HLbL direct lattice calculation from RBC-UKQCD

Luchang Jin

University of Connecticut

April 16, 2024

Muon $g-2$ theory initiative virtual spring meeting

Schedule

- 8:00 - 8:13 AM HLbL direct lattice calculation from RBC-UKQCD (Luchang Jin)
- 8:13 - 8:26 AM HLbL direct lattice calculation from Mainz-CLS (Harvey Meyer)
- 8:26 - 8:39 AM HLbL direct lattice calculation from BMW
- 8:39 - 8:52 AM HLbL direct lattice calculation charm quark contribution (En-Hung Chao)
- 8:52 - 9:05 AM Pion (eta, eta') pole contribution to HLbL from Mainz-CLS (Jonna Koponen)
- 9:05 - 9:18 AM Pion (eta, eta') pole contribution to HLbL from BMW (Antoine Gerardin)
- 9:18 - 9:31 AM Pion (eta, eta') pole contribution to HLbL from ETM
- 9:31 - 9:44 AM Pion (eta, eta') pole contribution to HLbL from RBC-UKQCD (Xu Feng)
- 9:44 - 10:00 AM General discussion

- RBC-UKQCD 2023 T. Blum et al 2023 (arXiv:2304.04423 [hep-lat])

Hadronic light-by-light contribution to the muon anomaly from lattice QCD with infinite volume QED at physical pion mass

Thomas Blum,[1,*] Norman Christ,[2] Masashi Hayakawa,[3,4] Taku Izubuchi,[5,6]
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(RBC and UKQCD Collaborations)

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(Dated: April 11, 2023)

The hadronic light-by-light scattering contribution to the muon anomalous magnetic moment, $(g-2)/2$, is computed in the infinite volume QED framework with lattice QCD. We report $a_\mu^{\mathrm{HLbL}} = 12.47(1.15)(0.99) \times 10^{-10}$ where the first error is statistical and the second systematic. The result is mainly based on the 2+1 flavor Möbius domain wall fermion ensemble with inverse lattice spacing $a^{-1} = 1.73$ GeV, lattice size $L = 5.5$ fm, and $m_\pi = 139$ MeV, generated by the RBC-UKQCD collaborations. The leading systematic error of this result comes from the lattice discretization. This result is consistent with previous determinations.

- Earlier work by RBC-UKQCD:

  RBC-UKQCD 2019 T. Blum et al 2020 (PRL 124, 13, 132002)

- RBC-UKQCD 2023 T. Blum et al 2023 (arXiv:2304.04423 [hep-lat])

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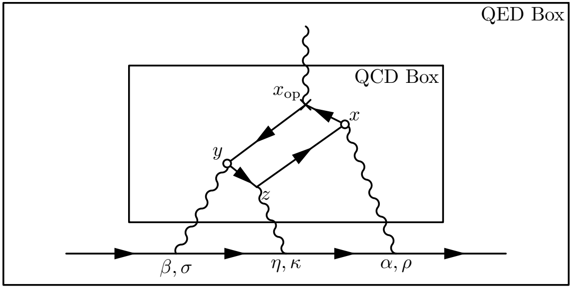
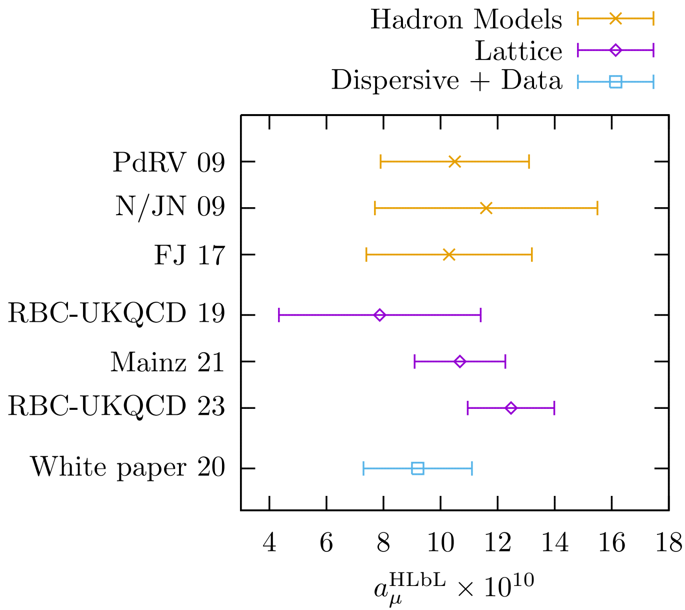
- Earlier work by RBC-UKQCD:
  RBC-UKQCD 2019 T. Blum et al 2020 (PRL 124, 13, 132002)

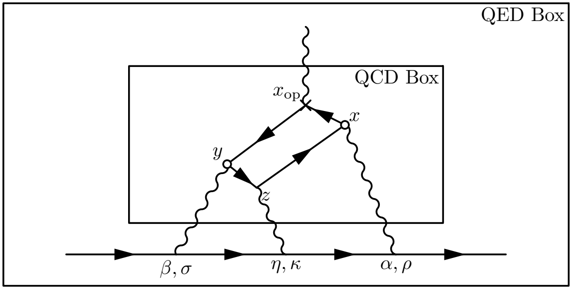
- This talk: mainly describe the recent RBC-UKQCD 2023 calculation.
  - Try to explain all major choices made in the formulation of the calculation.
  - Compare with Mainz's calculation E.H. Chao et al. 2021 (EPJC 81, 7, 651)

HLbL: lattice calculation setup



- RBC-UKQCD 2019: $L_{\rm QED} = L_{\rm QCD}$: $4.67 \sim 6.22$ fm, m_π: $135 \sim 144$ MeV, Domain wall fermion. T. Blum et al 2020 (PRL 124, 13, 132002)
- Mainz 2021: $L_{\rm QED} = \infty$, m_π: $200 \sim 422$ MeV, Wilson fermion. E.H. Chao et al. 2021 (EPJC 81, 7, 651)
- RBC-UKQCD 2023: $L_{\rm QED} = \infty$: 5.5 fm, $m_\pi = 139$ MeV, $a^{-1} = 1.73$ GeV, Domain wall fermion. T. Blum et al 2023 (arXiv:2304.04423 [hep-lat])


Hadron Models
Lattice
Dispersive + Data
PdRV 09
N/JN 09
FJ 17
RBC-UKQCD 19
Mainz 21
RBC-UKQCD 23
White paper 20
4
6
8
10
12
14
16
18
$a_\mu^{\mathrm{HLbL}} \times 10^{10}$



- The muon propagate in the lattice time direction. We do NOT average the muon propagation direction as in Mainz's work [E.H. Chao et al. 2021 (EPJC 81, 7, 651)].

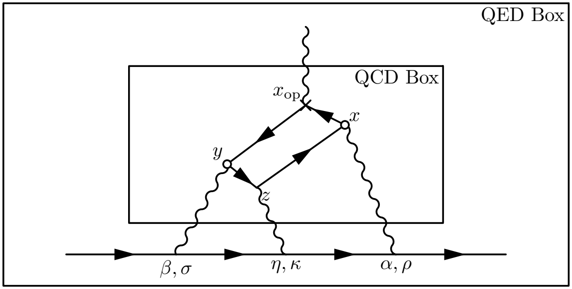
$$a_\mu^{\rm HLbL} = \frac{2me^2}{3}\frac{1}{VT}\sum_{x_{\rm op}}\sum_{x,y,z}\frac{1}{2}\epsilon_{i,j,k}\left(x_{\rm op} - x_{\rm ref}(x,y,z)\right)_j (6e^4)\mathcal{H}_{k,\rho,\sigma,\kappa}(x_{\rm op},x,y,z)\mathcal{M}_{i,\rho,\sigma,\kappa}(x,y,z)$$

$$(6e^4)\mathcal{H}_{k,\rho,\sigma,\kappa}(x_{\rm op},x,y,z) = \langle T J_k(x_{\rm op})J_\rho(x)J_\sigma(y)J_\kappa(z)\rangle_{\rm QCD}$$

$$\mathcal{M}_{i,\rho,\sigma,\kappa}(x,y,z) = \frac{1}{2}{\rm Tr}\left[\frac{1}{6}i^3\mathcal{G}_{\rho,\sigma,\kappa}(x,y,z)\Sigma_i\right].$$

T. Blum et al 2017. (PRD 96 3, 034515)

HLbL QED$_\infty$: formulation subtracted QED weighting function



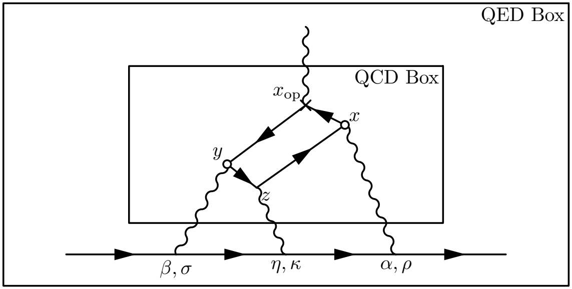
$$
\begin{aligned}
i^3\mathcal{G}_{\rho,\sigma,\kappa}(x,y,z) =& \mathfrak{G}_{\rho,\sigma,\kappa}(x,y,z) + \mathfrak{G}_{\sigma,\kappa,\rho}(y,z,x) + \mathfrak{G}_{\kappa,\rho,\sigma}(z,x,y) \\
&+ \mathfrak{G}_{\kappa,\sigma,\rho}(z,y,x) + \mathfrak{G}_{\rho,\kappa,\sigma}(x,z,y) + \mathfrak{G}_{\sigma,\rho,\kappa}(y,x,z), \\
\mathfrak{G}_{\sigma,\kappa,\rho}(y,z,x) =& \lim_{t_{\rm src}\to-\infty, t_{\rm snk}\to\infty} e^{m_\mu(t_{\rm snk}-t_{\rm src})} \int_{\alpha,\beta,\eta} G(x,\alpha)G(y,\beta)G(z,\eta) \\
&\times \int_{\vec{x}_{\rm snk},\vec{x}_{\rm src}} S_\mu(x_{\rm snk},\beta)\, i\gamma_\sigma S_\mu(\beta,\eta) i\gamma_\kappa S_\mu(\eta,\alpha) i\gamma_\rho S_\mu(\alpha,x_{\rm src}),
\end{aligned}
$$

Subtraction to (1) remove infrared divergence; **(2) reduce discretization and finite volume effects.**

$$
\begin{aligned}
\mathfrak{G}^{(1)}_{\sigma,\kappa,\rho}(y,z,x) =& \frac{1}{2}\mathfrak{G}_{\sigma,\kappa,\rho}(y,z,x) + \frac{1}{2}[\mathfrak{G}_{\rho,\kappa,\sigma}(x,z,y)]^\dagger, \\
\mathfrak{G}^{(2)}_{\sigma,\kappa,\rho}(y,z,x) =& \mathfrak{G}^{(1)}_{\sigma,\kappa,\rho}(y,z,x) - \mathfrak{G}^{(1)}_{\sigma,\kappa,\rho}(z,z,x) - \mathfrak{G}^{(1)}_{\sigma,\kappa,\rho}(y,z,z).
\end{aligned}
$$

T. Blum et al 2017. (PRD 96 3, 034515)

HLbL QED$_\infty$: consequence of subtraction

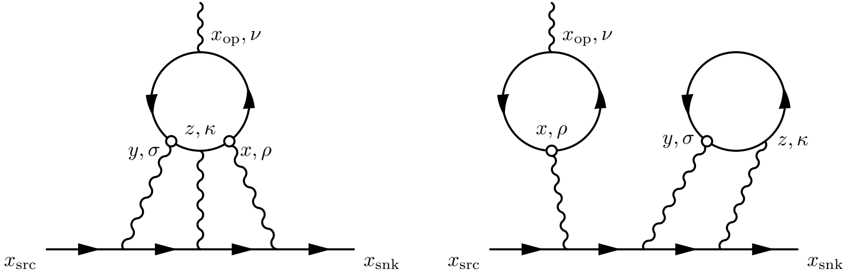


$$\mathfrak{G}^{(2)}_{\sigma,\kappa,\rho}(y,z,x) = \mathfrak{G}^{(1)}_{\sigma,\kappa,\rho}(y,z,x) - \mathfrak{G}^{(1)}_{\sigma,\kappa,\rho}(z,z,x) - \mathfrak{G}^{(1)}_{\sigma,\kappa,\rho}(y,z,z).$$

- Due to the current conservation condition must satisfied by the hadronic four-point function.
- Change discretization error (when use local current). Change finite volume error.
- **Change the integrand.** Later plots depends on the choice of subtraction scheme.
- Does not change the final results (infinite volume & continuum).
- Mainz's work [E.H. Chao et al. 2021 (EPJC 81, 7, 651)] uses a different subtraction scheme.

T. Blum et al 2017. (PRD 96 3, 034515)

HLbL QED$_\infty$: formulation - $x_{\text{ref}}(x, y, z)$



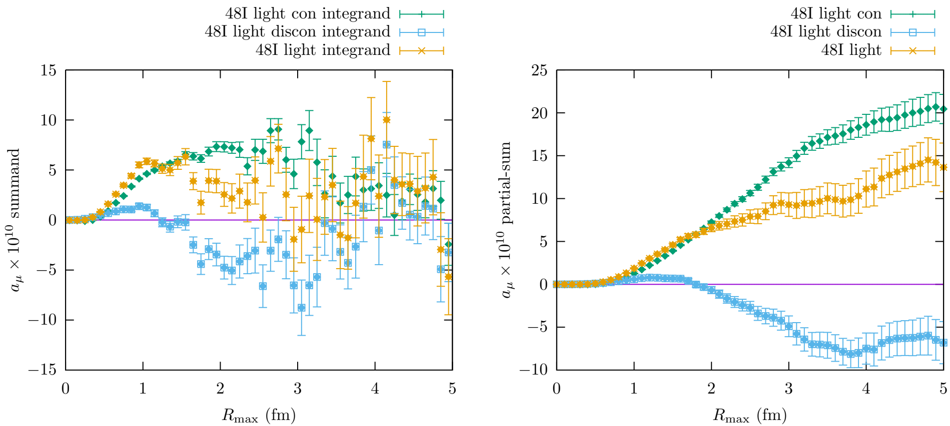
$$a_\mu^{\text{HLbL}} = \frac{2me^2}{3} \frac{1}{VT} \sum_{x_{\text{op}}} \sum_{x,y,z} \frac{1}{2} \epsilon_{i,j,k} \left(x_{\text{op}} - x_{\text{ref}}(x, y, z)\right)_j (6e^4) \mathcal{H}_{k,\rho,\sigma,\lambda}(x_{\text{op}}, x, y, z) \mathcal{M}_{i,\rho,\sigma,\lambda}(x, y, z)$$

$$x_{\text{ref}}(x, y, z) = x_{\text{ref-far}}(x, y, z) \tag{1}$$

$$= \begin{cases} x & \text{if } |y - z| < \min(|x - y|, |x - z|) \\ y & \text{if } |x - z| < \min(|x - y|, |y - z|) \\ z & \text{if } |x - y| < \min(|x - z|, |y - z|) \\ \frac{1}{3}(x + y + z) & \text{otherwise} \end{cases}$$

$$x_{\text{ref-discon}} = x. \tag{2}$$

T. Blum et al 2023 (arXiv:2304.04423 [hep-lat])



$$R_{\max} = \max(|x-y|, |y-z|, |x-z|)$$

- Already, we have $a_\mu^{\text{hlbl,light-quark}}(R_{\max} < 4 \text{ fm}) = 11.11(2.11) \times 10^{-10}$.
- Contributions mostly come from $1 \sim 3$ fm distance (depends on the choice of subtraction scheme).
- Significant cancellation between the connected and disconnected diagrams (almost statistically independent).

HLbL: long distance - π^0 exchange

- Try to take advantage of the known theoretical ratio of the connected diagram and disconnected diagram contribution at long distance.

$$R_{\text{max}} = \max(|x-y|, |y-z|, |x-z|),$$

- Use $a_\mu(R_{\text{max}} > R_{\text{max}}^{\text{cut}})$ denote contribution in the region where R_{max} larger than $R_{\text{max}}^{\text{cut}}$. We have:

$$\lim_{R_{\text{max}}^{\text{cut}} \to \infty} \frac{a_\mu^{\text{discon}}(R_{\text{max}} > R_{\text{max}}^{\text{cut}})}{a_\mu^{\text{con}}(R_{\text{max}} > R_{\text{max}}^{\text{cut}})} = -\frac{25}{34}.$$

- Note that we also need to use
  - the same QED weighting function $\mathcal{M}$.
  - the same choice of x_{ref} (in the long distance region).

$$x_{\text{ref-con}}(x,y,z) = x_{\text{ref-far}}(x,y,z) \qquad (x \text{ if } |y-z| < |x-y|, |x-z|)$$

$$x_{\text{ref-discon}}(x,y,z) = x$$

- Try to take advantage of the known theoretical ratio of the connected diagram and disconnected diagram contribution at long distance.

$$R_{\text{max}} = \max(|x-y|, |y-z|, |x-z|),$$

- Use $a_\mu(R_{\text{max}} > R_{\text{max}}^{\text{cut}})$ denote contribution in the region where R_{max} larger than $R_{\text{max}}^{\text{cut}}$. We have:

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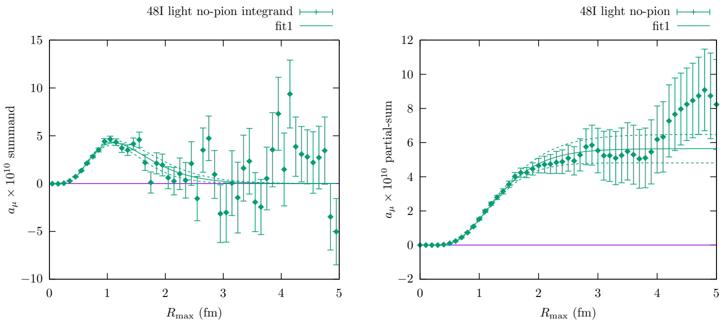
- Define contribution:

$$a_\mu^{\text{no-pion}} = a_\mu^{\text{discon}} + \frac{25}{34} a_\mu^{\text{con}},$$

- We have:

$$a_\mu^{\text{discon}} = a_\mu^{\text{no-pion}} - \frac{25}{34} a_\mu^{\text{con}},$$
$$a_\mu^{\text{total}} = a_\mu^{\text{no-pion}} + \frac{9}{34} a_\mu^{\text{con}}.$$



- Fit function (fit range 0.5 fm to 4 fm.)

$$f(R_{\text{max}}) = A\frac{R_{\text{max}}^6}{R_{\text{max}}^3 + C^3} e^{-BR_{\text{max}}}$$

Best fit parameters: $A \times 10^{10} = 130.58\ \text{fm}^{-1},\ B = 0.63\ \text{GeV},\ C = 0.66\ \text{fm}.$

- Based on fit (assign 100% systematic error):

$$a_\mu^{\text{no-pion}}(R_{\text{max}} > 2.5\ \text{fm}) \times 10^{10} = 0.31(0.22)_{\text{stat}}(0.31)_{\text{syst}}\ [0.38]$$

HLbL: compilation - light quark

Contribution name	a_μ	$\times 10^{10}$
48I light con $R_{\max} < 4$fm	18.61	$(1.22)_{\text{stat}}$
48I light no-pion $R_{\max} > 2.5$fm	0.31	$(0.22)_{\text{stat}}(0.31)_{\text{syst}}$ [0.38]
48I light discon $R_{\max} < 4$fm	-7.49	$(1.82)_{\text{stat}}$
48I light discon $R_{\max} < 4$fm hybrid-2.5fm	-8.28	$(1.31)_{\text{stat}}(0.31)_{\text{syst}}$ [1.35]
48I light $R_{\max} < 4$fm	11.11	$(2.11)_{\text{stat}}$
48I light $R_{\max} < 4$fm hybrid-2.5fm	10.32	$(0.99)_{\text{stat}}(0.31)_{\text{syst}}$ [1.04]
48I light $R_{\max} > 4$fm	2.00	$(0.11)_{\text{stat}}(0.28)_{\text{syst}}$ [0.30]
long distance π^0 exchange		Norman Christ & Cheng Tu
48I light FV-corr (LMD model)	-0.47	$(0.11)_{\text{syst}}$
48I light m_π-corr (340 MeV ensemble)	0.35	$(0.07)_{\text{stat}}(0.17)_{\text{syst}}$ [0.19]
48I light a^2-corr (48I/64I strange)	0.00	$(0.83)_{\text{syst}}$
light total	12.99	$(2.11)_{\text{stat}}(0.90)_{\text{syst}}$ [2.29]
light total hybrid-2.5fm	12.20	$(1.01)_{\text{stat}}(0.95)_{\text{syst}}$ [1.38]

Contribution	a_μ	$\times 10^{10}$
light con	25.70	$(1.33)_{\text{stat}}(1.99)_{\text{syst}}$ [2.39]
light discon	-12.71	$(1.87)_{\text{stat}}(1.17)_{\text{syst}}$ [2.20]
light discon hybrid-2.5fm	-13.50	$(1.36)_{\text{stat}}(1.21)_{\text{syst}}$ [1.82]
light total	12.99	$(2.11)_{\text{stat}}(0.90)_{\text{syst}}$ [2.29]
light total hybrid-2.5fm	12.20	$(1.01)_{\text{stat}}(0.95)_{\text{syst}}$ [1.38]
strange con	0.35	$(0.01)_{\text{stat}}$
strange discon hybrid-2.5fm	-0.36	$(0.22)_{\text{stat}}(0.03)_{\text{syst}}$ [0.22]
strange total hybrid-2.5fm	-0.00	$(0.22)_{\text{stat}}(0.03)_{\text{syst}}$ [0.23]
sub-leading discon	0.00	$(0.07)_{\text{syst}}$ [Mainz 2021]
charm total	0.28	$(0.05)_{\text{syst}}$ [Mainz 2022]
con	26.36	$(1.33)_{\text{stat}}(1.99)_{\text{syst}}$ [2.39]
discon	-13.12	$(2.30)_{\text{stat}}(1.18)_{\text{syst}}$ [2.59]
discon hybrid-2.5fm	-13.89	$(1.47)_{\text{stat}}(1.22)_{\text{syst}}$ [1.91]
total	13.24	$(2.53)_{\text{stat}}(0.90)_{\text{syst}}$ [2.68]
total hybrid-2.5fm	12.47	$(1.15)_{\text{stat}}(0.95)_{\text{syst}}$ [1.49]

- Domain wall fermion, Iwasaki gauge ensemble by RBC-UKQCD collaborations.
- $L_{\text{QED}} = \infty$, $L_{\text{QCD}} = 5.5$ fm, $m_\pi = 139$ MeV, $a^{-1} = 1.73$ GeV.
- Muon in the time direction of the Euclidean space-time lattice.
- The **subtracted** infinite volume QED weighting function.
- Use $x_{\text{ref}}(x, y, z) = x_{\text{ref-far}}(x, y, z)$ (details explained in previous slides).
- Rearrange the connected and disconnected diagrams to form $a_\mu^{\text{no-pion}}$.
- Separate lattice calculation of the long distance π^0 exchange contribution ($R_{\text{max}} > 4$ fm).
- Finite volume correction for $R_{\text{max}} < 4$ fm use π^0-pole amplitude via LMD model.
- Correction $m_\pi = 139$ MeV $\to 135$ MeV from the 24DH (341 MeV) and 32D (142 MeV) ensembles ($a^{-1} \approx 1$ GeV).
- Subleading disconnected diagrams and charm quark contribution from Mainz 21.

$$a_\mu^{\text{HLbL}} \times 10^{10} = 12.47(1.15)_{\text{stat}}(0.95)_{\text{syst}}\ [1.49],$$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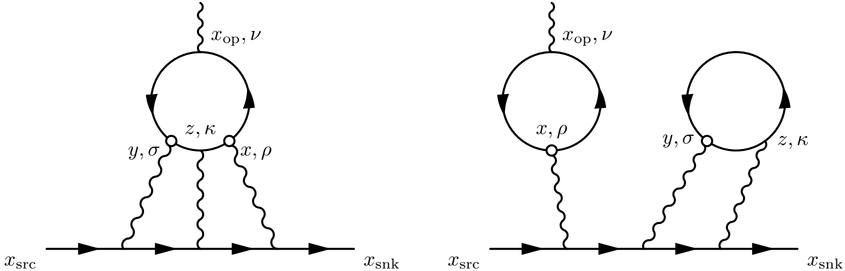
Thank You!

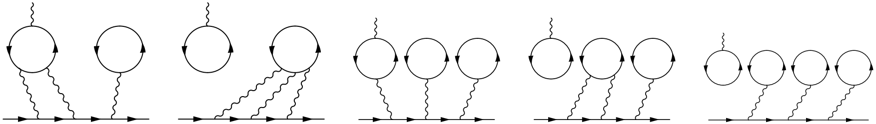
HLbL: comparison with QED_L

	a_μ	$\times 10^{10}$
all con QED_L	24.46	$(2.35)_{\text{stat}}(5.11)_{\text{syst}}$ [5.62]
all con diff	1.90	$(2.76)_{\text{stat}}(5.48)_{\text{syst}}$ [6.14]
all discon QED_L	-16.45	$(2.09)_{\text{stat}}(3.99)_{\text{syst}}$ [4.50]
all discon diff	2.56	$(2.57)_{\text{stat}}(4.17)_{\text{syst}}$ [4.90]
total QED_L	8.17	$(3.03)_{\text{stat}}(1.77)_{\text{syst}}$ [3.51]
total diff	4.30	$(3.25)_{\text{stat}}(2.01)_{\text{syst}}$ [3.82]

- Note: diff $=$ QED_∞ results $-$ QED_L results.
- Charm quark contribution is added to the previous QED_L results.



- We use two point sources quark propagators to calculate the hadronic part of the diagram. (Two point sources locations denoted as small circle.)
- The following sub-leading disconnected diagrams are suppressed by flavor SU(3). Mainz 2021 [E.H. Chao et al. 2021 (EPJC 81, 7, 651)]: explicitly calculated these diagrams and obtained $\pm 0.07 \times 10^{-10}$.



HLbL: some lattice calculation details

- RBC-UKQCD 48I ensemble ($48^3 \times 96$). 5.5 fm, $m_\pi = 139$ MeV, $a^{-1} = 1.73$ GeV.
- Uniformly random sample 2048 point locations per config. Calculate point source light quark propagators for each point. Overall, we calculated 113 configs.
- Same set of propagators used in the calculation of the connected and disconnected diagrams.
- Computational techniques:
  - locally-coherent Lanczos approach (arXiv:1710.06884 [hep-lat])
  - ZMobius (arXiv:1701.07792 [hep-lat])
  - AMA (arXiv:1208.4349 [hep-lat])

- RBC-UKQCD 48I ensemble ($48^3 \times 96$). 5.5 fm, $m_\pi = 139$ MeV, $a^{-1} = 1.73$ GeV.
- Uniformly random sample 2048 point locations per config. Calculate point source light quark propagators for each point. Overall, we calculated 113 configs.
- Connected diagrams: sample two-point-pairs (x, y) formed using these 2048 points based on the empirical probability:

$$p(r) = \begin{cases} 1 & \text{if } 8 \geq r > 0 \\ \frac{1}{(r/8)^3} & \text{if } L \geq r > 8 \\ 0 & \text{if } r > L \end{cases},$$

Compute 57,000 pairs per config on average. We also do stochastic sparsening for the other two points x_{op}, z with ratio $1/16$, which saves both computational time and storage (more important).

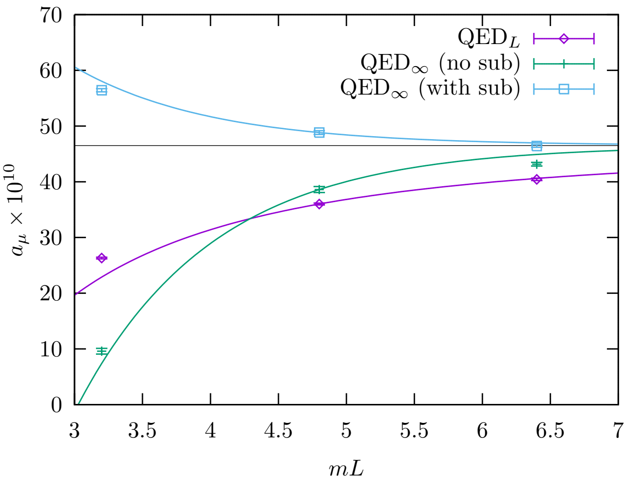
HLbL: some lattice calculation details

- RBC-UKQCD 48I ensemble ($48^3 \times 96$). 5.5 fm, $m_\pi = 139$ MeV, $a^{-1} = 1.73$ GeV.
- Uniformly random sample 2048 point locations per config. Calculate point source light quark propagators for each point. Overall, we calculated 113 configs.
- Disconnected diagrams: calculate all possible two-point-pairs formed with these 2048 points. To make it affordable, we aggressively sparsen when summing over z with "adaptive sampling". Note that the procedure is NOT biased!

$$n(z,y) = \sum_{\kappa,\sigma} \Big| \mathrm{Tr}\Big(\gamma_\kappa S_q(z,y)\gamma_\sigma S_q(y,z) - \langle \gamma_\kappa S_q(z,y)\gamma_\sigma S_q(y,z)\rangle_{\mathrm{QCD}}\Big)\Big|^2$$

$$p_y(z) = \begin{cases} 1 & \text{if } n(z,y) \geq t_0^2 \text{ and } |z-y| \leq L \\ \sqrt{n(z,y)}/t_0 & \text{if } n(z,y) < t_0^2 \text{ and } |z-y| \leq L \\ 0 & \text{if } |z-y| > L \end{cases},$$

where $t_0 = 5 \times 10^{-5}$. In short, we sample z with probability determined based on the magnitude of the value quark loop evaluated for this config and point source location y.

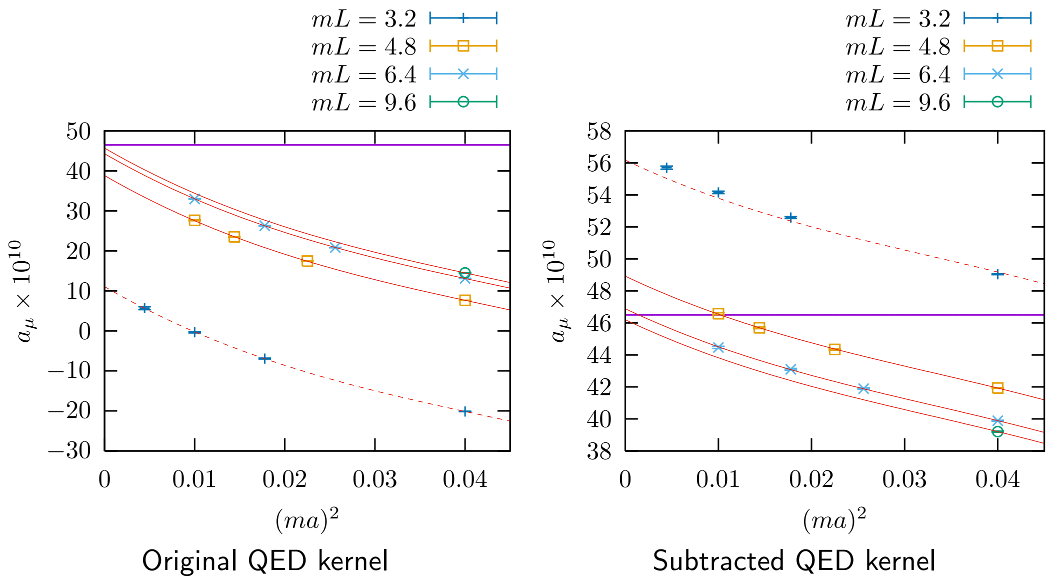


- QED_L: $\mathcal{O}(1/L^2)$ finite volume effects
- QED_∞ (no sub) $\mathfrak{G}^{(1)}$: $\mathcal{O}(e^{-mL})$ finite volume effects
- QED_∞ (with sub) $\mathfrak{G}^{(2)}$: smaller $\mathcal{O}(e^{-mL})$ finite volume effects

T. Blum et al 2017. (PRD 96 3, 034515)

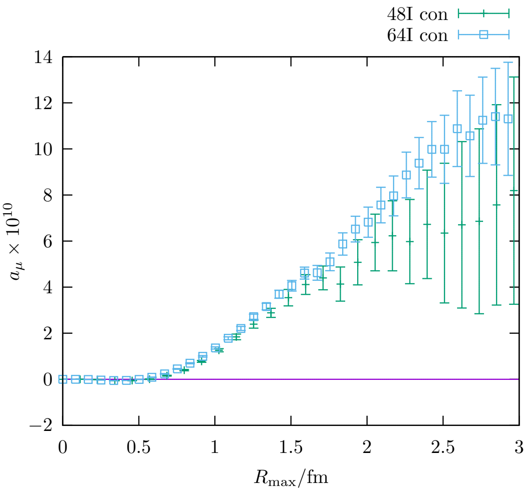
Muon leptonic LbL QED_∞ discretization effects

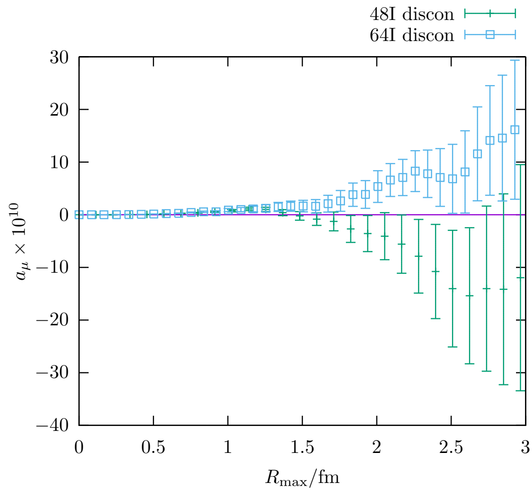
- Compare the two $\mathfrak{G}_{\rho,\sigma,\kappa}(x, y, z)$ in **pure QED computation**.



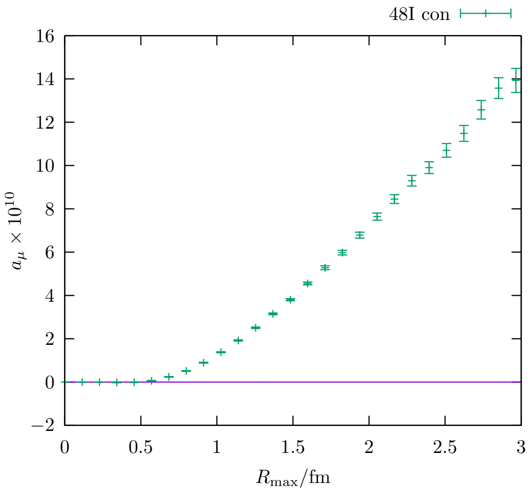
- Notice the vertical scales in the two plots are different.

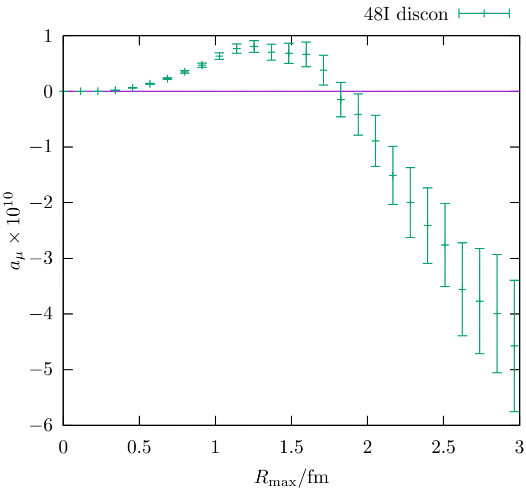
T. Blum et al 2017. (PRD 96 3, 034515)





- 48I: $a = 0.114$ fm. 64I: $a = 0.084$ fm.
- $R_{\text{max}} = \max(|x-y|, |x-z|, |y-z|)$. Partial sum up to R_{max}.
- Was planing to generate more statistics for the 48I ensemble.





- 48I: $a = 0.114$ fm. Only the light quark contributions.
- $R_{\rm max} = \max(|x - y|, |x - z|, |y - z|)$. Partial sum up to $R_{\rm max}$.
- Much more statistics for the 48I ensemble with our 2021-2022 ALCC allocation at SUMMIT.

HLbL: QED_∞ - break down of changes

- Code and hardware: Peter Boyle's Grid and Christoph Lehner's GPT are now used on SUMMIT's NVIDIA V100 GPUs. Also port the contraction code to work on GPU. Previously, we were using Peter Boyle's BFM on MIRA's IBM BG/Q system.
- Calculated with 113 48I configurations and each with 2048 propagators. Previously, we had 34 configurations and each with 1024 point source propagators. 6.65 times increase of the number of propagators.
- Connected diagram:
  - Increase the number of point pairs of x and y to $\sim 57,000$ per configuration. 8 times increase per propagator.
- Disconnected diagram:
  - Use of all possible combinations of point pairs of x and y. 2 times increase per propagator.
  - More efficient adaptive sampling scheme for the third vertex z. Sampling probability determined based on $\sum_{\mu,\nu} |\Pi_{\mu,\nu}(z-y) - \langle \Pi_{\mu,\nu}(z-y)\rangle|^2$ instead of |z - y|. 7 times more effective.

For the four-point-function, when its two ends, x and y, are far separated, but x' is close to x and y' is close to y, the four-point-function is dominated by π^0 exchange.

Both the connected and the disconnected diagram will contribute in these region. We can find a connection between the connnected diagram and the disconnected diagram by first investigating the η correlation function.

$$\langle \bar{u}\gamma_5 u(x)(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d)(y)\rangle \sim e^{-m_\eta|x-y|} \quad (24)$$

$$\langle \bar{u}\gamma_5 u(x)(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d)(y)\rangle + 2\langle \bar{u}\gamma_5 u(x)\bar{d}\gamma_5 d(y)\rangle \sim e^{-m_\eta|x-y|} \quad (25)$$

That is

$$\langle \bar{u}\gamma_5 u(x)\bar{d}\gamma_5 d(y)\rangle = -\frac{1}{2}\langle \bar{u}\gamma_5 u(x)(\bar{u}\gamma_5 u - \bar{d}\gamma_5 d)(y)\rangle + \mathcal{O}\big(e^{-m_\eta|x-y|}\big) \quad (26)$$

Above is a relation between disconnected diagram π^0 exchange (left hand side) and connected diagram π^0 exchange (right hand side).

The nearby two current operater can be viewed as an interpolating operator for π^0, just like $\bar{u}\gamma_5 u$ or $\bar{d}\gamma_5 d$ with appropriate charge factors.

Multiplied by appropriate charge factors:

$$\text{Connected contribution} \quad \left[\left(\frac{2}{3}\right)^4 + \left(-\frac{1}{3}\right)^4\right] = \frac{17}{81} \tag{27}$$

$$\text{Disconnected contribution} \quad \left[\left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2\right]^2 \left(-\frac{1}{2}\right) = \frac{25}{81}\left(-\frac{1}{2}\right) \tag{28}$$

$$\text{Connected} : \text{Disconnected} = 34 : -25 \tag{29}$$

Different approach by J. Bijnens and J. Relefors: **JHEP 1609 (2016) 113**.