Pseudoscalar transition form factors and the muon $g-2$

Virtual spring meeting of the Muon g-2 Theory Initiative

Antoine Gérardin

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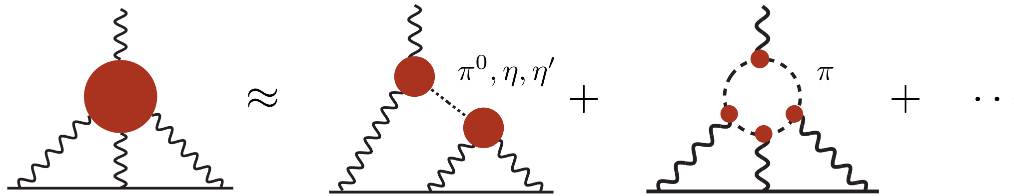
$a_\mu^{\mathrm{HLbL}} =$



Dispersive framework ('21)	$a_\mu \times 10^{11}$
π^0, η, η'	93.8 ± 4
pion/kaon loops	-16.4 ± 0.2
S-wave $\pi\pi$	-8 ± 1
axial vector	6 ± 6
scalar + tensor	-1 ± 3
q-loops / short. dist. cstr	15 ± 10
charm + heavy q	3 ± 1
sum	92 ± 19

Pseudoscalar TFFs on the lattice :

- Input for the dispersive framework

- Input for the direct lattice calculation

$$a_\mu^{\mathrm{HLbL};\pi^0} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{1} d\tau \; w_1(Q_1, Q_2, \tau)\, \mathcal{F}_{P\gamma^*\gamma^*}(-Q_1^2, -(Q_1+Q_2)^2)\, \mathcal{F}_{P\gamma^*\gamma^*}(-Q_2^2, 0) + w_2(Q_1, Q_2, \tau)\, \mathcal{F}_{P\gamma^*\gamma^*}(-Q_1^2, -Q_2^2)\, \mathcal{F}_{P\gamma^*\gamma^*}(-(Q_1+Q_2)^2, 0)$$

→ $w_{1,2}(Q_1, Q_2, \tau)$ are model-independent weight functions [Jegerlehner & Nyffeler '09]

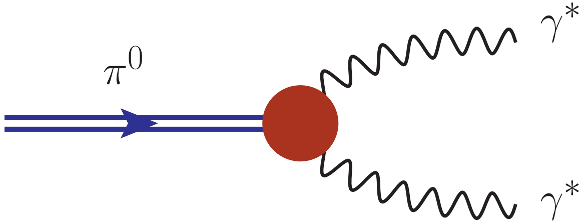
Calculation based on Budapest-Marseille-Wuppertal (BMW) gauge ensembles

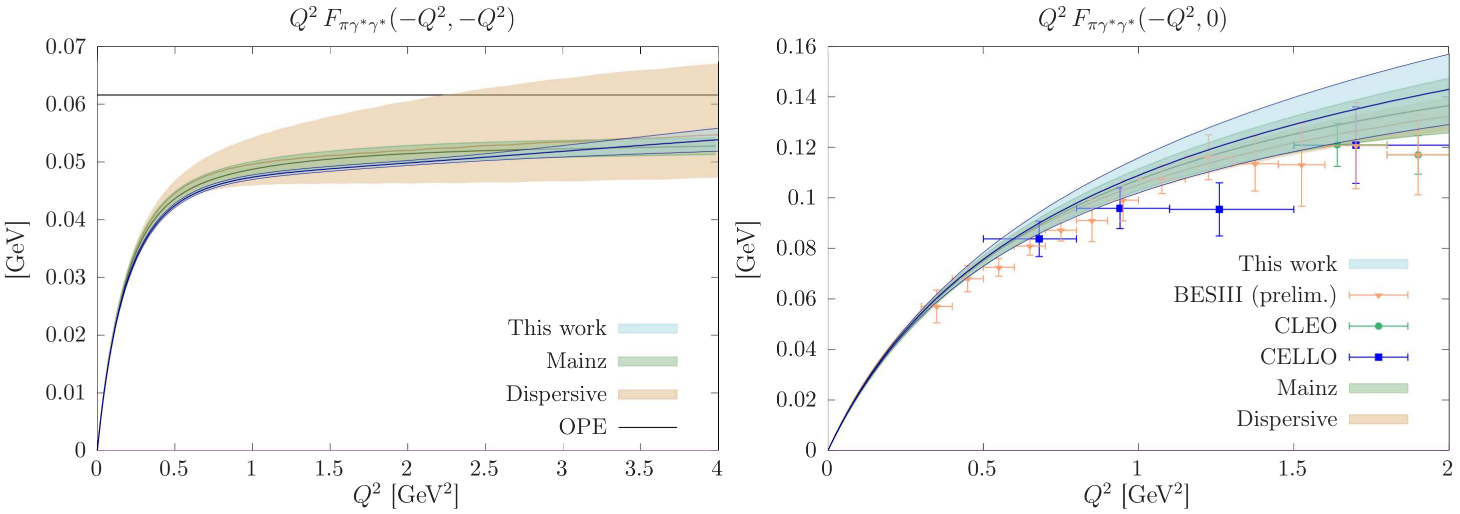
- Goldstone pion/kaon are tuned to their physical pion/kaon masses
  $\rightarrow$ $N_f = 2+1+1$ dynamical staggered fermions with four steps of stout smearing

- up to 6 lattice spacings from 0.13 fm down to 0.064 fm

- goal : determinations of the η and η' TFFs
  $\rightarrow$ first step : pion TFF

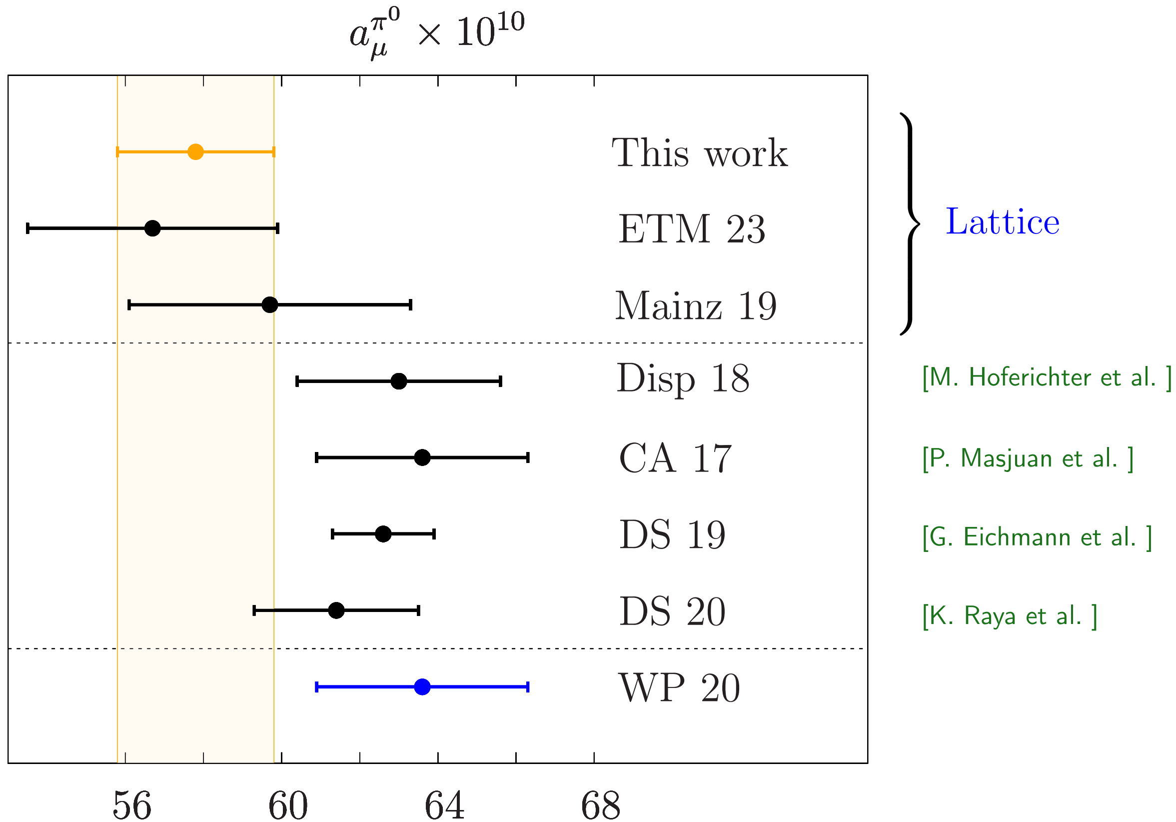
The pion transition form factor





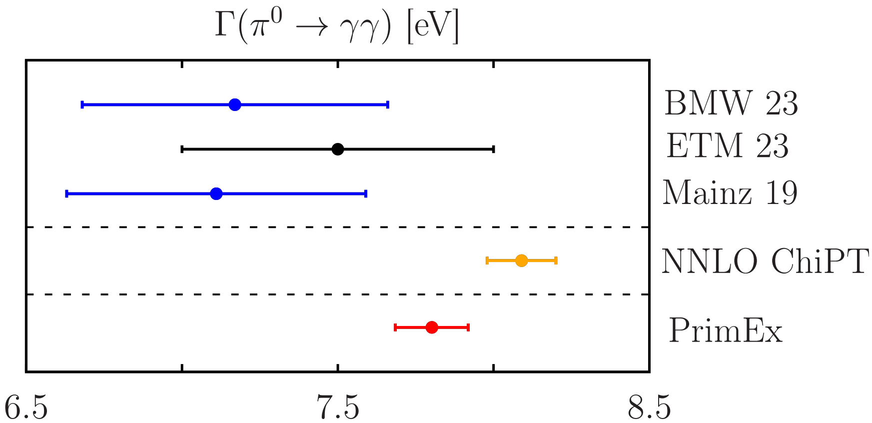
$$\Gamma(\pi^0 \to \gamma\gamma) = 7.11(0.44)_{\rm stat}(0.21)_{\rm syst}\ \text{eV}$$

$$a_\mu^{\rm HLbL;\pi^0} = 57.8(1.8)_{\rm stat}(0.9)_{\rm syst} \times 10^{-11}$$


$a_\mu^{\pi^0} \times 10^{10}$
This work
ETM 23
Mainz 19
Disp 18
CA 17
DS 19
DS 20
WP 20
56
60
64
68
Lattice
[M. Hoferichter et al.]
[P. Masjuan et al.]
[G. Eichmann et al.]
[K. Raya et al.]

▶ two-photon decay width :

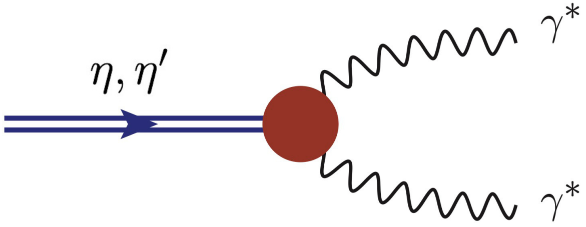
$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{\pi\alpha^2 m_{\pi^0}^3}{4} \mathcal{F}^2_{\pi^0\gamma\gamma}(0,0)$$

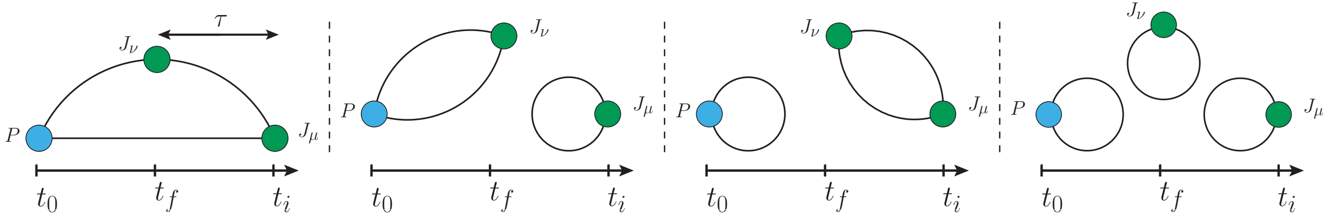


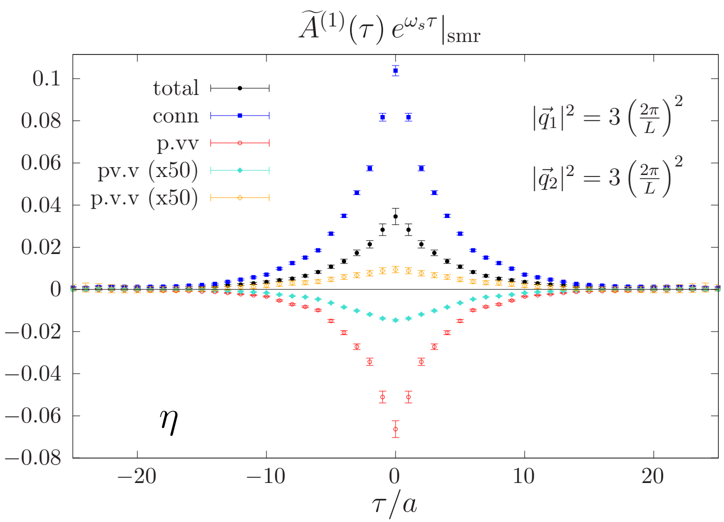
- lattice results tend to be smaller than exp. value, but compatible within 1 - 1.5 σ

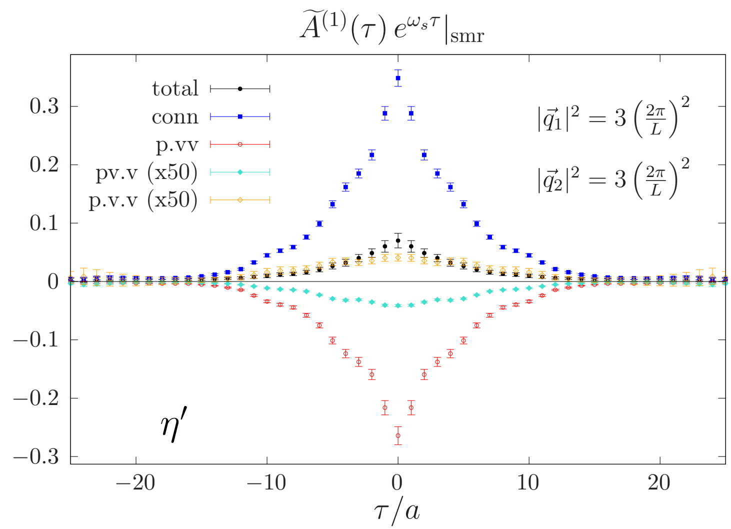
  $\to$ error dominated by statistics

The η and η' transition form factors

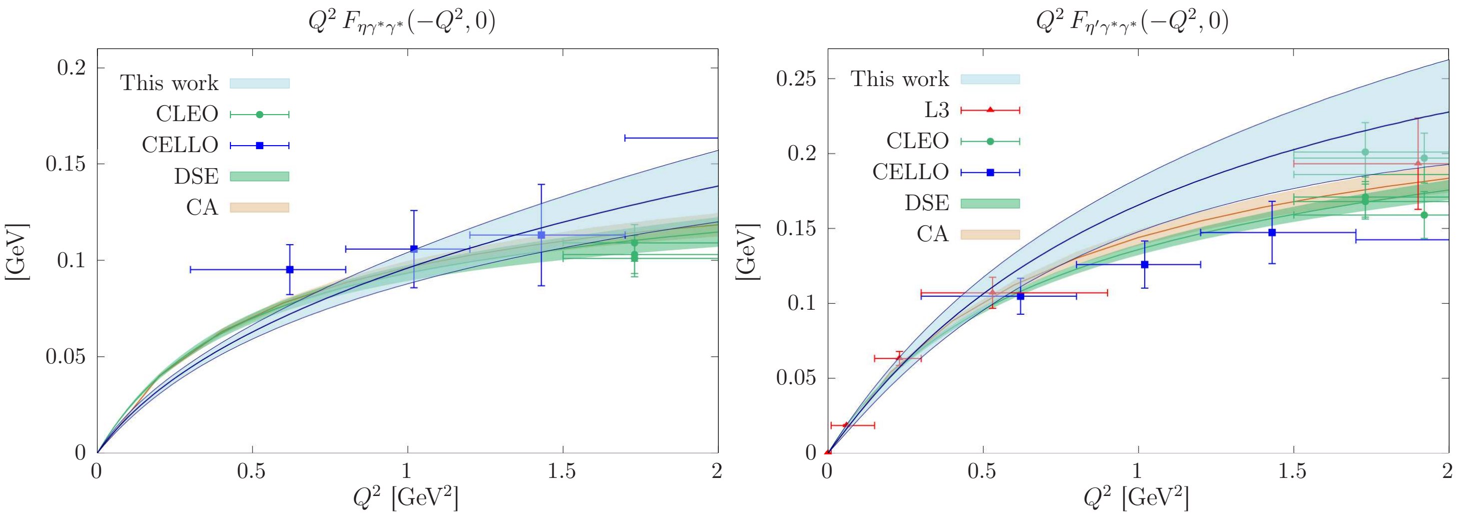








$$\mathcal{F}_{P\gamma\gamma}(q_1^2, q_2^2) \;\propto\; \int_{-\infty}^{\infty} \mathrm{d}\tau\, \widetilde{A}_{\mu\nu}(\tau)\, e^{\omega_1 \tau}$$



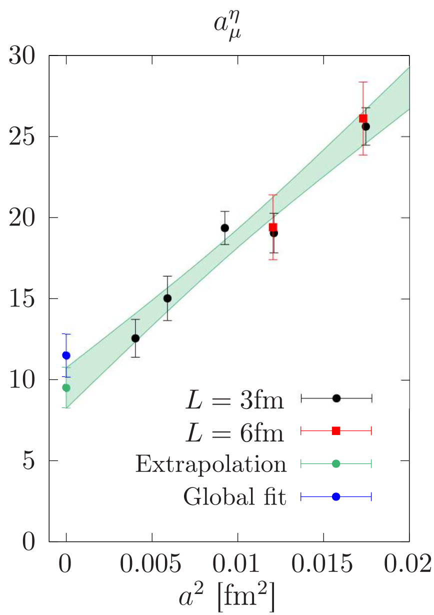
- two-photon decay width : $\Gamma(\pi^0 \to \gamma\gamma) = \dfrac{\pi\alpha^2 m_{\pi^0}^3}{4} \mathcal{F}^2_{\pi^0\gamma\gamma}(0,0)$

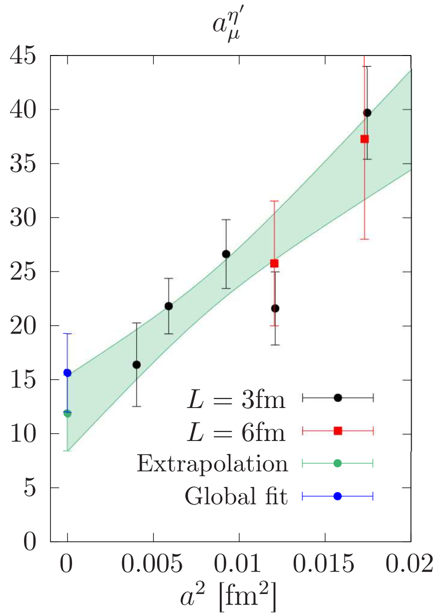
$$\Gamma(\eta \to \gamma\gamma) = 338(94)_{\rm stat}(35)_{\rm syst}\ \mathrm{eV}$$
$$\Gamma(\eta' \to \gamma\gamma) = 3.4(1.0)_{\rm stat}(0.4)_{\rm syst}\ \mathrm{keV}$$

$\to$ PDG : $\Gamma(\eta \to \gamma\gamma) = 516(18)$ eV

$\to$ PDG : $\Gamma(\eta' \to \gamma\gamma) = 4.28(19)$ keV

$$a_\mu^{\mathrm{HLbL};P} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^1 d\tau \; w_1(Q_1,Q_2,\tau)\; \mathcal{F}_{P\gamma^*\gamma^*}(-Q_1^2, -(Q_1+Q_2)^2)\; \mathcal{F}_{P\gamma^*\gamma^*}(-Q_2^2, 0) + w_2(Q_1,Q_2,\tau)\; \mathcal{F}_{P\gamma^*\gamma^*}(-Q_1^2, -Q_2^2)\; \mathcal{F}_{P\gamma^*\gamma^*}(-(Q_1+Q_2)^2, 0)$$



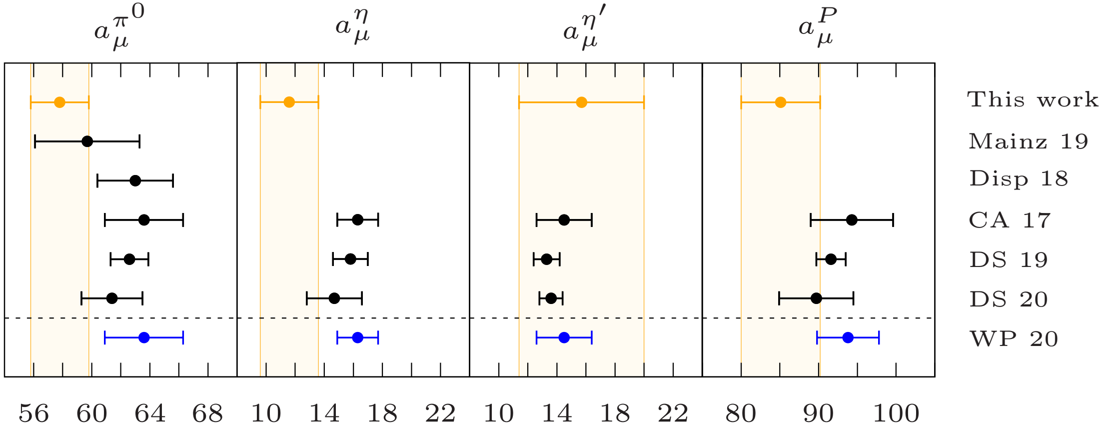


$$a_\mu^{\mathrm{HLbL};\eta} = 11.6(1.6)_{\mathrm{stat}}(0.5)_{\mathrm{syst}}(1.1)_{\mathrm{FSE}} \times 10^{-11}$$
$$a_\mu^{\mathrm{HLbL};\eta'} = 15.7(3.9)_{\mathrm{stat}}(1.1)_{\mathrm{syst}}(1.3)_{\mathrm{FSE}} \times 10^{-11}$$

Canterbury approximants [PRD 95, 054026 (2017)]

$\rightarrow a_\mu^{\mathrm{HLbL};\eta} = 16.3(1.4) \;\; \times 10^{-11}$

$\rightarrow a_\mu^{\mathrm{HLbL};\eta'} = 14.5(1.9) \;\; \times 10^{-11}$



- Our final estimate

$$a_\mu^{\mathrm{HLbL;ps-poles}} = (85.1 \pm 4.7_{\mathrm{stat}} \pm 2.3) \times 10^{-11}\,.$$

- Pion transition form factor

  $\rightarrow$ good agreement with Mainz'19, ETM'23 and with exp. data for $\mathcal{F}_{\pi^0\gamma\gamma}(Q^2, 0)$

- $\eta - \eta'$ transition form factors

  $\rightarrow$ first ab-initio calculation - error dominated by statistics

  $\rightarrow$ some tensions for the η TFF at very low virtualities