Status of the isosymmetric-HVP section

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Section content

The aim of this section is to review and combine lattice QCD results for the isosymmetric $a_\mu^{\rm HVP}$ and related observables.

This section, which is expected to be about 2 pages long, will cover:

- Single-flavour/disconnected contributions to isosymmetric $a_\mu^{\rm W}$.
- Short-distance window $a_\mu^{\rm SD}$.
- One-sided windows.
- Isosymmetric HVP $a_\mu^{\rm HVP}$.

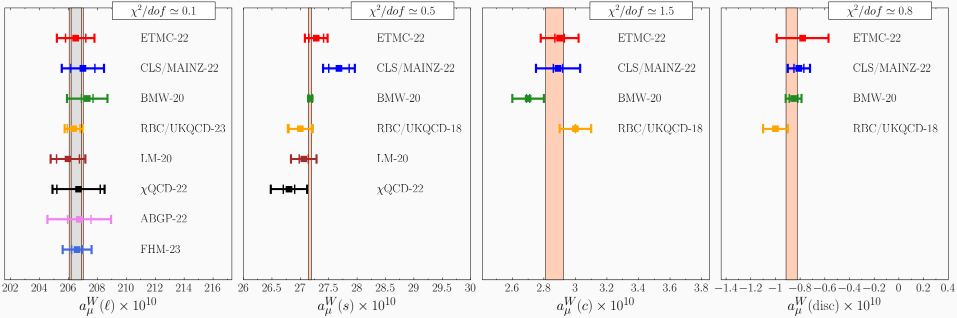
Averages of lattice results to be performed using the prescription adopted by the TI, which is briefly discussed in current version of the WP:

To combine results from different lattice calculations, we adopt a version of the procedure used by the FLAG group for averaging [86]. We assume that statistical errors from different calculations are uncorrelated, except in cases where the two calculations share the same gauge configurations, in which case we conservatively assume 100% correlation. Systematic errors that are shared between calculations, for example scale-setting uncertainty arising from dependence on the same physical scale, is also taken to be 100% correlated.

Single-flavour/disconnected contributions to isosymmetric a_μ^{W}

We assume $+100\%$ correlation in the stat. errors between groups which fully/partially share gauge configurations, and $+100\%$ correlation in the syst. errors if groups used same discretization in both sea and valence sectors.

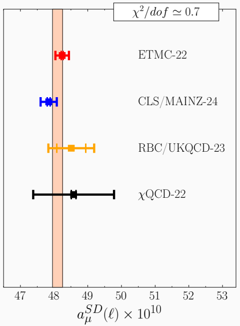
Quality criterion: average includes results from simulations with at least three β's (or two β's and more than one regularization), $M_\pi L \geq 3$, and at least one p.p. ensemble.

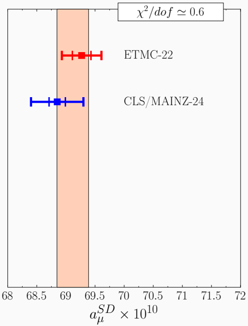


Assumed 100% correlation between stat. and syst. errors of FHM-LM-ABGP, and between stat. errors of χQCD and both RBC/UKQCD and FHM-LM-ABGP, this leads to a $\sim 40\%$ increase in final error for $a_\mu^{\mathrm{W}}(\ell)$ w.r.t. the case of uncorr. errors.

Short-distance window a_μ^{SD}

We employed for a_μ^{SD} the same average criterion used for a_μ^{W}. Since last TI-meeting CLS/MAINZ-24 results appeared.



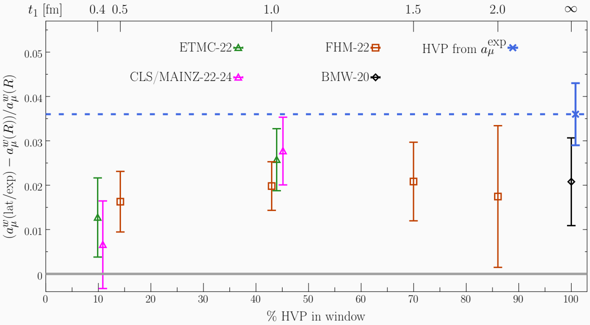


SD term	ETMC-22	CLS/MAINZ-24	χQCD-22	RBC/UKQCD-23
ℓ	48.24(3)(20)(20)	47.84(4)(24)(24)	48.6(0.1)(1.2)(1.2)	48.51(43)(53)(68)
s	9.074(14)(62)(64)	9.072(10)(58)(60)	9.18(1)(25)(25)	
c	11.61(9)(25)(27)	11.53(13)(26)(30)		
disc	−0.006(5)(2)(5)	0.0013(2)(5)(5)		
total	69.27(16)(30)(34)	68.85(14)(42)(45)		

Errors are stat., syst. and total, respectively.

One-sided windows

$$a_\mu(t_1) = \int_0^\infty dt\ K(t, m_\mu)\ C(t)\ \Theta(t; t_1, \Delta)\ , \qquad \Theta(t; t_1, \Delta = 0.15\ \text{fm}) = 1 - \frac{1}{1 + e^{-2(t-t_1)/\Delta}}$$



- Plot shows evolution of the relative difference between latt. and disp. results (baseline) as a function of t_1, from $a_\mu(0.4\ \text{fm}) = a_\mu^{\text{SD}}$ to $a_\mu(\infty) = a_\mu^{\text{HVP}}$.
- For ETMC and CLS/MAINZ, $a_\mu(1\ \text{fm}) = a_\mu^{\text{SD}} + a_\mu^{\text{W}}$, obtained here assuming $+100\%$ correlation between a_μ^{SD} and a_μ^{W}.

Continuum/mass-extrapolation plots included in the Section

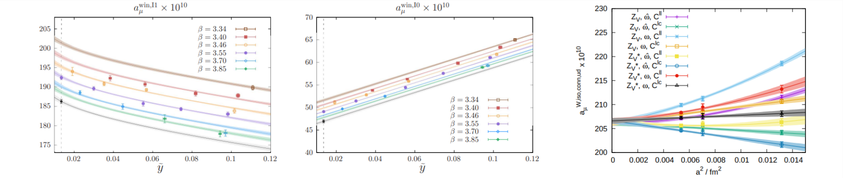


FIG. 6. Left and central panel: Extrapolation to the continuum limit and the physical mass point of the $I = 1$ and (charmless) $I = 0$ isospin components of $a_\mu^{\rm W}$ from CLS/MAINZ-22: $\tilde{y} = m_\pi^2/(8\pi f_\pi^2)$ and six lattice spacings are used ranging from $a = 0.099$ fm ($\beta = 3.84$) to $a = 0.039$ fm ($\beta = 3.85$), see Ref.[109] for details. Right panel: Extrapolation of $a_\mu^{\rm W}(\ell)$ (connected) to the continuum limit from RBC/UKQCD-23, with eight lattice variants of the observable of interest and three lattice spacings, down to $a = 0.073$ fm: see Ref.[111] for details.

SD term	ETMC22	CLS/Mainz24	χQCD22	RBC/UKQCD23
ℓ	48.24(3)(20)(20)	47.84(4)(24)(24)	48.6(0.1)(1.2)(1.2)	48.51(43)(53)(68)
s	9.074(14)(62)(64)	9.072(10)(58)(60)	9.18(1)(25)(25)	
c	11.61(9)(25)(27)	11.53(13)(26)(30)		
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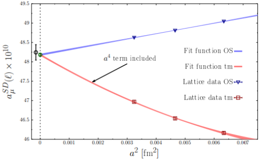


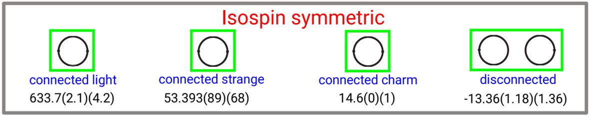
FIG. 7. Left panel: results for $a_\mu^{\rm SD} \cdot 10^{10}$ with $(t_0, t_1) = (0, 0.4)$ fm from ETMC-22[110], χQCD-22[113], RBC/UKQCD-23[111] and CLS/Mainz-24 [115]. The error in parenthesis are in the order: statistical, systematic and total. Right panel: quality of continuum extrapolation for the ℓ-quark contribution to $a_\mu^{\rm SD}$ in ETMC22, with data at three lattice spacings and two different valence quark regularizations. Tree level perturbative cutoff effects on lattice correlators were subtracted from the non-perturbative data, in order to avoid dangerous $O(a^2 \log a)$ artifacts.

Todo list and points for discussions

- We assumed no correlation between systematics errors when two groups use different discretizations. However, not clear if significant correlations still exist due to common choices of scale-setting parameters (a small effect because of $|\frac{\Delta_a a_\mu^{\mathrm{W}}}{a_\mu^{\mathrm{W}}}| < |\frac{\Delta a}{a}|$?) or to similar treatment of FV uncertainties.
- Slightly different prescriptions often used to define the isospin-symmetric world. How do we cope with this issue? Some groups provide derivatives w.r.t. input parameters. According to RBC/UKQCD-23 effect expected to be small on a_μ^{W}:

> For the intermediate-distance window a_μ^{W} in the isospin-symmetric limit with $t_0 = 0.4$ fm, $t_1 = 1.0$ fm, and $\Delta = 0.15$ fm, we find the up and down quark-connected contribution to be
>
> $$a_\mu^{\mathrm{W,iso,conn,ud}} = 206.36(44)_{\mathrm{S}}(42)_{\mathrm{C}}(01)_{\mathrm{FV}}(00)_{m_\pi\ \mathrm{FV}}(08)_{\partial_m\ \mathrm{C}}(00)_{\mathrm{WF\ order}}(03)_{m_{\mathrm{res}}} \times 10^{-10} \quad (42)$$
>
> in the BMW20 world and
>
> $$a_\mu^{\mathrm{W,iso,conn,ud}} = 206.46(53)_{\mathrm{S}}(43)_{\mathrm{C}}(01)_{\mathrm{FV}}(01)_{m_\pi\ \mathrm{FV}}(09)_{\partial_m\ \mathrm{C}}(00)_{\mathrm{WF\ order}}(03)_{m_{\mathrm{res}}} \times 10^{-10} \quad (43)$$

- Some content may be moved to other sections: e.g. mention of smeared R?
- Should the result of BMW-20 for isosymm. a_μ^{HVP} be discussed in this section?



Thank you for the attention!

Method for averages taken from FLAG

- Estimate $x_i \pm \sigma_i$ from group $i \in [1, M]$ weighted by

$$\omega_i = \frac{\sigma_i^{-2}}{\sum_{j=1}^{M} \sigma_j^{-2}}$$

- We then build covariance matrix C_{ij}

$$C_{ii} = \sigma_i^2, \qquad C_{ij} = \sigma_{i;j}\sigma_{j;i} \qquad i \neq j$$

- $\sigma_{i;j}$ is defined as

$$\sigma_{i;j} = \sqrt{\sum_{\alpha} [\sigma_i^{(\alpha)}]^2}$$

  where α runs over all sources of errors on x_i that are correlated with those on x_j.

- Final central value and error obtained using:

$$\bar{x} = \sum_i \omega_i x_i, \qquad \bar{\sigma}^2 = \sum_i \sum_j \omega_i \omega_j C_{ij}$$