# Realistic Shell Model Study of Electroweak Decays: a Chiral Perturbation Theory Approach

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Electroweak Physics InterseCtions - EPIC 2024

# Quenching of $\sigma\tau$ matrix elements

Quenching of $\sigma\tau$ matrix elements is quite a general phenomenon in nuclear-structure physics.

Gamow-Teller transitions ($\beta$-decay, *EC*, $2\nu\beta\beta$, charge-exchange) are hindered from expected values based on sum rules derived by nuclear structure models.

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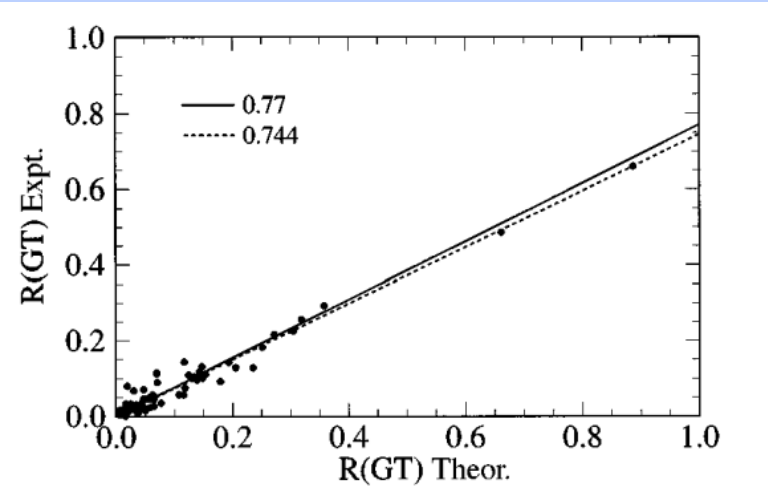


G. Martinez Pinedo et al., Phys. Rev. C 53, R2602 (1996)

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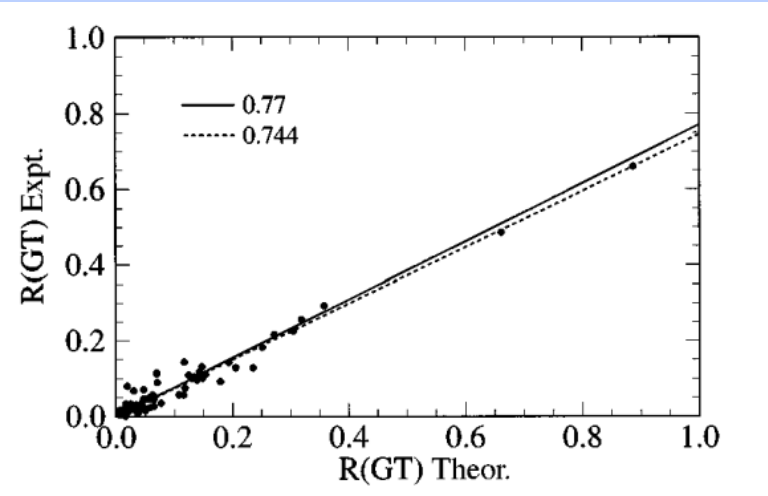


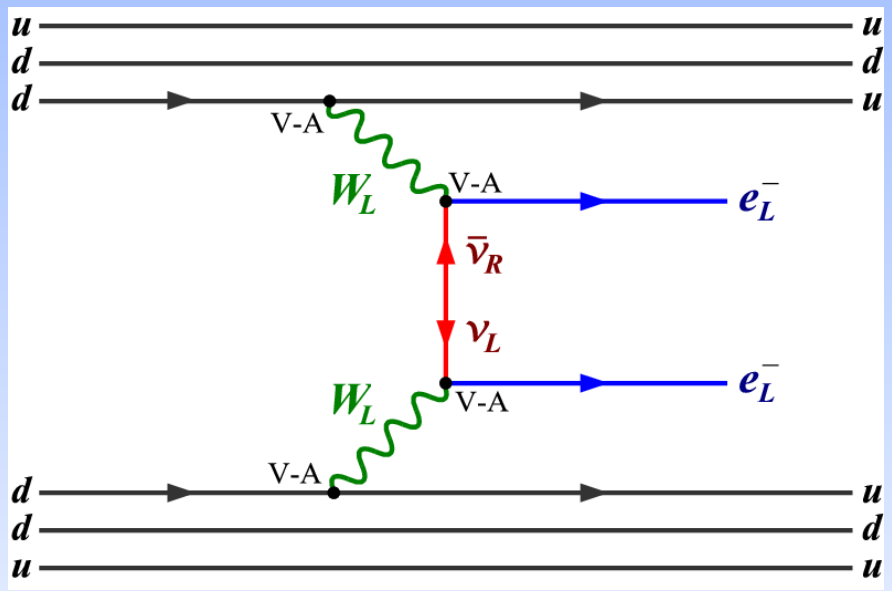

G. Martinez Pinedo et al., Phys. Rev. C 53, R2602 (1996)

$$g_A \Rightarrow g_A^{eff} = q g_A$$

*J. T. Suhonen*
*Frontiers in Physics* **5** , *55 (2017)*

# Quenching of $\sigma\tau$ matrix elements & $0\nu\beta\beta$ decay

The inverse of the $0\nu\beta\beta$-decay half-life is proportional to the squared nuclear matrix element (NME).
This evidences the relevance to calculate the NME




$$\left[T^{0\nu}_{1/2}\right]^{-1} = G^{0\nu}\left|M^{0\nu}\right|^2 \langle m_\nu\rangle^2 \propto g_A^4$$

- $G^{0\nu} \rightarrow$ phase-space factor
- $\langle m_\nu\rangle = \left|\sum_k m_k U_{ek}^2\right|$ effective mass of the Majorana neutrino, $U_{ek}$ being the lepton mixing matrix

# Quenching of $\sigma\tau$ matrix elements: theory

Two main sources $\Rightarrow$ missing degrees of freedom

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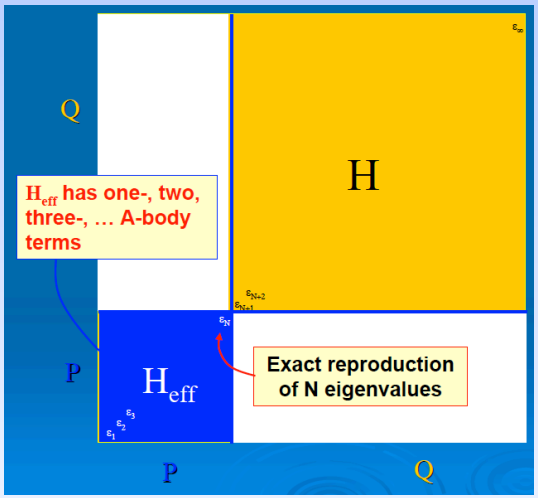
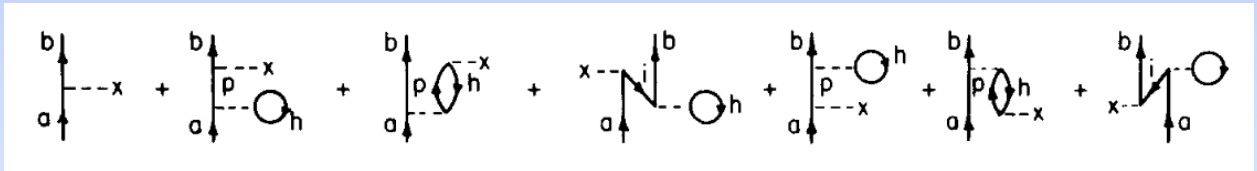
## Limited model space

Nuclear-structure calculations are carried out in truncated model spaces $\Rightarrow$ effective Hamiltonians and operators



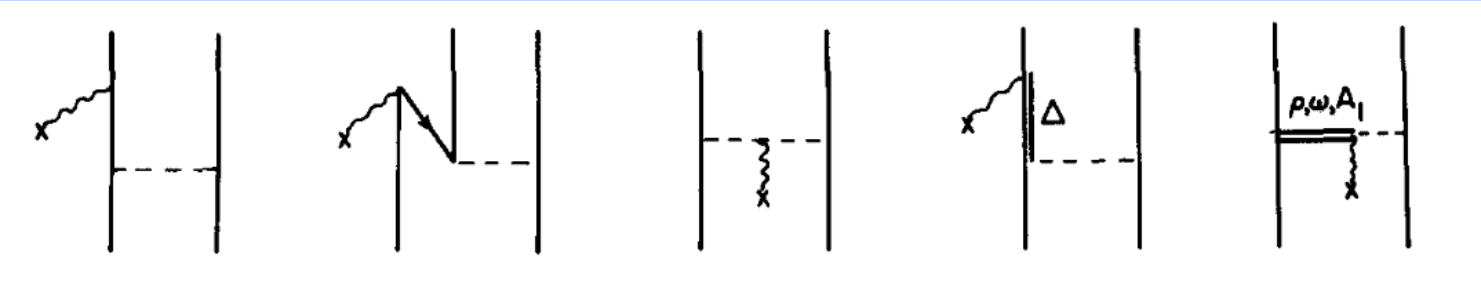



# Quenching of $\sigma\tau$ matrix elements: theory

Two main reasons $\Rightarrow$ missing degrees of freedom

## Non-nucleonic degrees of freedom

Processes in which the weak probe prompts a meson to be exchanged between two nucleons $\Rightarrow$ meson-exchange two-body currents (2BC)




*H. Hyuga and A. Arima, J. Phys. Soc. Jpn. Suppl. 34, 538 (1973)*

# Quenching of $\sigma\tau$ matrix elements: truncated model space

## Reduced model space $P$

$$H \Rightarrow H_{\text{eff}}$$

$$H|\Psi_\nu\rangle = E_\nu|\Psi_\nu\rangle \Rightarrow H_{\text{eff}}|\Phi_\alpha\rangle = E_\alpha|\Phi_\alpha\rangle$$

$|\Phi_\alpha\rangle$ = eigenvectors obtained diagonalizing $H_{\text{eff}}$ in the reduced model space $\Rightarrow |\Phi_\alpha\rangle = P|\Psi_\alpha\rangle$

$$\langle\Phi_\alpha|\hat{\Theta}|\Phi_\beta\rangle \neq \langle\Psi_\alpha|\hat{\Theta}|\Psi_\beta\rangle$$

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# Quenching of $\sigma\tau$ matrix elements: truncated model space

## Effective operator $\hat{\Theta}_{\rm eff}$: definition

$$\langle \Phi_\alpha | \hat{\Theta}_{\rm eff} | \Phi_\beta \rangle = \langle \Psi_\alpha | \hat{\Theta} | \Psi_\beta \rangle$$

$$\hat{\Theta}_{\rm eff} = \sum_{\alpha\beta} |\Phi_\alpha\rangle \langle \Psi_\alpha | \hat{\Theta} | \Psi_\beta \rangle \langle \Phi_\beta |$$

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## Gamow-Teller operator

$$\hat{\Theta}^{GT} = g_A \boldsymbol{\sigma\tau}^{\pm} \Rightarrow \hat{\Theta}^{GT}_{\text{eff}} = g_A^{\text{eff}} \boldsymbol{\sigma\tau}^{\pm}$$

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$$\hat{\Theta}^{GT} = g_A \sigma\tau^{\pm} \Rightarrow \hat{\Theta}^{GT}_{\rm eff} = g_A^{\rm eff} \sigma\tau^{\pm}$$

## Electric quadrupole operator

$$\hat{\Theta}^{E2} = e r^2 Y^2_\mu \Rightarrow \hat{\Theta}^{E2}_{\rm eff} = e^{\rm eff} r^2 Y^2_\mu$$

# Quenching of $\sigma\tau$ matrix elements: meson exchange currents

- In the 80s starting from OBEP models two-nucleon meson-exchange current operators have been constructed consistently as required by the continuity equation for vector currents and the PCAC.
- Nowadays, EFT provides a powerful approach where both nuclear potentials and two-body electroweak currents (2BC) may be consistently constructed, the latter appearing as subleading corrections to the one-body GT operator $\sigma\tau^{\pm}$

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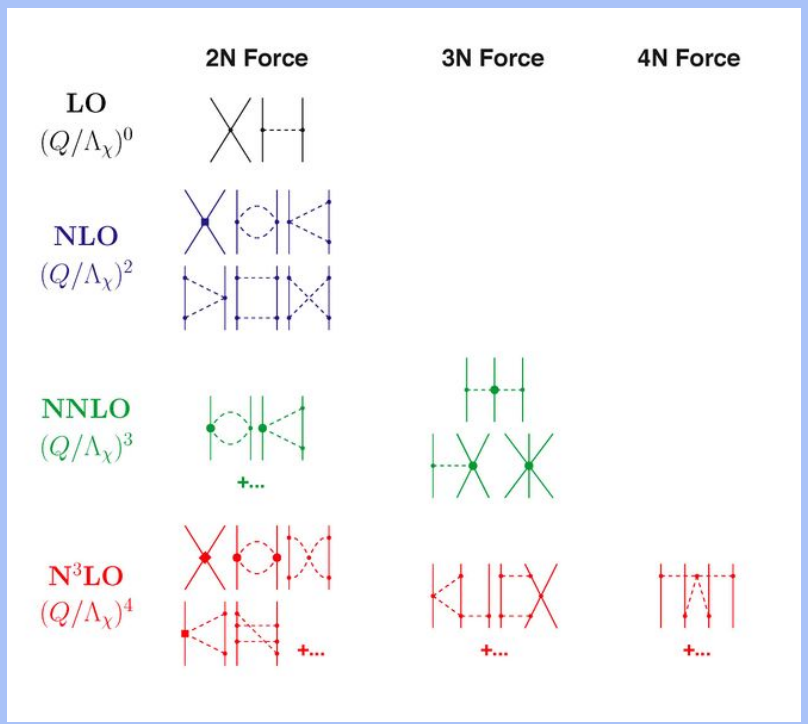
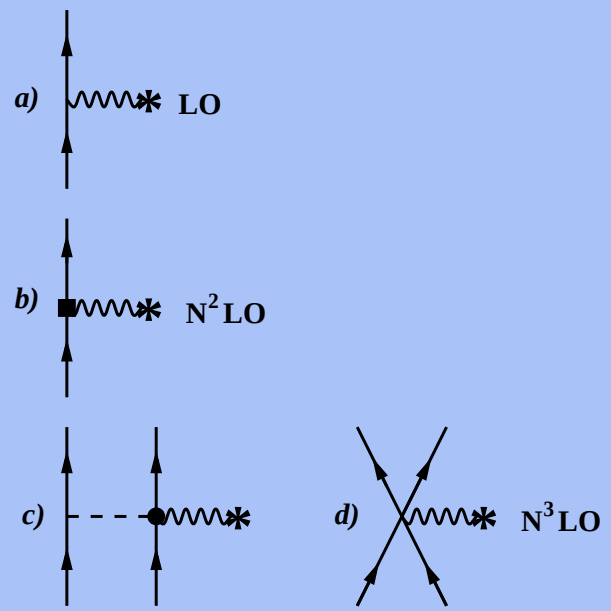
## Nuclear potential

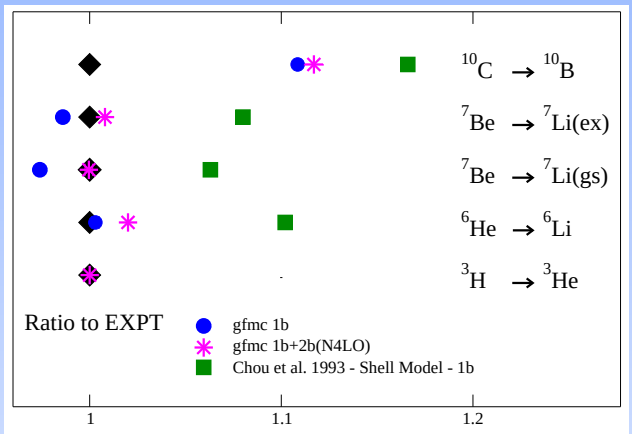



## Electroweak axial currents

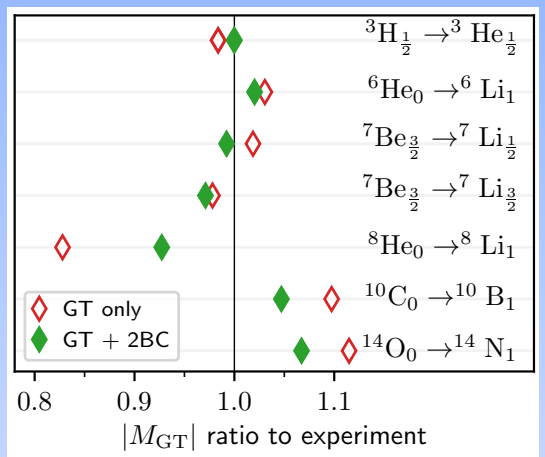


# Two-body e.w. currents effects: light nuclei

GT nuclear matrix elements of the $\beta$-decay of *p*-shell nuclei have been calculated with Quantum Monte Carlo (QMC) and no-core shell model (NCSM) methods, including contributions from 2BC

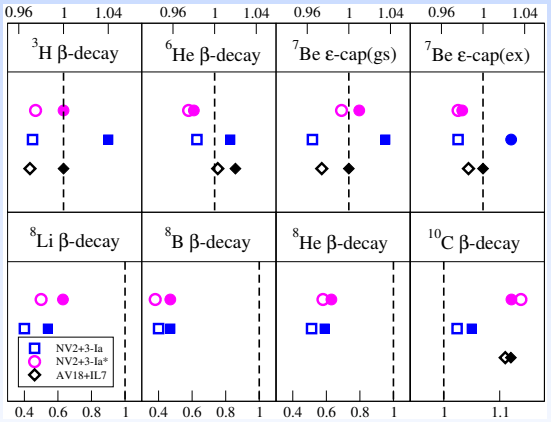



*S. Pastore et al., Phys. Rev. C* **97** *022501(R) (2018)*




*P. Gysbers et al., Nat. Phys.* **15** *428 (2019)*

The contribution of 2BC improves systematically the agreement between theory and experiment




*G. B. King et al., Phys. Rev. C* **102** *025501 (2020)*

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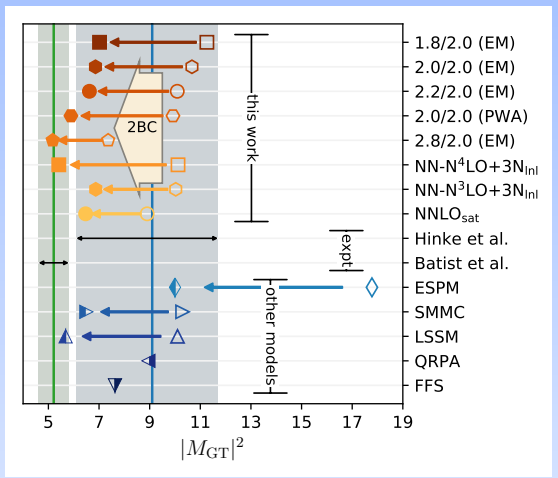
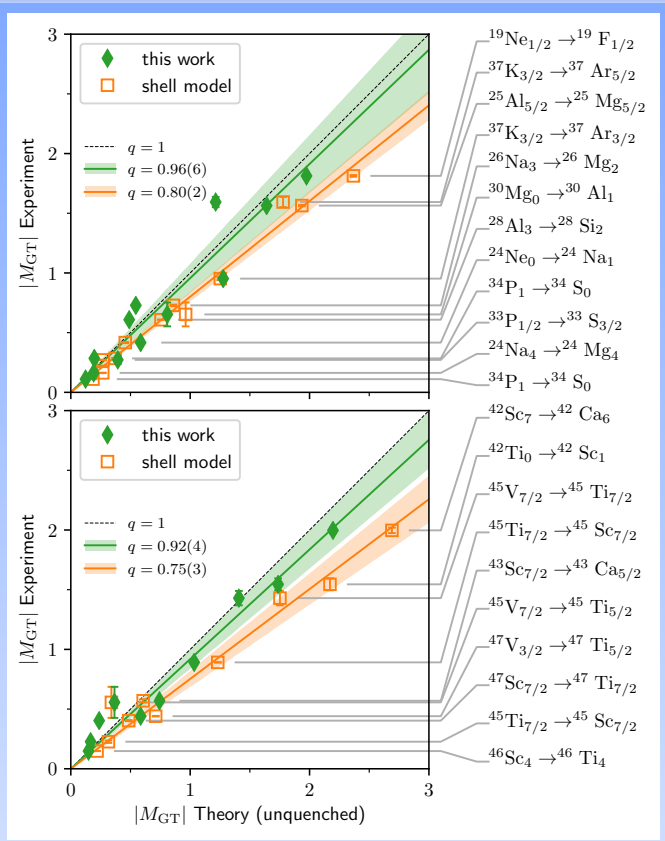
# Two-body e.w. currents effects: medium mass nuclei

Coupled-cluster method CCM and in-medium SRG (IMRSG) calculations have recently performed to overcome the quenching problem $g_A$ to reproduce $\beta$-decay observables in heavier systems

*P. Gysbers et al., Nat. Phys.* **15** *428 (2019)*




Coupled-Cluster Method




In-Medium SRG

A proper treatment of nuclear correlations and consistency between GT two-body currents and 3N forces, derived in terms of ChPT, describes microscopically the “quenching puzzle”

# Realistic Shell-Model Calculations

Realistic shell-model calculations starting from a nuclear Hamiltonian and electroweak axial currents derived consistently by way of $\chi$PT.

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- G. De Gregorio
- A. Gargano
- N. I.

**INFN-Pisa**

- M. Viviani

**Peking University**

- Z. H. Cheng
- Y. Z. Ma
- F. R. Xu

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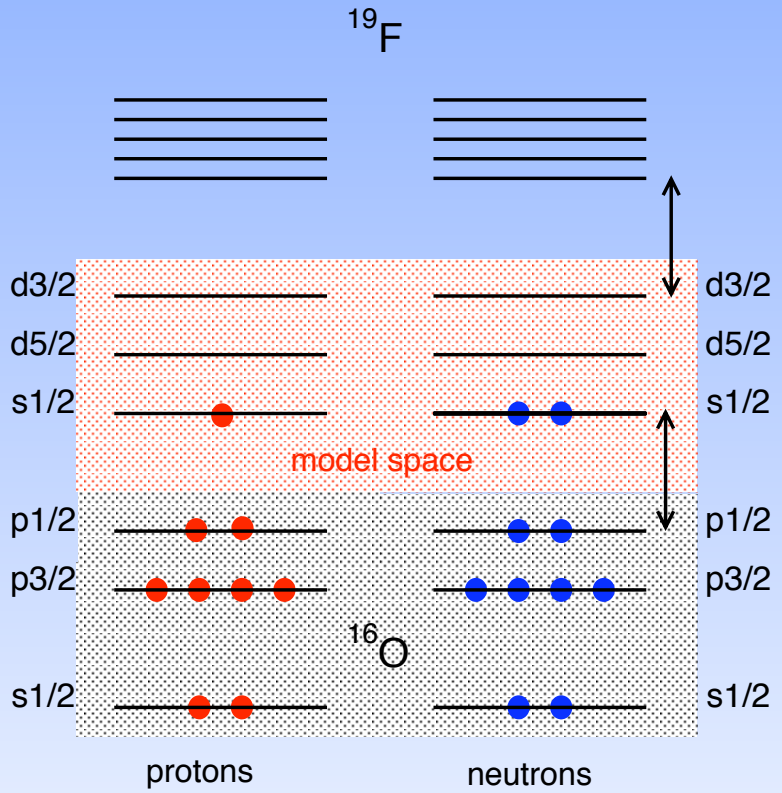
# Realistic Shell-Model Calculations

Shell model $\Rightarrow$ well-established approach to obtain a microscopic description of both collective and single-particle properties of nuclei




The degrees of freedom of the core nucleons and the excitations of the valence ones above the model space are not considered explicitly.

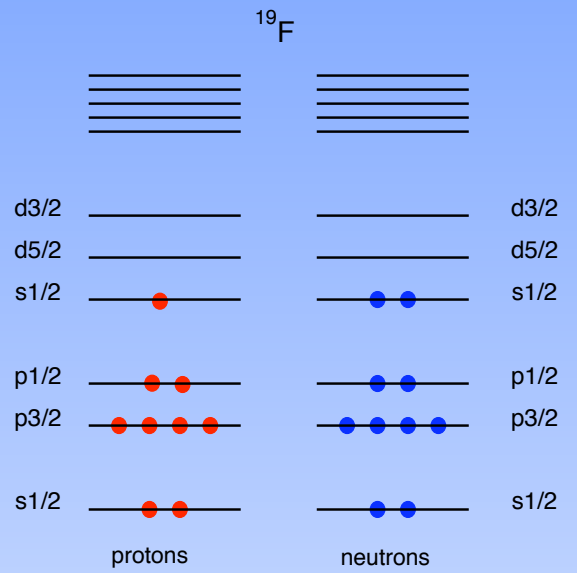
$$V_{NN} \ (+V_{NNN}) \Rightarrow \text{many-body theory} \Rightarrow H_{\text{eff}}$$

# Effective shell-model hamiltonian

## A-nucleon system Schrödinger equation

$$H|\Psi_\nu\rangle = E_\nu|\Psi_\nu\rangle$$

$$H = H_0 + H_1 = \sum_{i=1}^{A}(T_i + U_i) + \sum_{i<j}(V_{ij}^{NN} - U_i)$$



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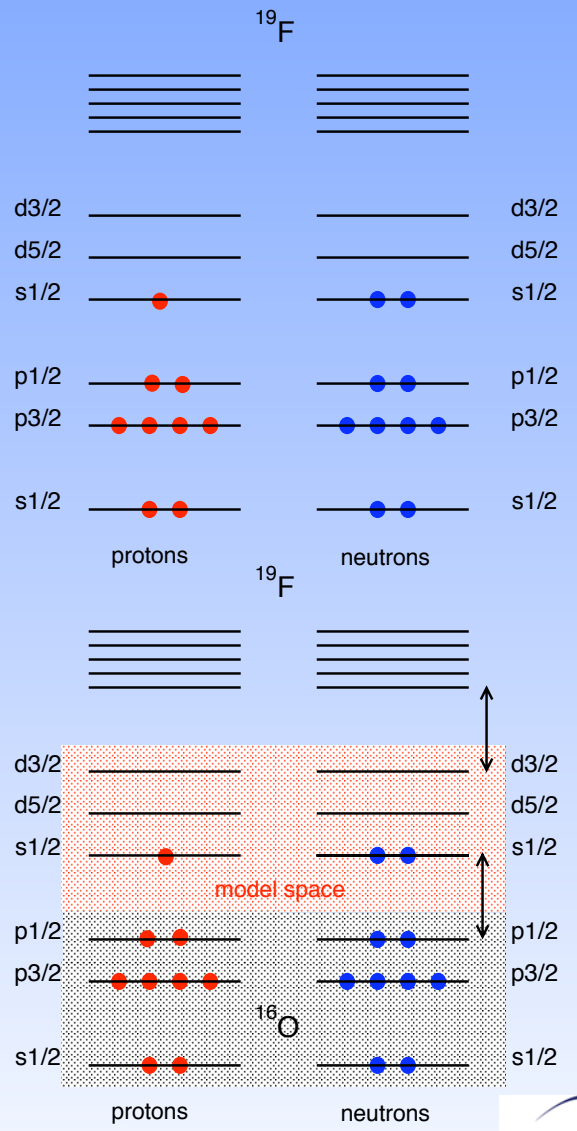
## Model space

$$|\Phi_i\rangle = [a_1^\dagger a_2^\dagger \ldots a_n^\dagger]_i|c\rangle \Rightarrow P = \sum_{i=1}^{d}|\Phi_i\rangle\langle\Phi_i|$$

## Model-space eigenvalue problem

$$H_{\rm eff}P|\Psi_\alpha\rangle = E_\alpha P|\Psi_\alpha\rangle$$



# Effective shell-model Hamiltonian

$$\left(\begin{array}{c|c} PHP & PHQ \\ \hline QHP & QHQ \end{array}\right) \quad \mathcal{H} = \Omega^{-1} H \Omega \quad \Longrightarrow \quad Q\mathcal{H}P = 0 \quad \left(\begin{array}{c|c} P\mathcal{H}P & P\mathcal{H}Q \\ \hline 0 & Q\mathcal{H}Q \end{array}\right)$$

$$H_{\text{eff}} = P\mathcal{H}P$$

$$\text{Suzuki \& Lee} \Rightarrow \Omega = e^{\omega} \text{ with } \omega = \left(\begin{array}{c|c} 0 & 0 \\ \hline Q\omega P & 0 \end{array}\right)$$

$$H_1^{\text{eff}}(\omega) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P - PH_1Q\frac{1}{\epsilon - QHQ}\omega H_1^{\text{eff}}(\omega)$$

# The shell-model effective hamiltonian

## Folded-diagram expansion

$\hat{Q}$-box vertex function

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P$$

$\Rightarrow$ Recursive equation for $H_{\text{eff}} \Rightarrow$ iterative techniques (Krenciglowa-Kuo, Lee-Suzuki, ...)

$$H_{\text{eff}} = \hat{Q} - \hat{Q}' \int \hat{Q} + \hat{Q}' \int \hat{Q} \int \hat{Q} - \hat{Q}' \int \hat{Q} \int \hat{Q} \int \hat{Q} \cdots ,$$

generalized folding

# The perturbative approach to the shell-model $H^{\text{eff}}$

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P$$

The $\hat{Q}$-box can be calculated perturbatively

$$\frac{1}{\epsilon - QHQ} = \sum_{n=0}^{\infty} \frac{(QH_1Q)^n}{(\epsilon - QH_0Q)^{n+1}}$$

The diagrammatic expansion of the $\hat{Q}$-box

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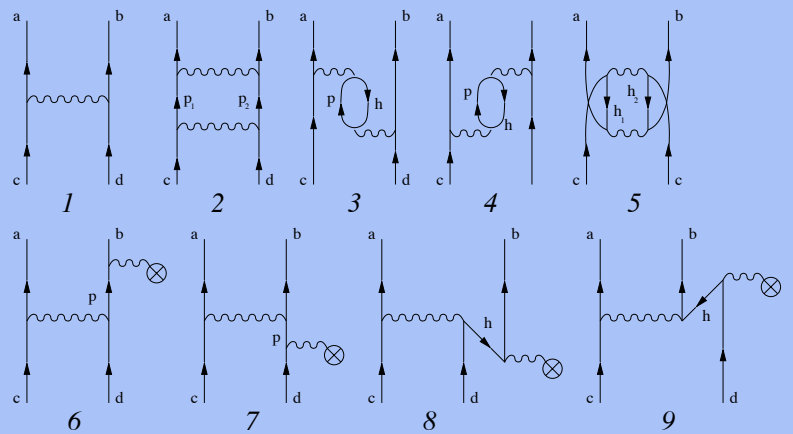

# The shell-model effective operators

$\hat{\Theta}_{\text{eff}}$ can be derived consistently in the MBPT framework

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$$\Theta_{\rm eff} = (P + \hat{Q}_1 + \hat{Q}_1\hat{Q}_1 + \hat{Q}_2\hat{Q} + \hat{Q}\hat{Q}_2 + \cdots)(\chi_0 + \chi_1 + \chi_2 + \cdots)$$

where the functions $\chi_n$ are defined as

$$\begin{aligned} \chi_0 &= (\hat{\Theta}_0 + h.c.) + \Theta_{00} \ , \\ \chi_1 &= (\hat{\Theta}_1\hat{Q} + h.c.) + (\hat{\Theta}_{01}\hat{Q} + h.c.) \ , \\ &\cdots \end{aligned}$$

in terms of the vertex functions and their derivatives

$$\begin{aligned} \hat{\Theta}(\epsilon) &= P\Theta P + P\Theta Q\frac{1}{\epsilon - QHQ}QH_1P \ , \\ \hat{\Theta}(\epsilon_1,\epsilon_2) &= PH_1Q\frac{1}{\epsilon_1 - QHQ}Q\Theta Q\frac{1}{\epsilon_2 - QHQ}QH_1P \end{aligned}$$

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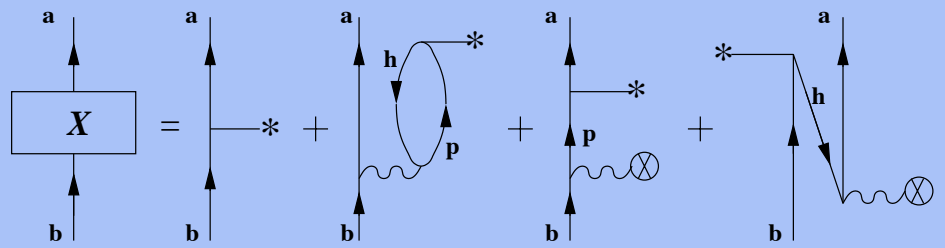
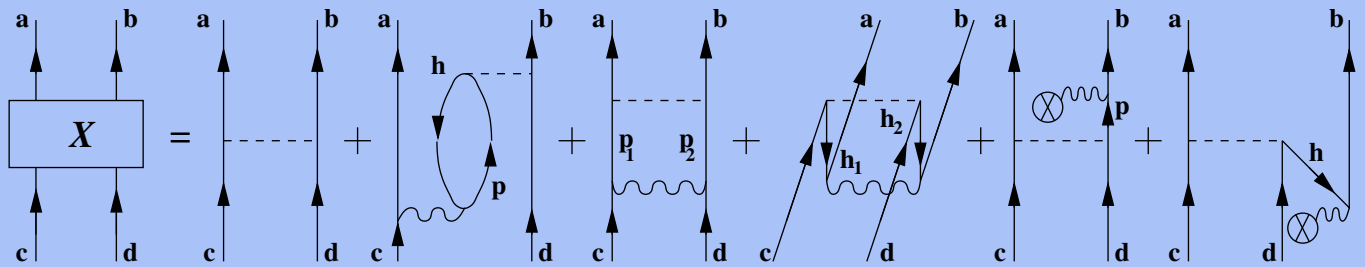
## One-body operator



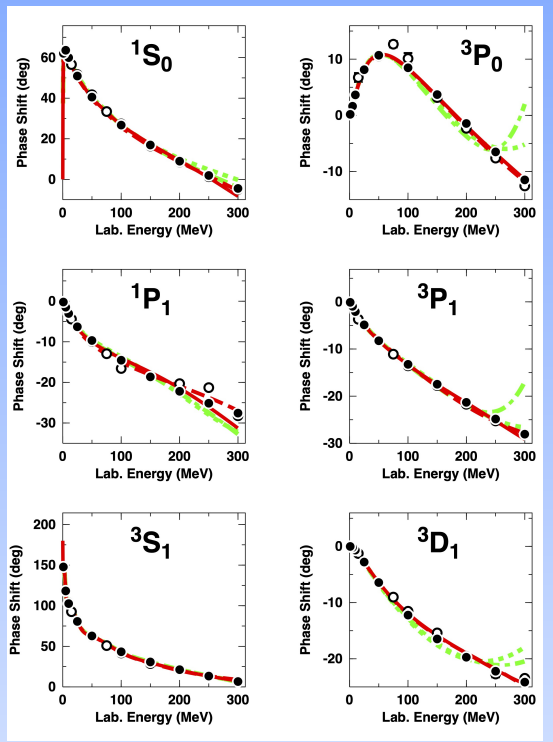

## Two-body operator




*K. Suzuki and R. Okamoto, Prog. Theor. Phys.* **93** *, 905 (1995)*

# Realistic Shell-Model Calculations

- Nuclear Hamiltonian: Entem-Machleidt $N^3LO$ two-body potential plus $N^2LO$ three-body potential ($\Lambda$ = 500 MeV)
- Axial current $\mathbf{J}_A$ calculated at $N^3LO$ in ChPT




- $H_{\text{eff}}$ obtained calculating the $Q$-box up to the 3rd order in $V_{NN}$ (up to 2p-2h core excitations) and up to the 1st order in $V_{NNN}$
- Effective operators are consistently derived by way of the MBPT

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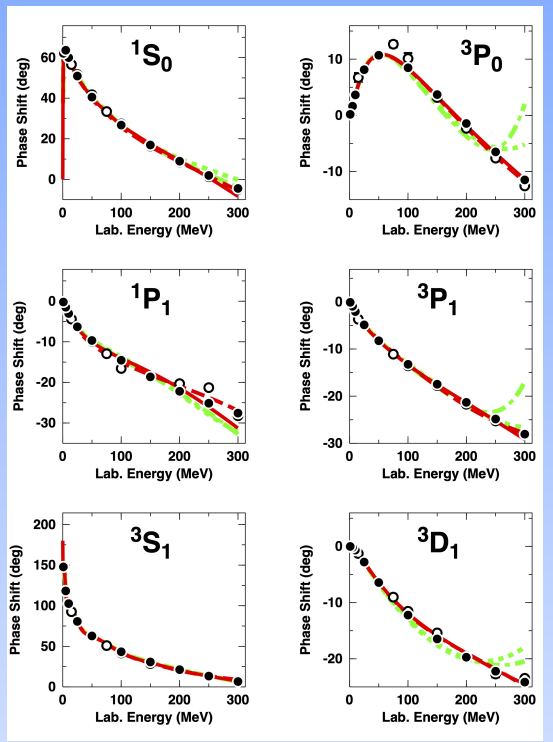


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# The axial current $\mathbf{J}_A$

The matrix elements of the axial current $\mathbf{J}_A$ are derived through a chiral expansion up to N$^3$LO, and employing the same LECs as in 2NF and 3NF

$$\mathbf{J}^{\rm LO}_{A,\pm} = -g_A \sum_i \boldsymbol{\sigma}_i \tau_{i,\pm} \ ,$$

$$\mathbf{J}^{\rm N^2LO}_{A,\pm} = \frac{g_A}{2m_N^2} \sum_i \mathbf{K}_i \times (\boldsymbol{\sigma}_i \times \mathbf{K}_i)\, \tau_{i,\pm} \ ,$$

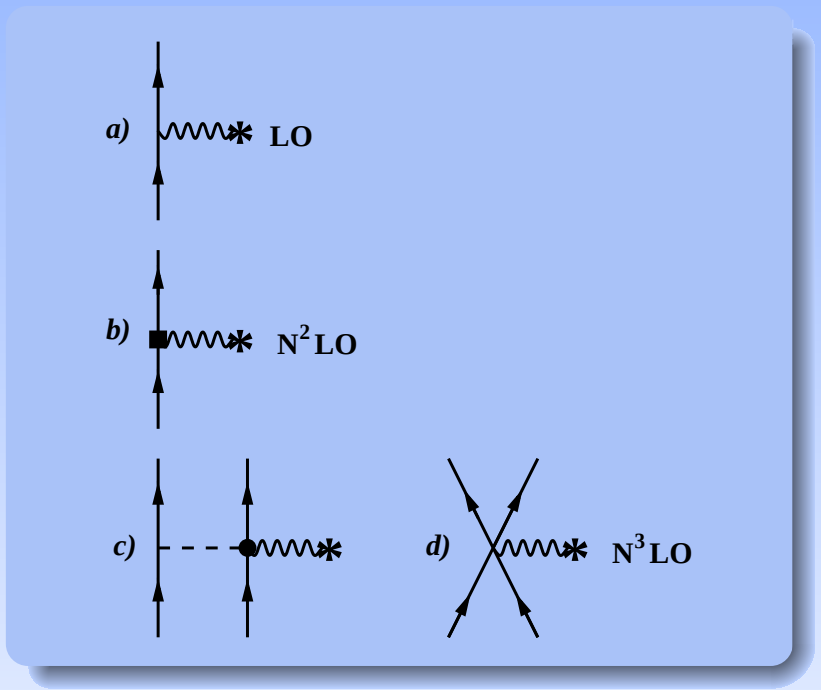
$$\mathbf{J}^{\rm N^3LO}_{A,\pm}({\rm 1PE};\mathbf{k}) = \sum_{i<j} \frac{g_A}{2f_\pi^2} \left\{ 4c_3 \tau_{j,\pm} \mathbf{k} + (\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)_\pm \right.$$

$$\left. \times \left[ \left( c_4 + \frac{1}{4m} \boldsymbol{\sigma}_i \times \mathbf{k} - \frac{i}{2m} \mathbf{K}_i \right) \right] \right\} \boldsymbol{\sigma}_j \cdot \mathbf{k} \frac{1}{\omega_k^2} \ ,$$

$$\mathbf{J}^{\rm N^3LO}_{A,\pm}({\rm CT};\mathbf{k}) = \sum_{i<j} z_0 (\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)_\pm (\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j) \ ,$$

where

$$z_0 = \frac{g_A}{2f_\pi^2 m_N} \left[ \frac{m_N}{4g_a \Lambda_\chi} c_D + \frac{m_N}{3} (c_3 + 2c_4) + \frac{1}{6} \right] \ .$$




*A. Baroni, L. Girlanda, S. Pastore, R. Schiavilla, and M. Viviani, Phys. Rev. C* **93**, *015501 (2016)*

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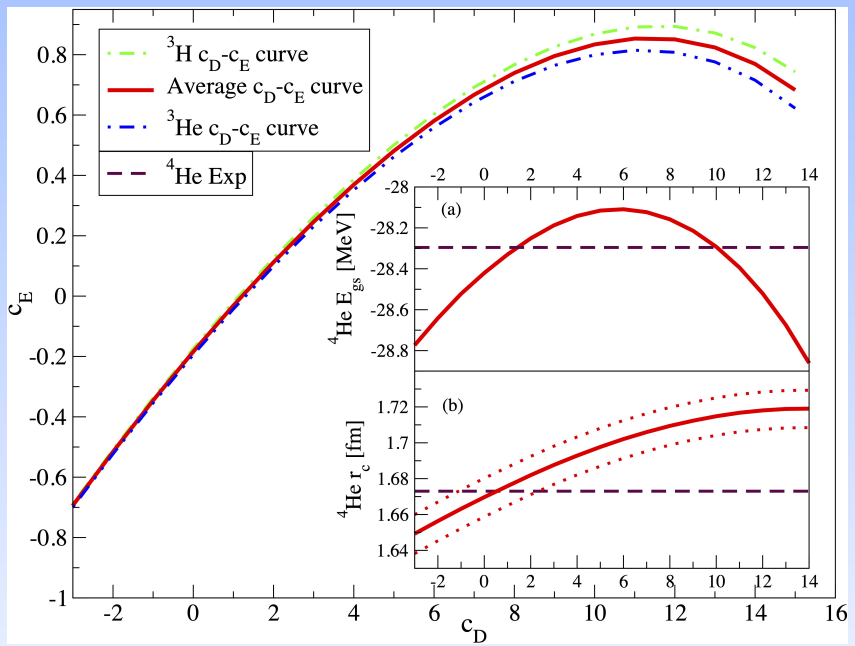
$$\left. \times \left[ \left( c_4 + \frac{1}{4m} \boldsymbol{\sigma}_i \times \mathbf{k} - \frac{i}{2m} \mathbf{K}_i \right) \right] \right\} \boldsymbol{\sigma}_j \cdot \mathbf{k} \frac{1}{\omega_k^2} \ ,$$

$$\mathbf{J}^{\rm N^3LO}_{A,\pm}({\rm CT};\mathbf{k}) = \sum_{i<j} z_0 (\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)_\pm (\boldsymbol{\sigma}_i \times \boldsymbol{\sigma}_j) \ ,$$

where

$$z_0 = \frac{g_A}{2f_\pi^2 m_N} \left[ \frac{m_N}{4 g_a \Lambda_\chi} c_D + \frac{m_N}{3} (c_3 + 2c_4) + \frac{1}{6} \right] \ .$$



# Realistic Shell-Model Calculations

- *fp*-shell nuclei: four proton and neutron orbitals outside *LS* core
  $^{40}$Ca $\rightarrow$ $0f_{7/2}0f_{5/2}, 1p_{3/2}, 1p_{1/2}$
- *fpg*-shell nuclei: four proton and neutron orbitals outside *jj* core
  $^{56}$Ni $\rightarrow$ $0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2}$

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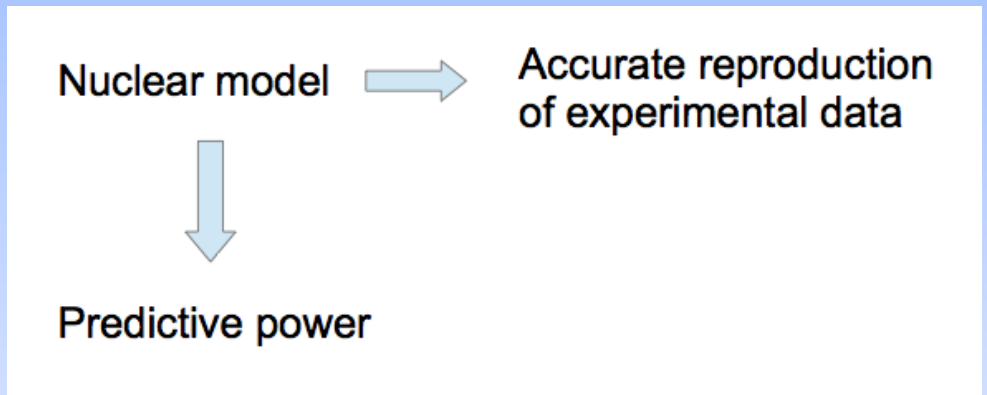
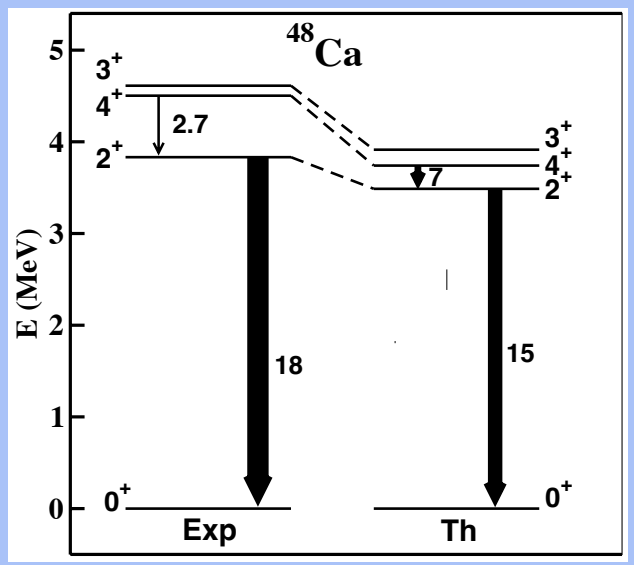
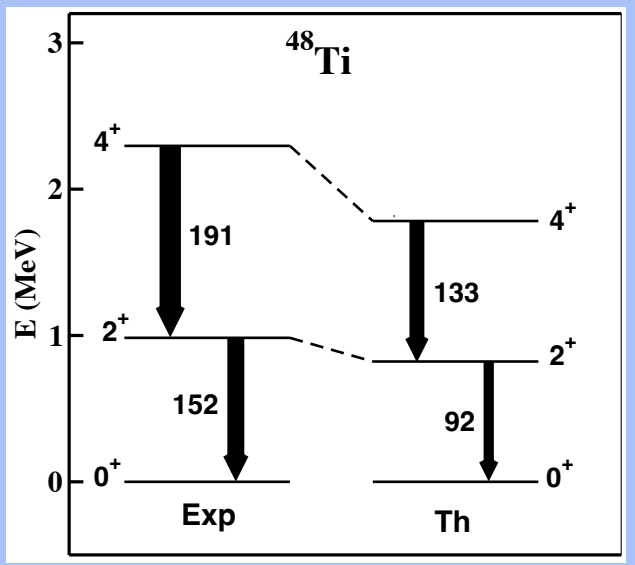



RSM calculations, starting from ChPT two- and three-body potentials and two-body electroweak currents for spectroscopic and spin-isospin dependent observables of $^{48}$Ca, $^{76}$Ge, $^{82}$Se

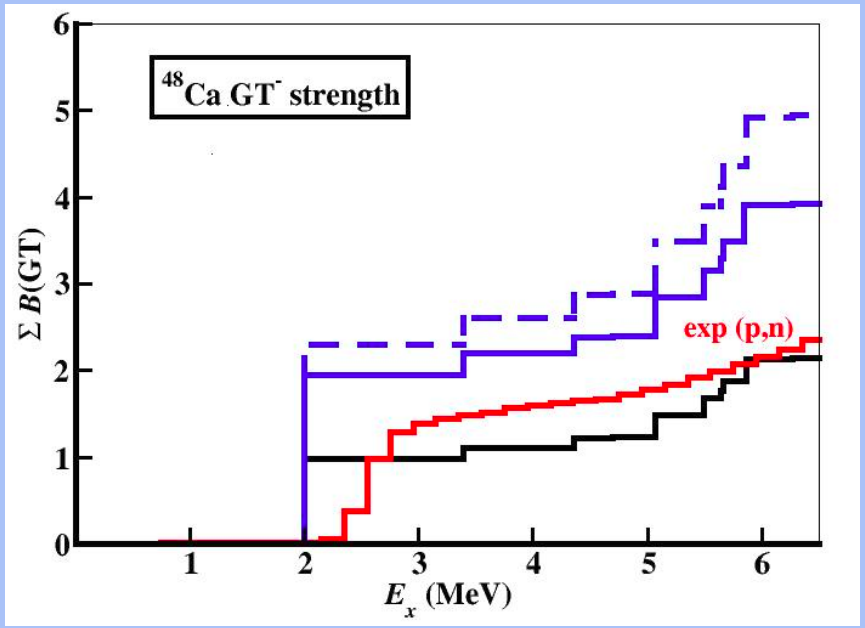
*L. Coraggio, N. Itaco, G. De Gregorio, A. Gargano, Z. H. Cheng, Y. Z. Ma, F. R. Xu, and M. Viviani, Phys. Rev. C* **109**, *014301 (2024)*

# *fp*-shell nuclei: spectroscopic properties



# $^{48}$Ca GT strength distribution




## Charge-exchange experiments

$$\left[\frac{d\sigma}{d\Omega}(q=0)\right] = \hat{\sigma} B_{exp}(GT)$$

$$B_{Th}(GT) = \frac{|\langle \Phi_f || \mathbf{J}_A || \Phi_i \rangle|^2}{2J_i + 1}$$

- - - bare $\mathbf{J}_A$ at LO in ChPT
(namely the GT operator $g_A \boldsymbol{\sigma} \cdot \boldsymbol{\tau}$)
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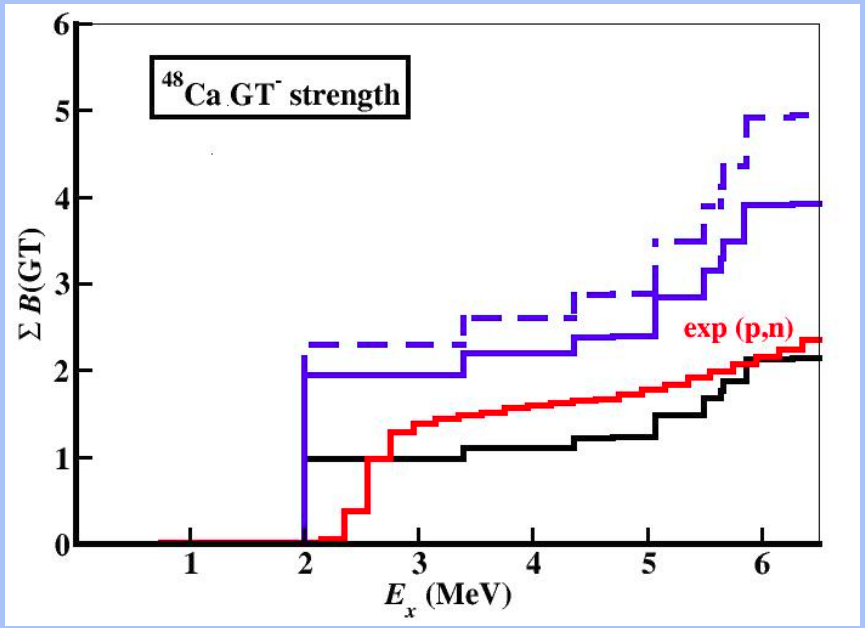


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Total GT$^-$ strength

| | (a) | (b) | (c) | Expt |
|---|---|---|---|---|
| $\sum B(GT^-)$ | 24.0 | 17.5 | 11.2 | $15.3 \pm 2.2$ |

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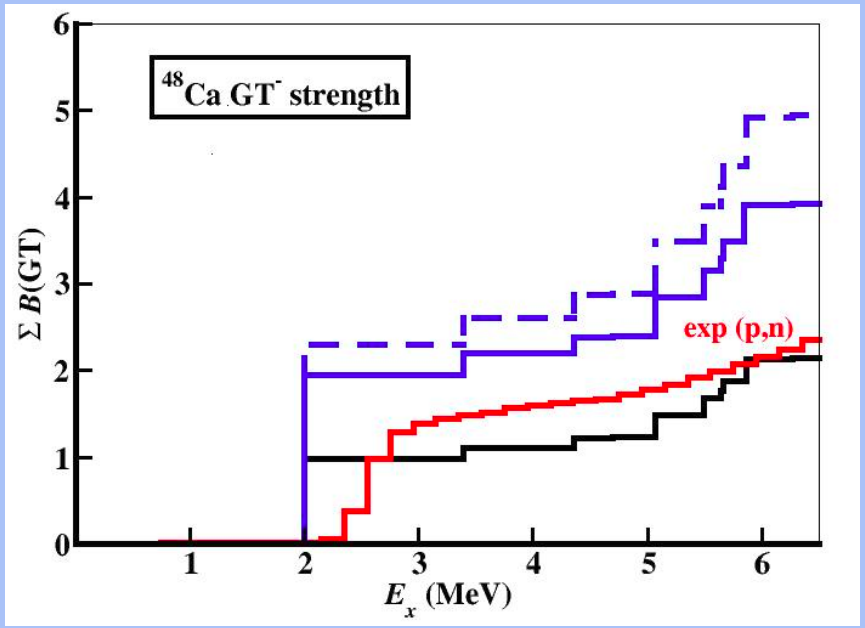


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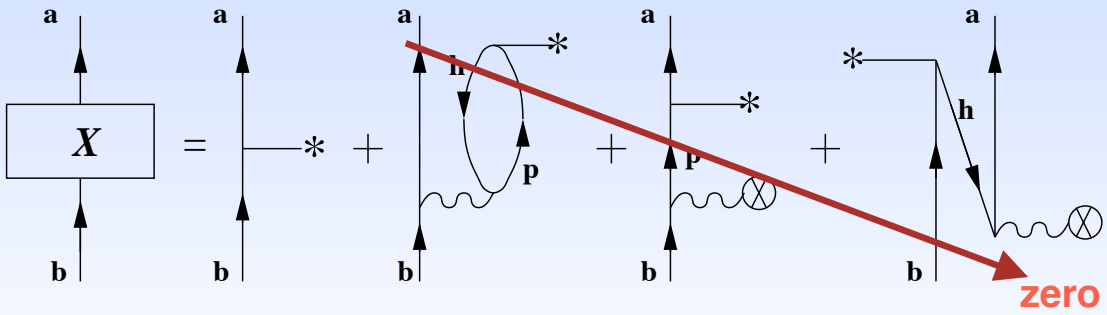
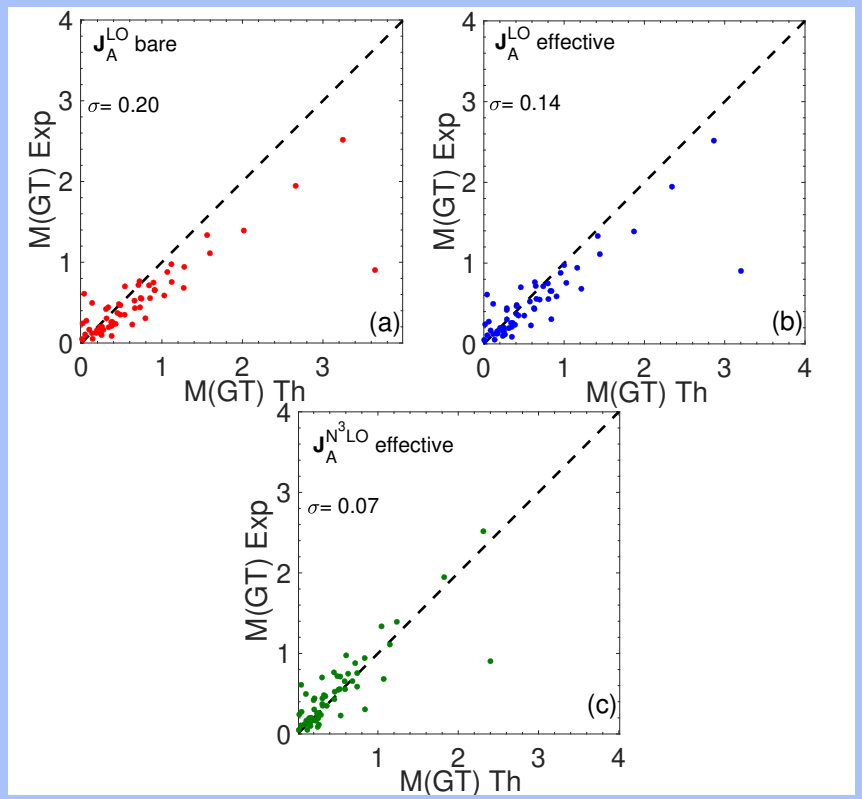


# *fp*-shell nuclei: GT matrix elements



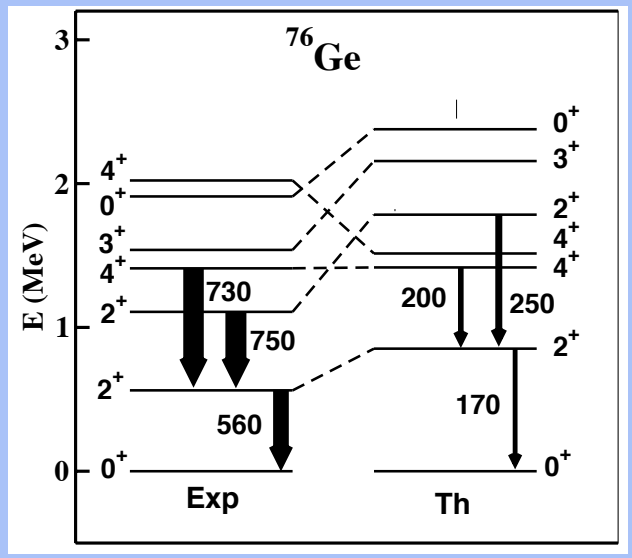

GT matrix elements of 60 experimental decays of 43 *fp*-shell nuclei

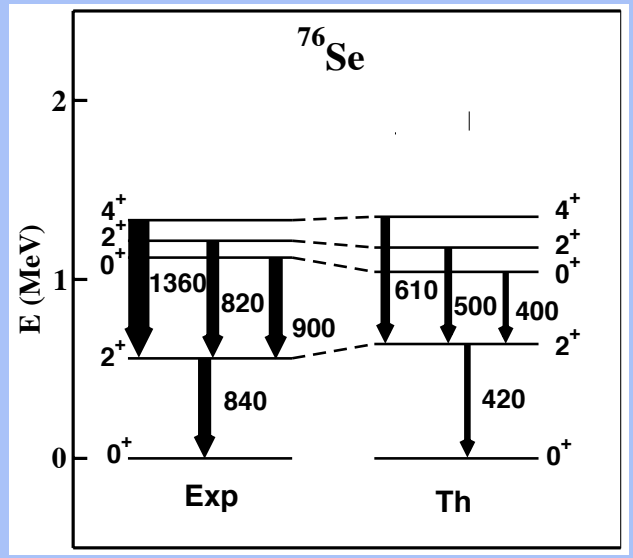
- (a) bare $\mathbf{J}_A$ at LO in ChPT (namely the GT operator $g_A\boldsymbol{\sigma}\cdot\boldsymbol{\tau}$);
- (b) effective $\mathbf{J}_A$ at LO in ChPT;
- (c) effective $\mathbf{J}_A$ at $N^3LO$ in ChPT.

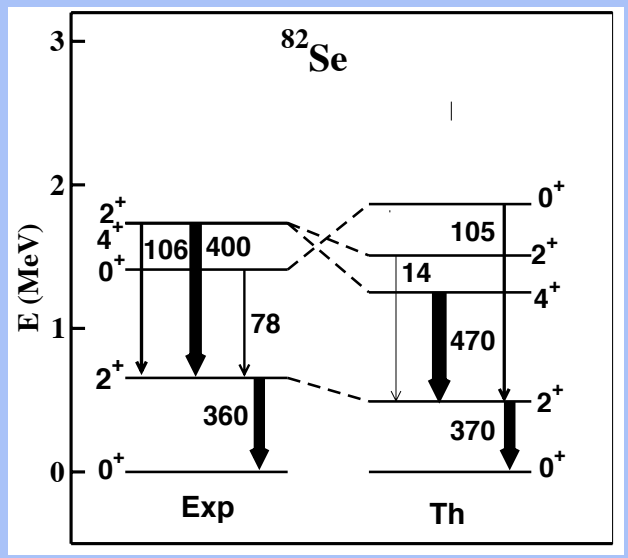
$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \hat{x}_i)^2}{n}}$$

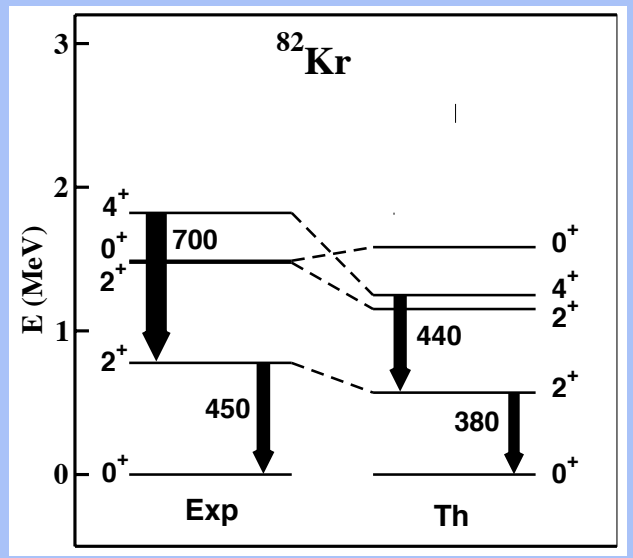
| | (a) | | (b) | | (d) |
|---|---|---|---|---|---|
| $\sigma$ | 0.20 | $\rightarrow$ | 0.14 | $\rightarrow$ | 0.07 |

# $0f_{5/2}1p0g_{9/2}$-shell nuclei spectroscopic properties



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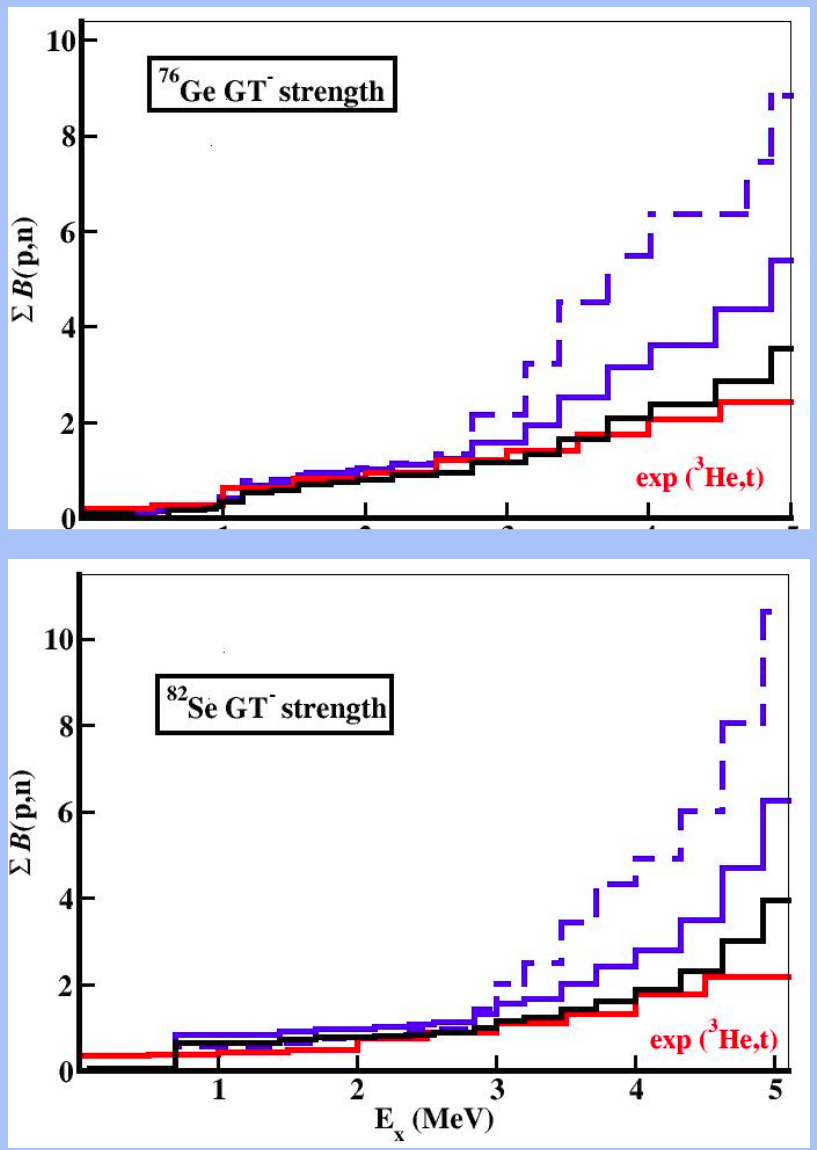


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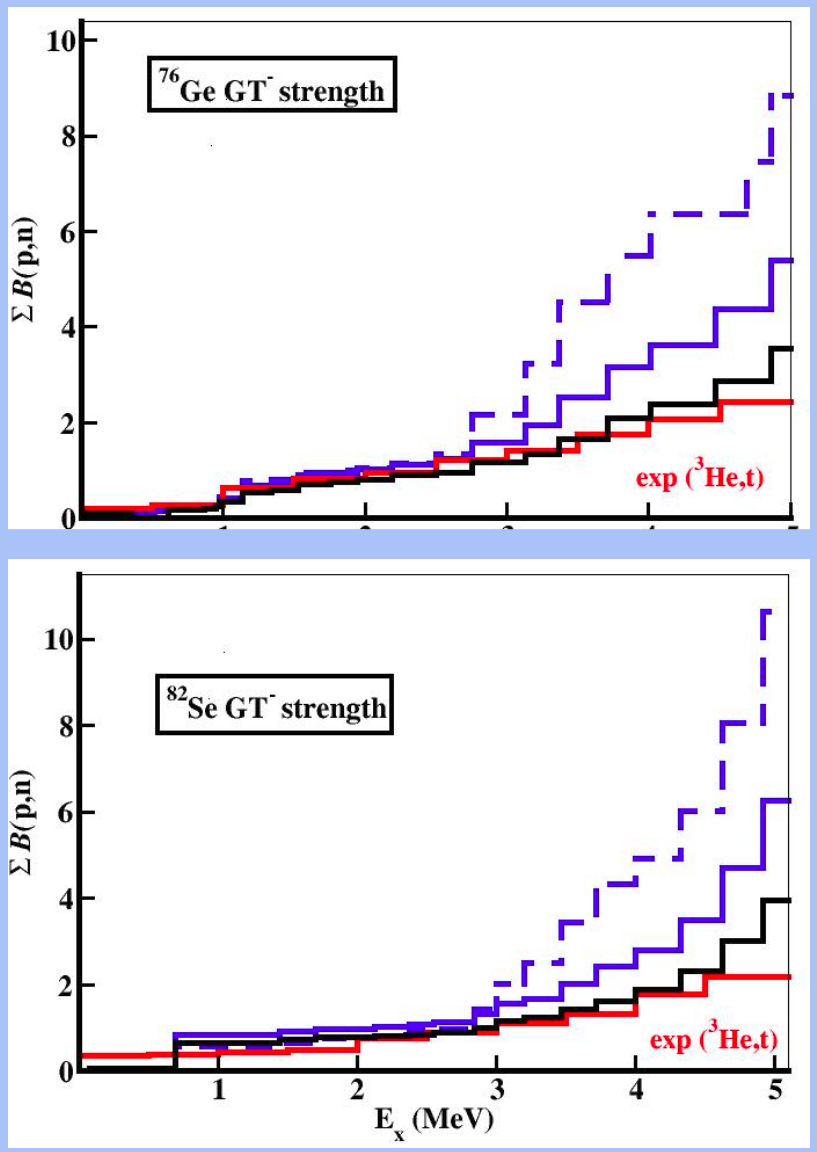


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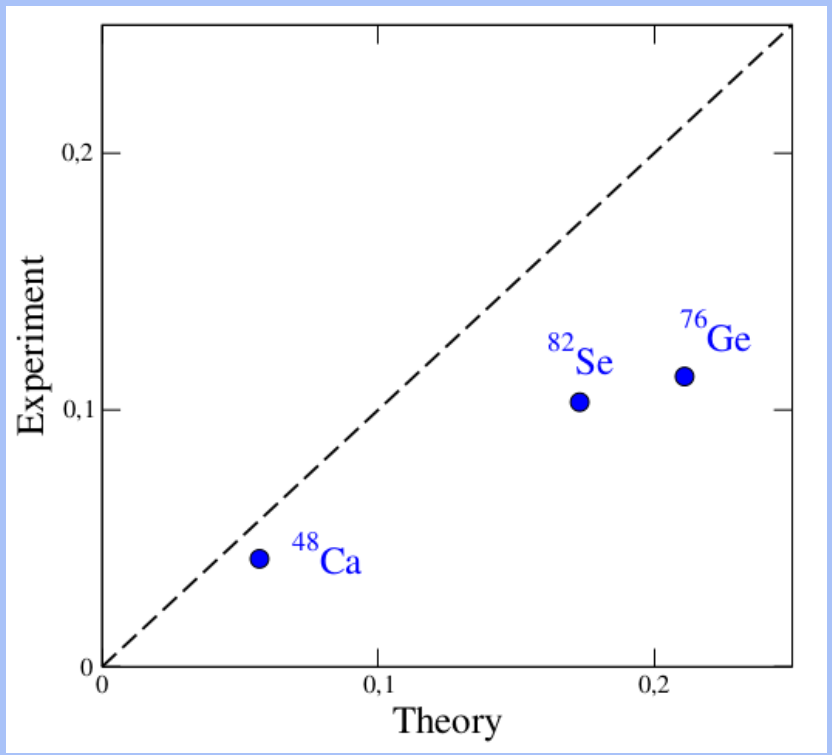
Total GT$^-$ strength

| | (a) | (b) | (c) | Expt |
|---|---|---|---|---|
| $^{76}$Ge | 15.8 | 10.6 | 7.1 | $\sim$ |
| $^{82}$Se | 19.0 | 11.9 | 7.6 | $\sim$ |

# $2\nu\beta\beta$ nuclear matrix elements

$$M_{2\nu}^{\mathrm{GT}} = \sum_n \frac{\langle 0_f^+ || \mathbf{J}_A || 1_n^+ \rangle \langle 1_n^+ || \mathbf{J}_A || 0_i^+ \rangle}{E_n + E_0}$$




**Blue dots:**
bare $\mathbf{J}_A$ at LO in ChPT (namely the GT operator $g_A \boldsymbol{\sigma} \cdot \boldsymbol{\tau}$)

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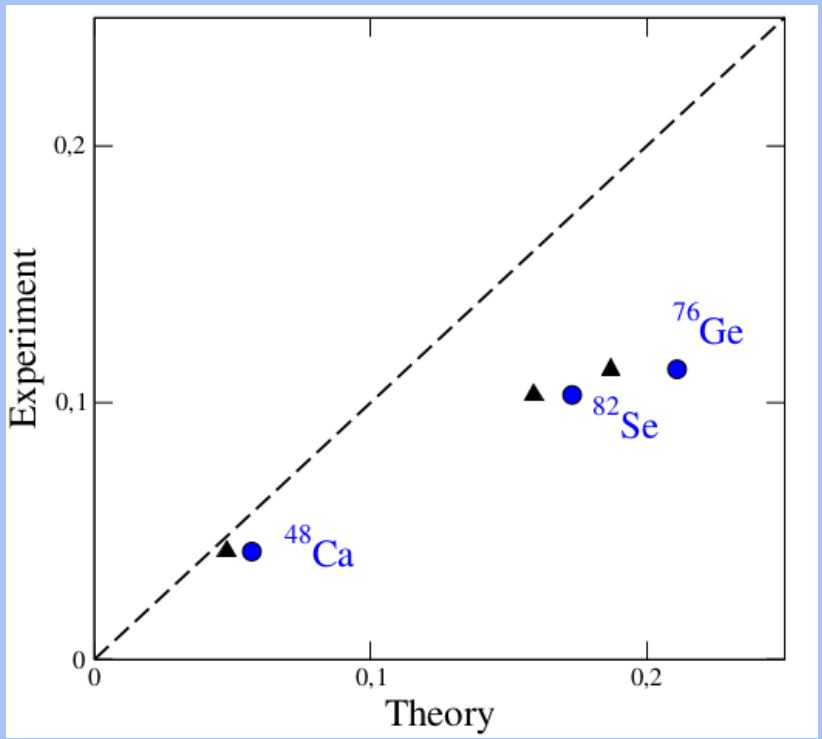


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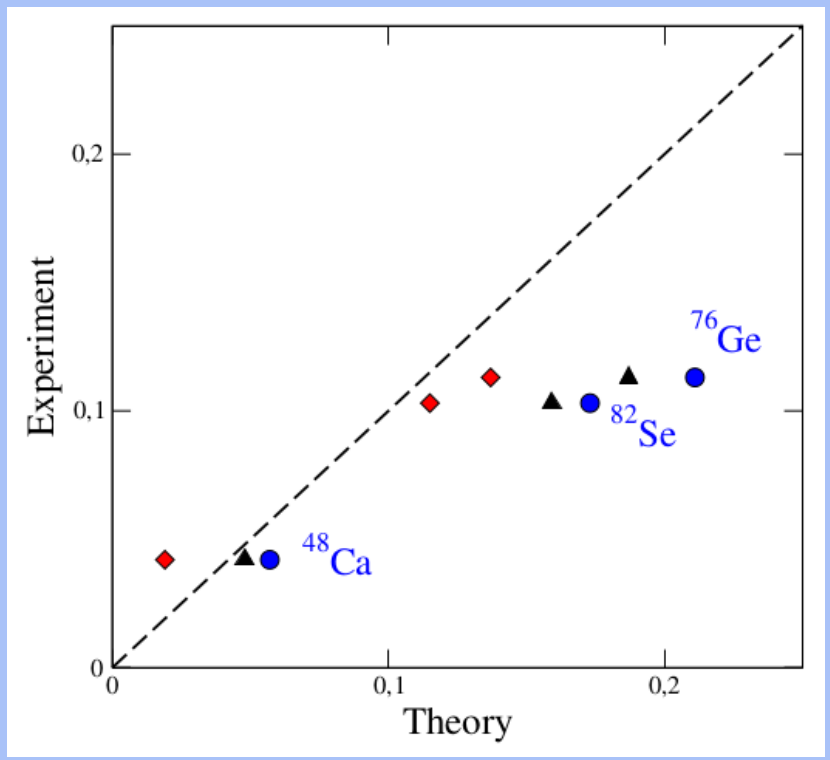


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**Black triangles:**
effective $\mathbf{J}_A$ at LO in ChPT

**Red diamonds:**
effective $\mathbf{J}_A$ at N$^3$LO in ChPT

# Conclusions & Perspectives

- Correlations + electroweak 2BC $\Rightarrow$ quite good description of $\sigma\tau$ observables
- 2BC introduce $\simeq$ 20% reduction of GT matrix elements

- Two-body currents for the $M1$ transitions
- Calculations for heavier-mass systems ($^{100}$Mo, $^{130}$Te, $^{136}$Xe)
- Calculating $0\nu\beta\beta$ decay $M^{0\nu}$ in the same $\chi PT$ framework: LO contact term, three-body currents, ..

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# Question for the discussion

- Is it possible to robustly quantify the error due to the truncation in the EFT expansion?

# Realistic Shell Model Study of Electroweak Decays: a Chiral Perturbation Theory Approach

Nunzio Itaco

Università della Campania “Luigi Vanvitelli”
Istituto Nazionale di Fisica Nucleare - Sezione di Napoli

Electroweak Physics InterseCtions - EPIC 2024