

Coalescence studies for light nuclei

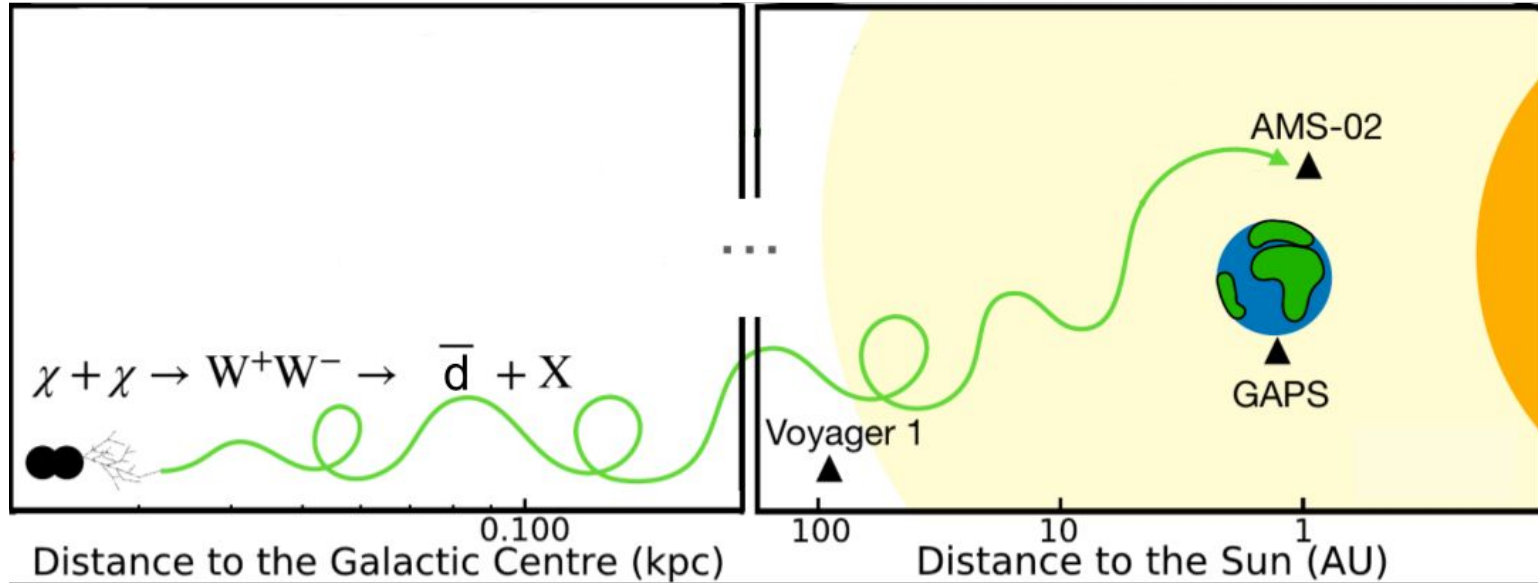
Maximilian Mahlein, Laura Fabbietti, Bhawani
Singh, Chiara Pinto, Michele Viviani

Based on: [arXiv:2404.03352](https://arxiv.org/abs/2404.03352)

Technical University Munich

Cosmic Rays

Antinuclei in Cosmic Rays

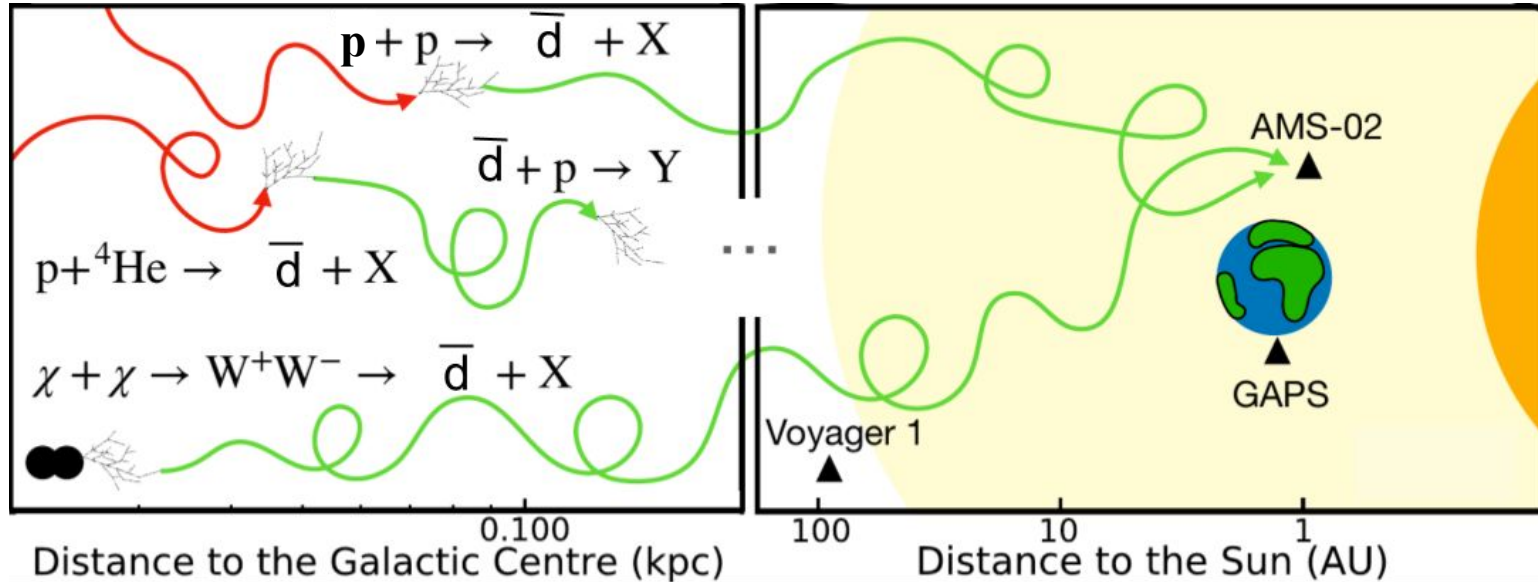


ALICE Collaboration, Nat. Phys. 19, 61–71 (2023)

- Antinuclei could be a probe for indirect Dark Matter searches

Cosmic Rays

Antinuclei in Cosmic Rays

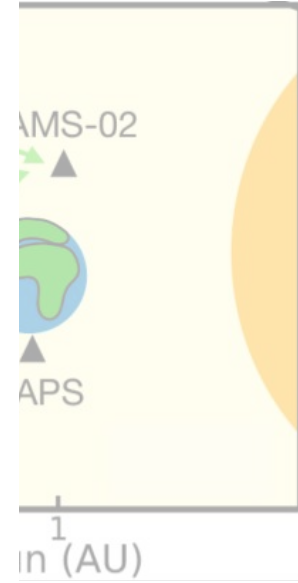
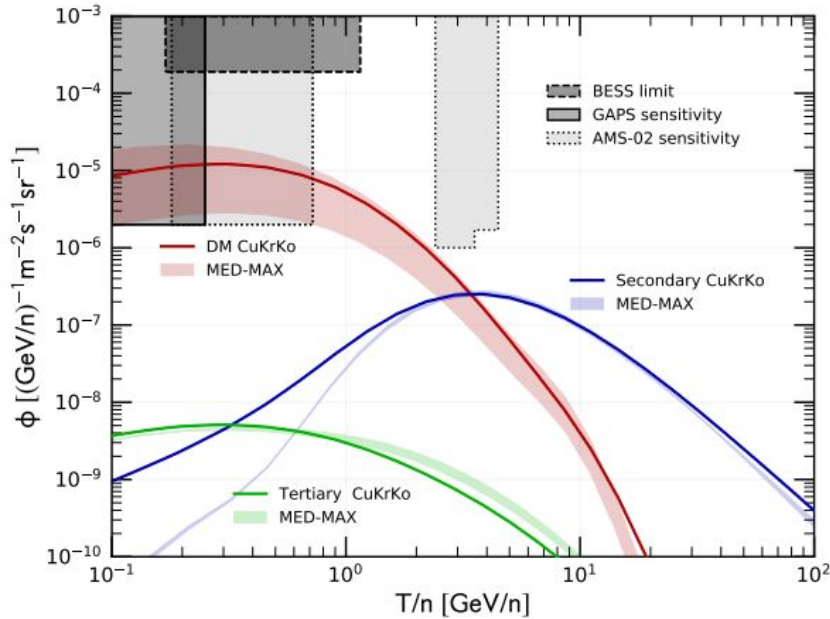
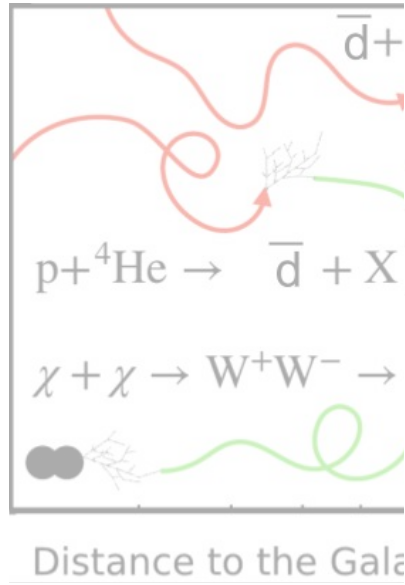


ALICE Collaboration, Nat. Phys. 19, 61–71 (2023)

- Antinuclei could be a probe for indirect Dark Matter searches
- However: Astrophysical background from cosmic rays expected

Cosmic Rays

Antinuclei in Cosmic Rays



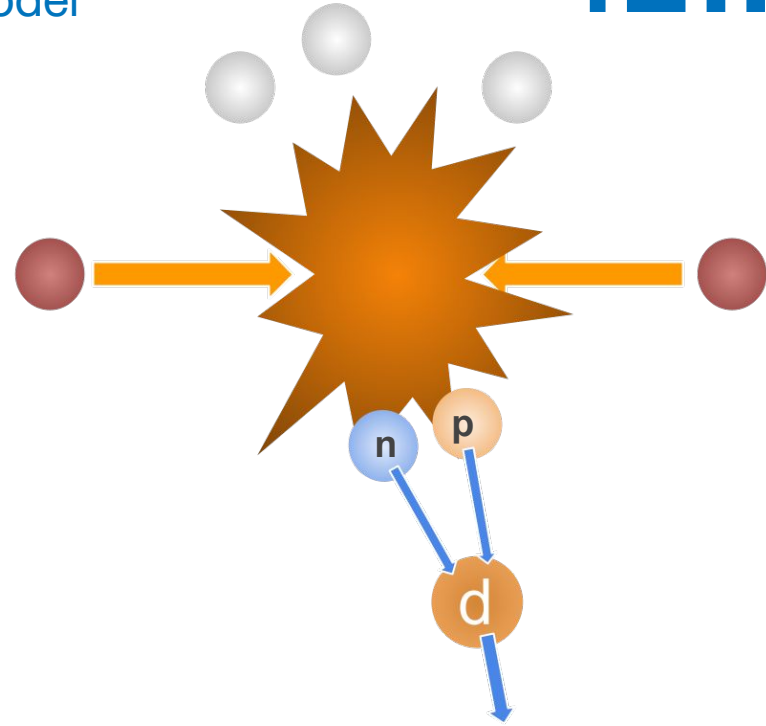
ation, Nat. Phys. 19, 61–71 (2023)

- Antinuclei could be a probe for indirect Dark Matter searches
- However: Astrophysical background from cosmic rays expected
- High Signal/Noise ratio ($\sim 10^2$ - 10^4) at low E_{kin} expected by many models!

Modelling (Anti)nuclei Production

The Coalescence Model

- Nucleons bind after freeze-out if they are close in phase-space

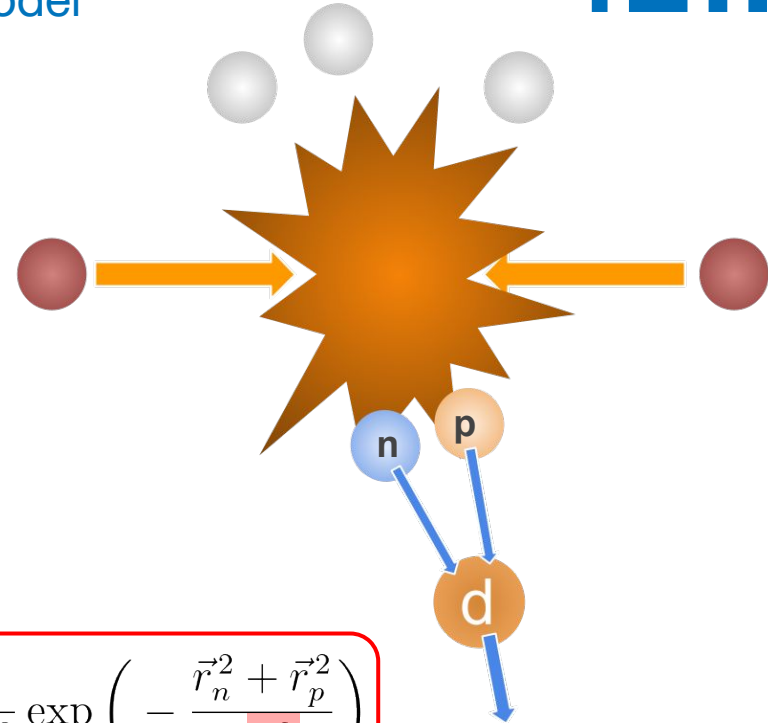


Modelling (Anti)nuclei Production

The Coalescence Model

- Nucleons bind after freeze-out if they are close in phase-space
- Wigner function formalism:

$$\frac{dN_d}{d^3P} = S_d \int d^3x_1 \int d^3x_2 \int d^3x'_1 \int d^3x'_2 \Psi_d^*(\vec{x}'_1, \vec{x}'_2) \times \Psi_d(\vec{x}_1, \vec{x}_2) \langle \Psi_2^\dagger(\vec{x}'_2) \Psi_1^\dagger(\vec{x}'_1) \Psi_1(\vec{x}_1) \Psi_2(\vec{x}_2) \rangle$$



$$\mathcal{P}(q, \sigma) = \frac{S_2}{(2\pi)^3 \sigma^6} \int d^3r_p d^3r_n D(q, r) e^{-\frac{r_p^2 + r_n^2}{2\sigma^2}}$$

$$= \int d^3\zeta \Psi(\vec{r} + \vec{\zeta}/2) \Psi^*(\vec{r} - \vec{\zeta}/2) \exp(i\vec{q} \cdot \vec{\zeta})$$

$$= \frac{1}{(2\pi\sigma^2)^3} \exp\left(-\frac{\vec{r}_n^2 + \vec{r}_p^2}{2\sigma^2}\right)$$

Nucleus wave function

Relative momenta of nucleons

Source size

Kachelriess et al EPJA (2020)56: 4, MM et al .Eur.Phys.J.C 83 (2023) 9, 804

Modelling (Anti)nuclei Production

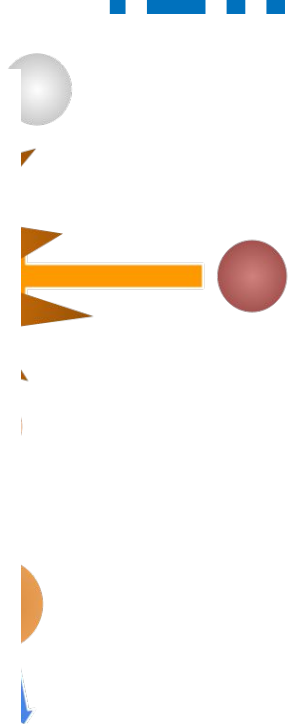
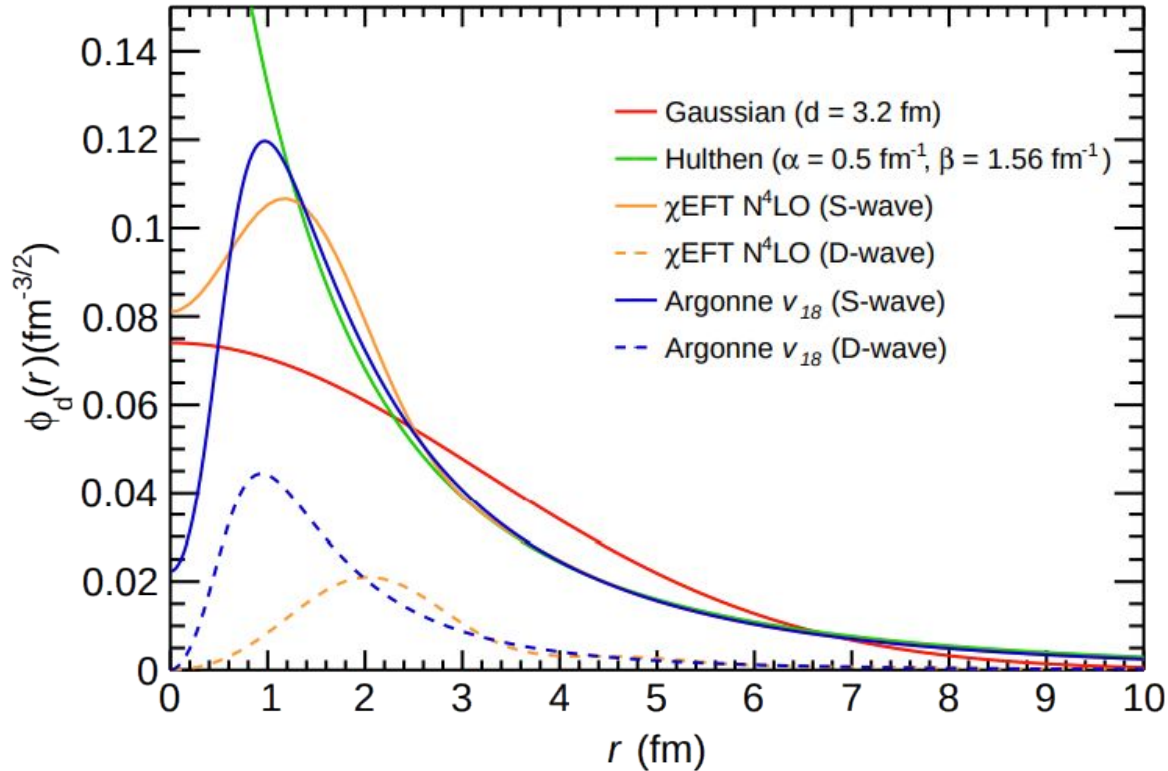
The Coalescence Model

- Nucleons bind are close in phase space
- Wigner function

$$\frac{dN_d}{d^3P} = S_d \int d^3x_1 \int d^3x_2 \dots \times \Psi_d(\vec{x}_1, \dots, \vec{x}_d)$$

$$\mathcal{P}(q, \sigma) = \frac{S_2}{(2\pi)^3} \int d^3\zeta \Psi(\vec{r} + \vec{\zeta}/2) \Psi^*(\vec{r} - \vec{\zeta}/2)$$

$$\int d^3\zeta \Psi(\vec{r} + \vec{\zeta}/2) \Psi^*(\vec{r} - \vec{\zeta}/2)$$



Kachelriess et al EPJA (2020)56: 4, MM et al .Eur.Phys.J.C 83 (2023) 9, 804

Modelling (Anti)nuclei Production

The Coalescence Model

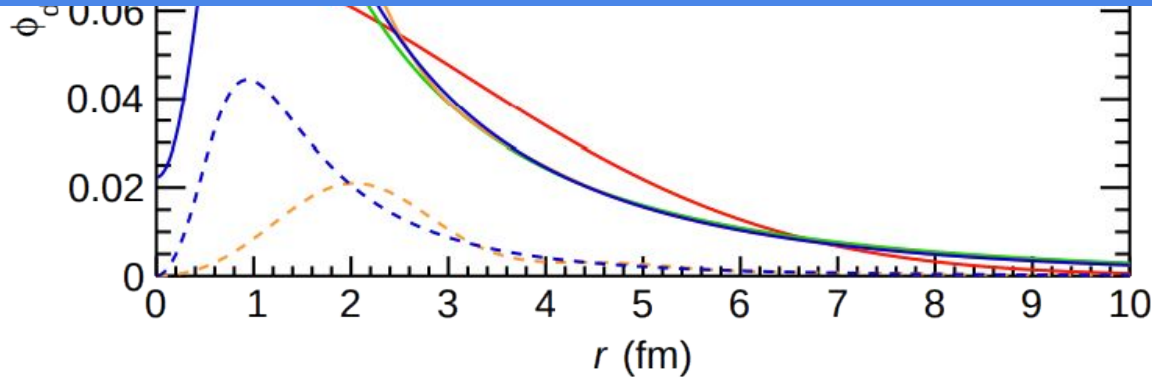
- Nucleons bind are close in phase space
- Wigner function

$$\frac{dN_d}{d^3p} = S_d \int d^3x_1 \int d^3x_2 \dots$$



Use a General Purpose Event Generator (EPOS, PYTHIA) for the phase-space input (momenta, source size)

$$\mathcal{P}(q, \sigma) = \frac{\sigma^2}{(2\pi)^3} \int d^3x_1 \int d^3x_2 \dots$$

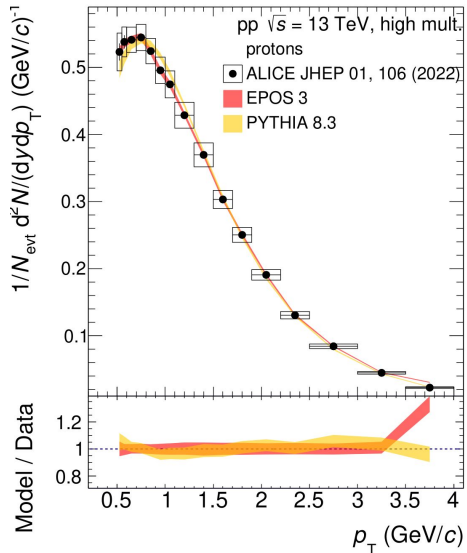


$$\int d^3\zeta \Psi(\vec{r} + \vec{\zeta}/2) \Psi^*(\vec{r} - \vec{\zeta}/2)$$

Nucleus wave function

Kachelriess et al EPJA (2020)56: 4, MM et al .Eur.Phys.J.C 83 (2023) 9, 804

- Corrections to Protons

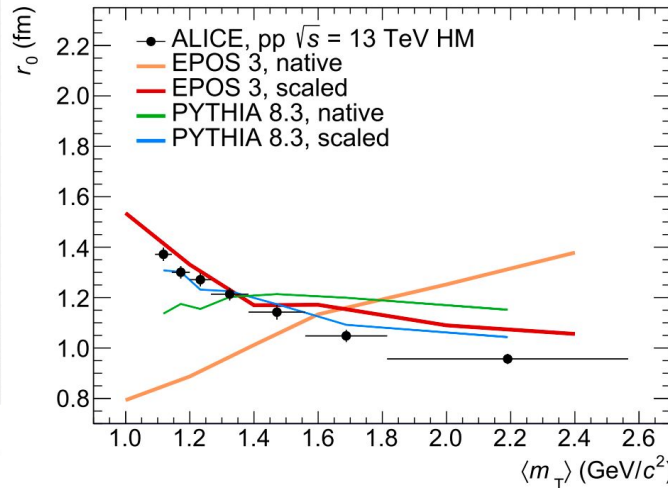
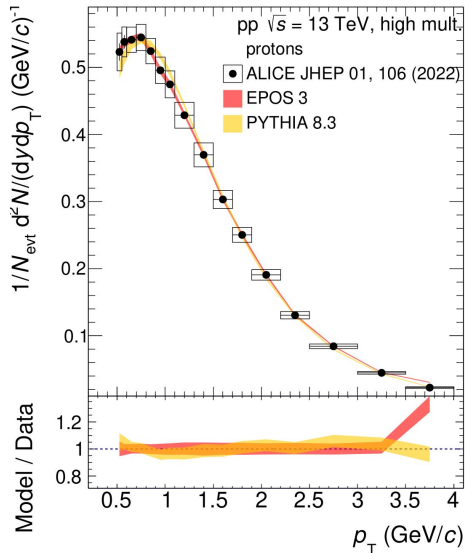


Coalescence Results EPOS & Pythia

Deuteron spectra

MM et al. Eur.Phys.J.C 83 (2023) 9, 804

- Corrections to Protons, Source, Multiplicity

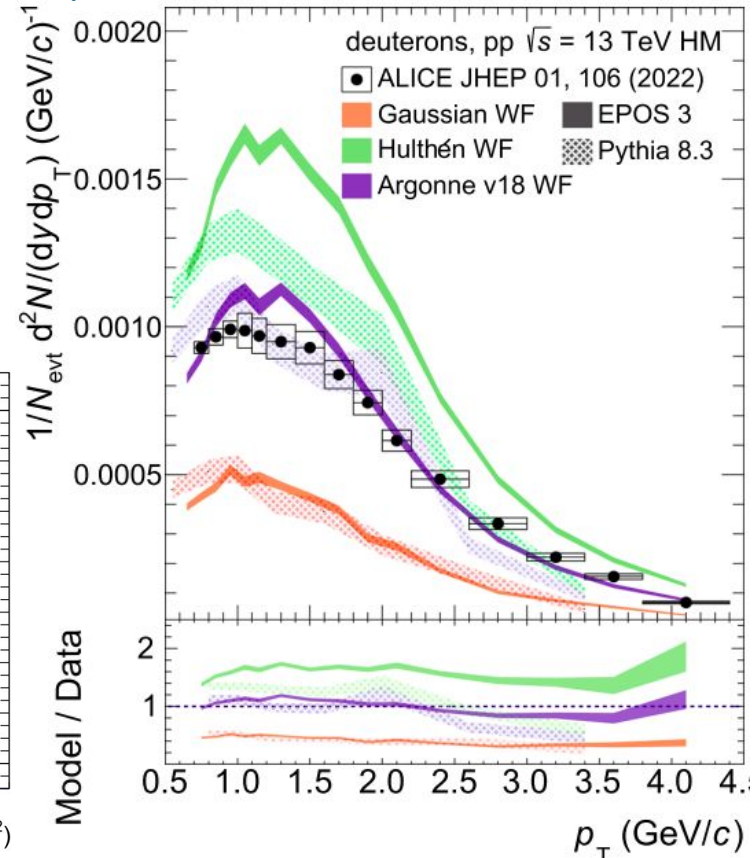
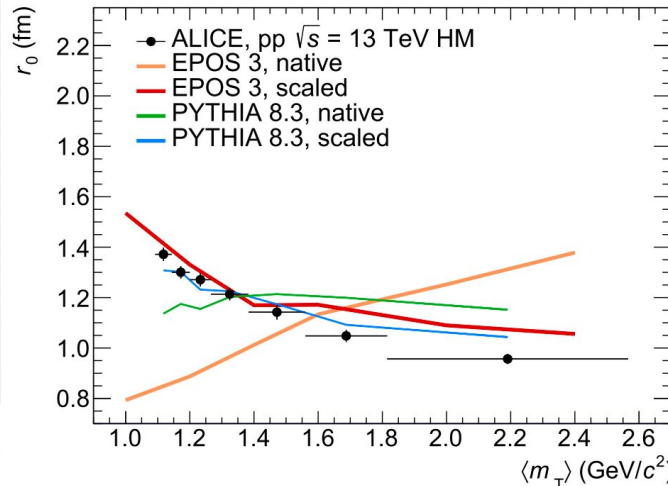
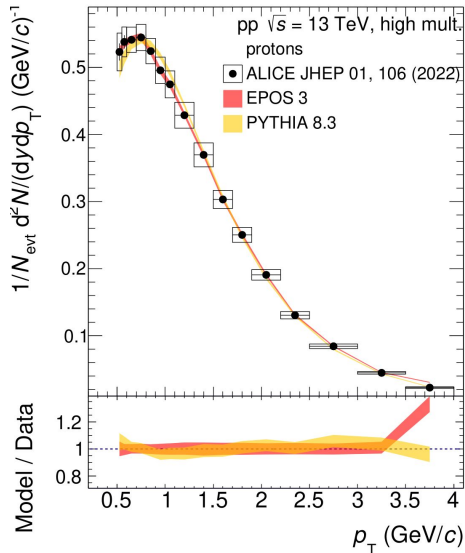


Coalescence Results EPOS & Pythia

Deuteron spectra

MM et al. Eur.Phys.J.C 83 (2023) 9, 804

- Corrections to Protons, Source, Multiplicity
- Wavefunctions: Gaussian, Hulthén and Argonne v_{18}
- AV_{18} reproduces data to $\sim 10\%$

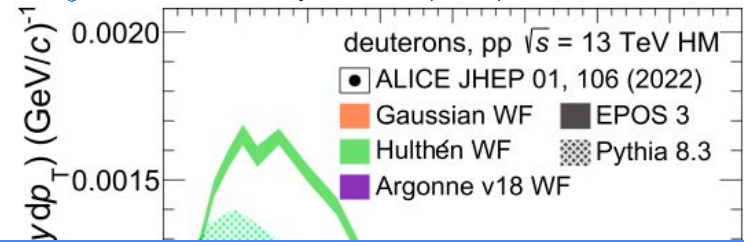


Coalescence Results EPOS & Pythia

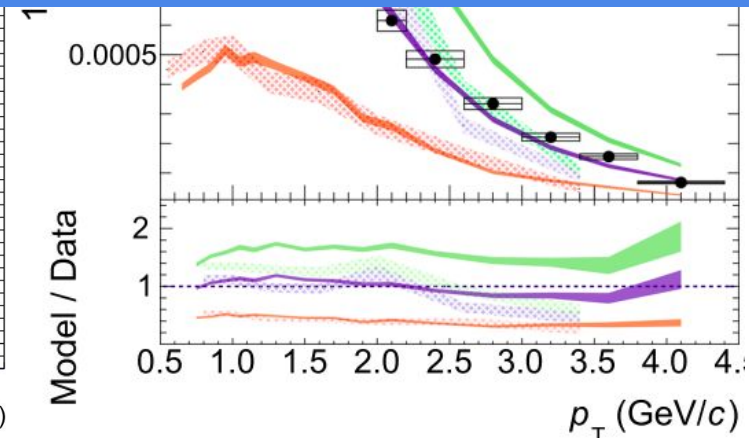
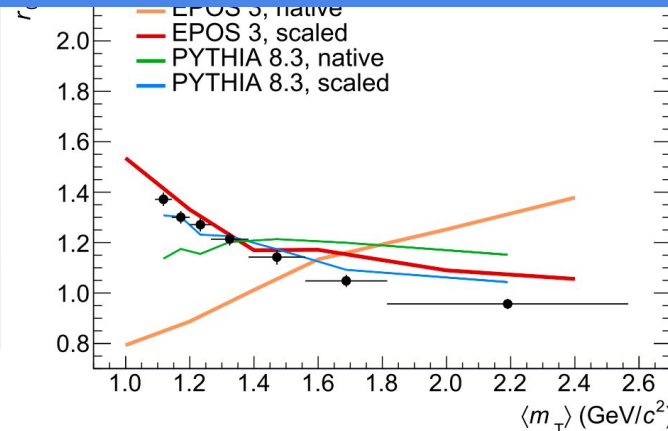
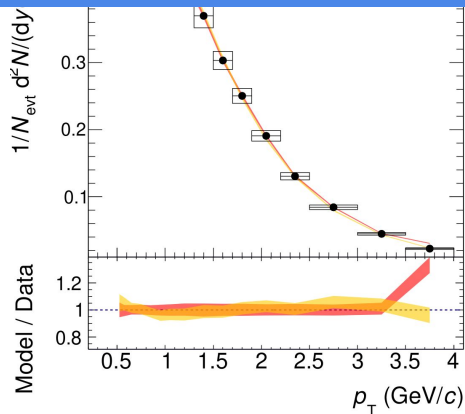
Deuteron spectra

- Corrections to Protons, Source, Multiplicity
- Wavefunctions: Gaussian, Hulthén and Argonne v_{18}
- AV_{18} reproduces data to $\sim 10\%$

MM et al. Eur.Phys.J.C 83 (2023) 9, 804



First event-by-event coalescence model with realistic wave function!



Advantages:

- Model extremely complex phenomena and particle correlations
- Easy to use ('Plug and play')
- Trivial extrapolation to different energies, multiplicities (and Collision systems)

Disadvantages:

- Convoluted Code, hard to adjust
- Hard to distill influence of single mechanism on the final result
- Long simulation times
- No nuclei production

Advantages:

- Model extremely complex phenomena and particle correlations
- Easy to use ('Plug and play')
- Trivial extrapolation to different energies, multiplicities (and Collision systems)

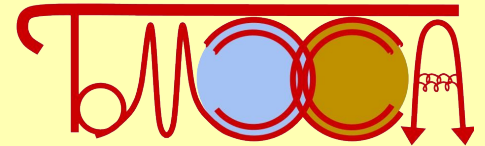
Disadvantages:

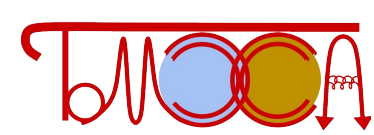
- Convoluted Code, hard to adjust
- Hard to distill influence of single mechanism on the final result
- Long simulation times
- No nuclei production

➔ Build Toy Monte Carlo that uses only the necessary mechanisms for nuclei production

Requirements: Fast simulation, easy to adjust to end-users needs

Toy **M**onte **C**arlo **C**oalescence **A**fterburner: ToMCCA





Speed:

Slowest parts of Event generators: *Hadronization, Hadronic Cascade*

- ➔ Fully omit Hadronization, start from a statistical distribution of nucleons (no mesons)
- ➔ No Rescattering, Flow, Jets, ...

Correlations:

No ab-initio correlations, built in fully by hand

- ➔ can be easily deactivated or adjusted

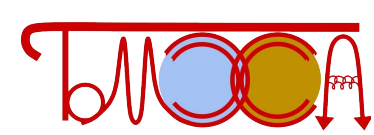
User-Friendly:

All of ToMCCA is ~800 lines of Code

- ➔ Easy to find code responsible for specific effect

Run-in-place configuration

- ➔ Download (<https://github.com/HorstMa/ToMCCA-Public>) and run immediately



Speed:

Slowest parts of Event generators: *Hadronization, Hadronic Cascade*

- ➔ Fully omit Hadronization, start from a statistical distribution of nucleons (no mesons)
- ➔ No Rescattering, Flow, Jets, ...

But a Toy Model needs measured inputs...

User-Friendly:

All of ToMCCA is ~800 lines of Code

- ➔ Easy to find code responsible for specific effect

Run-in-place configuration

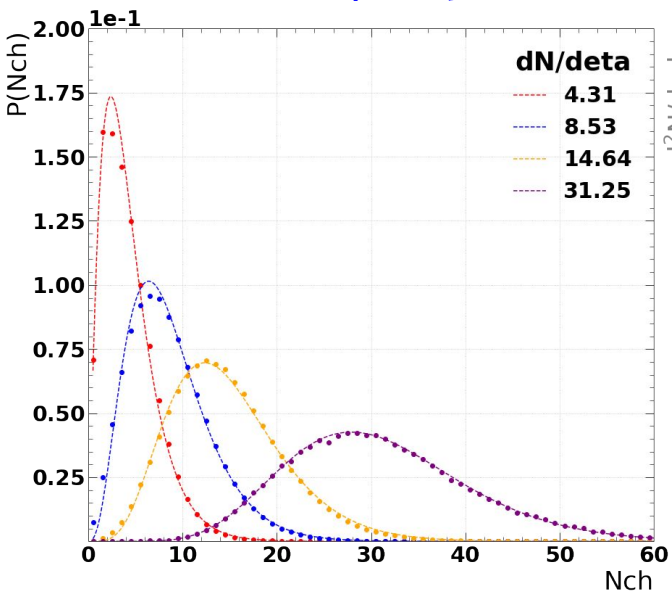
- ➔ Download (<https://github.com/HorstMa/ToMCCA-Public>) and run immediately

The ToMCCA Model

A Toy Monte Carlo Coalescence Afterburner

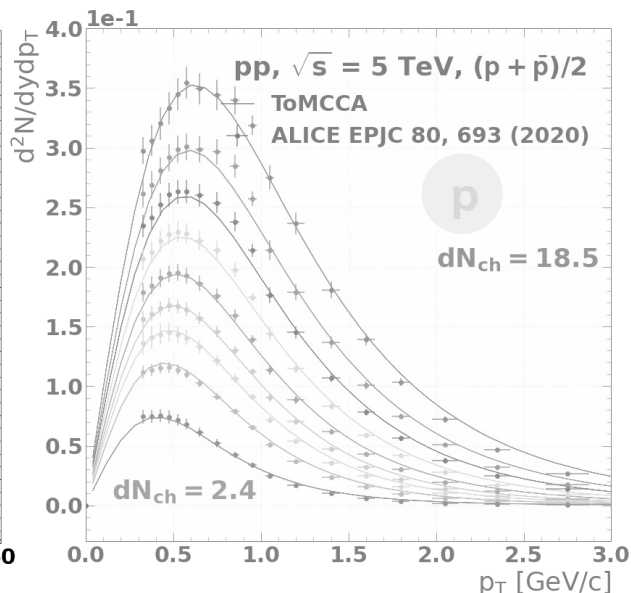
Main Inputs: Multiplicity, momentum distributions, source size

Multiplicity



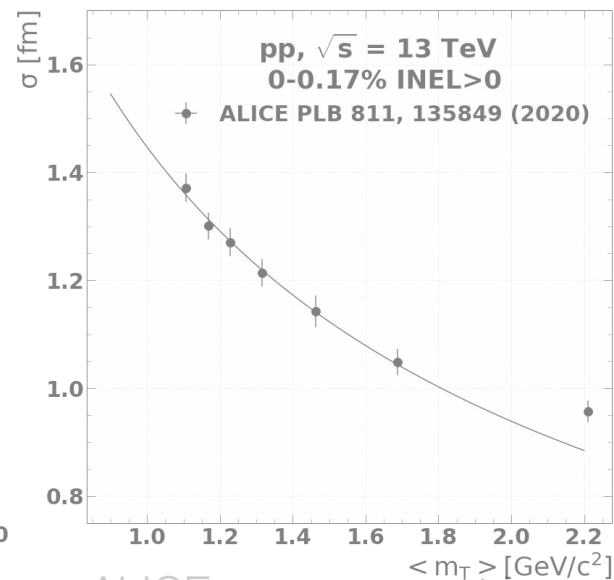
Erlang distribution fitted to EPOS3 simulations

Momentum distribution



ALICE measurements over a large multiplicity range

Source size



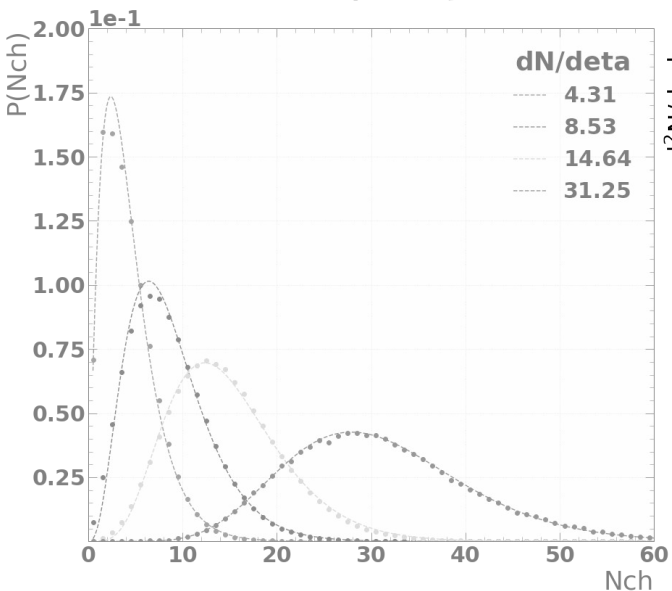
ALICE measurement in high-multiplicity collisions

The ToMCCA Model

A Toy Monte Carlo Coalescence Afterburner

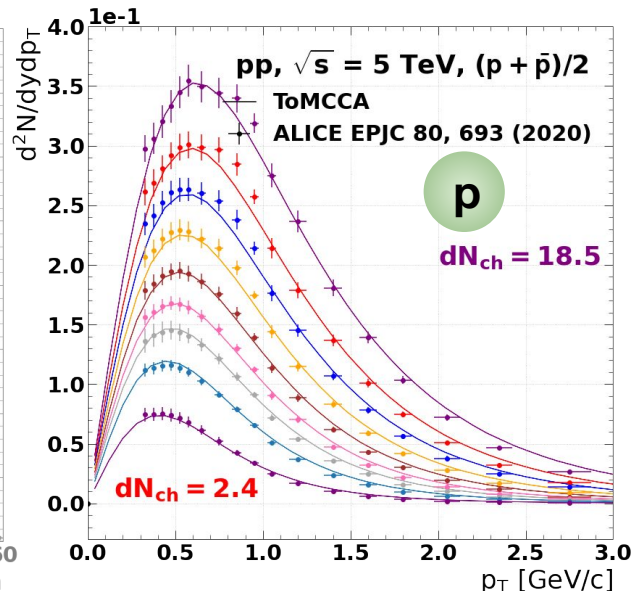
Main Inputs: Multiplicity, momentum distributions, source size

Multiplicity



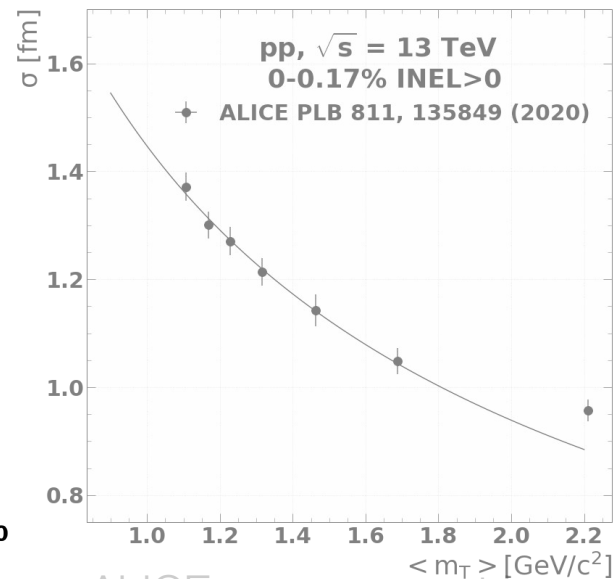
Erlang distribution fitted to EPOS3 simulations

Momentum distribution

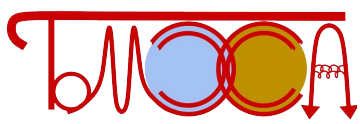


ALICE measurements over a large multiplicity range

Source size



ALICE measurement in high-multiplicity collisions



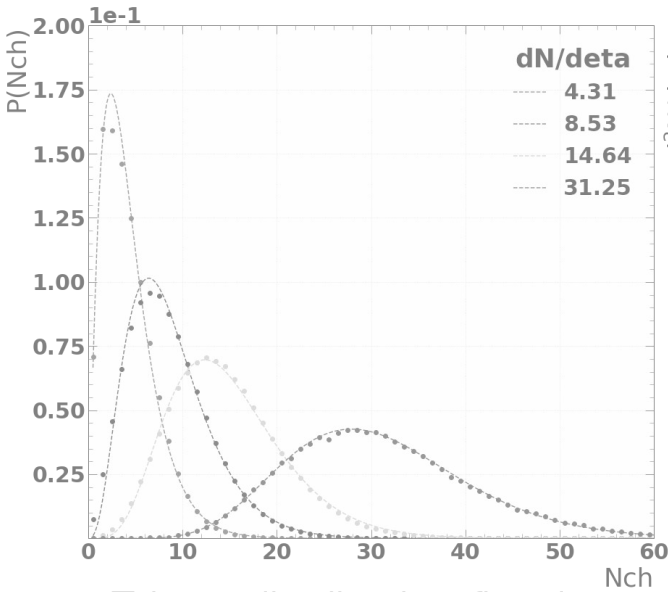
The ToMCCA Model

A Toy Monte Carlo Coalescence Afterburner



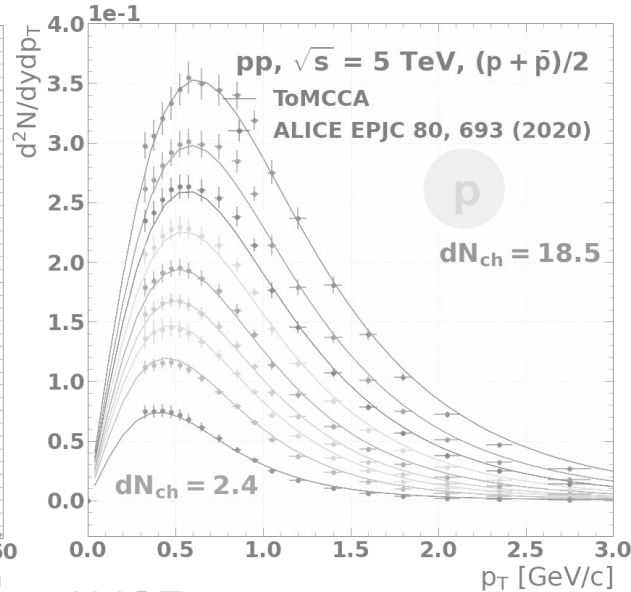
Main Inputs: Multiplicity, momentum distributions, source size

Multiplicity



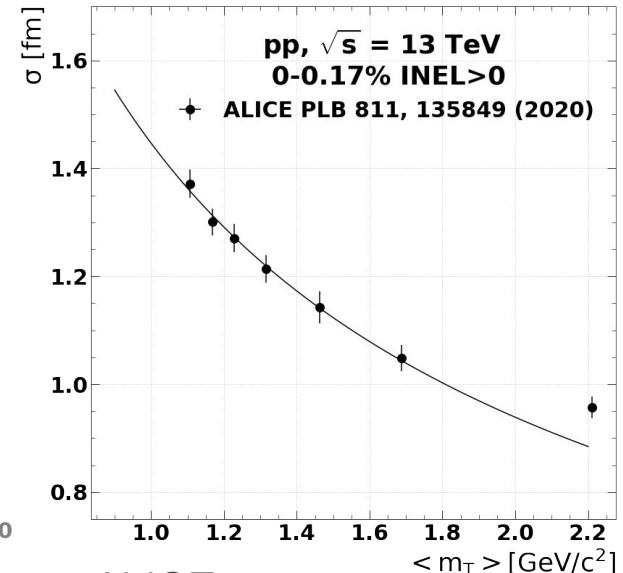
Erlang distribution fitted to EPOS3 simulations

Momentum distribution



ALICE measurements over a large multiplicity range

Source size



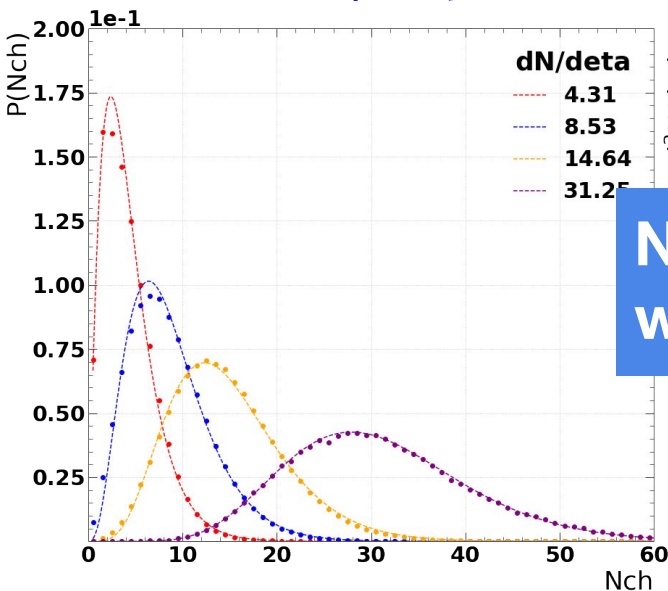
ALICE measurement in high-multiplicity collisions

The ToMCCA Model

A Toy Monte Carlo Coalescence Afterburner

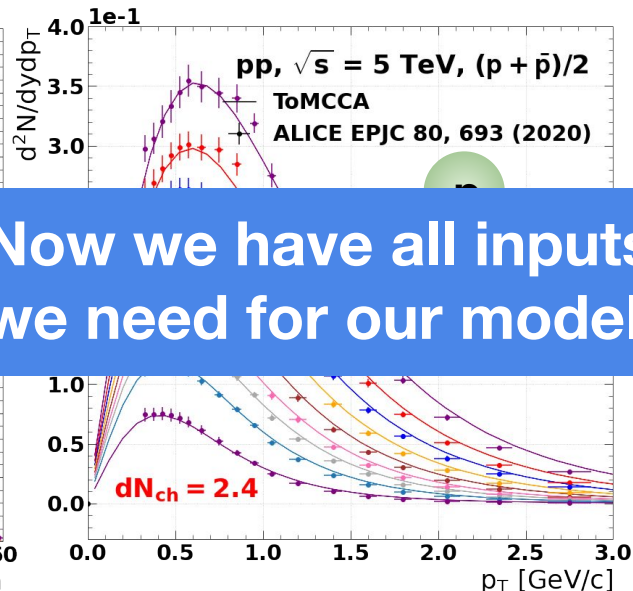
Main Inputs: Multiplicity, momentum distributions, source size

Multiplicity



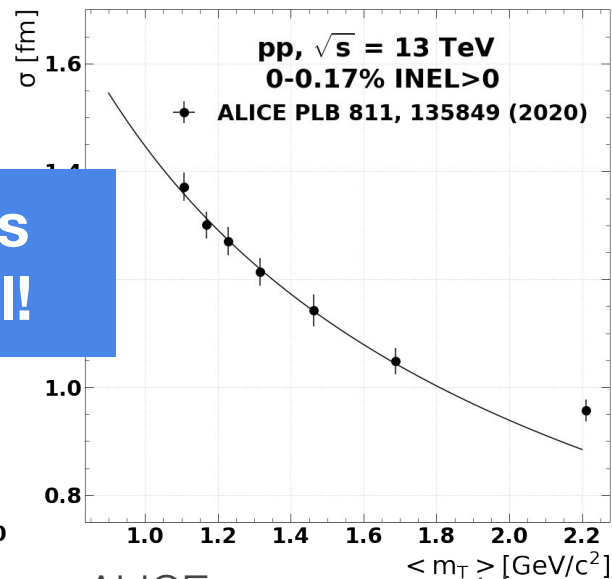
Erlang distribution fitted to EPOS3 simulations

Momentum distribution



ALICE measurements over a large multiplicity range

Source size

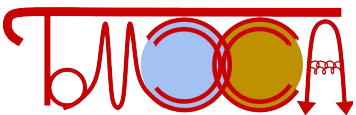


ALICE measurement in high-multiplicity collisions

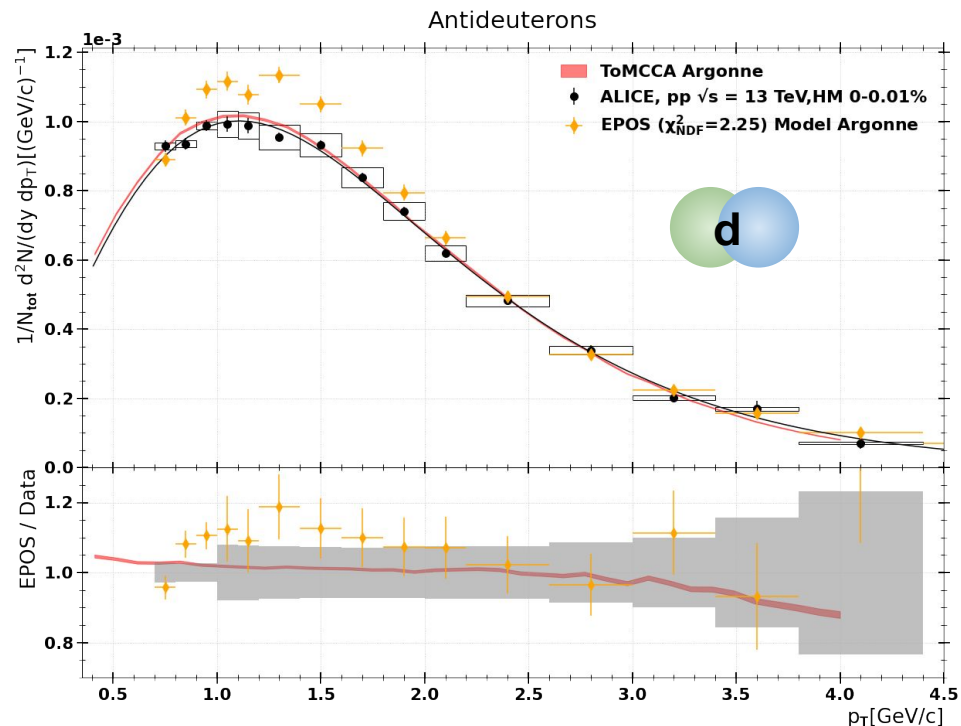
Now we have all inputs we need for our model!

Deuteron Spectra

ToMCCA Model in HM pp Collisions

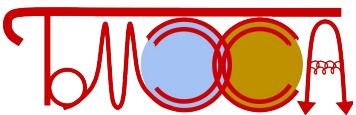


- Using ToMCCA for 13 TeV HM collisions ($(dN_{ch}/dn)_{|\eta|<0.8} \sim 31$) we can reproduce measured spectra
- No free parameters!



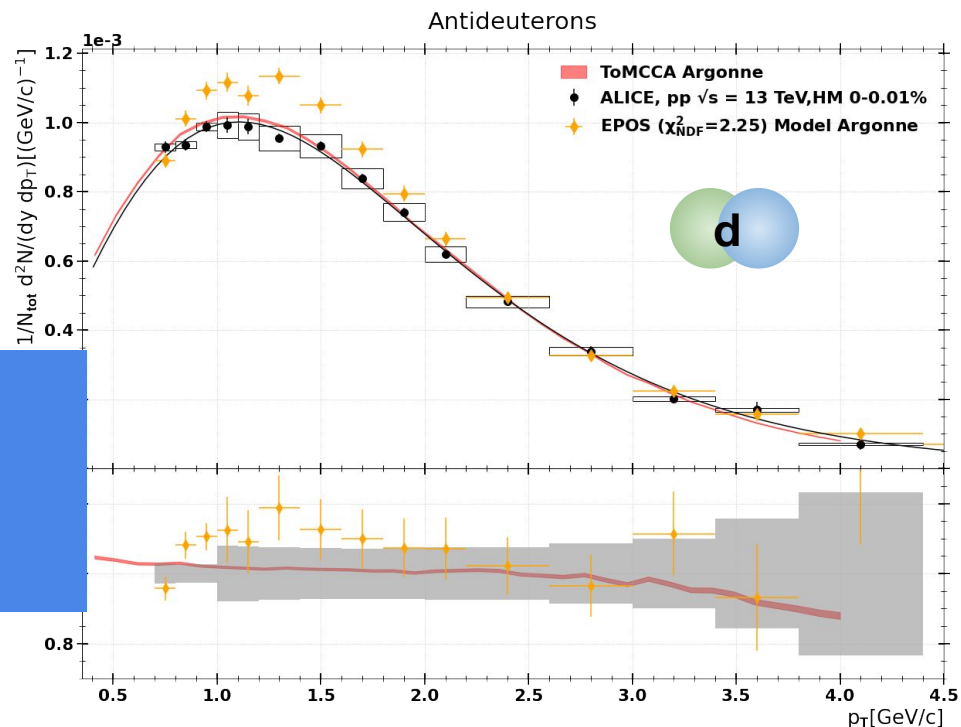
Deuteron Spectra

ToMCCA Model in HM pp Collisions



- Using ToMCCA for 13 TeV HM collisions ($(dN_{ch}/dn)_{|\eta|<0.8} \sim 31$) we can reproduce measured spectra
- No free parameters!

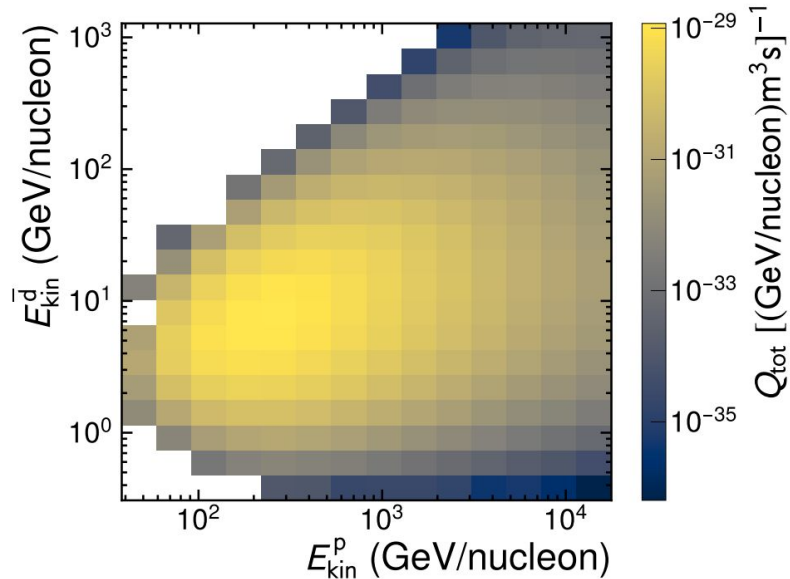
Does this help us in predicting Cosmic Ray fluxes?



Cosmic Rays

Production energy of antinuclei

- Antideuteron production predominantly for protons of $E_{\text{kin}} \sim 200\text{-}500$ GeV ($\sqrt{s} \sim \mathbf{19\text{-}30}$ GeV for p-H)



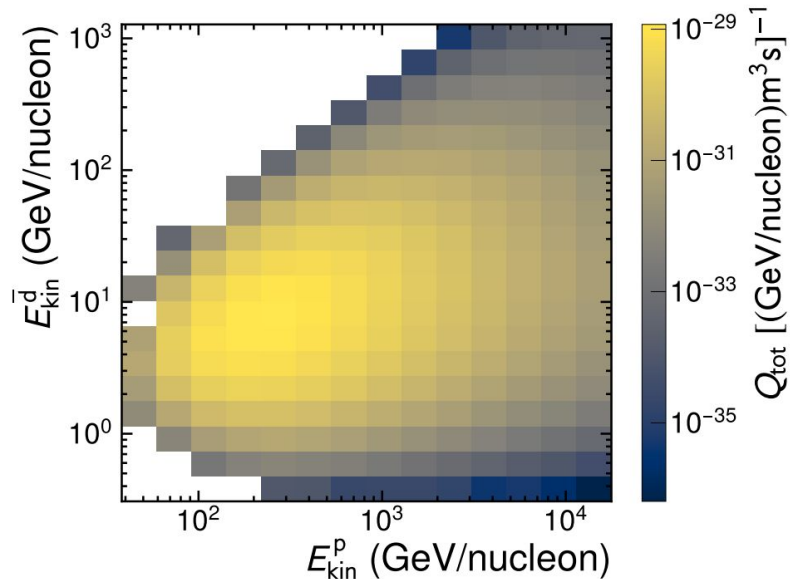
 Šerkšnytė, et al. PRD 105, 083021 (2022)

Cosmic Rays

Production energy of antinuclei

- Antideuteron production predominantly for protons of $E_{\text{kin}} \sim 200\text{-}500$ GeV ($\sqrt{s} \sim \mathbf{19\text{-}30}$ GeV for p-H)

- Extrapolation to lower energies via event multiplicity

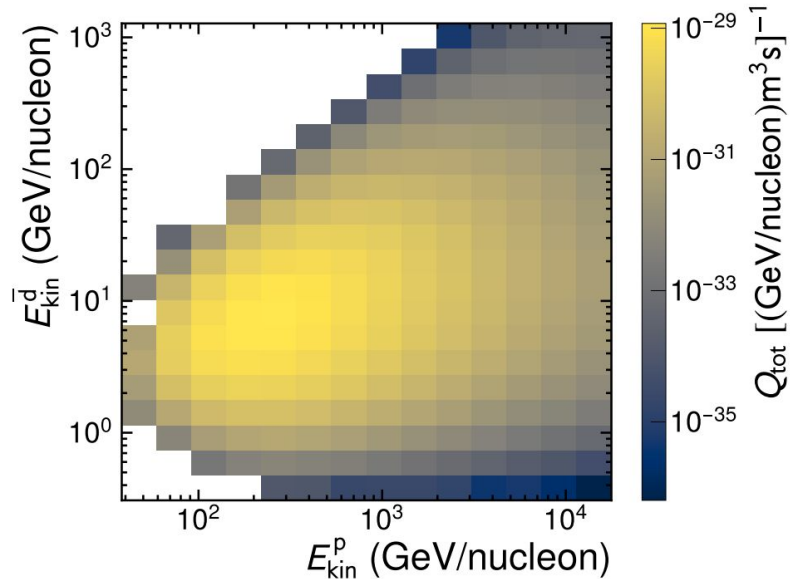


 Šerkšnytė, et al. PRD 105, 083021 (2022)

Cosmic Rays

Production energy of antinuclei

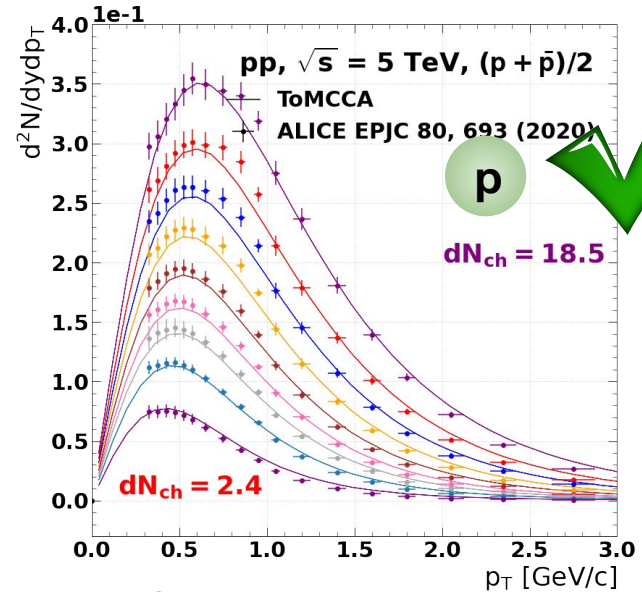
- Antideuteron production predominantly for protons of $E_{kin} \sim 200-500$ GeV ($\sqrt{s} \sim \mathbf{19-30}$ GeV for p-H)



Šerkšnytė, et al. PRD 105, 083021 (2022)

- Extrapolation to lower energies via event multiplicity

Momentum distribution

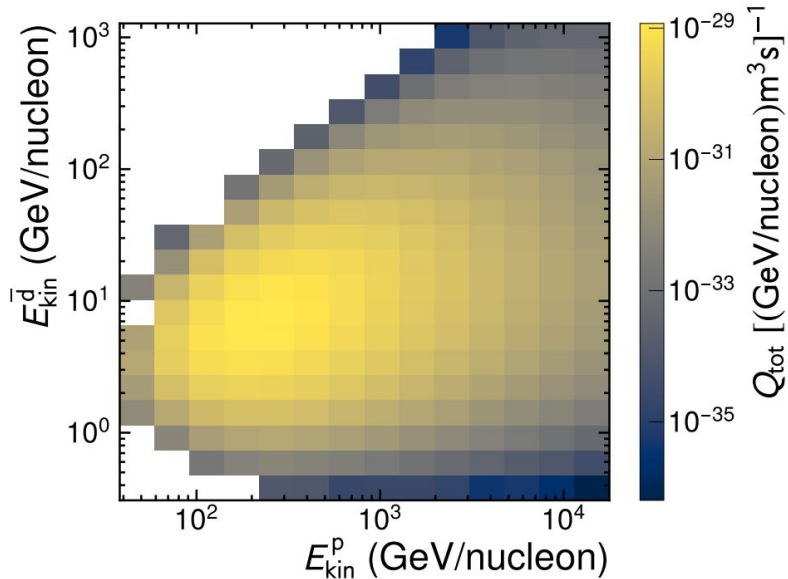


ALICE measurements over a large range in multiplicity

Cosmic Rays

Production energy of antinuclei

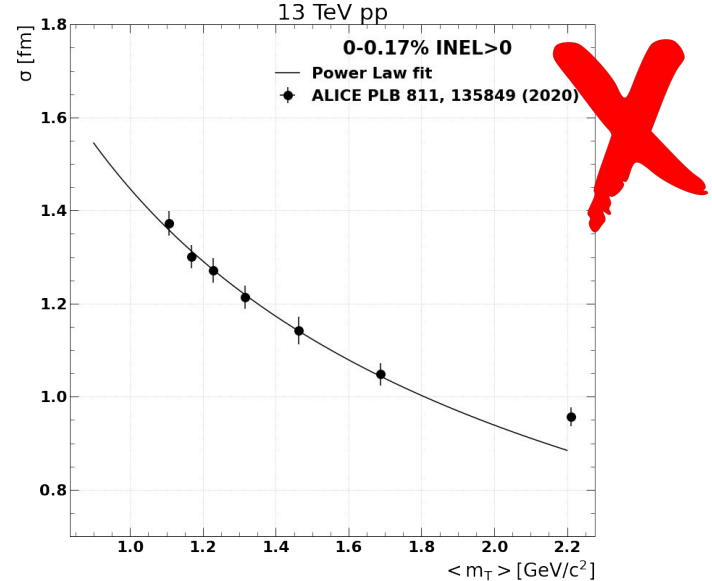
- Antideuteron production predominantly for protons of $E_{kin} \sim 200-500$ GeV ($\sqrt{s} \sim \mathbf{19-30}$ GeV for p-H)



Šerkšnytė, et al. PRD 105, 083021 (2022)

- Extrapolation to lower energies via event multiplicity

Source size

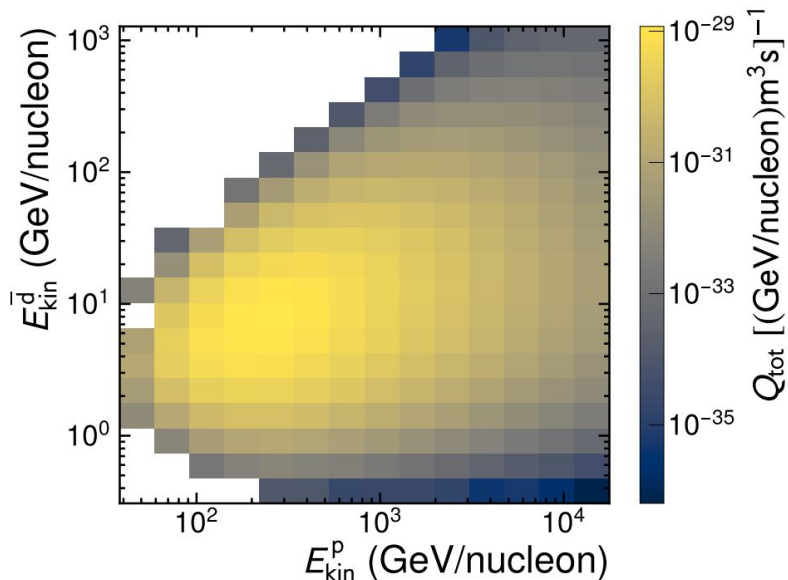


ALICE measurement in high-multiplicity collisions

Cosmic Rays

Production energy of antinuclei

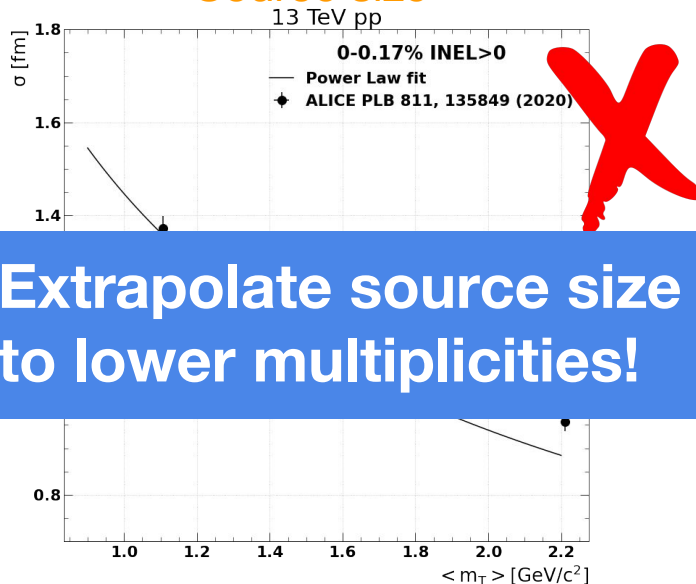
- Antideuteron production predominantly for protons of $E_{kin} \sim 200-500$ GeV ($\sqrt{s} \sim \mathbf{19-30}$ GeV for p-H)



Šerkšnytė, et al. PRD 105, 083021 (2022)

- Extrapolation to lower energies via event multiplicity

Source size

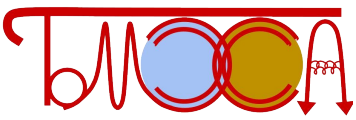


Extrapolate source size to lower multiplicities!

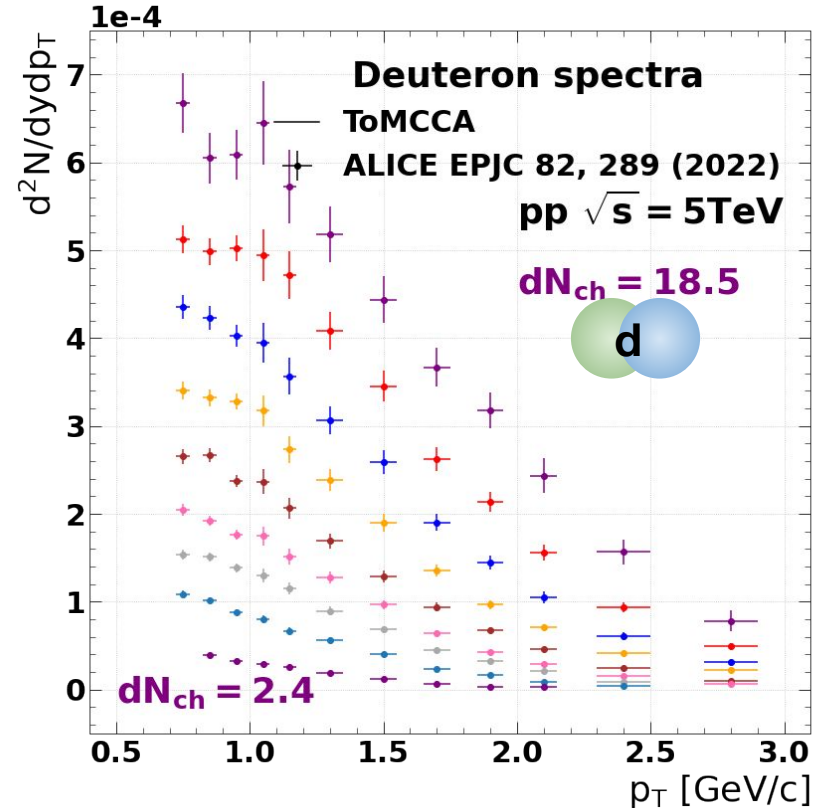
ALICE measurement in high-multiplicity collisions

Extrapolating the Source

Using ToMCCA as a fitting tool



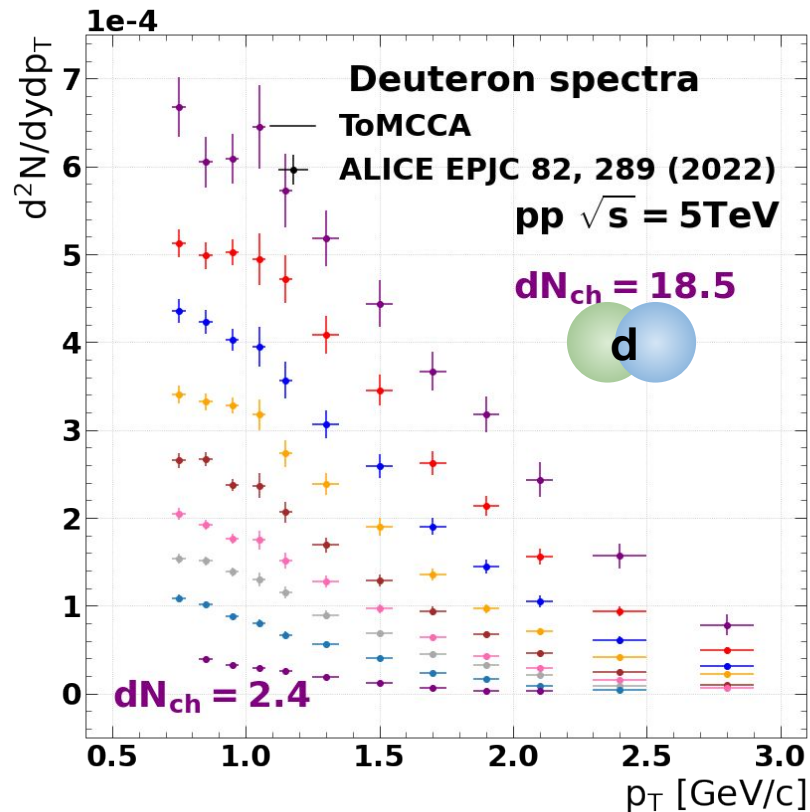
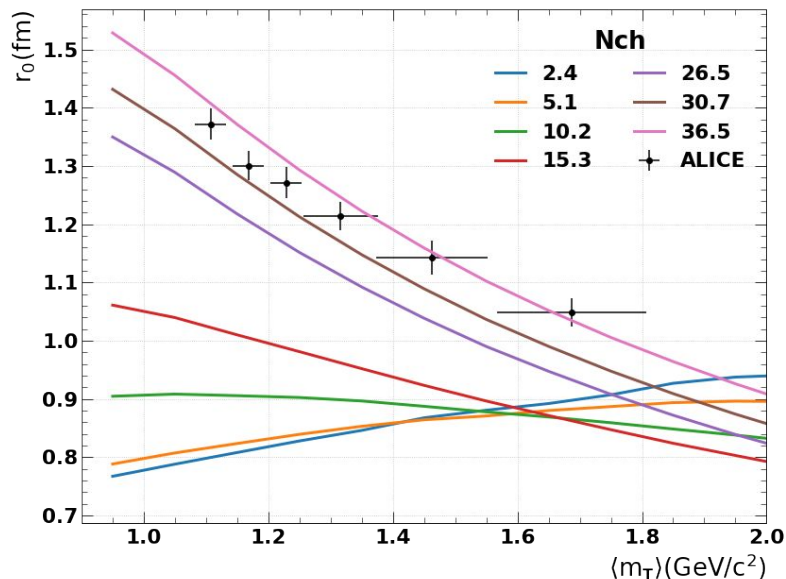
- Deuterons were also measured by ALICE Collab. for different multiplicities
- Fit source size and scaling with m_T to measured data
- Cross check at different energies



Extrapolating the Source

Using ToMCCA as a fitting tool

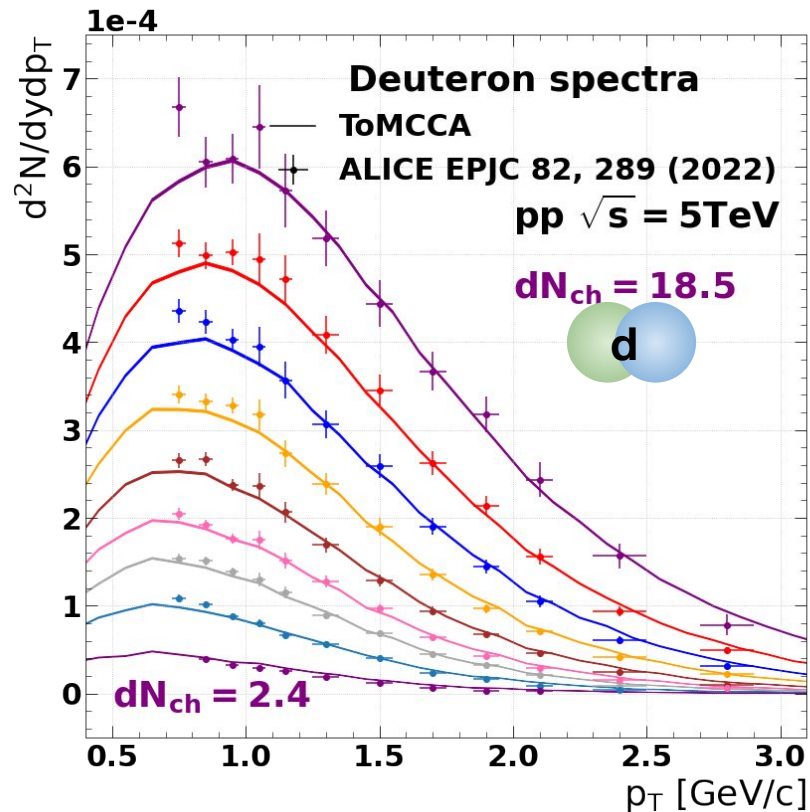
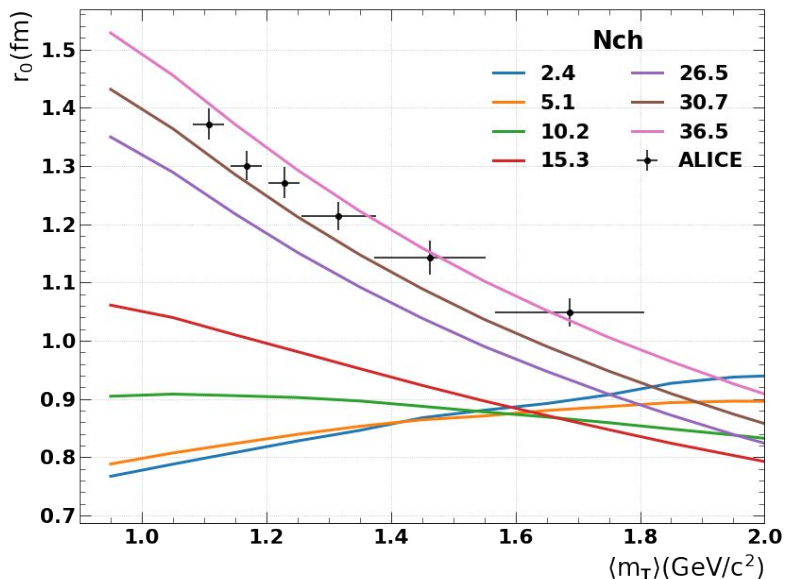
- Deuterons were also measured by ALICE Collab. for different multiplicities
- Fit source size and scaling with m_T to measured data
- Cross check at different energies



Extrapolating the Source

Using ToMCCA as a fitting tool

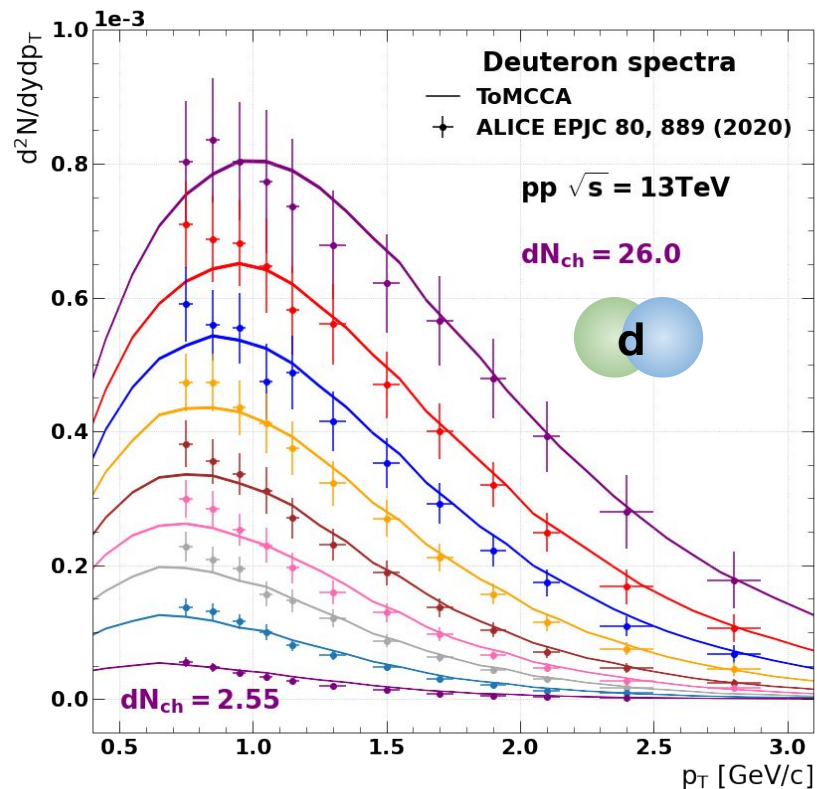
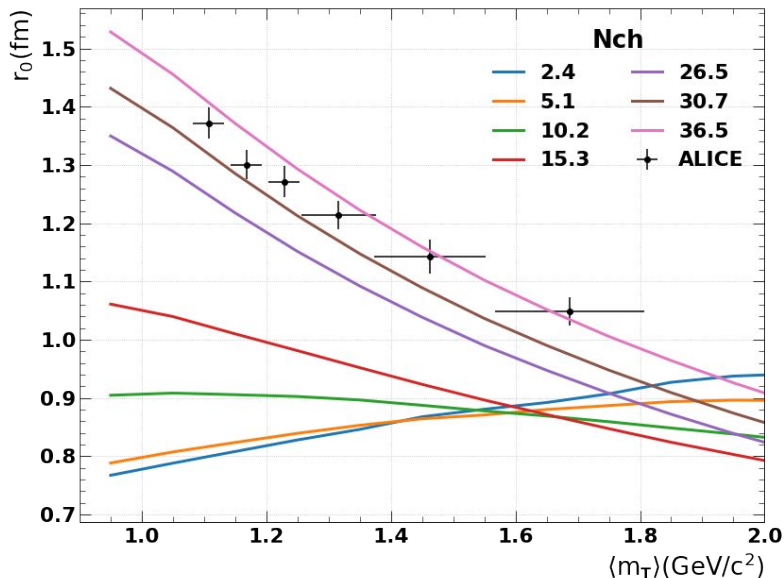
- Deuterons were also measured by ALICE Collab. for different multiplicities
- Fit source size and scaling with m_T to measured data
- Cross check at different energies



Extrapolating the Source

Using ToMCCA as a fitting tool

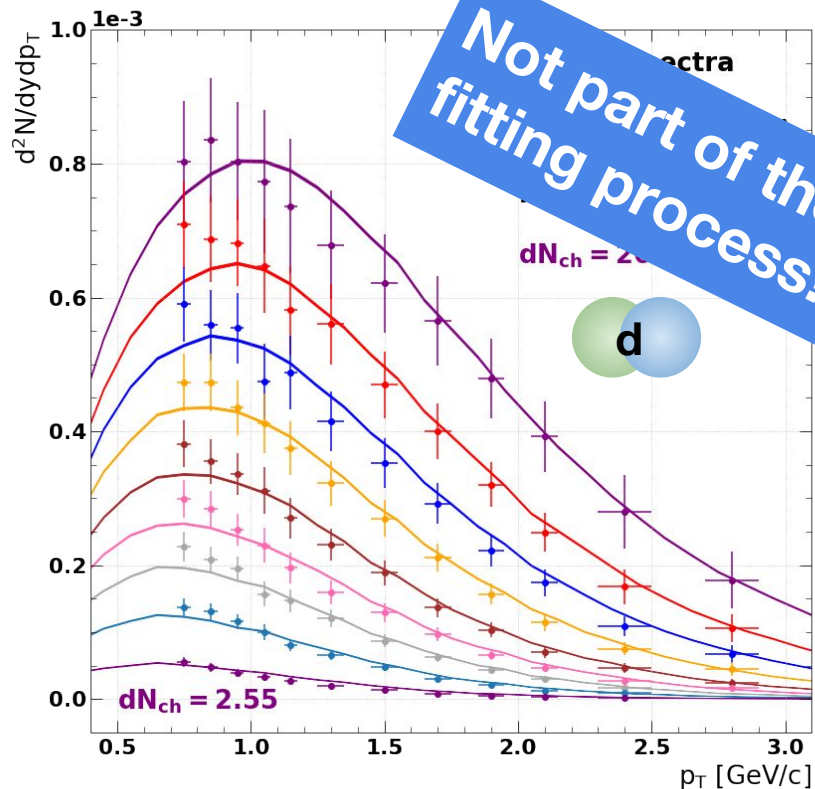
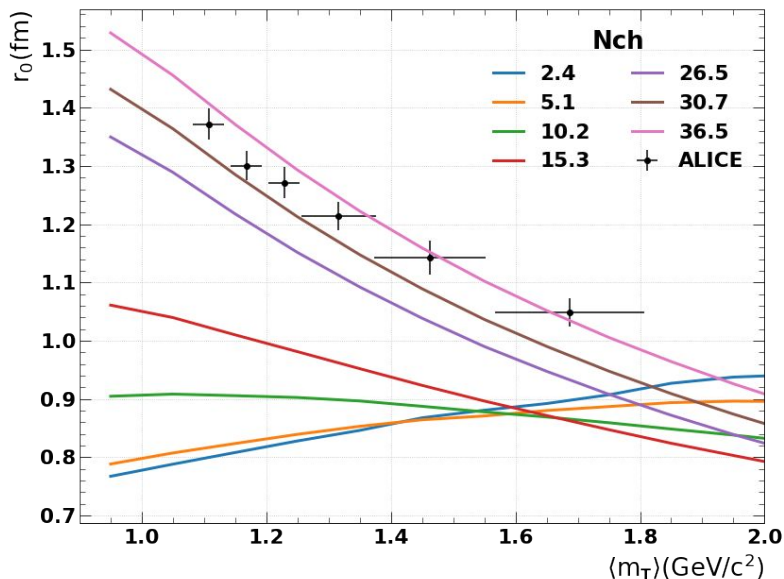
- Deuterons were also measured by ALICE Collab. for different multiplicities
- Fit source size and scaling with m_T to measured data
- Cross check at different energies



Extrapolating the Source

Using ToMCCA as a fitting tool

- Deuterons were also measured by ALICE Collab. for different multiplicities
- Fit source size and scaling with m_T to measured data
- Cross check at different energies



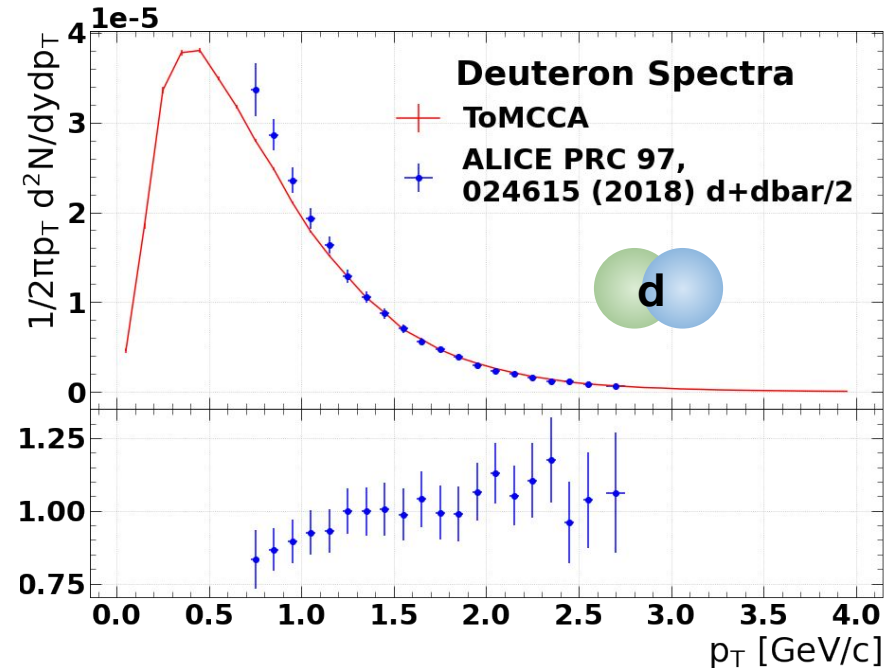
Not part of the fitting process!



Deuteron results

Minimum bias 7 TeV

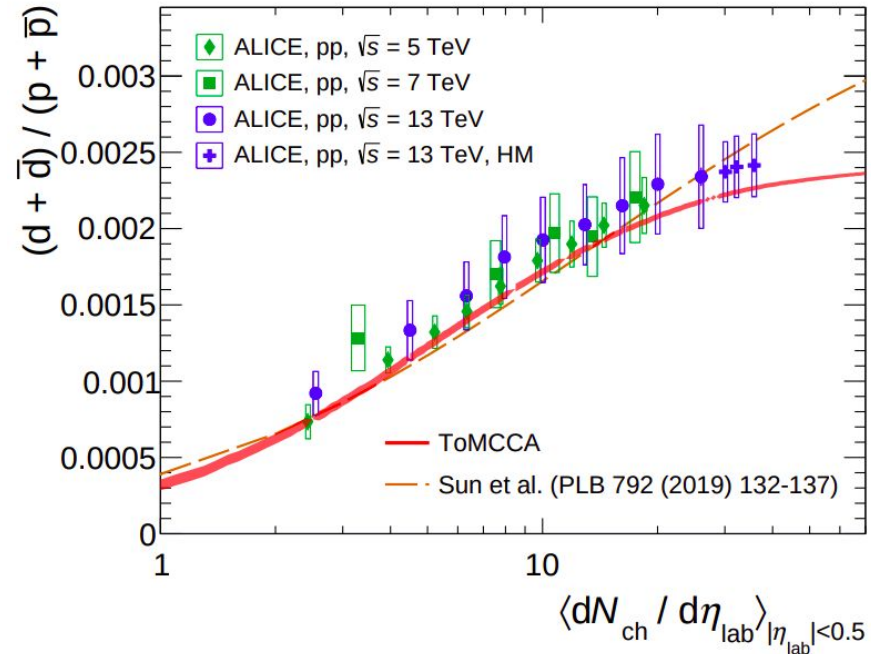
- Deuterons were also measured by ALICE Collab. for different multiplicities
- Fit source size and scaling with m_T to measured data
- Cross check at different energies
- Minimum Bias works well



Deuteron results

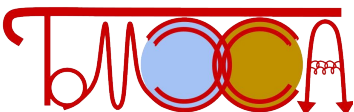
d/p ratio

- Deuterons were also measured by ALICE Collab. for different multiplicities
- Fit source size and scaling with m_T to measured data
- Cross check at different energies
- Minimum Bias works well
- d/p ratio reproduces data well, tension to previous predictions at high multiplicity



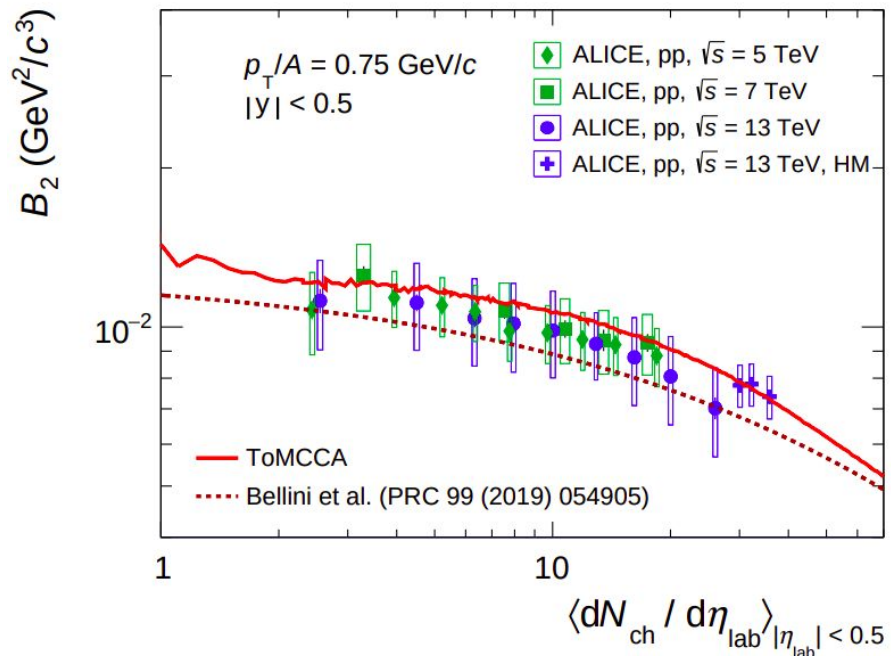
Deuteron results

B_2 parameter



- Deuterons were also measured by ALICE Collab. for different multiplicities
- Fit source size and scaling with m_T to measured data
- Cross check at different energies
- Minimum Bias works well
- d/p ratio reproduces data well, tension to previous predictions at high multiplicity
- B_2 also reproduced well

$$B_A(p_T^p) = E_A \frac{d^3 N_A}{dp_A^3} \Big/ \left(E_p \frac{d^3 N_p}{dp_p^3} \right)^A$$



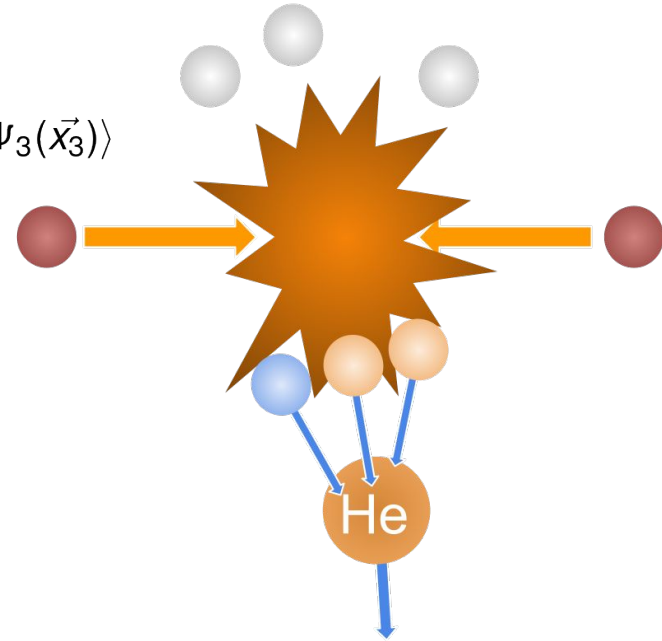
Add 3rd particle to basic formalism

$$\frac{dN_{He}}{d^3P} = S_{He} \int d^3x_1 \int d^3x_2 \int d^3x_3 \int d^3x'_1 \int d^3x'_2 \int d^3x'_3$$

$$\times \Psi_{He}^* (\vec{x}'_1, \vec{x}'_2, \vec{x}'_3) \Psi_{He} (\vec{x}_1, \vec{x}_2, \vec{x}_3) \langle \Psi_3^\dagger(\vec{x}'_3) \Psi_2^\dagger(\vec{x}'_2) \Psi_1^\dagger(\vec{x}'_1) \Psi_1(\vec{x}_1) \Psi_2(\vec{x}_2) \Psi_3(\vec{x}_3) \rangle$$

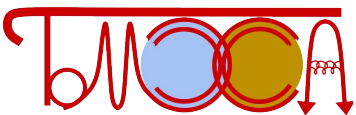
Similarly the probability can be expressed as

$$\mathcal{P}(q_1, q_2, \sigma) = \frac{S_d}{(2\pi)^3 2^3 \sigma^6} \int d^3r_1 d^3r_2 \mathcal{D}(q_1, q_2, r_1, r_2) e^{-\frac{r_1^2 + r_2^2}{4\sigma^2}}$$



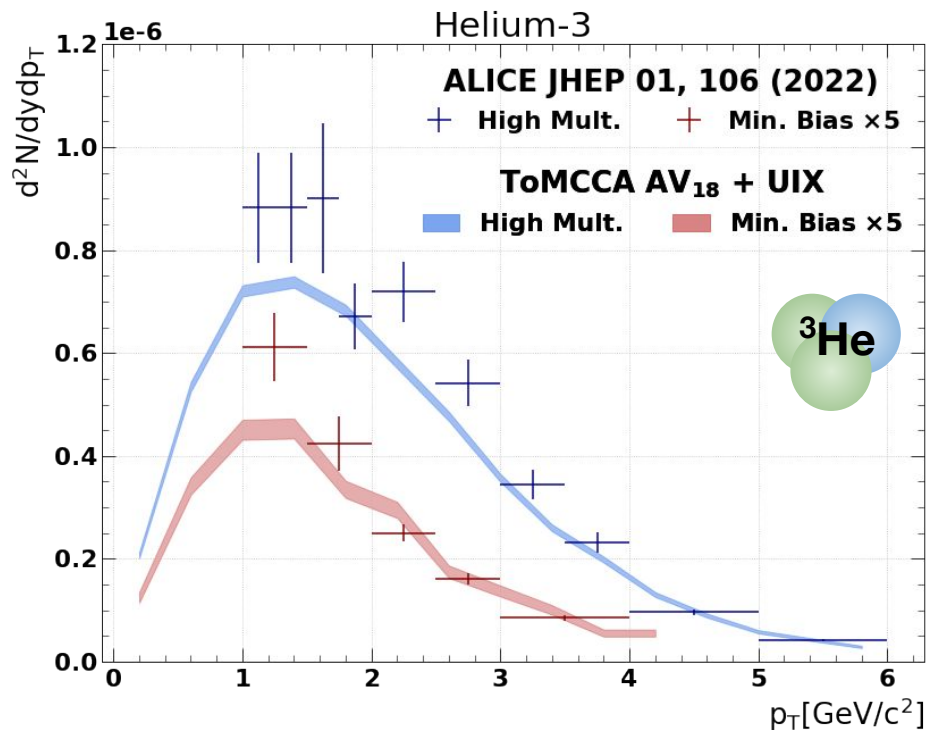
Extension to A=3

Helium-3



Extension to A=3 coalescence

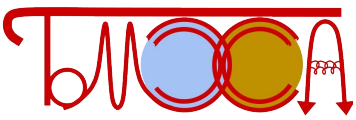
- Use 2-body source size
 - Assign every pair a distance
 - Geometric mean of distance for coalescence probability
- 3-body angular correlations built from 2-body
- Wavefunction based on Argonne v_{18} (2-body) + Urbana IX (3-body)¹
- Fully numeric calculation of Probability



¹ Provided by Michele Viviani, INFN Pisa

Extension to $A=3$

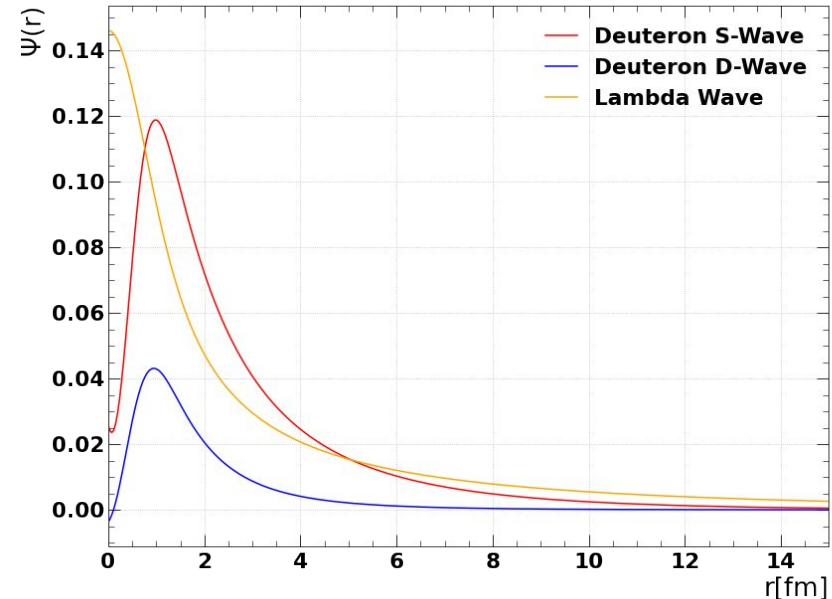
Hypertriton



- Congleton¹ wavefunction

$$\psi_{\Lambda}(q) = N \frac{\exp[-(q/\Lambda)^2]}{q^2 + \alpha^2}$$

- Assumes factorization of Hypertriton wavefunction into deuteron+ Λ
- Scattering parameters retuned to latest Hypertriton formfactor calculations² by Hildenbrand & Hammer³

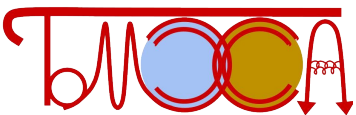


¹ J G Congleton 1992 J. Phys. G: Nucl. Part. Phys. 18 339

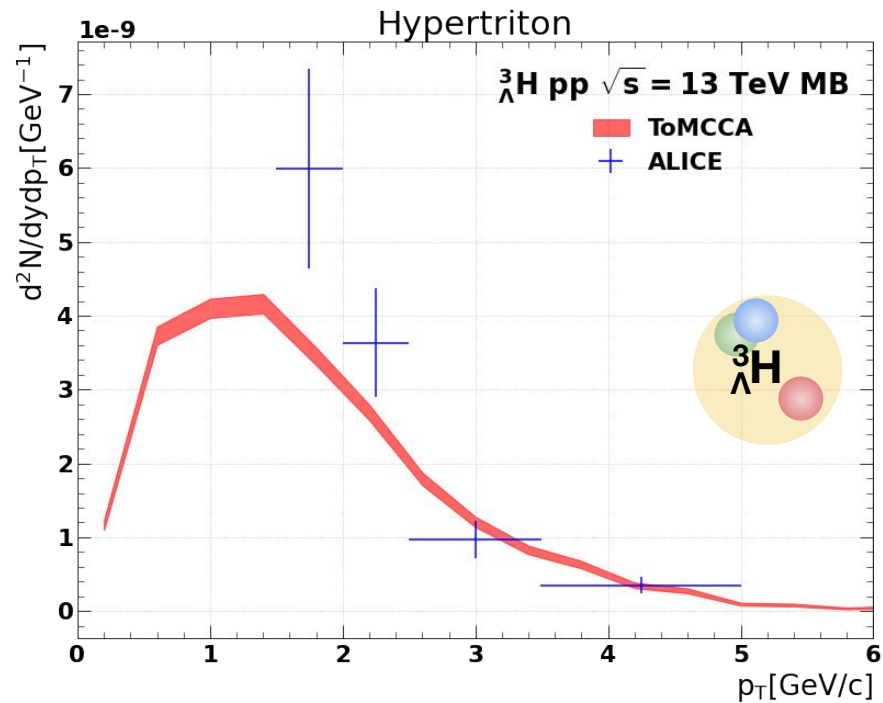
² F. Bellini et al.: Phys.Rev.C 103, 1 (2021)

³ F. Hildenbrand and H.-W. Hammer: Phys. Rev. C 100, 034002

Extension to $A=3$ Hypertriton



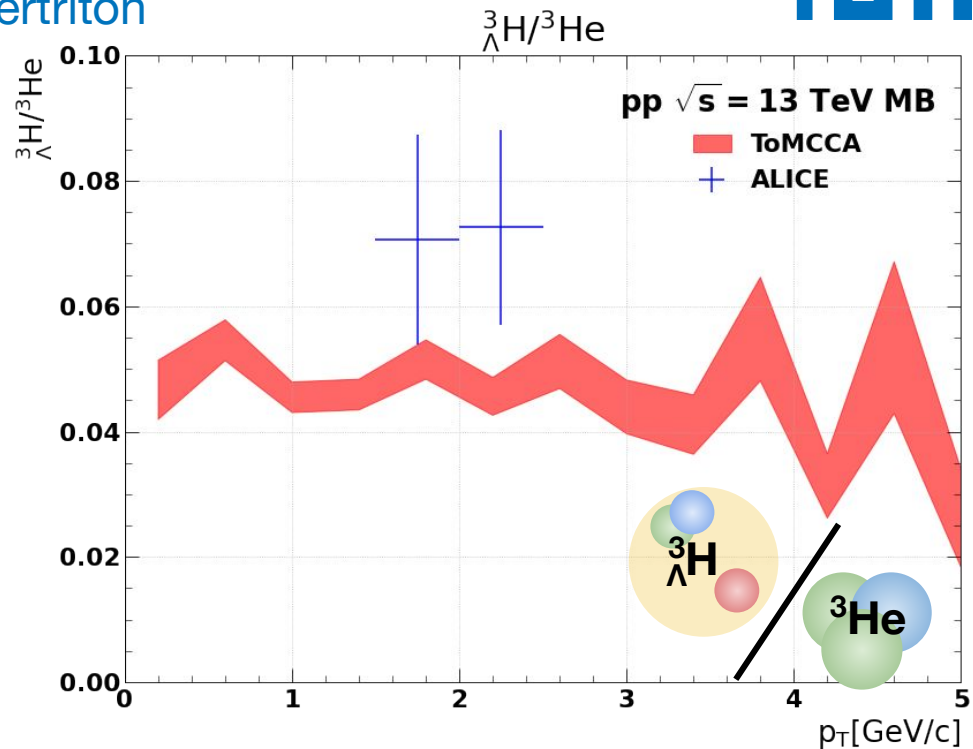
- Latest ALICE measurements of ${}^3_{\Lambda}\text{H}$ in 13 TeV MB



Extension to $A=3$

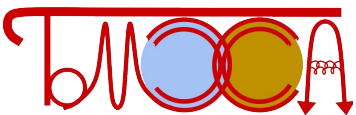
Hypertriton

- Latest ALICE measurements of ${}^3_{\Lambda}\text{H}$ in 13 TeV MB
- ${}^3_{\Lambda}\text{H}/{}^3\text{He}$ Ratio flat in p_T



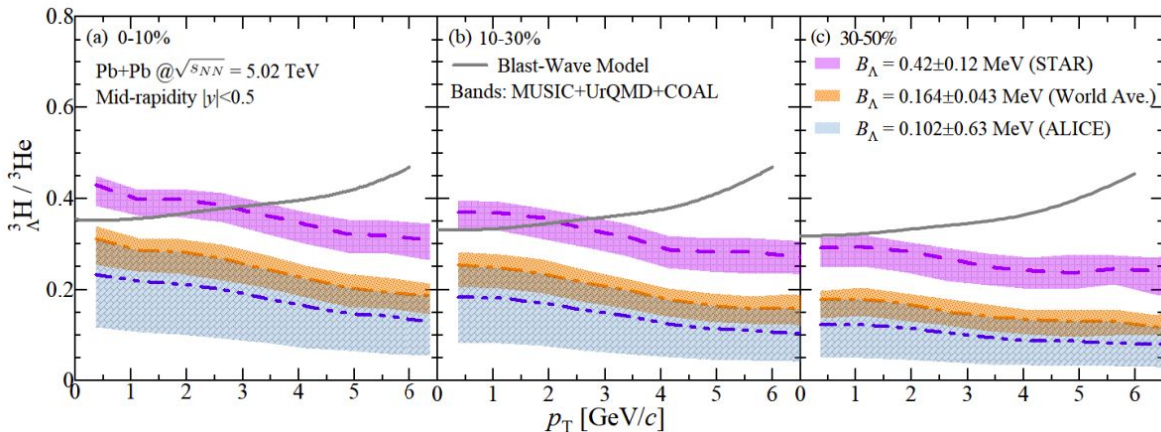
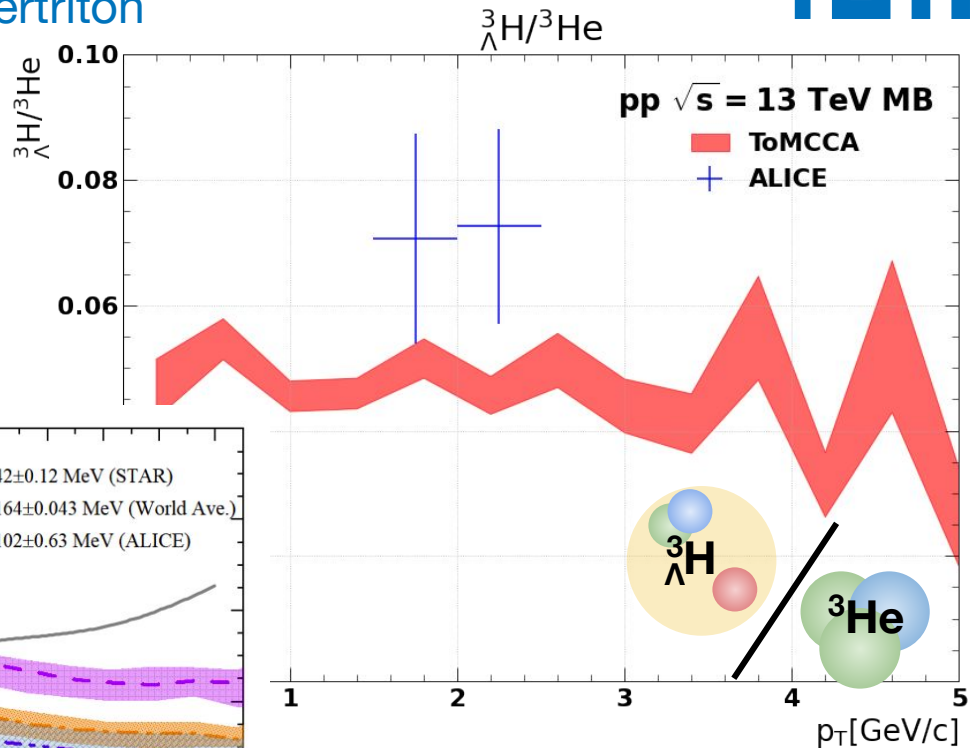
Extension to A=3

Hypertriton



- Latest ALICE measurements of ${}^3_{\Lambda}\text{H}$ in 13 TeV MB
- ${}^3_{\Lambda}\text{H}/{}^3\text{He}$ Ratio flat in p_T

Only available predictions in Heavy Ions¹:



¹K.-J. Sun et.al. arXiv:2404.02701

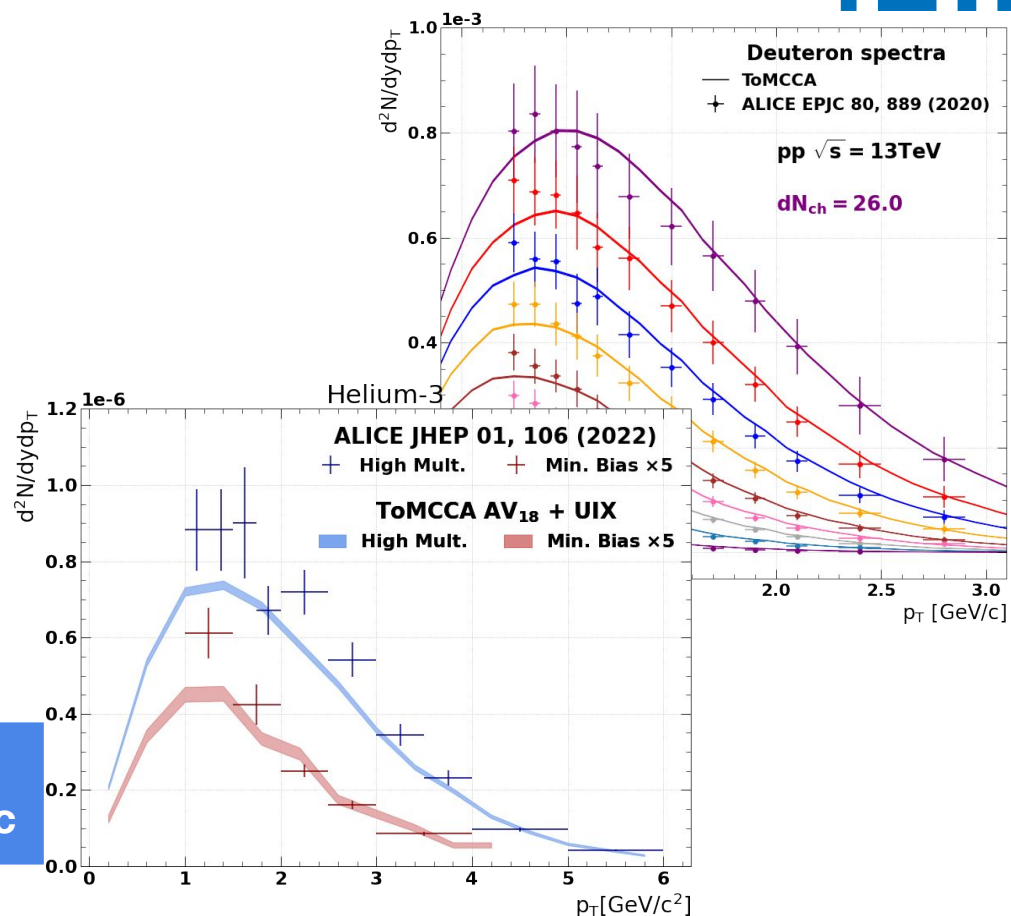
Deuterons:

- Coalescence model reproduces data with no free parameters
- Realistic wavefunction required
- ToMCCA allows for an extension to arbitrary multiplicities

A=3 Coalescence

- Successful extension of the model to A=3
- Nuclei and *Hyper*nuclei
- Realistic wavefunctions required

ToMCCA is available under:
<https://github.com/horstma/tomcca-public>

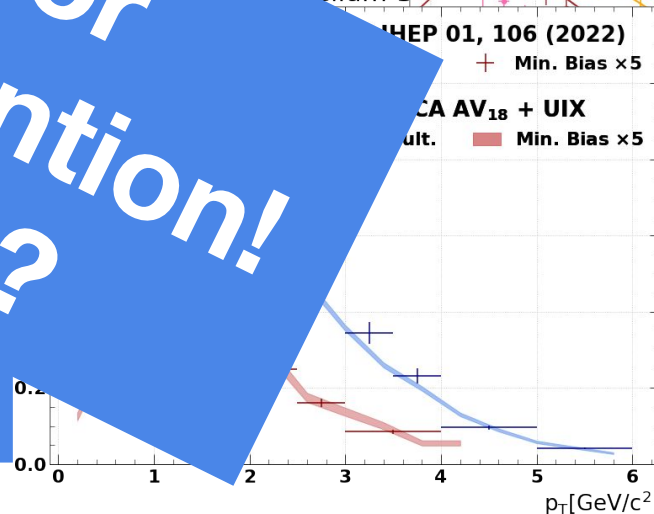
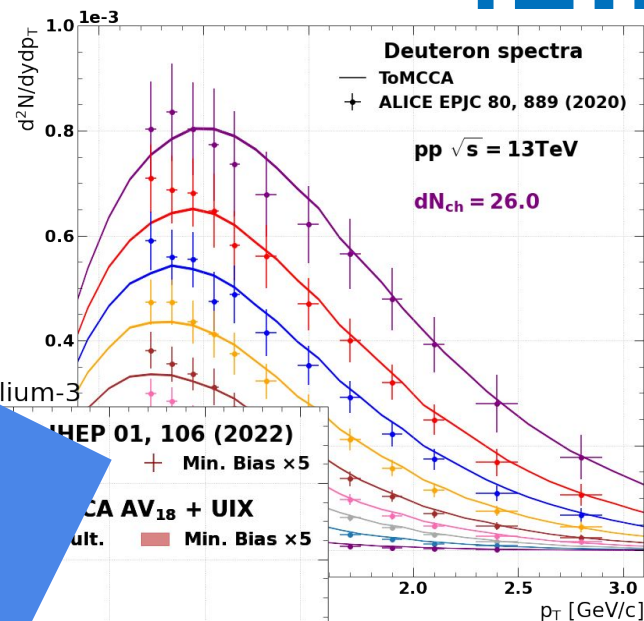


Deuterons:

- Coalescence model describes data with no free parameters
 - Realistic wavefunctions
 - ToMCCA allows arbitrary multiplicity
- A=3 Coalescence
- Successful description of A=3
 - Nuclei and Hypernuclei
 - Realistic wavefunctions

Thanks for your attention!
Questions?

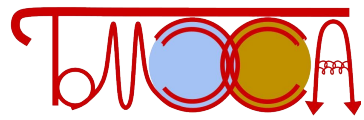
ToMCCA is available under:
<https://github.com/horstma/tomcca-public>



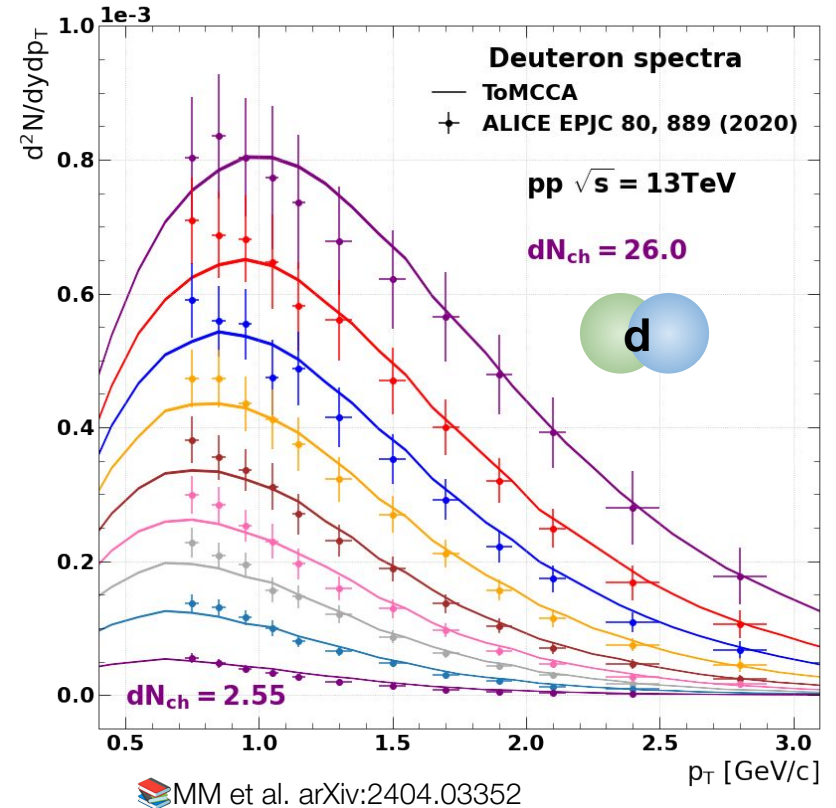
BACKUP

Conclusion

Deuteron production



- Understanding nuclei formation on earth can open a window to **indirect dark matter** searches
- **Wigner function formalism** can predict nuclei yields with no free parameters
- ToMCCA allows us to extrapolate to arbitrary multiplicities

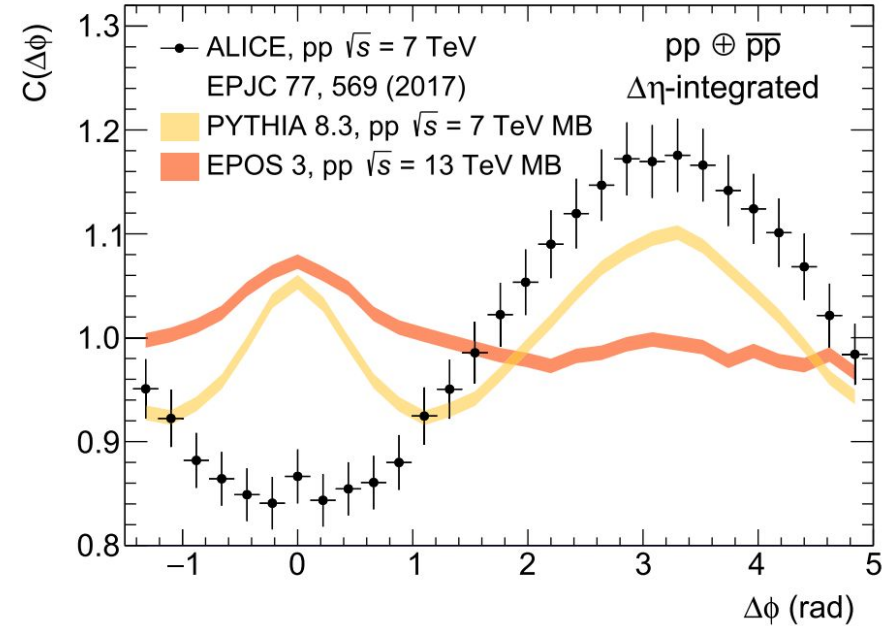


Coalescence Results EPOS

Angular correlations

- $\Delta\phi$ of pp (pn) pairs
- Not reproduced by EPOS or Pythia
- No real control over these behaviours in general purpose event generators

MM et al. Eur.Phys.J.C 83 (2023) 9, 804



Comparison to previous predictions

- Important observable in accelerator measurements: \mathbf{B}_A

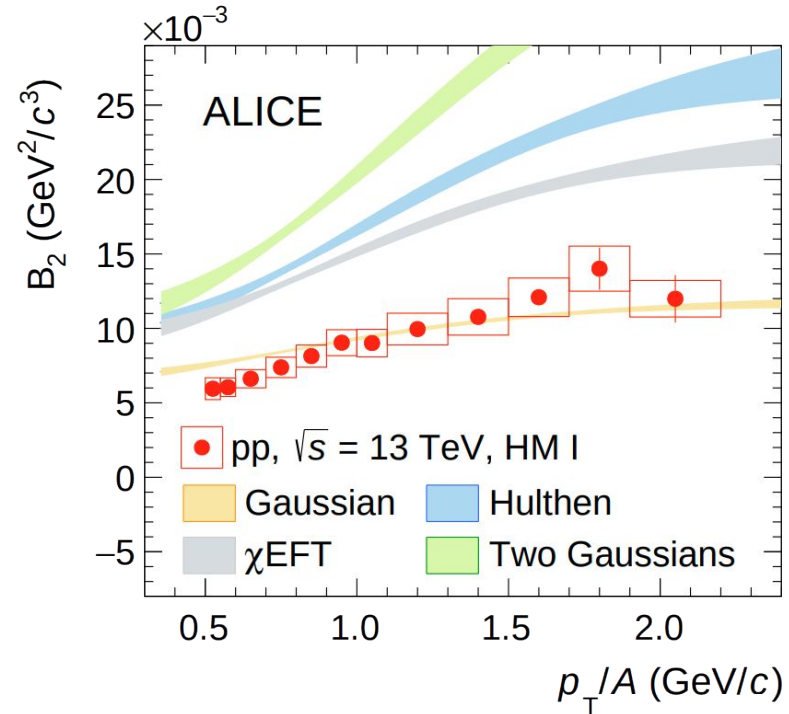
$$B_A(p_T^p) = E_A \frac{d^3 N_A}{dp_A^3} \bigg/ \left(E_p \frac{d^3 N_p}{dp_p^3} \right)^A$$

- Theoretical prediction [1]

$$B_2(\vec{p}) \approx \frac{3}{2m} \int d^3 q D(\vec{q}) e^{-R^2(p_T) q^2}$$

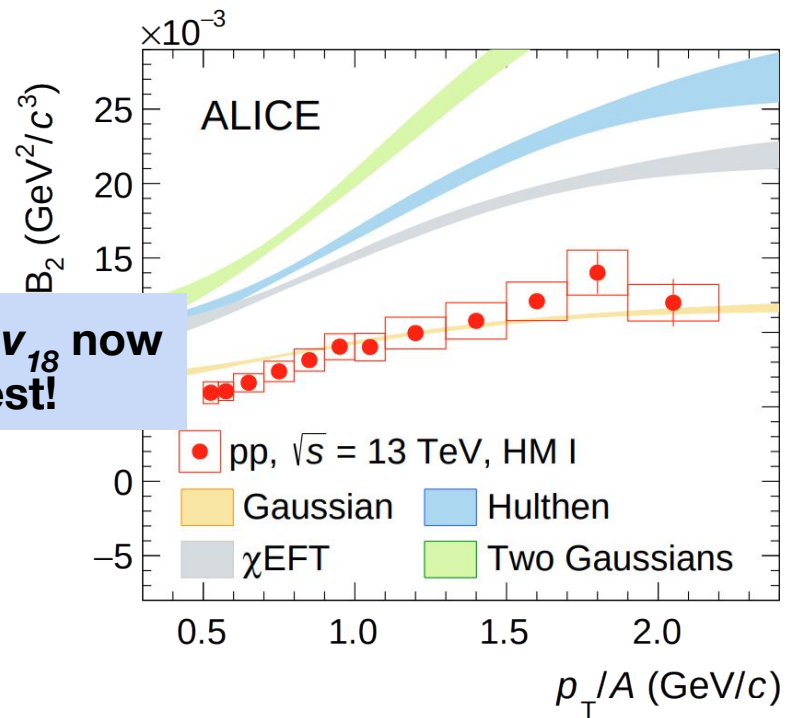
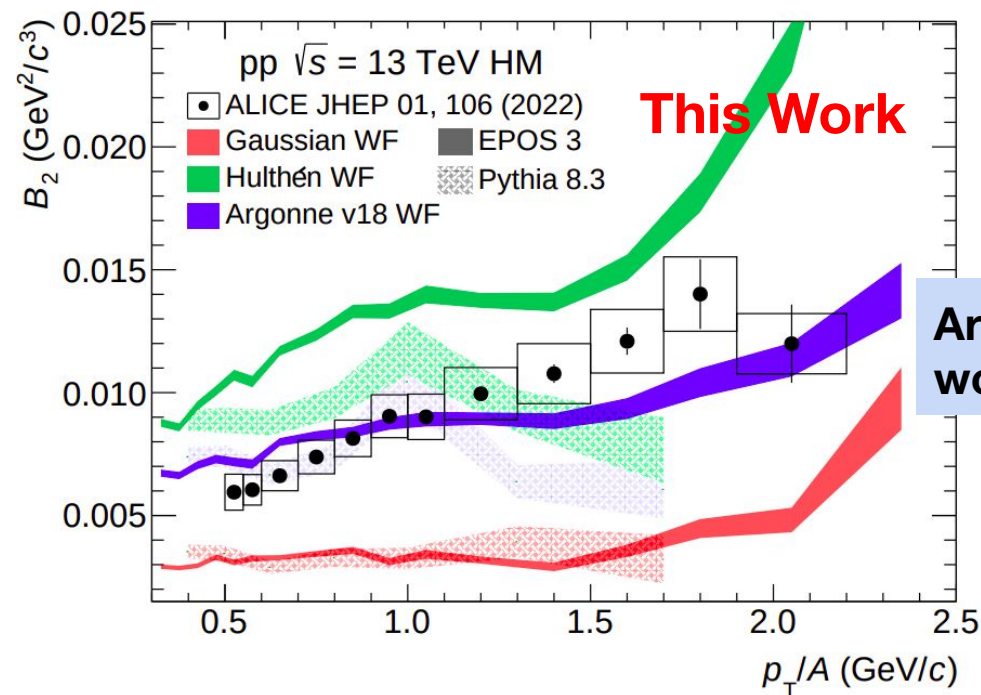
$$D(\vec{q}) = \int d^3 r |\phi_d(\vec{r})|^2 e^{-i\vec{q}\cdot\vec{r}}$$

- This neglects momentum difference between Nucleons
- approximate to 10% in Pb–Pb, factor 2 in pp



[1] Blum, Takimoto, PRC 99 (2019) 044913

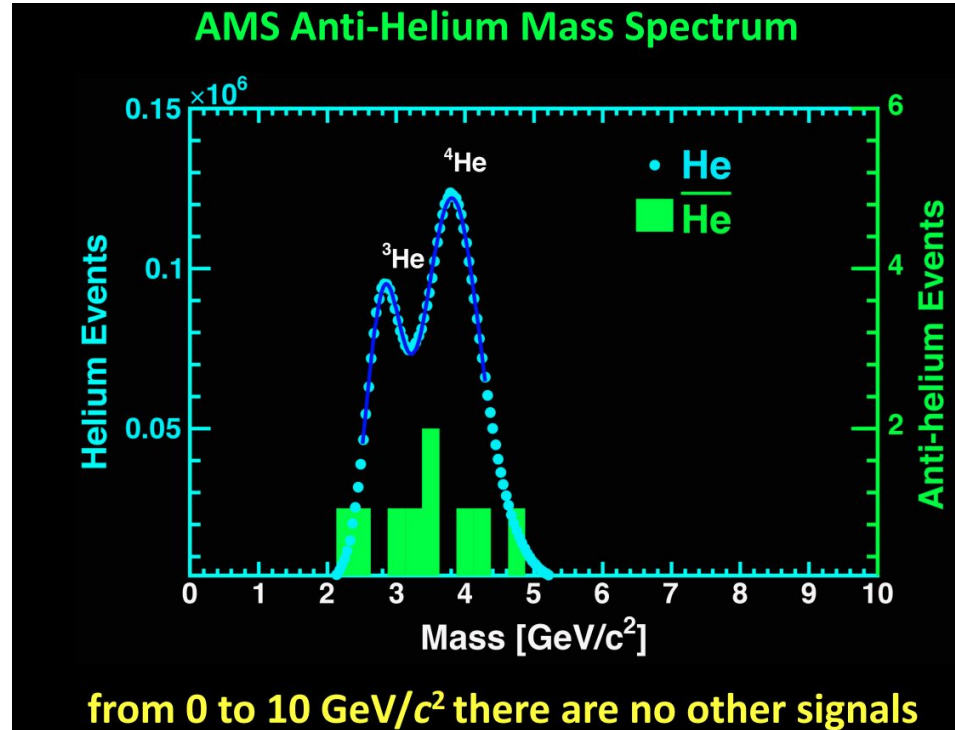
Comparison to previous predictions



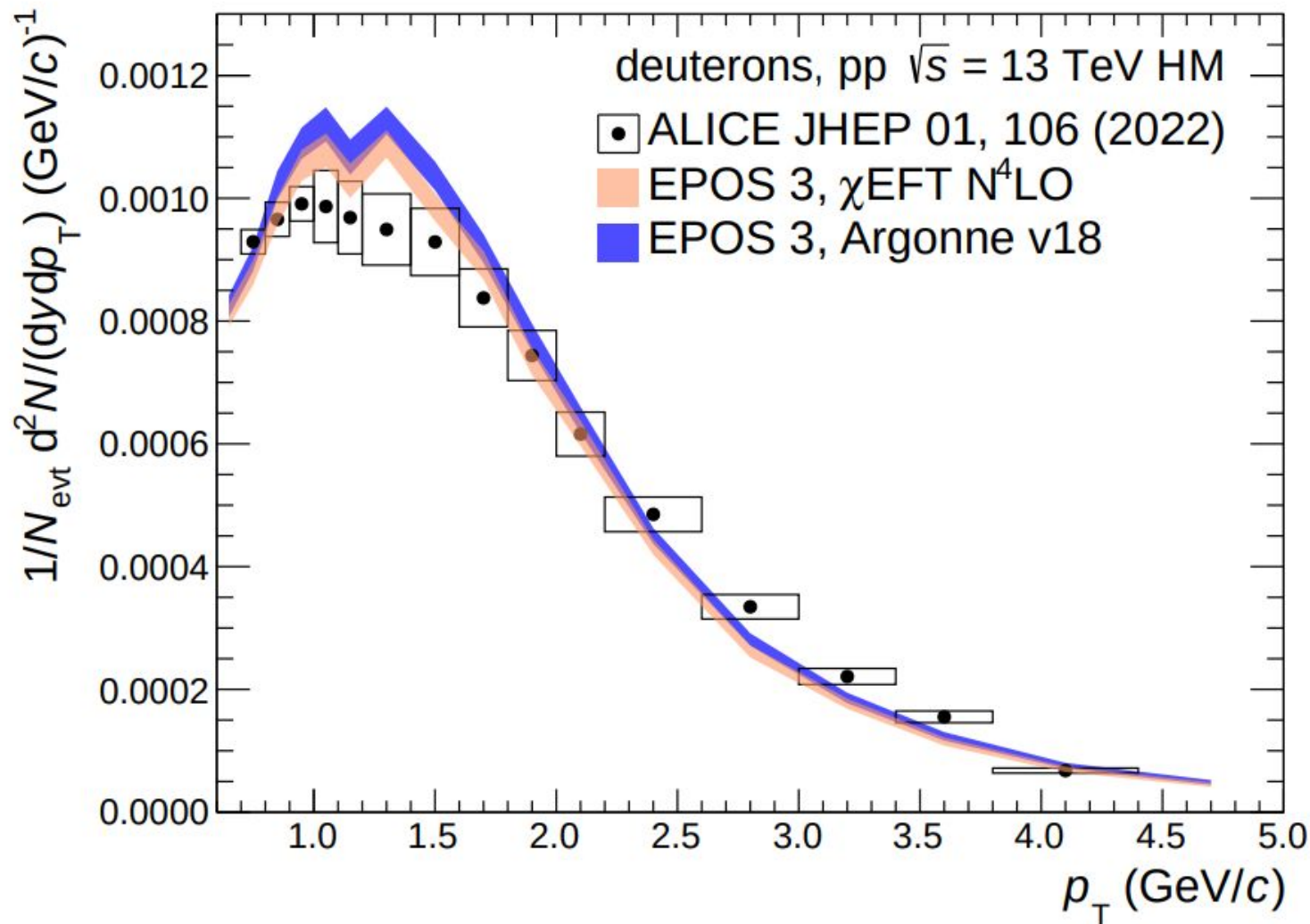
Cosmic Rays

Antinuclei in Cosmic Rays?

- AMS-02 @ ISS has measured 9 antihelium candidates
- Not yet published
- What could be the origin of these antinuclei?

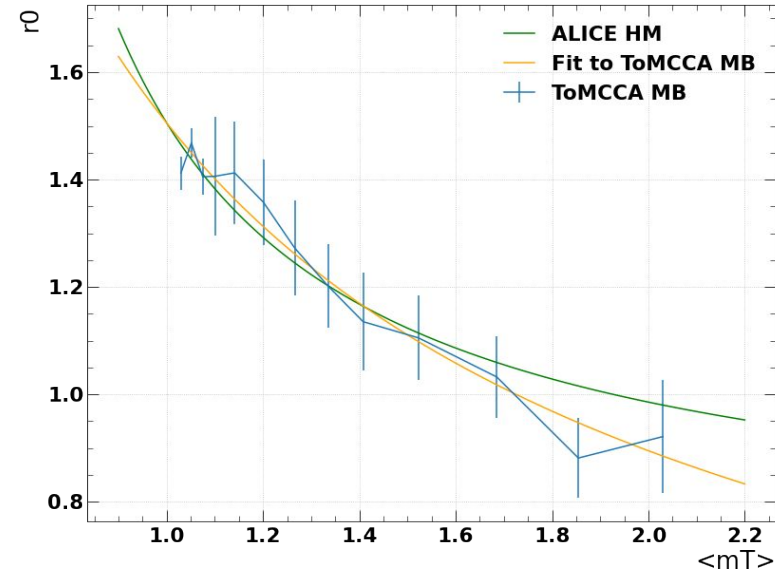
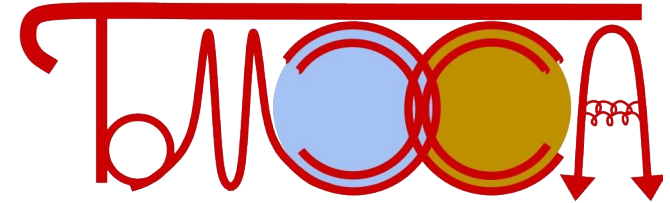
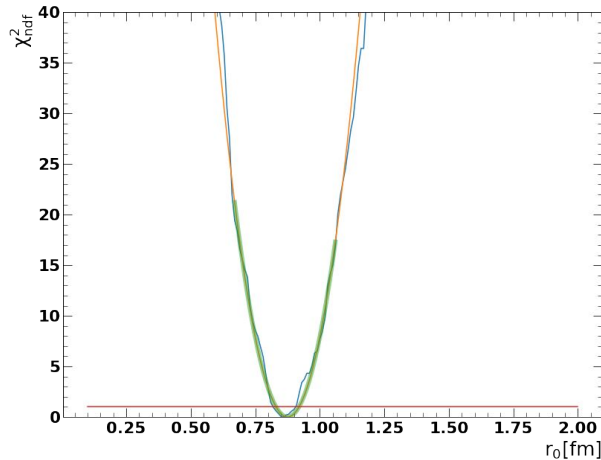


Pauolo Zuccon for AMS-02 Collaboration at MIAPP workshop 2022

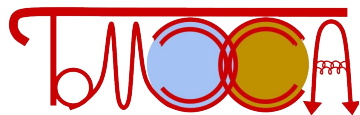


Fitting Procedure:

- Run ToMCCA with a fixed source size (e.g. 1.8 fm, flat in m_T)
- For the resulting deuteron spectra calculate the χ^2 for each bin and save it
- Reduce source size
- Repeat until source size is 0



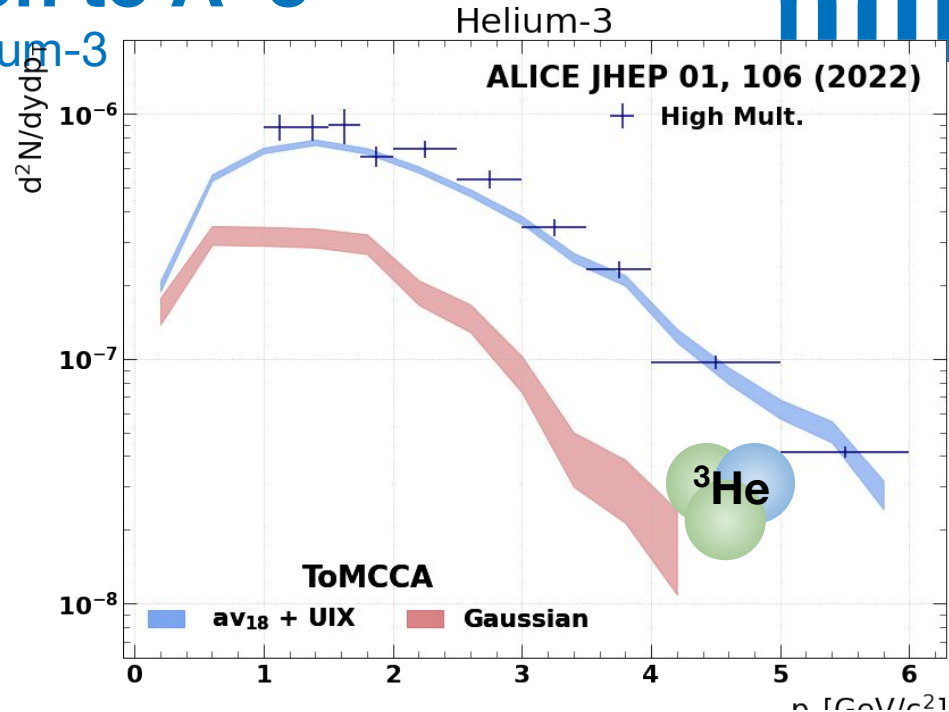
Extension to A=3



Extension to A=3 coalescence

- Use 2-body source size
 - Assign every pair a distance
 - Geometric mean of distance for coalescence probability
- For now only Gaussian wave function:
 - Yield *~50% lower* than data
 - Shape at large p_T deviates

Helium-3

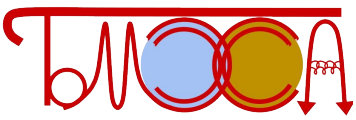


$$\mathcal{P}(k, q, \sigma) = \frac{S 64 b^6}{(b^2 + 2\sigma^2)^3} \exp[-b^2(k^2 + q^2)]$$

p_T [GeV/c²]

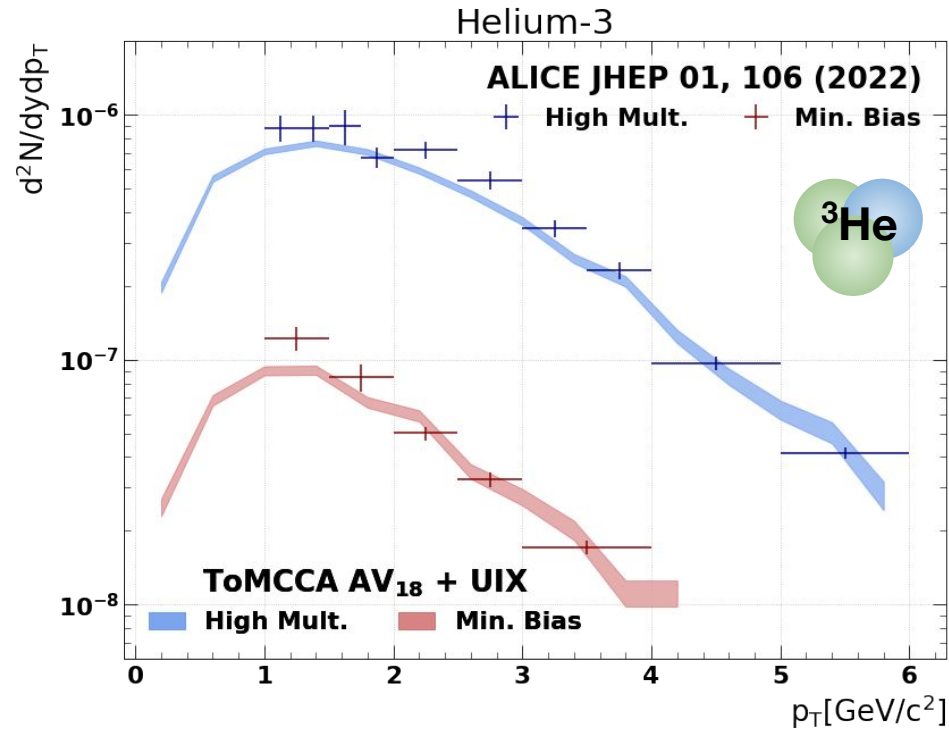
Extension to A=3

Helium-3



Extension to A=3 coalescence

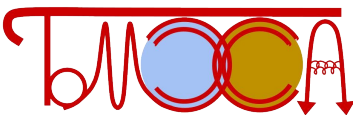
- Use 2-body source size
 - Assign every pair a distance
 - Geometric mean of distance for coalescence probability
- 3-body angular correlations built from 2-body
- Wavefunction based on Argonne v_{18} (2-body) + Urbana IX (3-body)¹
- Fully numeric calculation of Probability



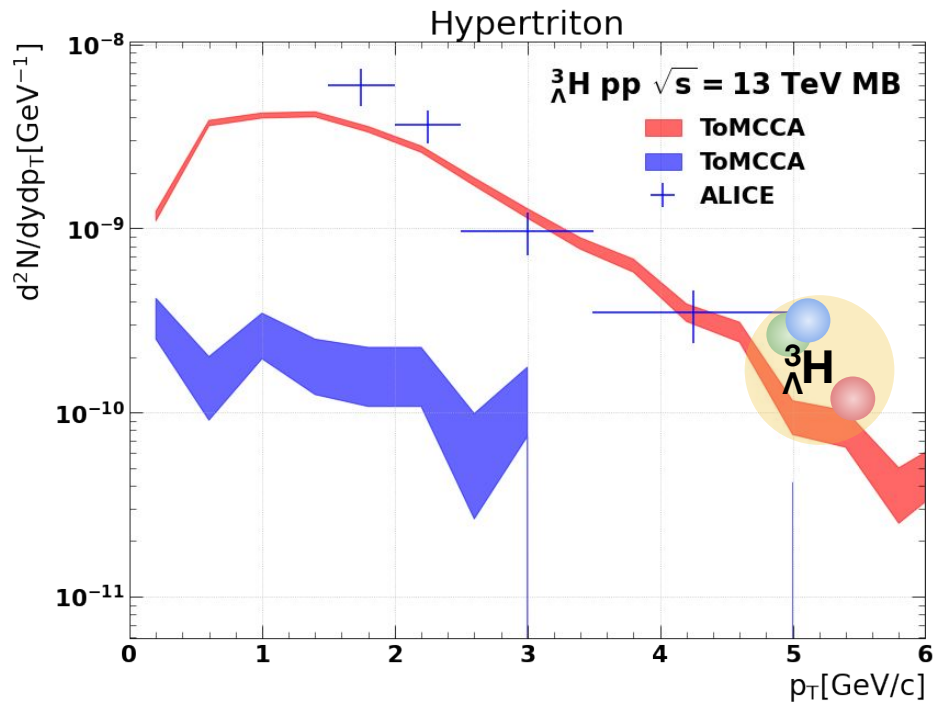
¹ Provided by Michele Viviani, INFN Pisa

Extension to $A=3$

Hypertriton

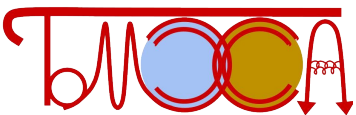


- Latest ALICE measurements of LH3 in 13 TeV MB



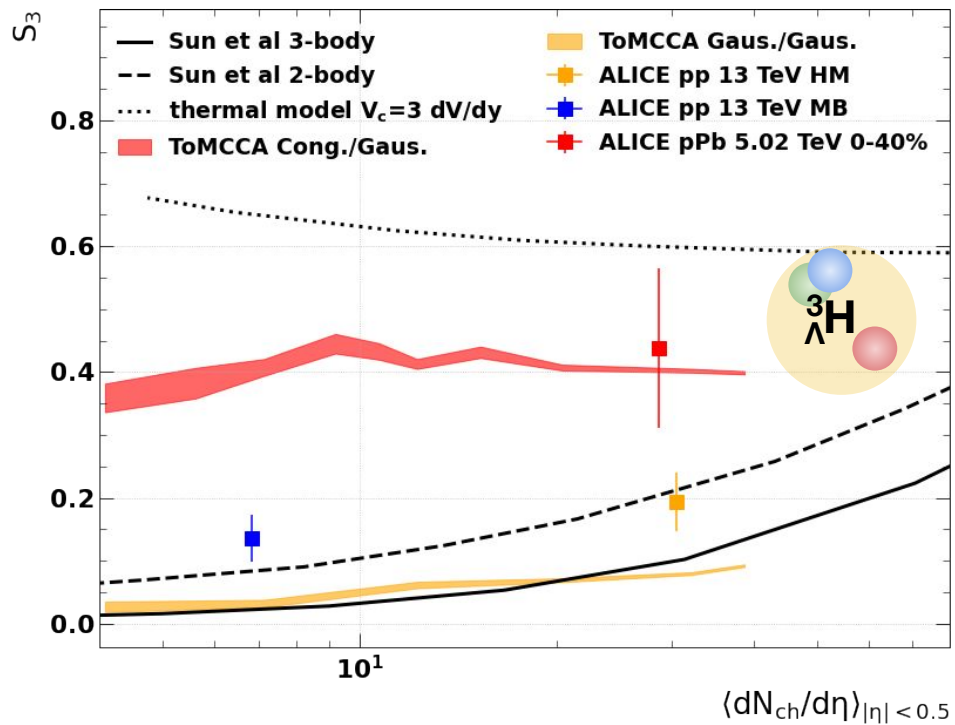
Extension to $A=3$

Hypertriton



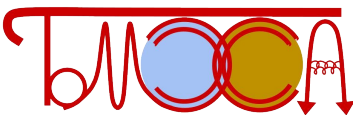
- S_3 observable is expected to be very sensitive to production mechanism
- Using Gaussian for LH3 and He-3 gives comparable results to Sun et al.
- Using Congleton for LH3 overestimates S_3

$$S_3 = (\Lambda^3\text{H}/^3\text{He}) / (\Lambda/p)$$



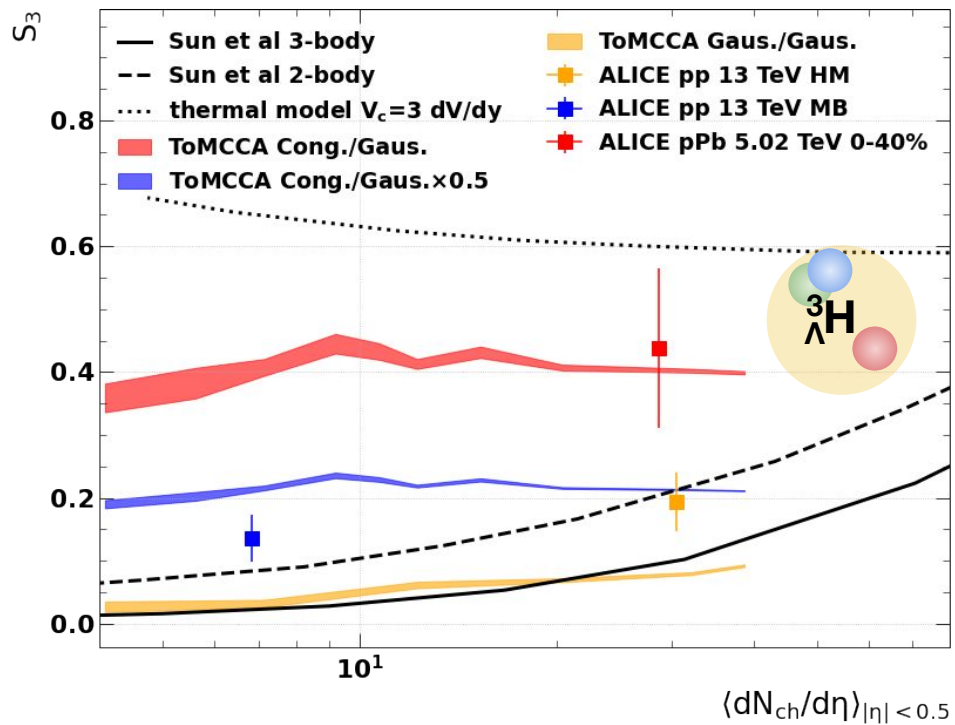
Extension to A=3

Hypertriton



- S_3 observable is expected to be very sensitive to production mechanism
- Using Gaussian for LH3 and He-3 gives comparable results to Sun et al.
- Using Congleton for LH3 overestimates S_3
- He-3 yield is underestimated \rightarrow Scale by 0.5

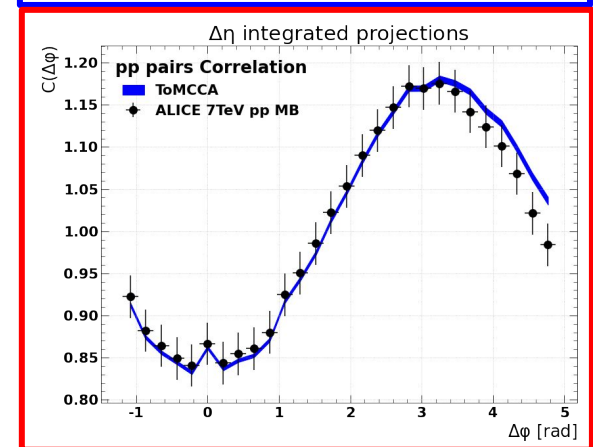
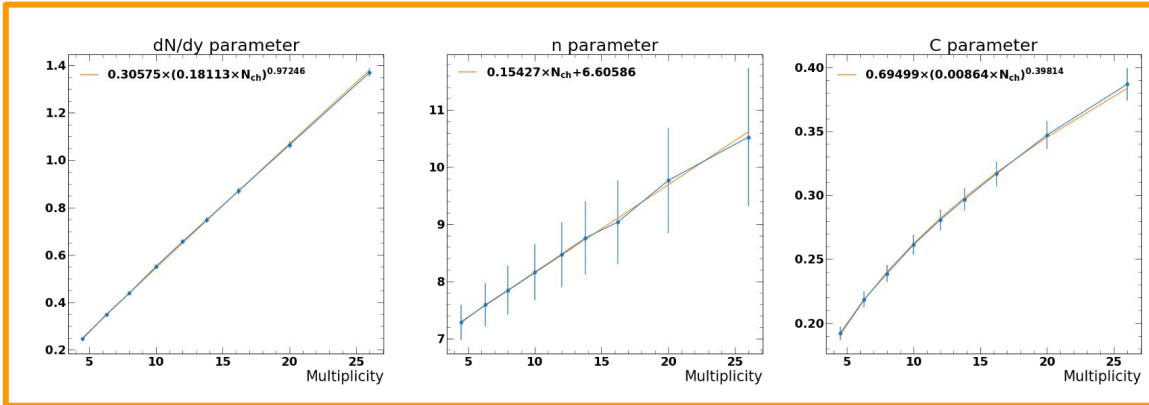
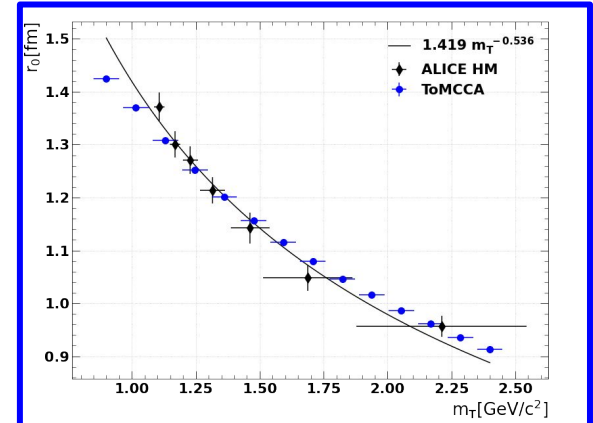
$$S_3 = (\Lambda^3\text{H}/\Lambda^3\text{He}) / (\Lambda/p)$$



Recap: ToMCCA

Inputs

- ToMCCA is a Toy Monte Carlo → it requires everything as an **input**:
 - **Momentum distribution** → Fully parameterized
 - **Multiplicity** → Poissonian/Event Generator
 - **Angular distribution** → From Measurement
 - **Source Size** → ALICE Measurement



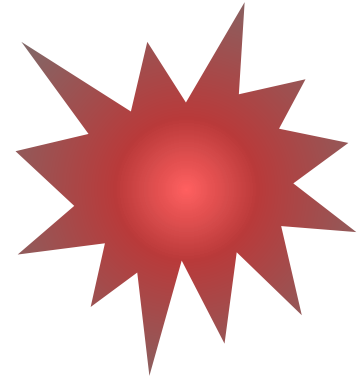
$$\frac{d^2N}{dydp_T} = \frac{dN}{dy} \frac{p_T(n-1)(n-2)}{nC[nC+m_p(n-2)]} \left(1 + \frac{m_T - m_p}{nC}\right)^{-n}$$

Using a toy MC for Coalescence

Basics of ToMCCA



Event Loop:



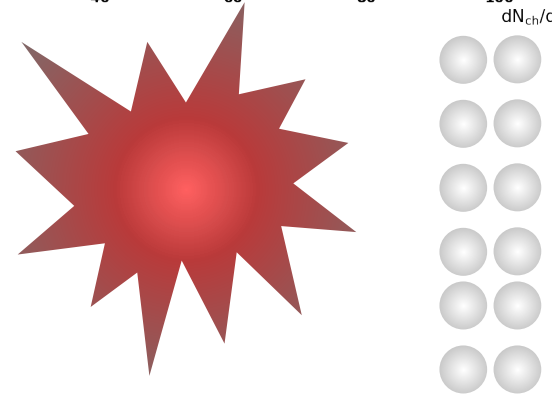
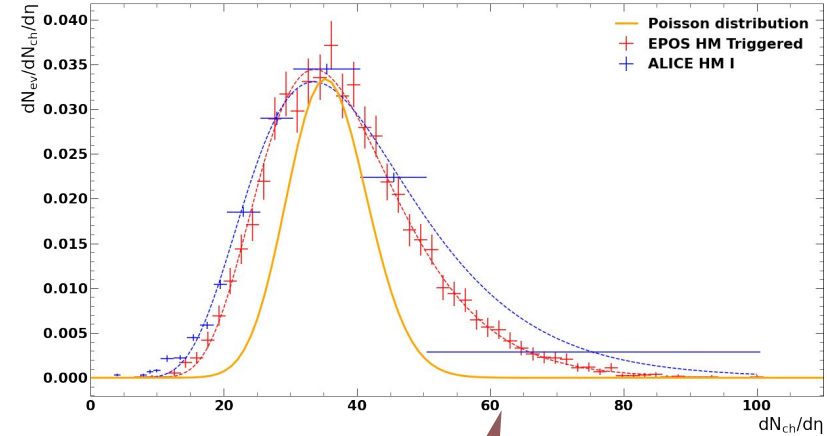
Using a toy MC for Coalescence

Basics of ToMCCA

Event Loop:

Get number of charged particles

1. Poissonian distribution with given mean
2. $dN/d\eta$ measurements by ALICE
3. Event generator output



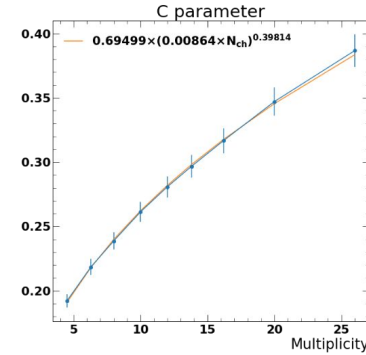
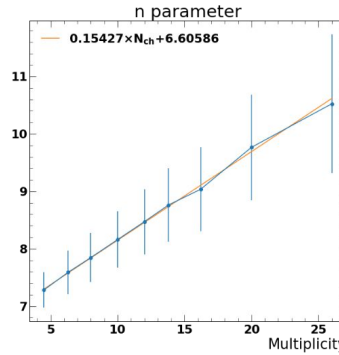
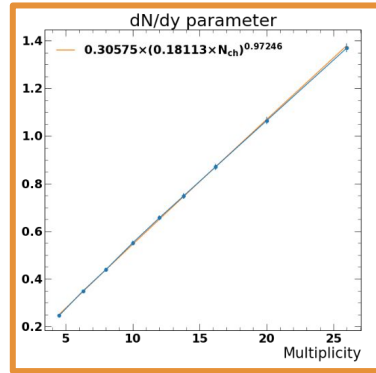
Using a toy MC for Coalescence

Basics of ToMCCA

Event Loop:

- Get number of charged particles
- Get proton yield
- Get neutron yield

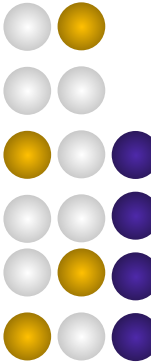
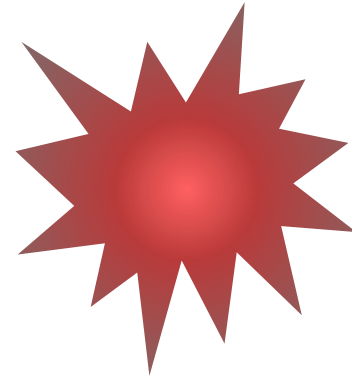
Fit all proton spectra for 13TeV using a Lévy Tsallis:



$$\frac{d^2N}{dydp_T} = \frac{dN}{dy} \frac{p_T(n-1)(n-2)}{nC[nC+m_p(n-2)]} \left(1 + \frac{m_T-m_p}{nC}\right)^{-n}$$

Yield parameter!

Full parameterization as a function of multiplicity

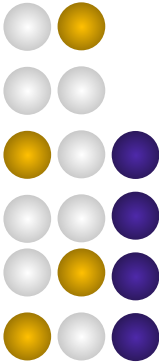
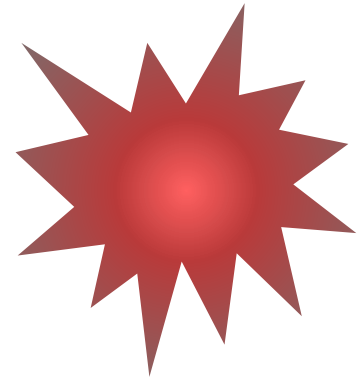


Using a toy MC for Coalescence

Basics of ToMCCA

Event Loop:

- Get number of charged particles
- Get proton yield
- Get neutron yield
- ↻ Loop over all protons



Using a toy MC for Coalescence

Basics of ToMCCA

Event Loop:

Get number of charged particles

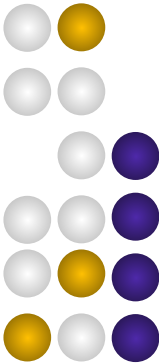
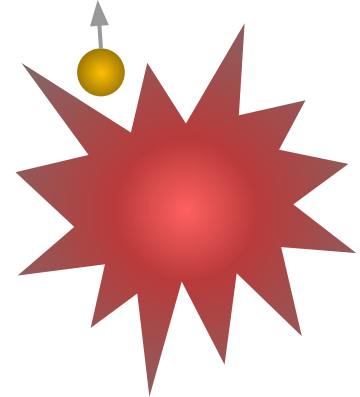
Get proton yield

Get neutron yield

↻ Loop over all protons

Get 3D momentum of proton

- Draw p_T from parameterization
- Draw flat rapidity $y=[-0.5,0.5]$
- Draw random $\phi=[0,2\pi)$

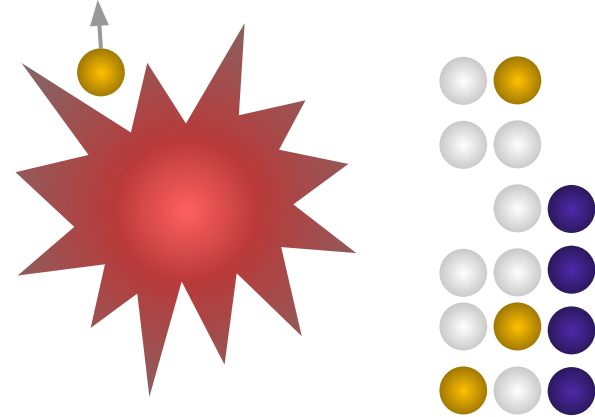


Using a toy MC for Coalescence

Basics of ToMCCA

Event Loop:

- Get number of charged particles
- Get proton yield
- Get neutron yield
- ↻ Loop over all protons
 - Get 3D momentum of proton
- ↻ Loop over all neutrons

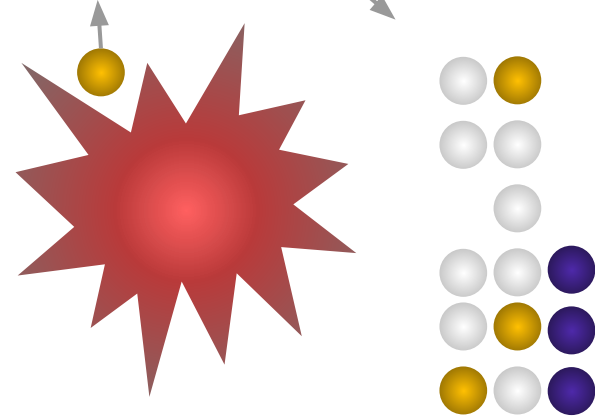
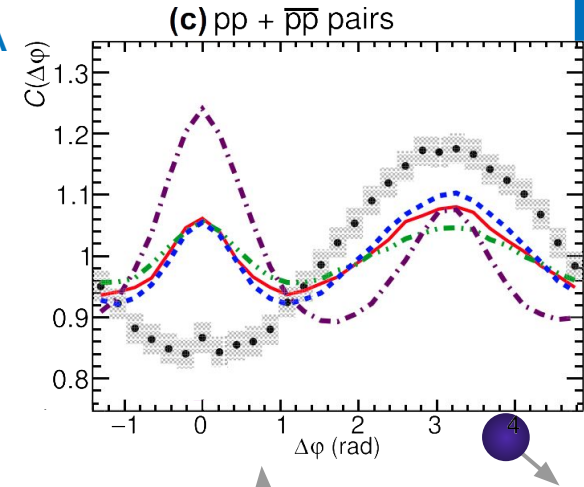


Using a toy MC for Coalescence

Basics of ToMCCA

Event Loop:

- Get number of charged particles
- Get proton yield
- Get neutron yield
- ↻ Loop over all protons
 - Get 3D momentum of proton
- ↻ Loop over all neutrons
 - Get 3D momentum of neutron
 - Draw p_T from parameterization
 - Draw flat rapidity $y=[-0.5,0.5]$
 - Draw random $\Delta\phi$ from ALICE measurement

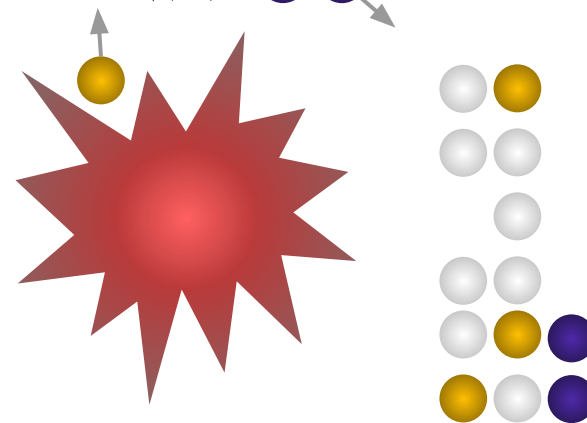
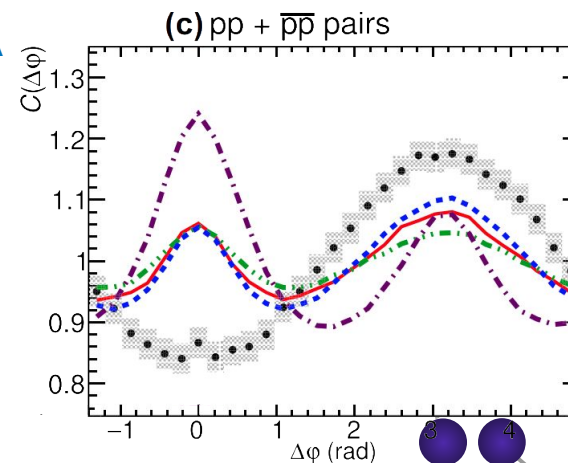


Using a toy MC for Coalescence

Basics of ToMCCA

Event Loop:

- Get number of charged particles
- Get proton yield
- Get neutron yield
- ↻ Loop over all protons
 - Get 3D momentum of proton
- ↻ Loop over all neutrons
 - Get 3D momentum of neutron
 - Draw p_T from parameterization
 - Draw flat rapidity $y=[-0.5,0.5]$
 - Draw random $\Delta\phi$ from ALICE measurement

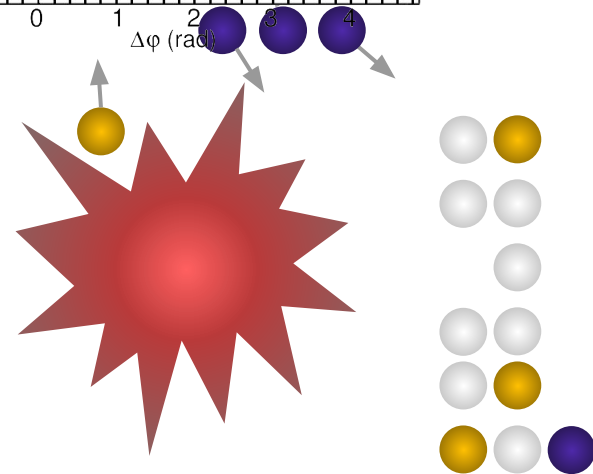
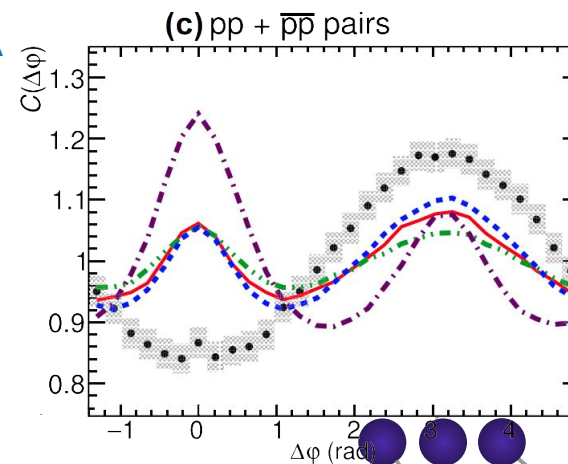


Using a toy MC for Coalescence

Basics of ToMCCA

Event Loop:

- Get number of charged particles
- Get proton yield
- Get neutron yield
- ↻ Loop over all protons
 - Get 3D momentum of proton
- ↻ Loop over all neutrons
 - Get 3D momentum of neutron
 - Draw p_T from parameterization
 - Draw flat rapidity $y=[-0.5,0.5]$
 - Draw random $\Delta\phi$ from ALICE measurement

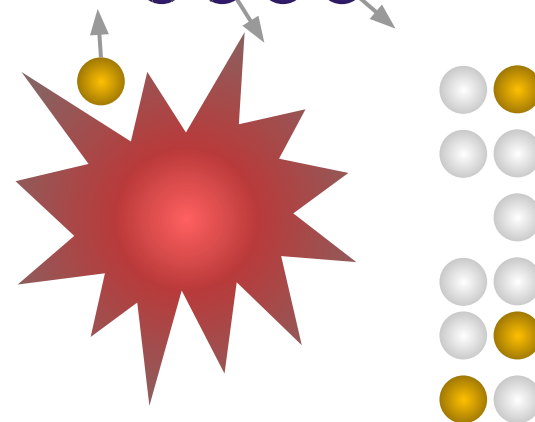
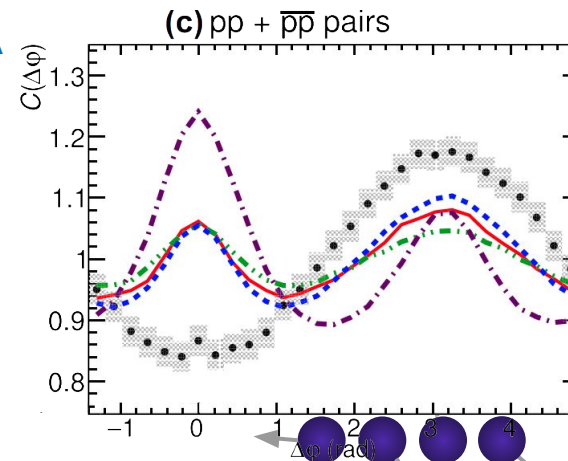


Using a toy MC for Coalescence

Basics of ToMCCA

Event Loop:

- Get number of charged particles
- Get proton yield
- Get neutron yield
- ↻ Loop over all protons
 - Get 3D momentum of proton
- ↻ Loop over all neutrons
 - Get 3D momentum of neutron
 - Draw p_T from parameterization
 - Draw flat rapidity $y=[-0.5,0.5]$
 - Draw random $\Delta\phi$ from ALICE measurement

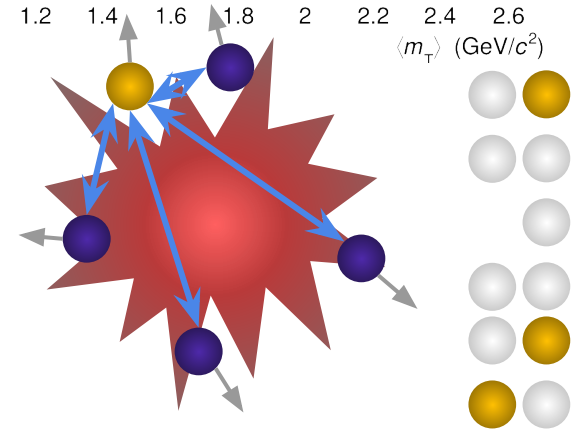
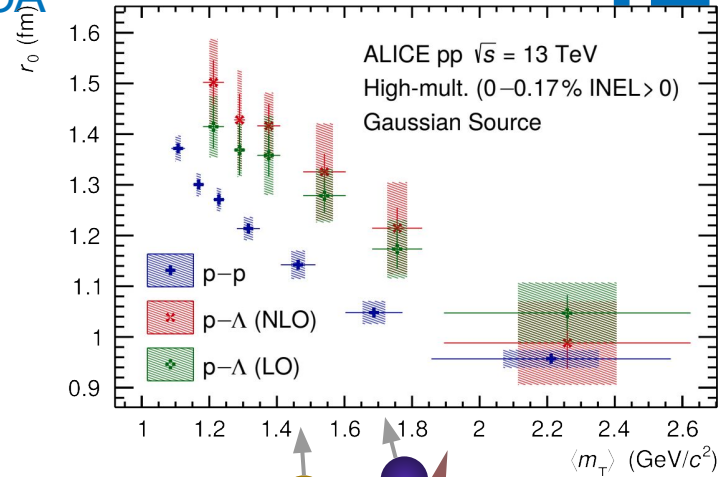


Using a toy MC for Coalescence

Basics of ToMCCA

Event Loop:

- Get number of charged particles
- Get proton yield
- Get neutron yield
- ↻ Loop over all protons
 - Get 3D momentum of proton
- ↻ Loop over all neutrons
 - Get 3D momentum of neutron
 - Get source size

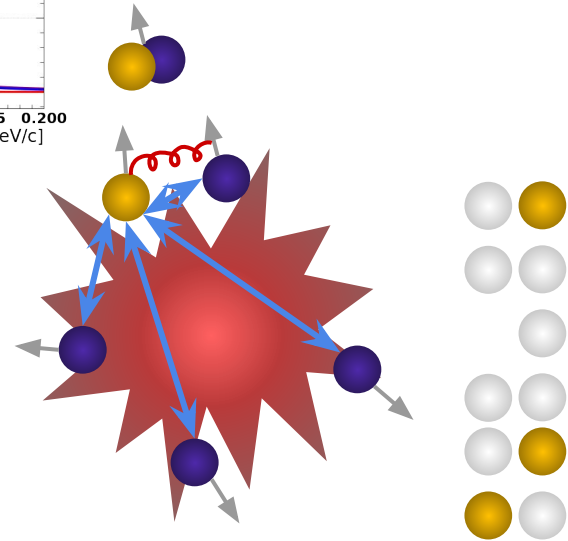
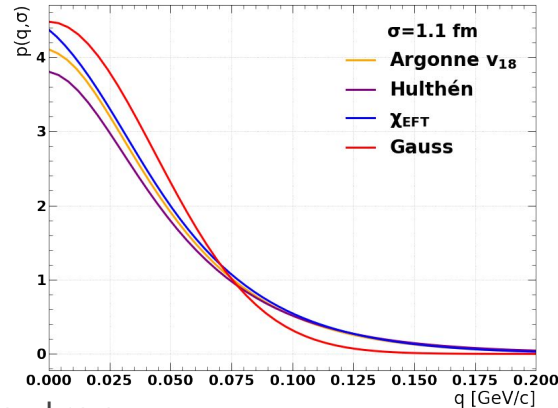


Using a toy MC for Coalescence

Basics of ToMCCA

Event Loop:

- Get number of charged particles
- Get proton yield
- Get neutron yield
- ↻ Loop over all protons
 - Get 3D momentum of proton
- ↻ Loop over all neutrons
 - Get 3D momentum of neutron
- Get source size
- Apply coalescence condition

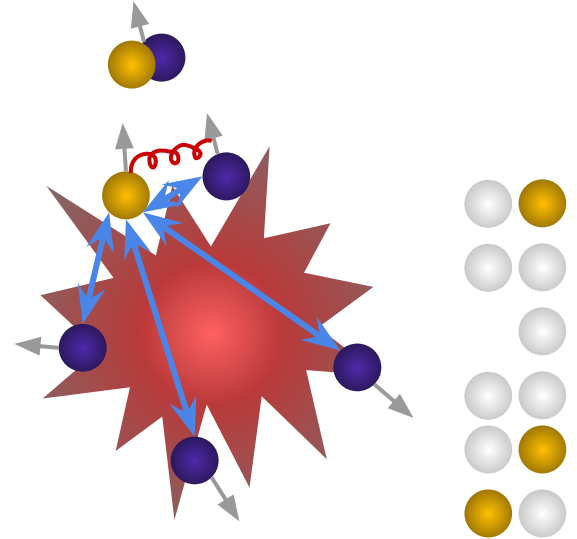


Using a toy MC for Coalescence

Basics of ToMCCA

Event Loop:

- Get number of charged particles
- Get proton yield
- Get neutron yield
- ↻ Loop over all protons
 - Get 3D momentum of proton
- ↻ Loop over all neutrons
 - Get 3D momentum of neutron
- Get source size
- Apply coalescence condition



Using a toy MC for Coalescence

Basics of ToMCCA

Event Loop:

Get number of charged particles

Get proton yield

Get neutron yield

↻ Loop over all protons

Get 3D momentum of proton

↻ Loop over all neutrons

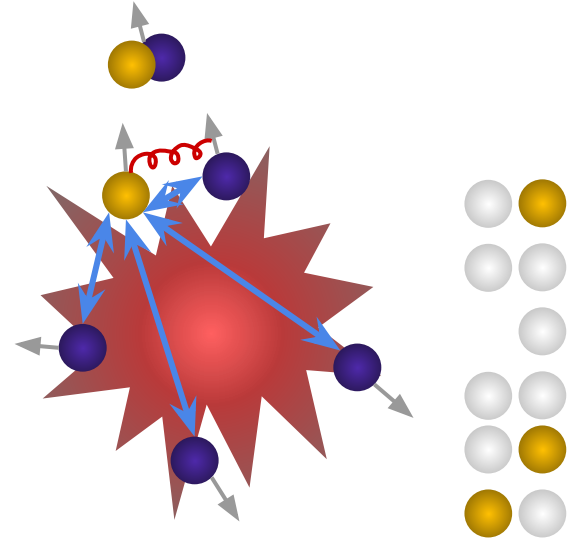
Get 3D momentum of neutron

Get source size

Apply coalescence condition

make deuteron, number of neutrons -1

try next neutron



Using a toy MC for Coalescence

Basics of ToMCCA

Event Loop:

Get number of charged particles

Get proton yield

Get neutron yield

↻ Loop over all protons

Get 3D momentum of proton

↻ Loop over all neutrons

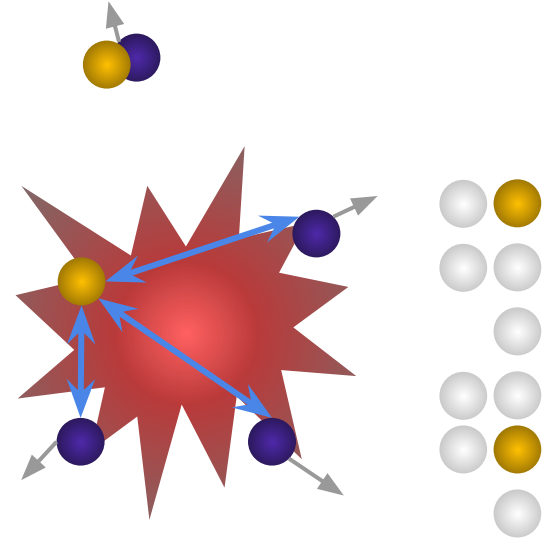
Get 3D momentum of neutron

Get source size

Apply coalescence condition

✓ make deuteron, number of neutrons -1

✗ try next neutron



Using a toy MC for Coalescence

Basics of ToMCCA

Event Loop:

Get number of charged particles

Get proton yield

Get neutron yield

↻ Loop over all protons

Get 3D momentum of proton

↻ Loop over all neutrons

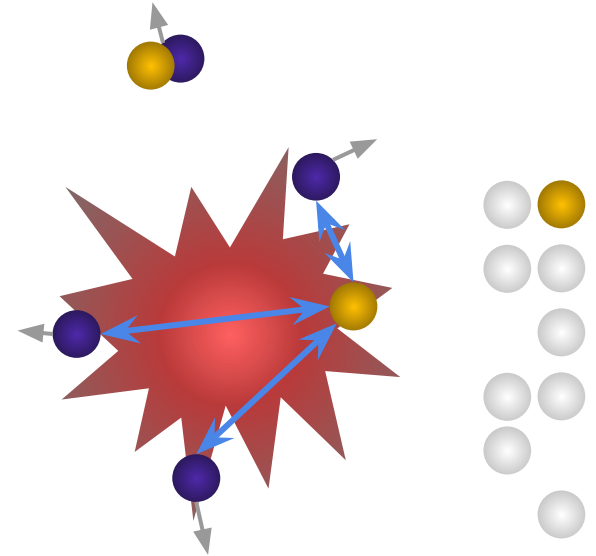
Get 3D momentum of neutron

Get source size

Apply coalescence condition

✓ make deuteron, number of neutrons -1

✗ try next neutron



Using a toy MC for Coalescence

Basics of ToMCCA

Event Loop:

Get number of charged particles

Get proton yield

Get neutron yield

↻ Loop over all protons

Get 3D momentum of proton

↻ Loop over all neutrons

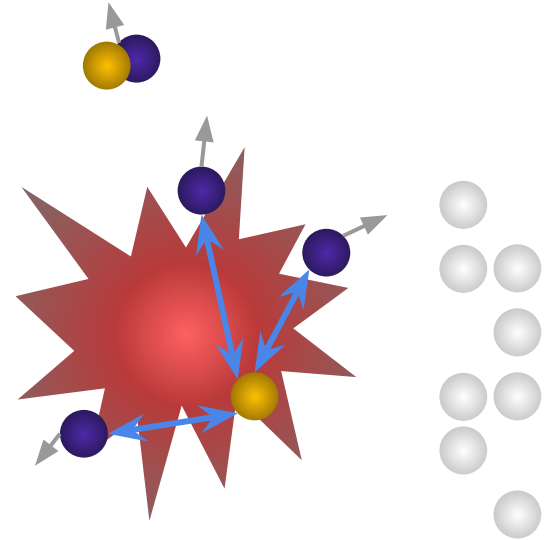
Get 3D momentum of neutron

Get source size

Apply coalescence condition

✓ make deuteron, number of neutrons -1

✗ try next neutron

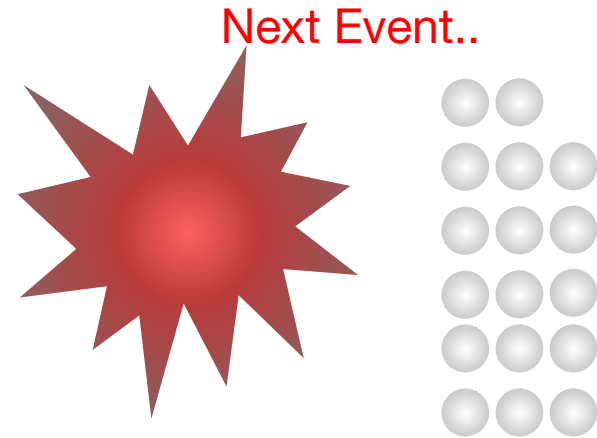


Using a toy MC for Coalescence

Basics of ToMCCA

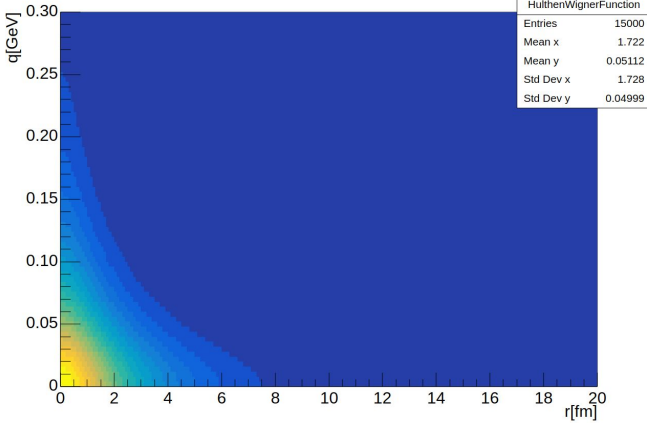
Event Loop:

- Get number of charged particles
- Get proton yield
- Get neutron yield
- ↻ Loop over all protons
 - Get 3D momentum of proton
- ↻ Loop over all neutrons
 - Get 3D momentum of neutron
 - Get source size
 - Apply coalescence condition
 - ✓ make deuteron, number of neutrons -1
 - ✗ try next neutron

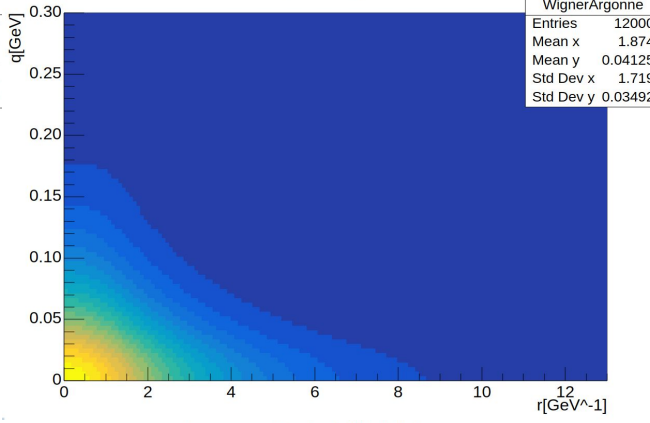


New Wigner functions/Probabilities

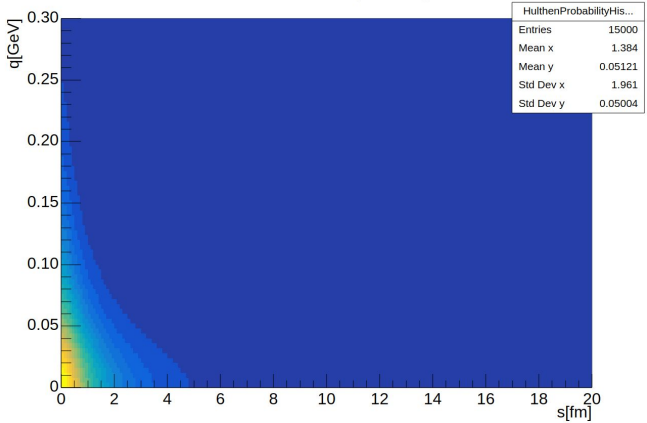
HulthenWignerFunction



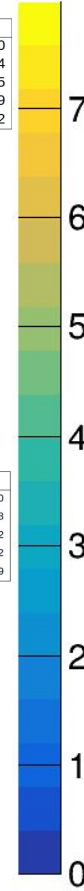
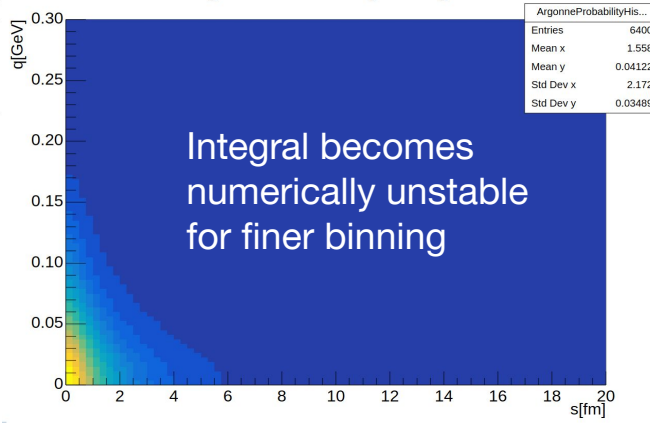
WignerArgonne



HulthenProbabilityHistogram

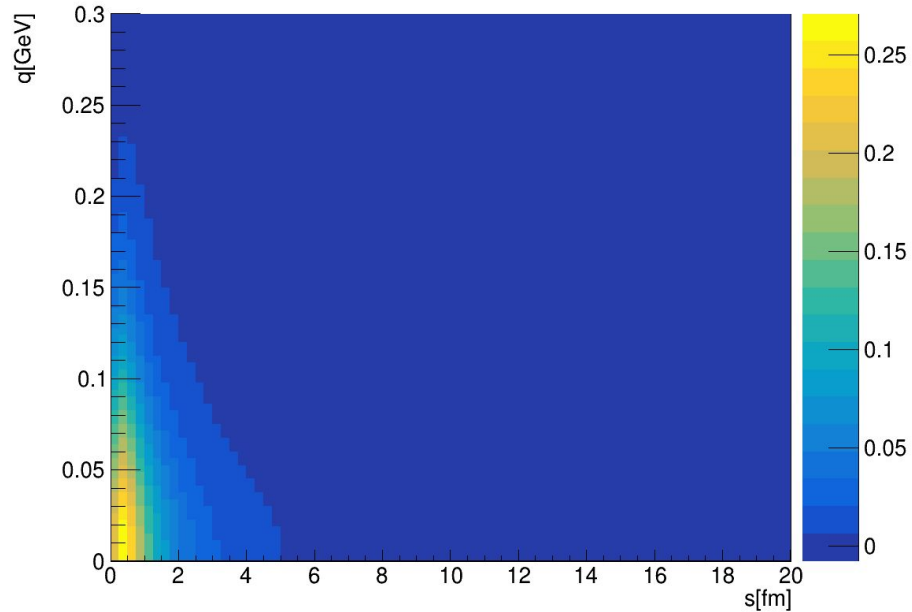


ArgonneProbabilityHistogram

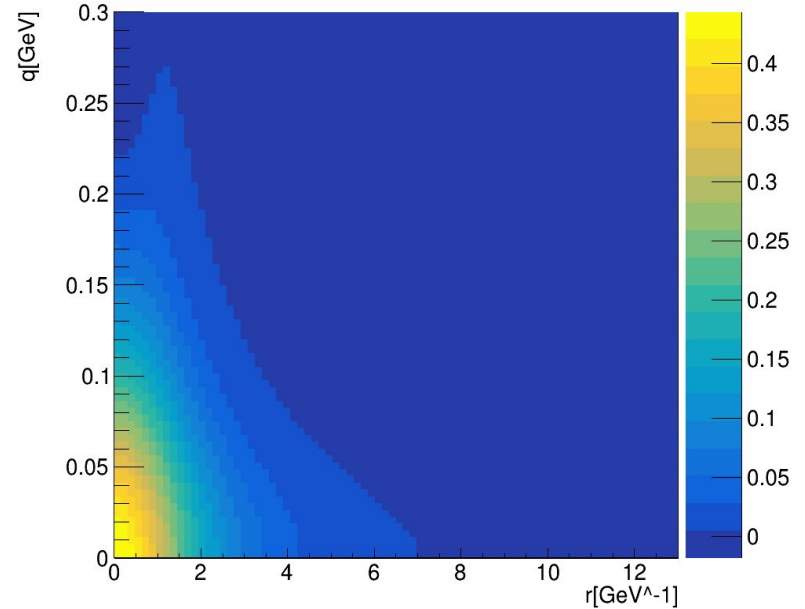


Argonne D-State probability

ArgonneProbabilityHistogramDWave



WignerArgonne_D_Wave



D-State probability is 6% → Maximum ~11% effect