HVP cross-checks

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PRiSMA+

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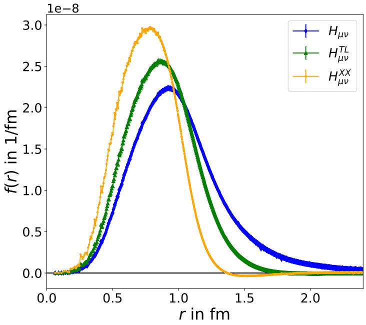
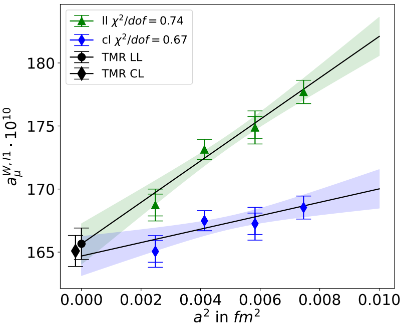
The intermediate window from the CCS representation



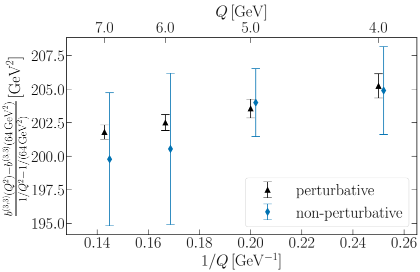


- calculation at $m_\pi = 350\,\text{MeV}$, $m_K = 450\,\text{MeV}$ with continuum extrapolation
- finite-volume effects corrected for with the Sakurai field-theoretic model $(\gamma, \rho, \pi^\pm)$; they are comparable in size to those in the TMR method.
- Test of Lorentz-symmetry restoration. Isovector contribution from two local j_μ:

$$\text{CCS}: \; a_\mu^{\text{win,I1}} = 165.17(157)_{\text{stat}}(99)_{\text{syst}} \qquad \text{TMR}: \; a_\mu^{\text{win,I1}} = 165.66(125)_{\text{stat}}$$

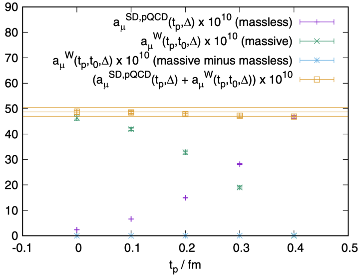
Chao, Parrino, HM 2211.15581 (PRD); TMR result: derived by S. Kuberski from Mainz-CLS 2206.06582 (PRD).

Short-distance contributions: lattice vs. perturbation theory



$$b(Q^2) \propto \Pi(Q^2) - \Pi(Q^2/4).$$

The quantity plotted would be constant in a scale-invariant theory; broken at O(α_s^2) in QCD.



Stability plot for handling the shortest distances in O(α_s^4) perturbation theory

At short distances, lattice QCD results are consistent with five-loop massless perturbation theory!

Figs. from Mainz-CLS 2401.11895 (PRD) and RBC/UKQCD 2301.08696 (PRD).

A check on theory-based volume corrections

Let $G_{\mathrm{TMR}}^{(lc)}(t)$ be the local-conserved discretisation of the TMR correlator.

On the infinite lattice, the following property holds exactly:

$$a \sum_{t=-\infty}^{\infty} G_{\mathrm{TMR}}^{(lc)}(t) = 0.$$

Therefore, there is a constraint on the finite-volume correction we apply (Hansen-Patella, chiral perturbation theory, ...):

$$a \sum_{t=-\infty}^{\infty} \underbrace{(G_{\mathrm{TMR}}^{(lc)}(t) - G_{\mathrm{TMR}}^{(lc)}(L,t))}_{=\text{ finite-vol. correction}} = -a \sum_{t=-\infty}^{\infty} \underbrace{G_{\mathrm{TMR}}^{(lc)}(L,t)}_{=\text{lattice data}} + \mathrm{O}(a^2).$$

Mostly a check on the correction applied at smallish $|t|$.

Testing a modification of the (pre-CMD3) R ratio compilation

$\sqrt{s}$ interval	$a_\mu^{\rm hvp}$	$(a_\mu^{\rm hvp})^{\rm SD}$	$(a_\mu^{\rm hvp})^{\rm ID}$	$(a_\mu^{\rm hvp})^{\rm LD}$	$\overline{\Pi}(1{\rm GeV}^2)$
below 0.6 GeV	15.5	1.5	5.5	23.5	8.2
0.6 to 0.9 GeV	58.3	23.1	54.9	65.4	52.6
above 0.9 GeV	26.2	75.4	39.6	11.1	39.2
Total	100.0	100.0	100.0	100.0	100.0

- an underestimate by an overall factor of $0.94(1)$ of the experimental R ratio [Colangelo et al 2205.12963] in the interval 600-900 MeV would explain
  - the WP'20 vs. FNAL a_μ anomaly
  - the dispersive vs. lattice $(a_\mu^{\rm hvp})^{\rm ID}$ anomaly.
- such an underestimate would still lead to an underestimate by a $1.4(4)\%$ of the short-distance window; the ETMC and Mainz-CLS results for $(a_\mu^{\rm hvp})^{\rm SD}$ are also consistent with this scenario.

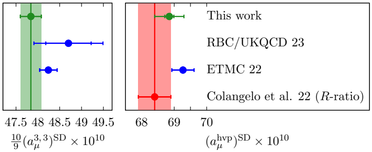


Table from Mainz-CLS 2206.06582 based on R from 1107.4388; Fig. from 2401.11895 (PRD).

Further challenging the scenario of the previous slide

$\sqrt{s}$ interval	a_μ^{hvp}	[1.0, 1.6]fm	[1.5, 1.9]fm
below 0.6 GeV	15.5	13.0	21.1
0.6 to 0.9 GeV	58.3	70.5	70.7
above 0.9 GeV	26.2	16.5	8.2
Total	100.0	100.0	100.0

- If the anomalies comes exclusively from the $\sqrt{s}$ interval 0.6–0.9GeV, expect 4.2(7)% deviation between lattice and dispersive determination of the window from 1.0 to 1.6 fm.
- That window receives 82% of its size from the $\pi\pi$ channel at $\sqrt{s} \leq 1$ GeV, and its size of 215.5 is very similar to the [0.4,1.0]fm window. [Colangelo et al 2205.12963]
- The window from 1.5 to 1.9 fm has been proposed in [Aubin et al 2204.12256].

The width of the smooth step-function is $\Delta = 0.15$fm in all cases above.

Isovector contribution: lattice vs. isospin separation of R

- the isospin separation performed in [Benton et al 2311.09523] finds a strong lattice vs. dispersive tension in the intermediate window in the light-quark connected sector; no evidence for tension in the complementary term.

- If $\frac{R^{(a,b)}}{12\pi^2}$ is the spectral function for correlator $\langle\bar\psi(x)\frac{\lambda^a}{2}\gamma_\mu\psi(x)\;\bar\psi(0)\frac{\lambda^b}{2}\gamma_\nu\psi(0)\rangle$, so that $R(s) = R^{(3,3)}(s) + \frac{1}{3}R^{(8,8)}(s) + \dots$, in pure QCD the (slowly convergent) sum rule

$$\int_0^\infty ds\,\Big(R^{33}(s) - R^{88}(s)\Big) = 0.$$

should hold [See Narison, de Rafael NPB169 (1980) 253]. Is this a useful check?