

HLbL direct lattice calculation from BMW

Virtual spring meeting of the Muon $g-2$ Theory Initiative

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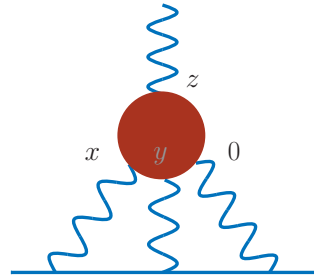
CPT - Marseille - April 16, 2024

- ▶ Based on the coordinate-space approach developed by the Mainz group

$$a_{\mu}^{\text{HLbL}} = \frac{me^6}{3} \int d^4y \int d^4x \mathcal{L}_{[\rho,\sigma],\mu\nu\lambda}(x, y) i\hat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x, y),$$

with

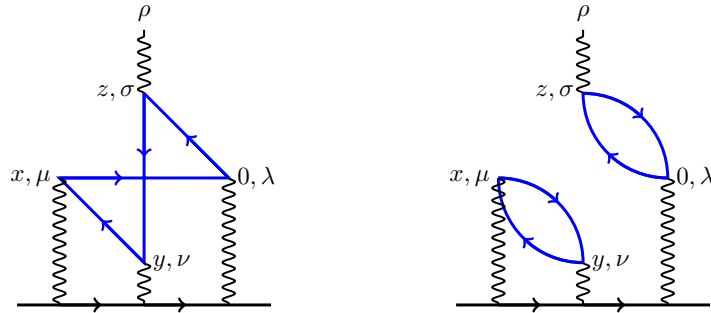
$$i\hat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x, y) = - \int d^4z z_{\rho} \langle j_{\mu}(x) j_{\nu}(y) j_{\sigma}(z) j_{\lambda}(0) \rangle$$



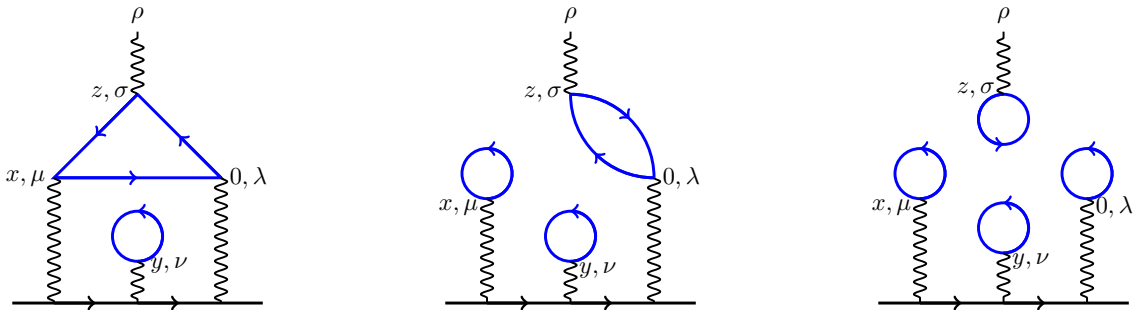
and $\mathcal{L}_{[\rho,\sigma],\mu\nu\lambda}(x, y)$ computed by Mainz group [JHEP 04 (2023) 040]

- ▶ Slightly different kernel compared to the Mainz calculation
 - **motivation** : simplification / optimization of the code
 - **direct comparison at the level of the integrand not possible** among groups
- ▶ Work at the physical pion mass
 - Challenge : **bad signal / noise ratio at long distances.**

- ▶ We focus on the connected and leading disconnected diagrams :
 - largest contribution - large cancellation between them



- ▶ Sub-leading diagrams : they are computed only on a subsets of ensembles

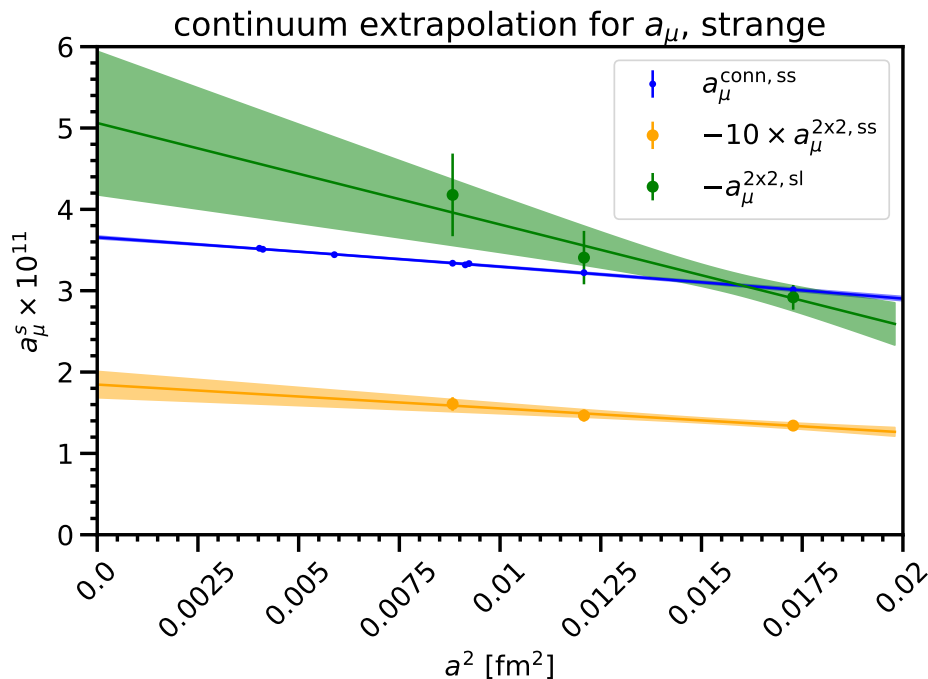


→ These contributions have been shown to be small by the Mainz group & RBC/UKQCD

Calculation based on Budapest-Marseille-Wuppertal (BMW) gauge ensembles

- Goldstone pion/kaon are tuned to their [physical pion/kaon masses](#)
 - $N_f = 2 + 1 + 1$ dynamical staggered fermions with four steps of stout smearing
- we have accumulated data at [3 values of the lattice spacing](#) from 0.13 fm down to 0.09 fm
 - analysis on-going (preliminary results presented by C. Zimmermann @Lattice 23)
- 2 additional lattice spacings down to 0.065 fm for some contributions
 - connected strange (very precise data)
- use the π^0 , η and η' transition form factors computed on the same set of ensembles
 - correct for finite-volume effects
 - better control on the tail of the integrand

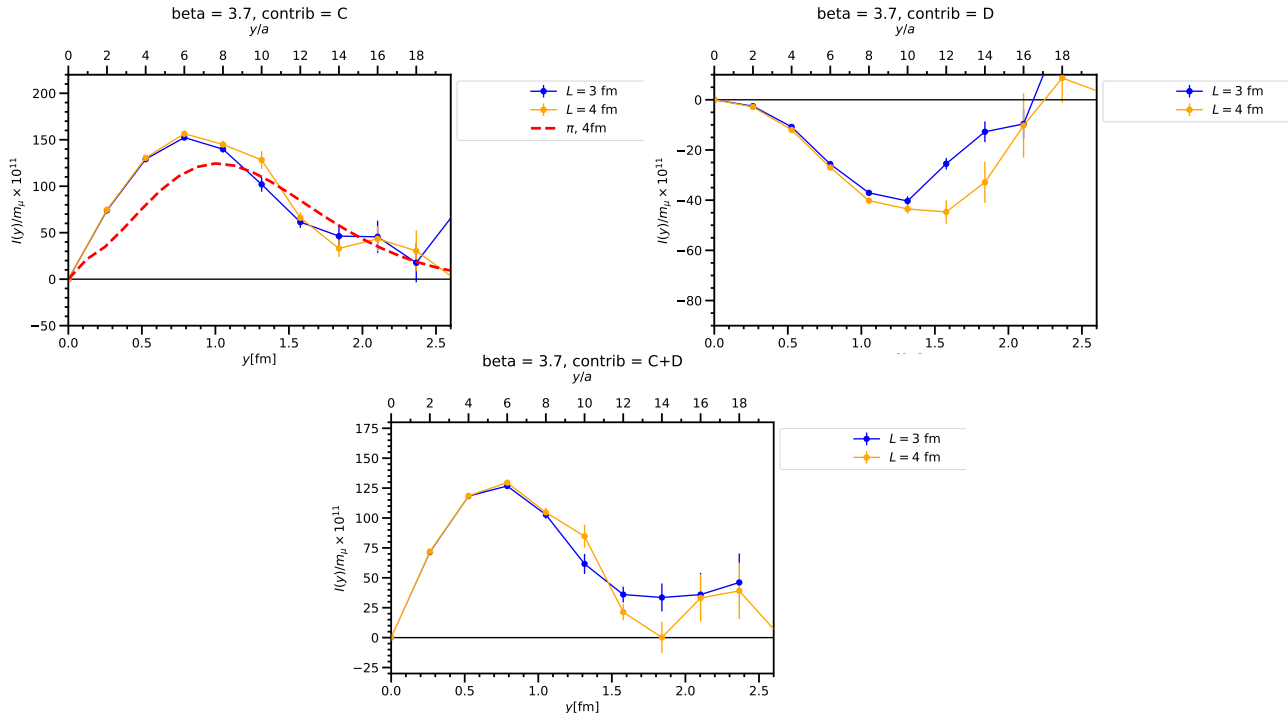
- ▶ Preliminary results presented by C. Zimmermann at Lattice 23
- ▶ Connected and 2+2 disconnected (preliminary)



- ▶ Individual contributions → cross-checks with other collaborations

► Connected and 2+2 disconnected (preliminary)

→ Results presented by C. Zimmermann @Lattice 23 ($a = 0.13$ fm)



→ Final results based on large-volume ensembles only

→ Current strategy : use the small volumes to test FSE correction.
then apply FSE on large volumes only.

- ▶ We have accumulated most of the statistics (3 lattice spacings).
- ▶ Analysis in on-going
 - focus on the dominant light-quark contribution (connected + leading disconnected)
- ▶ Use our calculation of the pseudoscalar transition form factors :
 - description of volume effects
 - tail of the integrand
- ▶ We expect first results by the end of the year