HLbL direct lattice calculation: charm-quark contribution

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Status of a_μ^{hlbl} from lattice QCD

- Subleading, entering at $O(\alpha_{\mathrm{QED}}^3) \Rightarrow$ needs to be determined to $O(10^{-1})$ relative precision from the Standard Model (SM).
- Recent $N_f = 2+1$ lattice calculations in good agreement with the 2021 White Paper (WP) value [Mainz/CLS '21, RBC/UKQCD '23].

	$a_\mu^{\mathrm{hlbl}}\ (u,d,s) \times 10^{11}$
WP	87(17)
Mainz/CLS '21	106.8(15.9)
RBC/UKQCD '23	124.7(15.2)

- Effects of the charm quark included in the 2021 WP:

	$a_\mu^{\mathrm{hlbl,c}} \times 10^{11}$
quark-loop	3.1(1)
$\eta_c(1S)$	0.8

- **Goal**: provide a first-principle determination of the charm-quark contribution to $a_\mu^{\mathrm{hlbl,c}}$ from lattice QCD.

Lattice QCD formalism for $a_\mu^{\rm hlbl}$

- Master formula of the Mainz x-space approach:

$$\boxed{a_\mu^{\rm hlbl} = \frac{m_\mu e^6}{3} \int_{x,y} \mathcal{L}_{[\rho,\sigma];\mu\nu\lambda}(x,y) i\hat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y)\,,}$$

$$i\hat{\Pi}_{\rho;\mu\nu\lambda\sigma} = -\int_z z_\rho \tilde{\Pi}_{\mu\nu\sigma\lambda}\,, \quad \tilde{\Pi}_{\mu\nu\sigma\lambda}(x,y,z) \equiv \langle j_\mu(x) j_\nu(y) j_\sigma(z) j_\lambda(0)\rangle_{\rm QCD}\,,$$

where $j_\mu = \sum_{i\in\text{flavor}} Q_i \bar{\psi}_i \gamma_\mu \psi_i$.

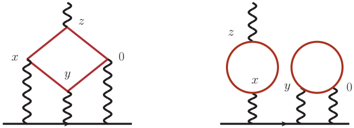
- The QED-kernel $\mathcal{L}$ is computed semi-analytically in the continuum and infinite-volume.

[Asmussen *et al* '23]

- O(4)-symmetry in the continuum and infinite-volume:

$$a_\mu^{\rm hlbl} = \lim_{|y|_{\rm max}\to\infty} a_\mu^{\rm hlbl}(|y|_{\rm max})\,, \quad a_\mu^{\rm hlbl}(|y|_{\rm max}) = \int_0^{|y|_{\rm max}} d|y| f(|y|)\,.$$

- For this work, considering only two Wick-contractions: fully-connected and (2+2)-disconnected. The other topologies are small in the $N_{\rm f} = 2+1$ case.



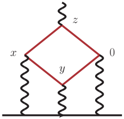
$a_\mu^{\mathrm{hlbl,c}}$ from LQCD

Calculation setup

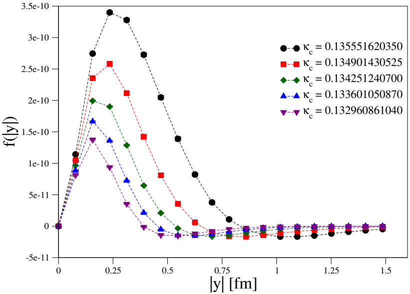
- Include six $N_{\mathrm{f}} = 3$ ensembles with Wilson-Clover fermions at $m_{\pi,K,\eta} \approx 415$ MeV and lattice spacing ranging from 0.039 to 0.099 fm.
- Charm quark at different masses introduced at partially-quenched level to each ensemble: sea-quark effects expected at order $\alpha_{\mathrm{s}}^2(m_c)$.
- Physical point defined by the (connected-part) η_c-mass
  $\Rightarrow$ Extrapolate $(a, m_{\eta_c}) \to (0, m_{\eta_c}^{\mathrm{phys.}})$ at fixed m_π.

a_μ^{hlbl} from LQCD

Connected integrand example: with $a = 0.039$ fm



- Fast fall-off of the integrand $\Rightarrow$ negligible finite-volume effects.
- Becomes shorter range as the bare charm quark mass increases $\Rightarrow$ large discretization effects expected.



$$am_c = \frac{1}{2}\left(\kappa_c^{-1} - \kappa_{\mathrm{critical}}^{-1}\right)$$

$a_\mu^{\rm hlbl}$ from LQCD

Extrapolation to the physical-point

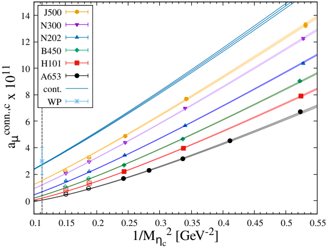
- Examples of fit ansätze scaling as $m_{\eta_c}^{-2}$ in the continuum:

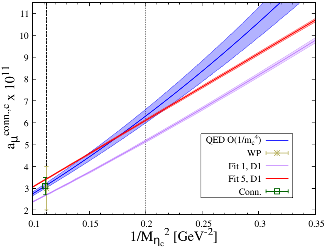
$$\text{Fit 1:} \qquad a_\mu(a, M_{\eta_c}) = \frac{A + BaM_{\eta_c}^2}{\frac{1}{4}(C + M_{\eta_c})^2 + (D + Ea^2 M_{\eta_c}^2)^2},$$

$$\text{Fit 5:} \qquad a_\mu(a, M_{\eta_c}) = Aa + \frac{B + Ca^2}{M_{\eta_c}^2} + Da^2 + E\frac{a^2}{M_{\eta_c}^2} \ln M_{\eta_c}.$$

  $\Rightarrow$ results dominated by the systematic error from the fit.

- The spread of the fit results is consistent with the expectation from the charm-quark loop when m_{η_c} approaches its physical value.
- Quark-disconnected part: small, done at physical κ_c from D_s-tuning [Gérardin *et al* '19].





Summary and conclusion

- Break-down of contributions from different Wick-contraction topologies

	$a_\mu^{\mathrm{hlbl,c}} \times 10^{11}$
Connected part	3.1(4)
Disconnected part	-0.30(23)
Total	2.8(5)

- Combined a_μ^{hlbl} with (u, d, s, c) quarks from Mainz/CLS:

$$\boxed{a_\mu^{\mathrm{hlbl}} = 109.6(15.9) \times 10^{-11}}$$