

Direct lattice calculation of a_{μ}^{HLbL} : Mainz results

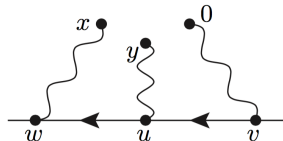
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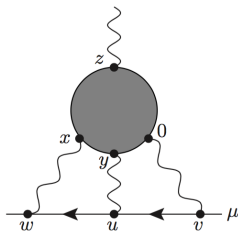


Hadronic light-by-light contribution: coordinate-space approach

QED kernel $\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)$



\Rightarrow

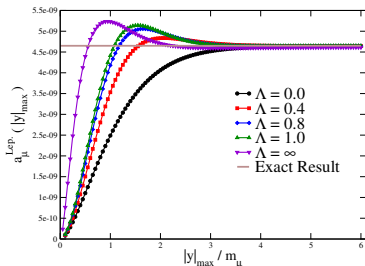


$$a_{\mu}^{\text{HLbL}} = \frac{me^6}{3} \underbrace{\int d^4 y}_{=2\pi^2|y|^3 d|y|} \left[\int d^4 x \underbrace{\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)}_{\text{QED}} \underbrace{i\hat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y)}_{=\text{QCD blob}} \right].$$

$$i\hat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y) = - \int d^4 z z_{\rho} \langle j_{\mu}(x) j_{\nu}(y) j_{\sigma}(z) j_{\lambda}(0) \rangle.$$

- ▶ $\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)$ computed in the continuum & infinite-volume
- ▶ no power-law finite-volume effects & only a 1d integral to sample the integrand in $|y|$.

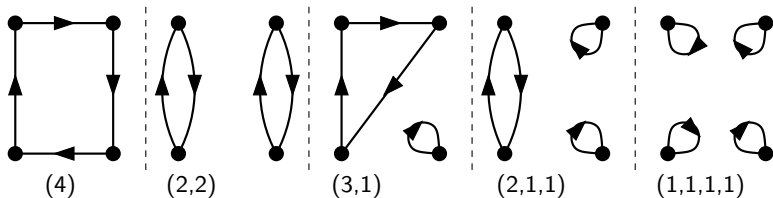
Tests of the framework



Integral for lepton loop

- ▶ The QED kernel $\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)$ is parametrized by six 'weight' functions of the variables $(x^2, x \cdot y, y^2)$.
- ▶ Using this kernel, we have reproduced (at the 1% level) known results for a range of masses for:
 1. the lepton loop (spinor QED, shown in the two plots);
 2. the charged pion loop (scalar QED);
 3. the π^0 exchange with a VMD-parametrized transition form factor.

Wick-contraction topologies in HLbL amplitude $\langle 0|T\{j_x^\mu j_y^\nu j_z^\lambda j_0^\sigma\}|0\rangle$



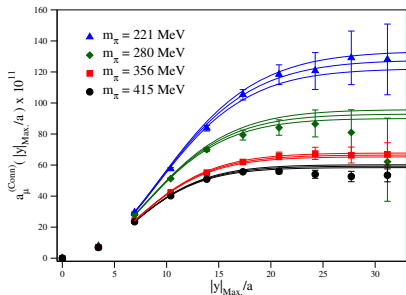
First two classes of diagrams turn out to be dominant, with a sizeable cancellation between them.

Large- N_c argument by J. Bijnens, 1608.01454.

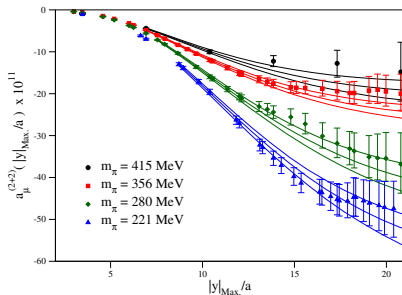
The connected and leading disconnected contribution

$$\text{Cumulated } a_\mu^{\text{HLbL}} = \int_0^{|y|_{\text{max}}} d|y| f(|y|)$$

Connected

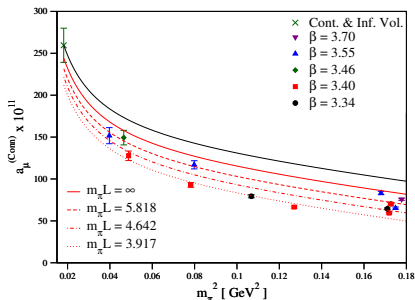


Leading disconnected

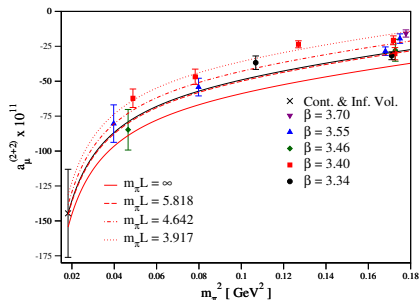


Chiral, continuum, volume extrapolation

Connected contribution



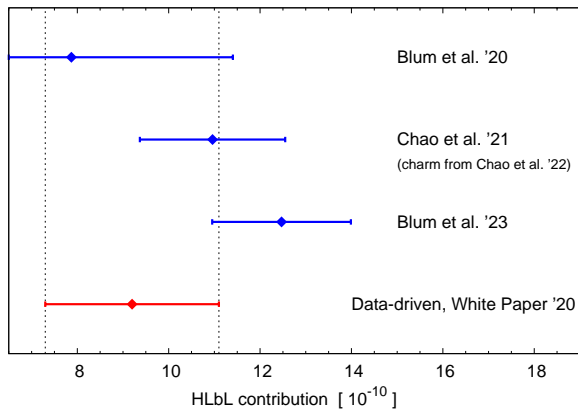
disconnected contribution



Contribution	Value $\times 10^{11}$
Light-quark fully-connected and (2 + 2)	107.4(11.3)(9.2)(6.0)
Strange-quark fully-connected and (2 + 2)	-0.6(2.0)
(3 + 1)	0.0(0.6)
(2 + 1 + 1)	0.0(0.3)
(1 + 1 + 1 + 1)	0.0(0.1)
Total	106.8(15.9)

[Chao, Hudspith, Gérardin, Green, HM, Ottnad, 2104.02632 (EPJC)]

Compilation of a_μ^{HLbL} determinations



Good consistency of different determinations.