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Pseudoscalar (π^0, η, η') pole contributions to HLbL

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on behalf of the Extended Twisted Mass Collaboration

Muon g-2 TI | Spring 2024 Meeting
April 15 - 23, 2024

PS-pole contribution $a_\mu^{P-\text{pole}}$ to muon g-2

$$a_\mu^{P-\text{pole},(1)} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{1} d\cos\theta \; w_1(Q_1, Q_2, \cos\theta)$$
$$\times F_{P\gamma\gamma}(-Q_1^2, -(Q_1+Q_2)^2)\, F_{P\gamma\gamma}(-Q_2^2, 0)$$

$$a_\mu^{P-\text{pole},(2)} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{1} d\cos\theta \; w_2(Q_1, Q_2, \cos\theta)$$
$$\times F_{P\gamma\gamma}(-Q_1^2, -Q_2^2)\, F_{P\gamma\gamma}(-(Q_1+Q_2)^2, 0)$$

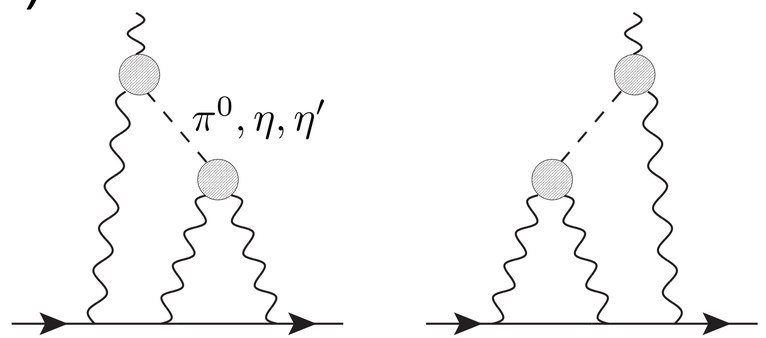
TFFs with DV x SV, spacelike structure

Known kinematic weight functions

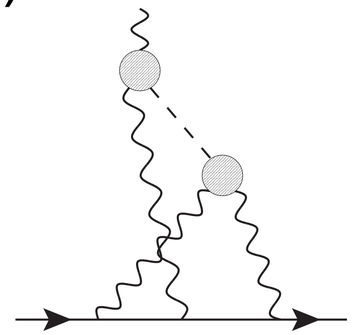
A. Nyffeler, PRD 94, 053006 (2016)

$$a_\mu^{P-\text{pole}} = \left(\frac{\alpha}{\pi}\right)^3 \left[a_\mu^{P-\text{pole},(1)} + a_\mu^{P-\text{pole},(2)}\right]$$

(1)



(2)



TFF kinematics

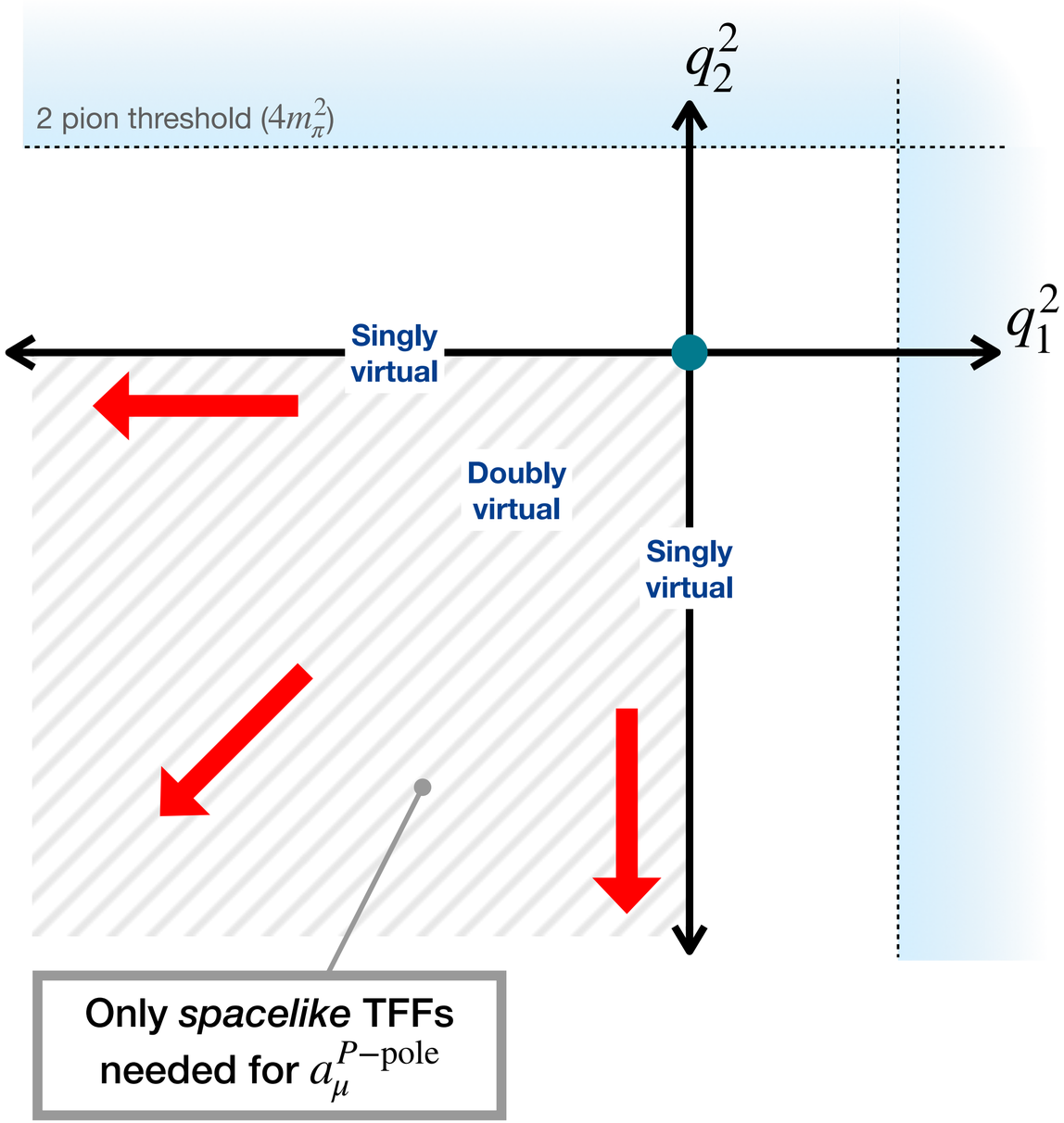
Related to decay width

$$\Gamma(P \to \gamma\gamma) = \left(\frac{\pi\alpha^2 M_P^3}{4}\right) |F_{P\gamma\gamma}(0,0)|^2$$

Large-Q^2 limits from pQCD

$$F_{P\gamma\gamma}(-Q^2,0) \to \frac{2F_P}{Q^2} \text{ (Brodsky-Lepage)}$$

$$F_{P\gamma\gamma}(-Q^2,-Q^2) \to \frac{2F_P}{3Q^2} \text{ (OPE)}$$

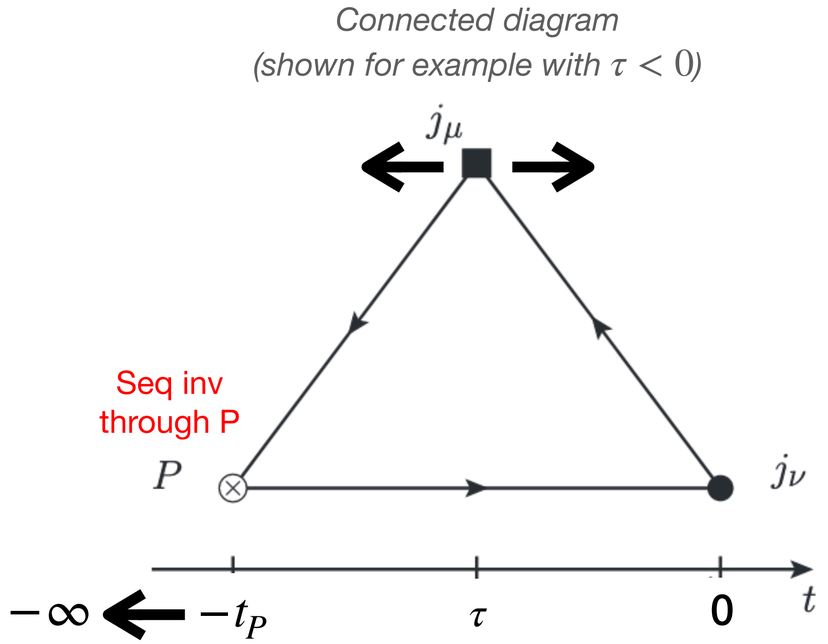


Lattice calculation of $F_{P\gamma\gamma}$

1. Euclidean transition amplitude

$$\tilde{A}_{\mu\nu}(\tau) = \sum_{\mathbf{x}} e^{-i\mathbf{q}_1\cdot\mathbf{x}} \left\langle 0 \left| j_\mu(\tau;\mathbf{x})\, j_\nu(0;\mathbf{0}) \right| P(\mathbf{p}) \right\rangle$$

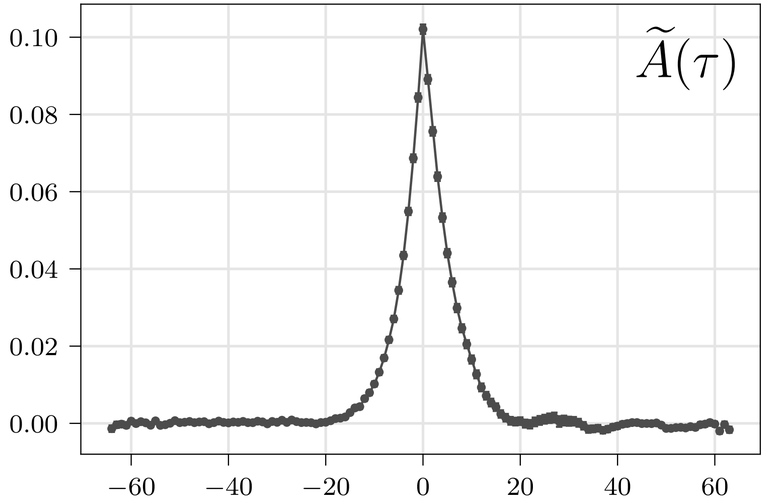
Note: renormalized currents
using precisely known Z_A, Z_V



2. Laplace transform

$$\epsilon^{\mu\nu\rho\sigma} q_{1\rho} q_{2\sigma} F_{P\gamma\gamma}(q_1^2, q_2^2) = -\, i^{n_0} \int_{-\infty}^{\infty} d\tau\; e^{\omega_1 \tau}\; \tilde{A}_{\mu\nu}(\tau)$$

where $\mathbf{q}_2 = \mathbf{p} - \mathbf{q}_1$ by momentum conservation



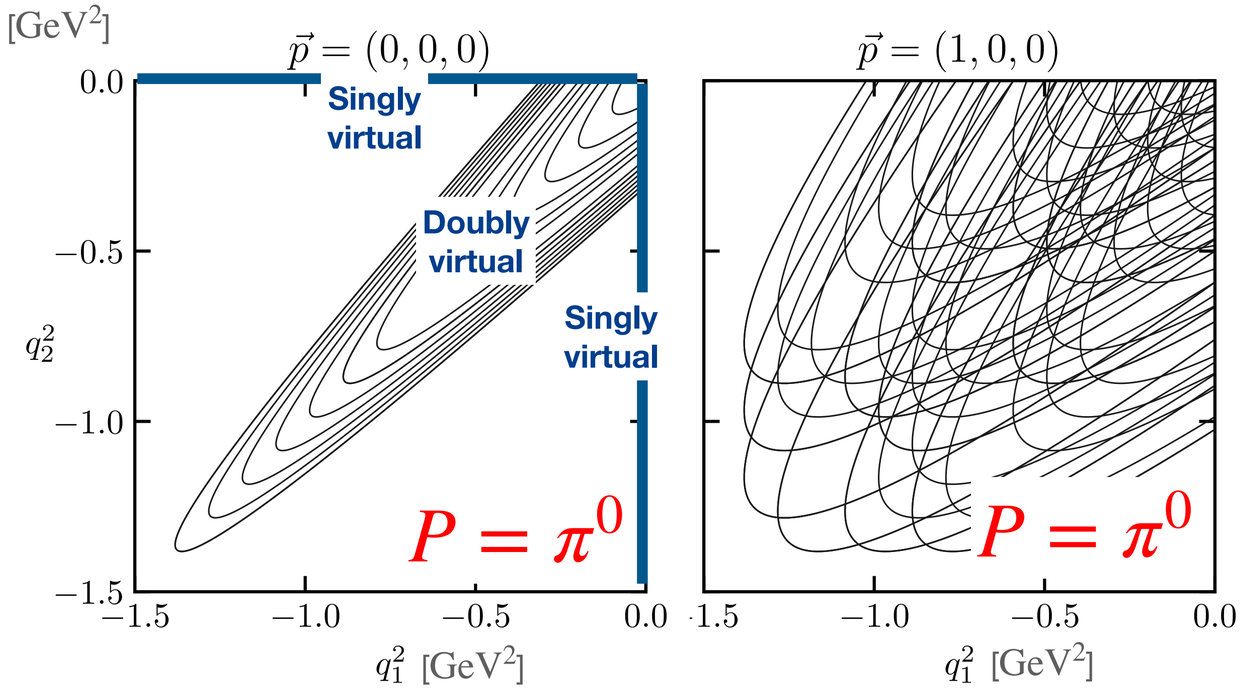
Extrapolating in (q_1^2, q_2^2) plane

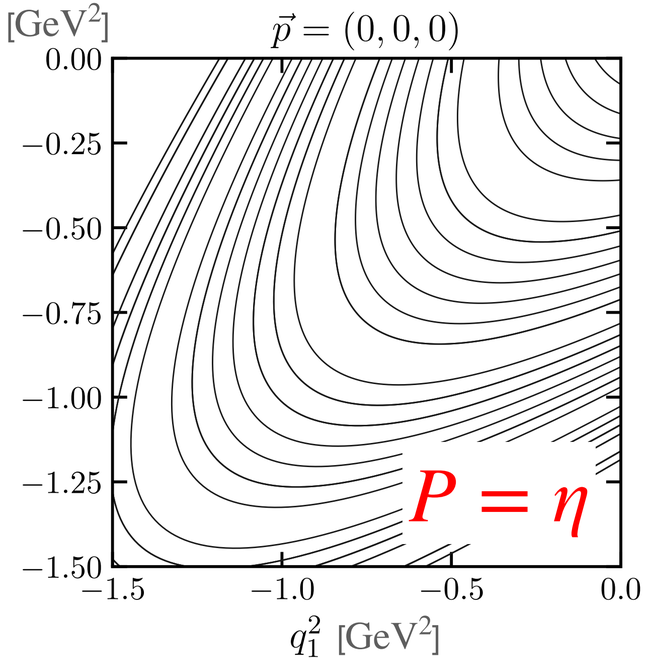
Finite volume momenta $\mathbf{q}_1$, $\mathbf{p}$, arbitrary ω_1

- Parabolic “orbits” of (q_1^2, q_2^2) accessible
- Significant dependence on $\mathbf{p}$ and m_P

z-expansion for full (q_1^2, q_2^2) dependence

$L \approx 5.1\text{fm}$





Summary of ETMC calculations

Action: Nf = 2+1+1 twisted clover, Iwasaki gauge action, *physical point* $\{l, s, c\}$

ensemble	$L^3 \cdot T/a^4$	m_π [MeV]	a [fm]	L [fm]	$m_\pi \cdot L$
cB072.64	$64^3 \cdot 128$	136.8(6)	0.082	5.22	3.6
cC060.80	$80^3 \cdot 160$	134.2(5)	0.069	5.55	3.8
cD054.96	$96^3 \times 192$	140.8	0.057	5.46	3.9

PRD 104, 074520 (2021)

Pion analysis:

All three lattice spacings + continuum extrapolation.

Pion Transition Form Factor from Twisted-Mass Lattice QCD and the Hadronic Light-by-Light π^0-pole Contribution to the Muon $g-2$

C. Alexandrou,[1,2] S. Bacchio,[2] G. Bergner,[3] S. Burri,[4] J. Finkenrath,[5,2] A. Gasbarro,[4] K. Hadjiyiannakou,[1,2] K. Jansen,[6] G. Kanwar,[4] B. Kostrzewa,[7] G. Koutsou,[2] K. Ottnad,[8] M. Petschlies,[9,10] F. Pittler,[2] F. Steffens,[9,10] C. Urbach,[9,10] and U. Wenger[4]

(Extended Twisted Mass Collaboration)

PRD 108, 094514 (2023)

Eta analysis:

Coarsest lattice spacing (cB) complete, finer cC under way.

$\eta \to \gamma^*\gamma^*$ transition form factor and the hadronic light-by-light η-pole contribution to the muon $g-2$ from lattice QCD

Constantia Alexandrou,[1,2] Simone Bacchio,[2] Sebastian Burri,[3] Jacob Finkenrath,[2] Andrew Gasbarro,[3] Kyriakos Hadjiyiannakou,[1,2] Karl Jansen,[4] Gurtej Kanwar,[3] Bartosz Kostrzewa,[5] Konstantin Ottnad,[6] Marcus Petschlies,[7,8] Ferenc Pittler,[2] Carsten Urbach,[7,8] and Urs Wenger[3]

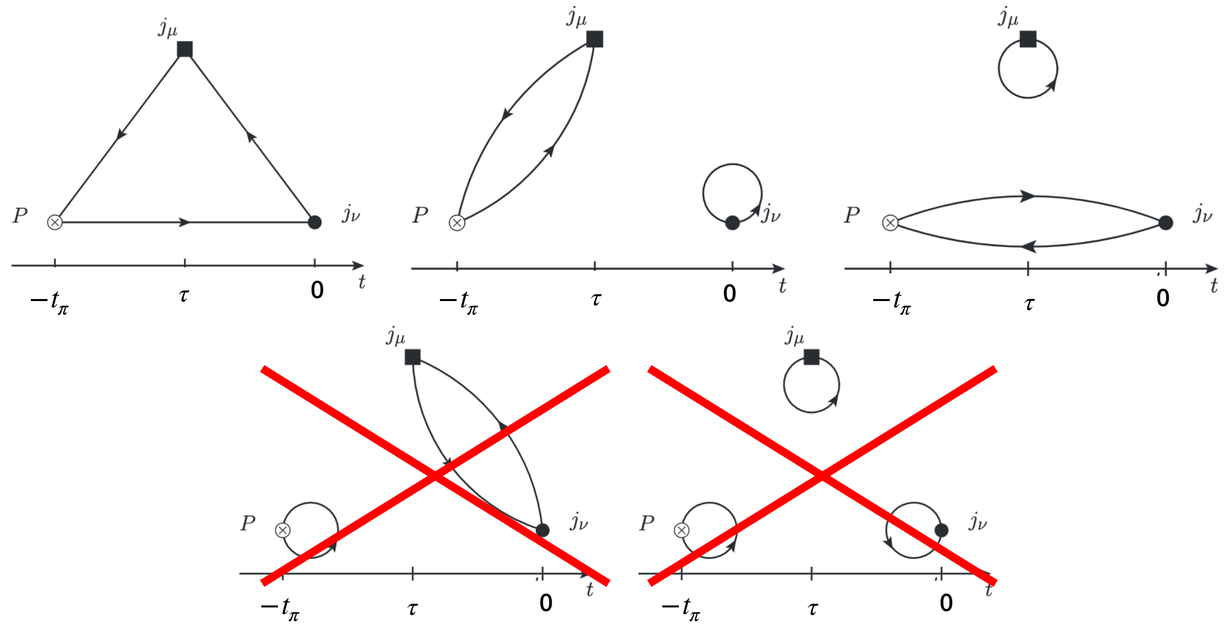
(Extended Twisted Mass Collaboration)

PRD 108, 054509 (2023)

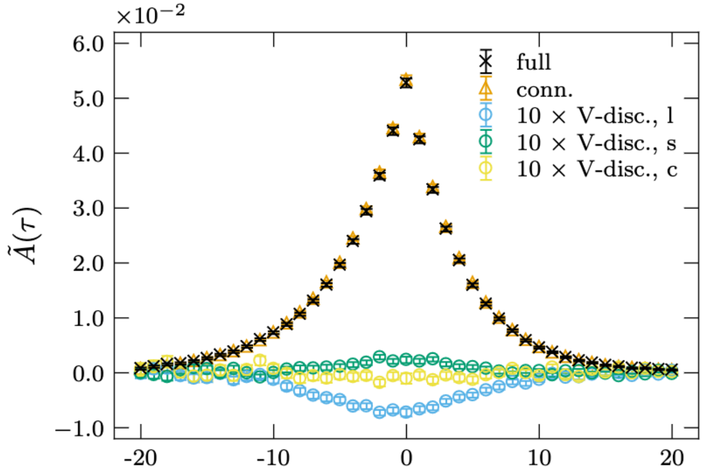
Pion analysis

Wick contractions

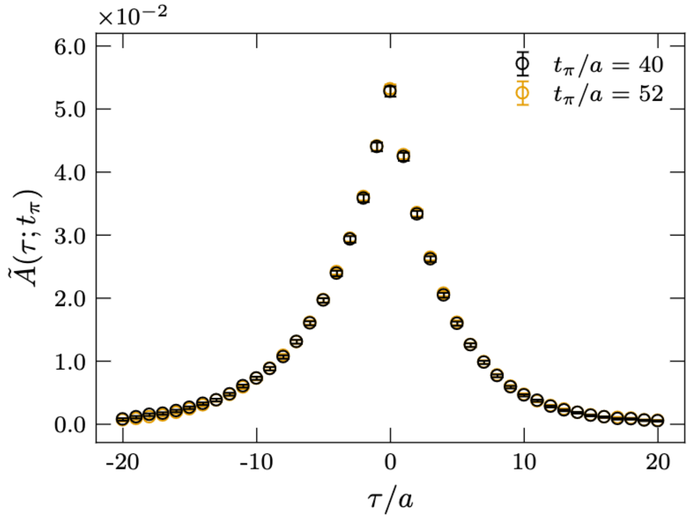
Isospin symmetry allows removing
P-disconnected diagrams:



Twisted mass isospin breaking introduces additional $O(a^2)$ artifact
$\Longrightarrow$ extrapolated away in continuum limit



Contributions to amplitude on cD ens ($a \approx 0.06$fm)



Check of excited state effects on cD ens ($a \approx 0.06$fm)

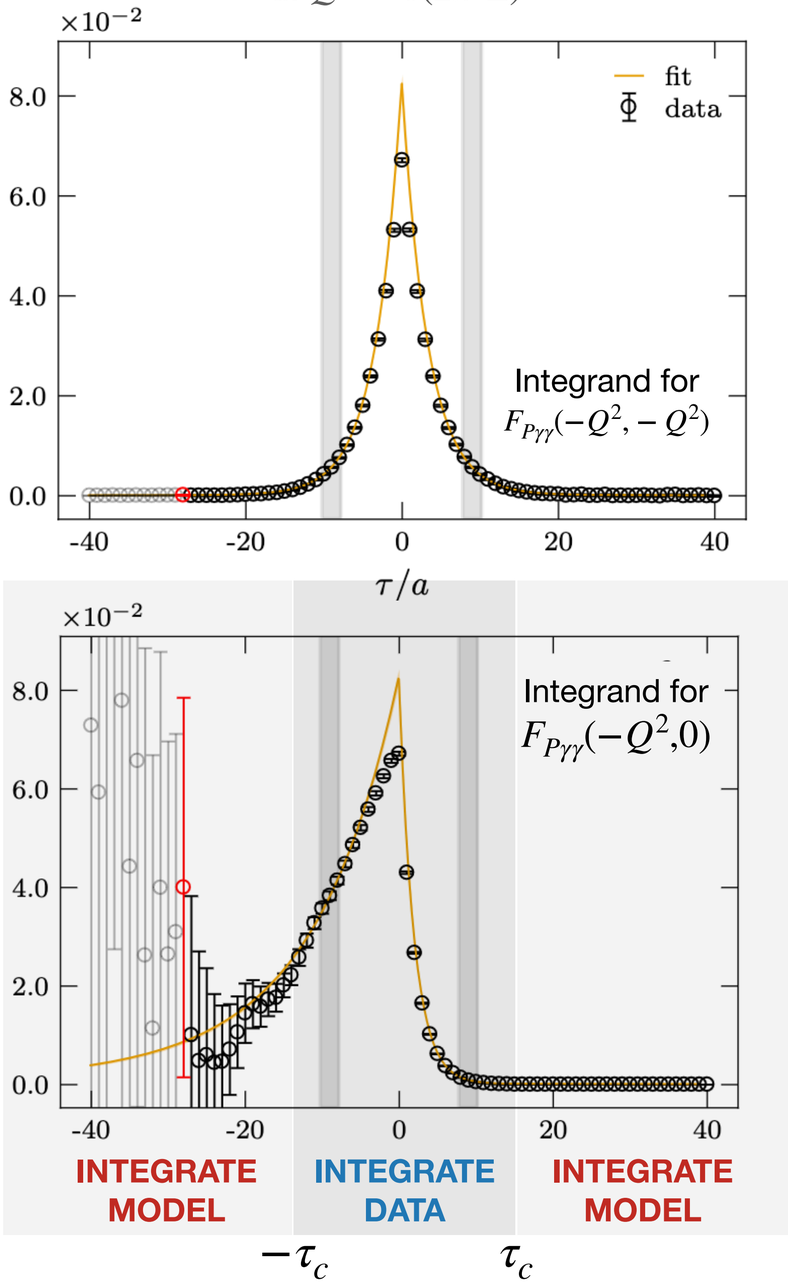
Tail fitting and integration

- Exponentially growing noise in the tails of $\tilde{A}_{\mu\nu}(\tau)$
  - Probed in the Laplace transform

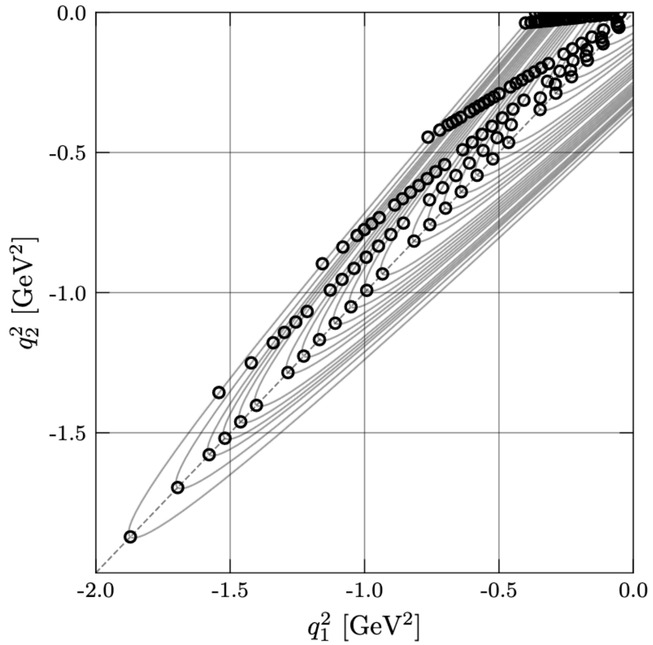
$$\epsilon^{\mu\nu\rho\sigma} q_{1\rho} q_{2\sigma} F_{P\gamma\gamma}(q_1^2, q_2^2) = -\, i^{n_0} \int_{-\infty}^{\infty} d\tau\, e^{\omega_1 \tau}\, \tilde{A}_{\mu\nu}(\tau)$$

  - Fit Vector Meson Dominance (VMD) or Lowest Meson Dominance (LMD) models to the tails
  - Data content = $F^{\text{data}}/(F^{\text{data}} + F^{\text{model}})$

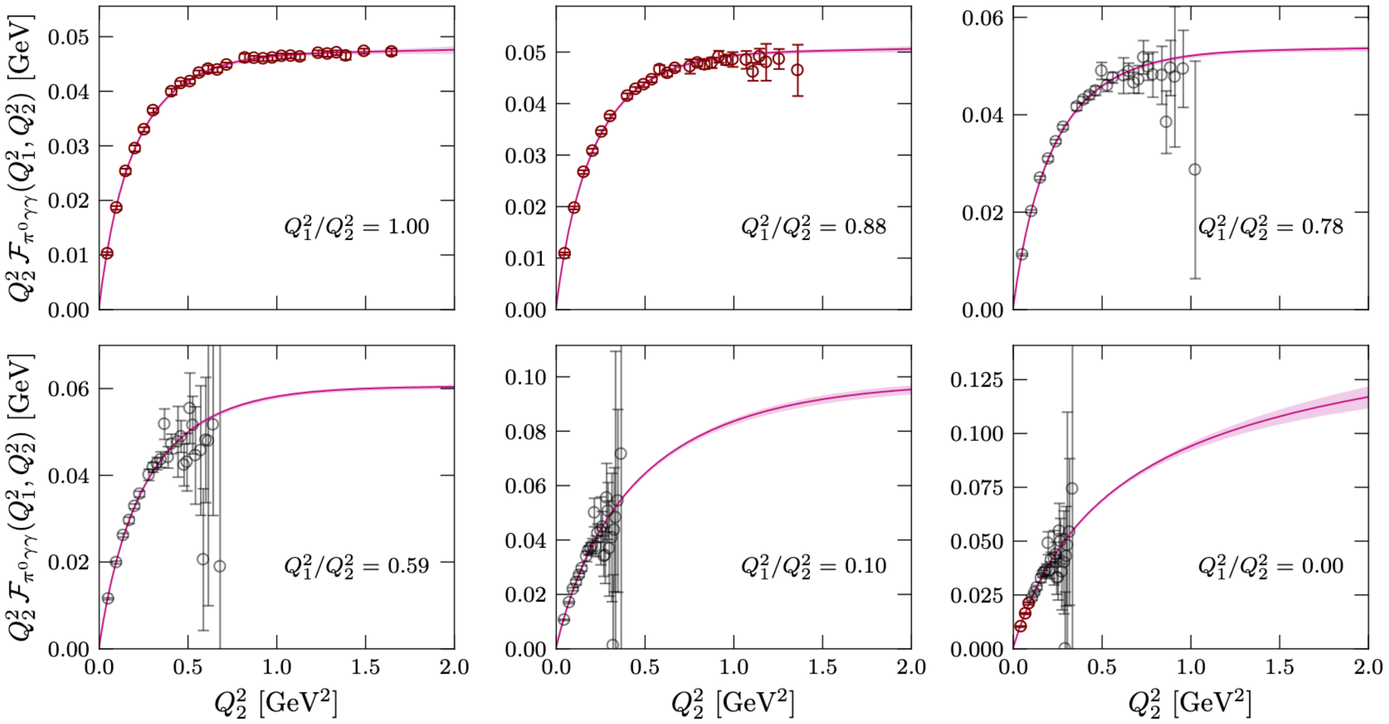
Example tail fits on cB ens ($a \approx 0.08\text{fm}$) at $Q^2 = 6(2\pi/L)^2$



Kinematics and global fits



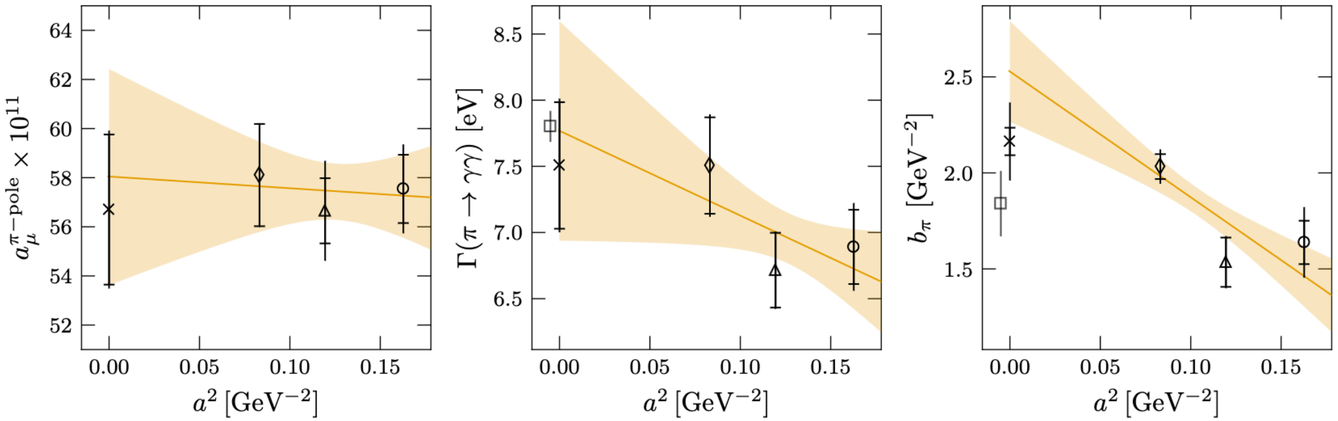
Choices of ω_1 per orbit on cB ens ($a \approx 0.08\text{fm}$) determining kinematic points used as inputs to global fit



Data along each slice and results of global fits

- Input data: points with $\geq 95\,\%$ data content to minimize model dependence
- Correlated global fits to all data points

Continuum extrapolation

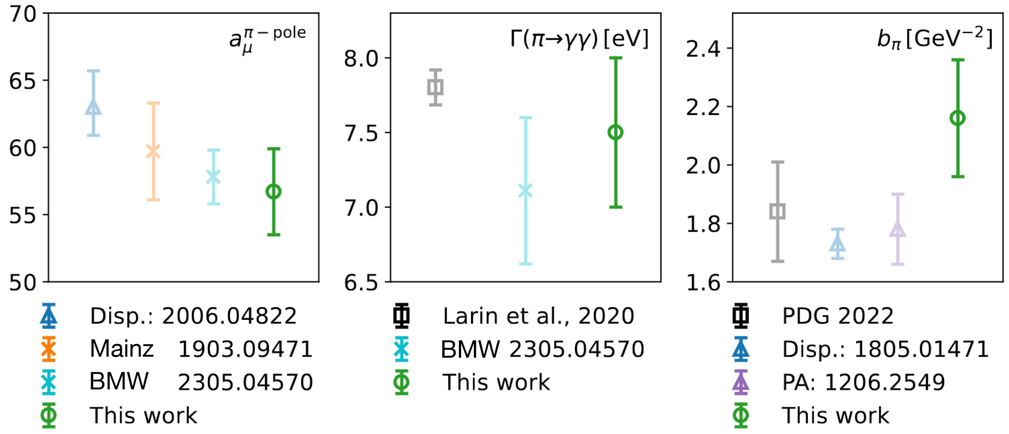


- Preferred extrapolation (black pt):
  Combined z-exp fit to TFF kinematics across all 3 lattice spacings
- Excluded global fits with unphysical $Q^2 \to \infty$ behavior
- Comparison (orange): Linear $\mathcal{O}(a^2)$ extrapolation

- Systematic errors assigned via AIC-weighted averaging
- Statistical errors dominate except for slope parameter b_π

Lattice TFF data only

combined fit	$a_\mu^{\pi^0-\text{pole}} \times 10^{11}$	$\Gamma(\pi \to \gamma\gamma)$ [eV]	b_π [GeV^{-2}]
value	**56.7 (3.2)**	**7.50 (0.50)**	**2.16 (0.20)**
σ_{stat}	3.06	0.48	0.07
$\sigma_{\text{sys},\, z-\text{exp.}}$	0.91	0.14	0.19
σ_{sys}	0.35	0.07	0.01



Lattice + expt TFF data

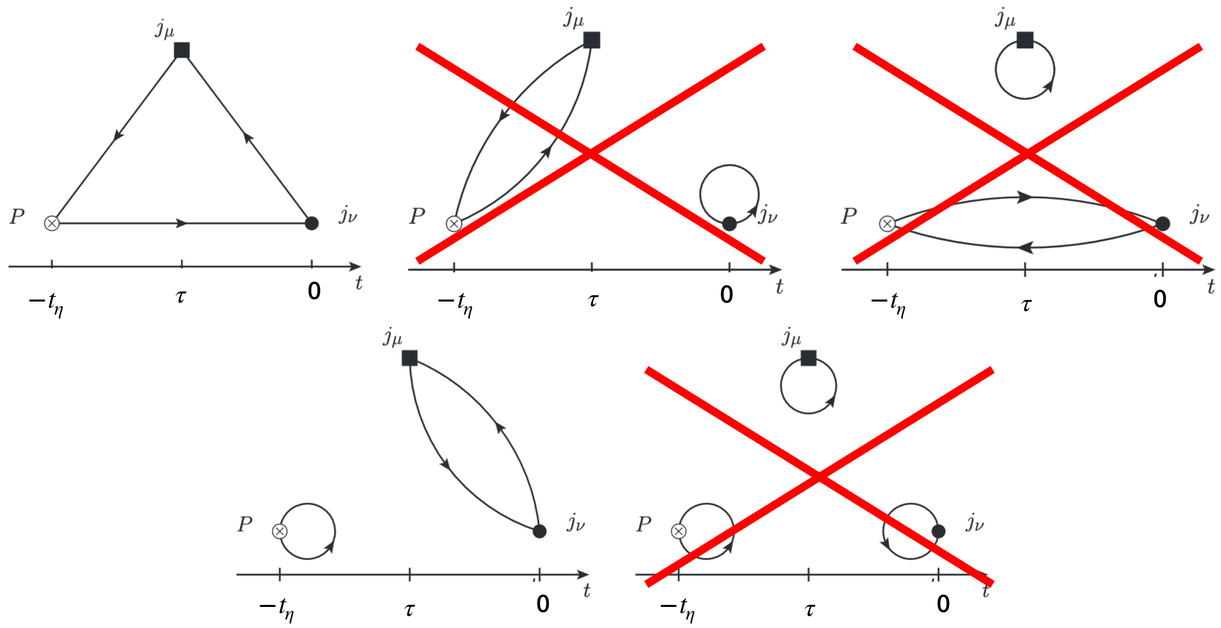
$$a_\mu^{\pi^0-\text{pole}} = 61.7(2.0)_{\text{stat}}(0.5)_{\text{sys}}[2.0]_{\text{tot}} \times 10^{-11},$$
$$\Gamma(\pi \to \gamma\gamma) = 7.97(0.35)_{\text{stat}}(0.11)_{\text{sys}}[0.37]_{\text{tot}}\ \text{eV},$$
$$b_\pi = 2.336(0.049)_{\text{stat}}(0.018)_{\text{sys}}[0.052]_{\text{tot}}\ \text{GeV}^{-2}.$$

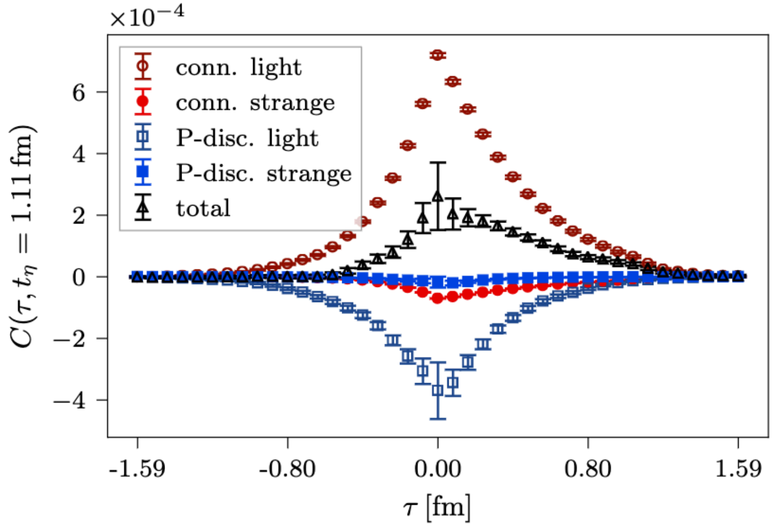
Eta analysis

Wick contractions

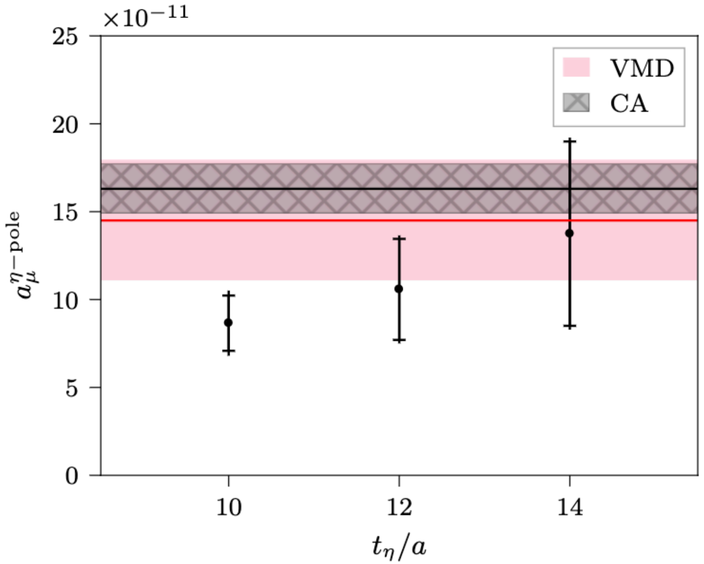
P-disconnected diagrams must be included.

Preliminary analysis based on dropping numerically small V-disconnected diagrams.





Contributions to amplitude on cB ens ($a \approx 0.08$fm)



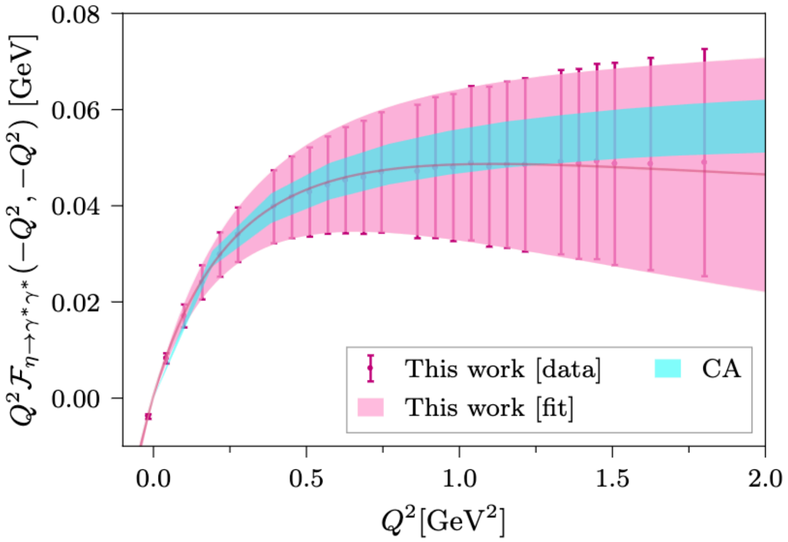
Check of excited state effects on cB ens ($a \approx 0.08$fm)

Global fit results

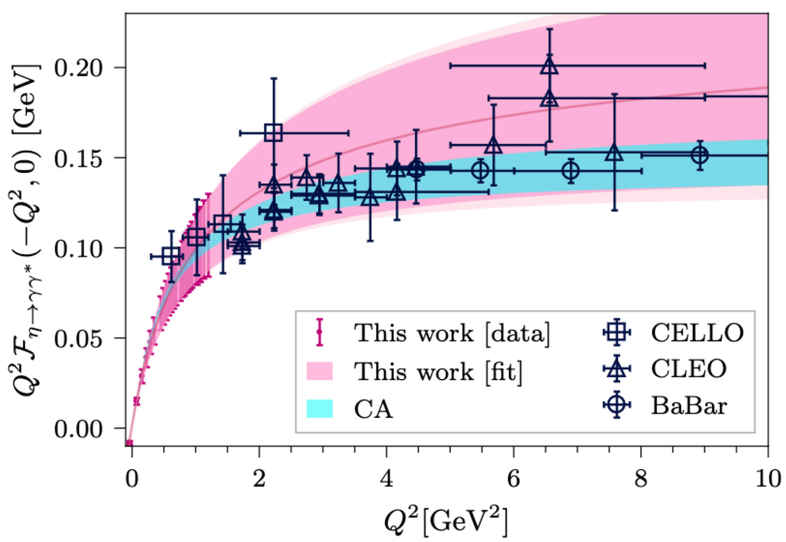
Similar analysis chain $\tilde{A}(\tau) \rightarrow \mathscr{F}_{\eta\rightarrow\gamma\gamma} \rightarrow z$-exp fits.

Statistical errors dominate.

Doubly virtual TFF on cB ens ($a \approx 0.08\text{fm}$)

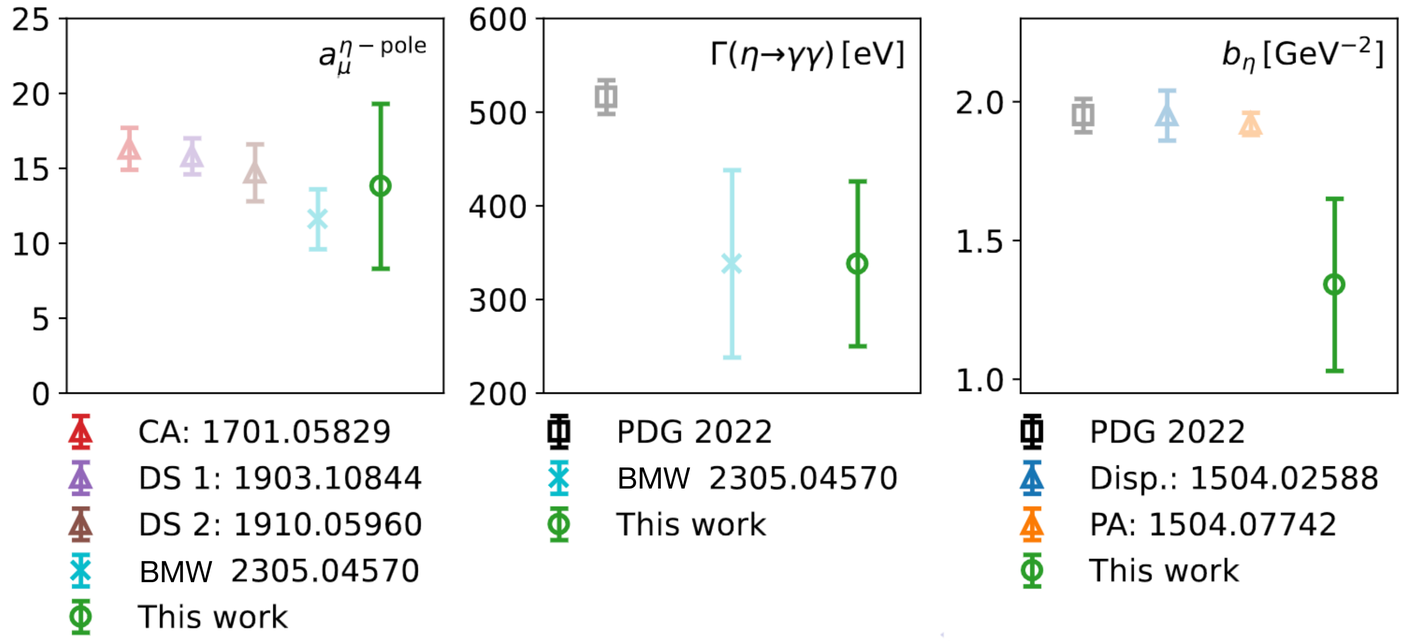


Singly virtual TFF on cB ens ($a \approx 0.08\text{fm}$)



Results at $a \approx 0.08\text{fm}$

	$a_\mu^{\eta\text{-pole}} \times 10^{11}$	$\Gamma(\eta \to \gamma\gamma)$ [eV]	b_η [GeV^{-2}]
value	**13.8 (5.5)**	**338 (88)**	**1.34 (0.31)**
σ_{stat}	5.24	86.7	0.279
Tail model vs data cut (τ_c)	0.22	10.1	0.020
Tail fit windows (t_i, t_f)	0.18	6.5	0.009
Fit model (VMD vs. LMD)	0.31	11.6	0.034
Conformal fit order (N)	1.44	1.8	0.123



Outlook

1. Evaluation of η TFF on finer lattices under way
   - cC ($a \approx 0.07\mathrm{fm}$) and cD ($a \approx 0.06\mathrm{fm}$) ensembles
   - Continuum limit
2. GEVP interpolators required for η', can also improve η TFFs
   - Note: η is the ground state of relevant channel, so no theoretical issues
3. TFF calculations may provide correlated input for full HLbL calculation
   - Remove finite volume effects in π^0-pole contribution