Axial-vector and tensor contributions in four- and three-point dispersive approaches

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in collaboration with M. Hoferichter and **M. Zillinger**

2402.14060 [hep-ph], to appear in JHEP,

with **J. Lüdtke** and M. Procura

JHEP **04** (2023) 125 and work in progress

and with **N. Geralis**, **E. Kaziukėnas**, and **J.-N. Toelstede**

work in progress

Muon $g-2$ Theory Initiative Spring 2024 meeting

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University of Zurich[UZH]

PAUL SCHERRER INSTITUT

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Outline

Analytic HLbL: main issues

- uncertainties dominated by parts that are not (yet) incorporated in dispersive framework
- required input for axial-vector & tensor **transition form factors** (TFFs)
- kinematic singularities & ambiguities for narrow resonances
- matching to short-distance constraints (SDCs)
  - cover everything that is not explicitly included as hadronic intermediate state
  - avoid double counting

White Paper estimate

→ T. Aoyama *et al.*, Phys. Rept. **887** (2020) 1-166

	$10^{11} \times a_\mu$	$10^{11} \times \Delta a_\mu$
π^0, η, η'-poles	93.8	4.0
pion/kaon box	-16.4	0.2
S-wave $\pi\pi$ rescattering	-8	1
scalars, tensors	-1	3
axials	6	6
light quarks, short distance	15	10
c-loop	3	1
HLbL total (LO)	92	19

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Some of the progress after White Paper

- scalar contributions in dispersive framework

  → Danilkin, Hoferichter, Stoffer, PLB **820** (2021) 136502

- first steps towards including axials in dispersive framework

  → Zanke, Hoferichter, Kubis, JHEP **07** (2021) 106; JHEP **08** (2023) 209, Colangelo, Hagelstein, Hoferichter, Laub, Stoffer, EPJC **81** (2021) 702

- holographic-QCD models point to rather large axial contribution

  → **talk by A. Rebhan**

- beyond spin 1: new dispersive framework in soft-photon kinematic limit

  → Lüdtke, Procura, Stoffer, JHEP **04** (2023) 125

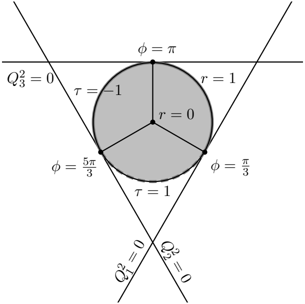
Kinematic singularities

- HLbL coefficient functions $\check{\Pi}_i$ free from kinematic singularities in Mandelstam variables $\Rightarrow$ enables dispersive treatment $\rightarrow$ Colangelo, Hoferichter, Procura, Stoffer, JHEP **04** (2017) 161
- not free from kinematic singularities in q_i^2, but **residues vanish** due to sum rules
- kinematic singularities can be subtracted, but introduce **ambiguities** if sum rules are violated
- narrow resonances (apart from pseudoscalars) do not fulfill sum rules individually

Optimized basis for resonances

→ Hoferichter, Stoffer, Zillinger, arXiv:2402.14060 [hep-ph] (to appear in JHEP)

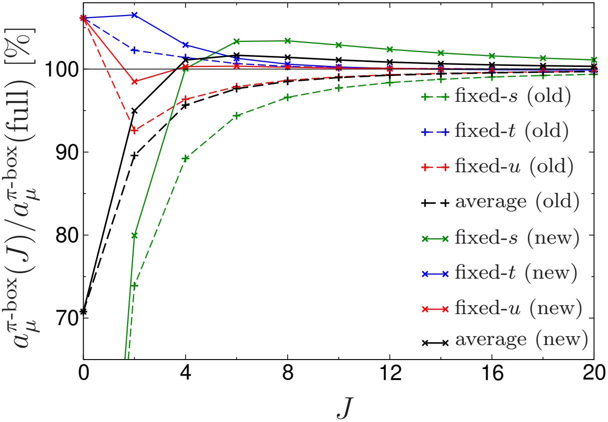
- **new basis** constructed **without singularities** for pseudoscalars, scalars, S-wave rescattering, axial-vectors
- remaining singularities **much simplified**: only $1/q_i^2$ poles appear (and $1/(q_i^2+q_j^2)$, outside $g-2$ integration region)



Optimized basis for resonances

→ Hoferichter, Stoffer, Zillinger, arXiv:2402.14060 [hep-ph] (to appear in JHEP)

- convergence of **partial-wave expansion** checked in new basis for pion box: found even slight improvement



Axial vectors in optimized basis

→ Hoferichter, Stoffer, Zillinger, arXiv:2402.14060 [hep-ph] (to appear in JHEP)
→ Colangelo, Hagelstein, Hoferichter, Laub, Stoffer, EPJC **81** (2021) 702

- **axial-vector poles** in **transverse part** of HLbL

  → Lüdtke, Procura, Stoffer, JHEP **04** (2023) 125

- **longitudinal part**: axial-vector pole in Mandelstam variable s cancels with numerator in $g-2$ limit $s \to q_3^2$, but leaves **non-pole contribution**

$$\bar{\Pi}_1^{\text{axial}} = \frac{G_2(q_1^2, q_2^2) G_1(q_3^2)}{M_A^6} \,,$$

$$G_1(q_3^2) = \mathcal{F}_1(q_3^2, 0) + \mathcal{F}_2(q_3^2, 0) \,,$$

$$G_2(q_1^2, q_2^2) = (q_1^2 - q_2^2)\mathcal{F}_1(q_1^2, q_2^2) + q_1^2 \mathcal{F}_2(q_1^2, q_2^2) + q_2^2 \mathcal{F}_2(q_2^2, q_1^2)$$

Axial vectors: TFF input

- **asymptotic constraints** on TFFs from light-cone expansion → Hoferichter, Stoffer, JHEP **05** (2020) 159
- f_1 TFFs: experimental constraints analyzed in a VMD representation

  → Zanke, Hoferichter, Kubis, JHEP **07** (2021) 106; JHEP **08** (2023) 209
- f_1' and a_1 TFFs could be related via $U(3)$ symmetry
- holographic-QCD models can provide useful input

  → **talk by A. Rebhan**

Axial vectors: TFF input

- with a given input for the axial-vector TFFs, we are now in a position to compute a_μ^{axials} in the established **four-point dispersive approach**
- numerical analysis in progress: **interplay with SDCs** is essential

Tensor mesons in optimized basis

→ Hoferichter, Stoffer, Zillinger, arXiv:2402.14060 [hep-ph] (to appear in JHEP)

- kinematic singularities **much simplified**: e.g., no singularities if only $\mathcal{F}^T_{1,3}$ or only $\mathcal{F}^T_{2,3}$ are present
- enables simple benchmark evaluation, e.g., with $\mathcal{F}^T_1$ from quark-model ($\mathcal{F}^T_{2,3,4,5} = 0$)
- even then: sum-rule violations lead to **basis dependence**

Tensor mesons in optimized basis

- tensor-meson contribution including all TFFs (and $\pi\pi$ D-wave contribution) affected by **kinematic singularities**
- for spin > 1, problem **cannot be solved** by basis change as for axials
- requires **new dispersive framework** in tree-point kinematics

  $\rightarrow$ Lüdtke, Procura, Stoffer, JHEP **04** (2023) 125

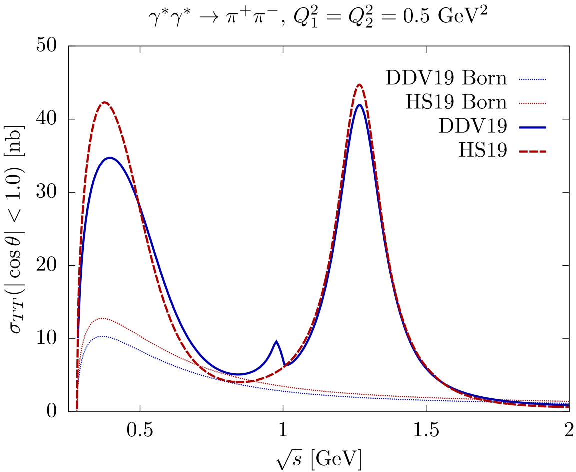
Input for tensor mesons

- **asymptotic constraints** on TFFs from light-cone expansion → Hoferichter, Stoffer, JHEP **05** (2020) 159
- similarity to $f_0(980)$ and S-waves: $f_2(1270)$ contribution should be compared in NWA and via $\pi\pi$ **rescattering**
- $\gamma^*\gamma^* \to \pi\pi$ helicity partial waves solved with Omnès methods including D-waves

  → Hoferichter, Stoffer, JHEP **07** (2019) 073

  → Danilkin, Deineka, Vanderhaeghen, PRD **101** (5) (2020) 054008
- future $\gamma^*\gamma \to \pi\pi$ single-tag measurements at BESIII will be useful to constrain q^2 dependence

Input for tensor mesons



→ T. Aoyama *et al.*, Phys. Rept. **887** (2020) 1-166

Master formula: HLbL contribution to $(g-2)_\mu$

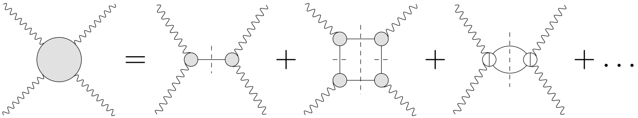
→ Colangelo, Hoferichter, Procura, Stoffer, JHEP **09** (2015) 074, JHEP **04** (2017) 161

$$a_\mu^{\mathrm{HLbL}} = \frac{2\alpha^3}{3\pi^2} \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \sqrt{1-\tau^2} Q_1^3 Q_2^3 \times \sum_{i=1}^{12} T_i(Q_1, Q_2, \tau) \bar{\Pi}_i(Q_1, Q_2, \tau)$$

- T_i: known integration kernels
- $\bar{\Pi}_i$: hadronic scalar functions
- Euclidean momenta: $Q_i^2 = -q_i^2$
- $Q_3^2 = Q_1^2 + Q_2^2 + 2Q_1Q_2\tau$

DR in four-point kinematics

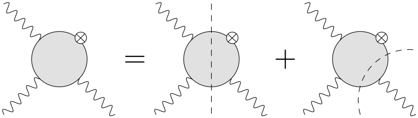
→ Colangelo, Hoferichter, Procura, Stoffer, JHEP **09** (2015) 074, JHEP **04** (2017) 161



- first write DR in four-point kinematics
- take $q_4 \to 0$ limit **in the very end**

DR in triangle kinematics

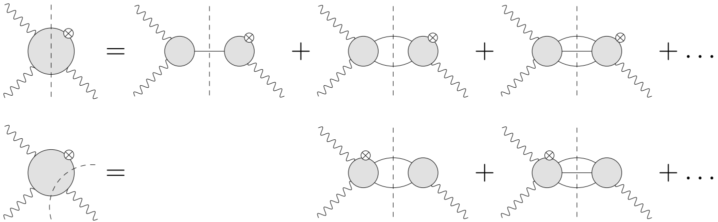
→ Lüdtke, Procura, Stoffer, JHEP **04** (2023) 125



- external photon at $q_4 \to 0$
- imaginary parts reconstructed for $\boldsymbol{g-2}$ **kinematics**

DR in triangle kinematics

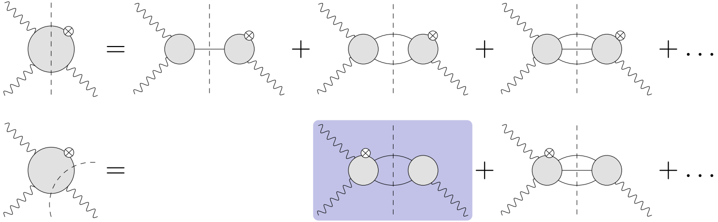
→ Lüdtke, Procura, Stoffer, JHEP **04** (2023) 125



- more complicated unitarity relation, more sub-processes
- redundancies and kinematic singularities **manifestly absent**
- combination of two dispersive approaches: assess **truncation errors**
- potentially **simplified matching to SDCs**

DR in triangle kinematics

→ Lüdtke, Procura, Stoffer, JHEP **04** (2023) 125



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- potentially **simplified matching to SDCs**

More sub-processes

- cancellation of soft divergences: solved for $\pi\pi \to \pi\pi\gamma$

  → Lüdtke, Procura, Stoffer, in preparation

- test case: understand reshuffling and truncation effects in $\gamma^*\gamma \to \pi\pi$

  → Geralis, Kaziukėnas, Stoffer, Toelstede, work in progress

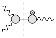
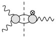
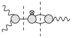
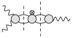
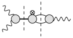
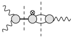
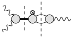
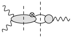
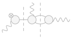
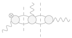
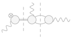
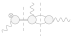
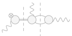
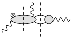
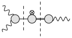
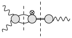
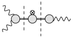
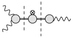
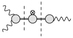
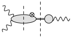
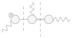
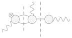
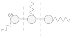
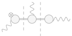
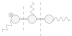
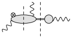
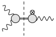
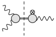
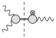
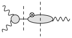
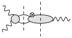
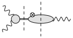
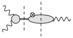
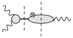
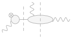
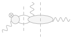
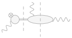
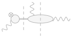
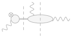
- apply same methods to $\gamma^*\gamma^*\gamma \to \pi\pi$

  → Lüdtke, Procura, Stoffer, JHEP **04** (2023) 125

  → Geralis, Kaziukėnas, Stoffer, Toelstede, work in progress

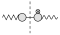
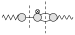
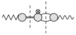
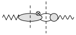
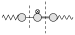
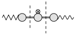
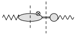
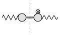
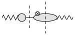
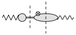
Reshuffling between two dispersive approaches

→ Lüdtke, Procura, Stoffer, JHEP **04** (2023) 125

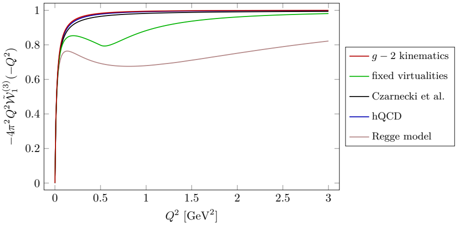
triangle-DR	DR in four-point kinematics					
	π^0, η, η'	2π	S	A	T	…
π^0, η, η'		×	×	×	×	×
2π	×		×	×	×	×
						
						
V						
						
S	×	×		×	×	×
A	×	×	×		×	×
T	×	×	×	×		×
…						…
						

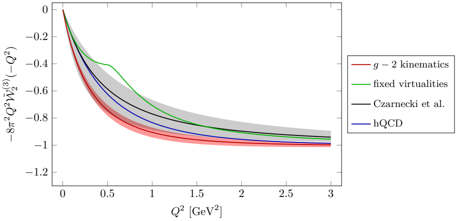
Proof of concept: VVA

→ Lüdtke, Procura, Stoffer, to appear

	DR for fixed photon virtualities		
$g-2$ DR	π^0, η, η'	A	$\dots$
π^0, η, η'		×	×
2π			
V			
A	×		×
$\dots$			$\dots$

- reshuffling much easier to understand in VVA
- side-product: improved prediction for EW contribution to a_μ





Conclusions

- conceptual obstacles for inclusion of **axial vectors** in NWA in dispersive framework resolved
- given data situation and asymptotic constraints, prospects best for a phenomenologically driven determination of $f_1(1285)$ contribution
- **tensor mesons**: compare NWA with **$\pi\pi$ rescattering**: $\gamma^*\gamma^* \to \pi\pi$ D-waves solved with Omnès methods
- full tensor contributions, assessment of overall uncertainties due to truncation and matching to SDCs: use combination with **new dispersive framework**

Backup

Narrow resonances

- in the NW limit, imaginary part from unitarity relation reduces to **δ-function**:

$$\mathrm{Im}_s \Pi^{\mu\nu\lambda\sigma} = \pi\delta(s - M^2)\mathcal{M}^{\mu\nu}(p \to q_1, q_2)^* \mathcal{M}^{\lambda\sigma}(p \to -q_3, q_4)\,,$$

$$\mathcal{M}^{\mu\nu}(p \to q_1, q_2) = i \int d^4x e^{iq_1\cdot x} \langle 0|T\{j^\mu_{\mathrm{em}}(x) j^\nu_{\mathrm{em}}(0)\}|p\rangle$$

- project onto tensor decomposition for HLbL and plug into dispersion relation for scalar functions:

$$\check{\Pi}_i(s) = \frac{1}{\pi}\int ds' \frac{\mathrm{Im}\check{\Pi}_i(s')}{s' - s}$$

- δ-function, Cauchy kernel, and polarization sum combine to propagator-like structure
- dispersive result may differ from propagator models by non-pole terms

Narrow resonances

- decompose $\mathcal{M}^{\mu\nu}$ into Lorentz structures $\times$ **transition form factors** (TFFs)
- in the NWA, dispersive definition only involves on-shell meson $\Rightarrow$ **only physical TFFs** enter

Sum rules and basis (in)dependence

- HLbL tensor basis involves structures of **different mass dimension**
- scalar coefficient functions of higher-dimension structures asymptotically fall off faster
- implies **sum rules** for those coefficient functions:

$$0 = \frac{1}{\pi} \int ds' \, \mathrm{Im}\check{\Pi}_i(s')$$

- guarantees **basis independence** of entire HLbL

Sum rules and basis (in)dependence

- sum-rule contribution of single-particle state (resonance):

$$\mathrm{Im}\check{\Pi}_i(s') \sim \pi\delta(s'-M^2)\mathcal{F}(q_1^2,q_2^2)\mathcal{F}(q_3^2,0)$$

$$\Rightarrow \frac{1}{\pi}\int ds'\,\mathrm{Im}\check{\Pi}_i(s') \sim \mathcal{F}(q_1^2,q_2^2)\mathcal{F}(q_3^2,0) \neq 0$$

- sum rules **not fulfilled** by resonances
  $\Rightarrow$ NW contribution to HLbL is **basis dependent**
- basis dependence only needs to cancel in sum over intermediate states
- only pseudoscalars do not contribute to sum rules
  $\Rightarrow$ unambiguous

Dispersive evaluation of $f_0(980)$ contribution

$\rightarrow$ Danilkin, Hoferichter, Stoffer, PLB **820** (2021) 136502

- $\pi\pi$ rescattering previously limited to $f_0(500)$

  $\rightarrow$ Colangelo, Hoferichter, Procura, Stoffer, JHEP **04** (2017) 161,

  PRL **118** (2017) 232001

- extension up to ~ 1.3 GeV by using coupled-channel $\gamma^*\gamma^* \rightarrow \pi\pi/\bar{K}K$ S-waves for $I = 0$

  $\rightarrow$ Danilkin, Deineka, Vanderhaeghen, PRD **101** (2020) 054008

- covers $f_0(980)$, dispersive description of resonance in terms of $\pi\pi/\bar{K}K$ rescattering

Dispersive evaluation of $f_0(980)$ contribution

→ Danilkin, Hoferichter, Stoffer, PLB **820** (2021) 136502

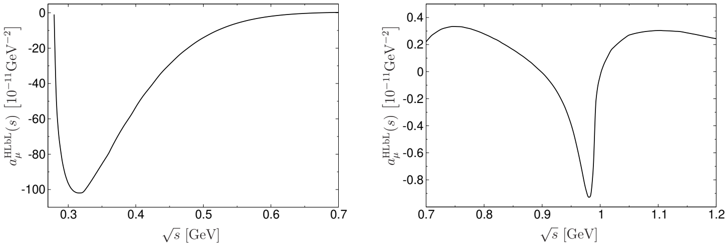
- sum-rule violations in S-wave rescattering are very small
- result largely **basis independent**
- together with $I = 2$ leads to

$$a_\mu^{\text{HLbL}}[S\text{-wave rescattering}] = -8.7(1.0) \times 10^{-11}$$

Dispersive evaluation of $f_0(980)$ contribution

→ Danilkin, Hoferichter, Stoffer, PLB **820** (2021) 136502

- dispersive $f_0(980)$ contribution estimated from deficit in shape of integrand:



$$a_\mu^{\text{HLbL}}[f_0(980)]_{\text{rescattering}} = -0.2(1) \times 10^{-11}$$

Dispersive evaluation of $f_0(980)$ contribution

$\rightarrow$ Danilkin, Hoferichter, Stoffer, PLB **820** (2021) 136502

- dispersive $f_0(980)$ contribution can be compared to **NWA in the same basis** for HLbL
- using TFFs from quark model $\rightarrow$ Schuler et al. (1998)

$$a_\mu^{\text{HLbL}}[f_0(980)]_{\text{NWA}} = -0.37(6) \times 10^{-11}$$

with $M_{f_0(980)} = 0.99$ GeV, $\Gamma_{\gamma\gamma}[f_0(980)] = 0.31(5)$ keV

- differences to NW estimates of $\rightarrow$ Knecht et al., PLB **787** (2018) 111 mainly due to propagator model, corresponding to a different HLbL basis
- comparison to $\rightarrow$ Pauk, Vanderhaeghen, EPJC **74** (2014) 3008 difficult due to kinematic singularities

Dispersive evaluation of $f_0(980)$ contribution

→ Danilkin, Hoferichter, Stoffer, PLB **820** (2021) 136502

- NWA for $a_0(980)$:

$$a_\mu^{\mathrm{HLbL}}[a_0(980)]_{\mathsf{NWA}} = -\left([0.4, 0.6]^{+0.2}_{-0.1}\right) \times 10^{-11},$$

  where TFF scale is given by $[M_\rho, M_S]$

- leads to

$$a_\mu^{\mathrm{HLbL}}[\mathsf{scalars}] = -9(1) \times 10^{-11}$$

- even heavier scalars: small contribution around -1×10^{-11}, but **very uncertain** two-photon coupling (not seen prominently in $\gamma\gamma$ reactions)
  ⇒ better treat in some form in asymptotic matching