Summarised introduction

Virtual spring meeting of the Muon g-2 Theory Initiative

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CPT - Marseille - April 15, 2024

- The time-momentum representation [Bernecker, Meyer '11]

$$a_\mu^{\rm HVP,LO} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty \mathrm{d}x_0\ \widetilde{f}(x_0)\ C(x_0)\ , \qquad C(x_0) = -\frac{1}{3}\sum_{k=1}^{3}\sum_{\vec{x}} \langle J_k(x) J_k(0)\rangle$$

$$J_k = \tfrac{2}{3}\overline{u}\gamma_k u - \tfrac{1}{3}\overline{d}\gamma_k d - \tfrac{1}{3}\overline{s}\gamma_k s + \tfrac{2}{3}\overline{c}\gamma_k c + \cdots$$

- The time-momentum representation [Bernecker, Meyer '11]

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- Window observables

  → Replace $\widetilde{f}(x_0)$ with $W(x_0)\widetilde{f}(x_0)$ above

  → Intermediate distance window : suppress long distance / remove very short distance

  → Easier to compute on the lattice

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- Time moments

$$G_{2n} \equiv \int_{-\infty}^{\infty} \mathrm{d}x_0\ x_0^{2n}\ C(x_0)\,, \quad \Pi_0 = -\frac{1}{2}G_2\,, \quad \Pi_n = -\frac{(-1)^n}{(2n+2)!}G_{2n+2}$$

→ In 2020, several groups presented results for the first and second time moments

→ Observable sensitive to the tail as n increases

- Original WP reviewed lattice calculations of the LO-HVP contribution to a_μ published before March 2020.

  $\rightarrow$ the 2020 result by the BMW collaboration were not included

- Up to 7 collaborations published high-quality calculations (but often only partial)

The total contribution was written as

$$a_\mu^{\mathrm{HVP,LO}} = a_\mu^{\mathrm{HVP,LO}}(ud) + a_\mu^{\mathrm{HVP,LO}}(s) + a_\mu^{\mathrm{HVP,LO}}(c) + a_{\mu,disc}^{\mathrm{HVP,LO}} + \delta a_\mu^{\mathrm{HVP}}$$

$\delta a_\mu^{\mathrm{HVP}}$: QED and SIB corrections

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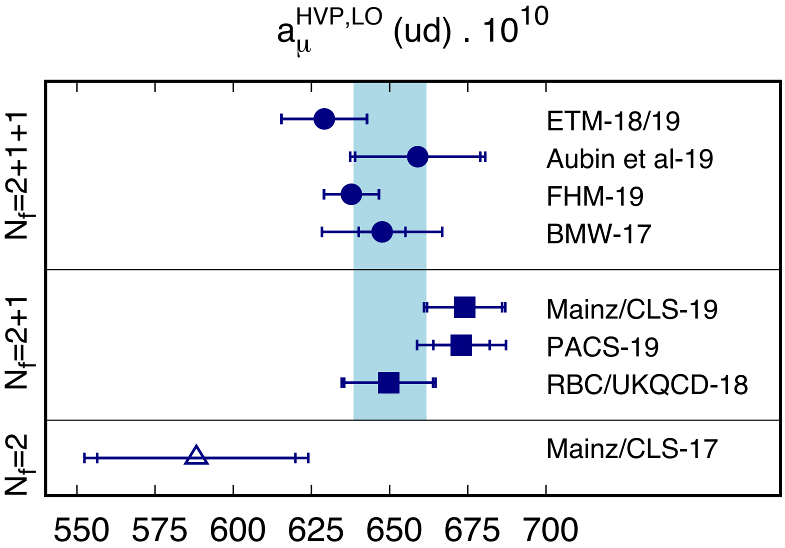
- Light-quark contribution



- 7 collaborations
- 1.5 - 3.3% precision
- error budget dominated by :

  $\rightarrow$ statistical error (tail)

  $\rightarrow$ finite-volume correction

  $\rightarrow$ discretization errors
- based on $\neq$ discretizations

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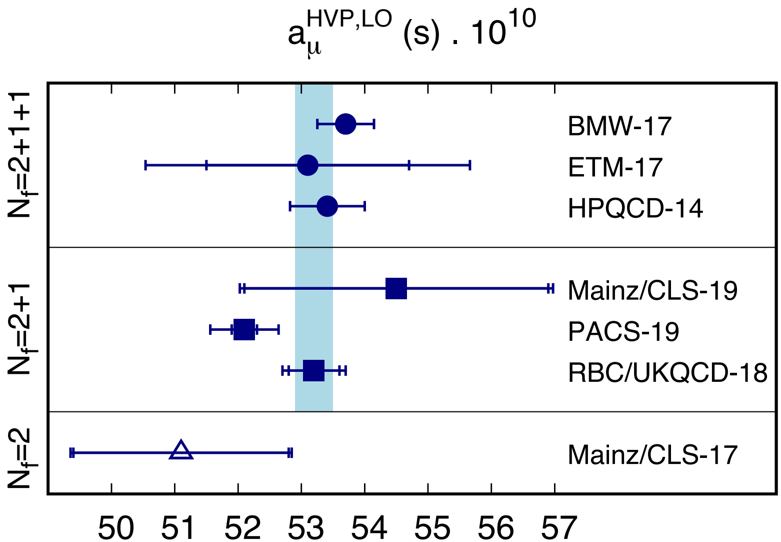
- Strange-quark contribution



- 6 collaborations
- final result dominated by 4 points
- uncertainty : 0.3×10^{-10}

  $\rightarrow$ under control ($\ll 1‰$)

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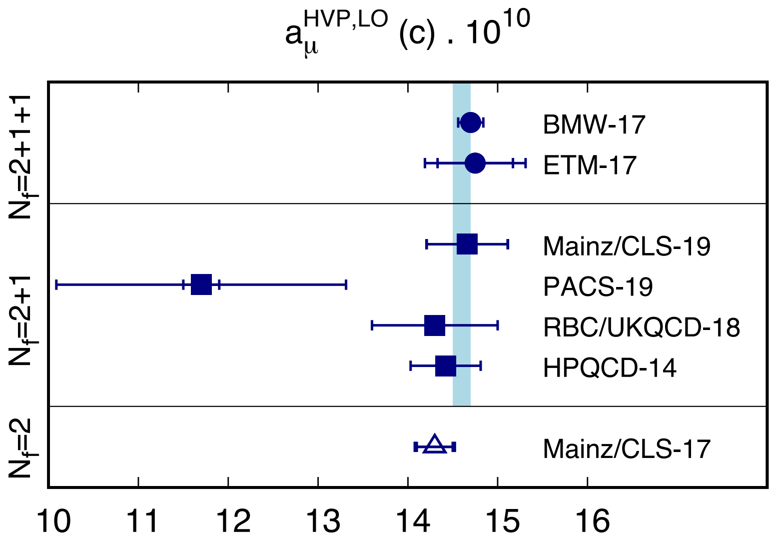
- Charm-quark contribution



- 6 collaborations
- uncertainty : 0.1×10^{-10}

  $\rightarrow$ under control ($\ll 1‰$)

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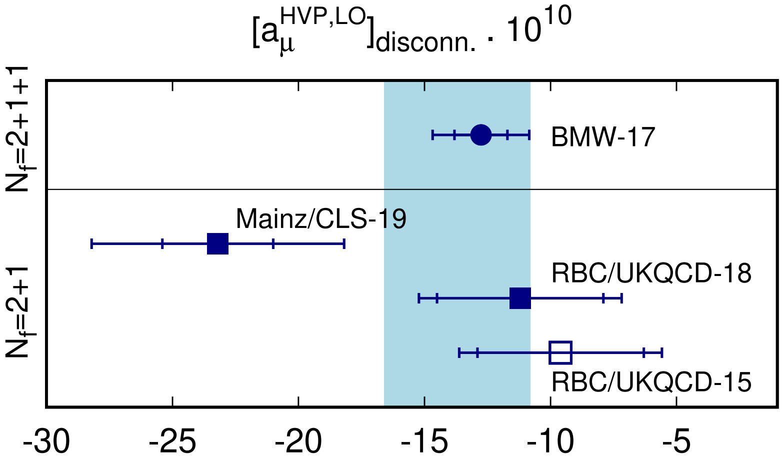
- Quark-disconnected contribution



- only 3 collaborations
- uncertainty : 2.9×10^{-10}

  $\rightarrow$ 4‰ on total contribution

- Original WP reviewed lattice calculations of the LO-HVP contribution to a_μ published before March 2020.

  $\rightarrow$ the 2020 result by the BMW collaboration were not included

- Up to 7 collaborations published high-quality calculations (but often only partial)

- Strong isospin breaking / QED corrections

  - only 2 collaborations : RBC/UKQCD-18 and ETM-19
    $\rightarrow$ electro-quenched approximation
    $\rightarrow$ not all diagrams were evaluated

  - FHM-17 : partial result for the SIB

  - BMW-20 : first complete calcualtion

- Individual flavor contributions in isospin limit + SIB/QED corrections :

$a_\mu^{\text{HVP, LO}}(ud)$	$a_\mu^{\text{HVP, LO}}(s)$	$a_\mu^{\text{HVP, LO}}(c)$	$a_{\mu,\text{disc}}^{\text{HVP, LO}}$	$\delta a_\mu^{\text{HVP}}$
650.2(11.6)	53.2(0.3)	14.6(0.1)	−13.7(2.9)	7.2(3.4)

$\rightarrow$ error budget : light-quark / SIB-QED / disconnected

$\rightarrow$ light-quark : FSE and cutoff effects were the largest systematics

- World average was obtained by adding all contributions listed above, assuming 100% correlation on the uncertainties

$$a_\mu^{\text{HVP,LO}}(\alpha^2) = 704.3(15) \times 10^{-10}$$

$$a_\mu^{\text{HVP,LO}} = 711.6(18.4) \times 10^{-10}$$

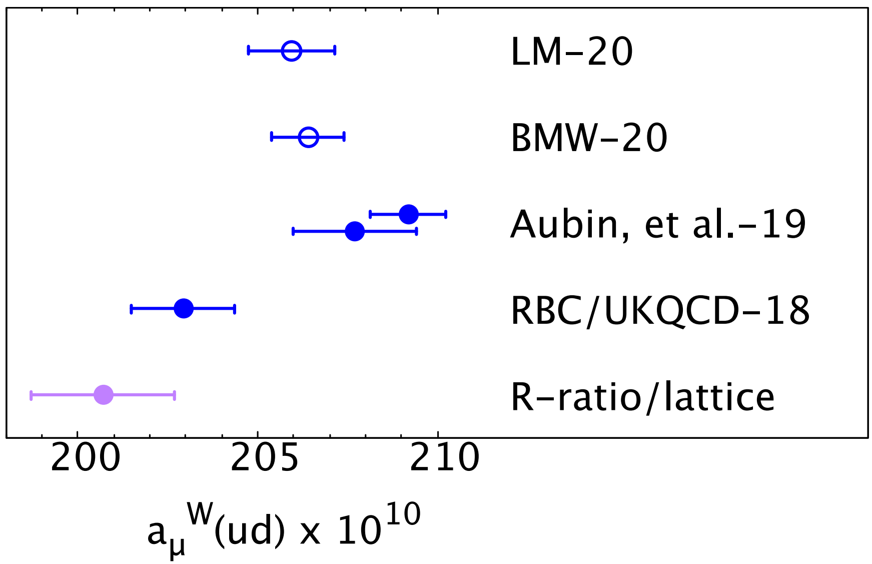
$\rightarrow$ total uncertainty of 2.6%

$\rightarrow$ compatible with the data driven estimate within 1σ

$\rightarrow$ but also with the "no new physics" scenario.

$\rightarrow$ lattice results where not included in the final average
(large uncertainties compared to data-driven)

- only two groups had published data : RBC/UKQCD-18 and Aubin et al.-19
- two other results appeared in 2020 (open circles)
- slight tension with RBC/UKQCD



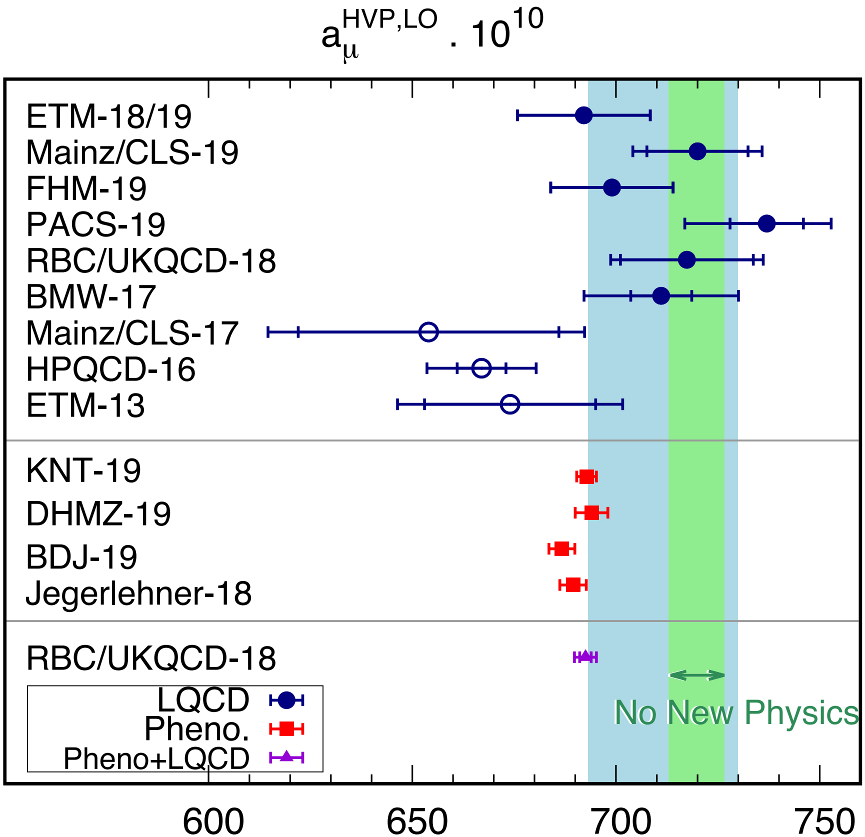
- This section should introduce a few notations (but 1 page)

  → should we focus on the time momentum representation ?

  → briefly introduce the window observables ? The precise definition would then be given in the corresponding section (C.2.)

  → time moments ? To my knowledge, no update since 2020. Should we mention them ?

Additional Slides


$a_\mu^{HVP,LO} \cdot 10^{10}$
ETM-18/19
Mainz/CLS-19
FHM-19
PACS-19
RBC/UKQCD-18
BMW-17
Mainz/CLS-17
HPQCD-16
ETM-13
KNT-19
DHMZ-19
BDJ-19
Jegerlehner-18
RBC/UKQCD-18
LQCD
Pheno.
Pheno+LQCD
No New Physics
600
650
700
750

Collaboration	N_f	$a_\mu^{\text{HVP, LO}}(ud) \times 10^{10}$	$a_\mu^{\text{HVP, LO}}(s) \times 10^{10}$	$a_\mu^{\text{HVP, LO}}(c) \times 10^{10}$	$a_{\mu,\text{disc}}^{\text{HVP, LO}} \times 10^{10}$
ETM-18/19 [17, 377]	2+1+1	629.1(13.7)	53.1(1.6)(2.0)	14.75(42)(37)	–
Aubin *et al.*-19 [16]	2+1+1	659 (22)	–	–	–
FHM-19 [14]	2+1+1	637.8(8.8)	–	–	−13(5)
BMW-17 [10]	2+1+1	647.6(7.5)(17.7)	53.73(04)(49)	14.74(04)(16)	−12.8(1.1)(1.6)
HPQCD-16 [376]	2+1+1	599.0(6.0)(11.0)†	–	–	0(9)(−)
HPQCD-14 [358]	2+1+1/2+1	–	53.41(00)(59)	14.42(00)(39)	–
Mainz/CLS-19 [15]	2+1	674(12)(5)	54.5(2.4)(0.6)	14.66(45)(6)	−23.2(2.2)(4.5)
PACS-19 [13]	2+1	673(9)(11)	52.1(2)(5)	11.7(0.2)(1.6)	–
RBC/UKQCD-18 [11]	2+1	649.7(14.2)(4.9)	53.2(4)(3)	14.3(0)(7)	−11.2(3.3)(2.3)
Mainz/CLS-17 [369]	2	588.2(31.7)(16.6)	51.1(1.7)(0.4)	14.3(2)(1)	–

Table 9: Flavor-specific contributions to $a_\mu^{\text{HVP, LO}}$, see also Fig. 45. The HPQCD-16 result for the light-quark connected contribution (marked by a †) is evaluated at the physical charged pion mass, and so includes some SIB and QED corrections. It is therefore not directly comparable with the other lattice results for $a_\mu^{\text{HVP, LO}}(ud)$. The ETM-19 result [17] is an update of ETM-18 [377] and uses the same gauge ensembles and analysis methods. The FHM-19 [14] entry for $a_{\mu,\text{disc}}^{\text{HVP, LO}}$ is a phenomenological (non-lattice) estimate obtained from a ChPT-inspired model. When results are displayed with two errors, the first is the statistical uncertainty and the second the systematic one. With only one quoted error, the statistical and systematic uncertainties are combined.