Short-distance in the Melnikov-Vainshtein corner

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JHEP 02 (2023) 167

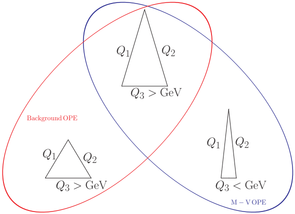
In progress

Data-driven HLbL: OPEs in a multi-scale problem

- HLbL involves the interplay of several regimes

$$a_\mu^{\mathrm{HLbL}} \sim \int_0^\infty dQ_{1,2,3}^{\lambda<0} \sum_i T_i'(m_\mu, Q_i)\, \overline{\Pi}_i(Q_i) \sim \sum_{\triangle} T_i''(m_\mu, \triangle) \cdot \overline{\Pi}_i(\triangle)$$

- Regions with large Q_i harder to address with nonperturbative methods
- OPEs can be applied in those regions ($Q_{1,2} \gtrsim \mathrm{GeV}$ and permutations)



- Background OPE Phys.Lett.B 798 134994, JHEP 10 (2020) 203, JHEP 04 (2021) 240
- Melnikov-Vainshtein (M-V) OPE limit Phys.Rev.D 70 (2004) 113006

Melnikov-Vainshtein OPE: aspects to consider

- $Q_3^2 = Q_1^2 + Q_2^2 + 2\tau Q_1 Q_2$ JHEP 04 (2017) 161

$$a_\mu^{\mathrm{HLbL}} = \frac{2\alpha^3}{3\pi^2} \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \sqrt{1-\tau^2}\, Q_1^3 Q_2^3 \sum_{i=1}^{12} T_i(Q_1, Q_2, \tau)\, \overline{\Pi}_i(Q_1, Q_2, \tau)$$

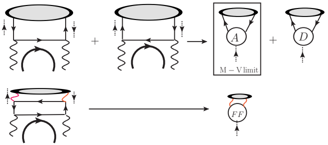
- M-V limit, $Q_1, Q_2 \gg \Lambda_{\mathrm{QCD}}, Q_3$

Questions we are trying to address

1. How the expansion works beyond M-V limit? What $\overline{\Pi}$ can we obtain?
2. If $Q_3 \gg \Lambda_{\mathrm{QCD}}$, can one recover background OPE?
3. Is leading nonzero M-V term for a $\bar{\Pi} \sim \hat{\Pi}$ (plus background OPE) enough to assess it in the whole $Q_{1,2} \gtrsim \mathrm{GeV}$ regions?
4. In the integrand, are there enhanced $\overline{\Pi}$ in this regime?
5. What can we do for the nonperturbative matrix elements?

How the expansion works beyond M-V limit? What $\overline{\Pi}$ can we obtain?

$$\underbrace{\Pi^{\mu_1\mu_2\mu_3\mu_4}}_{\text{dim}=2} = \sum_{j,k} \frac{ie_{q_j}e_{q_k}}{e^2} \int \frac{d^4q_4}{(2\pi)^4} \int d^4x_1 \int d^4x_2\, e^{-i(q_1x_1+q_2x_2)}$$
$$\times\ \langle 0|T(J_j^{\mu_1}(x_1)J_k^{\mu_2}(x_2))|\gamma^{\mu_3}(q_3)\gamma^{\mu_4}(q_4)\rangle$$
$$= \sum \underbrace{C_{i,D}^{\mu_1\mu_2,\cdots}(\hat{q})}_{\text{dim}=2-D} \langle 0|\mathcal{O}_{i,D,\ldots}|\gamma^{\mu_3}(q_3)\gamma^{\mu_4}(q_4)\rangle$$

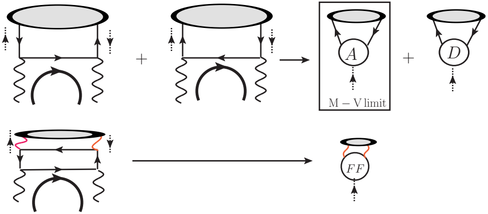


- $\overline{\Pi}_i = P_{i,\mu_1\cdots}(q_3,\hat{q})\Pi^{\mu_1,\cdots}$, $2\hat{q} = q_1 - q_2$
- Euclidean variables: $\overline{Q}_3 = Q_1 + Q_2 \approx 2\sqrt{-\hat{q}^2}$, $\delta_{12} = Q_1 - Q_2$, $Q_3 > |\delta_{12}|$
- Expansion in powers of $1/\overline{Q}_3$
- Keeping D gives (JHEP 12 (2023) 129, $D = 3$ in agreement with JHEP 03 (2020) 101)

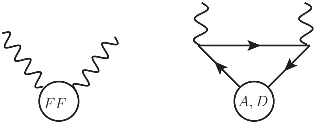
$$\hat{\Pi}_1 \to \frac{1}{\overline{Q}_3^{D-1}}\,, \quad \hat{\Pi}_4 \to \frac{1}{\overline{Q}_3^{D}}\,, \quad \hat{\Pi}_7 \to \frac{1}{\overline{Q}_3^{D+1}}\,, \quad \hat{\Pi}_{17} \to \frac{1}{\overline{Q}_3^{D}}\,, \cdots$$

If $Q_3 \gg \Lambda_{\mathrm{QCD}}$, can one recover background OPE? Leading

Up to $D = 4$ we have $\bar{q}\Gamma q$, $F \cdot F$ and $G \cdot G$ like operators

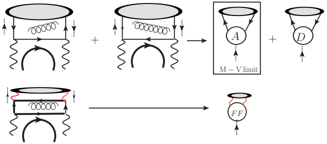


When $Q_3 \gg \Lambda_{\mathrm{QCD}}$ one can also compute these matrix elements perturbatively

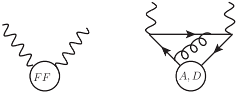


Fully matches quark loop from background OPE JHEP 02 (2023) 167

If $Q_3 \gg \Lambda_{\rm QCD}$, can one recover background OPE? Gluons



$Q_3 \gg \Lambda_{\rm QCD}$



- $D = 3$:

$$\gamma_{\mu_1}\gamma_\alpha\gamma_{\mu_2} - \gamma_{\mu_2}\gamma_\alpha\gamma_{\mu_1}$$

Agree with EPJC 80 (2020) 12, 1108

- $D = 4$:

$$\gamma_\beta \left[\overrightarrow{D}_\alpha - \overleftarrow{D}_\alpha\right]$$

$$(\gamma_\alpha\gamma_\beta\gamma_\gamma + \gamma_\gamma\gamma_\beta\gamma_\alpha)\left[\overrightarrow{D}^\gamma + \overleftarrow{D}^\gamma\right]$$

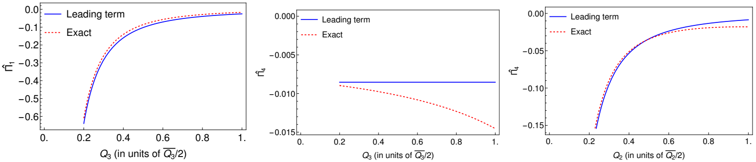
$$F_{\alpha\beta}F_{\gamma\delta}$$

Fully matches gluonic from background OPE in JHEP 04 (2021) 240, To appear

Is leading M-V term for a $\bar{\Pi} \sim \hat{\Pi}$ enough for $Q_{1,2} > Q_{\min} \approx \mathrm{GeV}$?

- Euclidean variables: $\overline{Q}_3 = Q_1 + Q_2$, $\delta_{12} = Q_1 - Q_2$, $Q_3 > |\delta_{12}|$
- Reasonable description up to near $Q_3 \approx \overline{Q}_3/2$ (symmetric point)?
- If it works at $Q_3 > Q_{\min}$ (quark loop), one expects it will also work below

Preliminary



- Good enough, given kinematic suppression

In the integrand, are there enhanced $\overline{\Pi}$ in this regime?

- Euclidean variables: $\overline{Q}_3 = Q_1 + Q_2$, $y = \frac{Q_1 - Q_2}{Q_3}$, Q_3

$$a_\mu^{\mathrm{HLbL}} \propto \int_0^\infty d\overline{Q}_3 \int_0^{\overline{Q}_3} dQ_3 \int_{-1}^1 dy \, \frac{1}{16} \left(\overline{Q}_3{}^2 - y^2 Q_3^2 \right) Q_3^3 \sqrt{\overline{Q}_3{}^2 - Q_3^2} \sqrt{1 - y^2} \sum_{i=1}^{12} T_i \, \overline{\Pi}_i$$

- Know at which $\overline{Q}_a$ order enters each $\overline{\Pi}_i$, so expand also the T_i.

Preliminary, in progress

- In principle first $\sum_i T_i \, \overline{\Pi}_i$ go as $1/\overline{Q}_a^5$, but appears to be odd in y
- First nonzero contributions of $\sum_i T_i \, \overline{\Pi}_i$ to the integral would be $\sim 1/\overline{Q}_a^6$

What can we do for the nonperturbative matrix elements? Preliminary

- $D = 3 \to$ axial current
- At $D = 4$ we have (similar for the singlet, but also some gluonic operators)

$$\frac{i\sum_j e_{q,j}}{3e^2} \lim_{q_4\to 0} \partial_{q_4}^{\nu_4} \left\langle \bar{q}_j(0) \left[\overrightarrow{D}^{\alpha} - \overleftarrow{D}^{\alpha}\right] \gamma^{\beta} q_j(0) \middle| \gamma^{\mu_3}(q_3)\gamma^{\mu_4}(q_4) \right\rangle = \sum_{i=1}^{6} \omega_{(8)}^{D,i}\, L_i^{\alpha\beta\mu_3\mu_4\nu_4}$$

- Separate the operator into irreps under Lorentz group.

$$\begin{aligned}
\omega_D^1 &= \omega_{D,S}^1 + \omega_{D,\delta}^1 = \omega_{D,S}^1\,, \\
\omega_D^2 &= \omega_{D,S}^2 + \omega_{D,A}^2 = \omega_{D,S}^2 - \frac{Q_i^2\omega_T}{8\pi^2}\,, \\
\omega_D^3 &= \omega_{D,S}^2 - \omega_{D,A}^2 = \omega_{D,S}^2 + \frac{Q_i^2\omega_T}{8\pi^2}\,, \\
\omega_D^4 &= \omega_{D,S}^4 + \omega_{D,A}^4 = \omega_{D,S}^4\,, \\
\omega_D^5 &= \omega_{D,S}^4 - \omega_{D,A}^4 = \omega_{D,S}^4\,, \\
\omega_D^6 &= \omega_{D,S}^6 = -d\omega_{D,S}^1 - \omega_{D,S}^2 - \omega_{D,S}^3 + \omega_{D,S}^4 + \omega_{D,S}^5 = -d\omega_{D,S}^1 - 2\omega_{D,S}^2 + 2\omega_{D,S}^4\,.
\end{aligned}$$

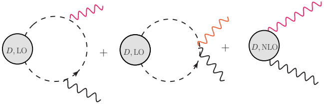
- Only symmetric traceless part of the tensor brings new form factors. $\overline{\Pi}$ simple functions of them

What can we do for the nonperturbative matrix elements? Preliminary

- One can obtain the low-energy/chiral form of the matrix elements

$$\langle 0| \left.\frac{\delta S_\chi[v_\mu = -eQA_\mu, \mathcal{J}_\mathcal{O}]}{\delta \mathcal{J}_\mathcal{O}}\right|_{\mathcal{J}_\mathcal{O}=0} e^{iS_\chi[v_\mu=-eQA_\mu, \mathcal{J}_\mathcal{O}=0]} \,|\gamma^{\mu_3}(q_3)\gamma^{\mu_4}(q_4)\rangle$$

- $D=3$ at leading order one easily recovers the correct ω_L
- $D=4$. $\chi\mathrm{pT}$ with the new external source. Leading nonzero contributions (chiral limit) Preliminary, in progress



$$\omega^1_{D,S} = \omega^4_{D,S} = \frac{1}{9\pi^2}\left[-2\pi^2 c_{\mathrm{NLO}}(\nu_\chi^2 = \mu^2) + \frac{c_L^S}{2}\ln\frac{Q_3}{\mu} - \frac{1}{6} - \frac{5}{12}c_L^S\right],$$

$$\omega^2_{D,S} = -\frac{1}{9\pi^2}\left[-2\pi^2 c_{\mathrm{NLO}}(\nu_\chi^2 = \mu^2) + \frac{c_L^S}{2}\ln\frac{Q_3}{\mu} - \frac{1}{6} - \frac{1}{6}c_L^S\right].$$

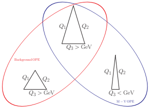
One may match SD at some fixed $Q_3 \sim \Lambda_\chi$. Instead one may interpolate by adding resonances

Conclusions

- HLbL involves the interplay of several regimes

$$a_\mu^{\rm HLbL} \sim \int_{0,\lambda<0}^{\infty} dQ_{1,2,3} \sum_i T_i'(m_\mu, Q_i)\, \overline{\Pi}_i(Q_i) \sim \sum_{\triangle} T_i''(m_\mu, \triangle) \cdot \overline{\Pi}_i(\triangle)$$

- Large momenta harder to address with nonperturbative methods: use OPEs



- Some progress in a few points regarding M-V OPE

  1. How the expansion works beyond M-V limit? What $\overline{\Pi}$ can we obtain?
  2. If $Q_3 \gg \Lambda_{\rm QCD}$, can one recover background OPE?
  3. Is leading nonzero M-V term for a $\bar{\Pi} \sim \hat{\Pi}$ (plus background OPE) enough to assess it in the whole $Q_{1,2} \gtrsim \mathrm{GeV}$ regions?
  4. In the integrand, are there enhanced $\overline{\Pi}$ in this regime?
  5. What can we do for the nonperturbative matrix elements?

- So far no reason to expect any new unexpectedly large contribution