

Intermediate-distance Window

Steven Gottlieb
Indiana University

Muon $g-2$ Theory Initiative Spring Meeting
Zoomland
April 15, 2024

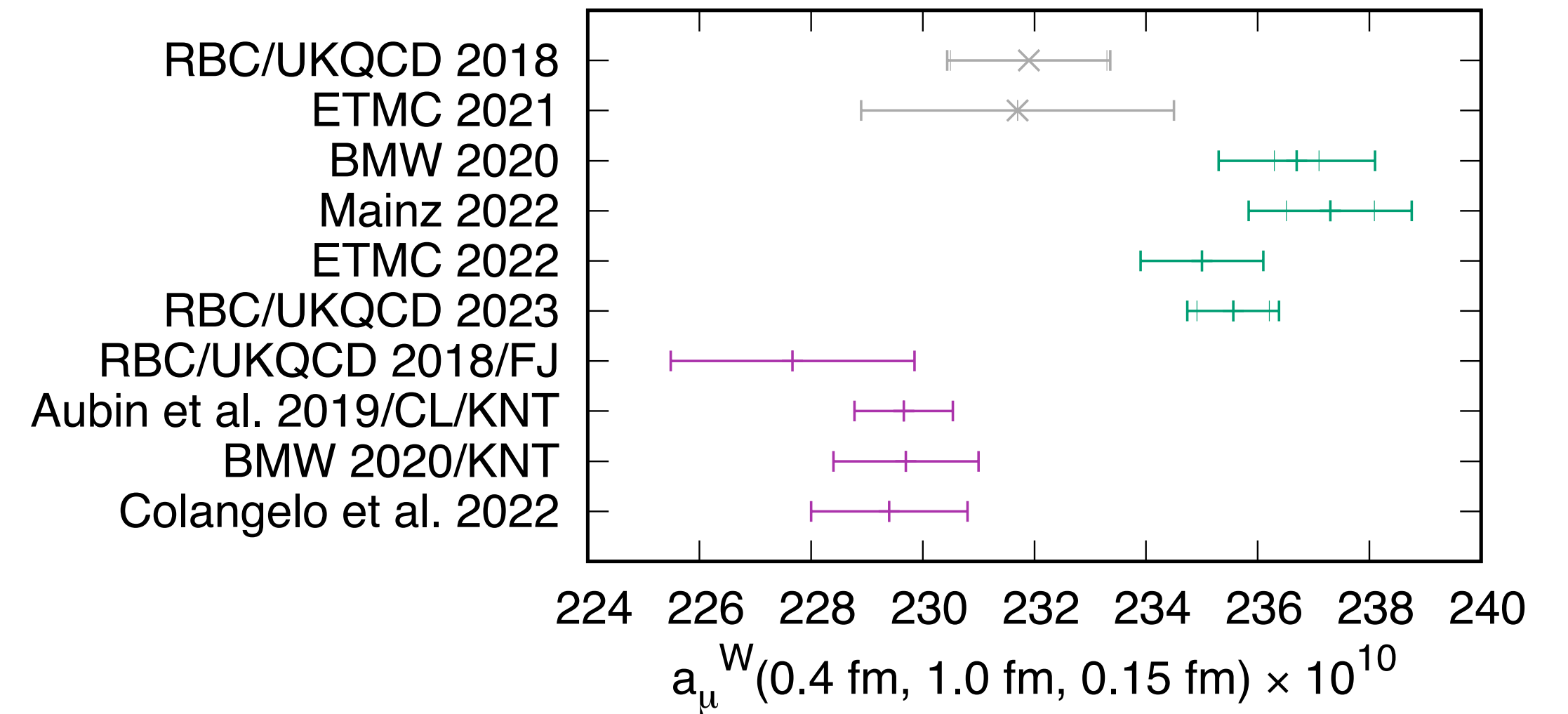
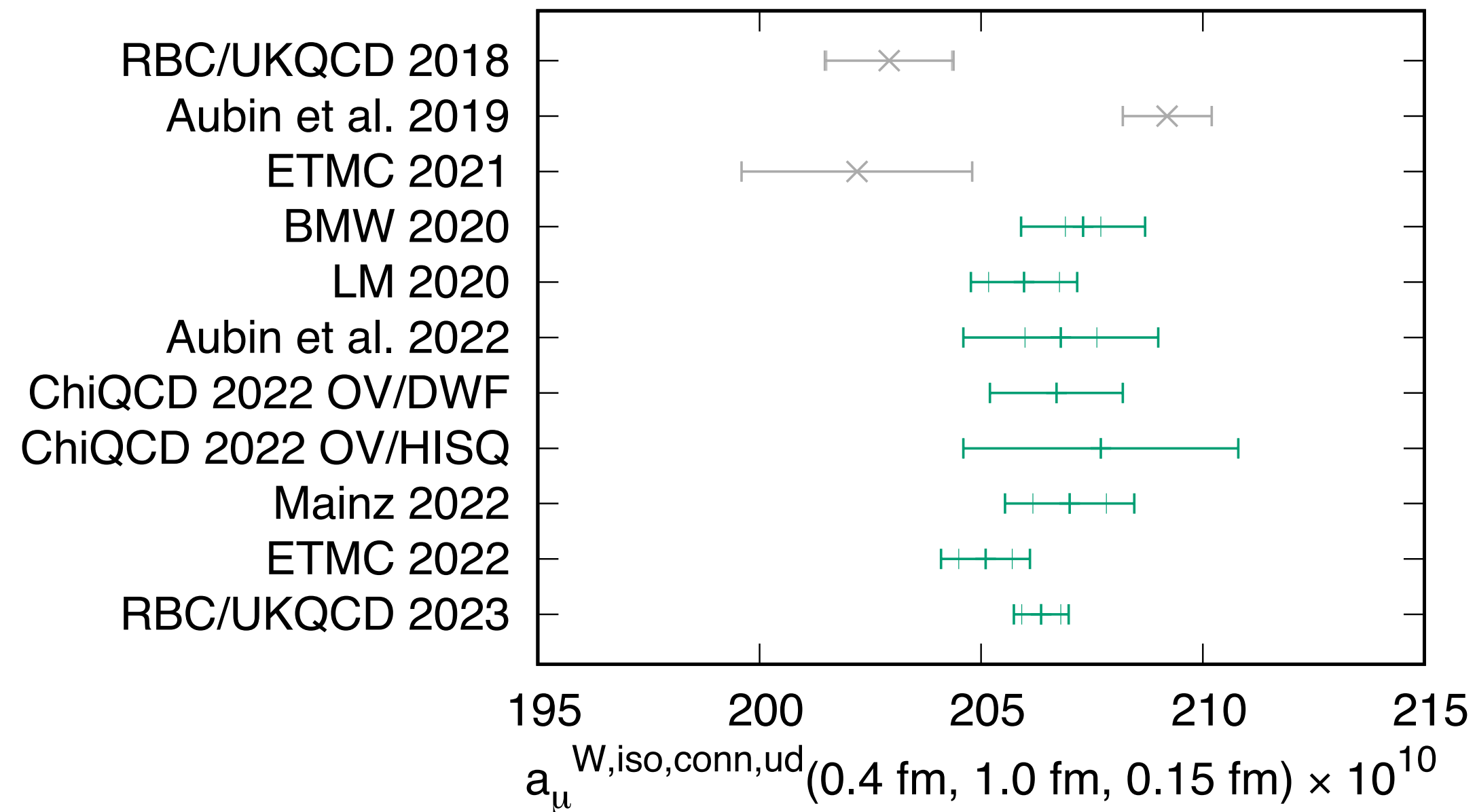
Why Windows?

- ◆ RBC/UKQCD collaboration introduced filters in Euclidean time to reduce contribution from long time tail. PRL121, 022003 (2018)
 - allows precise comparisons among different lattice calculations
 - allows comparison with data driven approach to explore apparent current difference of 2.2σ
 - Initially three filters: short distance (SD), intermediate distance (W), and long distance (LD)
 - Here we consider W
 - *Should the white paper also consider SD?*
- ◆ $\mathcal{W}(t, t_0, t_1, \Delta) = \frac{1}{2} \left[\tanh\left(\frac{t - t_0}{\Delta}\right) - \tanh\left(\frac{t - t_1}{\Delta}\right) \right] + (t \rightarrow -t)$ defines the window function, plotted on future slides

Literature Search

- ◆ Found 8 archival papers and 4 lattice proceedings from mid-2022 to present
 - Wang:2022lkq, Muon $g-2$ with overlap valence fermions
 - Aubin:2022hgm, Muon anomalous magnetic moment with staggered fermions: Is the lattice spacing small enough?
 - Ce:2022kxy, Window observable for the hadronic vacuum polarization contribution to the muon $g-2$ from lattice QCD
 - ExtendedTwistedMass:2022jpw, Lattice calculation of the short and intermediate time-distance hadronic vacuum polarization contributions to the muon magnetic moment using twisted-mass fermions
 - FermilabLattice:2022izv, Windows on the hadronic vacuum polarization contribution to the muon anomalous magnetic moment
 - Chao:2022ycy, Coordinate-space calculation of the window observable for the hadronic vacuum polarization contribution to $g-2$
 - FermilabLatticeHPQCD:2023jof, Light-quark connected intermediate-window contributions to the muon $g-2$ hadronic vacuum polarization from lattice QCD
 - RBC:2023pvn, Update of Euclidean windows of the hadronic vacuum polarization

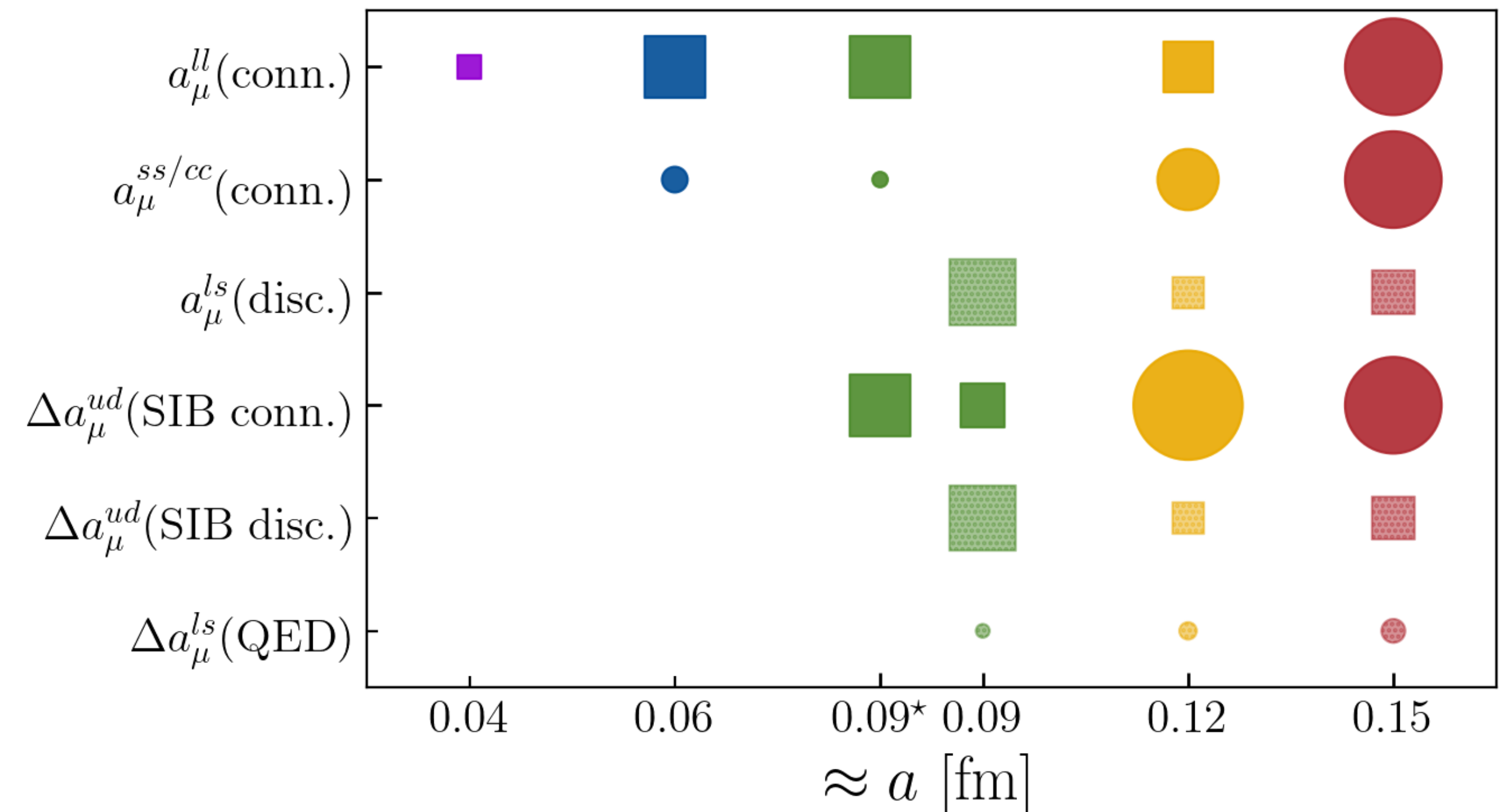
Light, Intermediate Contribution



- (Left) results from various groups for light connected contribution
- (Right) Complete contribution to the intermediate window. Includes strange, and charm quarks, disconnected contributions, isospin breaking and electromagnetic corrections. [10.1103/PhysRevD.108.054507](https://arxiv.org/abs/2301.08696); arXiv:2301.08696

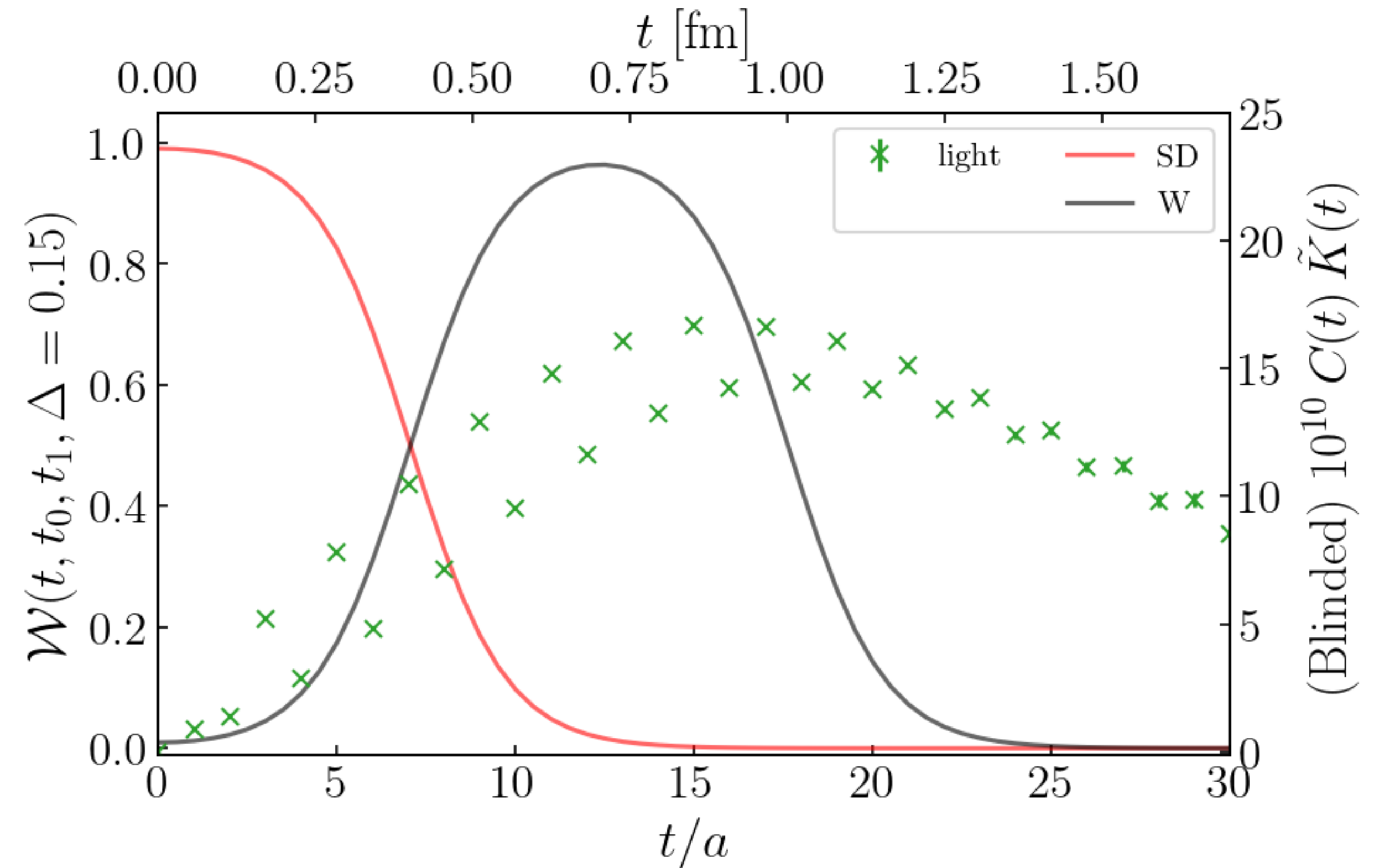
Ensembles Used by Fermilab/HPQCD/MILC

- Circles denote truncated solver method (TSM)
- Squares are for low mode improved (LMI)
- Area of symbol proportional to statistics. Several smaller symbols are runs in progress.
- Solid colors for local vector current; hatched for 1-link current
- 0.09* is a better tuned ensemble started by CalLat, extended by MILC



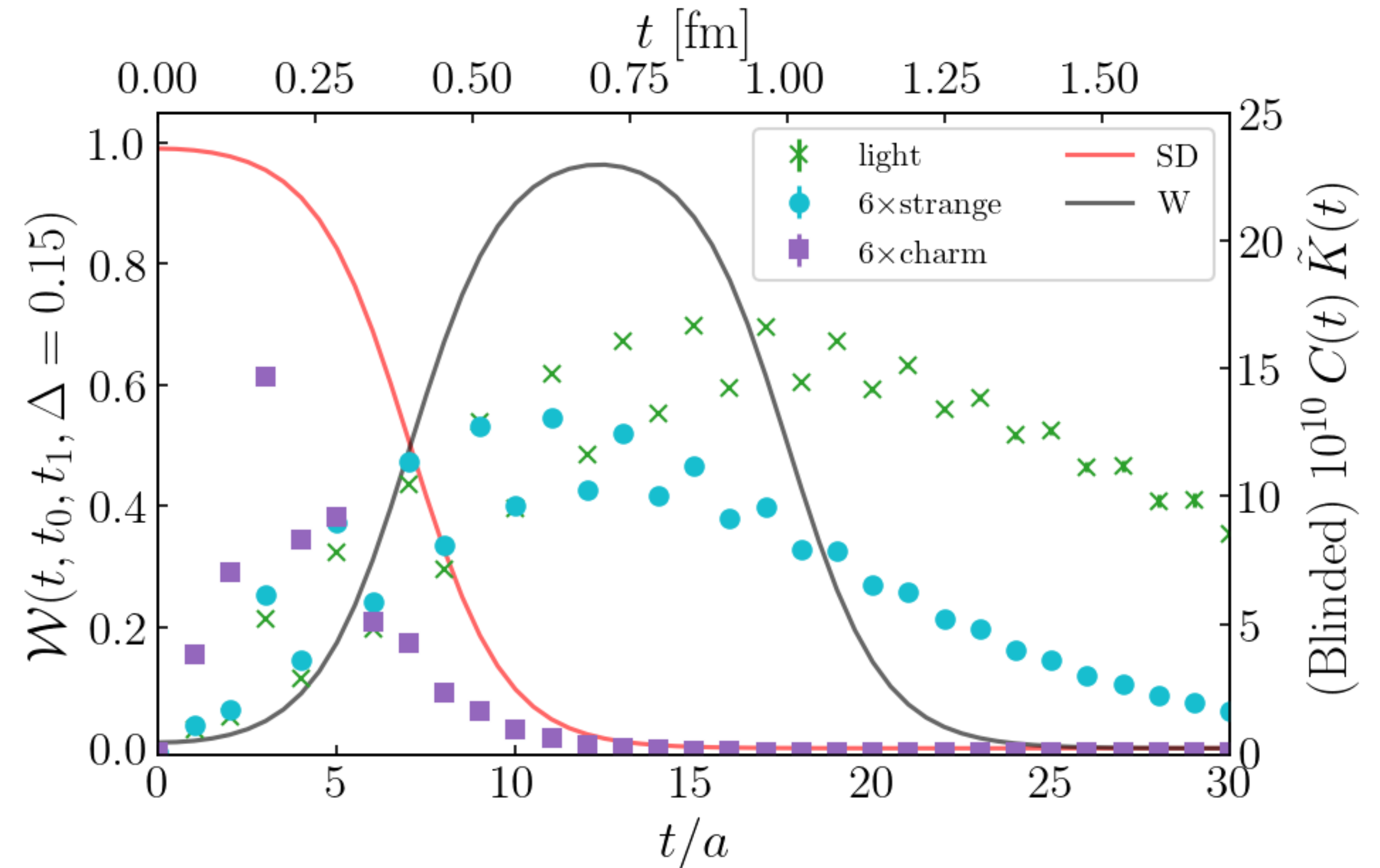
Example of Windows

- Results for $a \approx 0.06$ fm
- Red curve is SD filter out to 0.4 fm
- Black curve is W filter 0.4 to 1 fm
- In each case width is 0.15 fm
 - scale is on left
- Data for light quarks shows effect of staggering
- Data is blinded & preliminary
 - scale is on right
- Light contribution extends well beyond W, i.e., long distance contribution to g-2 is substantial.

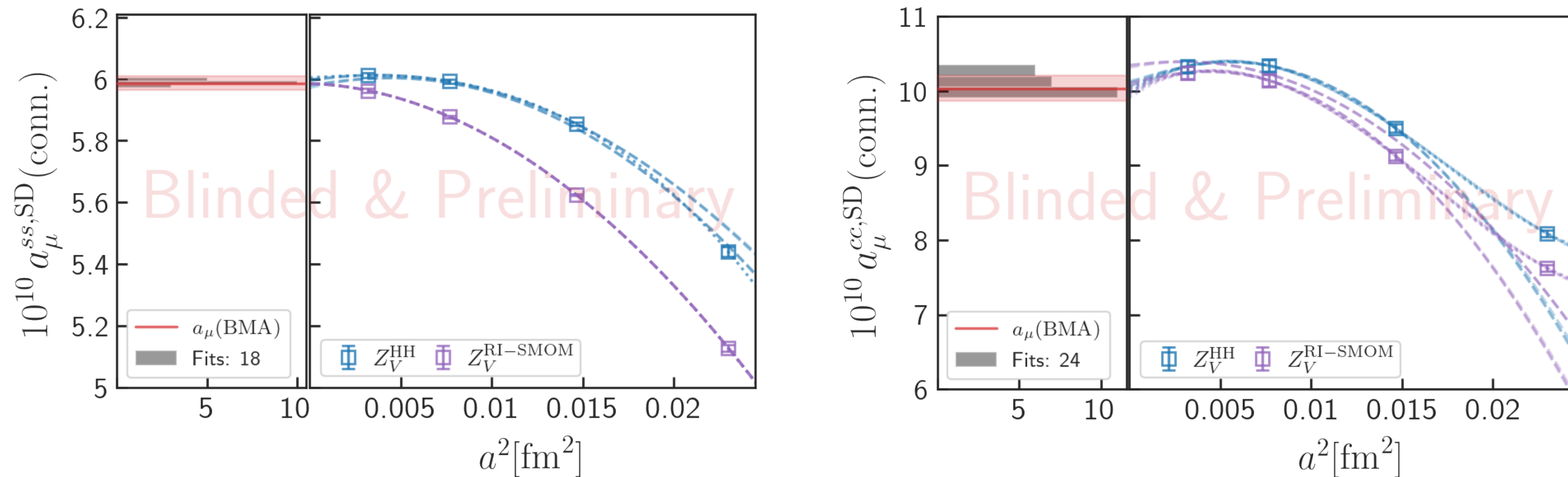


Example of Windows II

- Results for $a \approx 0.06$ fm
- Red curve is SD filter out to 0.4 fm
- Black curve is W filter 0.4 to 1 fm
- In each case width is 0.15 fm
- We add results for strange (blue), and charm (purple) quarks. They are both multiplied by 6 as they are small.
- Strange is mostly within W
- Charm mostly within SD

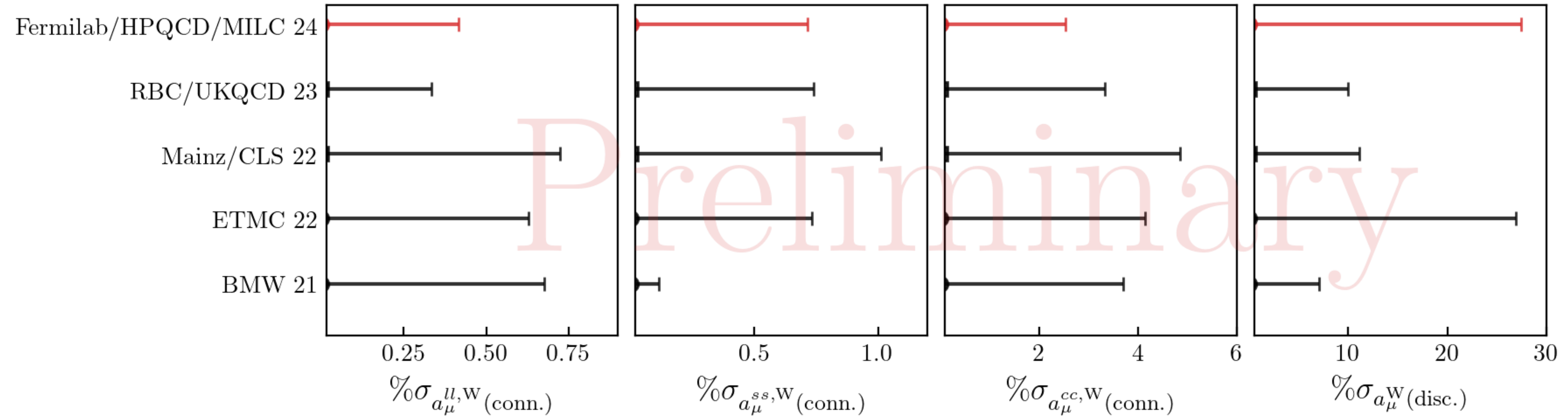


Short Distance Window for Heavier Quarks



- Contribution of strange quark (L) and charm quark (R). Note difference in size.
- Charm contribution to SD somewhat larger than strange.
- The dependence on lattice spacing is notably different than for W where continuum limit is approached from above.

Preliminary Error Comparison from FHM



- We compare our preliminary errors with four other recent calculations for connected contributions in the W window for light, strange, and charm quarks.
- The fourth panel shows the disconnected contribution summed over flavors.
- Note that the highest precision is for the largest contribution, light connected.

Conclusions

- ◆ There have been a number of papers published since the first white paper.
 - We are expected to produce three pages for this section
 - Should be try to cover both intermediate and short distance windows?
 - We plan to cover both flavor specific connected contributions, and complete contribution to the window(s).
- ◆ Please let us know if we are missing references.
 - We are looking forward to more contributions that can be added to the comparisons.

Additional Slides

Light, Connected Intermediate Contribution

- Similar to left graph on slide 4, but includes result from Fermilab Lattice, HPQCD, and MILC Collaborations.
- Light gray points near bottom have been superseded.
- For the top point, inner error bar is the statistical error.

