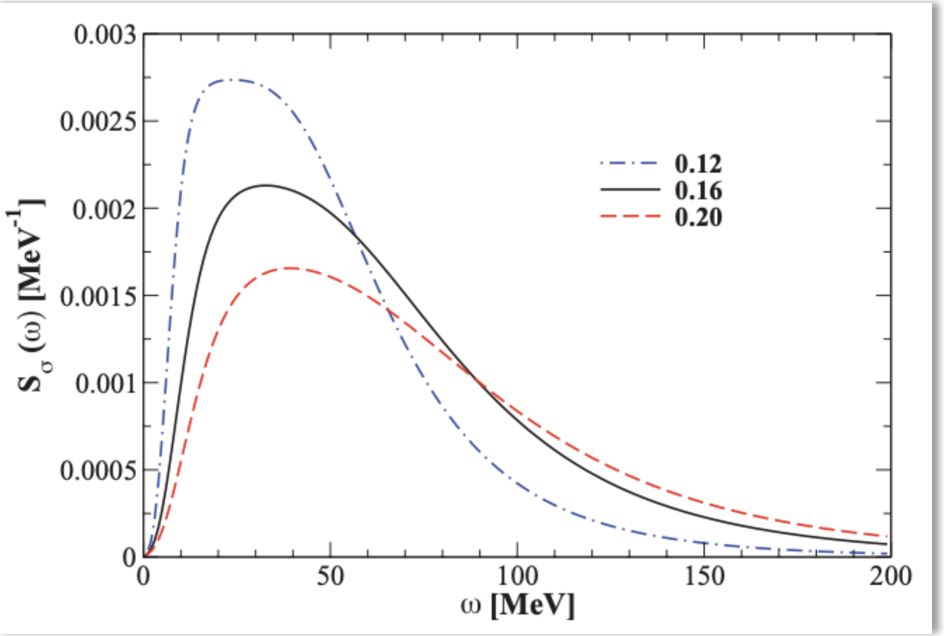
St. Louis: Samuel J. Novari

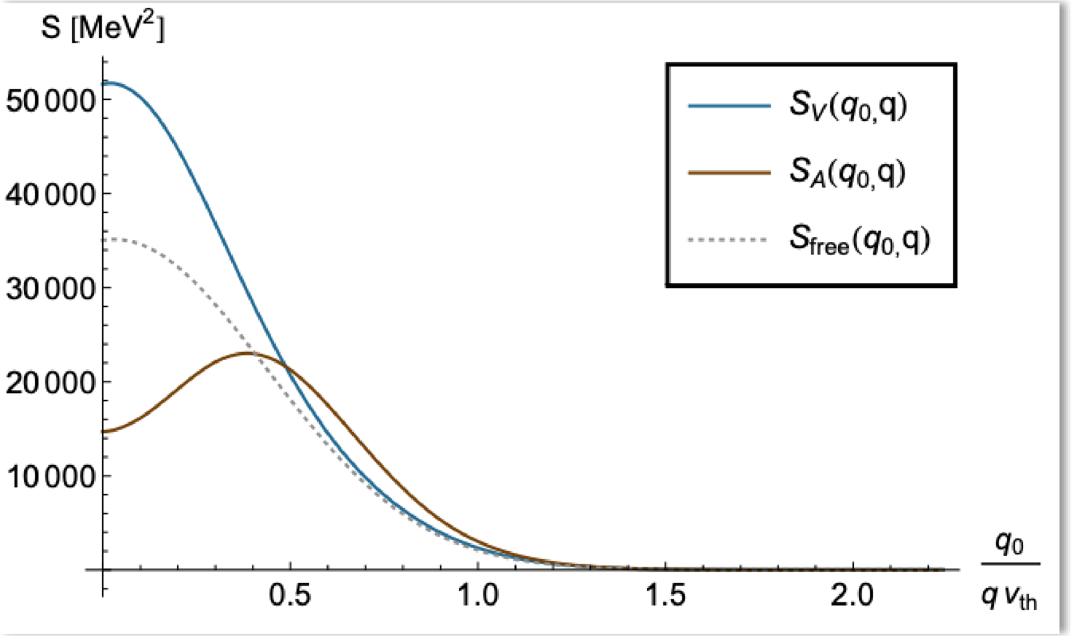
Trento: Alessandro Rogger

Motivation – Previous studies

- Quantum Monte Carlo method + realistic AV8 potential, reconstruct the dynamical spin response from three energy-weighted sum rules. *
- Virial expansion + pseudopotential, construct the responses at low fugacity, low density, and/or high temperature. **

Our goal: To obtain an explicit response function with robust many-body methods and modern chiral interaction.





* *Phys. Rev. C 87, 025802 (2013)*

*** Phys. Rev. C 98, 015802 (2018)*

Method – Integral transform

Response:

$$R^{\mu\nu} = \sum\!\!\!\!\!\!\int_f \langle 0|J^{\mu\dagger}(q)|f\rangle\langle f|J^{\nu}(q)|0\rangle\,\delta(E_0+\omega-E_f)$$

See Immo C. Reis's poster

^{16}O: Lorentz kernel
neutron matter: Gaussian kernel

Integral transform → $$I(\nu;\lambda) = \int d\omega K(\nu,\omega;\lambda)R(\omega)$$

Lorentz kernel

$$K^{(L)}(\nu,\omega;\lambda) = \frac{1}{\pi\lambda}\frac{\lambda^2}{(\omega-\nu)^2+\lambda^2}$$

Gaussian kernel

$$K^{(G)}(\nu,\omega;\lambda) = \frac{1}{\sqrt{2\pi}\lambda}\exp\left(-\frac{(\omega-\nu)^2}{2\lambda^2}\right)$$

Expansion in a complete basis of orthogonal polynomials → $$K(\nu,\omega;\lambda) = \sum_k^{\infty} c_k(\nu;\lambda)T_k(\omega)$$

$$I(\nu;\lambda) = \sum_k^{\infty} c_k(\nu;\lambda)m_k$$

Moments m_k
this we can calculate

$$m_k = \int d\omega T_k(\omega)R(\omega) = \frac{\langle\Phi_0|\hat{\Theta}T_k(\hat{H})\hat{\Theta}|\Phi_0\rangle}{\langle\Phi_0|\hat{\Theta}^2|\Phi_0\rangle}$$

Method – GIT Coupled Cluster (GIT-CC)

We can expand Gaussian kernel, in Chebyshev polynomials:

$$T_0(H) = 1$$
$$T_{-1}(H) = T_1(H) = H$$
$$T_{n+1}(H) = 2H \cdot T_n(H) - T_{n-1}(H)$$

$$|\Phi_1\rangle \equiv \hat{\Theta}|\Phi_0\rangle$$
$$|\Phi_n\rangle = \hat{H}|\Phi_{n-1}\rangle$$
$$m_0 = \langle\Phi_1|\Phi_1\rangle$$
$$m_1 = \langle\Phi_1|\Phi_2\rangle \equiv \langle\Phi_2|\Phi_1\rangle$$
$$m_{n+1} = 2\langle\Phi_1|\Phi_{n+1}\rangle - m_{n-1} \equiv 2\langle\Phi_{n+1}|\Phi_1\rangle - m_{n-1}$$

recurrence relation

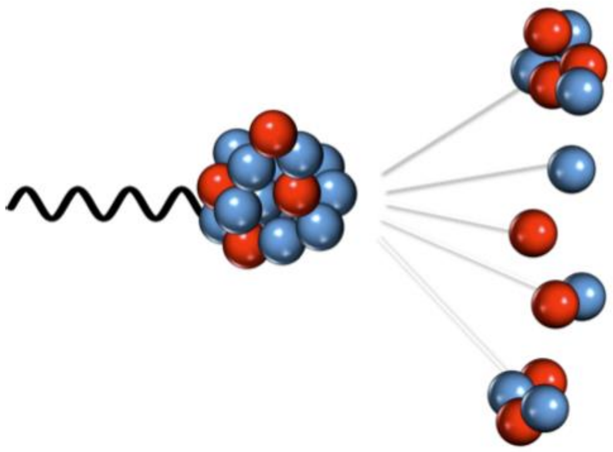
A numerical inversion of the resulting integral transform to recover the response function:

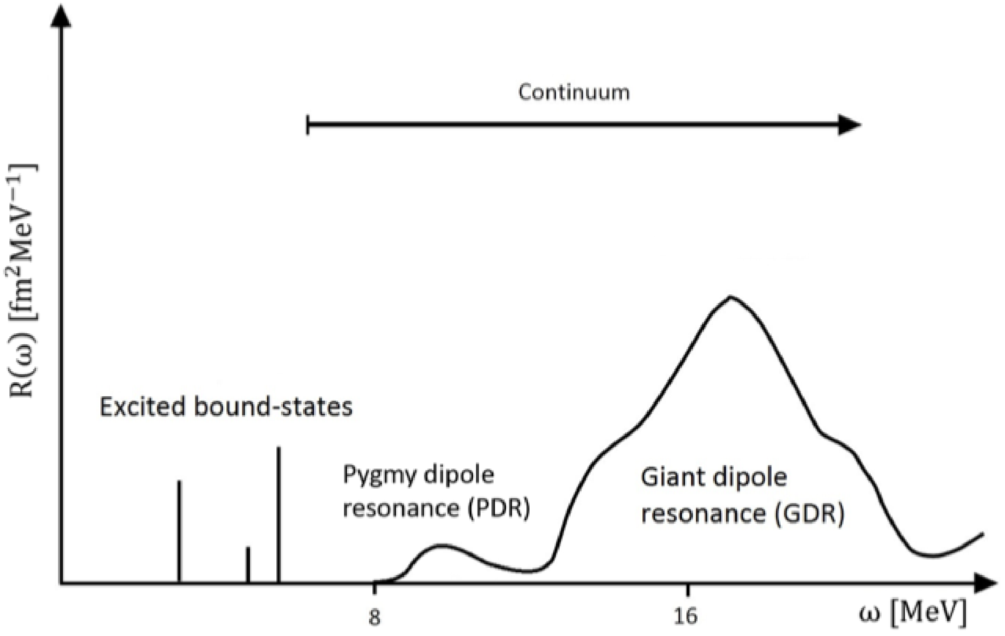
$$I(\nu;\lambda) = \int d\omega K(\nu,\omega;\lambda) R(\omega)$$

To evaluate the Chebyshev moments, we apply Coupled Cluster (CC) method:

$$\hat{H} = e^{-T} H e^{T}$$
$$\hat{\Theta} = e^{-T} \Theta e^{T}$$

Motivation – Photoabsorption





** Mirko Miorelli, PhD thesis (2017)*