Status of holographic results on HLbL

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Muon g-2 Theory Initiative Spring 2024 Meeting

Chiral hQCD results

2020, at time of WP, only hQCD results for a_μ from chiral models were available

- HW1 (Erlich-Katz-Son-Stephanov 2005, but $m_q = 0$)
- HW2 (Hirn-Satz 2005, simpler, inherently chiral)
- (Witten-)Sakai-Sugimoto (2004, top-down string theory construction, inherently chiral, low energy limit only because of Kaluza-Klein circle)

Pion TFF and $a_\mu^{\pi^0}$ first fully evaluated* by Leutgeb, Mager, AR, 1906.11795, following partial/hybrid evaluation of Cappiello, Cata, D'Ambrosio, 1009.1161 (m_π inserted by hand)

Extended to axial TFF and a_μ by Leutgeb & AR, 1912.01596 and independently by Cappiello, Cata, D'Ambrosio, Greynat & Iyer, 1912.02779 (HW2 only, different extrapolation to $\eta^{(\prime)}/f_1^{(\prime)}$ sector)

$$\text{LR: } a_\mu^{a_1+f_1+f_1'} = 4a_\mu^{a_1} \text{ (flavor symmetric)} \approx (29\ldots41)\times10^{-11}$$

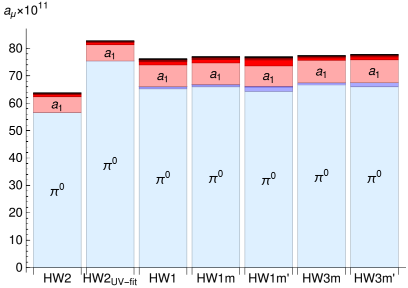
$$\text{CCDGI: } a_\mu^{a_1+f_1+f_1'} \approx 3.5a_\mu^{a_1} \text{ (non-uniform model)} \approx 28\times10^{-11}$$

model	LSDC	m_ρ	$m_{a_1(1260)}$	$a_\mu^{\pi^0}$	$a_\mu^{a_1}$	$a_\mu^{a_1}$+tower
HW1 chiral	100%	775	1375	65.2*	7.85	10.15
HW2(UV-fit)/CCDGI-Set2	100%	987	1573	75	5.75	7.2
HW2(IR-fit)/CCDGI-Set1	62%	775	1235	57	5.9	7.35
Sakai-Sugimoto	0%	775	1187	48.3	3.45	3.7

* Erratum: LMR, PRD 104 (2021) 059903 ! (included in arxiv versions)

HW models with massive pions [Leutgeb & AR: 2108.12345]

Rigorous inclusion of quark masses in HW1 and HW3 (=HW1 w/ HW2 b.c.) models:
→ little difference to chiral model with manually inserted pion mass



(darker colors: excited π^{0*}, a_1^*)

- **HW1m:** HW1 with nonzero light quark mass and correct pion mass
- **HW1m':** HW1m with modified scaling dimension of bifundamental scalar, additionally correct $a_1(1230)$ mass, but not mass of $\pi(1300)$
- **HW3m:** HW1m with HW2 boundary conditions
- **HW3m':** HW3m with modified scaling dimension of bifundamental scalar, additionally correct $\pi(1300)$ mass, but not mass of $a_1(1230)$

Excited pions: don't decouple even in chiral limit, contribute $a_\mu^{\pi^{0*}} \approx (0.8 \ldots 1.8) \times 10^{-11}$

Short distance constraints on TFFs

Crucially, hQCD models with asymptotic AdS$_5$ geometry reproduce
asymptotic momentum dependence of LCE [Brodsky-Lepage 1979-81]
(HW1 model exactly with $g_5 = 2\pi$; HW2 model only at 62%)

- **Pseudoscalars** [Grigoryan & Radyushkin, PRD76,77,78 (2007-8)]:

$$F_{\pi^0\gamma^*\gamma^*}(Q_1^2, Q_2^2) \quad \to \quad \frac{2f_\pi}{Q^2}\sqrt{1-w^2}\int_0^\infty d\xi\, \xi^3 K_1(\xi\sqrt{1+w})K_1(\xi\sqrt{1-w})$$
$$= \frac{2f_\pi}{Q^2}\left[\frac{1}{w^2} - \frac{1-w^2}{2w^3}\ln\frac{1+w}{1-w}\right],$$

with $Q^2 = \frac{1}{2}(Q_1^2 + Q_2^2) \to \infty$, $w = (Q_1^2 - Q_2^2)/(Q_1^2 + Q_2^2)$,
corresponding to asymptotic behavior

$$F^\infty(Q^2, 0) = \frac{2f_\pi}{Q^2}, \qquad F^\infty(Q^2, Q^2) = \frac{2f_\pi}{3Q^2} \quad (\Leftarrow \text{OPE}).$$

- **Axial vector mesons** [J. Leutgeb & AR, 1912.01596]
(confirmed by pQCD result of Hoferichter & Stoffer 2004.06127):

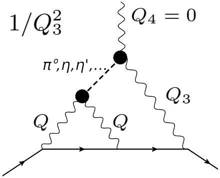
$$A_n(Q_1^2, Q_2^2) \to \frac{12\pi^2 F_n^A}{N_c Q^4}\frac{1}{w^4}\left[w(3-2w) + \frac{1}{2}(w+3)(1-w)\ln\frac{1-w}{1+w}\right]$$

Melnikov-Vainshtein short-distance constraint

Melnikov and Vainshtein [hep-ph/0312226, PRD70(2004)]:
nonrenormalization theorem for axial anomaly implies
short-distance constraint for 4-photon-amplitude (in BTT basis w/ 54 structure functions):

$$\lim_{Q_3\to\infty}\lim_{Q\to\infty} Q^2 Q_3^2 \bar{\Pi}_1(Q,Q,Q_3) = -\frac{2}{3\pi^2}$$

each single meson exchange contribution gives 0
because propagator $\sim 1/Q_3^2$ and the two form factors $\sim 1/Q^2$ and $1/Q_3^2$



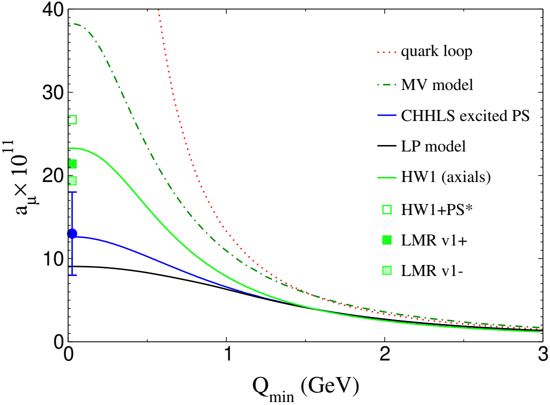
MV model: MV-SDC satisfied by replacing
external TFF by constant on-shell value, leading to
significant (almost +40%) increase of $a_\mu^{\pi^0,\eta,\eta'}$ by 38×10^{-11}

WP estimate for MV-SDC based on Regge model of infinite tower of excited PS states
constructed to saturate MV-SDC with $\Delta a_\mu^{\mathrm{PS}} = 13(6) \times 10^{-11}$ [Colangelo et al., 1910.11881]

HW models: infinite tower of axials saturates MV-SDC to 100% in HW1 models,
with $a_\mu^{A(L)} = 23.2 \times 10^{-11}$ in chiral model;
no contribution to MV-SDC from excited PS, $a_\mu^{\pi^{0*}} = (0.8 \ldots 1.8) \times 10^{-11}$

Comparison of results for the longitudinal SDC

Update of Fig. 69 in the WP / Fig. 5 of Colangelo et al. 2106.13222
(dropping the HW2 models which cannot fit UV and IR parameters simultaneously)



Contribution to a_μ for $Q_i \geq Q_{\mathrm{match}}$: the longitudinal part of the massless perturbative QCD quark loop (dotted red), the Melnikov-Vainshtein model (MV, dot-dashed dark green), the Lüdtke/Procura model (LP, solid black), the CHHLS Regge model of excited pseudoscalars (solid blue), and the contribution of axials in the chiral HW1 model (solid light green), with **squares indicating the final values including excited pseudoscalars** (LMR: in our 2022 model with quark masses and $\mathrm{U}(1)_A$ anomaly)

Massive HW1+U(1)$_A$-Anomaly Model [LMR, 2211.16562]

$N_f = 2+1$ with $m_s \approx 24.3 m_{u,d}$ and Witten-Veneziano mechanism for η' mass

Two version of UV fits:

a) $g_5 = 2\pi$ such that UV constraints on TFF satisfied to 100%

b) $g_5 = 5.94$ such that f_ρ is fitted ($\approx 90\%$ of asymptotic SDCs)

Tuning of gluon condensate Ξ (neglected by KS) $\rightarrow$ virtually exact fit of m_η and $m_{\eta'}$:

Version a) (OPE fit)

	m [MeV]	m–m^{exp} [%]	f^8	f^0	f_G	$\lvert F(0,0)\rvert$	$F - F^{\text{exp}}$
π^0	135	(input)	0	0	0	0.277	
η	557	+1.7%	0.101	0.027	-0.030	0.275	+1(2)% (!)
η'	950	-0.8%	-0.0385	0.113	-0.077	0.340	-0(2)% (!)
G/η''	1992	?	-0.027	0.005	0.053	0.116	
	m [MeV]	m–m^{exp} [%]	F^8_A/m_A	F^0_A/m_A	$A^8(0,0)$	$A^{0\vee 3}(0,0)$	
a_1	1363	+11%	0	0	0	20.96	
f_1	1481	+15%	0.176	0.0365	20.77	3.857	
f_1'	1810	+27%	-0.030	0.201	-3.842	20.07	

gluon condensate parameter $\lvert\Xi\rvert = 0.01051\ \text{GeV}^4$

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Version b) (our current "best guess" regarding a_μ)

	m [MeV]	m–$m^{\rm exp}$ [%]	f^8	f^0	f_G	$\lvert F(0,0)\rvert$	$F - F^{\rm exp}$
π^0	135	(input)	0	0	0	0.276	
η	561	+2.4%	0.103	0.030	-0.031	0.268	+2(2)%
η'	947	-1.1%	-0.039	0.121	-0.082	0.313	-8(2)%
G/η''	1943	?	-0.030	0.0076	0.048	0.111	
	m [MeV]	m–$m^{\rm exp}$ [%]	F_A^8/m_A	F_A^0/m_A	$A^8(0,0)$	$A^{0\vee 3}(0,0)$	
a_1	1278	+4%	0	0	0	19.46	
f_1	1410	+10%	0.176	0.029	19.58	2.69	
f_1'	1820	+28%	-0.017	0.219	-2.56	19.00	

gluon condensate parameter $|\Xi| = 0.01416$ GeV^4

PS: $f^{8,0}$'s within a few % of χPT values

AV: f_1-f_1' mixing angle $\phi_f - \phi_f^{\rm ideal}$ about twice as large as indicated by L3 data

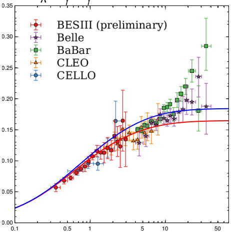
(ϕ_f strongly dependent on Ξ; but sum $a_\mu^{f_1} + a_\mu^{f_1'}$ rather insensitive)

a_μ in HW1+U(1)$_A$-Anomaly Model [LMR, 2211.16562]

comparing also to **Soft-Wall model of P. Colangelo, F. Giannuzzi, S. Nicotri**
with $m_s > m_{u,d}$, accurate η, η' masses, good $F(0,0)$, and correct U(1)$_A$ anomaly
(CGN 2301.06456: scalar sector with U(1)$_A$ anomaly;
CGN 2402.07579: axial vector contributions, but in a simpler, flavor symmetric set-up!)

$a_\mu^{\cdots} \times 10^{11}$	LMR(OPE fit)	LMR(F_ρ-fit)	CGN(OPE fit)	WP2020
π^0	66.1	63.4	75.2	$63.0^{+2.7}_{-2.1}$
η	19.3	17.6	21.2	16.3(1.4)
η'	16.9	14.9	12.3	14.5(1.9)
$PSGB/\eta''$	0.2	0.2	5.1	
$\sum_{PS^*}$	1.6	1.4	1.7	
PS poles total	104	97.5	115.5	93.8(4.0)

$Q^2 F_{\pi^0\gamma^*\gamma}(Q^2, 0)$ [GeV] in LMR 2211.16562



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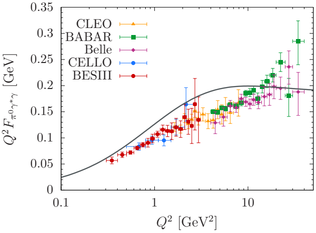

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$\sum_{PS^*}$	1.6	1.4	1.7	
PS poles total	104	97.5	115.5	93.8(4.0)
a_1	7.8	7.1	9.0	
$f_1 + f_1'$	20.0	17.9	3×9.0	
$\sum_{a_1^*}$	2.2	2.4	?	
$\sum_{f_1^{(\prime)*}}$	3.6	3.0	?	
AV+LSDC total	34	30.5	> 36	21(16)
total	138	128	??	115(16.5)

??: don't add results from different models here,
AV and PS sectors are coupled!!

Attempts for further improvements

Issues with LMR2022 model:

- equivalent photon decay rate of f_1, f_1' higher than L3 data indicate
- f_1-f_1' mixing angles unrealistic, too far from ideal mixing

To appear soon: LMR2024 with scalar-extended CS term (Quillen's superconnection)
(adaption of open-string-tachyon condensation model of Casero, Kiritsis & Paredes 2007)

preliminary results:

- f_1-f_1' mixing angle closer to ideal, lower equivalent photon rate
- $\rightarrow$ lower contribution from ground-state a_1, f_1's, but more from excited AV
- but less perfect fit of η and η', larger contribution from π^0
- total sum almost unchanged:

$a_\mu^{\cdots} \times 10^{11}$	LMR(OPE fit)	LMR(F_ρ-fit)	WP2020
PS poles	104$\rightarrow$ 112.5	97.5$\rightarrow$ 103.5	93.8(4.0)
AV+LSDC	34$\rightarrow$ 24.9	30.5$\rightarrow$ 24.7	21(16)
total	138$\rightarrow$ 137	128$\rightarrow$ 128	115(16.5)

- NEW: scalar nonet naturally couples to photons, unlike minimal model, with one of the terms (ζ_+) considered by Cappiello, Cata, D'Ambrosio 2110.05962

Scalar contributions

Cappiello, Cata, D'Ambrosio 2110.05962
have calculated a_μ contributions of $\sigma(500)$, $a_0(980)$, $f_0(990)$
by non-minimal *chiral* HW1 models (using different Lagrangians for each) with results

$a_\mu^{\cdots} \times 10^{11}$	$n=1$	$n=2$	all n
σ	-8.5(2.0)	-0.07(2)	-8.7(2.0)
a_0	-0.29(13)	-0.025(10)	-0.32(14)
f_0	-0.27(13)	-0.025(9)	-0.29(14)
sum	-9(2)	-0.12(4)	-9(2)

Issues:

- if tetraquarks, qualitatively different descriptions needed (certainly at large N)
- should not be added to previous CCDGI results due to different model
- asymptotics of TFFs $\mathcal{F}_1^S(Q_1^2,Q_2^2) \sim Q^{-6}$, $\mathcal{F}_2^S(Q_1^2,Q_2^2) \sim Q^{-8}$
  instead of Q^{-2} and Q^{-4} in pQCD

LMR2024 model:

- a_0, f_0, f_0' somewhat too heavy ($a_0(1450), \ldots ?$), $a_\mu^S \neq 0$, but not yet evaluated
- asymptotics of TFFs $\mathcal{F}_1^S(Q_1^2,Q_2^2) \sim Q^{-4}$, $\mathcal{F}_2^S(Q_1^2,Q_2^2) \sim Q^{-6}$ with $m_q \neq 0$, consistent with OPE in symmetric limit, but not with LCE
  NB: Hoferichter-Stoffer result also consistent with OPE in symmetric result, since at $q_1 = -q_2$ leading terms cancel!

Tensor contributions

Following [Katz, Lewandowski & Schwartz hep-ph/0510388] CGN 2402.07579 have implemented tensor mesons in SW and HW models,

matching $f_2(1270) \to \gamma\gamma$ with $\Gamma_{\gamma\gamma} = 2.6(5)$ keV

with result $a_\mu^{f_2(1270)} = (0.61 \ldots 0.63) \times 10^{-11}$,

consistent with Pauk & Vanderhaeghen 2014

Issues:

- dual operator not only quark bilinear,
  but rather energy-momentum tensor $\leftrightarrow$ tensor glueball
- perhaps therefore: asymptotics of TFF with correct Q^{-4} behavior, but different $f(w)$ than Hoferichter & Stoffer 2020

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Tensor and (pseudo)scalar glueballs in Hechenberger, Leutgeb & AR, 2302.13379

for Witten-Sakai-Sugimoto model: $\Gamma(G^{S,P,T} \to \gamma\gamma) \sim \text{few keV}$ but $|a_\mu^G| \lesssim 10^{-12}$