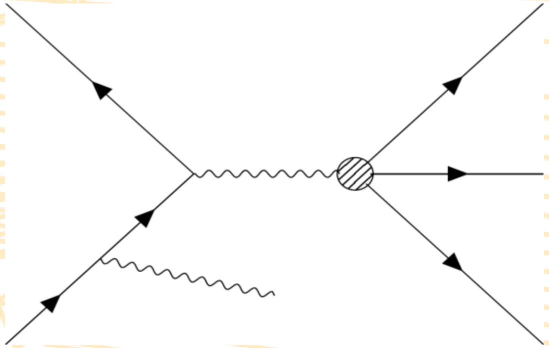
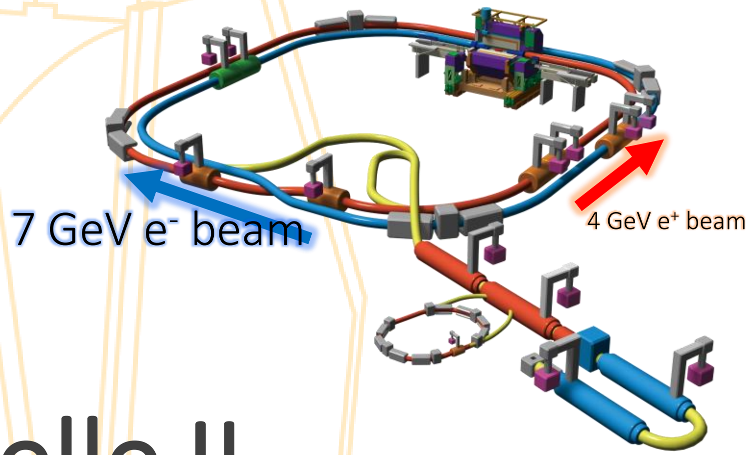
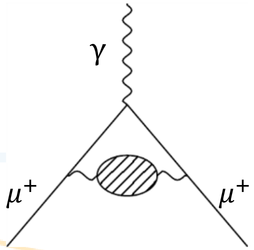




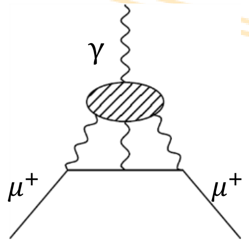
Status and plans regarding g-2 at Belle II
---Measurement of the $e^+e^-\to\pi^+\pi^-\pi^0$ cross section with the Belle II detector---

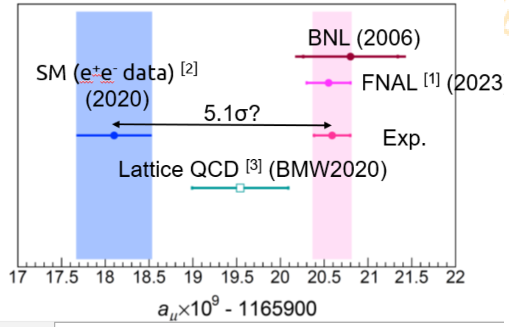


Hisaki Hayashii, Nara Women's University,
for Yuki Sue and on behalf of Belle II collaboration

2024.4.22

g-2 Theory Initiative mini-Workshop



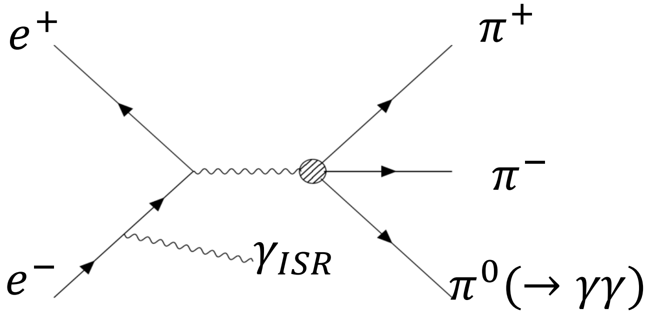


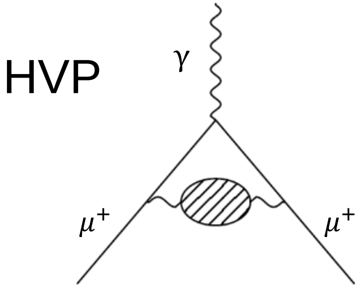
Introduction(1)

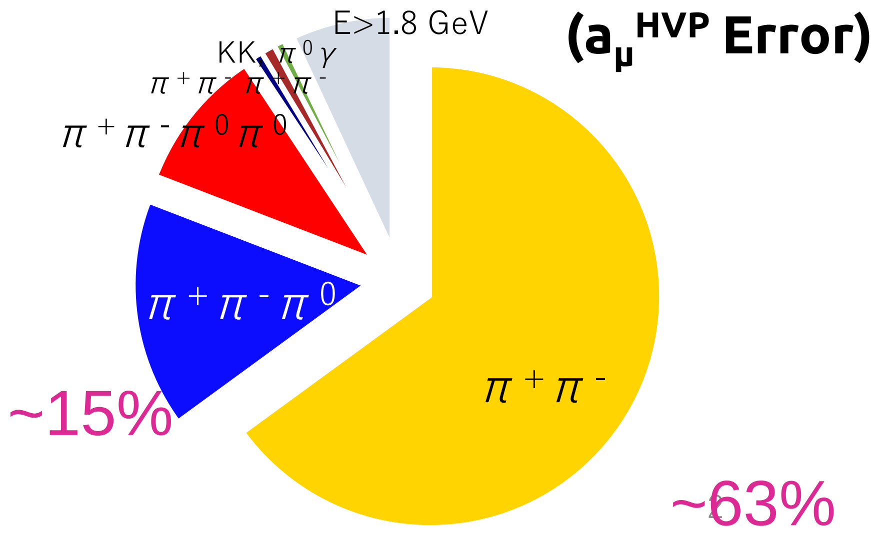
- We present our new results on a measurement of the $e^+e^- \to \pi^+\pi^-\pi^0$ cross section in the energy range 0.62- 3.50 GeV with the Belle II detector at the SuperKEKB collider
- We measure the cross section using ISR technique for the $e^+e^- \to \pi^+\pi^-\pi^0\gamma_{ISR}$ events, taken at the fixed c.m.s $\sqrt{s}$=10.58 GeV.

$$a_\mu^{\rm SM} = \frac{g_\mu - 2}{2} = a_\mu^{\rm QED} + a_\mu^{\rm EW} + a_\mu^{\rm HVP} + a_\mu^{\rm HLbL}$$

- $\pi^+\pi^-\pi^0$ is the second largest source on the uncertainty in the theoretical evaluation of muon g-2.







Introduction(2)

- We measure the $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ cross section with 2.2 % accuracy at the ω mass region.
- The accuracy of $a_\mu^{3\pi}$ is $\delta a_\mu^{3\pi}/a_\mu^{3\pi}$ ~2%

- My task is to explain how this accuracy is achieved.
- Detailed description is available at arXiv:2404.04915. The same draft is just submitted to PRD.

Key items

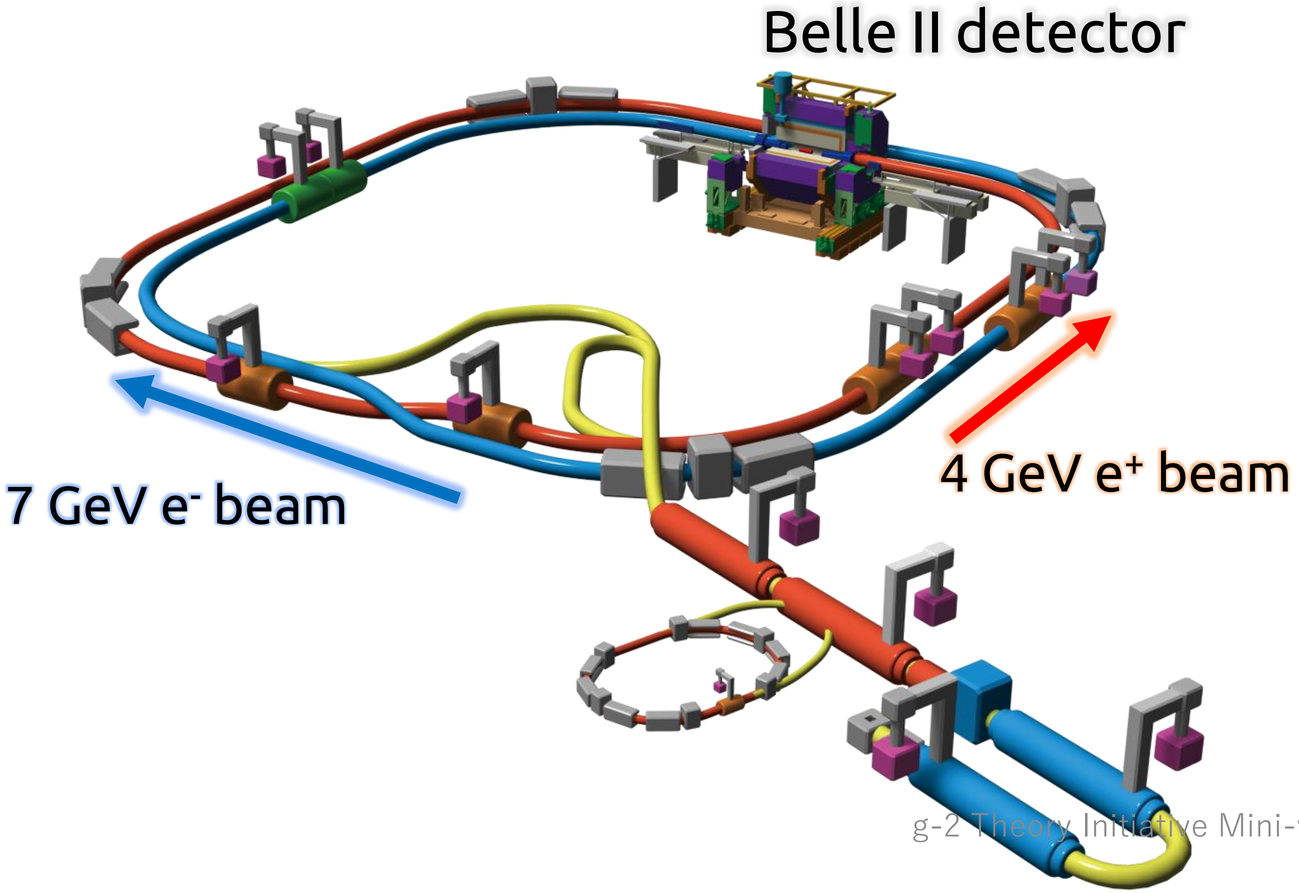
- SuperKEKB/Belle II
- Trigger, ISR technique
- Event selection to extract $e^+e^-\to\pi^+\pi^-\pi^0\gamma_{ISR}$ process
- Background estimation
- Signal extraction and Unfolding
- Efficiency corrections
- Cross section and $a_\mu^{3\pi}$ calculation

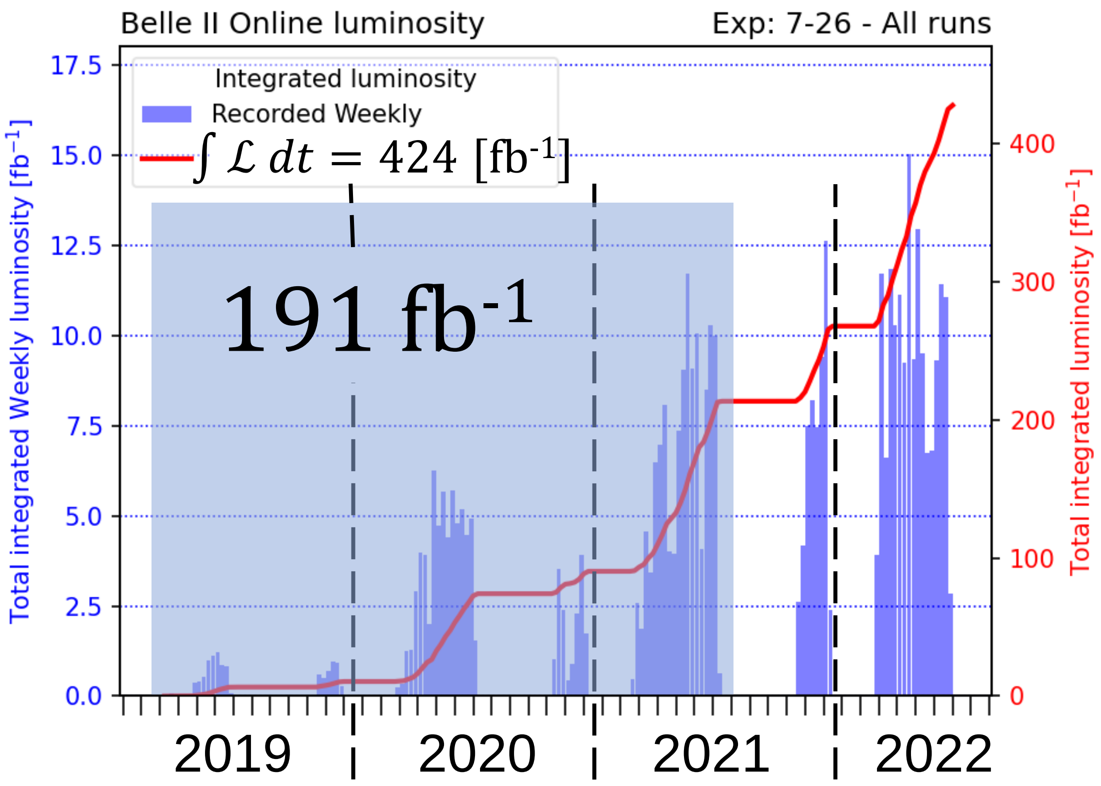
--Blind analysis--

- Study/optimization of analysis methods and corrections have been fixed using MC/control data, before examining the signal.

SuperKEKB

- Asymmetric e^+e^- collider at KEK
  - $\sqrt{s} = M(\Upsilon(4S)) = 10.58$ GeV
  - World record instantaneous luminosity : 4.7×10^{34} /cm²/s
  - ~90% data taking efficiency : 1-2 fb^{-1}/day
- Used dataset in this analysis
  - 2019 - 2021 Summer dataset
  - Integrated luminosity: 191 fb^{-1}



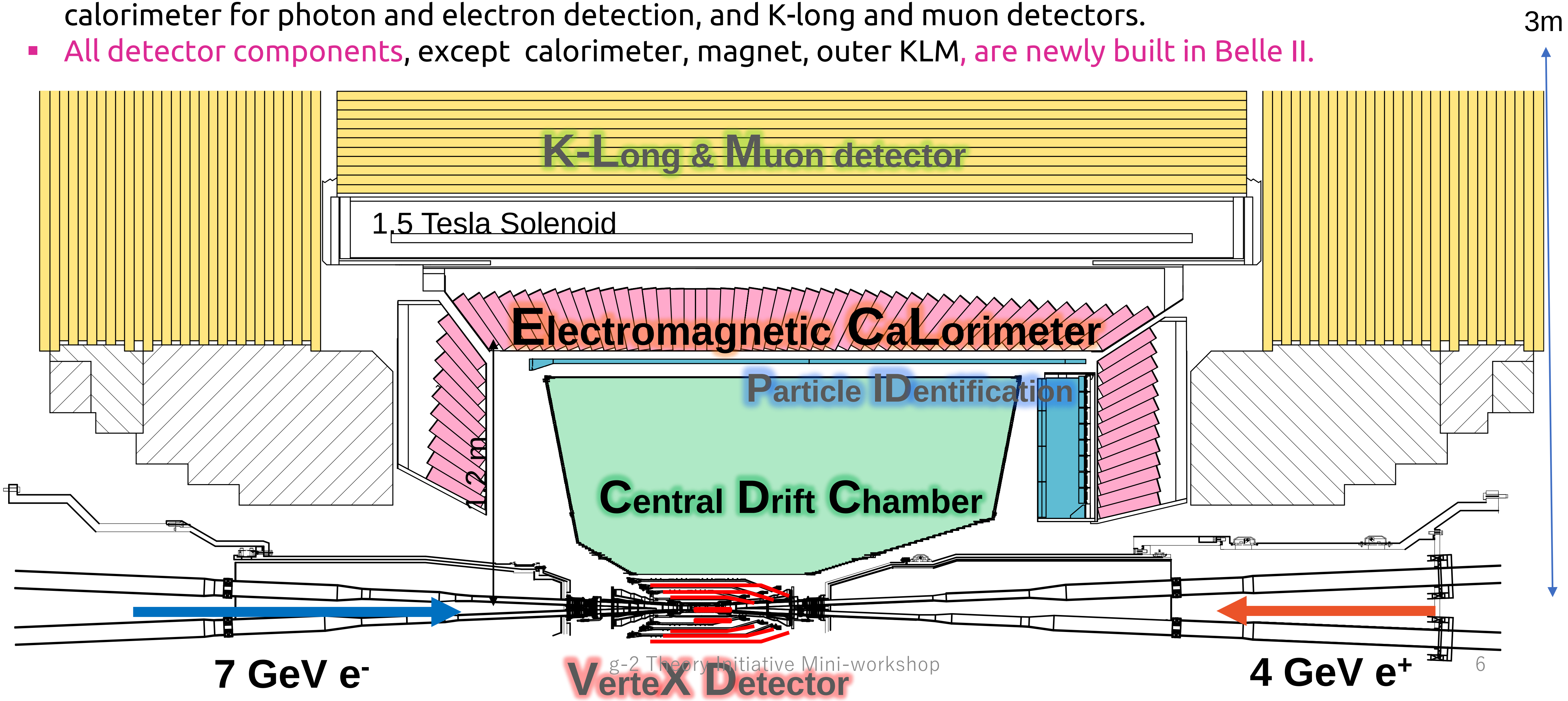


Belle II detector

Trigger & DAQ

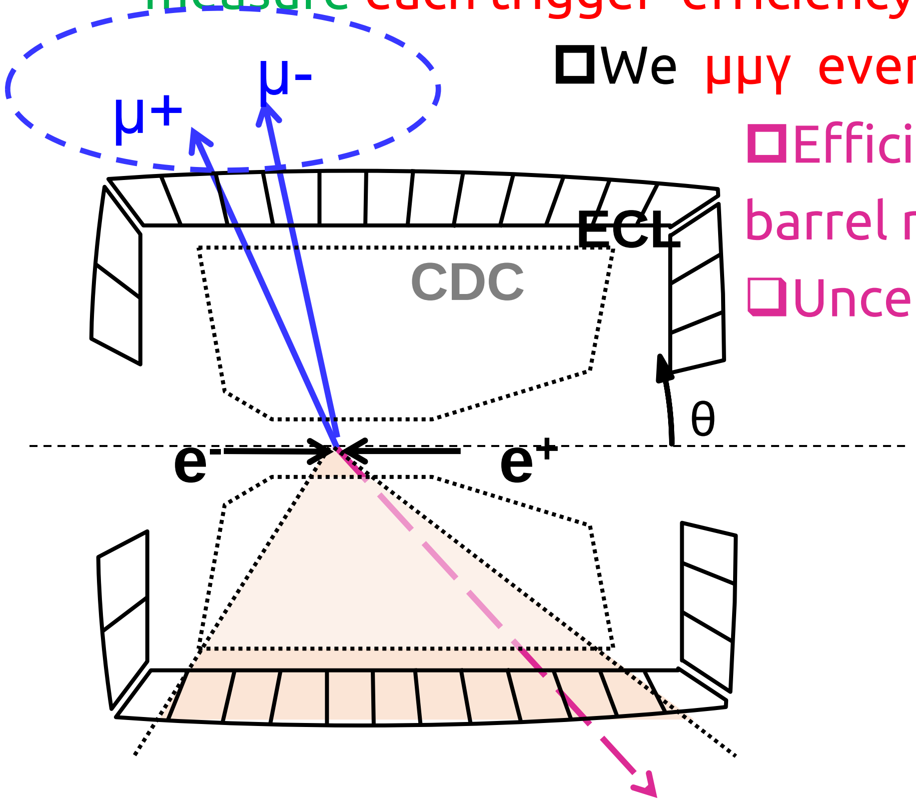
Upper half cross sectional view of the Belle II detector.

- Belle II consists of vertex and central drift chamber for tracking, Cherenkov for Particle ID, calorimeter for photon and electron detection, and K-long and muon detectors.
- All detector components, except calorimeter, magnet, outer KLM, are newly built in Belle II.

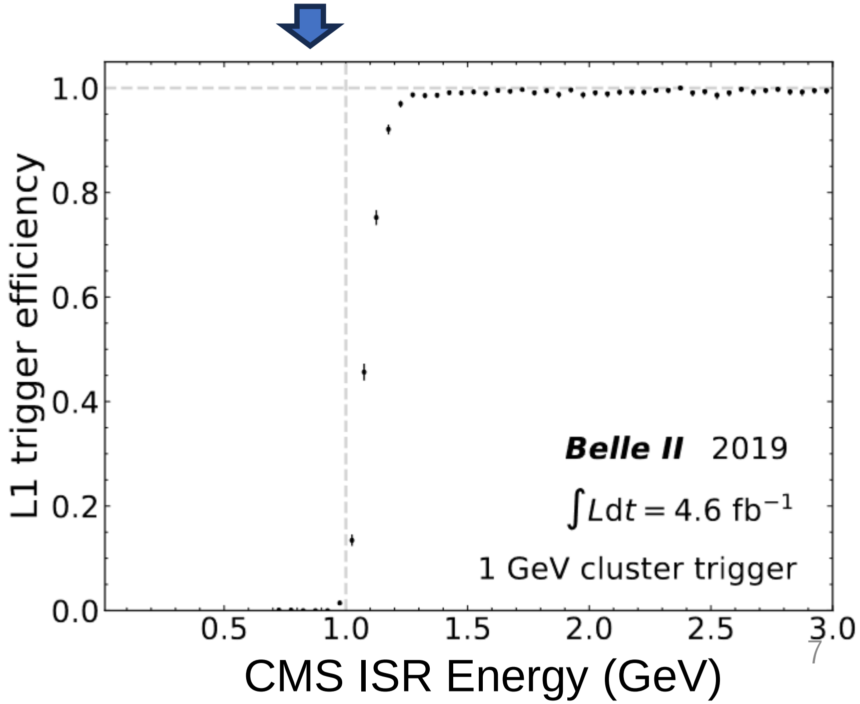


Trigger for ISR events

- Special trigger lines are prepared, in Belle II, to detect low-multiplicity events such as **ISR , $\tau' s$**
- This analysis uses **Dark matter.**
  - Single (≥ 1) high energy clusters in calorimeter with ≥ 2.0 GeV.
  - (This trigger line is scaled down by factor of 2 in high luminosity runs.)
- Independence between energy triggers and track triggers (Single track trigger etc.) allows us to measure each trigger efficiency in data.



- We μμγ events to measure the enery trigger efficiency in data.
  - Efficiency for energetic ISR in barrel region: 99.9%.
  - Uncertainty is 0.1%.



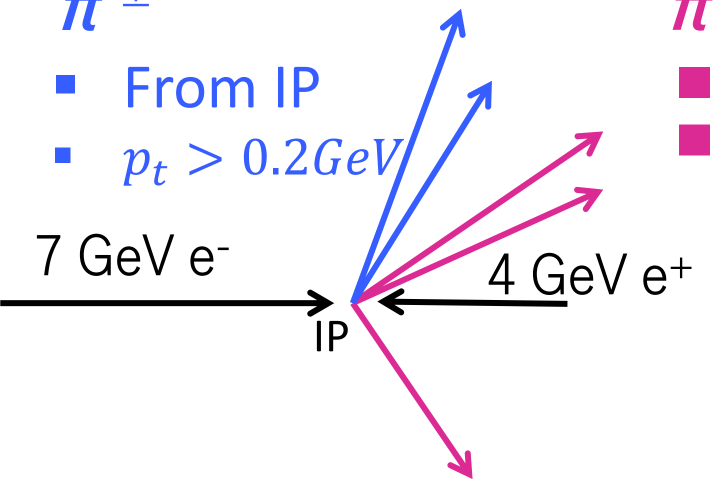
$e^+e^- \to \pi^+\pi^-\pi^0\gamma_{ISR}$ selection

Reconstruct two tracks + three photons : $e^+e^- \to \pi^+\pi^-\pi^0\gamma_{ISR} \to \pi^+\pi^-\gamma\gamma\gamma_{ISR}$

- Four-mom. kinematic fit (4C-KFit)
  - Fit to positions and momenta
  - Constrain to initial e^+e^- four-momentum
  - Select small χ^2 to extract signal-like event, $\chi^2(2\pi 3\gamma) \leq 50$.

$\pi^{\pm}$

- From IP
- $p_t > 0.2 GeV$



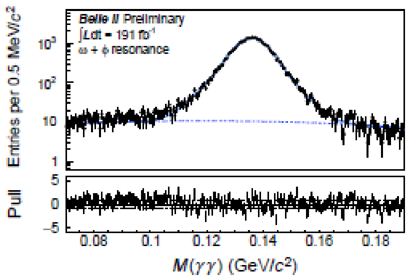
π^0-decay photons

- $E > 0.1$ GeV
- $M(\gamma\gamma) < 1$ GeV/c²
  [Wide mass range for π^0 mass fit]

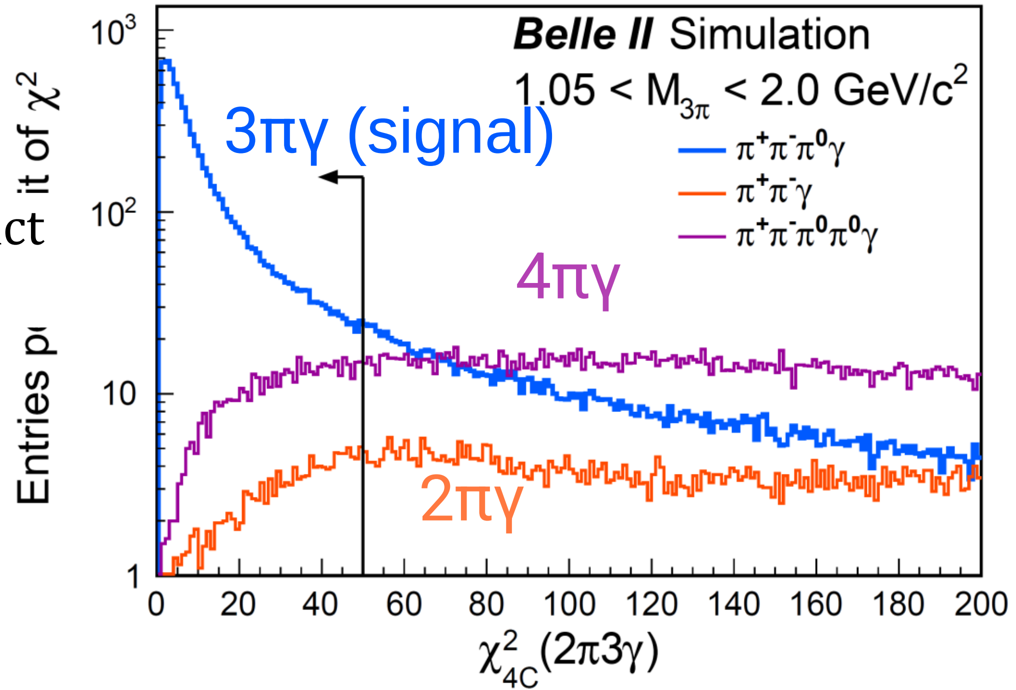
- (Fit $M_{\gamma\gamma}$ dis. in each $M_{3\pi}$ to extract the signal

ISR photon

- $E^{CMS} > 4$ GeV
- In barrel ECL for trigger



4C-Kfit χ^2 distribution (MC)



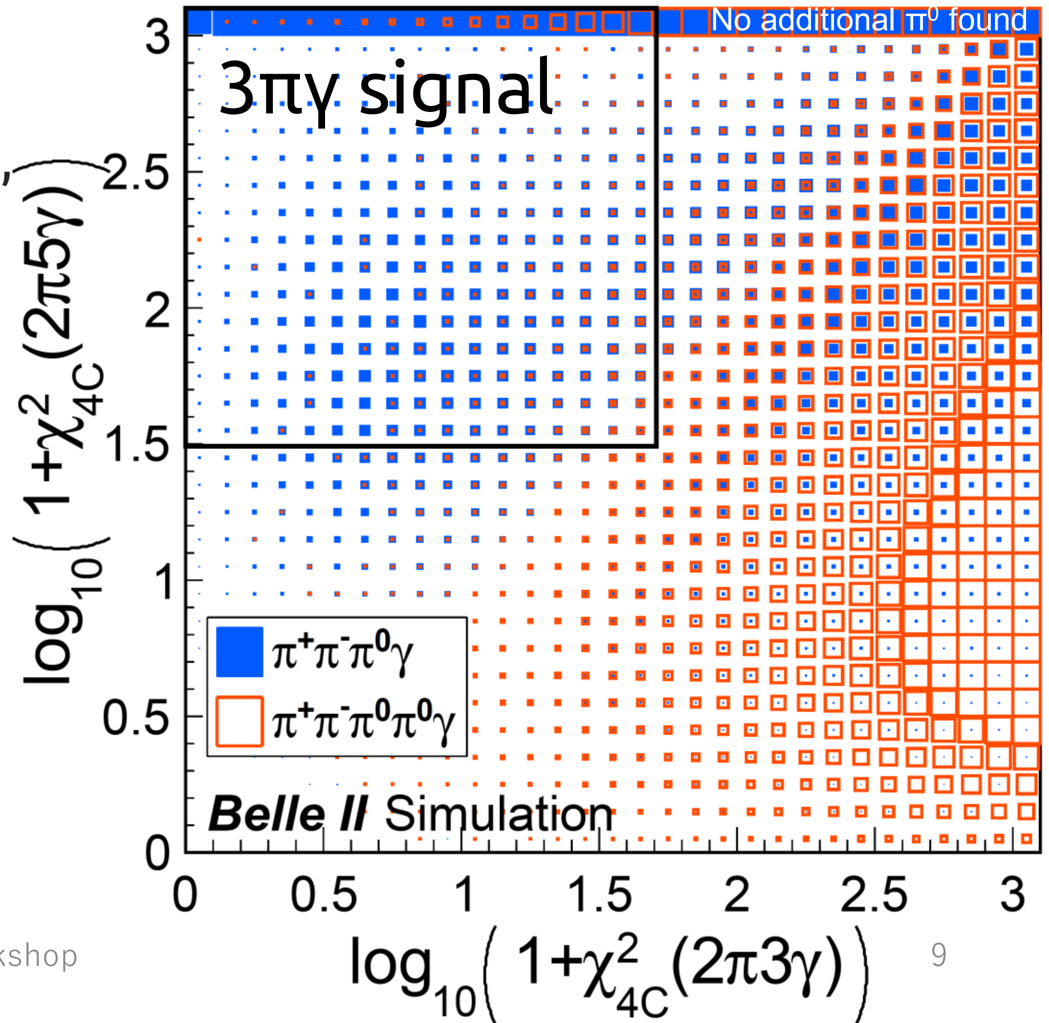
Background suppression (1)

A) Dominant background is from $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\gamma$, since we allow any number of photons in an event.

- Reconstruct $\pi^+\pi^-\pi^0\pi^0\gamma$ (with additional π^0)
- 4C kinematic fit under $\pi^+\pi^-\pi^0\pi^0\gamma$ (2π5γ) hypothesis, and $\chi^2_{4C}(2\pi5\gamma) > 30$,

can reduce this background effectively.

$\chi^2_{4C}(2\pi3\gamma)$ versus $\chi^2_{4C}(2\pi5\gamma)$



Background suppression (2)

These kinematic selections are quite effective.
Remaining background is already quite small,

~1 % in the ω, ϕ resonance regions, and ~10 % at $M_{3\pi} \geq 1.05$ GeV.

B) Background not containing true π^0 : $e^+e^- \rightarrow e^+e^-\gamma$, $\pi^+\pi^-\gamma$, $\mu^+\mu^-\gamma$

- π/e ID L(π/e) > 0.1
- $M^2_{recoil}(\pi^+\pi^-) > 4\ \mathrm{GeV}^2/c^4$

Note that these background processes do not create any π^0 peak in $M_{\gamma\gamma}$

C) Charged kaon : $e^+e^- \rightarrow K^+K^-\pi^0\gamma$

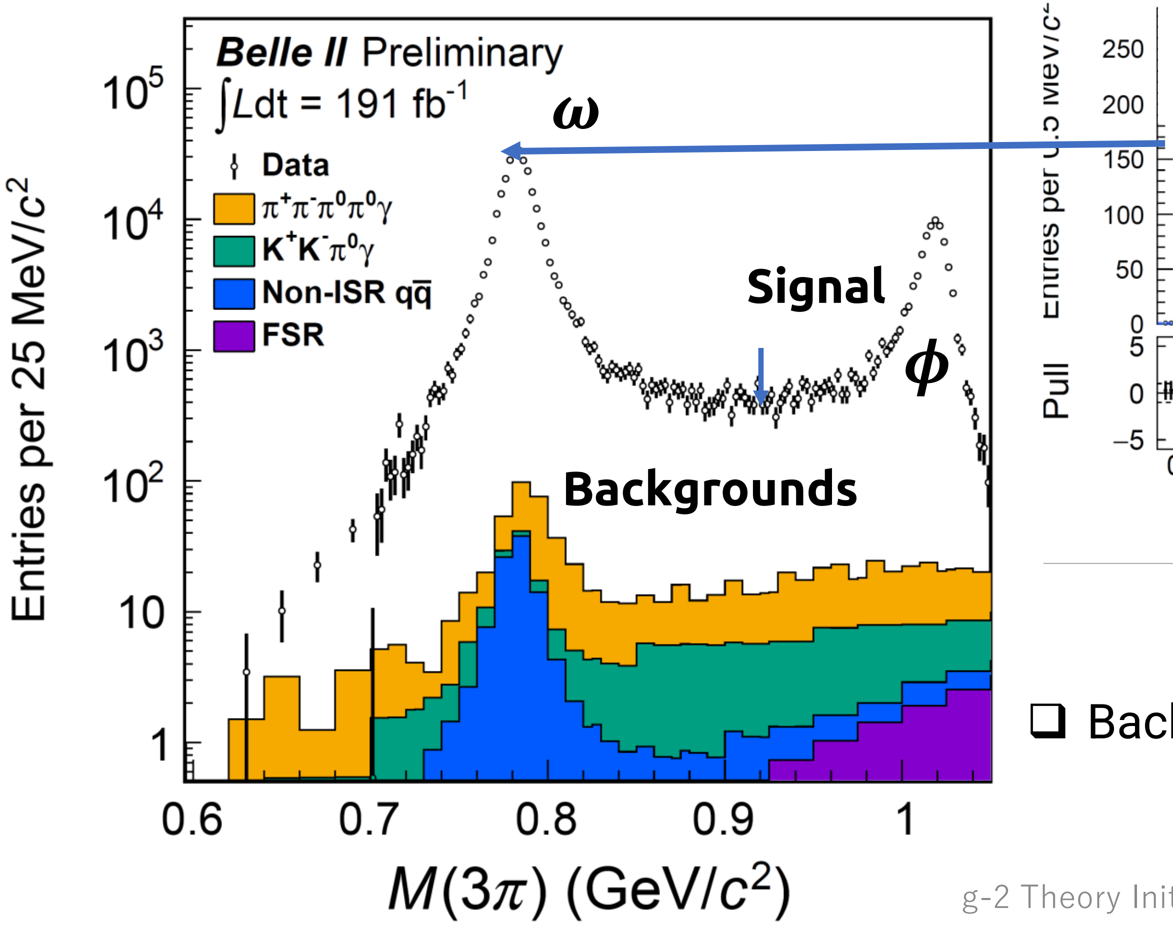
- π/K ID L(π/K) > 0.1

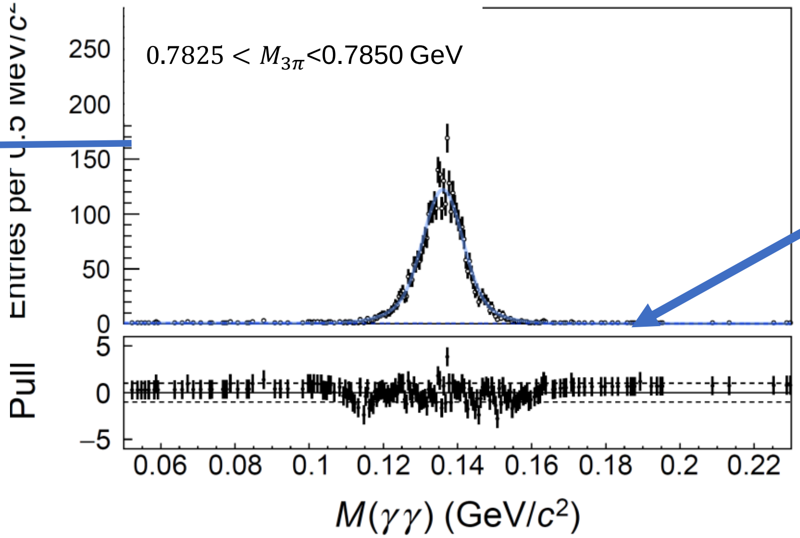
D) Remove events in which high momentum π^0 mis-identified as a ISR photon.

Those events are from non-ISR $q\bar{q}$ (dominated by $\pi^+\pi^-\pi^0\pi^0$) and $\tau^+\tau^-$ ($\tau^\pm \rightarrow \pi^\pm\pi^0\nu_\tau$) .

Extracted signal: $e^+e^- \to \pi^+\pi^-\pi^0\gamma_{ISR}$

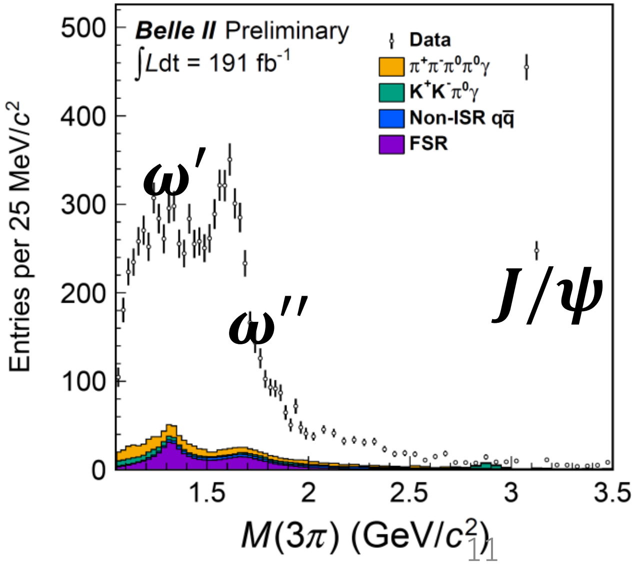
- We obtain the number of evens in each M(3π) bin by fitting M(γγ) with signal and background..
- Dotted points show the data.
- Histograms show the background expectation after applying data-MC correction factors.





Any combinatorial background can be removed by a fit.

- Background is 0.5% at the ω peak.

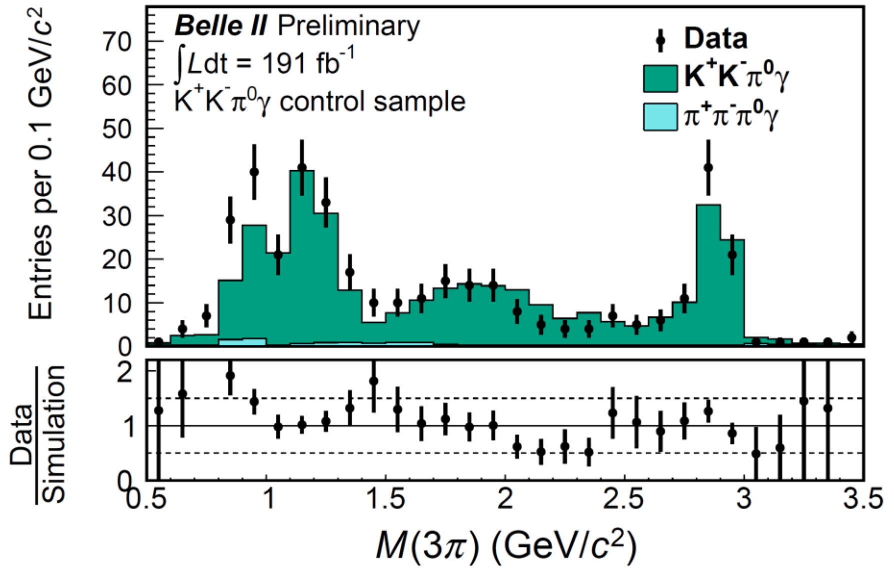


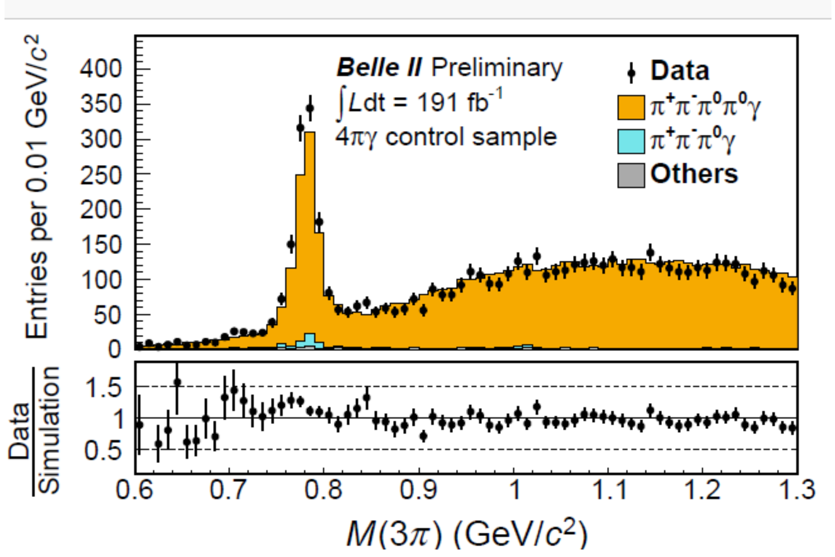
Validation of the background predictions

We prepare background enhanced data as a control sample to validate the background MC.

- $e^+e^- \to K^+K^-\pi^0\gamma$:
  Invert π/K-ID L(π/K) > 0.1 ⇒ L(π/K) < 0.1

- $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\gamma$:
  Reconstruct $\pi^+\pi^-\pi^0\pi^0\gamma$ and select $\chi^2(4\pi\gamma) < 30$





- The contamination from other processes is negligible..

A slight data-MC difference is assigned as correction factors for the background

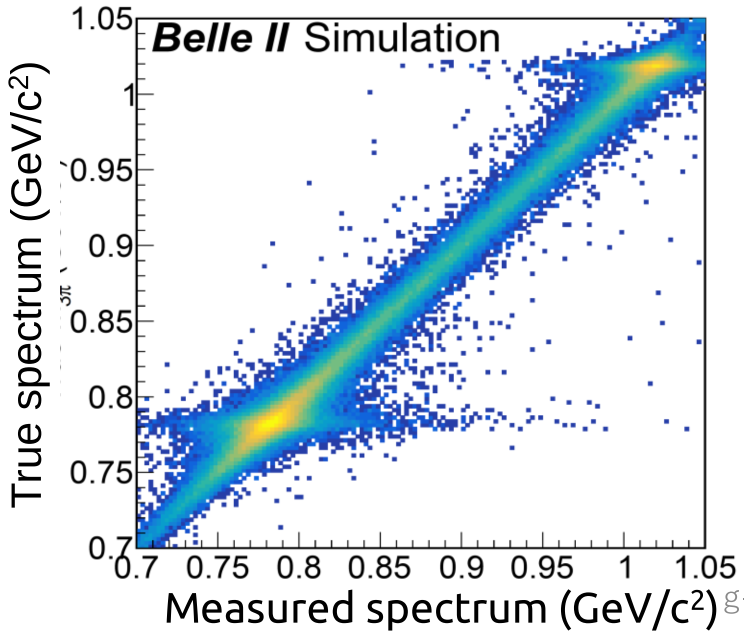
$$N^{\mathrm{data}}_{\mathrm{Signal}} = N^{\mathrm{MC}}_{\mathrm{Signal}} \cdot \frac{N^{\mathrm{data}}_{\mathrm{Control}}}{N^{\mathrm{MC}}_{\mathrm{Control}}}$$

Unfolding

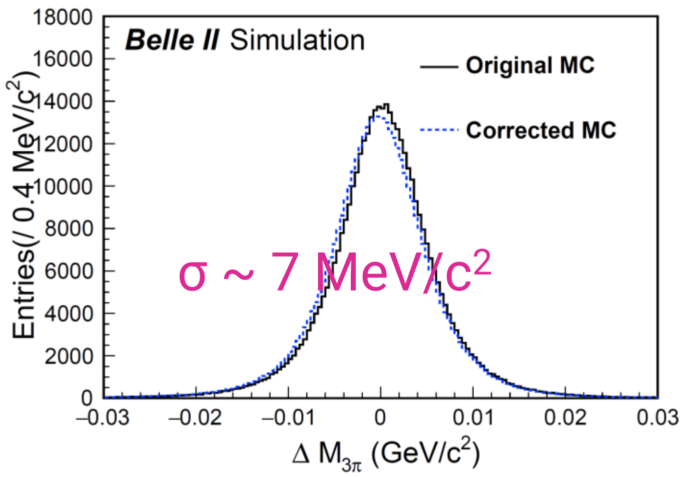
- The signal spectrum is unfolded to remove the effect of finite detector resolution
  - Typical mass resolution is around 7-10 MeV/c^2
- To check how well this detector resolution is simulated by MC, we use the narrow peaks at ω, Φ, and J/ψ in data.
  - In particular, J/ψ is useful since it's resolution is determined only by the detector effect.
  - We found about 1 MeV/c^2 difference on the resolution, and 0.5-1.5 MeV/c^2

mass shift between data and the MC and correct these differences in the MC.

Transfer matrix



Mass resolution



Cross section calculation

Unfolded signal spectrum

Cross section

$$\sigma_{ee\to3\pi}(M_i(3\pi)) = \frac{N_{\mathrm{unfolded},i}}{\varepsilon(M_i(3\pi)) \cdot L_{\mathrm{eff}}(M_i(3\pi)) \cdot r_{\mathrm{rad}}}$$

3π mass at i-th bin

Radiative correction

Effective luminosity

Corrected Efficiency

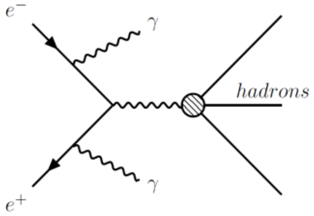
$r_{rad} = 1.0080 \pm 0.0007$

Correction is <1 %.

L_{eff}

$$\frac{\mathrm{d}L_{\mathrm{eff}}}{\mathrm{d}\sqrt{s'}} = L_{\mathrm{int}}\frac{2\sqrt{s'}}{s}\int_{\theta^*_{\mathrm{min}}}^{\pi-\theta^*_{\mathrm{min}}} W(s,s',\theta')\sin\theta'\mathrm{d}\theta'$$
$$= L_{\mathrm{int}}\frac{2\sqrt{s'}}{s}\frac{\alpha}{\pi}\left(\frac{s^2+s'^2}{s(s-s')}\ln\frac{1+C}{1-C}-\frac{s-s'}{s}C\right), \quad (4)$$

where L_{int} is the integrated luminosity of the data set, θ^*_{min} is the minimum polar angle of an ISR photon in the c.m. frame, and C is $\cos\theta^*_{\mathrm{min}}$.



Systematic uncertainty for $e^+e^- \to \pi^+\pi^-\pi^0$ cross section

- Major systematic uncertainty comes from π^0 efficiency and MC generator.

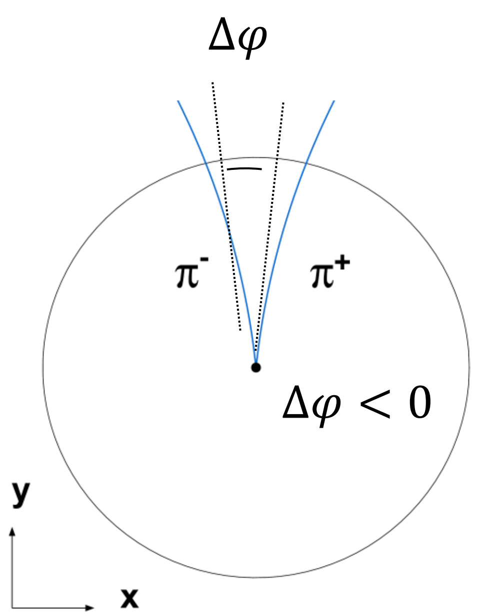
Source	Systematic uncertainty (%)	
	$\sqrt{s}$ < 1.05 GeV[2]	$\sqrt{s}$ > 1.05 GeV
Trigger efficiency	0.1	0.2
ISR photon efficiency	0.7	0.7
Tracking efficiency	0.8	0.8
π^0 efficiency	1.0	1.0
χ^2 criteria efficiency	0.6	0.3
Background suppression efficiency	0.2	1.9
MC generator	1.2	1.2
Radiative correction	0.5	0.5
Integrated luminosity	0.6	0.6
Total systematics	2.2	2.8

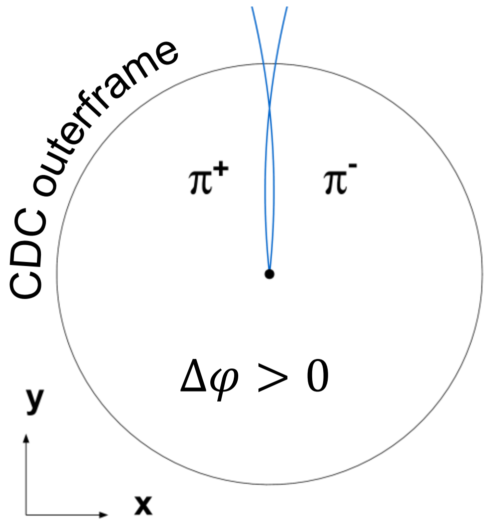
ISR photon &tracking efficiency

- We measure ISR photon efficiency using $e^+e^-\to\mu^+\mu^-\gamma$
- We measure charged track efficiency using $e^+e^-\to\tau^+\tau^-$ (0.3%/track).
- We check track loss due to shared hits in the drift chamber using $e^+e^-\to\pi^+\pi^-\pi^0\gamma$ events.
  - The Inefficiency, f, due to track loss is found to be 5.0% in data and 4.0% in MC

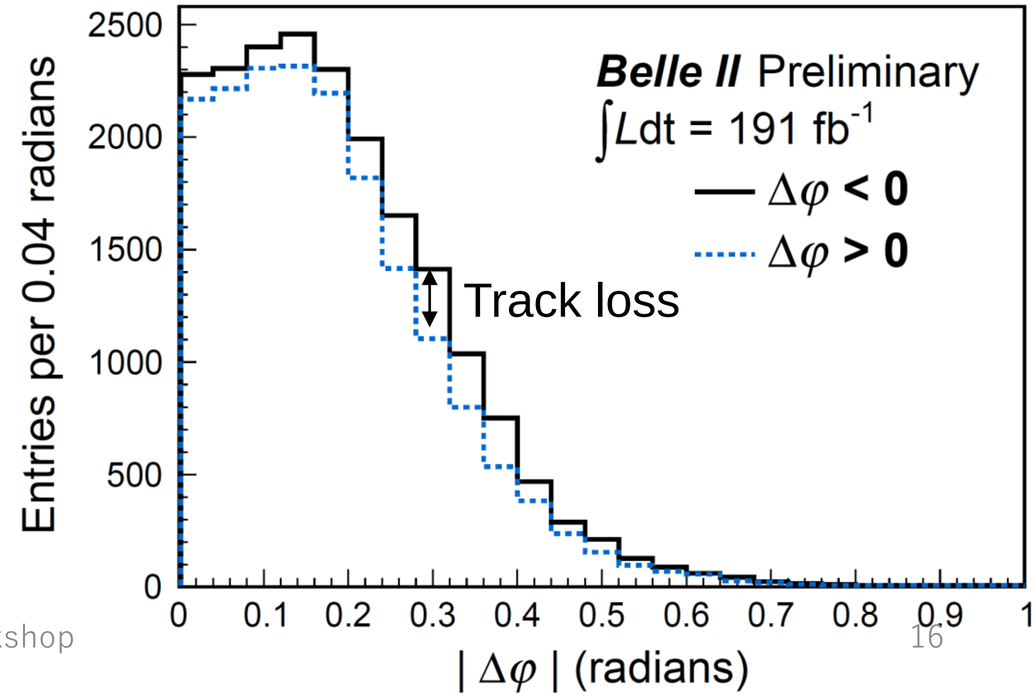
$$f = \frac{N(\Delta\varphi<0)-N(\Delta\varphi>0)}{2N(\Delta\varphi<0)}$$

- Total correction factor for tracking is (-1.4 ± 0.8)%.





Δφ distribution in data

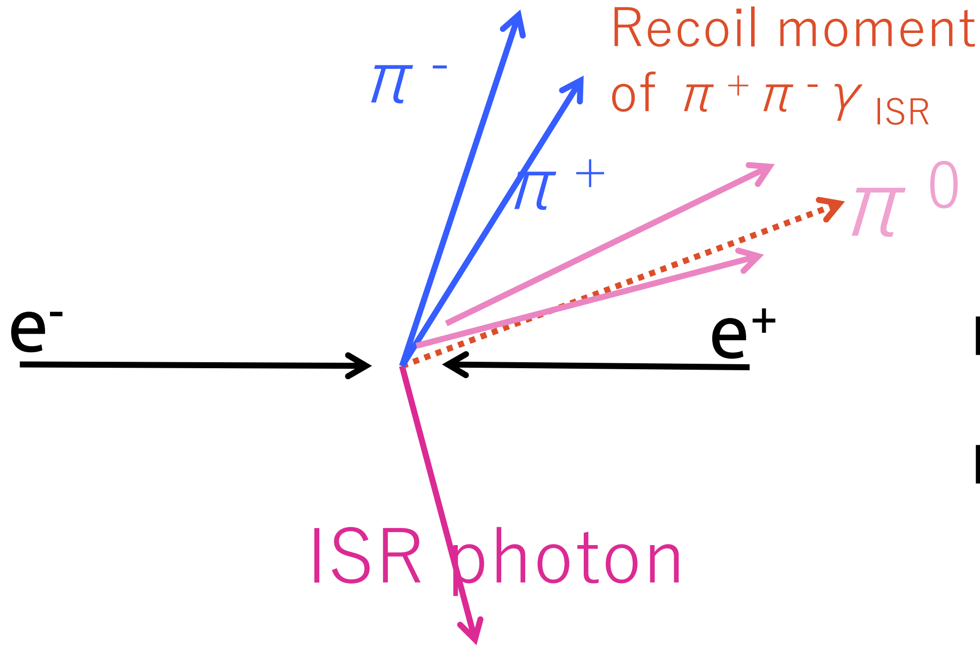


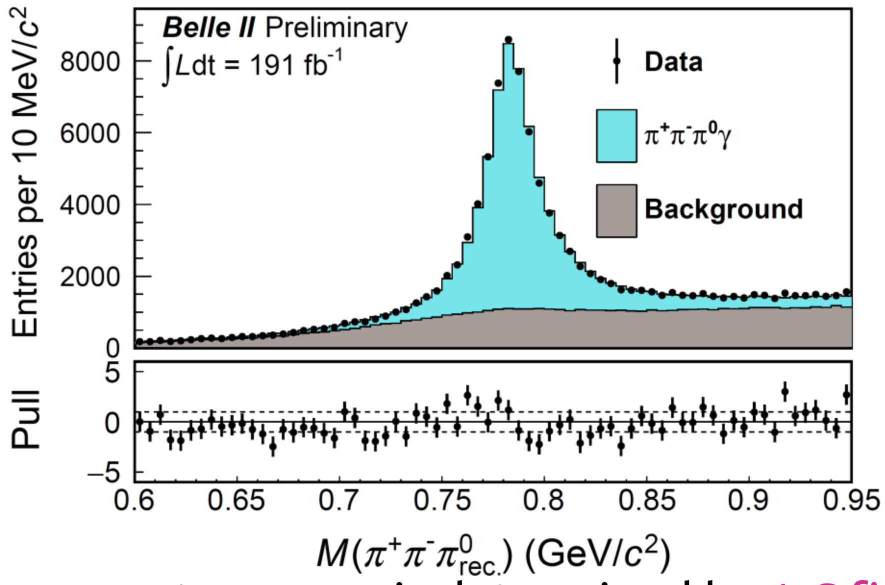
π^0 efficiency correction

- Measure π^0 efficiency using events in exclusive process $e^+e^- \to \omega\gamma \to \pi^+\pi^-\pi^0\gamma$

$$\varepsilon_{\pi^0} = \frac{N(\text{Full reconstruction} : \gamma_{\text{ISR}}\pi^+\pi^-\pi^0)}{N(\text{Partial reconstruction} : \gamma_{\text{ISR}}\pi^+\pi^-)}$$

- Reconstruct the process using only $\pi^+\pi^-\gamma_{ISR}$, without using π^0 information.
- Count $\omega \to \pi^+\pi^-\pi^0$ signal in $M(\pi^+\pi^-\pi^0_{rec})$





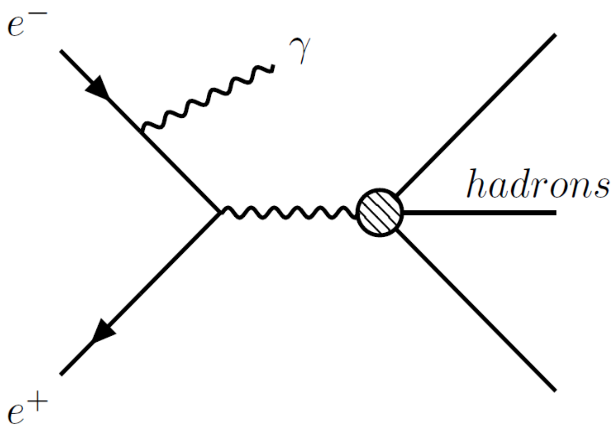
- Where π^0_{rec} momentum p_{recoil} is determined by 1-C fit to $\pi^+\pi^-\gamma_{ISR}$ with a hypothesis that recoil mass equals π^0 mass

- ε_{π^0} are independently evaluated by the data and MC
  - Data/MC ratio = $0.986 \pm 0.006_{stat}$
- Systematic uncertainty related to π^0 is 1.0%, dominated by the uncertainty of the background in ω signal.

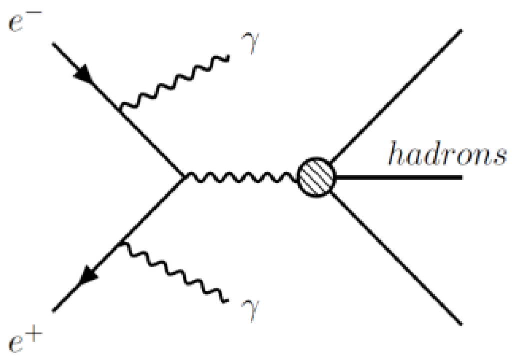
Higher-order ISR effects in MC

- Recently BaBar reports issues on the small-angle additional photon emission in PHOKHARA MC, (PRD 108 (2023)11, arxiv:2308.05233.
- We also confirm that, although the total radiative correction r_{rad} is small (1% level), 15-20% events contain additional ISR photons in the generator. This additional photon emission affects the efficiency of kinematic selection of events using χ2 .
- By taking into account additional NNLO effects, we assign 1.2% uncertainty on this issue as the uncertainty of the generator. (We need complete NNLO MCs.)

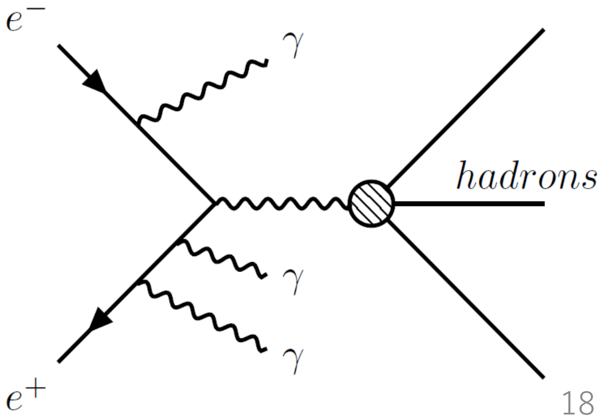
Leading-order (LO) ISR



Next-to-Leading-order (NLO) ISR

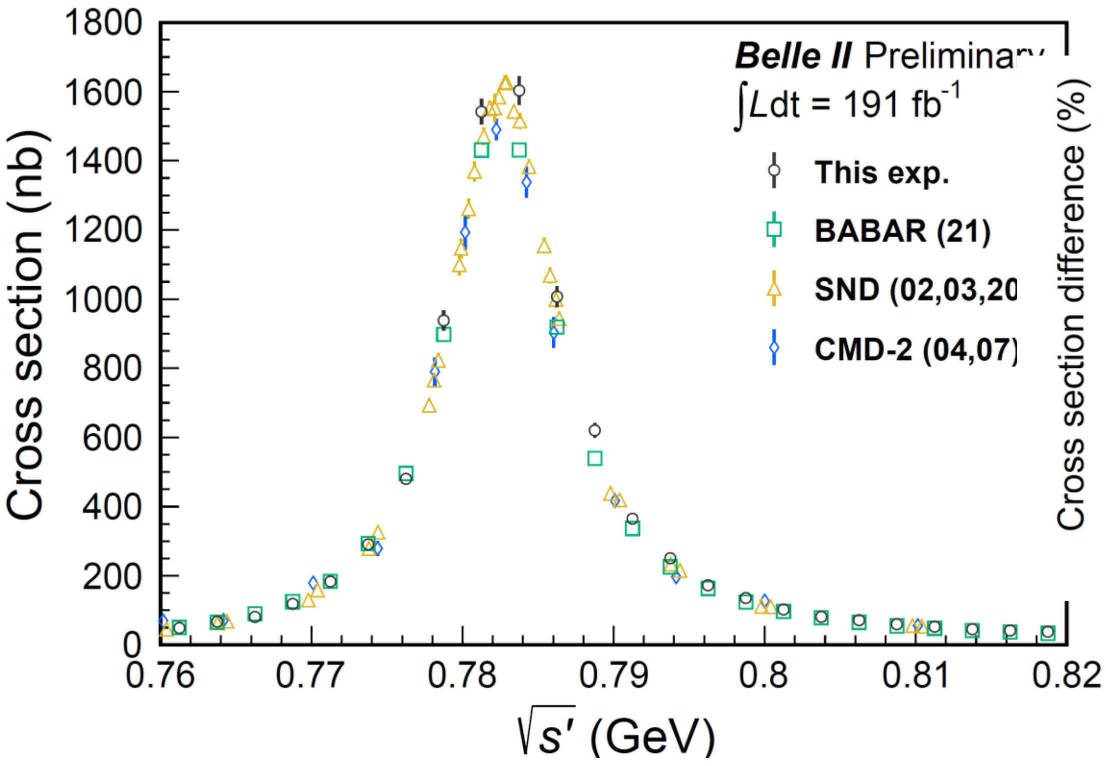


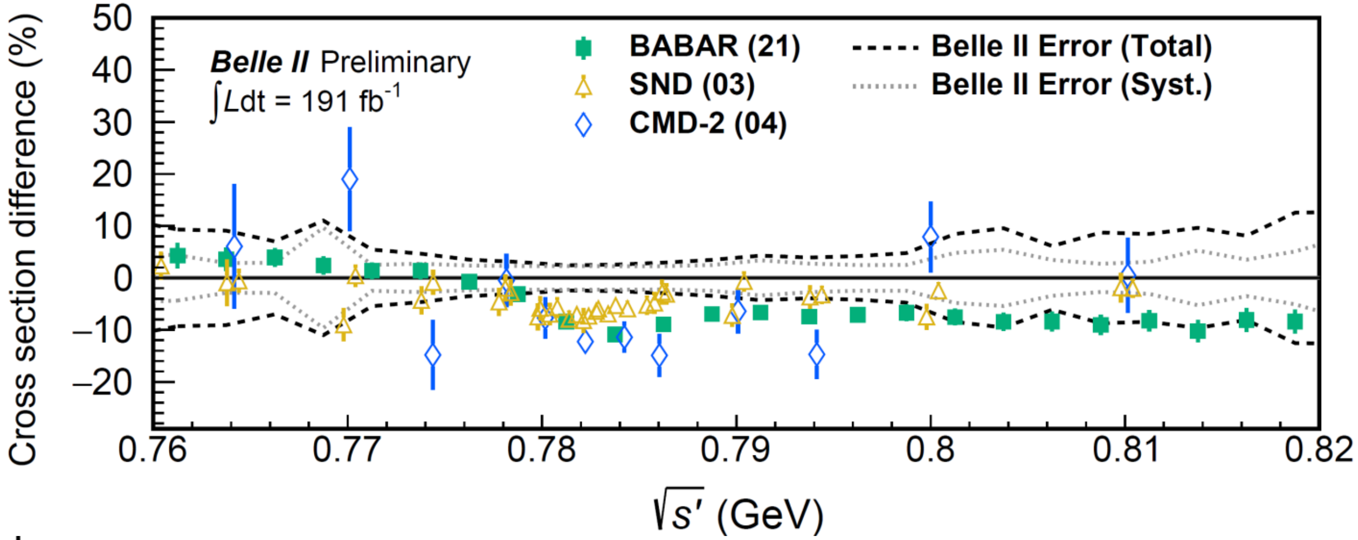
NNLO ISR



Result: cross section at ω

ω(782)



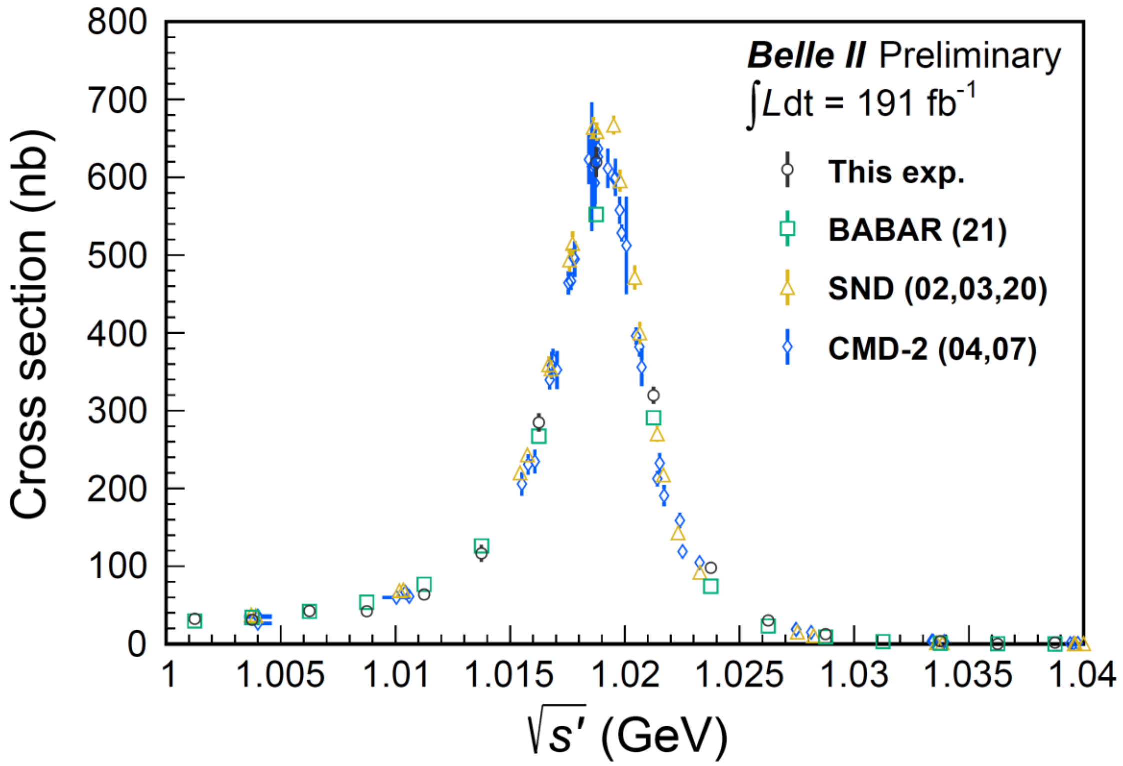


Belle II cross section at ω resonance is 5-10% higher than that from BABAR and SND.

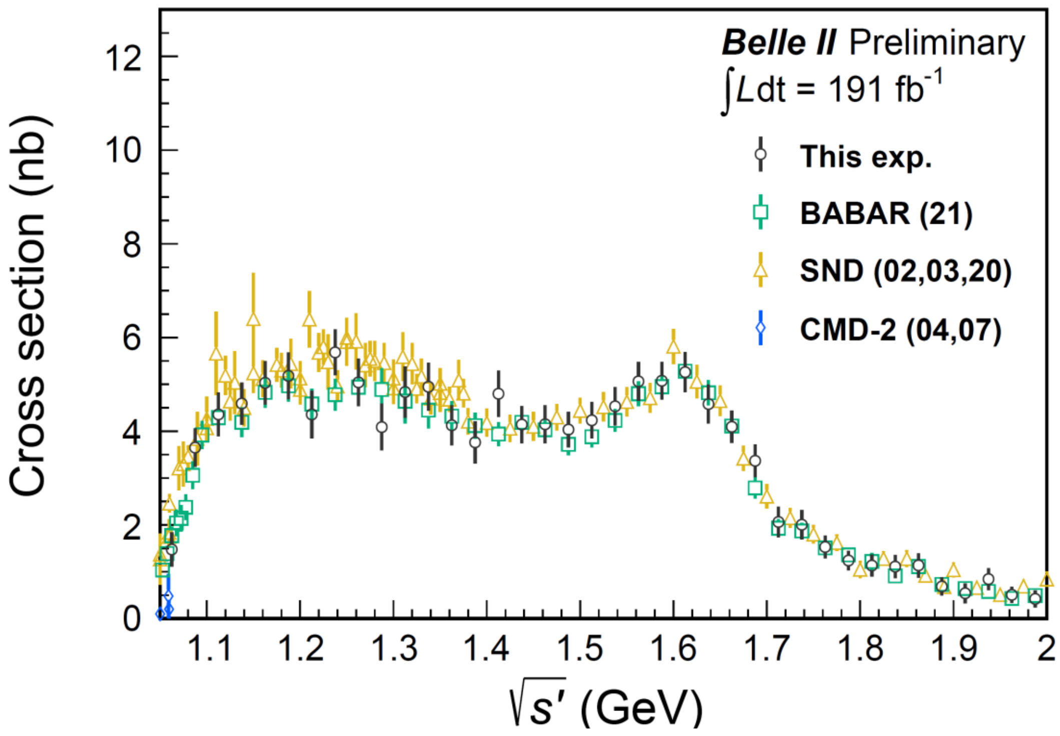
Result: cross section above 1.0 GeV

- Good agreement with BABAR results

Φ(1020)



$1.05\ \mathbf{GeV} < M(3\pi) < 2.0\ \mathbf{Gev}$



Results: 3π contribution to a_μ HVP

$$a_\mu^{\mathrm{LO,HVP,3\pi}}(0.62\text{-}\,1.8\ \mathrm{GeV}) = \left(48.91 \pm 0.25_{\mathrm{stat}} \pm 1.07_{\mathrm{syst}}\right) \times 10^{-10}$$

	$a_\mu(3\pi)\times10^{10}$	Difference$\times10^{10}$
BABAR alone [PRD104 11 (2021)]	45.86 ± 0.14 ± 0.58	-2.5σ (6.9%)
Global fit [JHEP08 208 (2023)]	45.91 ± 0.37 ± 0.38	-2.5σ (6.5%)

- 6.5% higher than the global fit result with 2.5σ significance
- This difference $3\mathrm{x}10^{-10}$ corresponds 10% of $\Delta a_\mu = a_\mu(\mathrm{Exp}) - a_\mu(\mathrm{SM}) = 25\mathrm{x}10^{-10}$

$$a_\mu^{\mathrm{HVP,LO}} = \frac{\alpha}{3\pi^2}\int_{m_\pi^2}^{\infty}\frac{K(s)}{s}R_{\mathrm{had}}(s)ds,$$

$$R_{\mathrm{had}}(s) = \frac{\sigma_0(e^+e^- \to \mathrm{hadrons})}{\sigma_{\mathrm{pt}}(e^+e^- \to \mu^+\mu^-)},$$

$$\sigma_0 = \sigma^{\mathrm{dressed}}|1-\Pi(s')|^2.$$

↑ measured

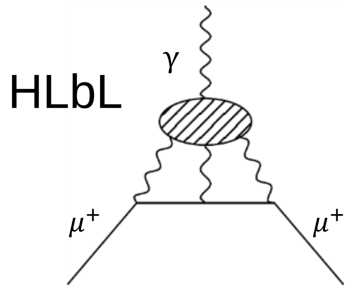
Systematic uncertainty for a_μ

Source	Systematic uncertainty (%)
Efficiency corrections	1.63
Monte Carlo generator	1.20
Integrated luminosity	0.64
Simulated sample size	0.15
Background subtraction	0.02
Unfolding	0.12
Radiative corrections	0.50
Vacuum polarization corrections	0.04
Total	2.19

Summary and Prospects

- We report a measurement of the $e^+e^- \to \pi^+\pi^-\pi^0$ cross section using ISR events with 2.2 % (1.6%(exp.)+1.2%(MC)) accuracy at ω. This is the first ISR results from Belle II.

  See arXiv:2404.04915 for details.
- Our results are about 2.5σ greater than BABAR and a global fit.
  - $a_\mu^{\mathrm{LO,HVP,}}(3\pi) = \left(48.91 \pm 0.25_{\mathrm{stat}} \pm 1.07_{\mathrm{syst}}\right) \times 10^{-10}$
- Statistical errors are still dominated except ω and ϕ peaks. Systematic errors can also be improved by adding more data.
- For further improvement, QED NNLO MC generators are crucial.
- We are planning to make precise measurements on the cross sections in other ISR processes, $e^+e^- \rightarrow \pi^+\pi^-$, as well as 4π, $\bar{p}$p, $\mathrm{KK}\pi$…
- In addition, We are planning π^0, η, η' production in single-tag processes $e^+e^- \to e(e)\pi^0, \eta, \eta'$, which are important for HLbL terms.



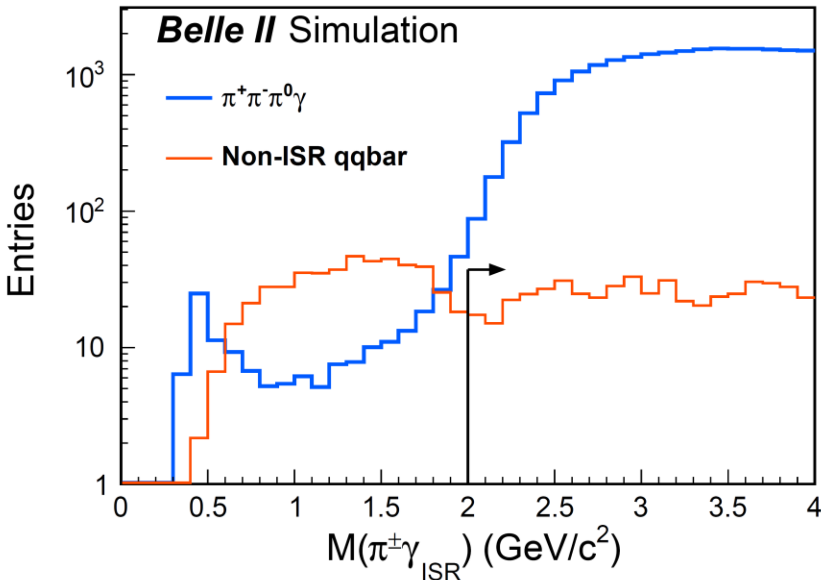
Backup

Background suppression (3)

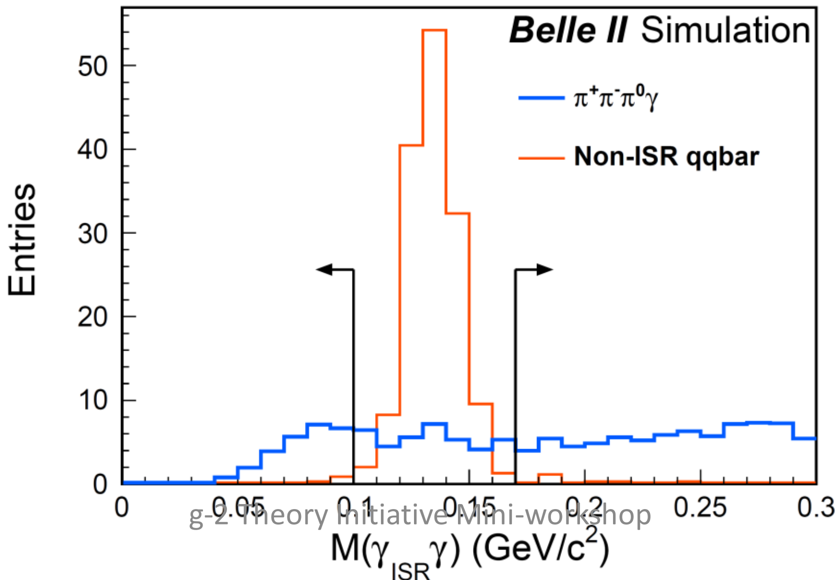
D)

i. $M(\pi^{\pm}\gamma_{ISR}) > 2$ GeV/c^2 to reduce high momentum $\rho^{\pm} \rightarrow \pi^{+}\pi^{0}$
ii. $M(\gamma_{ISR}\gamma)$ cut to reduce ISR candidate from π^0-decay photon
iii. Cluster shape cut to reduce ISR-like photon in which two photons from of π^0 are merged

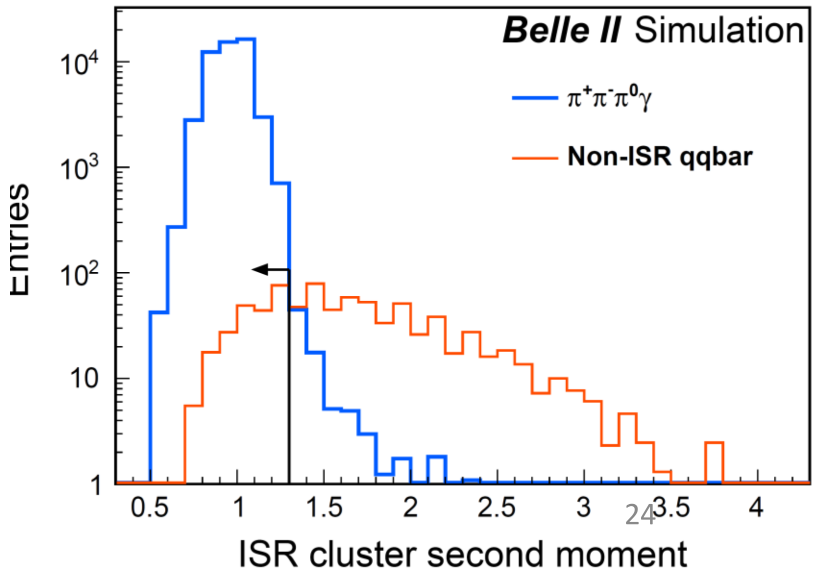
i) $M(\pi^{\pm}\gamma_{ISR})$ cut



ii) $M(\gamma_{ISR}\gamma)$ cut

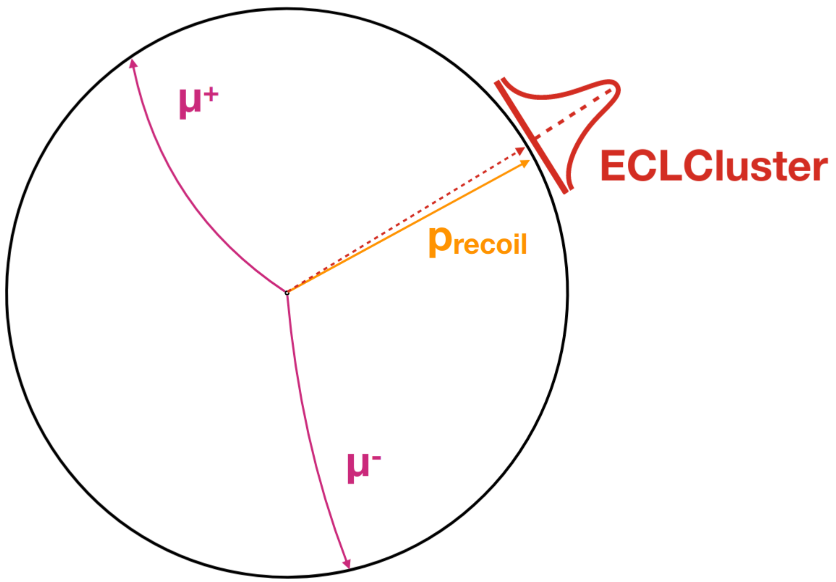


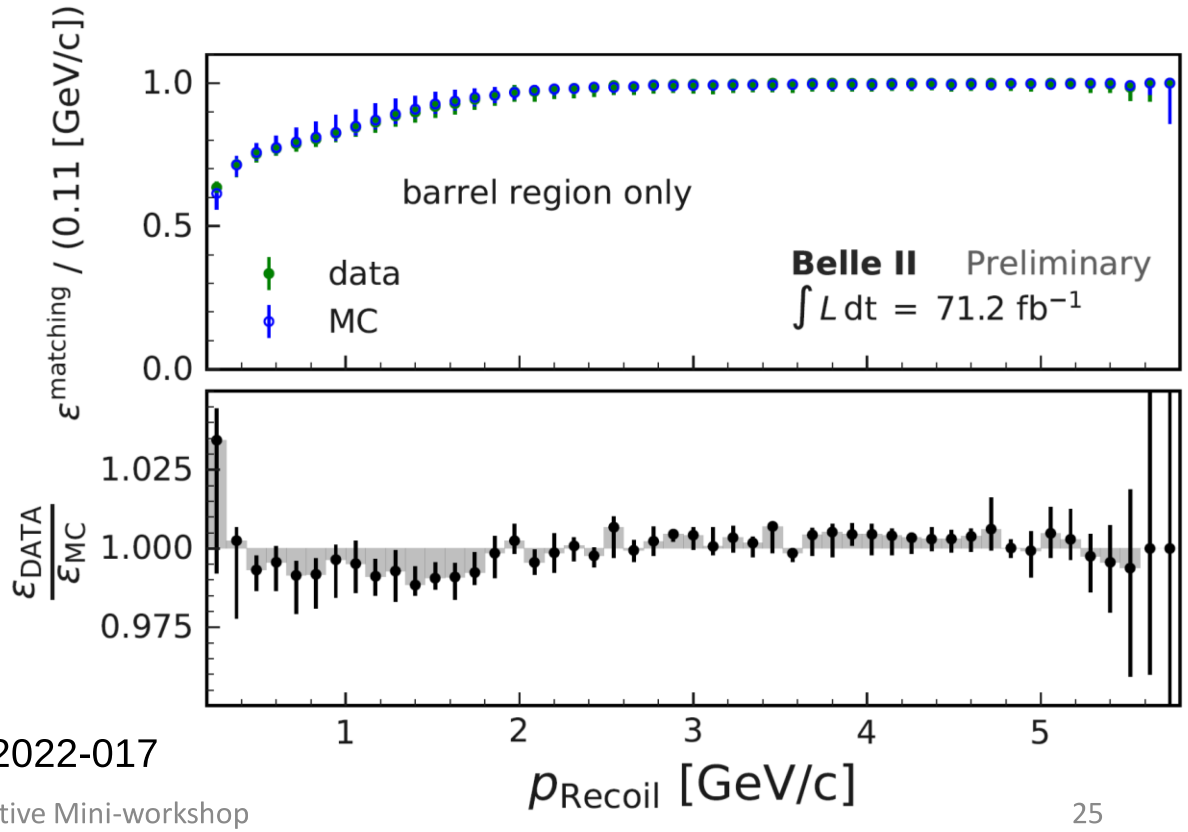
iii) ISR photon cluster shape cut



ISR photon detection efficiency

- Photon detection efficiency is measured using $e^+e^- \to \mu^+\mu^-\gamma$ events
  - Taking a match between a calorimeter cluster and the missing momentum of dimuon system
- Efficiency is in good agreement w





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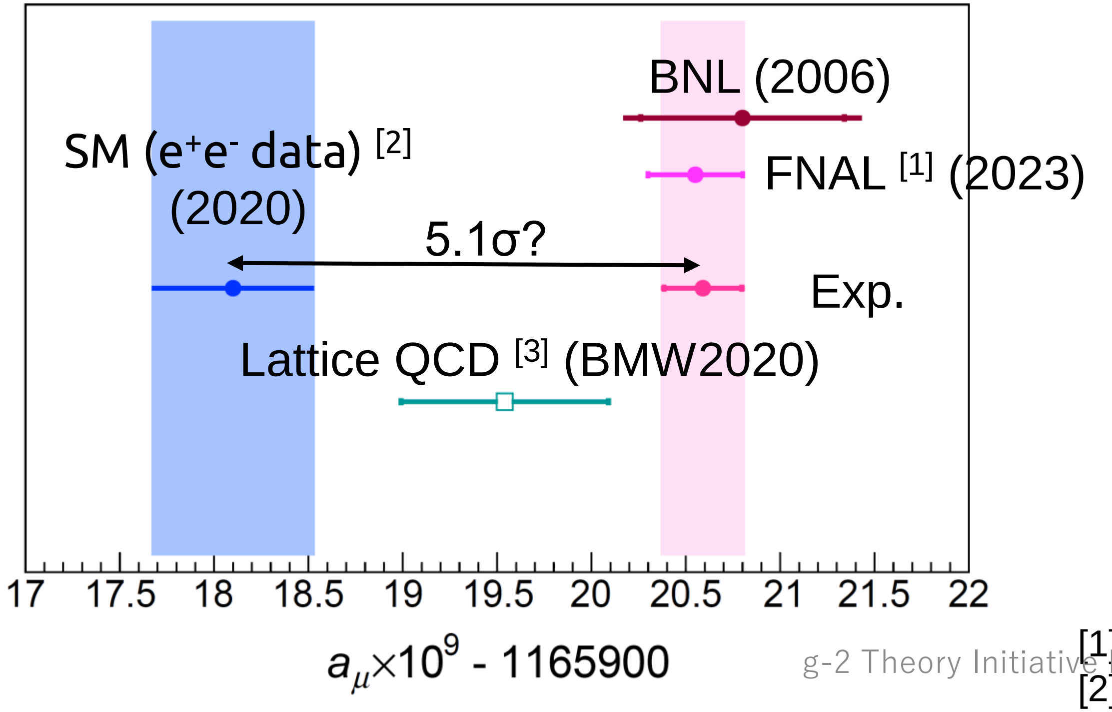
Higher-order ISR effects

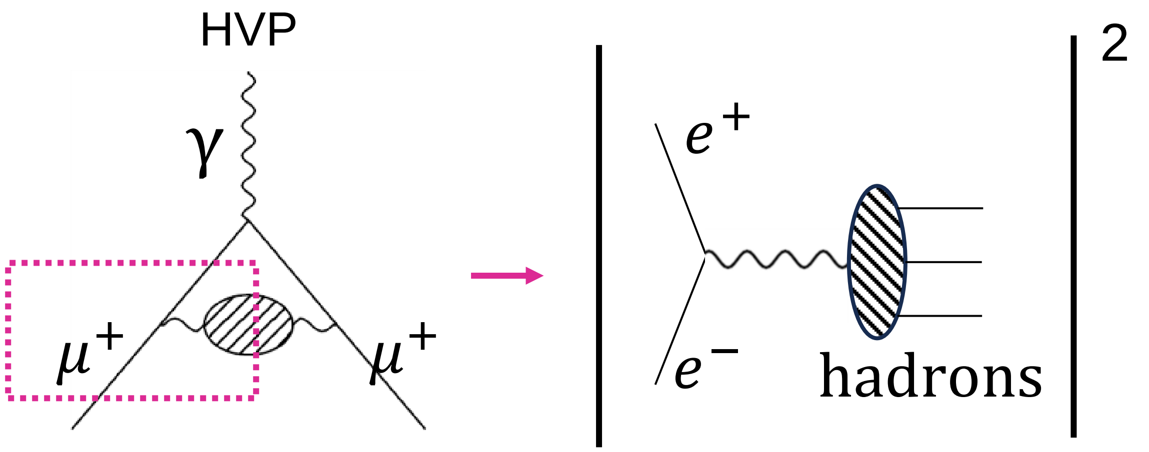
- 20% excess in the small angle additional photon events in the PHOKHARA MC is reported by BABAR
  - We verify the BABAR observation in PHOKHARA in the Belle II data.
  - Signal efficiency can change because most NLO additional photon events are rejected by χ2 criteria in kinematical fit.
  - Due to this effect, we confirm that the χ2 efficiency by PHOHARA is underestimated by (2.4 ± 0.7)%.
- Using KKMC, we estimate that efficiency will also change due to NNLO effects by 1.9% in the opposite direction.
- As a result of these consideration, we conclude
  - No correction is assigned to our results but,
  - 1.2% systematic uncertainty is accounted for as MC generator-derived error
    - 0.7%(NLO excess error) ⊕ 0.95% (half of NNLO effect) = 1.2%

Recent situation in muon g-2 anomaly

- 5σ significance through new direct measurements from Fermilab
- Non-negligible uncertainty in theoretical predictions
  - Major uncertainty is derived from Hadronic Vacuum Polarization (HVP) term
  - HVP predictions are different depending on methods: e^+e^- data vs Lattice QCD



HVP calculation with ee→hadrons data



[1] PRL 131 161802 (2023)
[2] Phys. Rept. 887, 1 (2020)
[3] Nature 593, 7857 (2021)

Recent situation in muon g-2 anomaly

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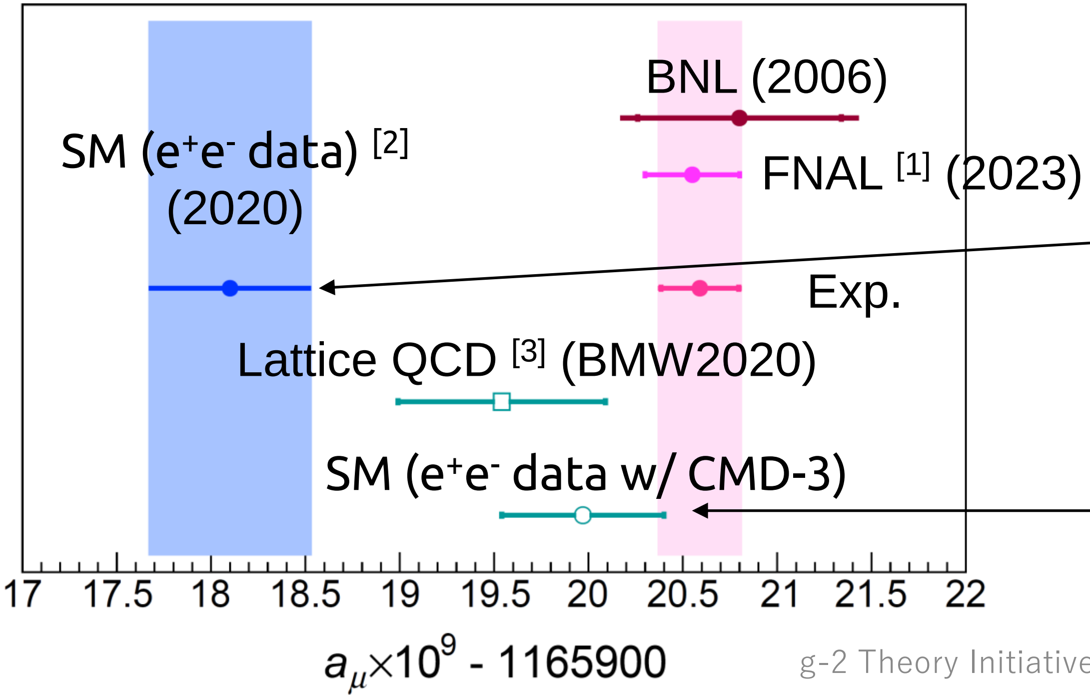
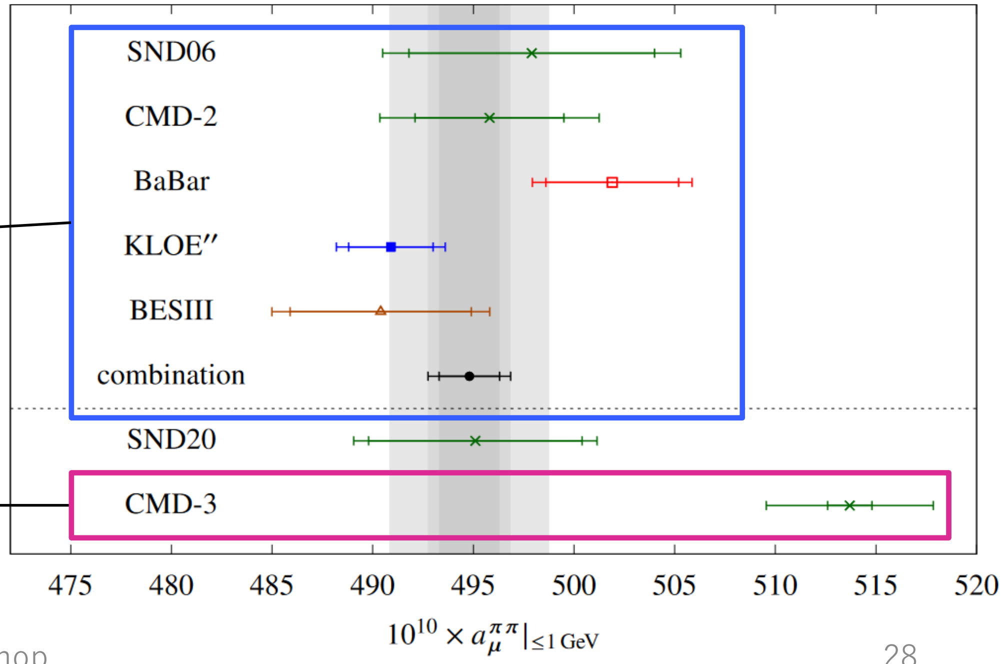
- Non-negligible uncertainty in theoretical predictions
  - Major uncertainty is derived from Hadronic Vacuum Polarization (HVP) term
  - HVP predictions are different depending on methods: e^+e^- data vs Lattice QCD
  - Differences among e^+e^- experiments are also non-negligible
- Validation by independent experiments is important in HVP prediction



ee→ππ contribution to HVP



G. Colangelo *et. al*, arXiv:2308.04217

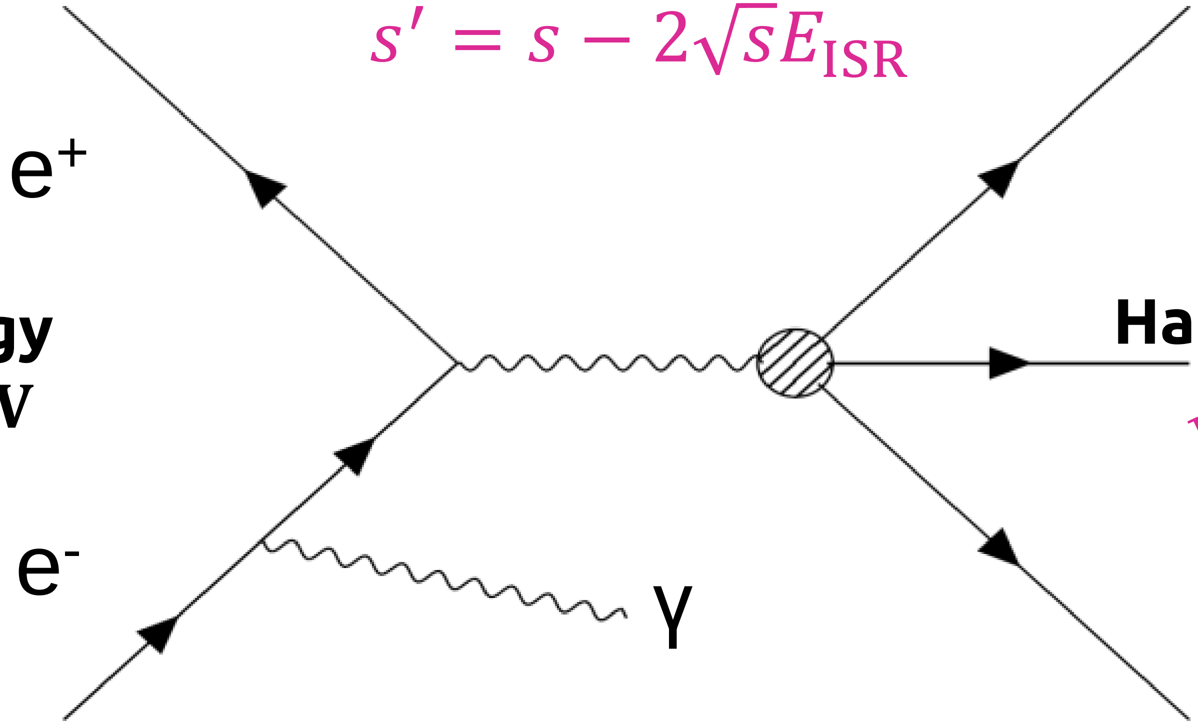
Radiative return method

- Measure the cross section in the energy range 0.4-3.5 GeV at fixed e^+e^- energy collision
- Use a process associated with energetic ISR emission
  - Only less than10% of ISR photons are emitted into detector acceptance

$$s' = s - 2\sqrt{s}E_{\mathrm{ISR}}$$

Initial e^+e^- energy

$\sqrt{s} = \mathbf{10.58\ GeV}$



Hadronic system energy

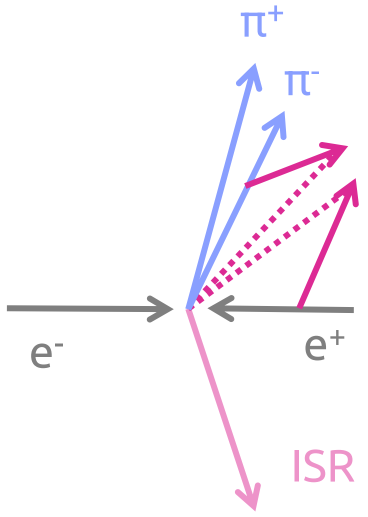
$\sqrt{s'} = \mathbf{0.4 - 3.5\ GeV}$

Initial-state radiation : ISR

$E_{\mathrm{ISR}} > 4.7 - 5.3\ \mathrm{GeV}$

$e^+e^- \to \pi^+\pi^-\pi^0\gamma_{ISR}$ selection: Background suppression

Apply background suppression criteria to reduce remaining backgrounds

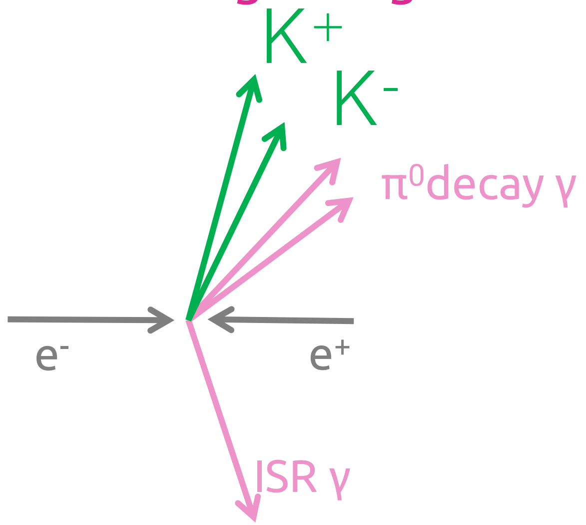


e.g. cluster from detector interaction

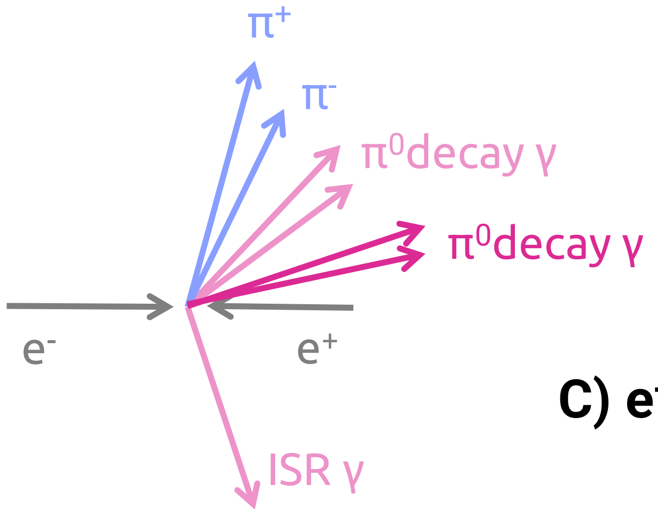
e.g. cluster from beam background

A) Background not containing real π^0:
e.g., $e^+e^- \to e^+e^-\gamma$, $\mu^+\mu^-\gamma$, $\pi^+\pi^-\gamma$

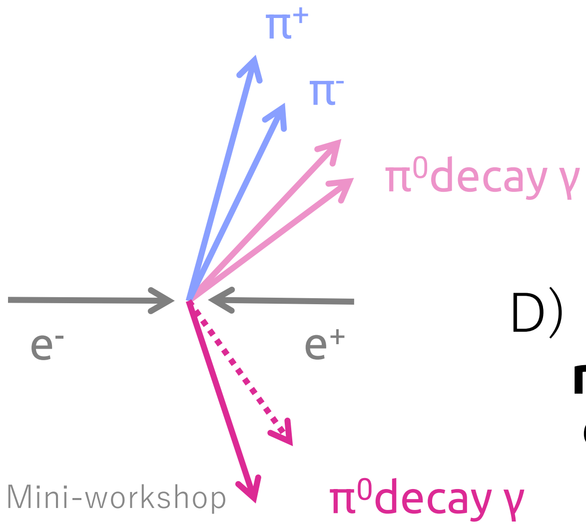
These bkg do not create any π^0 *peak in* $M_{\gamma\gamma}$



B) Kaon background:
$e^+e^- \to K^+K^-\pi^0\gamma_{ISR}$



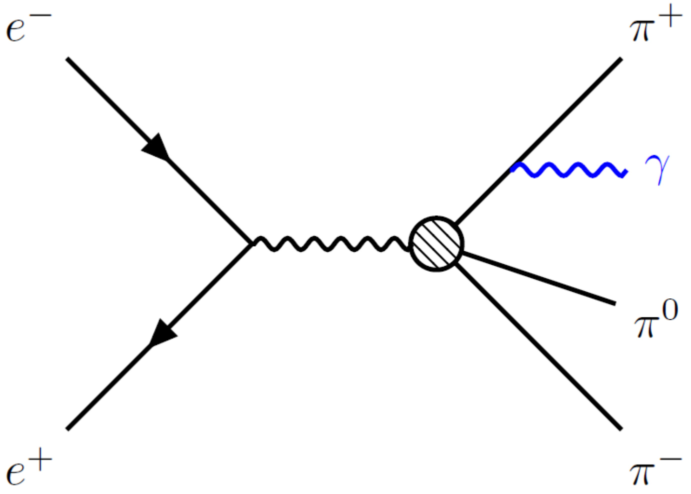
C) $e^+e^- \to \pi^+\pi^-\pi^0\pi^0\gamma_{ISR}$



D) **Background**
not containing real ISR:
$e^+e^- \to \pi^+\pi^-\pi^0\pi^0$, $\tau^+\tau^-$

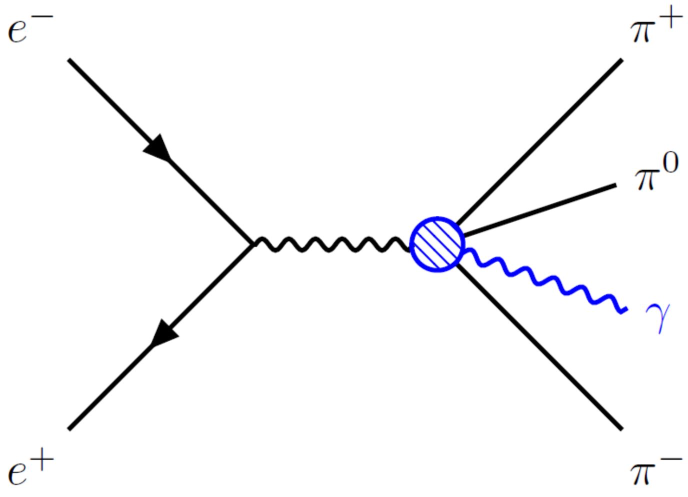
Final-state radiation background

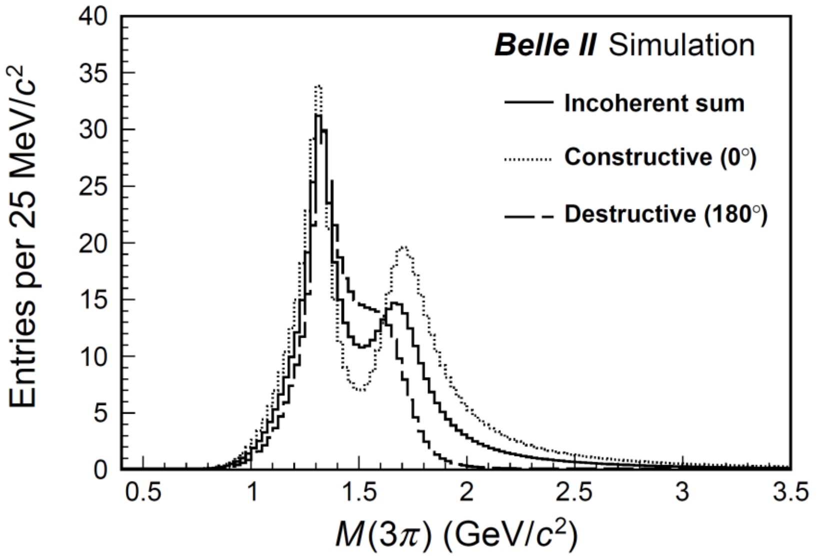
- Difficult to reject FSR background or extract control sample
- Estimate FSR background using pQCD prediction based on the BABAR previous analysis [PRD112003]



FSR emission from final-state pions

~ 0.001fb → < 1 event occur





FSR emission from the quark legs

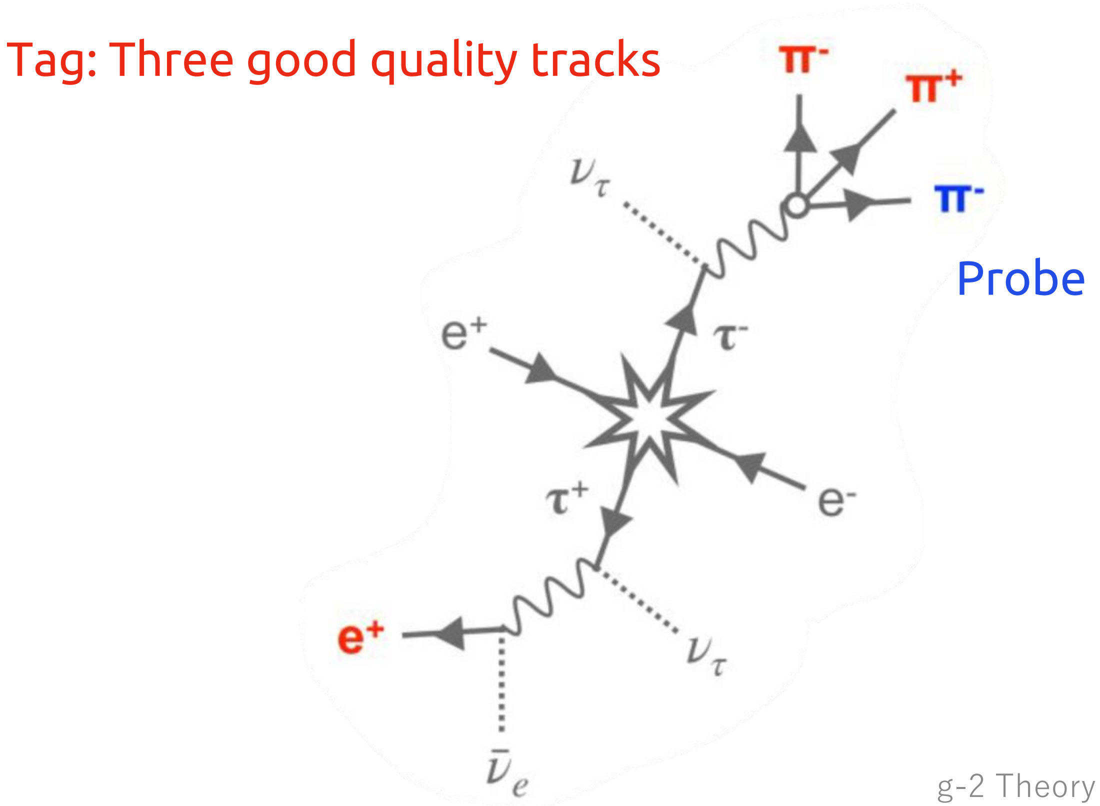
- $e^+e^- \to M\gamma_{FSR} \to \pi^+\pi^-\pi^0\gamma_{FSR}$;
  M= η, $a_1(1260)$, $a_2(1320)$, $a_1(1640)$, $a_2(1700)$, $a_1(1930)$, $a_2(2030)$

Efficiency correction : Summary

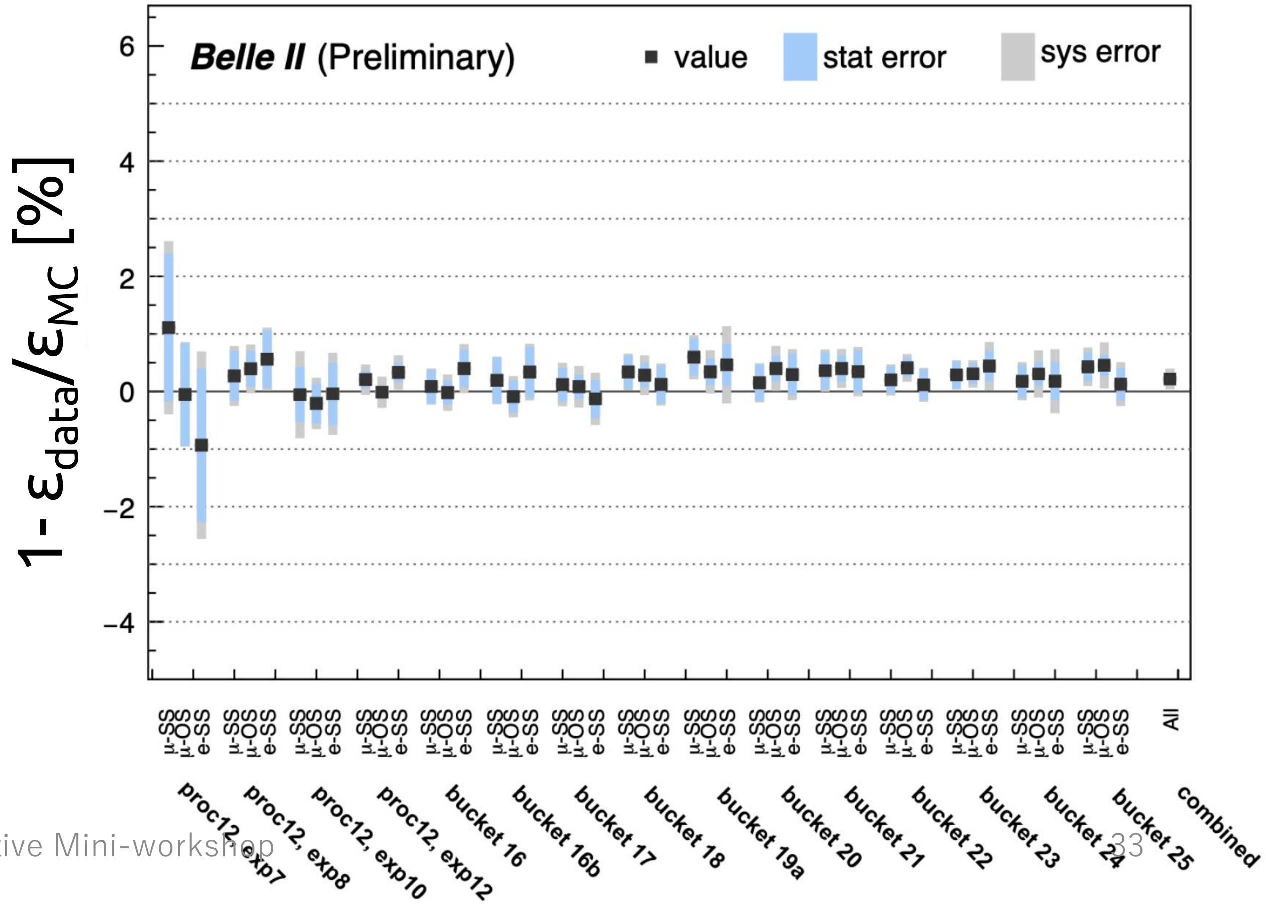
Source	Efficiency correction (%)	
	M < 1.05 GeV/c^2	M > 1.05 GeV/c^2
Trigger	-0.1 ± 0.1	-0.1 ± 0.1
ISR photon detection	0.2 ± 0.7	+0.2 ± 0.7
Tracking	-1.4 ± 0.8	-1.7 ± 0.8
π^0 detection	-1.4 ± 1.0	-1.4 ± 1.0
Background suppression	-1.9 ± 0.2	-1.8 ± 1.9
χ^2 distribution	0.0 ± 0.6	0.3 ± 0.3
MC generator	0.0 ± 1.2	0.0 ± 1.2
Total correction	-4.6 ± 2.0	-4.6 ± 2.0

Tracking efficiency

- Tracking efficiency for pions is studied with the $e^+e^-\rightarrow\tau^+\tau^-$ process.
- Data-MC differences are confirmed to be small with 0.3% uncertainty per track.



Data-MC discrepancy of tracking efficiency

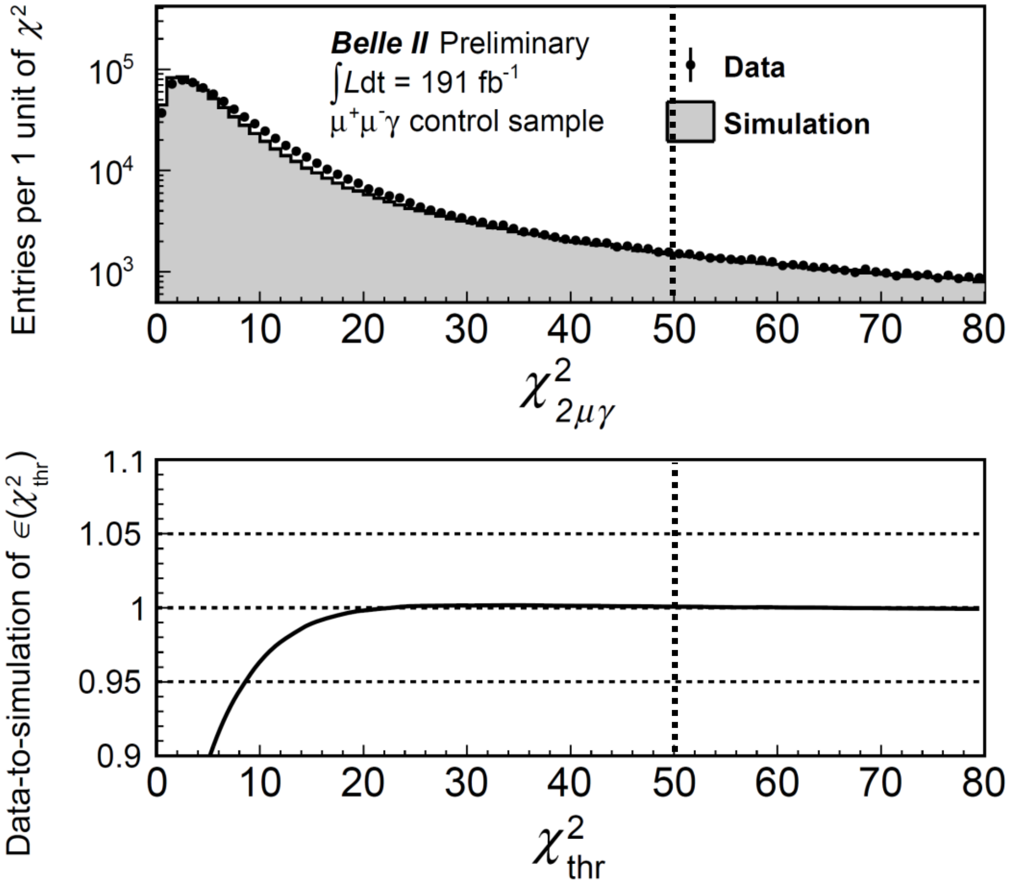


Uncertainty on background suppression

- Estimated by the ratio of signal yield before/after the criteria
- It is evaluated using ω,ϕ, and J/ψ resonances of good S/N
- In M(3π) < 1.05 GeV/c^2, efficiency is (89.5 ± 0.2)% for data
  - Data-MC difference is $\varepsilon_{data}/\varepsilon_{MC}$ -1 = (-1.90 ± 0.20)%
- M(3π) > 1.05 GeV/c^2 : the number of J/ψ was obtained by M(3π) fitting
  - Data-MC difference is $\varepsilon_{data}/\varepsilon_{MC}$ -1 =(-1.78 ± 1.85)%
  - Error is due to statistical errors in the sample

χ2 kinematic selection efficiency

- ISR and tracks χ^2-criteria efficiency is confirmed using $e^+e^-\to\mu^+\mu^-\gamma$ sample
- Confirm effects from differences in position, momentum, and energy of ISR and tracks
  - Agreement confirmed within ±0.6% uncertainty
- Dependence on multi-ISR photon calculations is discussed on the next page

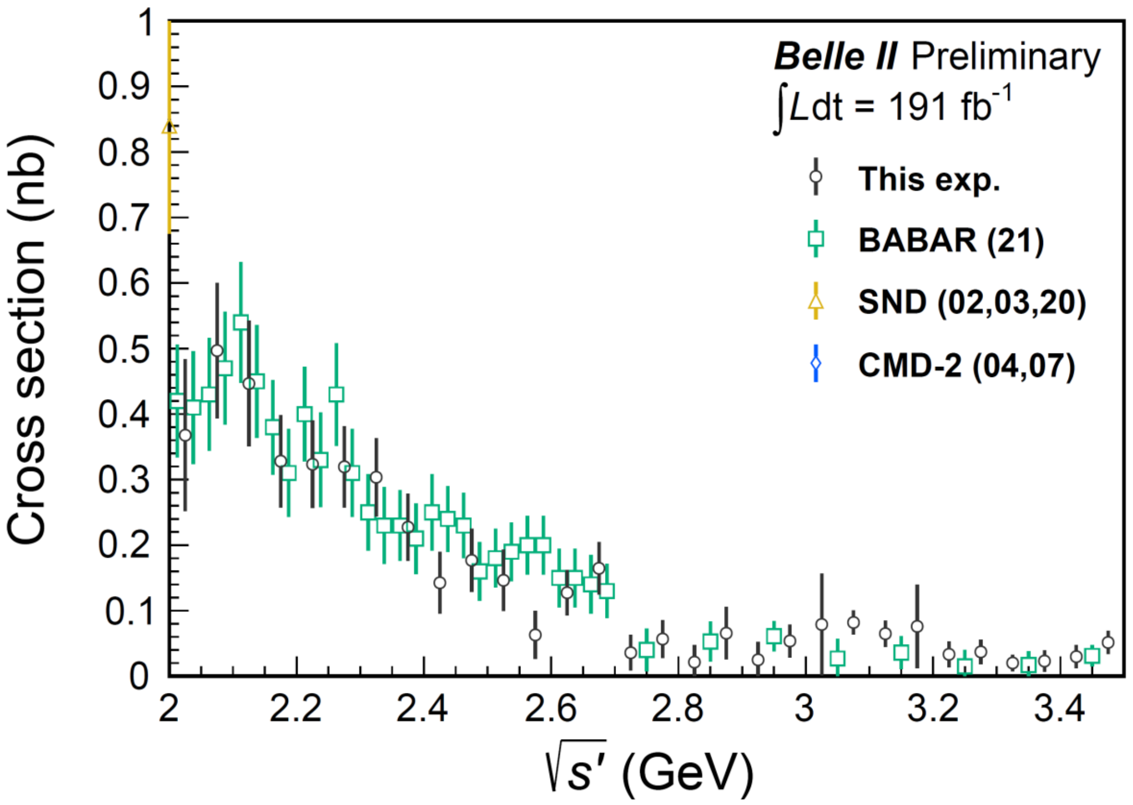


$$\varepsilon(\chi^2_{\text{thr}}) = \frac{N(\chi^2 < \chi^2_{\text{thr}})}{N_{\text{all}}}$$

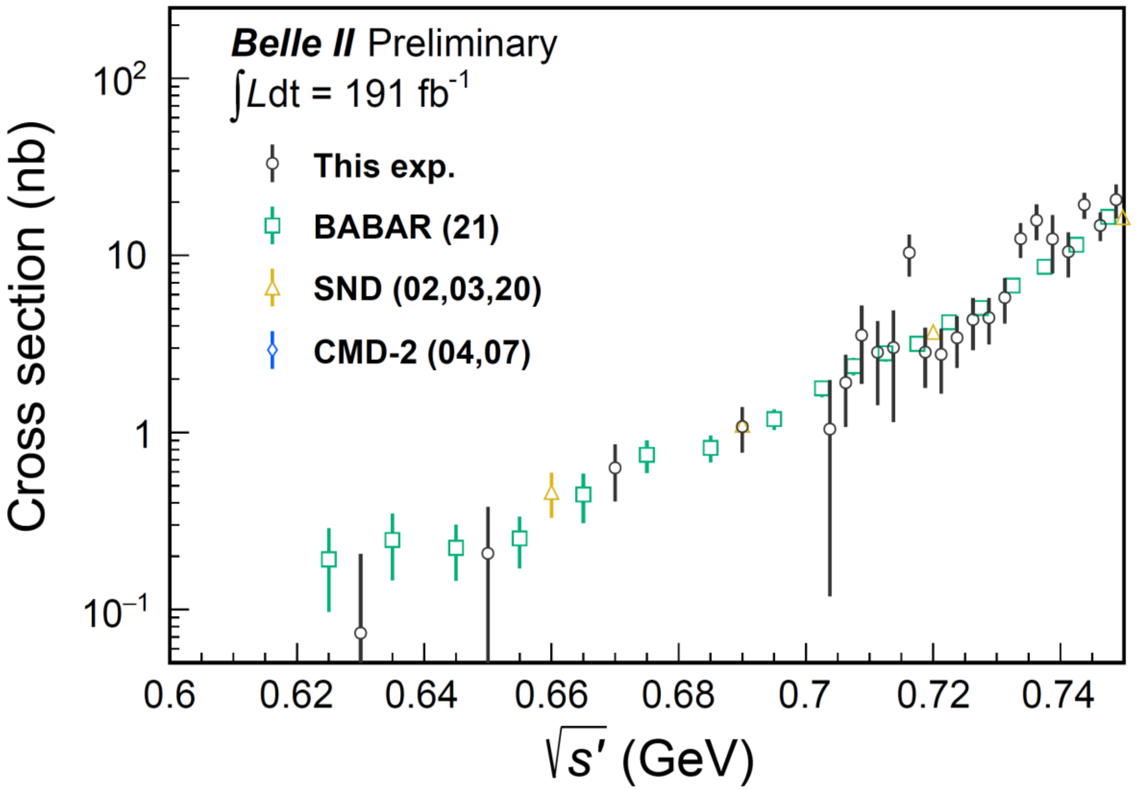
Data-MC ratio $\dfrac{\varepsilon_{\text{data}}(\chi^2_{\text{thr}})}{\varepsilon_{\text{MC}}(\chi^2_{\text{thr}})}$

Result: cross section at high and low masses

2.0 GeV < $M(3\pi) < 3.5$ Gev



0.62 GeV < $M(3\pi) < 0.75$ Gev



Comparison with BABAR 2021 measurement

- In quite a few respects, this analysis follows the BABAR method
- Systematic uncertainty is still nearly twice as large
  - NNLO generator is needed

	Belle II	BABAR (2021)
Dataset	191 fb^{-1}	469 fb^{-1}
Combinatorial γγ background	M(γγ) fit	MC
ISR energy in kinematic fit	Used	Not used
Generator	PHOKHARA	AfkQed
Generator uncertainty	1.2%	-
Detection efficiency uncertainty	1.6%	1.1%
Integrated luminosity	0.6%	0.3%
Total systematic uncertainty for $a_\mu(3\pi)$	2.2%	1.3%