

# Bayesian analysis of nuclear theory and astrophysics constraints for the dense matter equation of state

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## Constraining the Dense Matter Equation of State with New NICER Mass–Radius Measurements and New Chiral Effective Field Theory Inputs

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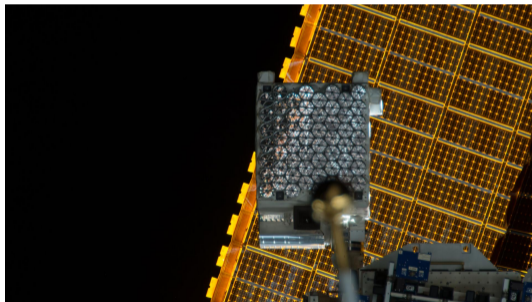
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Goal: to infer posterior probability distributions (PDFs) for the mass-radius ( $M$ - $R$ ) and pressure-density ( $P$ - $\epsilon$ ) of neutron stars using chiral effective field theory ( $\chi$ EFT) and astrophysical data

With data from ...

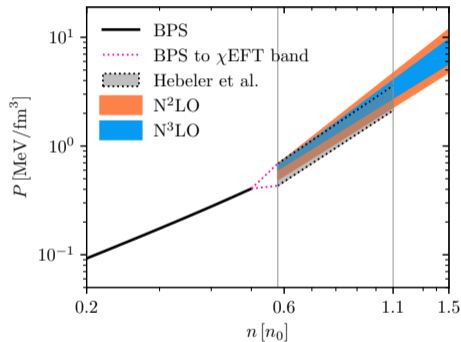
Gravitational wave data from neutron star mergers from LIGO/VIRGO

Mass-radius data of 3 neutron stars from NICER:



NASA's Neutron Star Interior Composition Explorer (NICER) aboard the ISS (image from NASA, obviously)

# Nuclear physics constraints at low densities

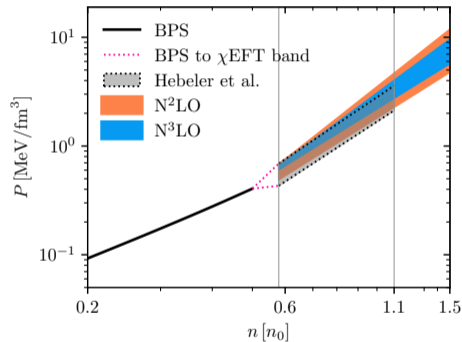


- Up to  $0.5n_0$  (saturation density): Baym-Pethick-Sutherland (BPS)<sup>a</sup> crust
- From  $\approx 0.6n_0$  to 1.1 or  $1.5n_0$ :  $\chi$ EFT at N<sup>2</sup>LO and N<sup>3</sup>LO<sup>b</sup>
- High-density parametrizations at higher densities

<sup>a</sup>Baym et al., ApJ **170** (1971)

<sup>b</sup>Keller et al., Phys. Rev. Lett., **130** (2023)

# Nuclear physics constraints at low densities



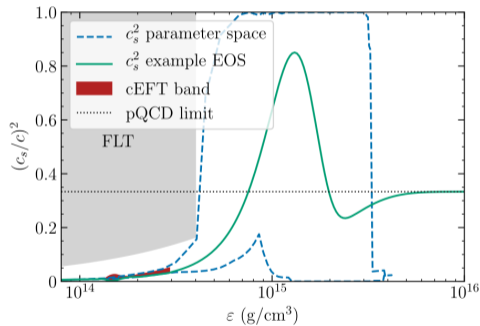
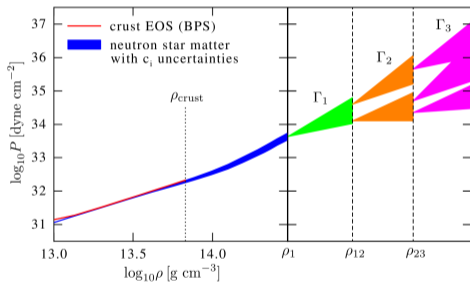
The new  $\chi$ EFT bands:

- New bands calculated directly in  $\beta$ -equilibrium; Hebel et al. bands use an empirical parametrization
- We trust  $\chi$ EFT to higher density ( $1.5n_0$ )
- Include muons in addition to electrons and neutrons/protons

# High-density parametrizations, $\approx 5$ unknown parameters

Piecewise polytropes (PP): 3  
independent polytropes with parameters  
 $\Gamma_{1-3}$ ,  $\rho_{12}$ , and  $\rho_{23}$

Speed of sound (CS): constrained by  
FLT, causality,  $c_s^2 \rightarrow 1/3$  from below at  
high densities



Hebeler et al., arXiv:1303.4662, ApJ **773**  
(2013)—also the reference for the "Hebeler et  
al."  $\chi$ EFT band

Greif et al., arXiv:1812.08188, MNRAS **485**  
(2019)

# Bayesian posterior for EOS parameters $\boldsymbol{\theta}$ and central energy densities $\boldsymbol{\varepsilon}$

$$\begin{aligned} p(\boldsymbol{\theta}, \boldsymbol{\varepsilon} \mid \mathbf{d}, \mathbb{M}) &\propto p(\boldsymbol{\theta} \mid \mathbb{M}) p(\boldsymbol{\varepsilon} \mid \boldsymbol{\theta}, \mathbb{M}) \\ &\times \prod_i p(\Lambda_{1,i}, \Lambda_{2,i}, M_{1,i}, M_{2,i} \mid \mathbf{d}_{\text{GW},i}) \\ &\times \prod_j p(M_j, R_j \mid \mathbf{d}_{\text{NICER},j}) \end{aligned}$$

- $\Lambda_{1,i}$  and  $\Lambda_{2,i}$  ( $M_{1,i}$  and  $M_{2,i}$ ): tidal deformabilities (source-frame component masses) given GW data  $\mathbf{d}_{\text{GW},i}$ <sup>1</sup>
- $\mathbf{d}_{\text{NICER},j}$ : mass-radius NICER data (folds in highest-mass data from radio)
- $\mathbb{M}$ : all modeling assumptions we make

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<sup>1</sup>References to all data at the end of the presentation.

# Software

All calculations are performed with N<sub>E</sub>oST, which is publicly available:

<https://xpsi-group.github.io/neost/>

N<sub>E</sub>oST uses MultiNest as its sampler: Feroz et al., MNRAS **495** (2009), Buchner et al., A&A **471** (2014)

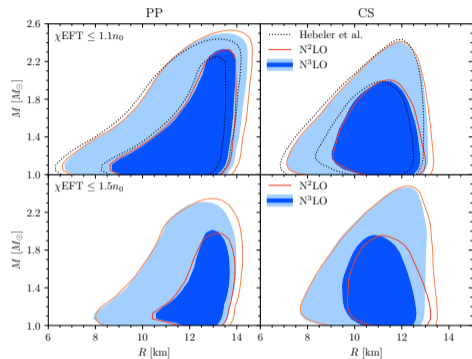
A full reproduction package for all our results is available:

<https://zenodo.org/records/10871354>

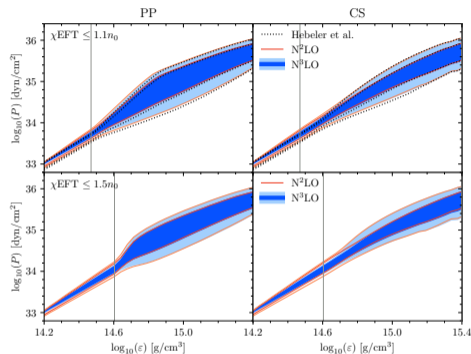


# Priors

Mass-radius:

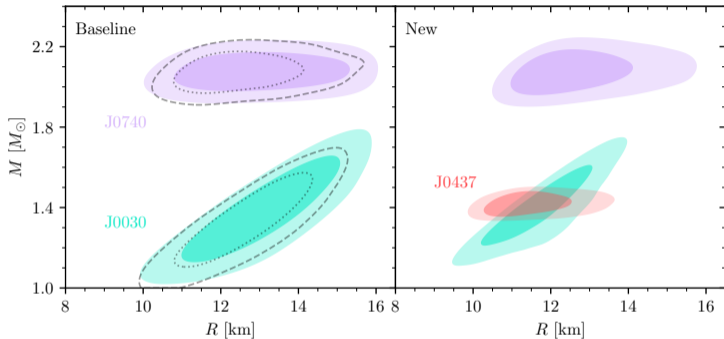


Pressure-energy density:



- PP and CS overall similar, CS is a bit more constraining
- Trusting  $\chi\text{EFT}$  to  $1.5n_0$  yields less uncertainty
- New  $\chi\text{EFT}$  bands slightly more constraining than the Hebel band
- Priors limits upper radius

# Data scenarios

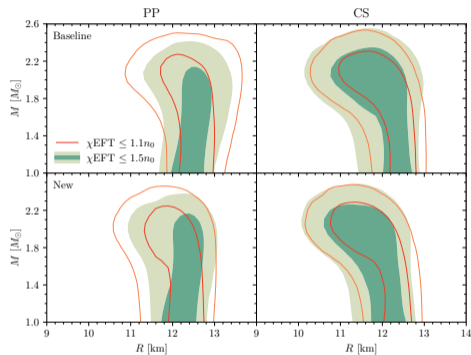


Two scenarios for comparison: "Baseline" with the (slightly updated) older NICER results, and "New" with the recent J0437 millisecond pulsar

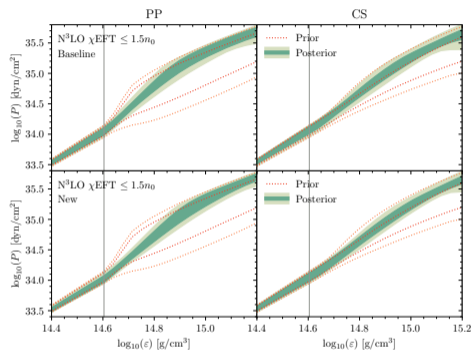
(In the paper we also consider two more scenarios, which I will not cover here)

# Posteriors

Mass-radius:

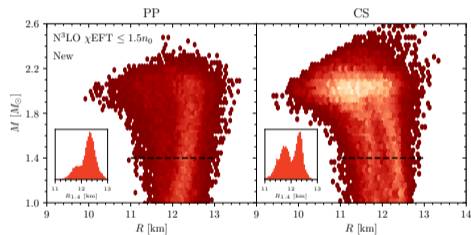


Pressure-energy density:



- The new NICER result favors smaller radii, especially for low-mass stars
- Trusting  $\chi\text{EFT}$  to higher densities disfavors high-mass, low-radius stars
- Data prefers high pressures

# Bimodal-like structure

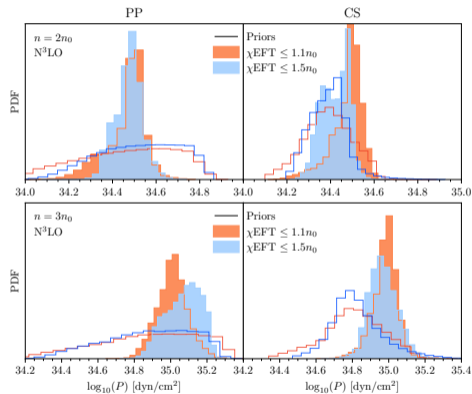


A **bimodal-like** structure appears in several posteriors, both at  $N^2\text{LO}$  and  $N^3\text{LO}$ , most clearly seen with the CS parametrization.

For masses below  $\approx 1.4M_{\odot}$ , our results favor radii lower or higher than—but not equal to—12 km

Origin is not completely clear—tension between data points

# Bimodal-like structure



Also visible in the pressure posterior, but less pronounced

The data prefers higher pressures

# Extended outlook: replace $\chi$ EFT band with Bayesian results?

- $\chi$ EFT: systematic expansion for low momentum
- all predictions **uncertain** due to (i) unknown low-energy constants (LECs), (ii) finite-order truncation (plus some other problems)
- Three-nucleon forces appear at  $N^2$ LO with two LECs  $c_D, c_E$ :

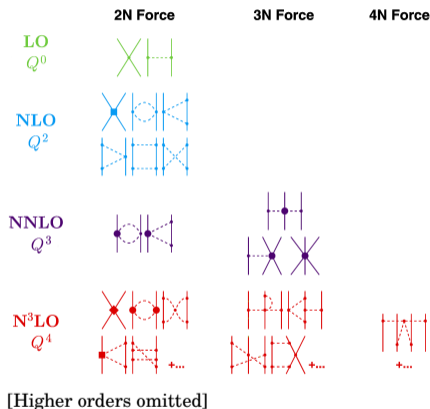
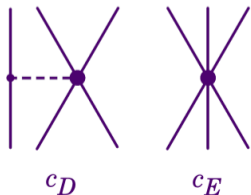
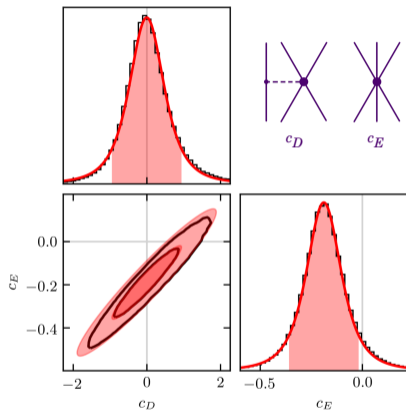


Figure adapted from Entem et al., Phys. Rev. C **96** (2017).

# Inferred posterior for $c_D$ and $c_E$ <sup>2</sup>

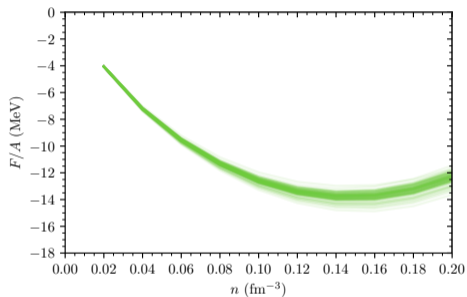


Posterior for the 3NF LECs.

- Infer  $c_D, c_E$  from data: mass and radius of  ${}^4\text{He}$ , mass and  $\beta$ -decay rate of  ${}^3\text{H}$ .
- Account for experimental errors and EFT truncation errors in the inference
- Truncation errors crucial

<sup>2</sup>Wesolowski, IS, et al., Phys. Rev. C **104** (2021)

# Posterior predictive distribution (PPD) for **symmetric** nuclear matter



PPD based on the LEC distribution on the previous slide.

Calculation of the free energy per particle as a function of density.<sup>a</sup>

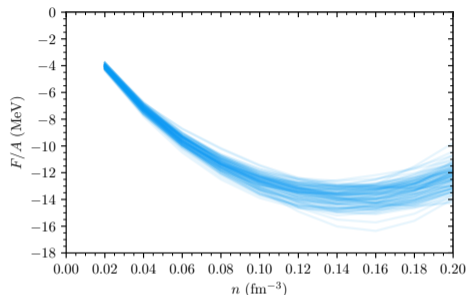
- Up to slightly above saturation density  $n_0 = 0.16 \text{ fm}^{-3}$ , can be extended
- Relatively unknown 3NF strengths yield fairly uncertain predictions
- Straightforward extension to arbitrary proton fraction (and temperature), or matter in  $\beta$ -equilibrium as required by N<sup>3</sup>EOST

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<sup>a</sup>Performed using many-body perturbation theory with code from Jonas Keller+Yannick Dietz, TU Darmstadt.



## With EFT truncation errors modeled with Gaussian processes



As before, but with correlated EFT  
truncation errors added on

### PRELIMINARY:

We also model EFT truncation errors  
(correlated across density) using  
Gaussian processes<sup>a</sup>

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<sup>a</sup>Melendez et al., Phys. Rev. C **100** (2019), Drischler et al., Phys. Rev. C **102** (2020).

Ongoing work with Hannah Götting, Alex Tichai, Kai Hebeler, Achim Schwenk.  
We plan to also account for errors in the MBPT calculation.

**Replace  $\chi$ EFT band in N<sup>2</sup>LoST with distribution based on this work?**

# Inferring three-nucleon forces from astro data

In collaboration with (among others) Rahul Somasundaram and Ingo Tews, LANL

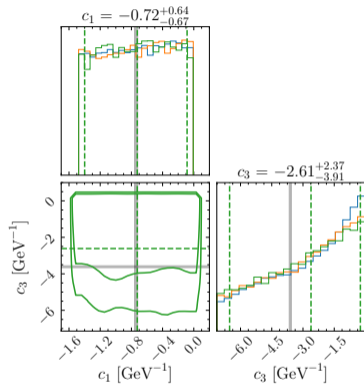
- The LEC  $c_3$  appears in the  $\pi N$  part of  $\chi$ EFT potentials
- Influences both 2- and 3-nucleon forces; we focus on  $3N$
- Typically fitted to  $\pi N$  scattering data

$c_3$  also contributes to the nuclear EOS in neutron matter, and can thus in principle be constrained by **neutron star observations**.

Until now, the computations have been prohibitively expensive, but new developments have overcome this roadblock.

We infer  $c_3$  from **current and next-generation astro data**

# Inferring three-nucleon forces from astro data



- $c_1$  (another  $\pi N$  LEC) is unconstrained
- Current data is not enough to precisely determine  $c_3$
- But: precise inference of  $c_3$  becomes **feasible with next-generation GW data**

(paper is on its way)

Constraints on  $c_1$  and  $c_3$  using **currently** available astro data.

## Thank you & data references

PSR J0437, PSR J0030, PSR J0740:

- Choudhury, Salmi, Vinciguerra et al., ApJ Lett. **971** (2024)
- Vinciguerra, Salmi, Watts et al., ApJ **961** (2024)
- Salmi, Choudhury, Kini et al., ApJ (2024, in press)

GW:

- Abbott, B. P., Abbott, R., Abbott, T. D., et al., Phys. Rev. X **9** (2019)
- Abbott, B. P., Abbott, R., Abbott, T. D., et al., ApJ Lett. **892** (2020)

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