(image source: https://www.quantamagazine.org/last-hope-experiment-finds-evidence-for-unknown-particles-20210407/)

Lattice HVP: methodology

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$(g\text{-}2)_\mu$ TI Virtual Meeting

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Outline

Goal: coordinate methodology for presentation of lattice HVP results, including combinations.

1. Common methods: model averaging/blinding

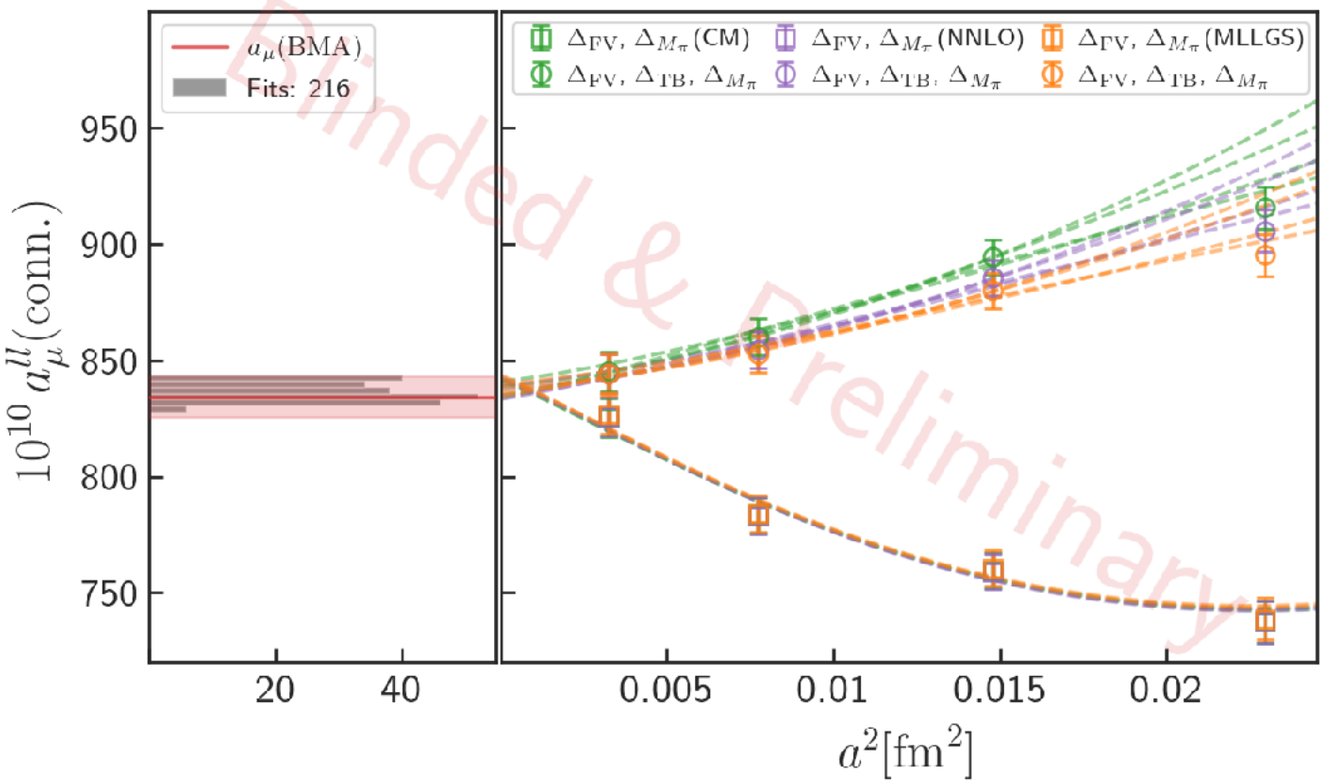
2. Scale setting and mass tuning

3. Combination procedure and example

Methods: model averaging/blinding

- **Model averaging** uses fit results weighted by model probability ($p \sim \exp(-AIC/2)$, $AIC = \chi^2 + 2k$) to estimate systematics due to model choice. AIC-based MA pioneered by BMW in lattice calculations and in (g-2) HVP; now widely used in lattice HVP.

(plot from S. Lahert, Fermilab/MILC/HPQCD preliminary)



- **Blinding** is now also common practice, following the example of the Fermilab (g-2) experiment. Unknown additive/multiplicative offsets are added to data in analysis pipeline; removed once analysis choices are finalized.

- Plot to the left shows both in action!

Scale setting

- Lattice produces dimensionless numbers: need to fix quark masses (next slide) and overall energy scale to get physics.

- Ω^- baryon as most common choice: M_{Ω^-} =1672.45(29) MeV, 0.02% experimental error (!), precise from lattice since only strange quarks. (QED effects matter at some point; <0.2% shift.) f_π another common precise choice.

- w_0 as a useful pure-glue intermediate quantity: calculate w_0/a on each ensemble, use $w_0 M_{\Omega^-}$ to fix w_0 (fm).

- This is just review again: I don't think any common choice or prescription is needed here…

Mass tuning

- Portelli prescription from TI meeting, Edinburgh ’22:

Pure QCD	*Iso-symmetric QCD*
$\hat{M}_{\pi^+} = 135.0$ MeV	$\bar{M}_{\pi} = 135.0$ MeV
$\hat{M}_{K^+} = 491.6$ MeV	$\bar{M}_{K} = 494.6$ MeV
$\hat{M}_{K^0} = 497.6$ MeV	$\bar{M}_{D_s} = 1967$ MeV
$\hat{M}_{D_s} = 1967$ MeV	
$\hat{M}_{\Omega^-} = \bar{M}_{\Omega^-} = 1672.45$ MeV	

(A. Portelli, https://indico.ph.ed.ac.uk/event/112/contributions/1663/attachments/999/1466/portelli.pdf)

- **Q:** Do we want to adopt this scheme for the whitepaper?

- **Q:** In principle, calculations with other prescriptions need to be corrected (see e.g. RBC/UKQCD ’23 where explicit results in two schemes are given.) Should this be included in combinations? Should we give required derivatives?

Combination method

- Following FLAG methodology (FLAG '21, 2.3.) Given: $\{(x_i, \sigma_i^{st}, \sigma_i^{sys})\}$

1. Compute total errors and weights ω_i:

$$\sigma_i = \sqrt{(\sigma_i^{st})^2 + (\sigma_i^{sys})^2} \qquad \omega_i = \frac{\sigma_i^{-2}}{\sum_{i=1}^{M} \sigma_i^{-2}}$$

2. Compute covariance, where errors are correlated (take 100%):

$$\sigma_{i;j} = \sqrt{\sum_{\alpha} [\sigma_i^{(\alpha)}]^2}$$

with **α** running over *correlated* errors.

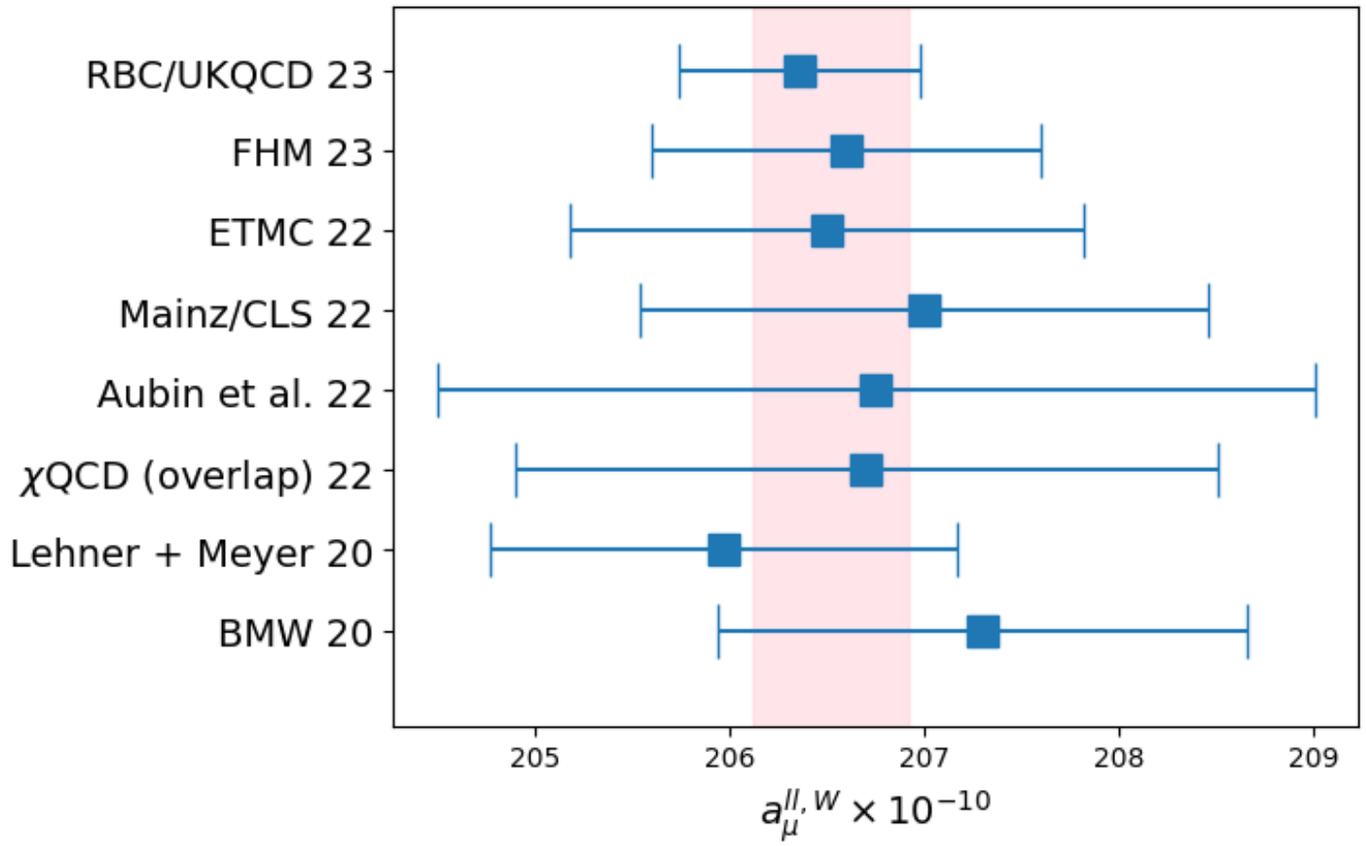
$$C_{ij} = \begin{cases} \sigma_i^2, & i = j; \\ \sigma_{i;j}\sigma_{j;i}, & i \neq j. \end{cases}$$

3. Weighted average:

$$\bar{x} = \sum_i \omega_i x_i \qquad \bar{\sigma}^2 = \sum_{i,j} \omega_i \omega_j C_{ij}$$

- **Q:** Policy for new results superseding old? (Just "newest result from each group" seems simplest.)

Combination example: window HVP



- **100% stat correlation** between Aubin ’22 and FHM ’23 taken (both MILC gauge configs). No scheme/“world” corrections; variations between groups sort of average over this above. Python code available on request. **Q:** include χQCD/RBC correlation?

- Result: $a_\mu^{ll,W}$=206.52(40) (0.2% rel. unc.)

- If I assume 100% correlation between all systematics instead, then I get 206.52(80); 0.4% rel. unc. (Too conservative?)

Some questions:

- Should we adopt and state an explicit convention for mass tuning in pure/iso-sym QCD?
- Should we worry about mass-scheme corrections in averages explicitly? Or let variation between groups deal with it in averages?
- How much correlation between systematics to assume? Getting too granular may be difficult since different groups break errors up in different ways…
- Policy for superseded results: just adopt newest for each group/quantity? Show older calculations not used in averages?

Thank you!