Overview on the three-body interactions with strangeness

Raffaele Del Grande[1,*]

[1] Physik Department E62, Technische Universität München, 85748 Garching, Germany

JENAA workshop on Nuclear Physics at the LHC and connections to astrophysics

Geneva, Switzerland

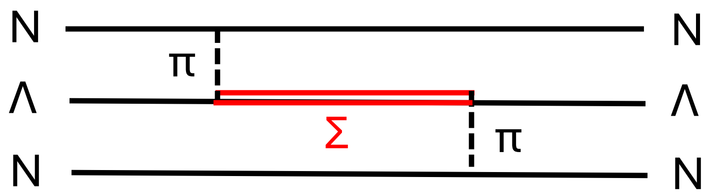
19th August 2024

***raffaele.del-grande@tum.de**

Three-body dynamics with hyperons

Dynamics of baryons involves formation of hadronic excitations

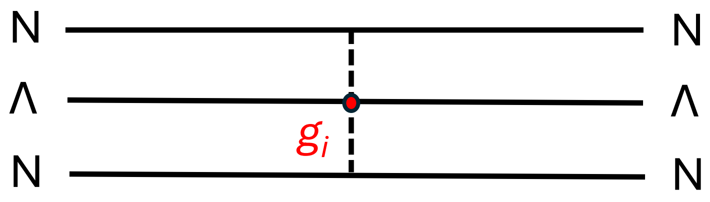
H.-W. Hammer, S. König, U. van Kolck RMP 92 (2020)



Short-range dynamics

Three-body forces in Effective Field Theories

E. Epelbaum, H.-W. Hammer, U.-G. Meißner, RMP 81, 1773 (2009)

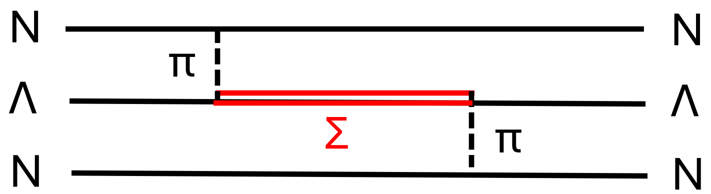


g_i constants to be fixed by the **experimental data**

Three-body dynamics with hyperons

Dynamics of baryons involves formation of hadronic excitations

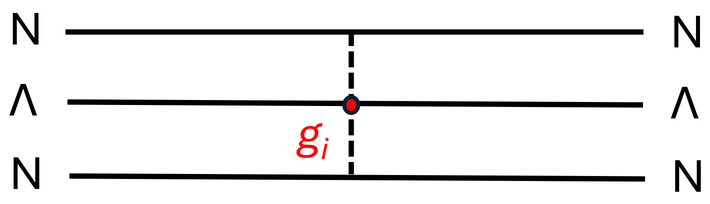
H.-W. Hammer, S. König, U. van Kolck RMP 92 (2020)



Short-range dynamics

Three-body forces in Effective Field Theories

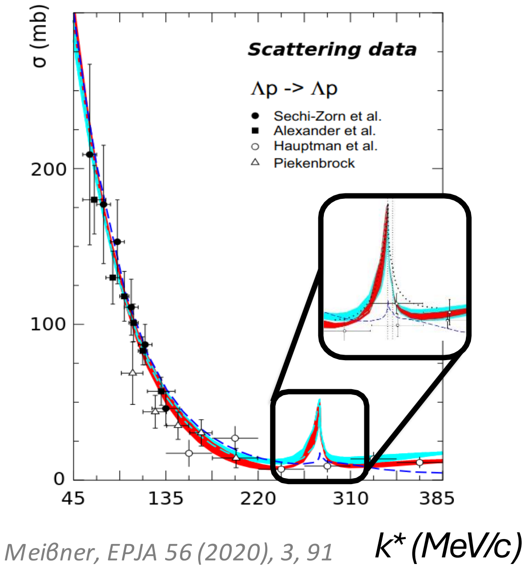
E. Epelbaum, H.-W. Hammer, U.-G. Meißner, RMP 81, 1773 (2009)



g_i constants to be fixed by the **experimental data**

Cusp structure:
ΛN-ΣN coupling

Talk by L. Serksnyte



J.Haidenbauer, U. Meißner, EPJA 56 (2020), 3, 91

Λ-Hypernuclei: 35 measurements

A. Gal, E. V. Hungerford, and D. J. Millener, RMP 88, 035004 (2016)

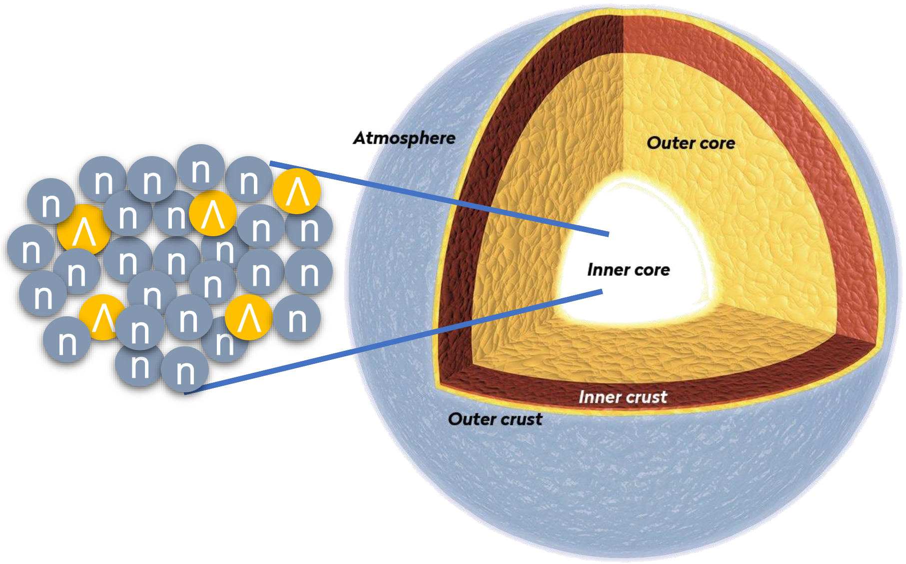
- **Strongly dependent on ΛN interaction**
- **Average distances**: about 2 fm (Hypertriton: Λ-d about 10 fm)

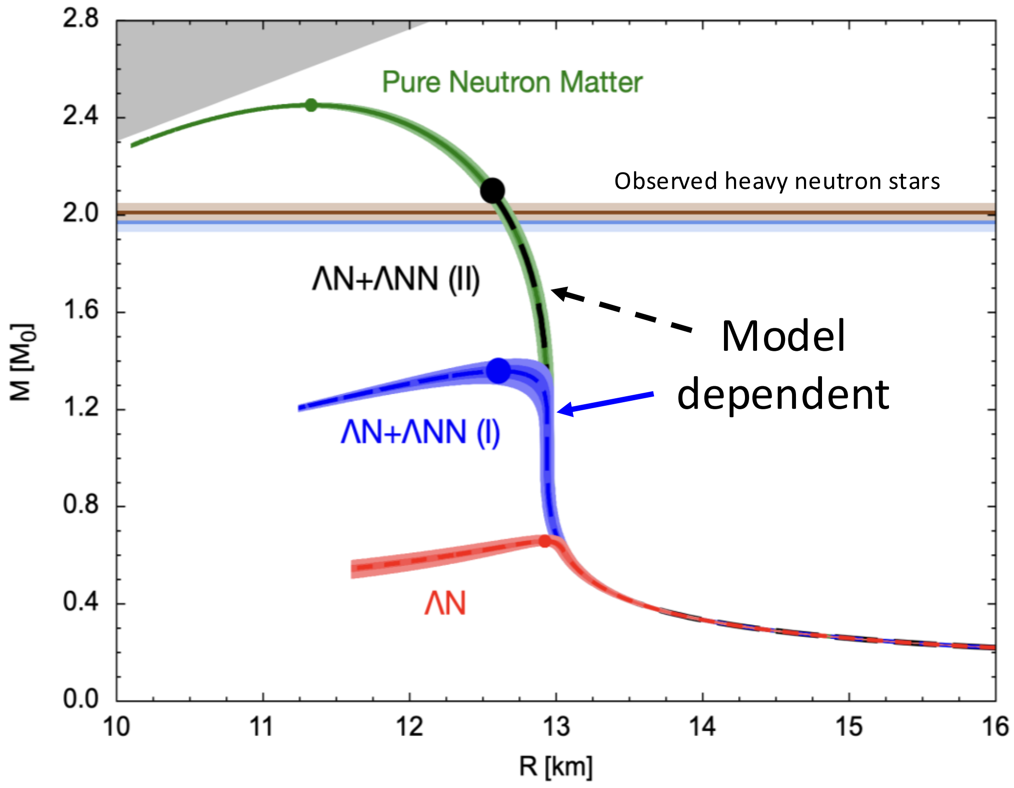
The NΛ and NNΛ interactions in neutron stars

Stronger impact on dense nuclear matter?

D. Logoteta et al., EPJA 55 (2019); D. Lonardoni et al., PRL 114 (2019)

Large densities (3-5ρ_0) → Small distances (0.8-1.0 fm)



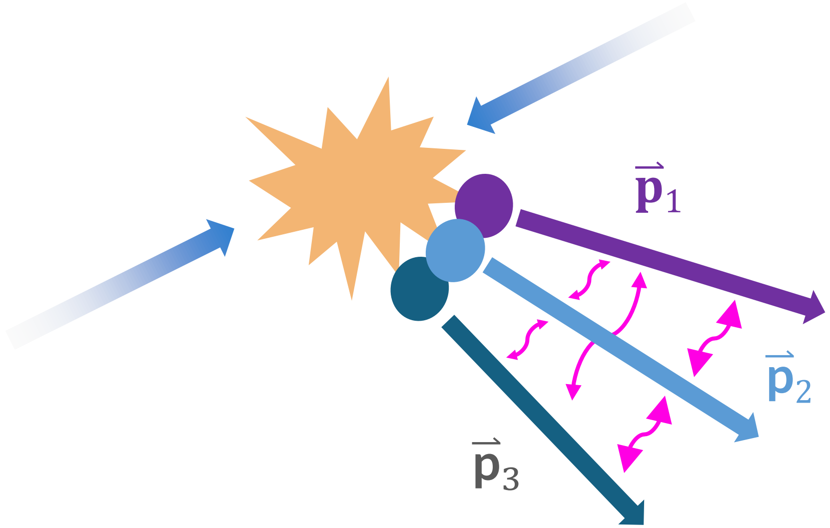


D. Lonardoni et al., PRL 114 (2019)

Small particle distances can be accessed using femtoscopy!

Talk by I. Vidana

Femtoscopy in three-particle system



Correlation function:

$$C(Q_3) = \int S(\rho)|\psi\,(Q_3, \rho\,)|^2\,\rho^5\,d\rho$$

Three-body scattering wave function

Hyper-momentum:

$$Q_3 = 2\sqrt{k_{12}^2 + k_{23}^2 + k_{31}^2}$$

R. Del Grande et al. EPJC 82 (2022) 244
ALICE Coll., EPJ A 59, 145 (2023)

Hyper-radius:

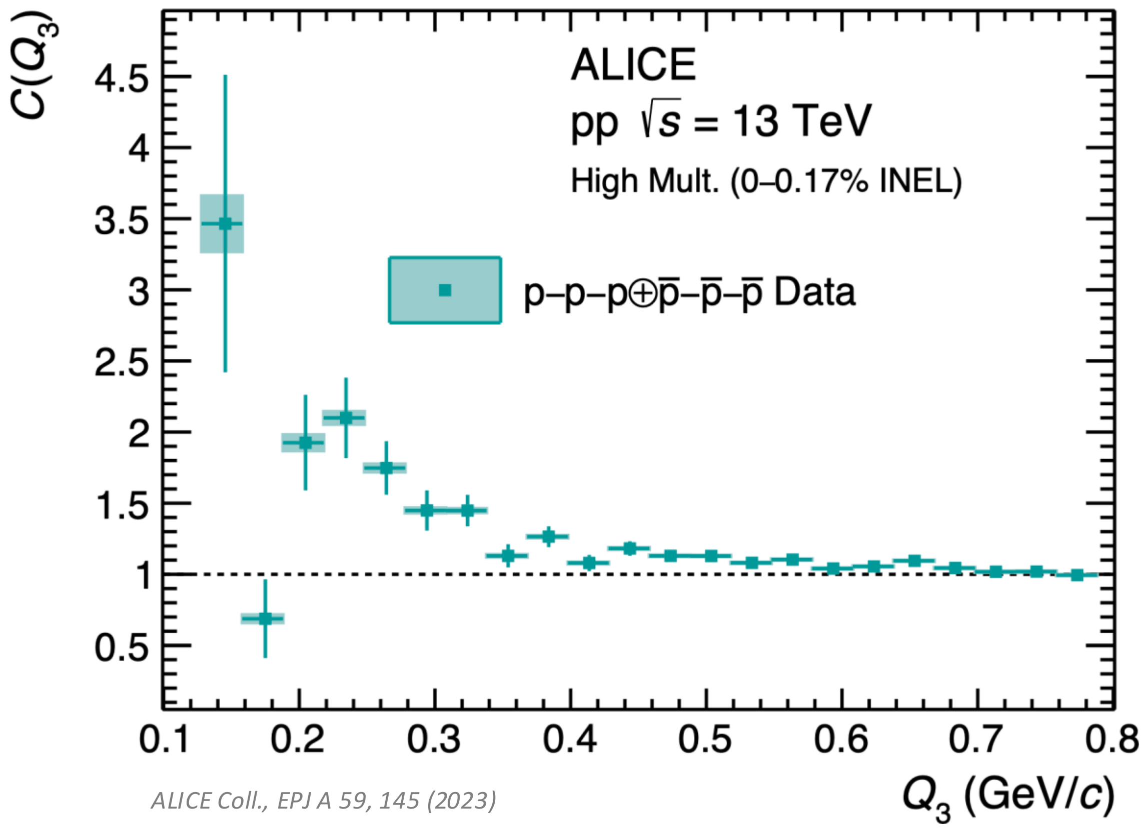
$$\rho = 2\sqrt{r_{12}^2 + r_{23}^2 + r_{31}^2}$$

L. E. Marcucci et al., Front. in Phys. 8, 69 (2020).

Extension to three-particle system

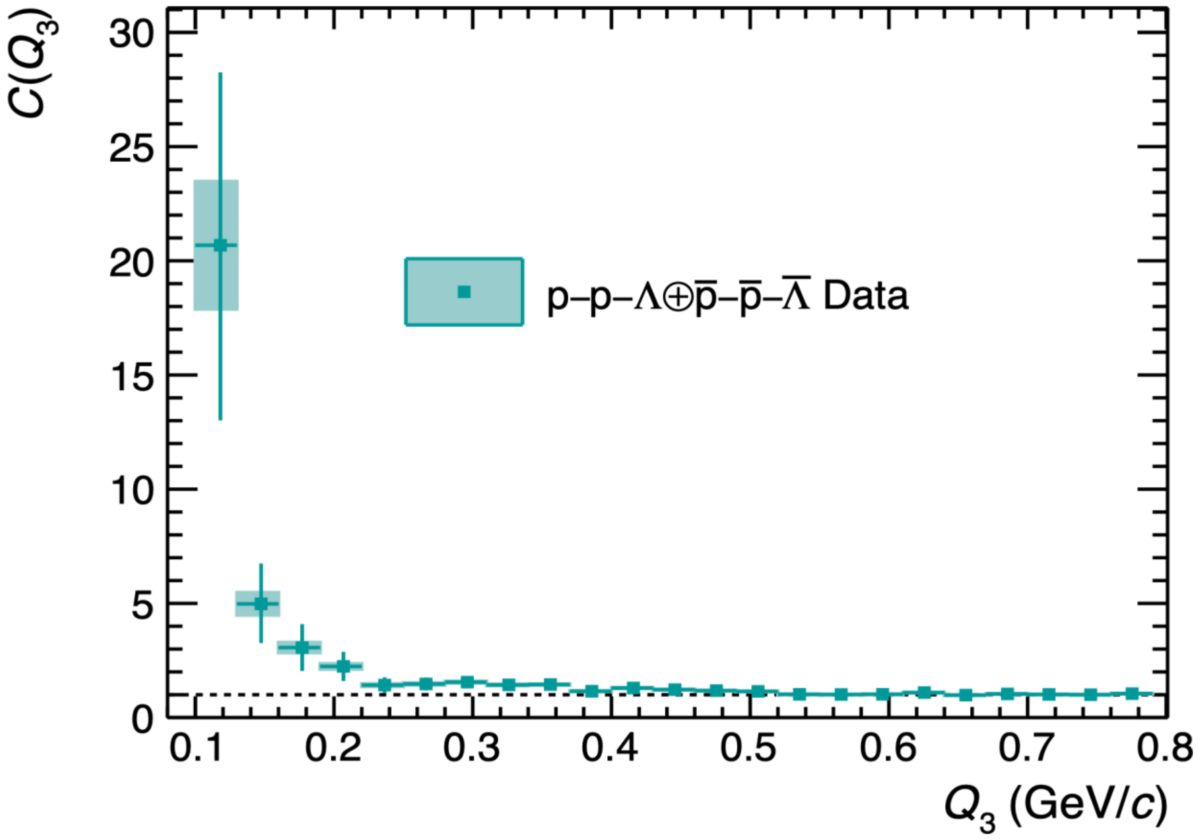
- First measurement of the free scattering of three hadrons
- Deviation from unity in p-p-p and p-p-Λ correlation functions

p-p-p



ALICE Coll., EPJ A 59, 145 (2023)

p-p-Λ



Source function for three particles in pp collisions

- Source function derived from three independent Gaussian emitters ALICE Coll., Phys. Lett. B 811, 135849 (2020) ALICE Coll., arXiv:2311.14527 (2023)

- In hypersperical coordinates

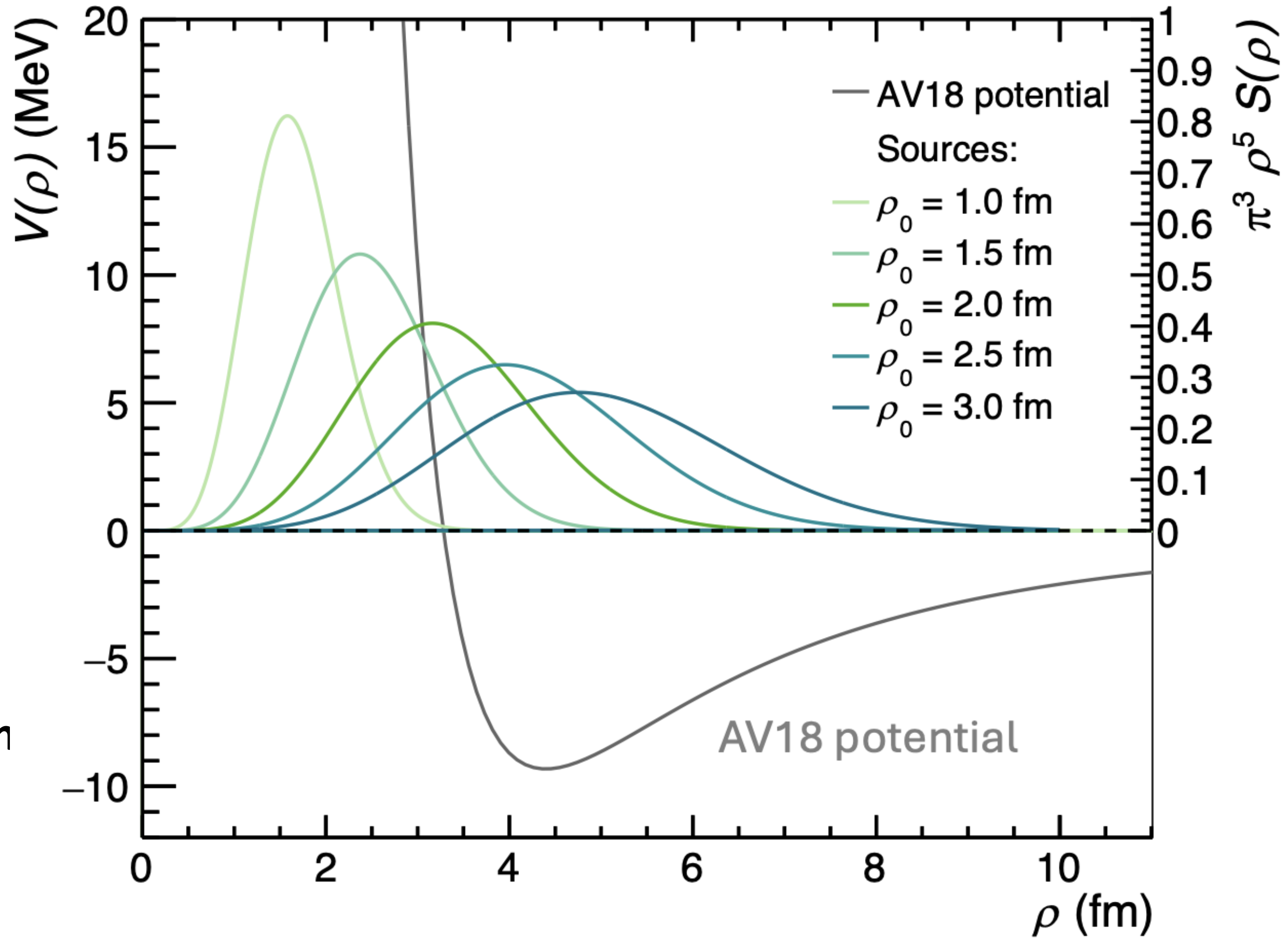
$$S(\rho) = \frac{1}{\pi^3 \rho_0^6} e^{-\left(\frac{\rho}{\rho_0}\right)^2}$$

with $\rho_0 = 2\, r_o$ and r_o is two-body source size.

A. Kievsky, et al., Phys.Rev.C 109 (2024) 3, 034006

- The value of ρ_0 is determinted from the m_T of the pairs in the triplets

- **In pp collsions at the LHC small source: ρ_0 = 1-3 fm**



A. Kievsky, et al., Phys.Rev.C 109 (2024) 3, 034006

p-p-p correlation function

- Wave function in hyperspherical harmonics

$$\Psi(\rho, Q_3) = \sum_K R_K(\rho) Y_K(\Omega)$$

- First ever full three-body correlation function calculations

three-proton wave function

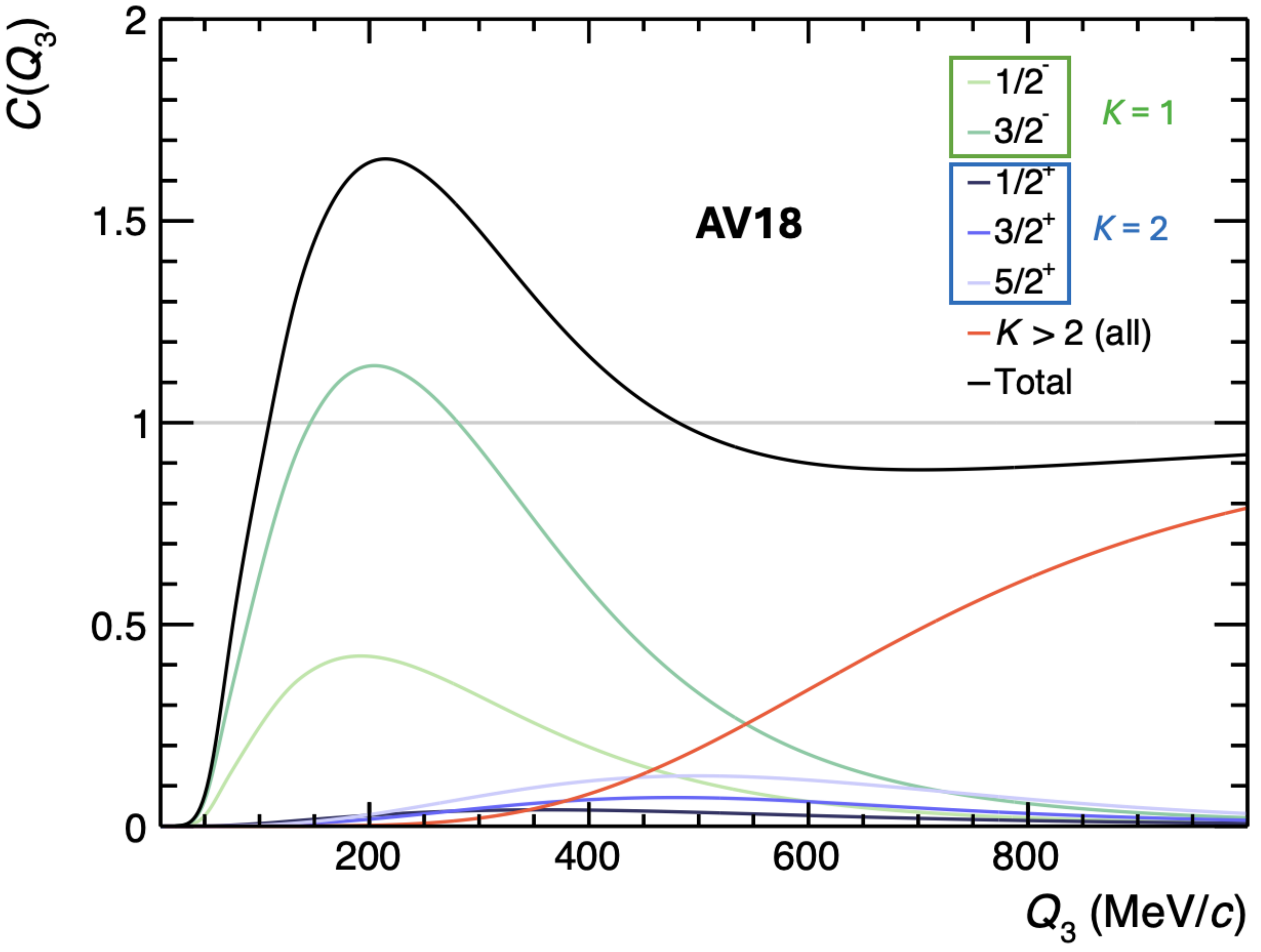
$$C(Q_3) = \int \rho^5 d\rho\, S(\rho, \rho_0) |\Psi(\rho, Q_3)|^2$$

hyperradius

- Interactions:
  - pp strong interaction (AV18)
  - Coulomb
  - No three-body forces

A. Kievsky, et al., Phys.Rev.C 109 (2024) 3, 034006

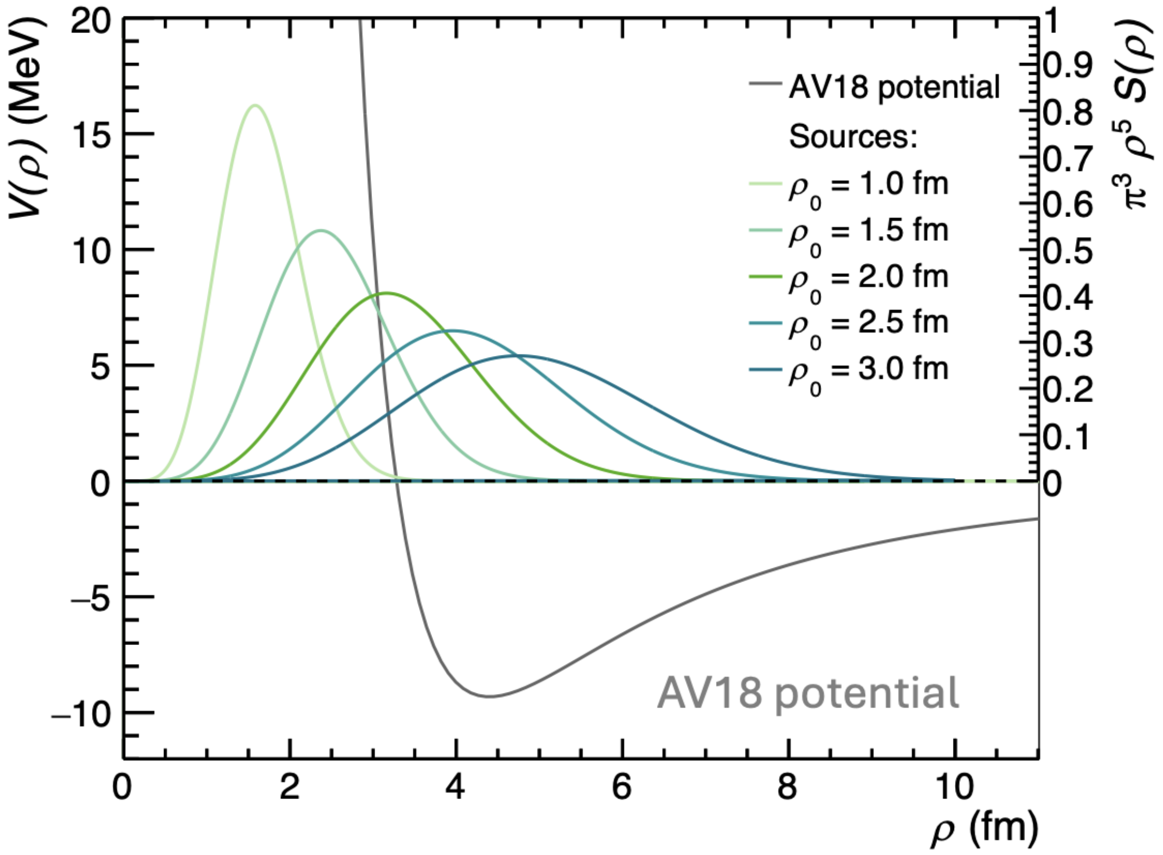
- p-p-p correlation function:
  superposition of many partial waves

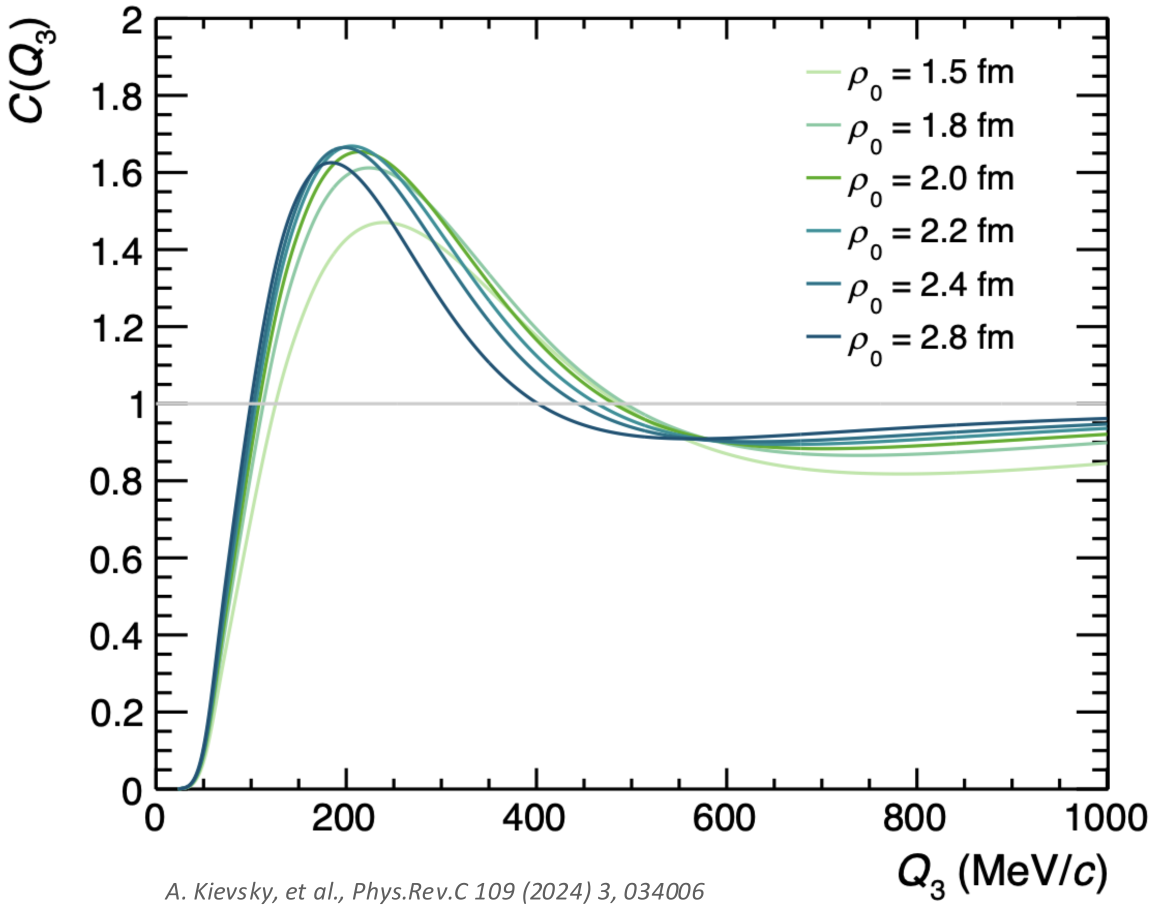


A. Kievsky, et al., Phys.Rev.C 109 (2024) 3, 034006

Influence of the source size

- Using m_T (source size) differential studies we can probe the interaction with the distances





A. Kievsky, et al., Phys.Rev.C 109 (2024) 3, 034006

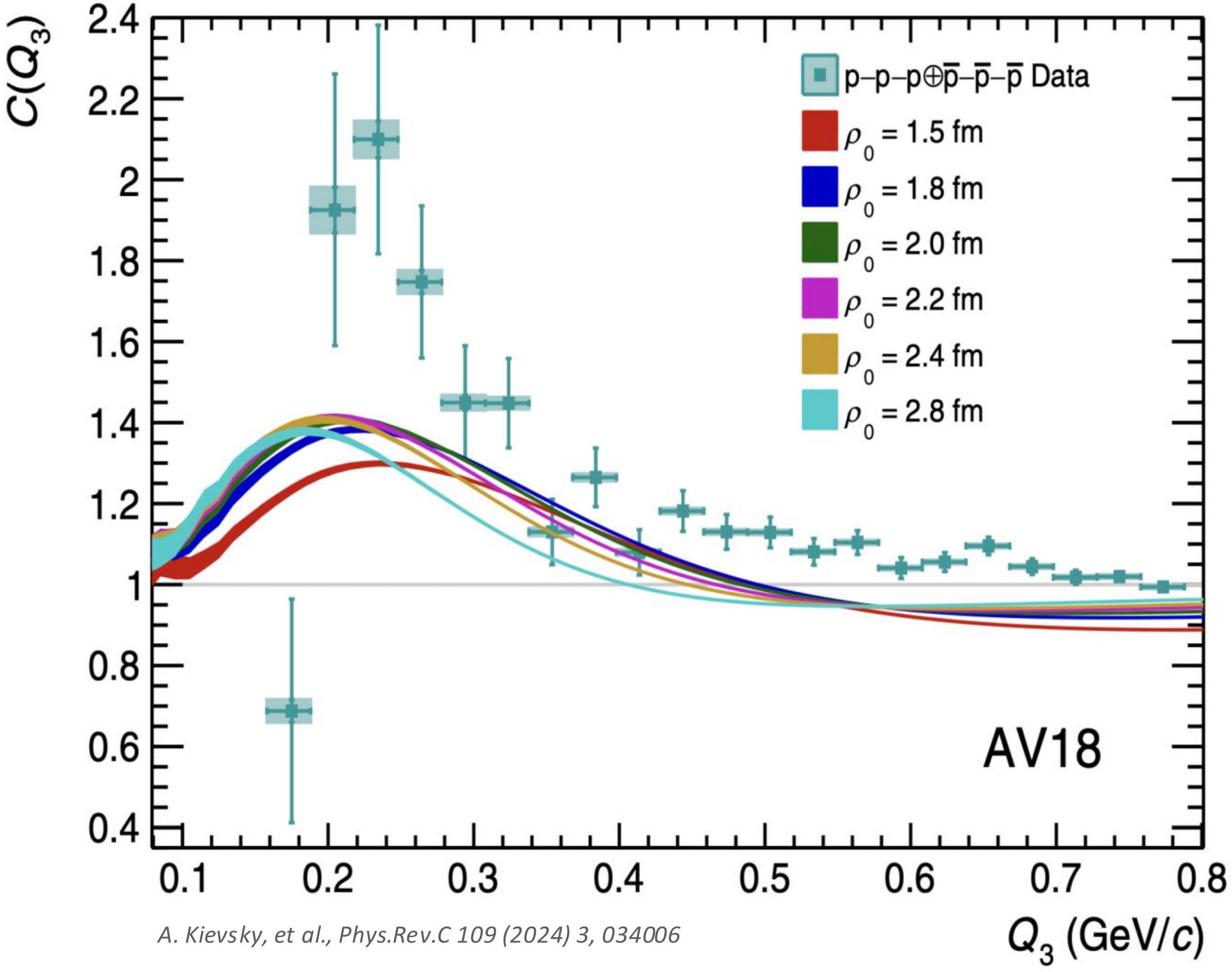
Comparison Run-2 data

Comparison with the ALICE Run-2 measurement:

- calculations can describe the shape observed in the data

Required improvements:

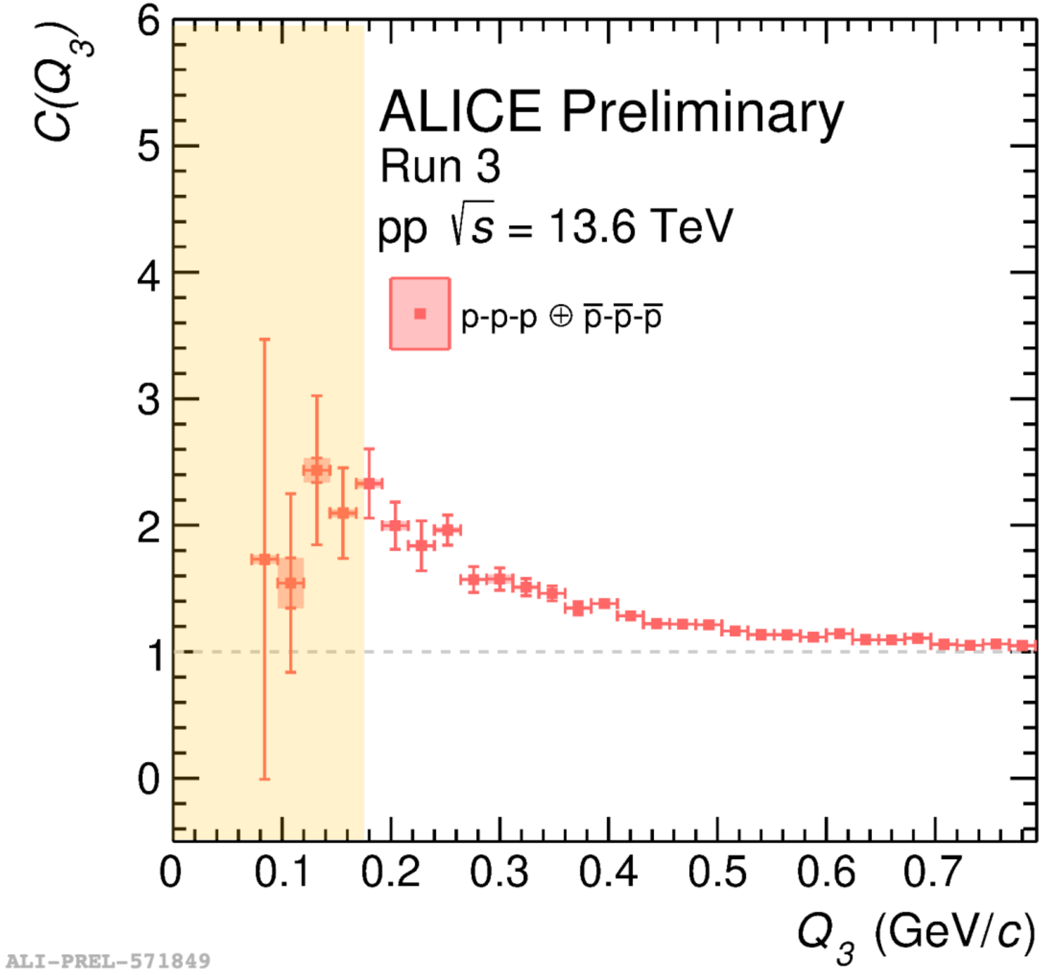
- source model based on two-body femtoscopy measurements
- feed-down contribution from p-p-Λ must be evaluated
- **More precision in the data is required**



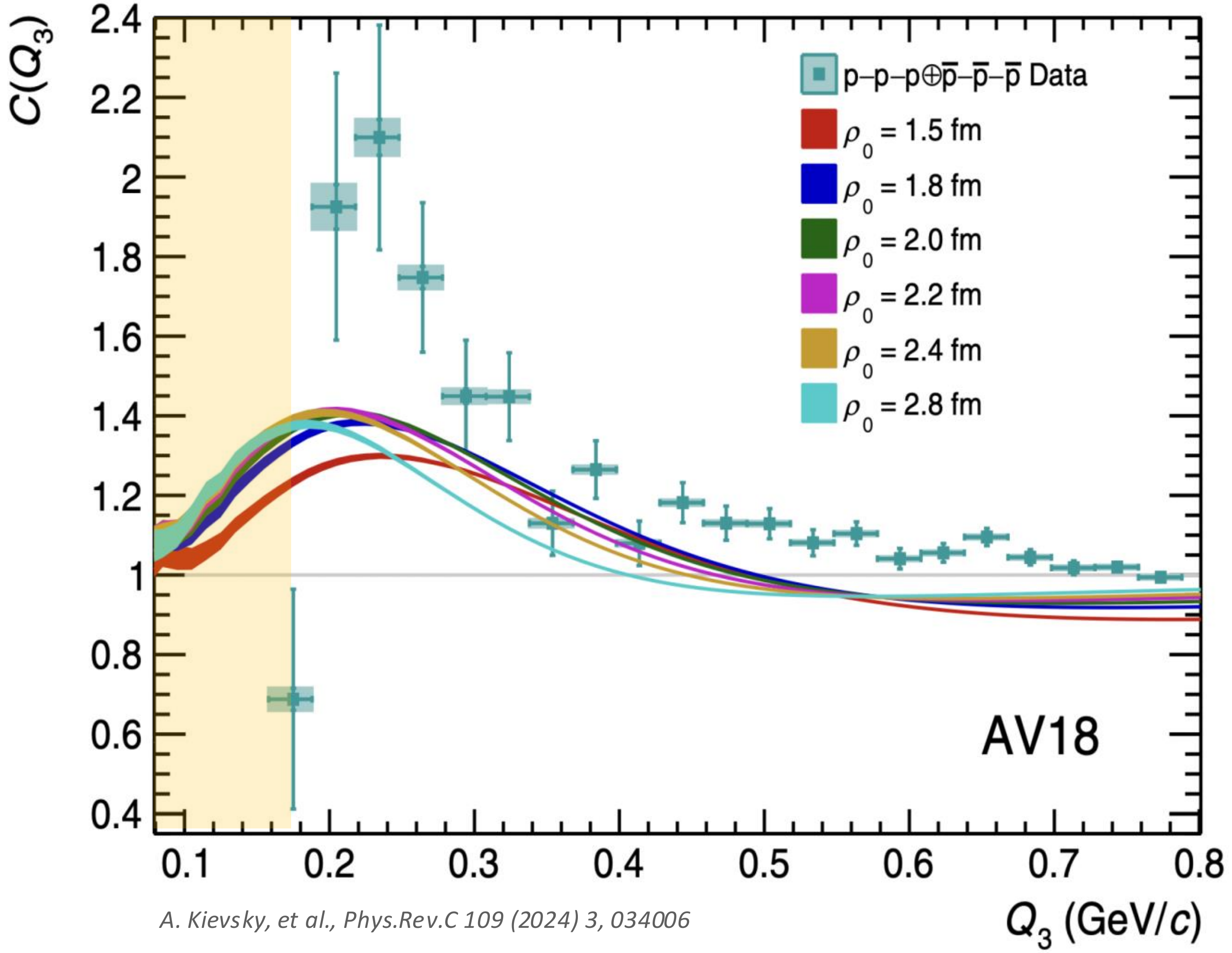
A. Kievsky, et al., Phys.Rev.C 109 (2024) 3, 034006

p-p-p correlation function in Run-3

- ALICE Run 3 data from 2022 are promising!



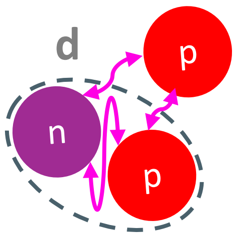
- ALICE Run-2 (2015-2018)

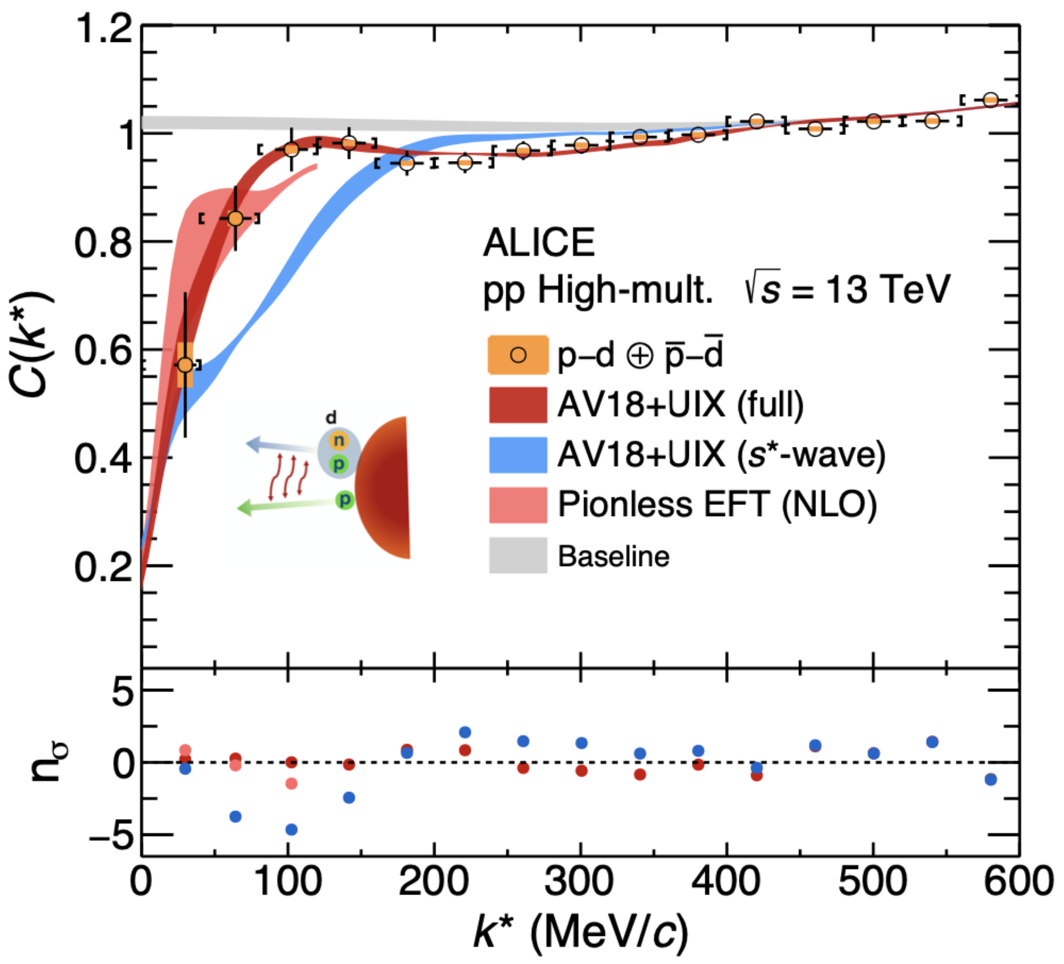


A. Kievsky, et al., Phys.Rev.C 109 (2024) 3, 034006

NNN using proton-deuteron correlations

- Full three-body calculations are required (NN + NNN + Quantum Statistics)
- Hadron-nuclei correlations at the LHC can be used to study many-body dynamics

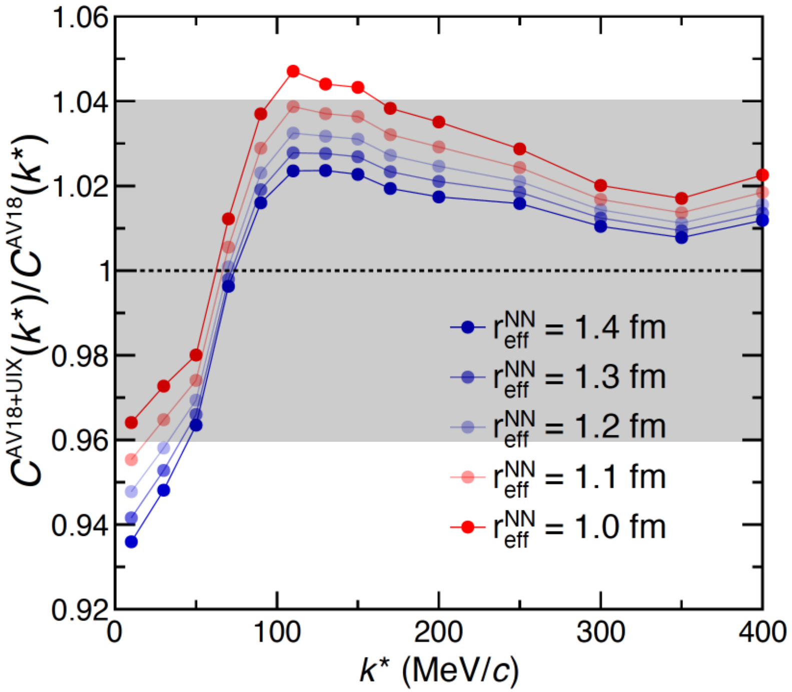


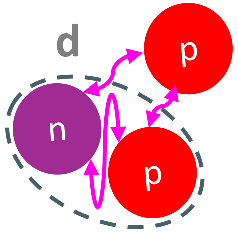


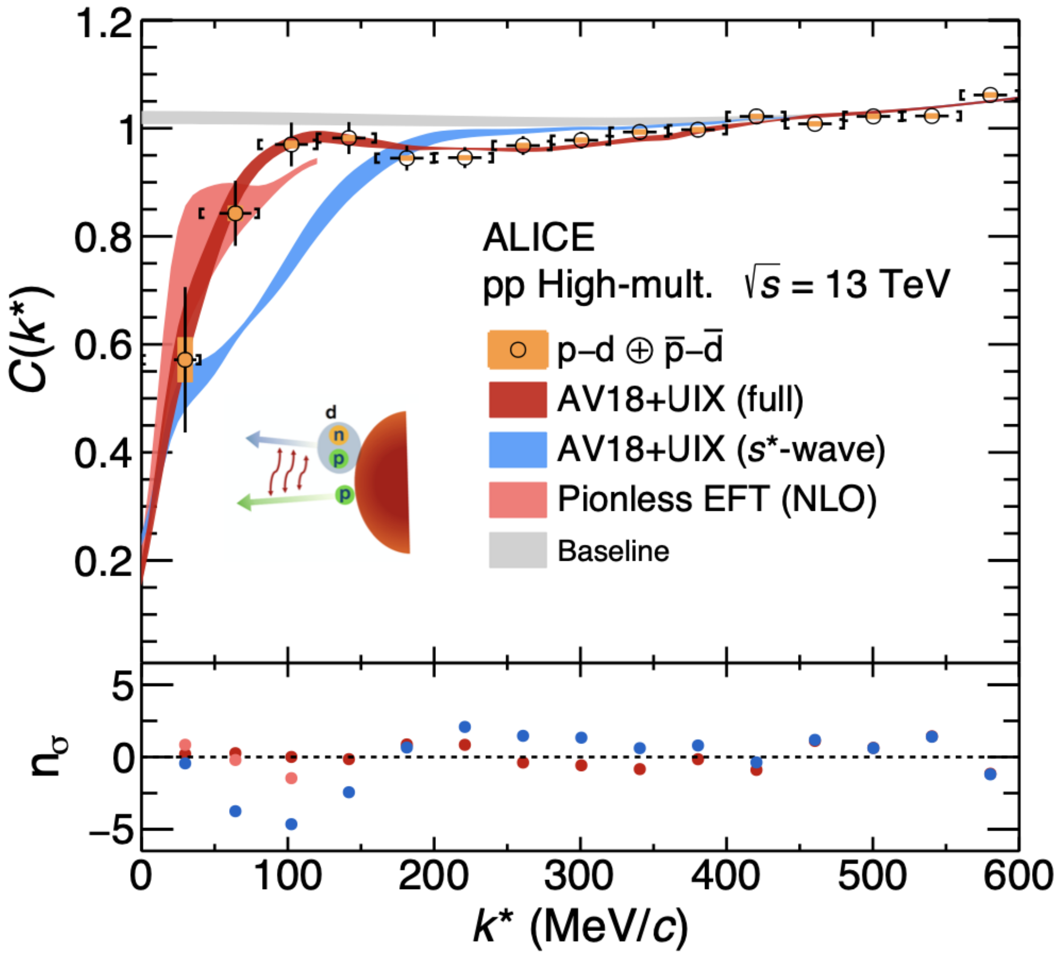
ALICE Coll. arXiv:2308.16120 (2023), accepted by PRX
M. Viviani et al, Phys.Rev.C 108 (2023) 6, 064002

NNN using proton-deuteron correlations

- Full three-body calculations are required (NN + NNN + Quantum Statistics)
- Hadron-nuclei correlations at the LHC can be used to study many-body dynamics
- Sensitivity to three-body forces up to 5%



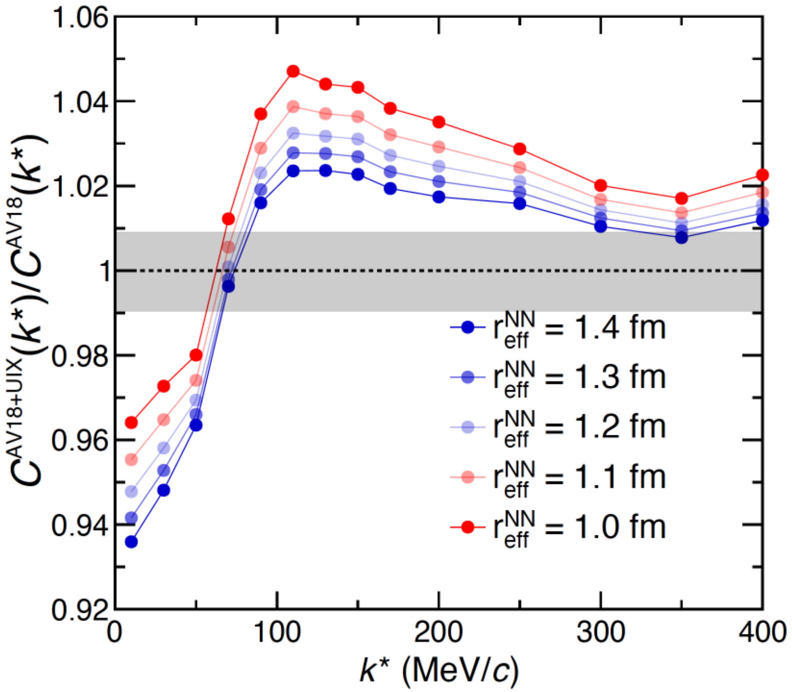


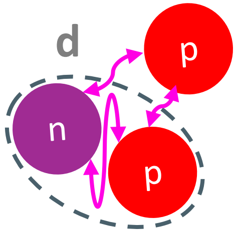


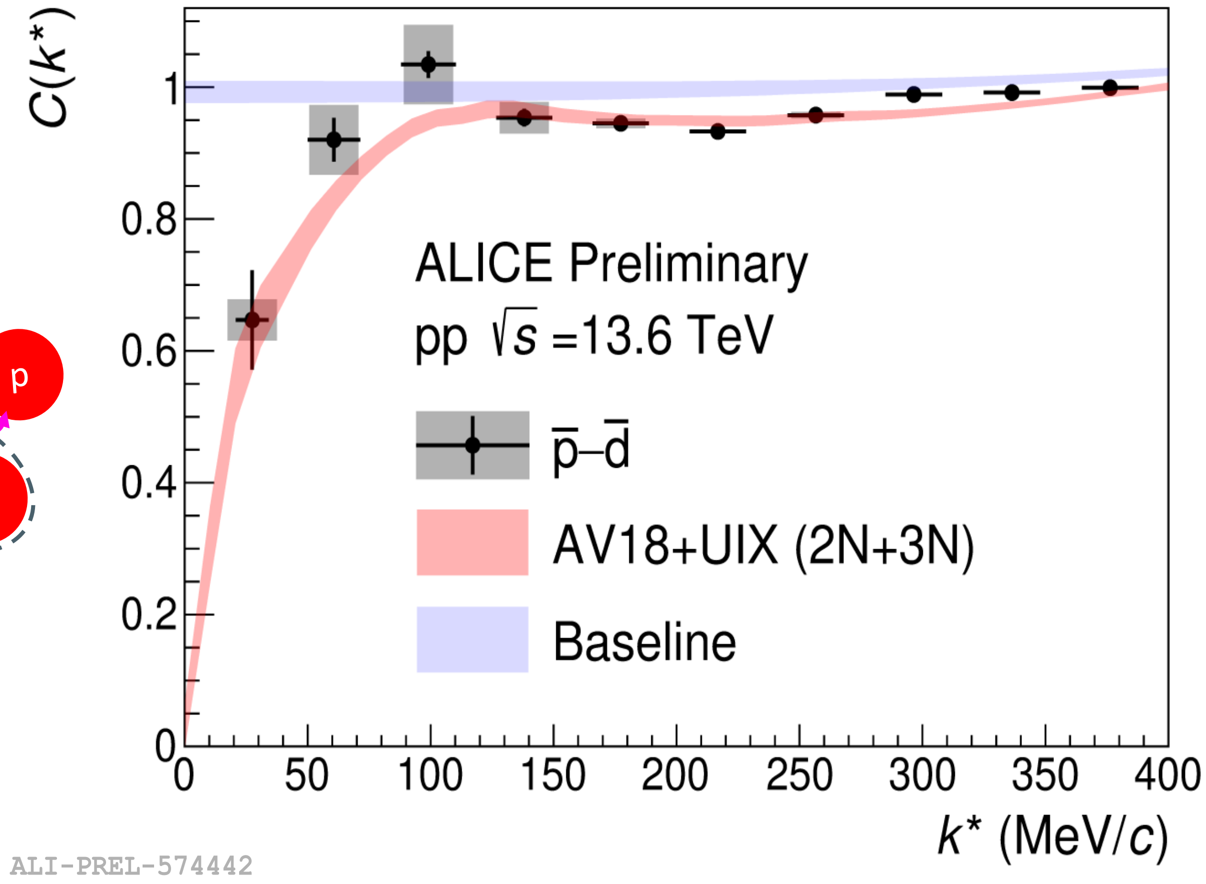
ALICE Coll. arXiv:2308.16120 (2023), accepted by PRX
M. Viviani et al, Phys.Rev.C 108 (2023) 6, 064002

NNN using proton-deuteron correlations

- Full three-body calculations are required (NN + NNN + Quantum Statistics)
- Run 3 data from 2022 already analysed and results are promising!
- In Run 3 expected uncertainty of 1%







pΛ and ppΛ interactions

Results from recent paper:
E. Garrido et al., arXiv: 2408.01750 (2024)

- ΛN interaction modelled using Gaussian potentials anchored to scattering parameters

$$V_{p\Lambda}(r) = \sum_S V_S e^{-(r/r_s)^2} \mathcal{P}_{0,S}$$

NLO19 (600): V_0 = -31.9 MeV r_0 = 1.4 fm
V_1 = -42.1 MeV r_1 = 1.1 fm

→ BE(${}^{3}_{\Lambda}\mathrm{H}$) = 2.904 MeV exp: 2.39 MeV

Binding energy from:
https://hypernuclei.kph.uni-mainz.de

- ΛNN interaction modelled using Gaussian potentials

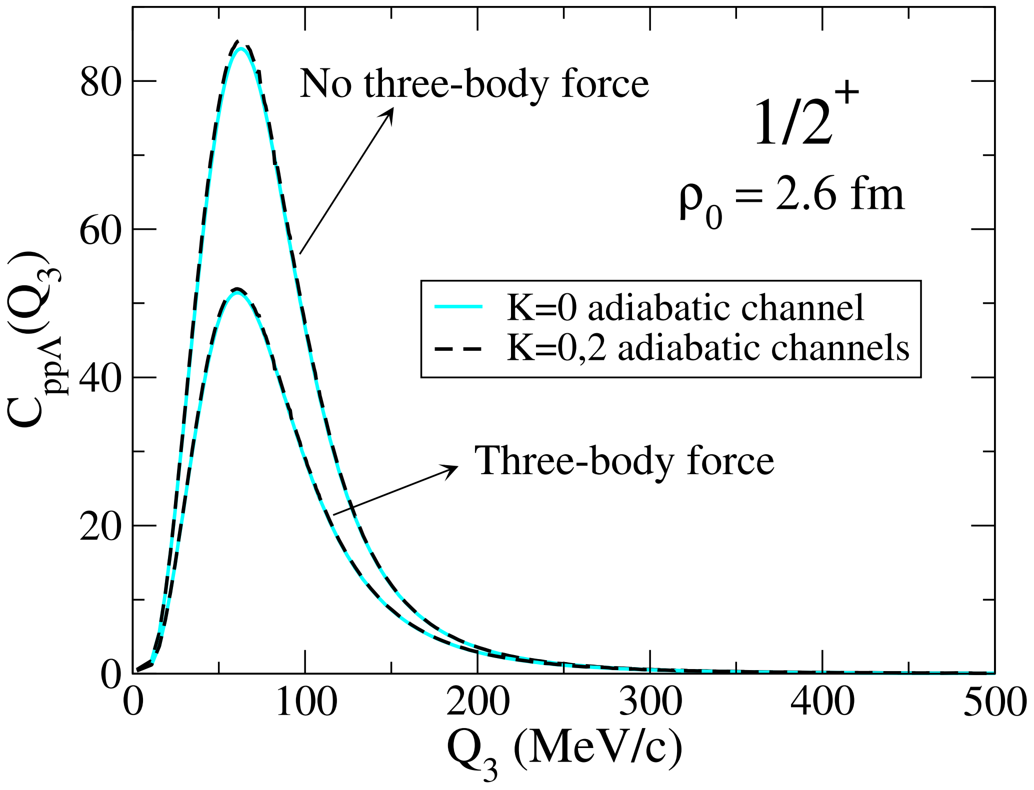
$$W(r_{13}, r_{23}) = W_3\, e^{-(r_{13}^2 + r_{23}^2)/\rho_3^2}$$

W_3 = 11.8 MeV (anchored to ${}^{3}_{\Lambda}\mathrm{H}$ binding energy)
ρ_3 = 2.0 fm (anchored to four-body hypernuclei, ${}^{4}_{\Lambda}\mathrm{H}$ and ${}^{4}_{\Lambda}\mathrm{He}$)

p-p-Λ correlation function

Results from recent paper:
E. Garrido et al., arXiv: 2408.01750 (2024)

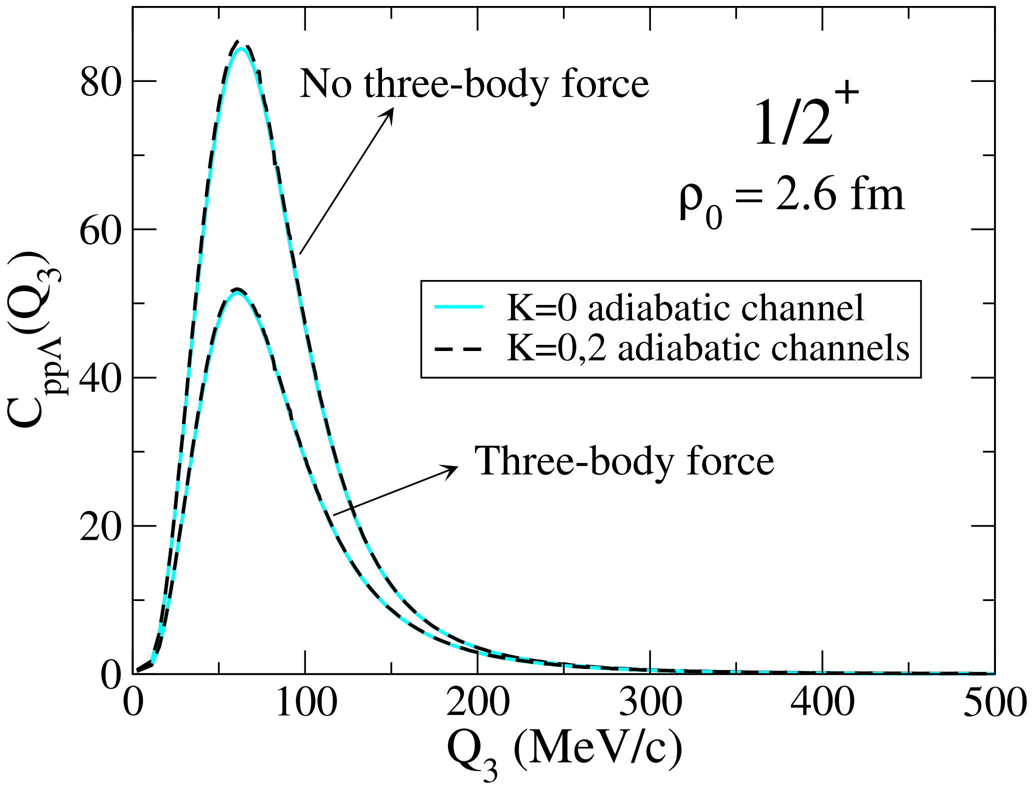
- Reference calculations with NLO19 (600)
- ΛNN interaction gives 50% effect:
  only one partial wave (K=0) significantly contributes

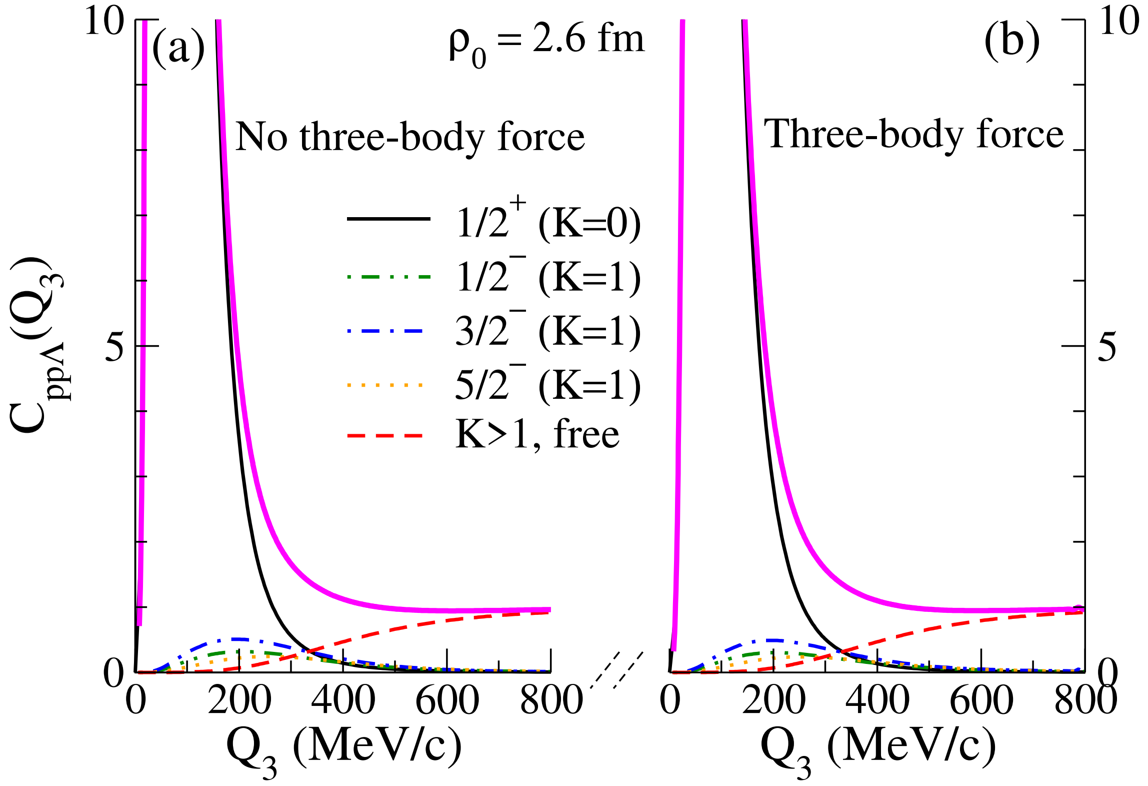


p-p-Λ correlation function

Results from recent paper:
E. Garrido et al., arXiv: 2408.01750 (2024)

- Reference calculations with NLO19 (600)
- ΛNN interaction gives 50% effect:
  only one partial wave (K=0) significantly contributes

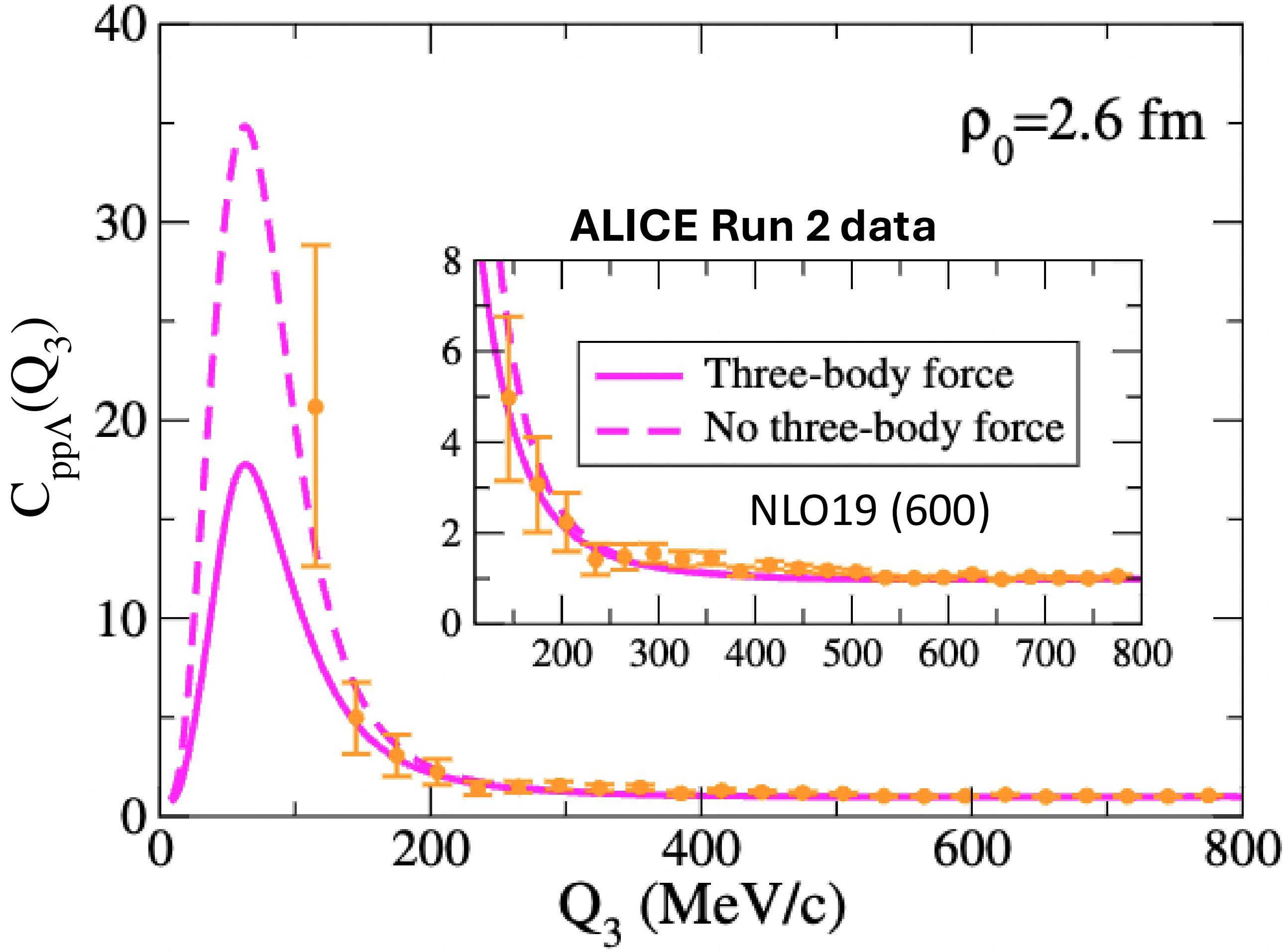




p-p-Λ correlation function

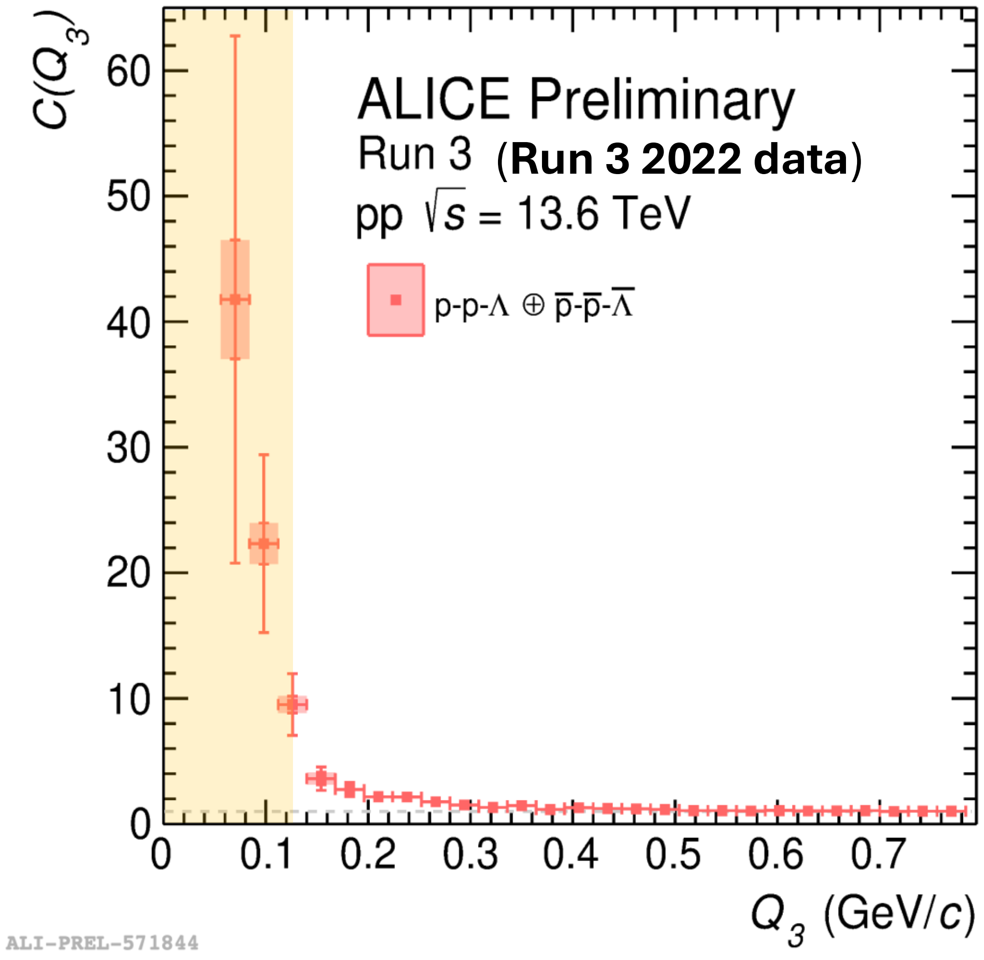
Results from recent paper:
E. Garrido et al., arXiv: 2408.01750 (2024)

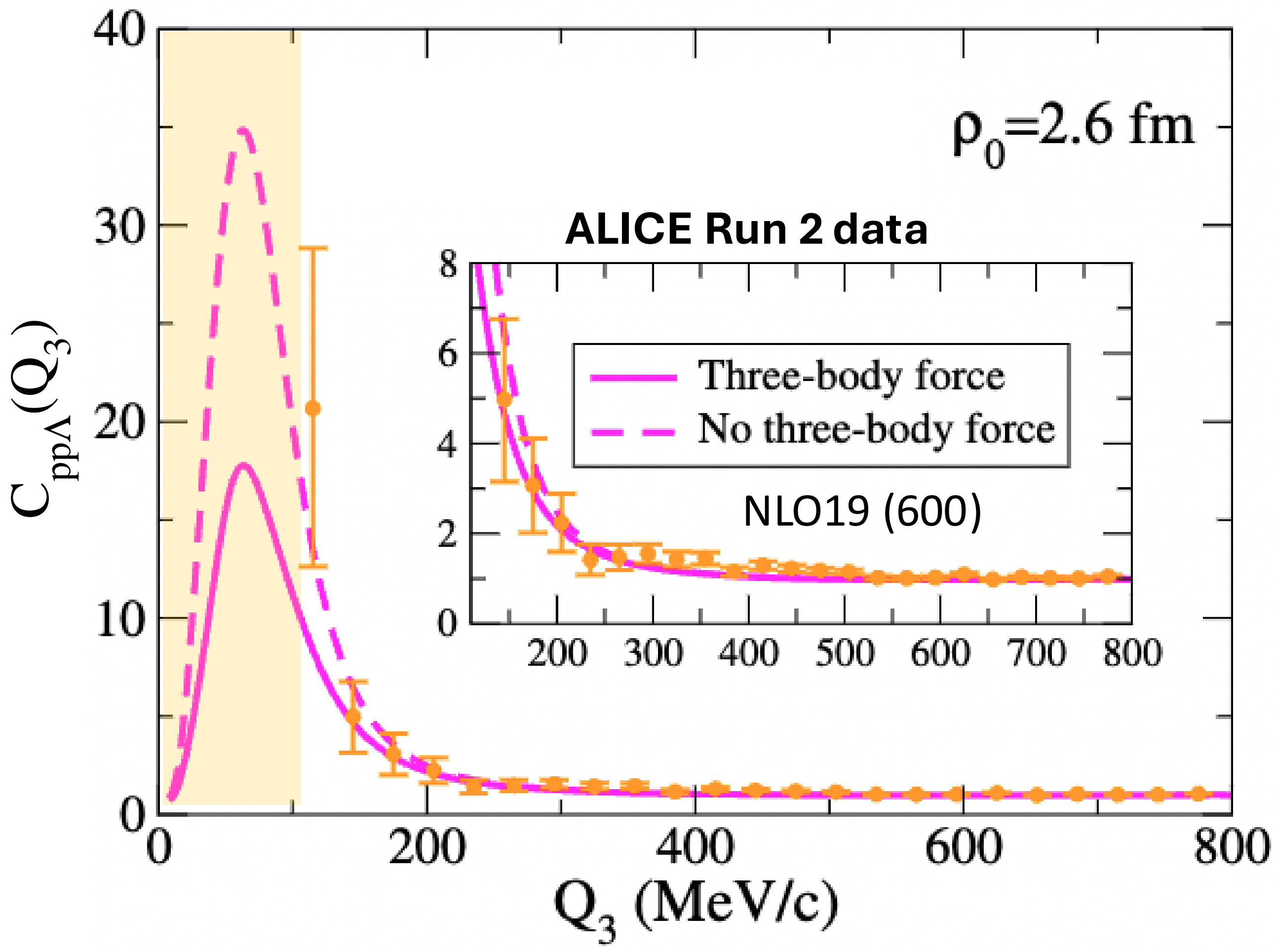
- Experimental corrections applied to the theory (feed-down from resonances and misidentifications)
- Gauss NLO19 (600): 50% effect of three-body interactions
- Run-2 data: one data point in the region of the maximum



p-p-Λ correlation function

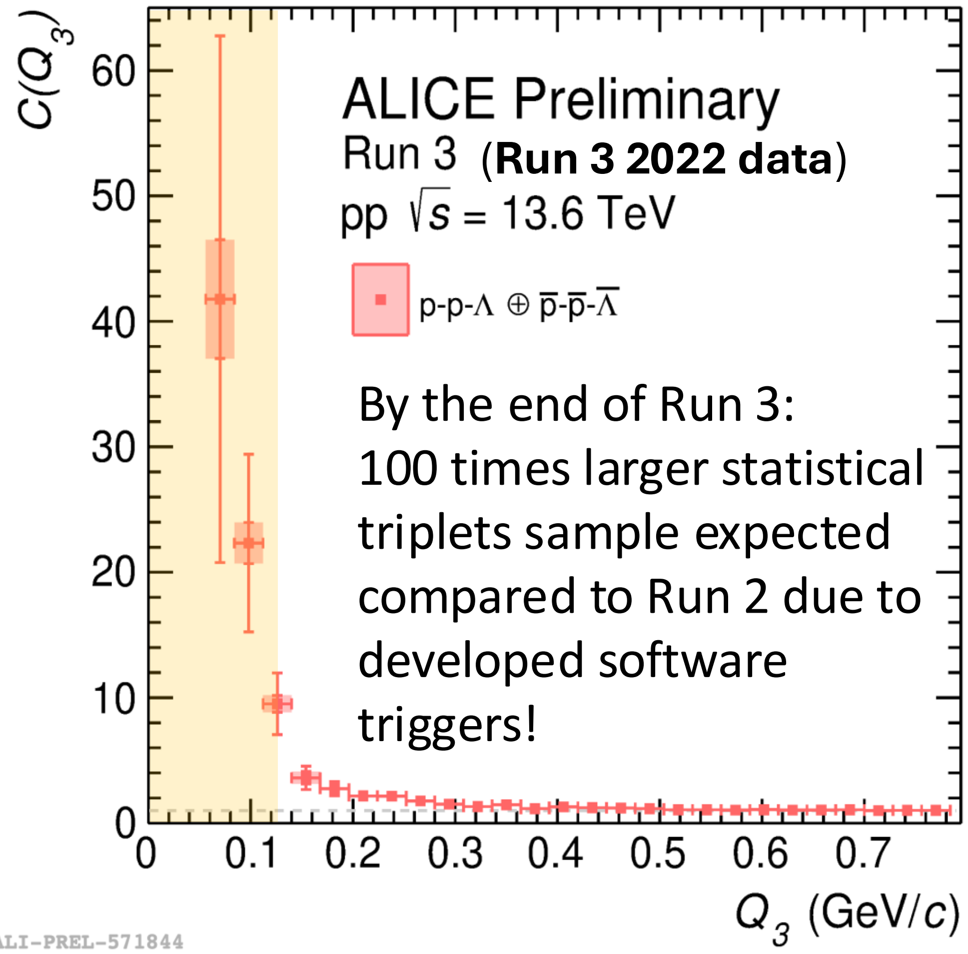
Results from recent paper:
E. Garrido et al., arXiv: 2408.01750 (2024)





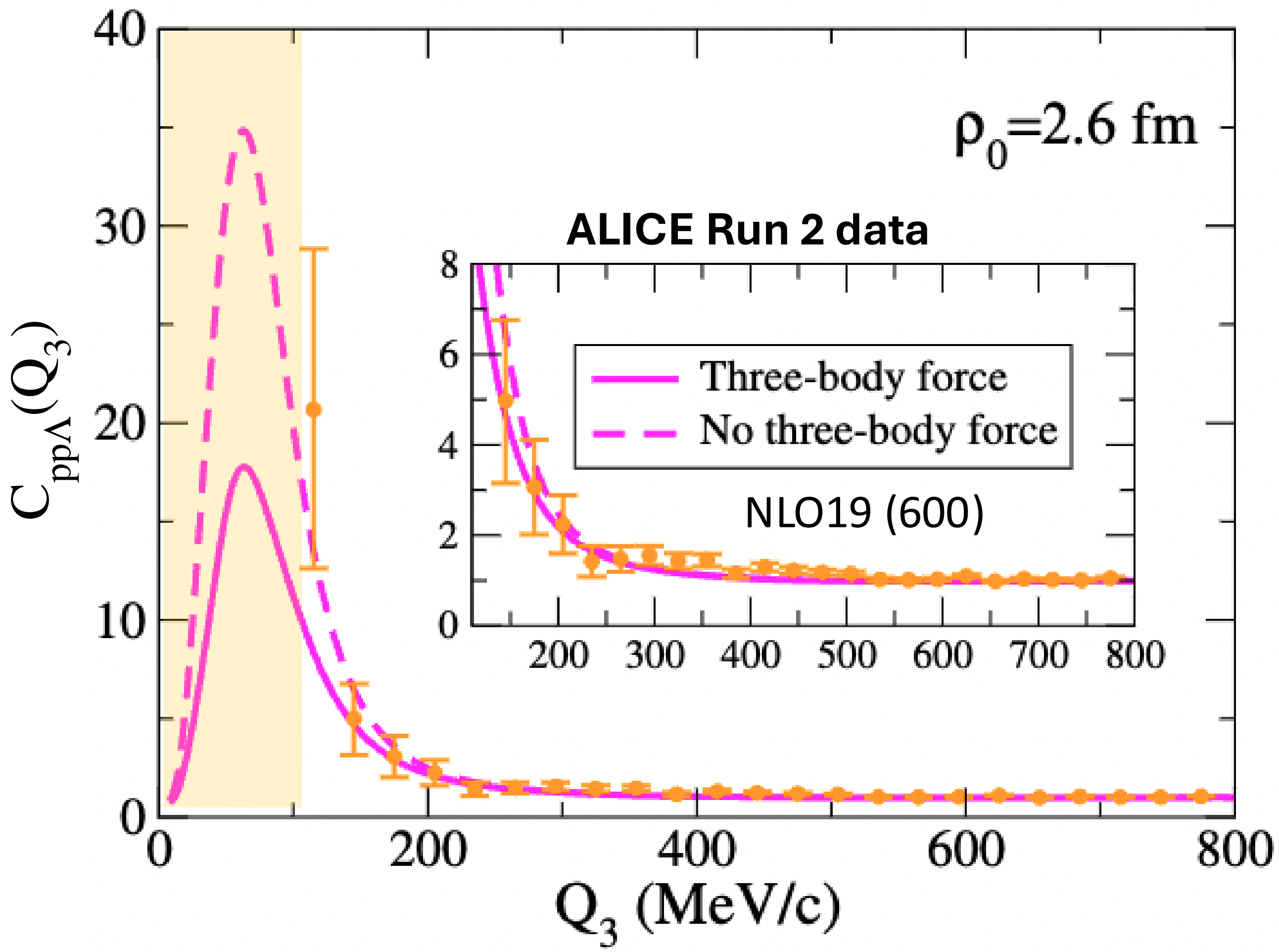
p-p-Λ correlation function

Results from recent paper:
E. Garrido et al., arXiv: 2408.01750 (2024)



By the end of Run 3:
100 times larger statistical triplets sample expected compared to Run 2 due to developed software triggers!

ALI-PREL-571844

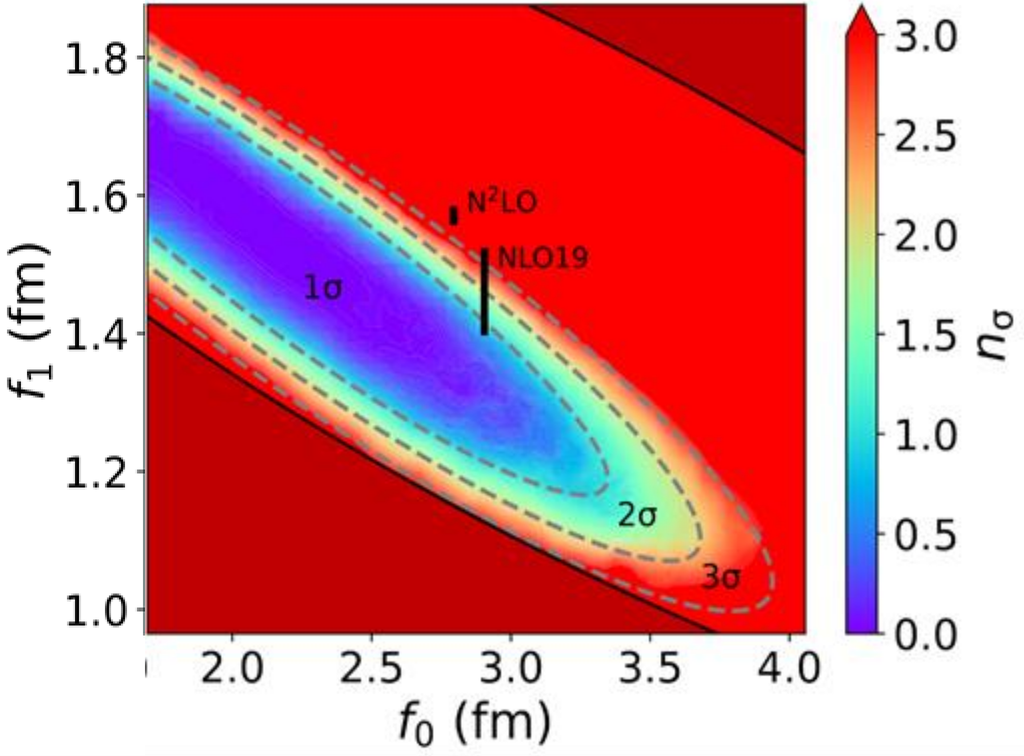


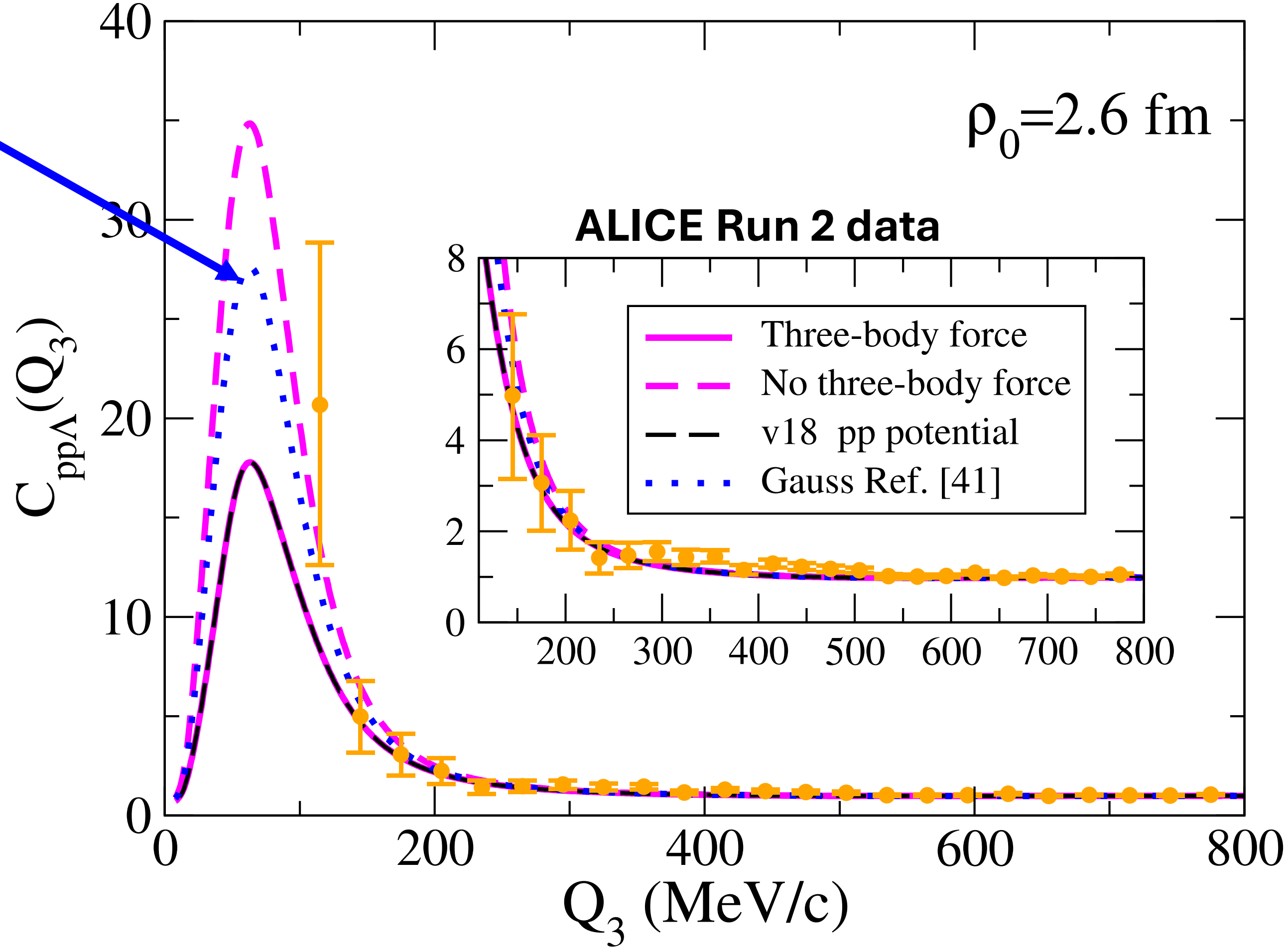
p-p-Λ correlation function

Results from recent paper:
E. Garrido et al., arXiv: 2408.01750 (2024)

- **pΛ interaction**: scattering + femto

D. Mihaylov, J. Haidenbauer and V. Mantovani Sarti, PLB 850 (2024) 138550





p-p-Λ correlation function

Results from recent paper:
E. Garrido et al., arXiv: 2408.01750 (2024)

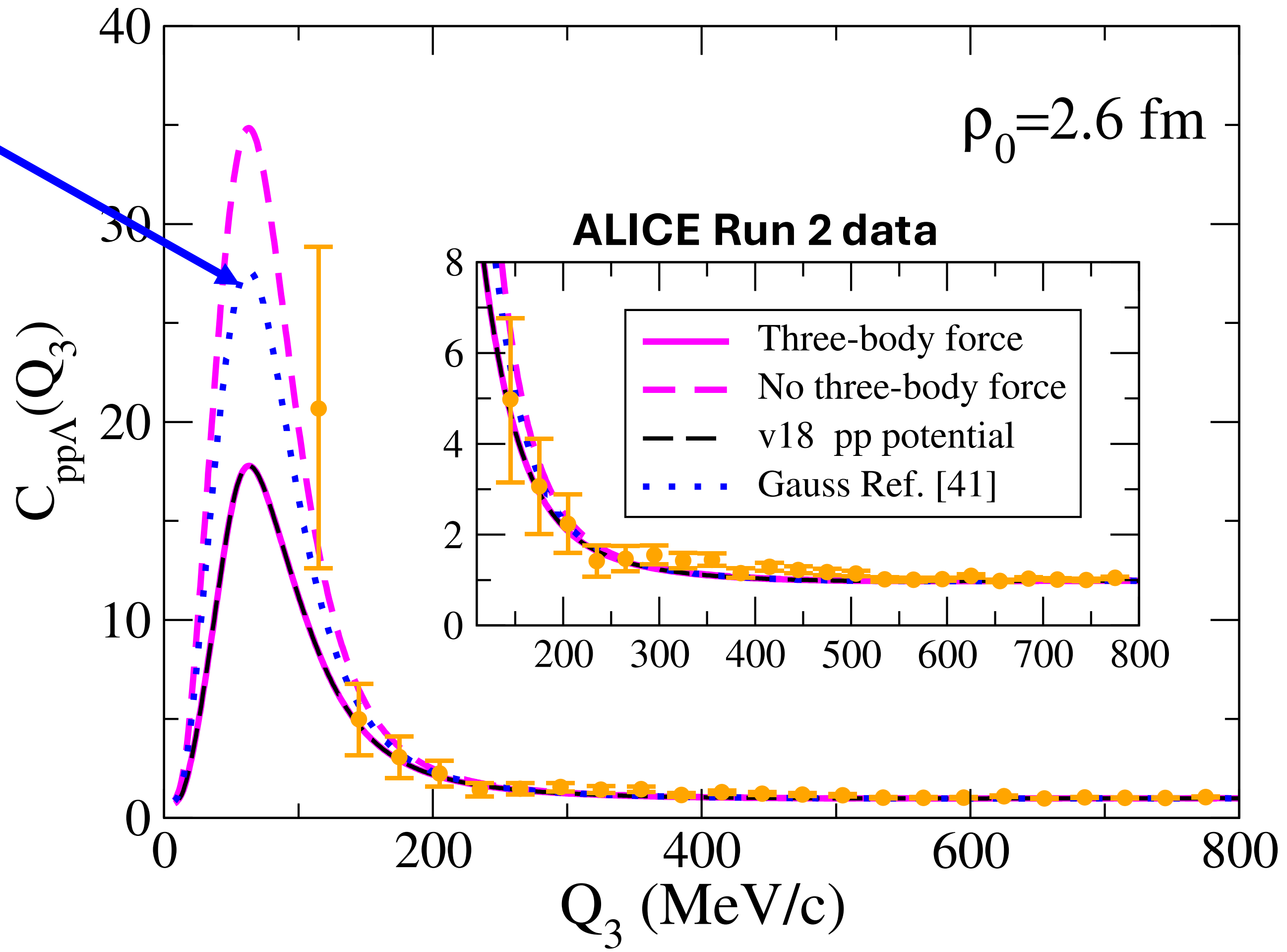
- **pΛ interaction**: scattering + femto
  D. Mihaylov, J. Haidenbauer and V. Mantovani Sarti, PLB 850 (2024) 138550
- Hypertriton binding energy can be reproduced using ONLY ΛN interaction

  BE = 2.41 MeV **exp: 2.39 MeV**

  E. Garrido et al., arXiv: 2408.01750 (2024)
- Four-body systems:

  BE = 13.37 MeV* **exp: 10.651 MeV ($^4_\Lambda$H)**
  exp: 10.064 MeV ($^4_\Lambda$He)

- Future:
  Combined analysis of scattering data + hypernuclei + femtoscopy data



*Private comunication A. Kievsky and E. Garrido

Conclusions and Outlook

- Exciting results from femtoscopy:
  - pp collisions at the LHC provides access to the interactions at short distances
  - p-p-p correlation function:
    - ✓ overlap of many partial waves
    - ✓ negligible effect of three-body forces (< 1%)
  - p-d correlation function:
    - ✓ Many-body dynamics can be studied using hadron-deuteron correlations
  - p-p-Λ correlations:
    - ✓ only one contributing wave in the signal region
    - ✓ 50 % effect of three-body forces

- On-going Run 3 and future Run 4
  - Access to precise data on three-particle correlations
  - Sensitivity to the effect of three-body forces in the correlation functions

- Future: combined analysis of femtoscopy + scattering data + hypernuclei

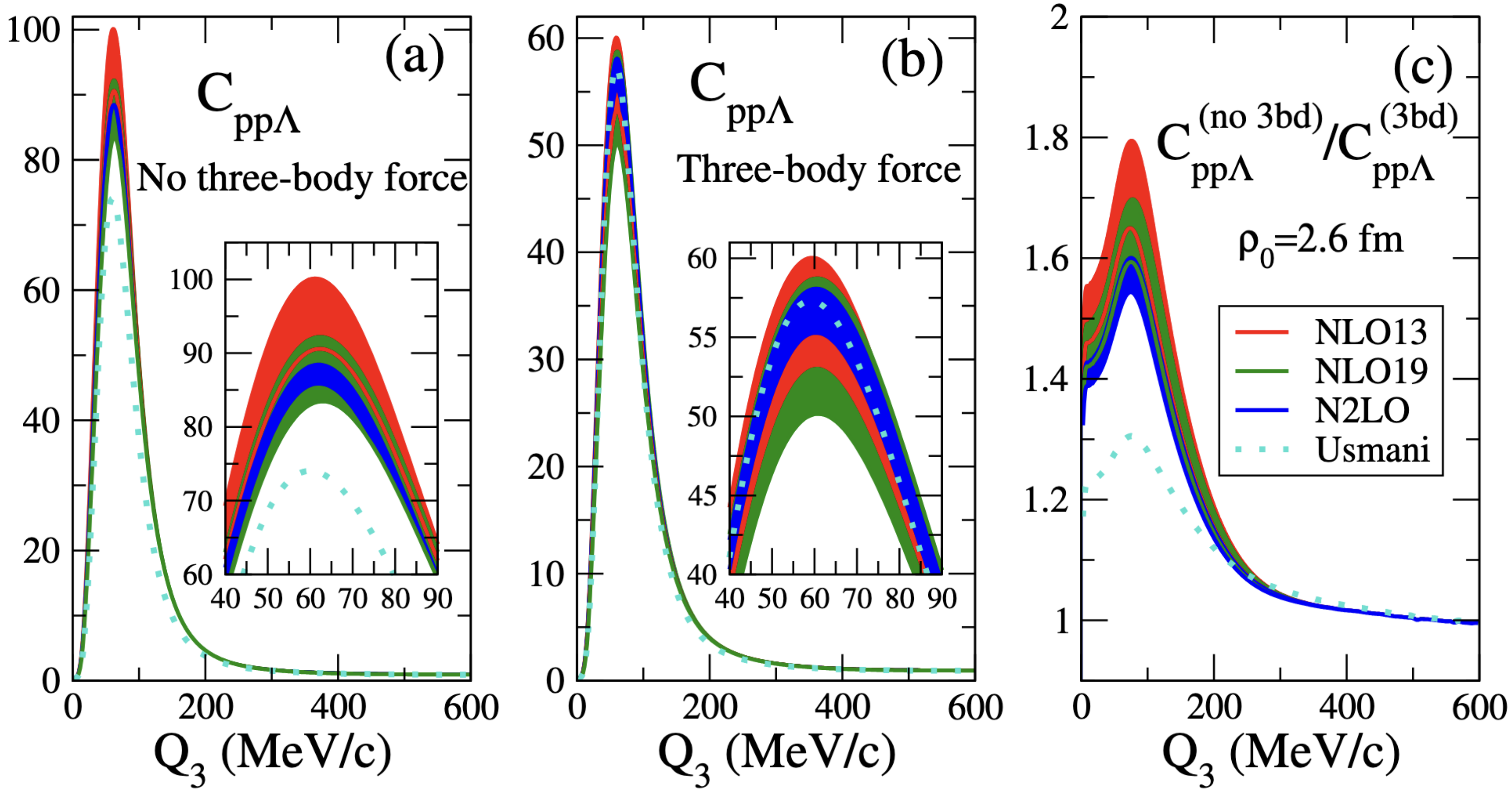
Backup

Effect of the three-body interaction

Results from recent paper:
E. Garrido et al., arXiv: 2408.01750 (2024)

- Effect of the three-body interaction for different interaction models: 30% (Usmani) - 80% (NLO13)

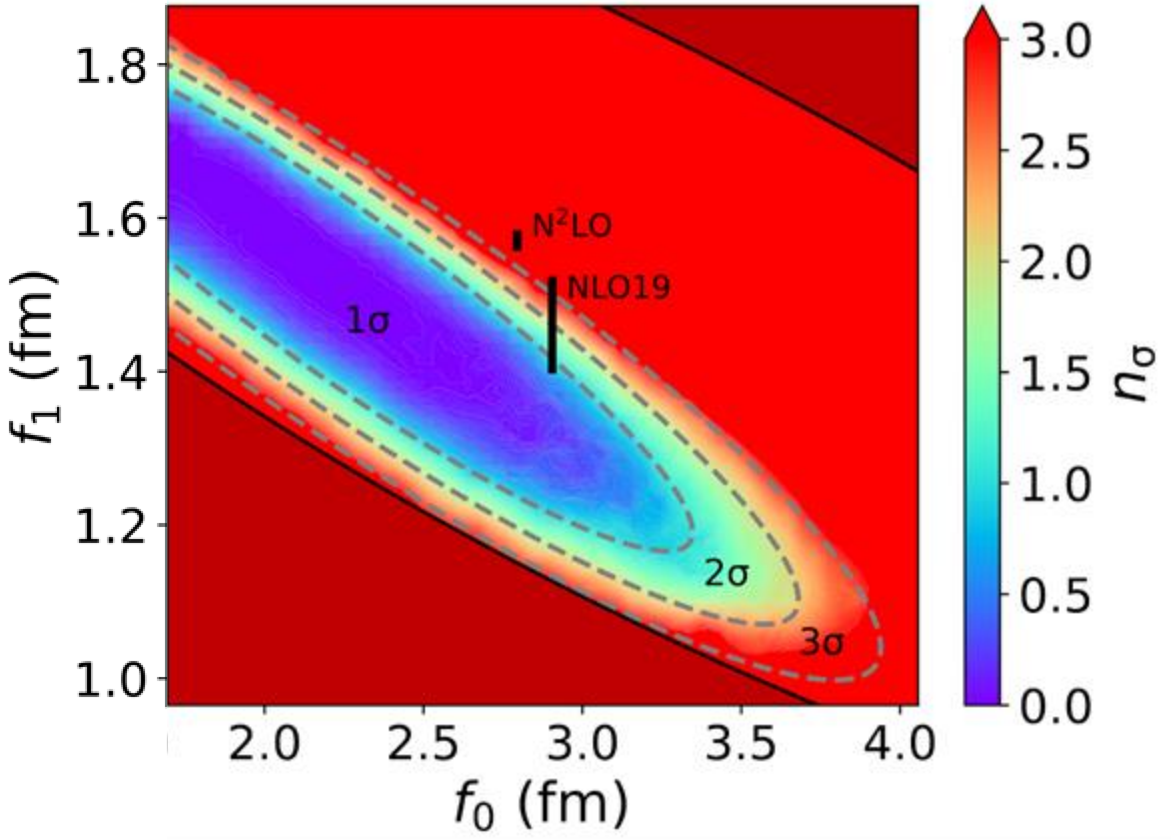
$C(Q_3)$



The pΛ interaction in the femtoscopy era

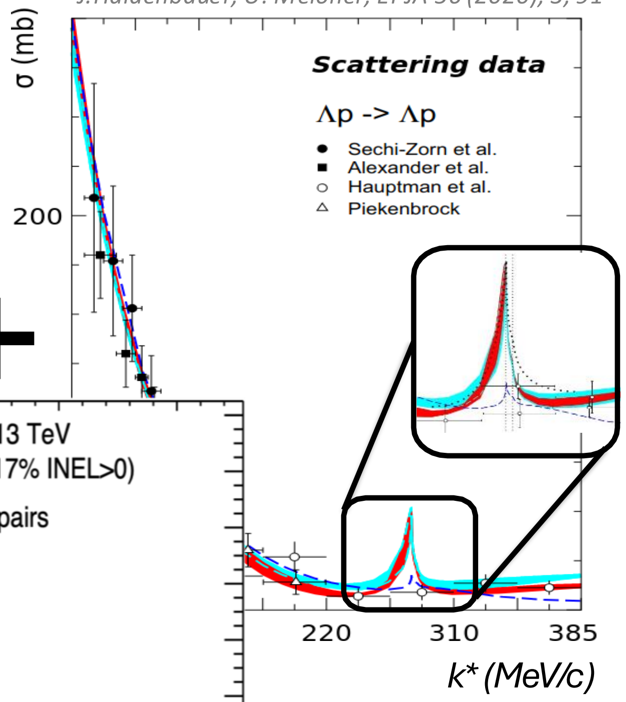
- **Improvement:** combined analysis of femtoscopic and scattering data

D. Mihaylov, J. Haidenbauer and V. Mantovani Sarti, PLB 850 (2024) 138550

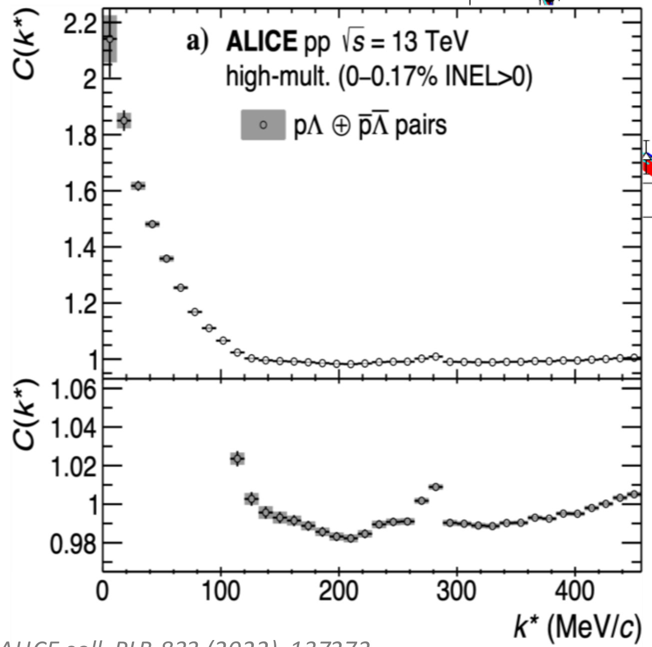


Talk by L. Serksnyte

J.Haidenbauer, U. Meißner, EPJA 56 (2020), 3, 91



+

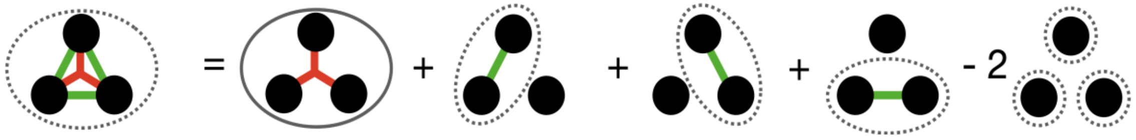


ALICE coll. PLB 833 (2022), 137272

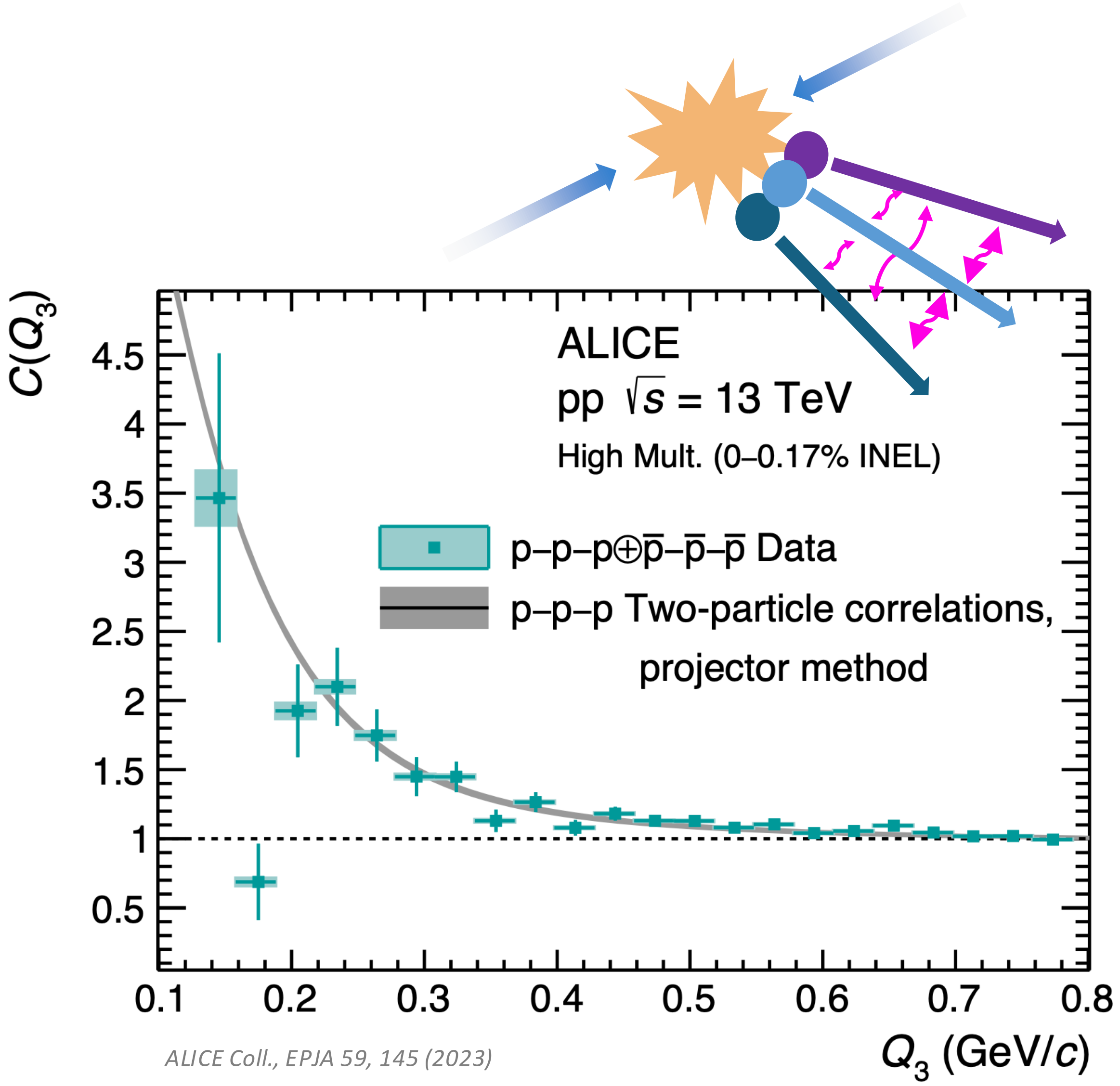
p-p-p correlation function

- Cumulant method provides first hint of effects beyond two-body correlations

R. Kubo, J. Phys. Soc. Jpn. 17, 1100-1120 (1962)
R. Del Grande et al. EPJC 82 (2022) 244



- A deviation of nσ = 6.7 from lower-order contributions
- Theoretical predictions necessary to understand the origin of the deviation further



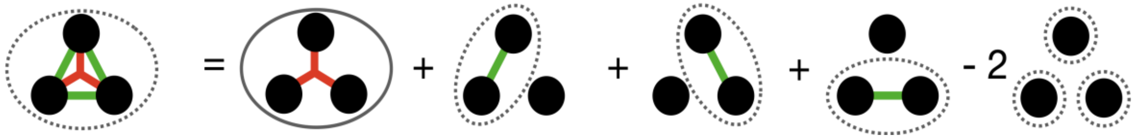
ALICE Coll., EPJA 59, 145 (2023)

p-p-Λ correlation function

- Cumulant method provides first hint of effects beyond two-body correlations

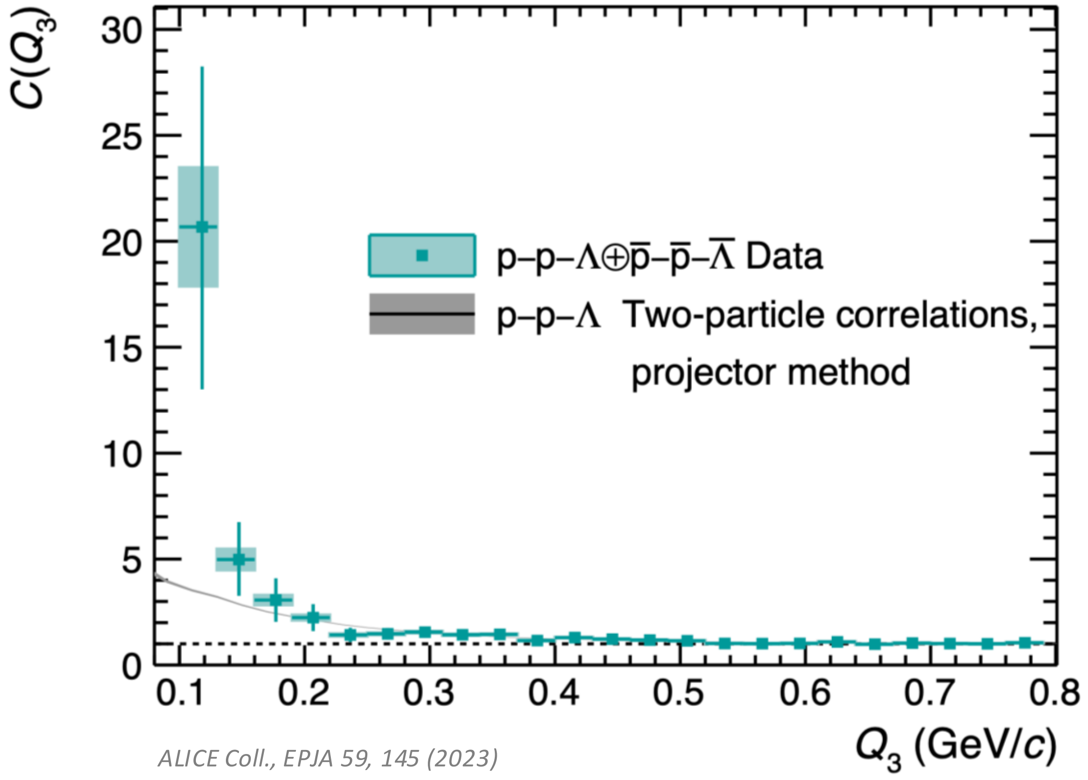
R. Kubo, J. Phys. Soc. Jpn. 17, 1100-1120 (1962)

R. Del Grande et al. EPJC 82 (2022) 244



Measured three-body correlation = Genuine three-body correlations + Two-body correlations - Single particle contribution

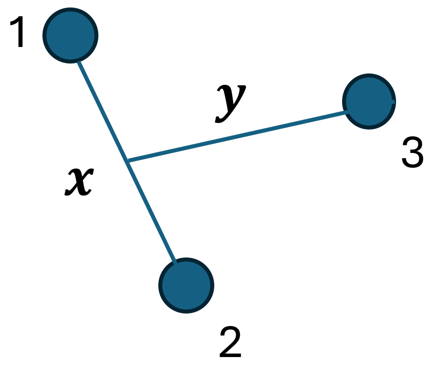
- Compatible with lower-order contributions (nσ = 0.8)



ALICE Coll., EPJA 59, 145 (2023)

Hypersperical Harmonics formalism

- The Jacobi coordinates:



$$\begin{cases} \boldsymbol{x} = \boldsymbol{r}_2 - \boldsymbol{r}_1 \\ \boldsymbol{y} = \sqrt{\frac{4}{(1+2m/M)}}\left(\boldsymbol{r}_3 - \frac{\boldsymbol{r}_1+\boldsymbol{r}_2}{2}\right) \end{cases}$$

- We introduce the hyperradius and hyperangle:

$$\rho = \sqrt{x^2 + y^2} \qquad \phi = \arctan\left(\frac{y}{x}\right)$$

- Now the 6 variables are: $(\rho, \phi, \hat{\mathbf{x}}, \hat{\mathbf{y}})$ → 1 radius, 5 angles

Hyperspherical Harmonics formalism

Defining the wave function as:

$$\psi = \sum_{[K]} \rho^{-5/2} u_{[K]}(\rho) Y_{[K]}(\Omega)$$

Schroedinger equation with the interaction:

$$\left(\frac{\partial^2 u_{[K]}(\rho)}{\partial \rho^2} - \frac{(K+3/2)(K+5/2)}{\rho^2} u_{[K]}(\rho) \right) + \sum_{[K']} U_{[K][K']}(\rho)\, u_{[K']}(\rho) = Q^2\, u_{[K]}(\rho)$$

Where the hypercentral potential is obtained as

$$U_{[K][K']}(\rho) = \int d\Omega\, Y^*_{[K]}(\Omega)[V_{12} + V_{23} + V_{31} + V_{123}]\, Y_{[K']}(\Omega)$$

Kaon/Proton-deuteron correlation

- Effective two-body system
  - Coulomb + Strong interactions via Lednický model; only s-wave
  - Anchored to scattering experiments
  - Emission source: from m_T scaling

R. Lednický, Phys. Part. Nucl. 40, 307(2009)

W. T. H. Van Oers, & K. W. Brockman Jr, NPA 561 (1967); J. Arvieux et al., NPA 221 (1973); E. Huttel et al., NPA 406 (1983); A. Kievsky et al., PLB 406 (1997); T. C. Black et al., PLB 471 (1999);

System	Spin averaged		$S = 1/2$		$S = 3/2$	
	a_0(fm)	d_0(fm)	a_0(fm)	d_0(fm)	a_0(fm)	d_0(fm)
p–d			$1.30^{+0.20}_{-0.20}$	—	$11.40^{+1.80}_{-1.20}$	$2.05^{+0.25}_{-0.25}$
			$2.73^{+0.10}_{-0.10}$	$2.27^{+0.12}_{-0.12}$	$11.88^{-0.10}_{+0.40}$	$2.63^{+0.01}_{-0.02}$
			4.0	—	11.1	—
			0.024	—	13.8	—
			$-0.13^{+0.04}_{-0.04}$	—	$14.70^{+2.30}_{-2.30}$	—
K^+–d	−0.470	1.75				
	−0.540	0.0				

**R. Lednicky and V. L. Lyuboshits Sov. J. Nucl. Phys. 35 (1982)

S = spin state

d_0^S = effective range

f_0^S = scattering length

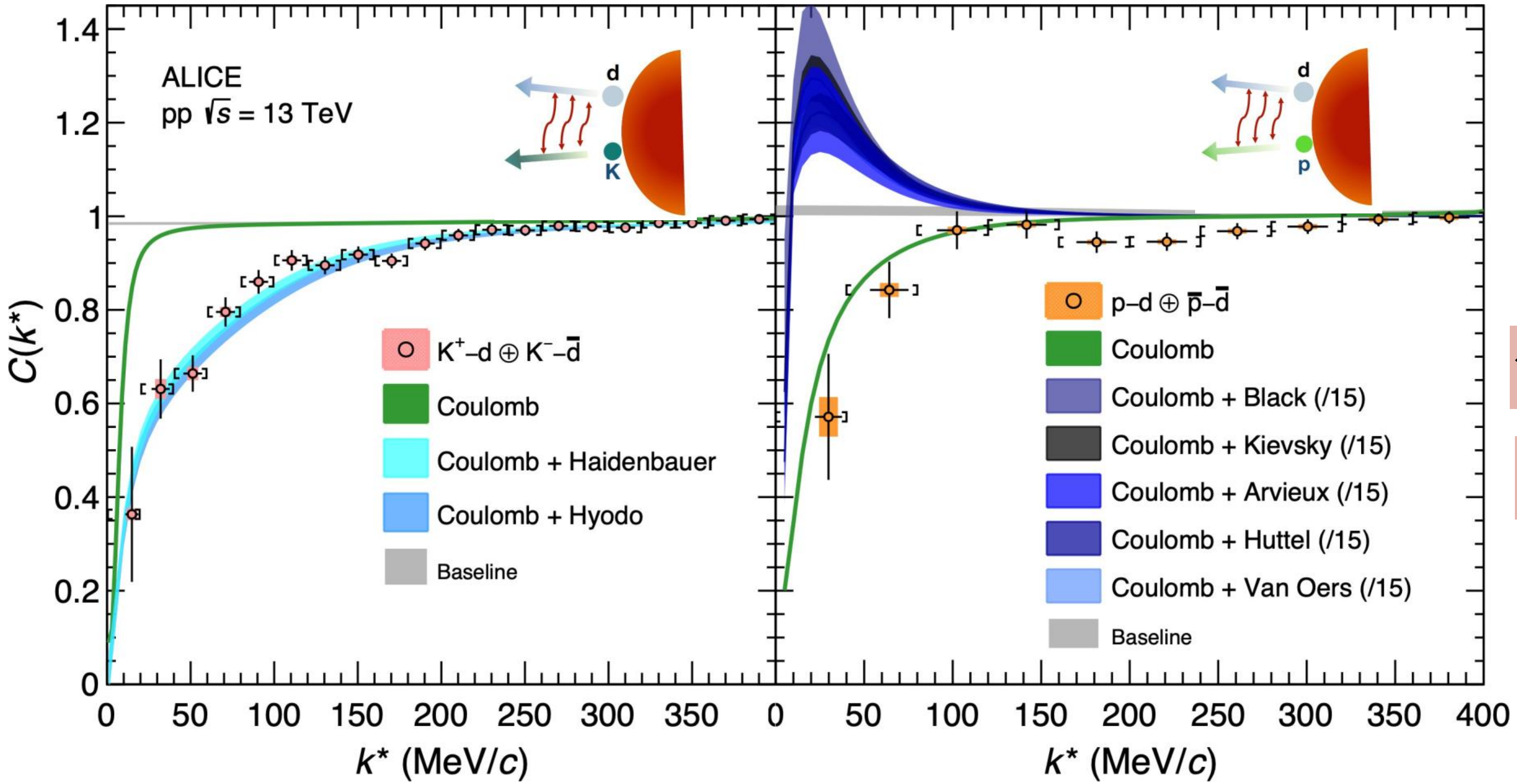
$$f(k^*)^S = \left(\frac{1}{f_0^S} + \frac{1}{2} d_0^S k^{*2} - ik^*\right)^{-1}$$

$$C(k^*) = 1 + \sum_S \rho_S \left[\frac{1}{2}\left|\frac{f(k^*)^S}{r_0}\right|^2 \left(1 - \frac{d_0^S}{2\sqrt{\pi} r_0}\right) + \frac{2\Re f(k^*)^S}{\sqrt{\pi} r_0} F_1(2k^* r_0) - \frac{2\Im f(k^*)^S}{\sqrt{\pi} r_0} F_2(2k^* r_0)\right]$$

$$S(r) = (4\pi r_0^2)^{-3/2} \cdot exp\left(-\frac{r^2}{4r_0^2}\right)$$

Kaon/Proton-deuteron correlation

ALICE Coll., arXiv:2308.16120 (2023)



$$r_{\mathrm{eff}}^{\mathrm{Kd}} = 1.41^{+0.03}_{-0.06}\ \mathrm{fm}$$

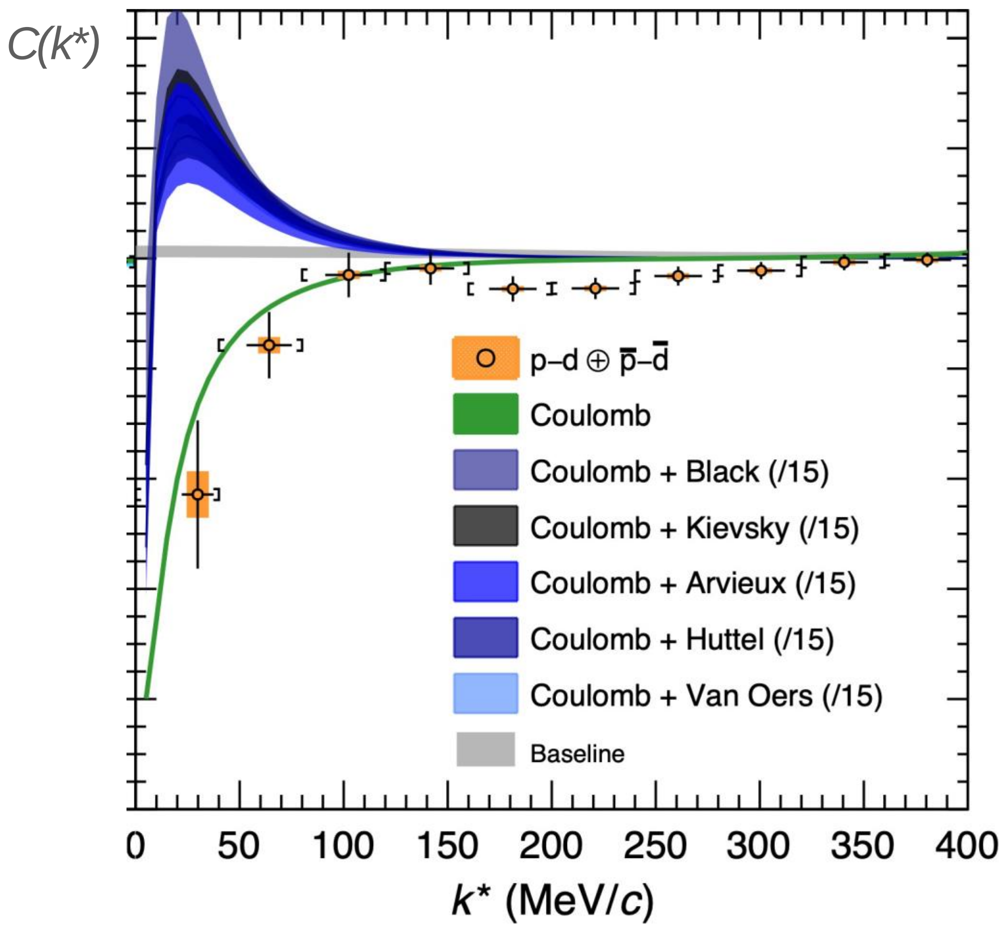
$$r_{\mathrm{eff}}^{\mathrm{pd}} = 1.059^{+0.04}_{-0.04}\ \mathrm{fm}$$

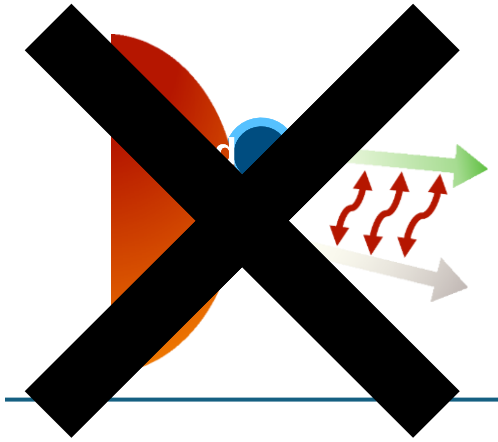
It works very well for k-d since this interaction is only repulsive and there are no features of the interaction that appears only at short distances. The asymptotic description is sufficient

Proton-deuteron correlation

- The picture of two point-like particles does not work for p-d
  - the deuteron is a composite object
  - Pauli blocking at work for p-(pn) at short distances
  - The asymptotic interaction is different from the short distance one
  - One need a full-fledged three-body calculation

ALICE Coll., arXiv:2308.16120 (2023)

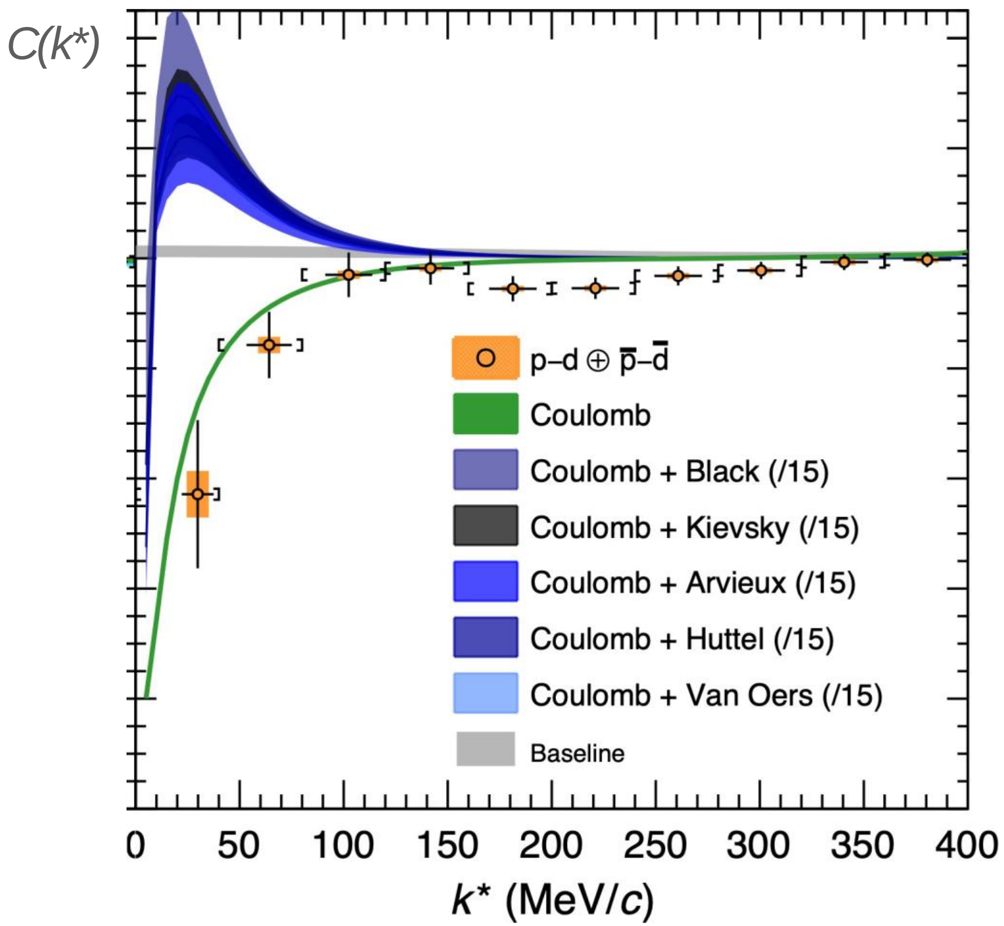


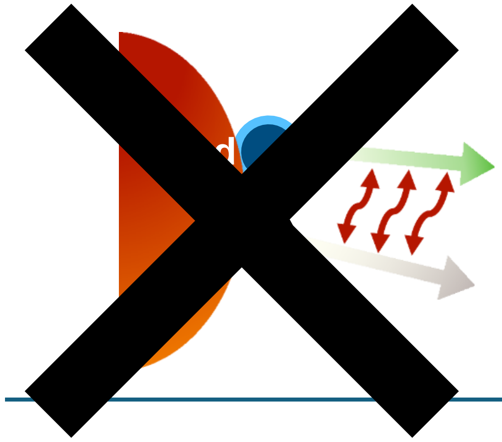


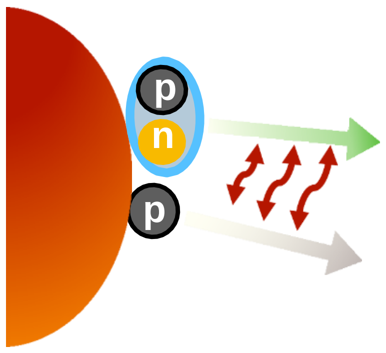
Proton-deuteron correlation

- The picture of two point-like particles does not work for p-d
  - the deuteron is a composite object
  - Pauli blocking at work for p-(pn) at short distances
  - The asymptotic interaction is different from the short distance one
  - One need a full-fledged three-body calculation

ALICE Coll., arXiv:2308.16120 (2023)







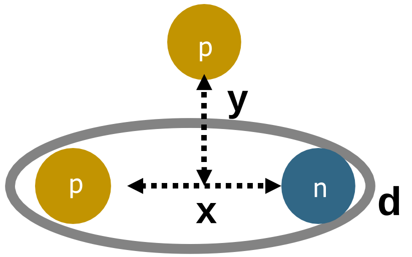
Pisa model: p-d as three-body system

- **Starting with the p-p-n state that goes into p-d state:**
  - Nucleons with the Gaussian sources distributions

Single-particle Gaussian emission source

$$A_d C_{pd}(k) = \frac{1}{6}\sum_{m_2,m_1}\int d^3r_1 d^3r_2 d^3r_3\, S_1(r_1)S_1(r_2)S_1(r_3)|\Psi_{m_2,m_1}|^2 ,$$

  - $\Psi_{m(,m)}(x,y)$ three-nucleon wave function asymptotically behaves as p–d state



Calculation done by PISA theory group: Michele Viviani, Alejandro Kievsky and Laura Marcucci

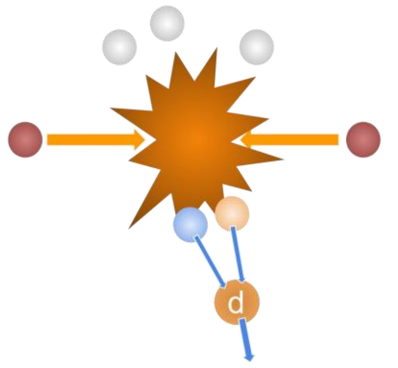
Pisa model: p-d as three-body system

- **Starting with the p-p-n state that goes into p-d state:**
  - Nucleons with the Gaussian sources distributions

Single-particle Gaussian emission source

$$A_d C_{pd}(k) = \frac{1}{6} \sum_{m_2,m_1} \int d^3r_1 d^3r_2 d^3r_3 \, S_1(r_1)S_1(r_2)S_1(r_3) |\Psi_{m_2,m_1}|^2 ,$$

- $\Psi_{m(,m)}(x,y)$ three-nucleon wave function asymptotically behaves as p–d state
- A_d is the deuteron formation probability using deuteron wavefunction



Mrówczyński et al *Eur. Phys. J. Special Topics 229, 3559 (2020)*

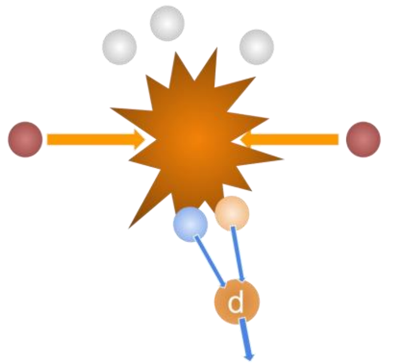
Pisa model: p-d as three-body system

- **Starting with the p-p-n state that goes into p-d state:**
  - Nucleons with the Gaussian sources distributions

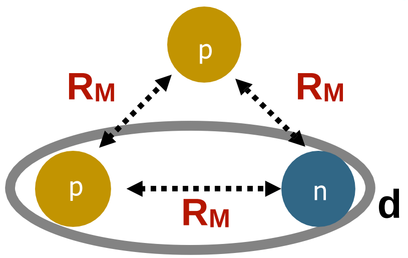
Single-particle Gaussian emission source

$$A_d C_{pd}(k) = \frac{1}{6} \sum_{m_2, m_1} \int d^3 r_1 d^3 r_2 d^3 r_3 \, S_1(r_1) S_1(r_2) S_1(r_3) |\Psi_{m_2,m_1}|^2 \, ,$$

  - $\Psi_{m_{(,m)}}(x, y)$ three-nucleon wave function asymptotically behaves as p–d state
  - A_d is the deuteron formation probability using deuteron wavefunction
  - Final definition of the correlation with p-p source size R_M :

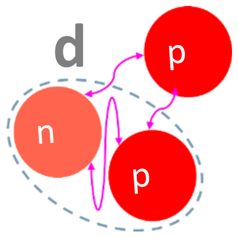


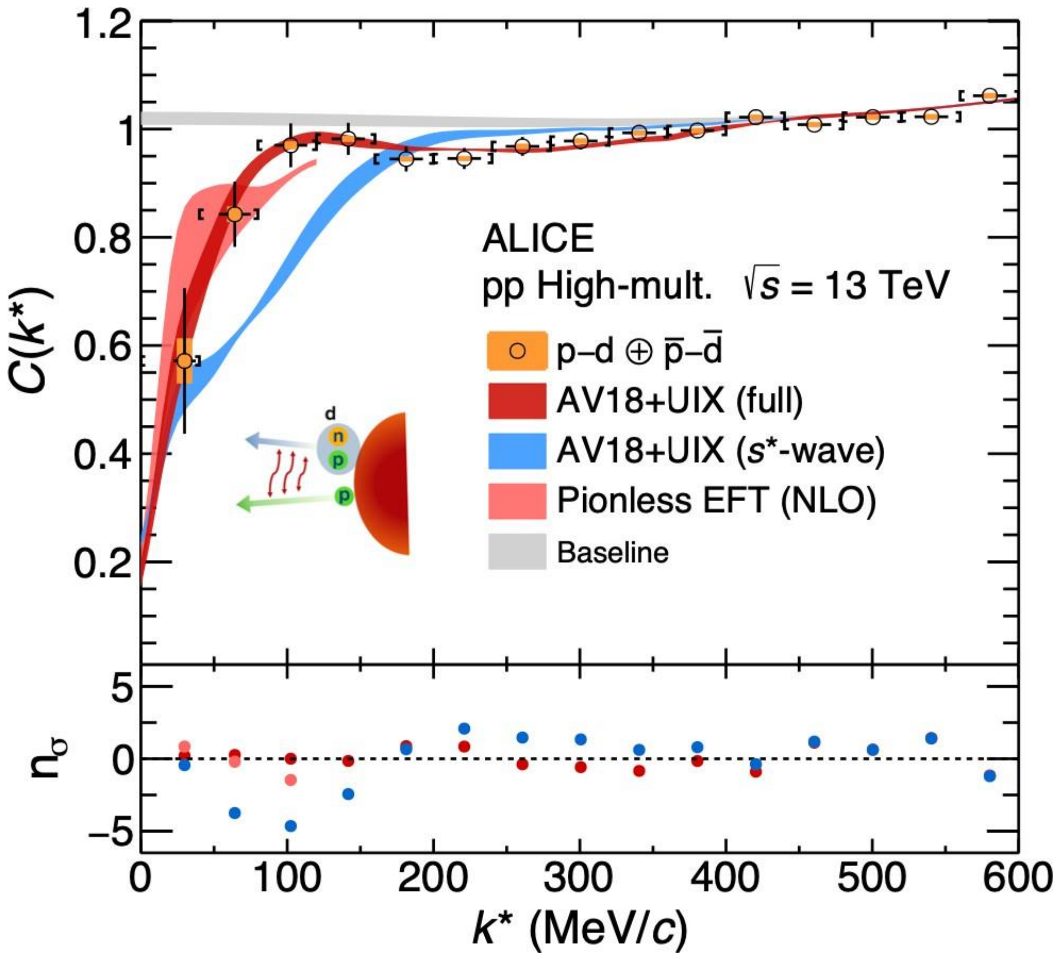
$$A_d C_{pd}(k) = \frac{1}{6} \sum_{m_2, m_1} \int \rho^5 d\rho d\Omega \, \frac{e^{-\rho^2/4R_M^2}}{(4\pi R_M^2)^3} |\Psi_{m_2,m_1}|^2 \, .$$



NNN using proton-deuteron correlations

- Full three-body calculations are required (NN + NNN + Quantum Statistics)
- Hadron-nuclei correlations at the LHC can be used to study many-body dynamics



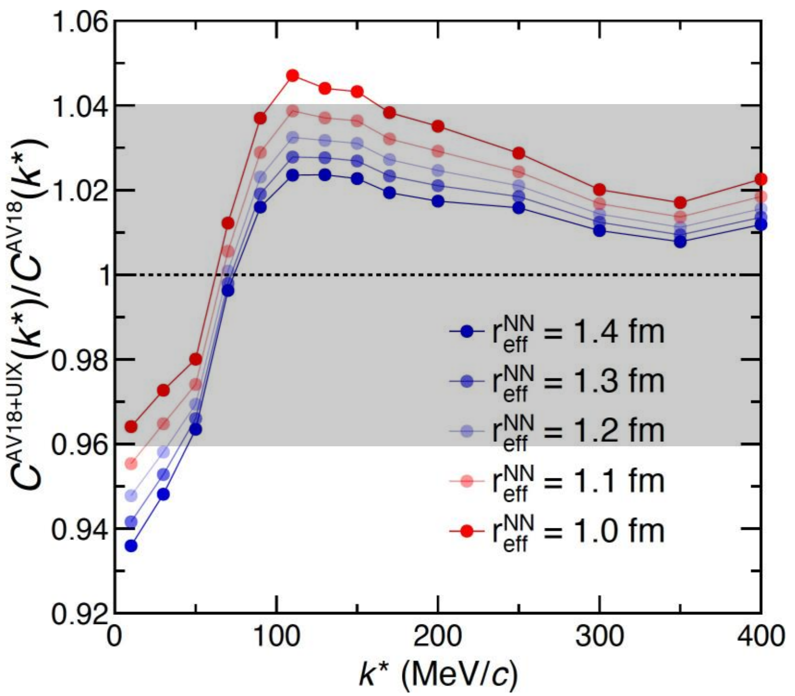


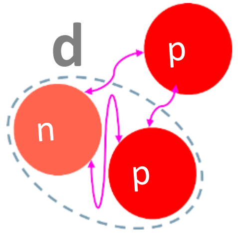
ALICE Coll., arXiv:2308.16120 (2023)

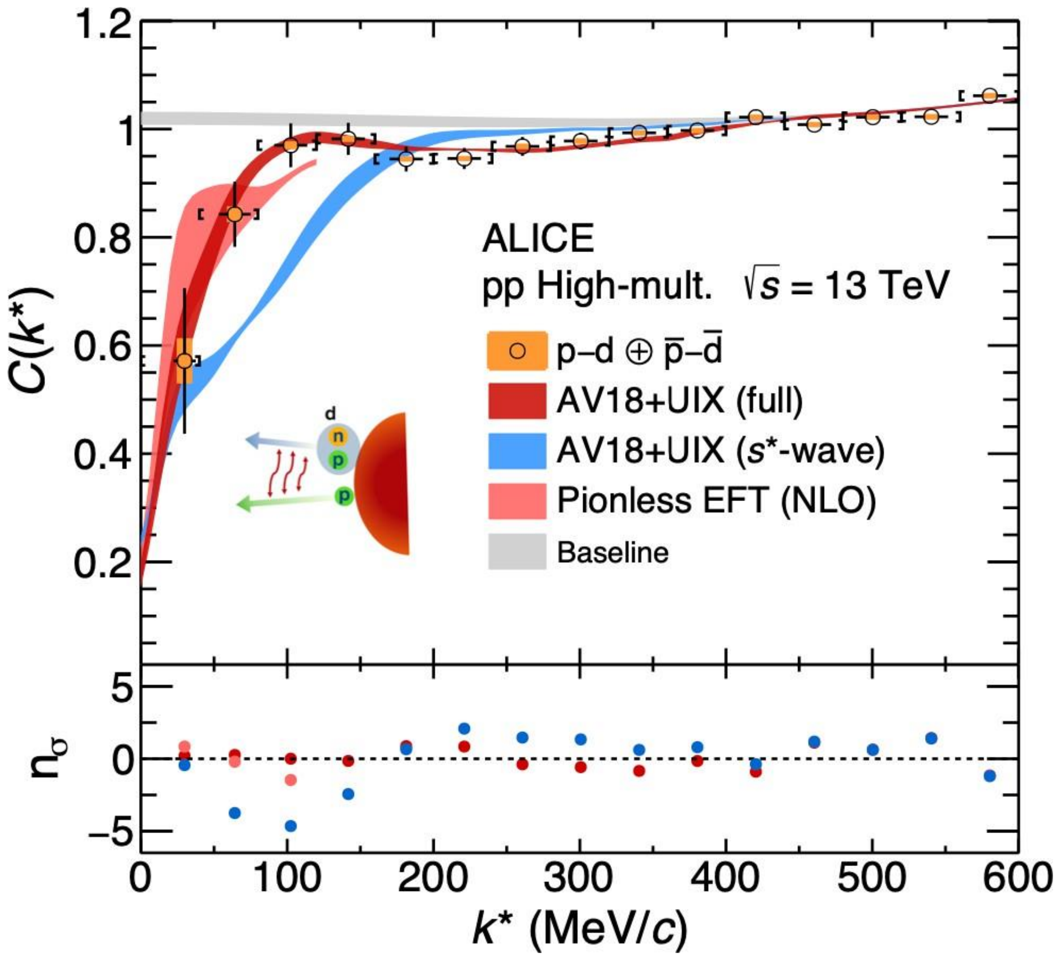
M. Viviani et al, Phys.Rev.C 108 (2023) 6, 064002

NNN using proton-deuteron correlations

- Full three-body calculations are required (NN + NNN + Quantum Statistics)
- Hadron-nuclei correlations at the LHC can be used to study many-body dynamics
- Sensitivity to three-body forces up to 5%





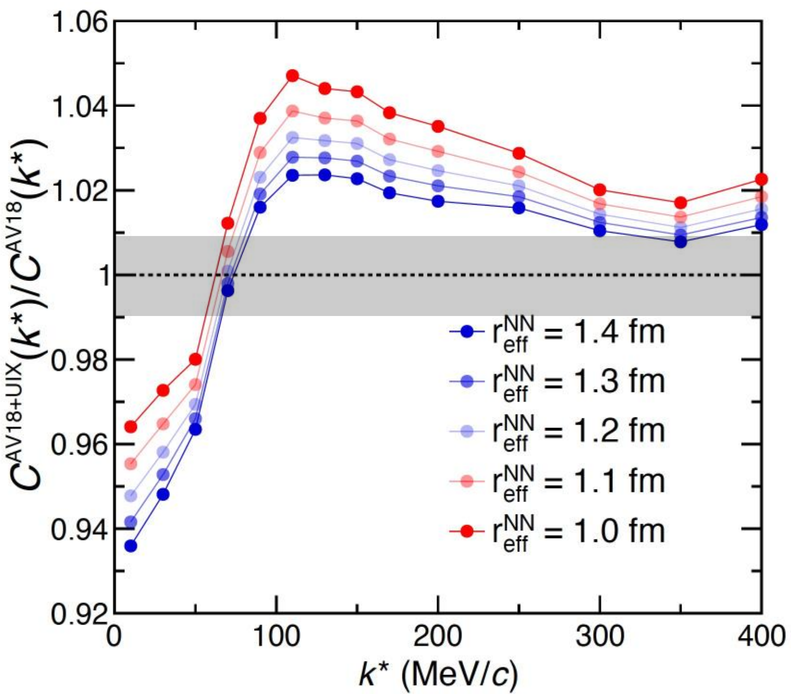


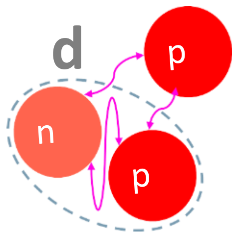
ALICE Coll., arXiv:2308.16120 (2023)

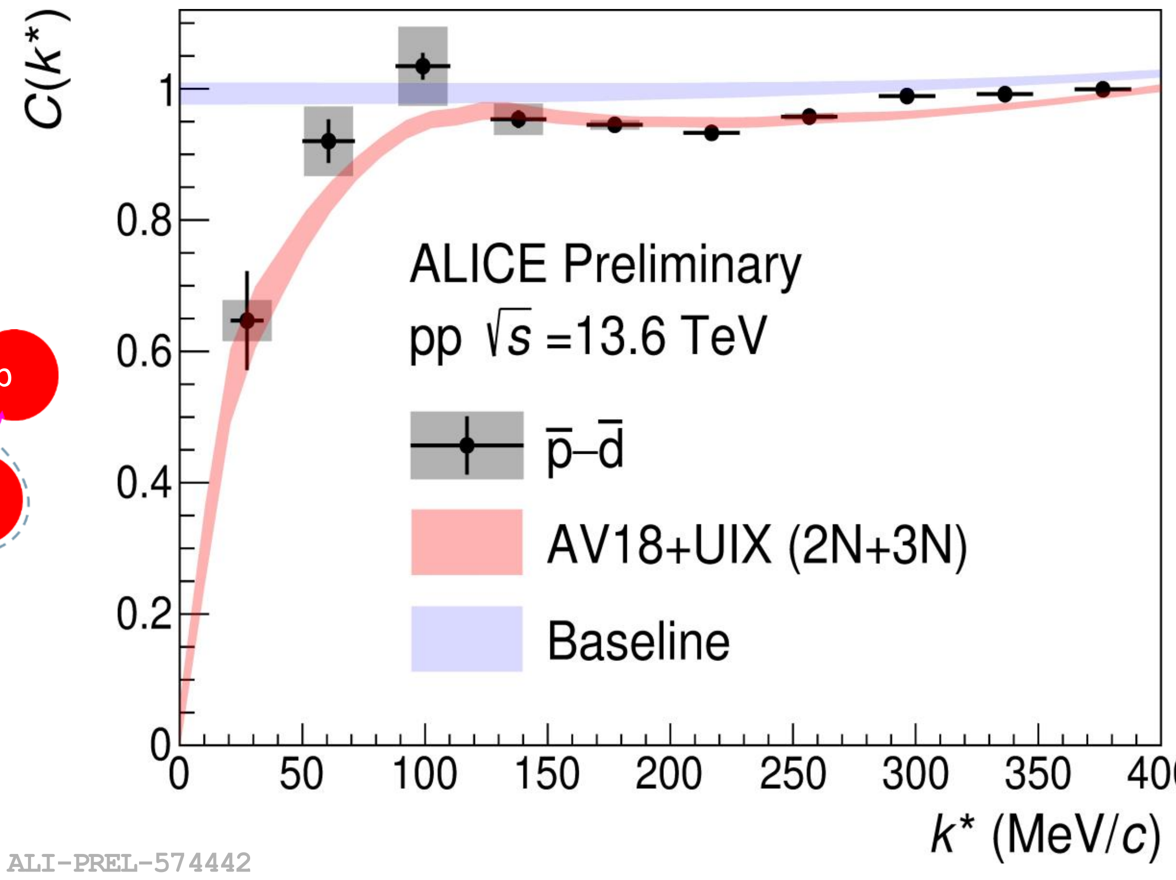
M. Viviani et al, Phys.Rev.C 108 (2023) 6, 064002

NNN using proton-deuteron correlations

- Full three-body calculations are required (NN + NNN + Quantum Statistics)
- Run 3 data from 2022 already analysed and results are promising!
- In Run 3 expected uncertainty of 1%



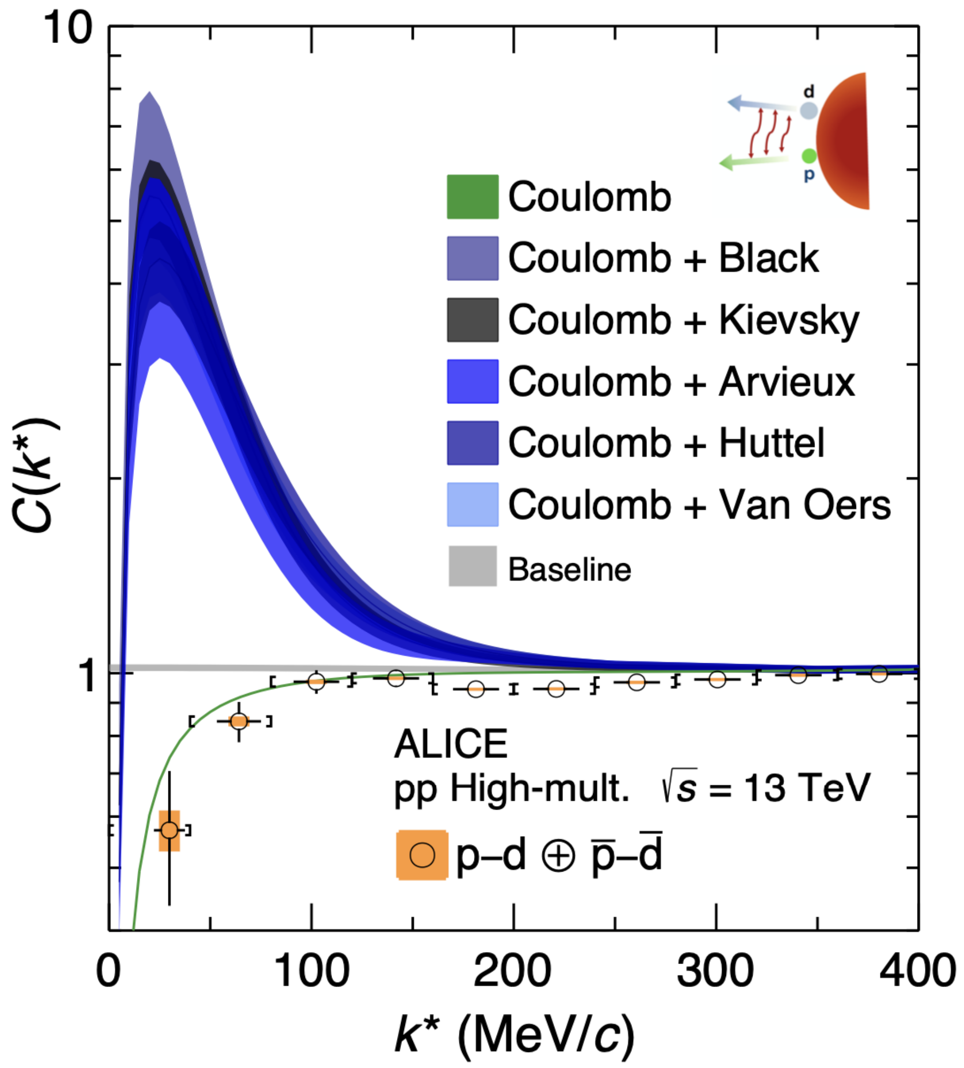




NNN using proton-deuteron correlations

- Point-like particle models anchored to scattering experiments
  W. T. H. Van Oers et al., NPA 561 (1967);
  J. Arvieux et al., NPA 221 (1973); E. Huttel et al., NPA 406 (1983);
  A. Kievsky et al., PLB 406 (1997); T. C. Black et al., PLB 471 (1999);
- Coulomb + strong interaction using Lednický model
  Lednický, R. Phys. Part. Nuclei 40, 307–352 (2009)
- Only s-wave interaction
- Source radius evaluated using the universal m_T scaling

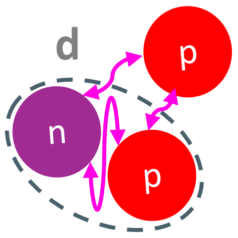
Point-like particle description doesn't work for p-d

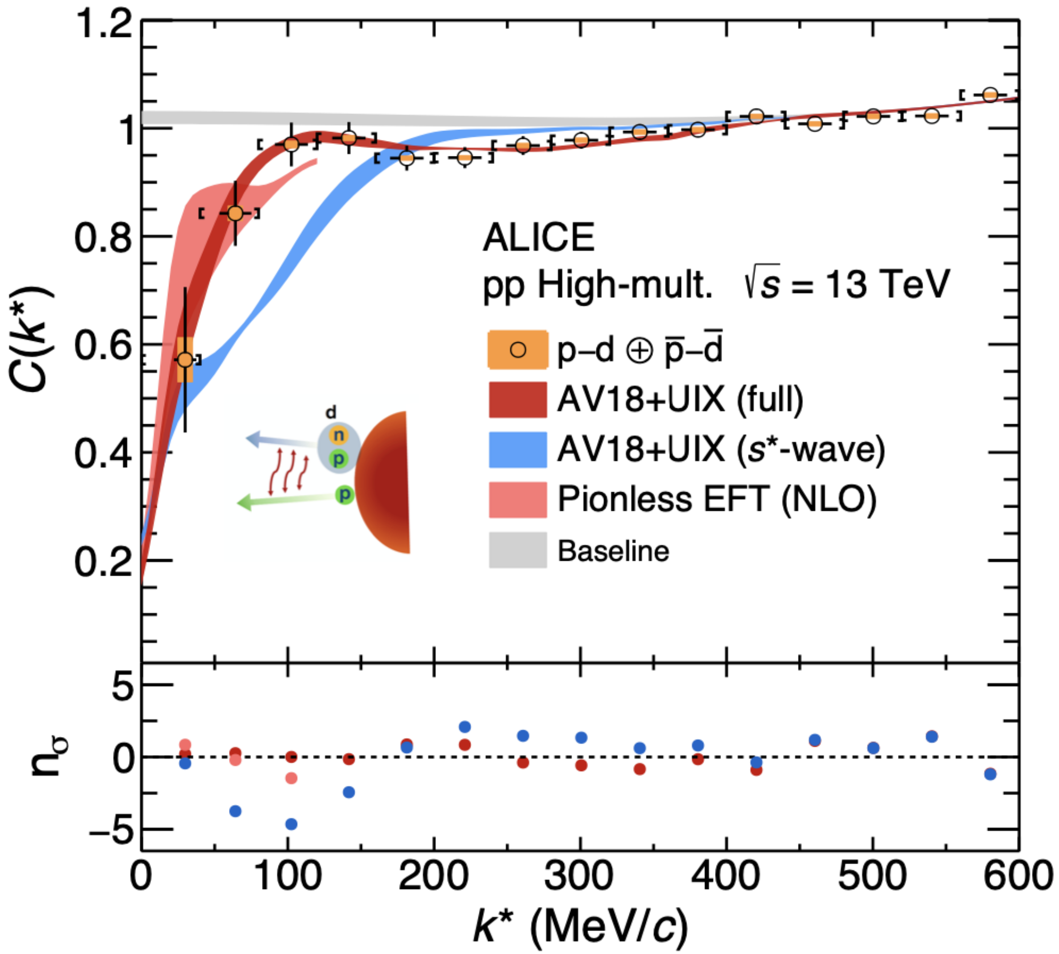


ALICE Coll. arXiv:2308.16120 (2023), accepted by PRX

NNN using proton-deuteron correlations

- Full three-body calculations are required (NN + NNN + Quantum Statistics)
- Hadron-nuclei correlations at the LHC can be used to study many-body dynamics

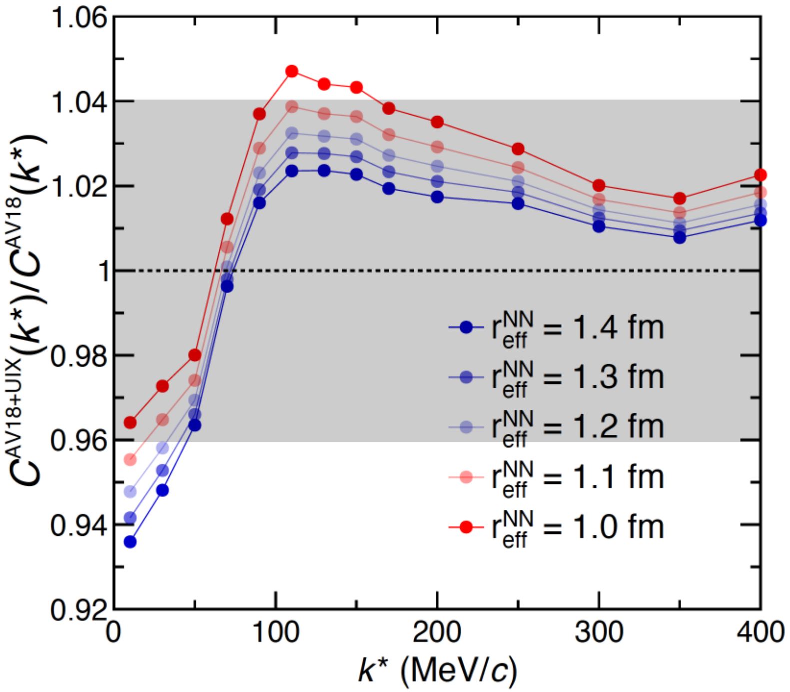


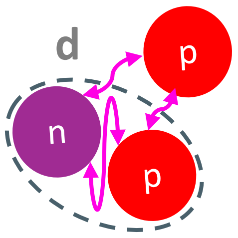


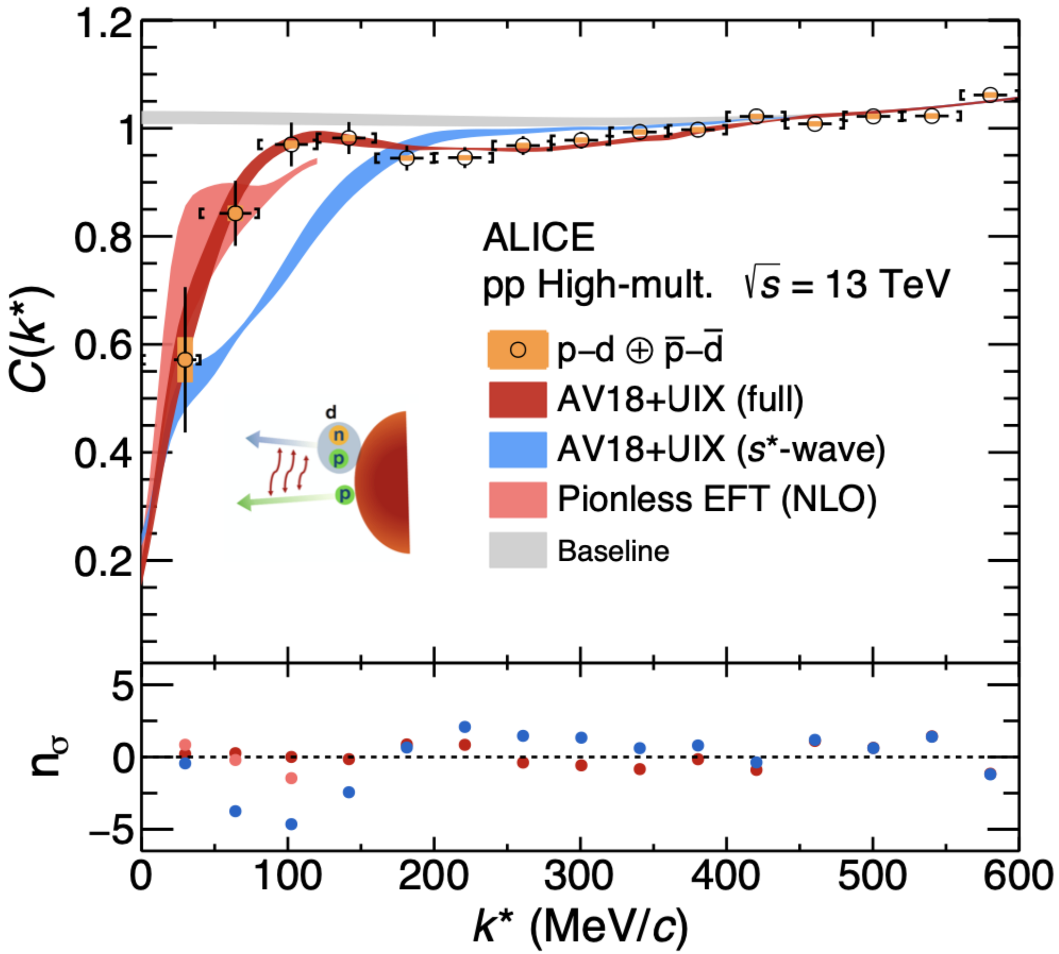
ALICE Coll., arXiv:2308.16120 (2023)
M. Viviani et al, Phys.Rev.C 108 (2023) 6, 064002

NNN using proton-deuteron correlations

- Full three-body calculations are required (NN + NNN + Quantum Statistics)
- Hadron-nuclei correlations at the LHC can be used to study many-body dynamics
- Sensitivity to three-body forces up to 5%



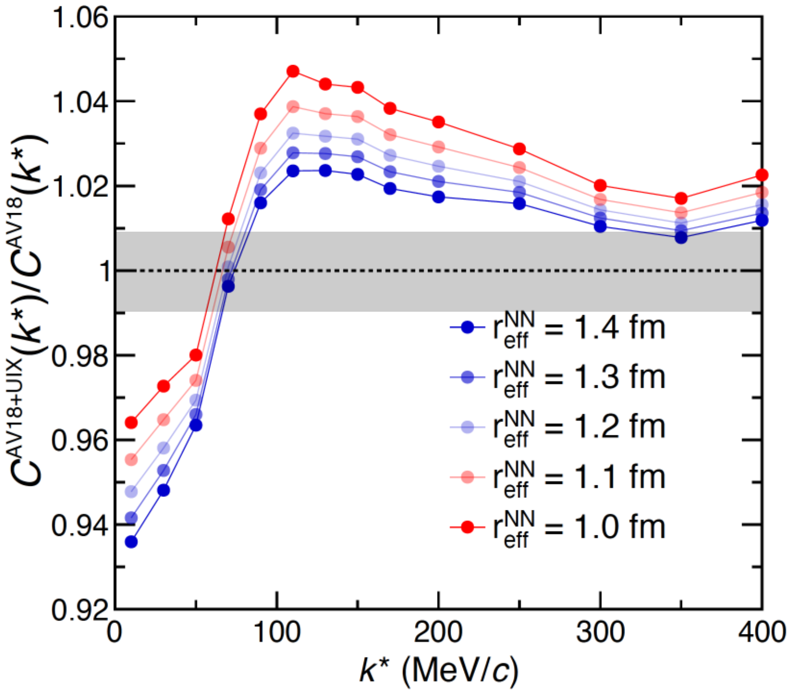


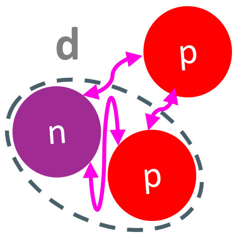


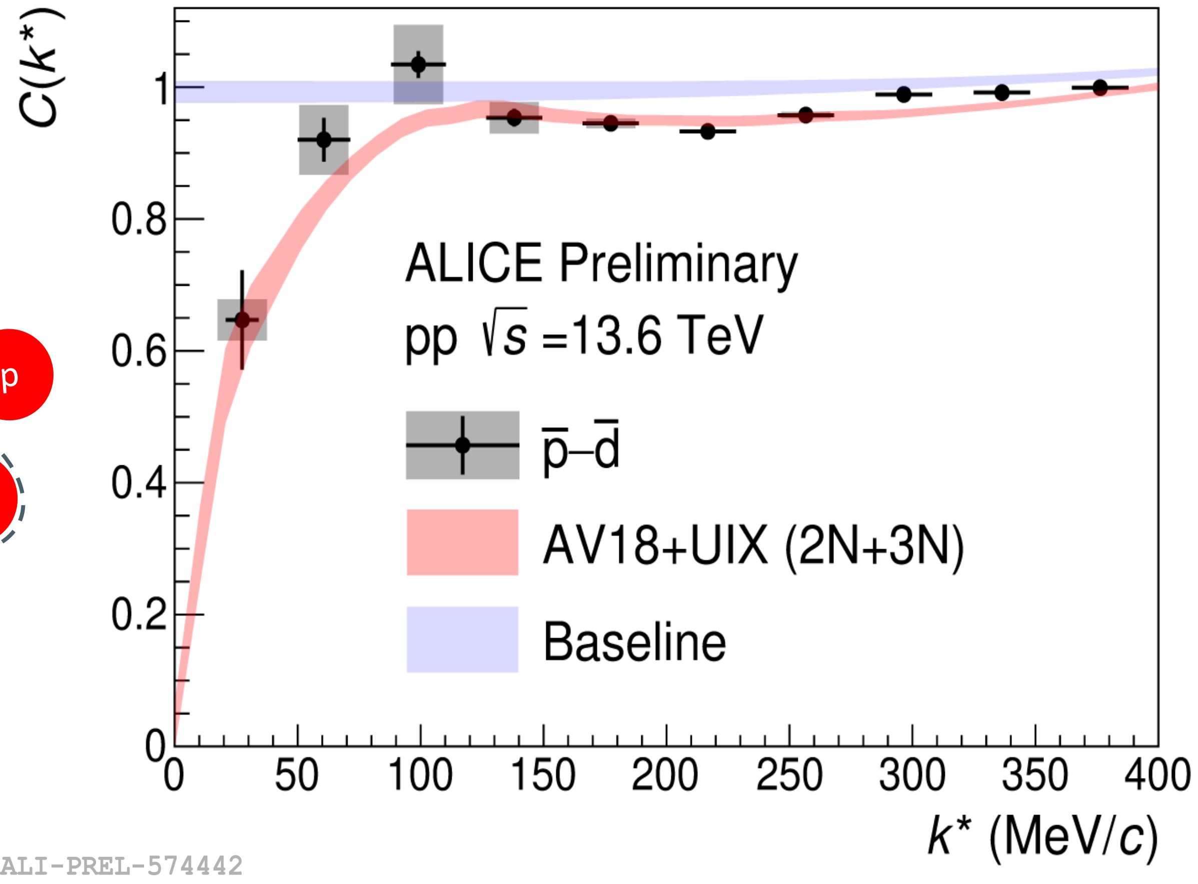
ALICE Coll., arXiv:2308.16120 (2023)
M. Viviani et al, Phys.Rev.C 108 (2023) 6, 064002

NNN using proton-deuteron correlations

- Full three-body calculations are required (NN + NNN + Quantum Statistics)
- Run 3 data from 2022 already analysed and results are promising!
- In Run 3 expected uncertainty of 1%





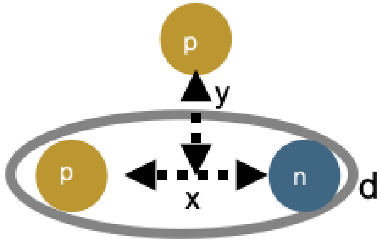


***Talk by** Laura Serksnyte/Anton Riedel 4 Jun, 17:30*

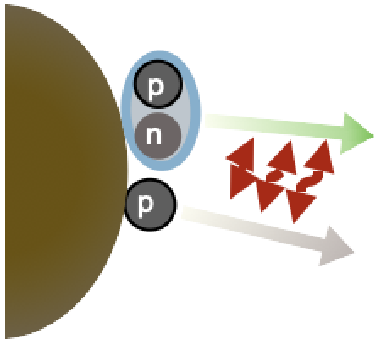
p–d correlation function: d as composite object

The three body wave function with proper treatment of 2N and 3N interaction at very short distances goes to a p–d state.

- **Three–body wavefunction for p–d:** $\Psi_{m_2,m_1}(x,y)$ describing three-body dynamics, anchored to p–d scattering observables.
  - x = distance of p-n system within the deuteron
  - y = p-d distance
  - m_2 and m_1 deuteron and proton spin
- $\Psi_{m_2,m_1}(x,y)$ three-nucleon wave function asymptotically behaves as p–d state:



$$\Psi_{m_2,m_1}(\boldsymbol{x},\boldsymbol{y}) = \underbrace{\Psi^{(\mathrm{free})}_{m_2,m_1}}_{\text{Asymptotic form}} + \sum_{LSJ}^{J\leq\bar{J}} \sqrt{4\pi} i^L \sqrt{2L+1} e^{i\sigma_L} \underbrace{(1 m_2 \tfrac{1}{2} m_1 | S J_z)(L 0 S J_z | J J_z) \widetilde{\Psi}_{LSJJ_z}}_{\text{Strong three-body interaction}} .$$



➡ $\widetilde{\Psi}_{LSJJ_z}$ describe the configurations where the three particles are close to each other

➡ $\Psi^{(\mathrm{free})}_{m_1,m_2}$ an asymptotic form of p-d wave function

Kievsky et al, Phys. Rev. C 64 (2001) 024002
Kievsky et al, Phys. Rev. C 69 (2004) 014002
Deltuva et al, Phys. Rev. C71 (2005) 064003

p–d correlation function

- **Starting with the PPN state that goes into pd state**:
  - Nucleons with the Gaussian sources distributions

Single-particle Gaussian emission source

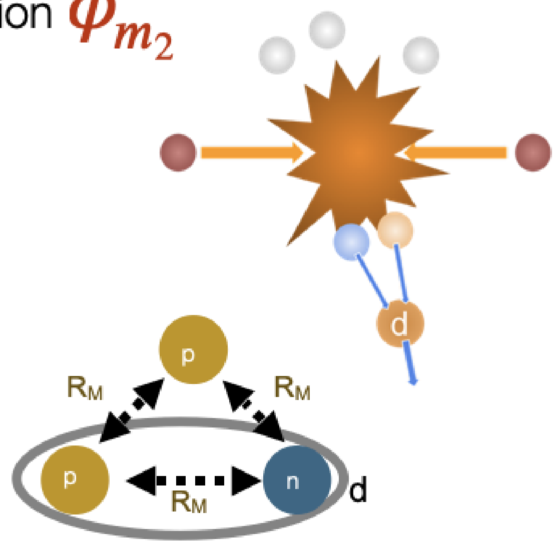
$$A_d C_{pd}(k) = \frac{1}{6}\sum_{m_2,m_1}\int d^3r_1 d^3r_2 d^3r_3\, S_1(r_1)S_1(r_2)S_1(r_3)|\Psi_{m_2,m_1}|^2 \,,$$

- Where A_d is the deuteron formation probability using deuteron wavefunction ϕ_{m_2}

$$A_d = \frac{1}{3}\sum_{m_2}\int d^3r_1 d^3r_2\, S_1(r_1)S_1(r_2)|\phi_{m_2}|^2 \,,$$

- Final definition of the correlation with p–p source size R_M:

$$A_d C_{pd}(k) = \frac{1}{6}\sum_{m_2,m_1}\int \rho^5 d\rho d\Omega\, \frac{e^{-\rho^2/4R_M^2}}{(4\pi R_M^2)^3}|\Psi_{m_2,m_1}|^2 \,.$$



Mrówczyński et al *Eur. Phys. J. Special Topics 229, 3559 (2020)*