A Simple Model of Pentaquarks

Davide Germani

Based on:

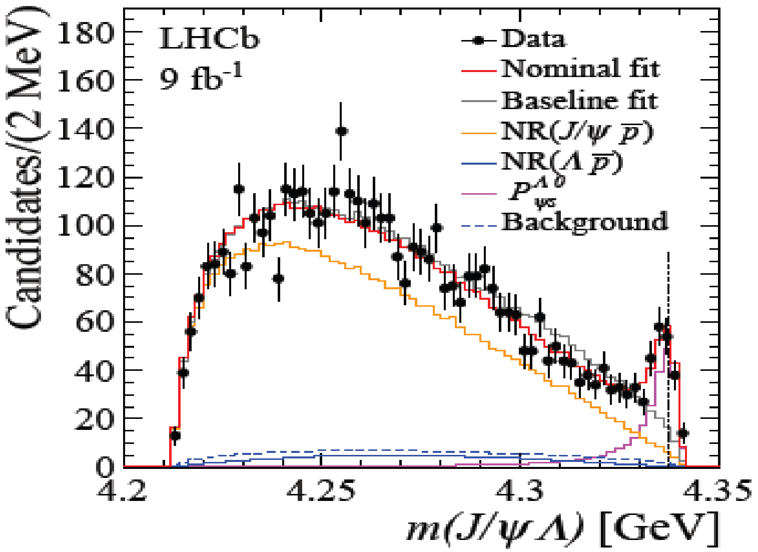
D. Germani, A. D. Polosa, F. Niliani, **A Simple Model of Pentaquarks (arXiv:2403.04068 [hep-ph])**

Outline

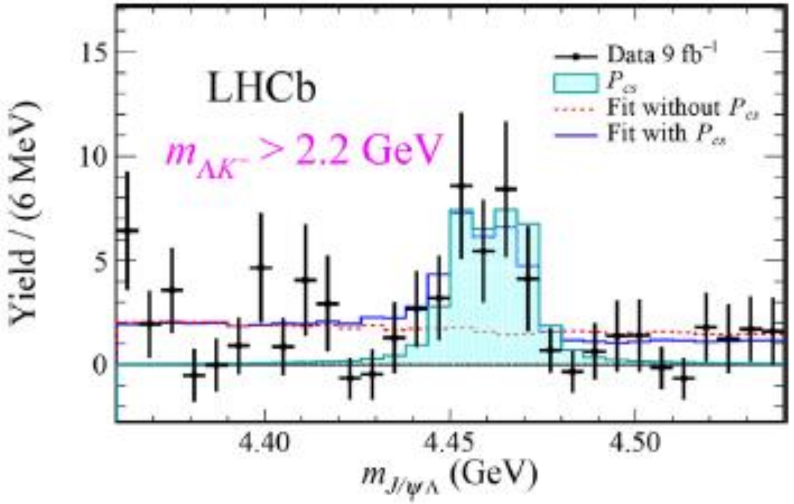
- Overview of the current situation;

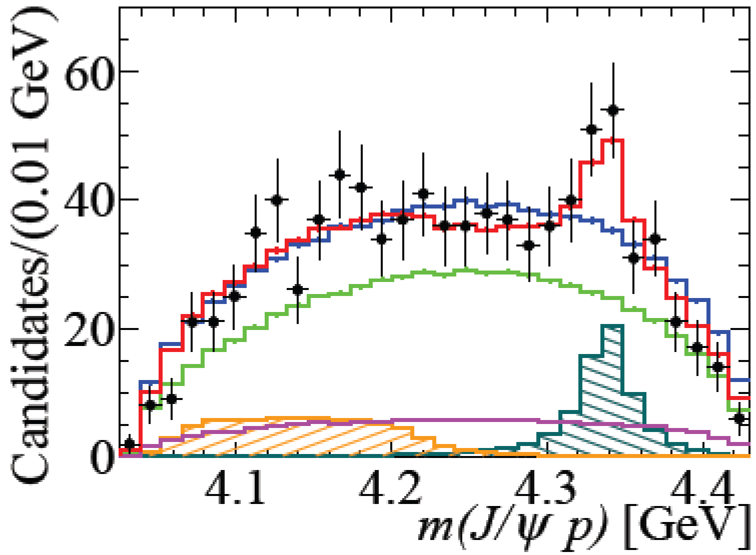
- A simple model for pentaquarks
    - A different point of view
    - Our ‘Hadro-Charmonia’ model: the role of the exchange interaction
    - Spin assignment and Mass prediction
    - **Possible new processes**
    - **Suppression Mechanism in Baryons**

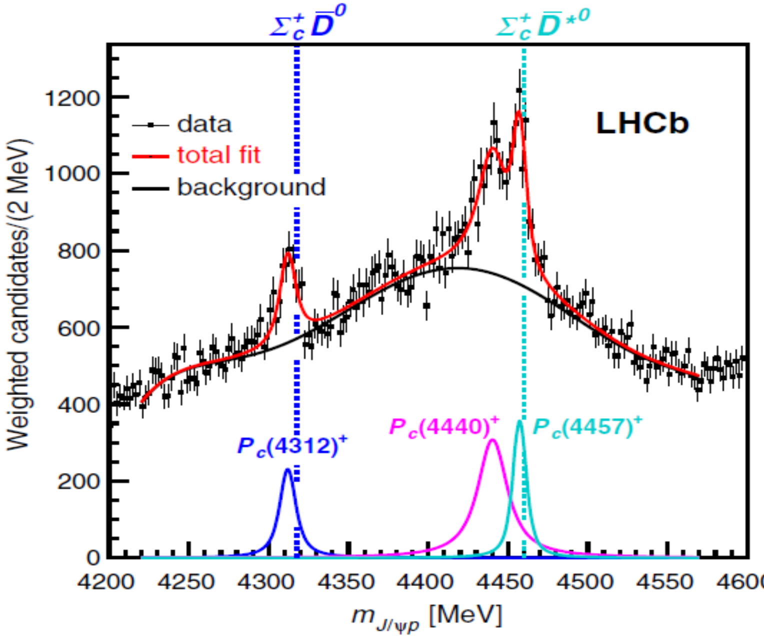
Current situation



State	Mass [MeV]	Width [MeV]	Observed Process	Year
$P_c(4312)$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$	$\Lambda_b^0 \to (J/\psi\, p) K^-$	2019
$\widetilde{P}_c(4337)$	$4337^{+7}_{-4}{}^{+2}_{-2}$	$29^{+26}_{-12}{}^{+14}_{-14}$	$B_s^0 \to (J/\psi p)\, \overline{p}$	2022
$P_c(4440)$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+8.7}_{-10.1}$	$\Lambda_b^0 \to (J/\psi\, p) K^-$	2019
$P_c(4457)$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$	$\Lambda_b^0 \to (J/\psi\, p) K^-$	2019
$\widetilde{P}_{cs}(4338)^{\frac{1}{2}^-}$	$4338.2 \pm 0.7 \pm 0.4$	$7.0 \pm 1.2 \pm 1.3$	$B^- \to (J/\psi\, \Lambda)\, \overline{p}$	2022
$P_{cs}(4459)$	$4458.9 \pm 2.9^{+4.7}_{-1.1}$	$17.3 \pm 6.5^{8.0}_{-5.7}$	$\Xi_b^- \to (J/\psi\, \Lambda) K^-$	2021





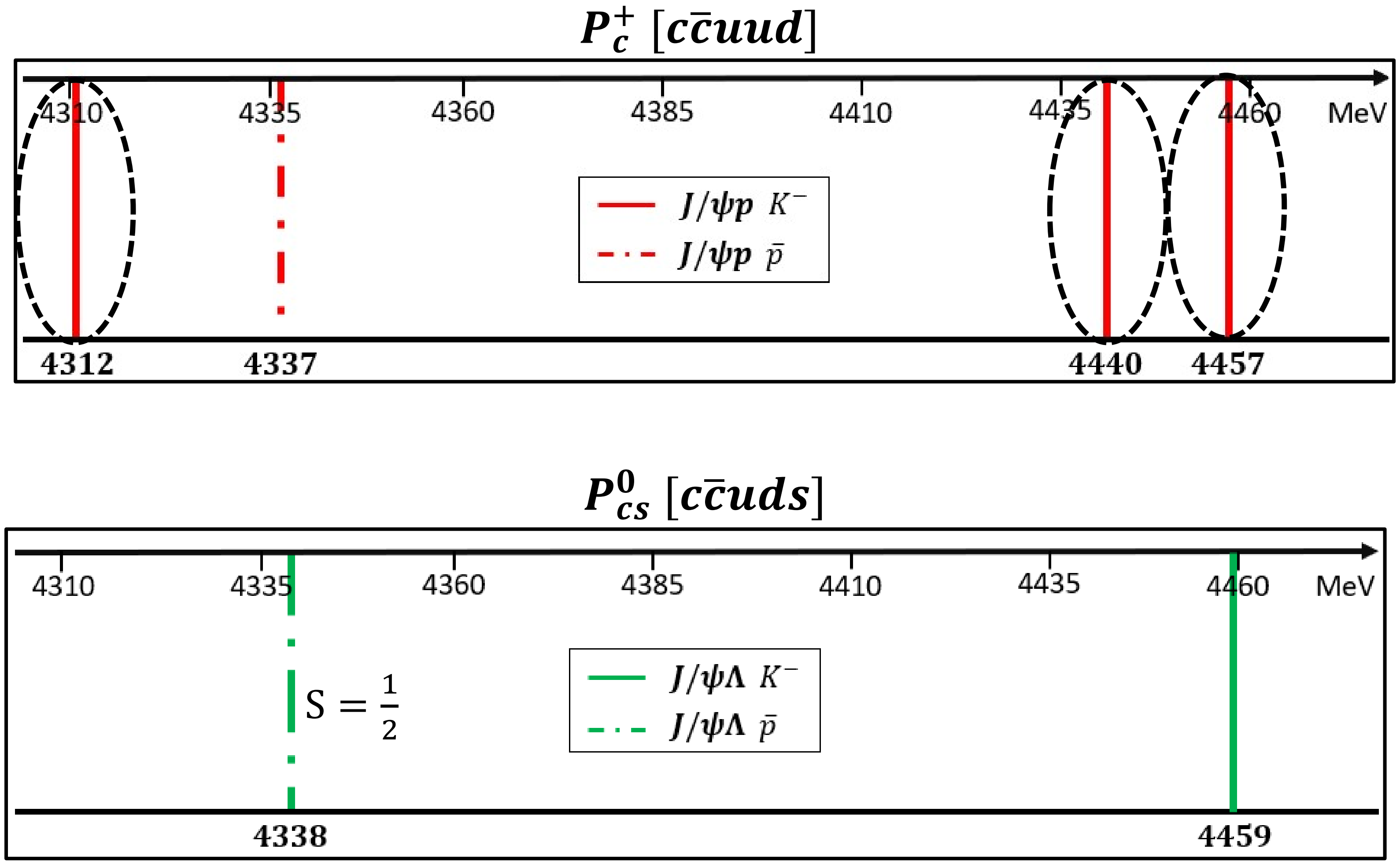


R. Aaij et al. arXiv:1904.03947 [hep-ex] (2019), R. Aaij et al. arXiv:2012.10380v2 [hep-ex] (2021), R. Aaij et al. arXiv:2210.10346 [hep-ex] (2021), R. Aaij et al. arXiv:2108.04720 [hep-ex] (2022)

A different point of view

- We divide the spectrum according to strangeness content.
- Data suggests two different type of production: in association with a K^- or with an $\bar{p}$.
- Pentaquark seems to appear in triplet

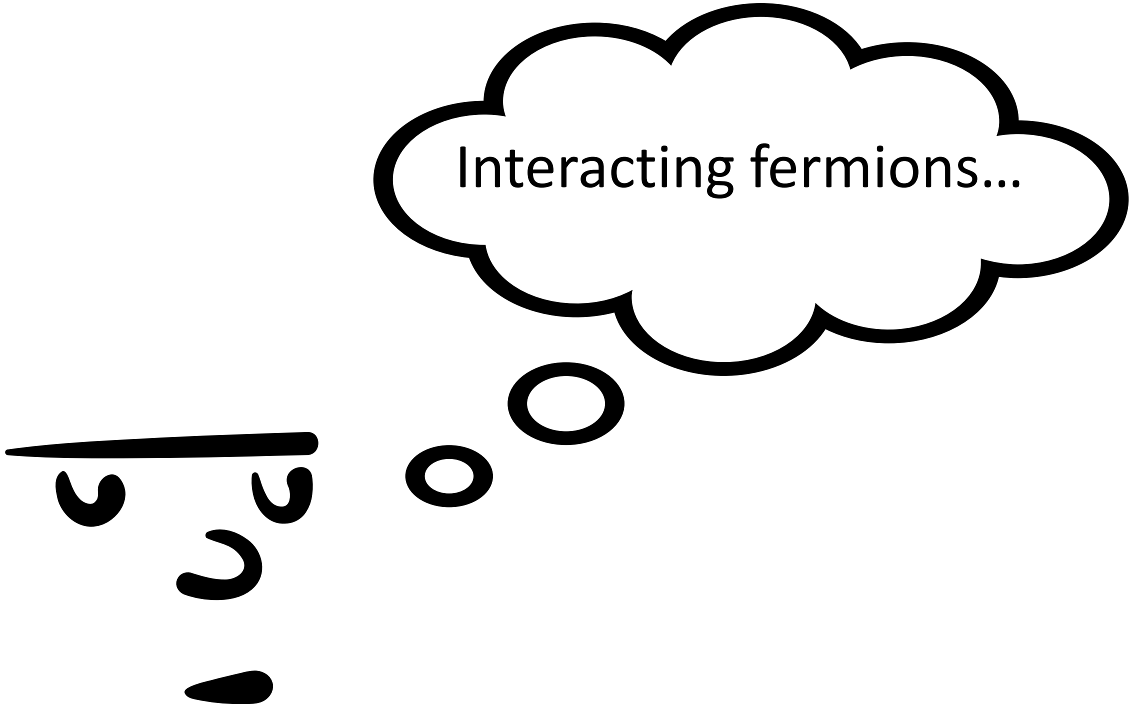
Can we build a model to account for these properties?

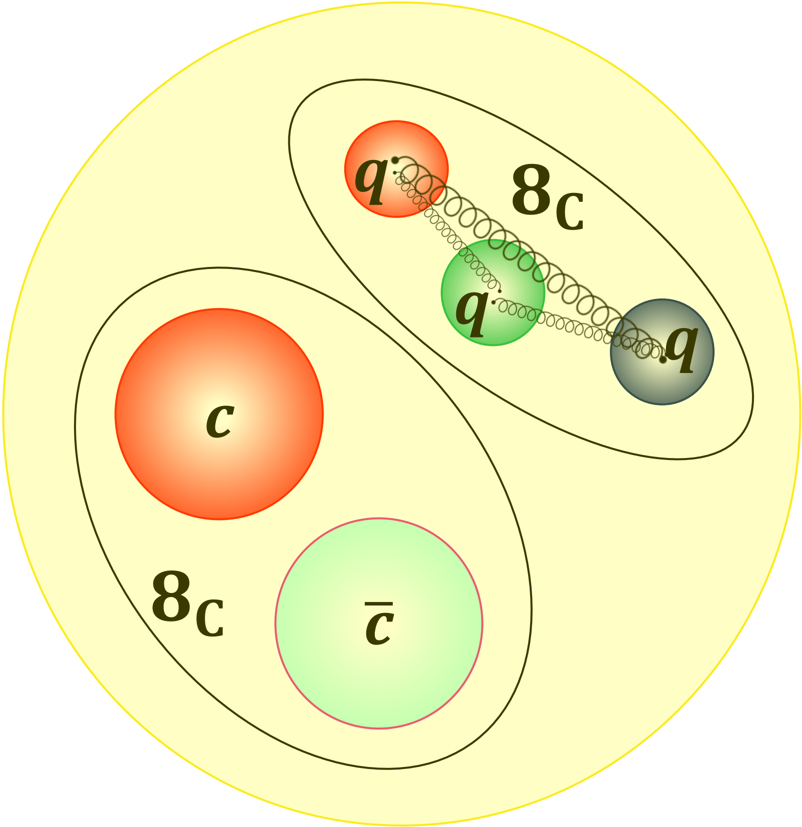


An ‘Hadro-Charmonia’ Model

- The starting point is more or less the same of the Born-Oppenheimer approach. We consider the $c\bar{c}$ system in a color octet as well as the three light quarks sysyem;

- **Fermi statistics for ligth quarks;**





Exchange Interaction! Three Fermions Case

$$V = -\sum_{\text{pairs}} J_{ab} \left(\frac{1}{2} + 2\mathbf{S}_a \cdot \mathbf{S}_b \right)$$

Generalization of the well known two fermions case

$$J_{ab} \equiv \int [\phi_a(\mathbf{r}_1)\phi_b(\mathbf{r}_2)]^* \, U(\mathbf{r}_1 - \mathbf{r}_2) \, [\phi_b(\mathbf{r}_1)\phi_a(\mathbf{r}_2)] \, d^3\mathbf{r}_1 \, d^3\mathbf{r}_2$$

The potential has **three** distinct eigenvalues

$$\Delta E_{1/2} = \pm\sqrt{J_{12}^2 + J_{13}^2 + J_{23}^2 - J_{12}J_{13} - J_{12}J_{23} - J_{13}J_{23}}$$

$$\Delta E_{3/2} = -J_{12} - J_{13} - J_{23}$$

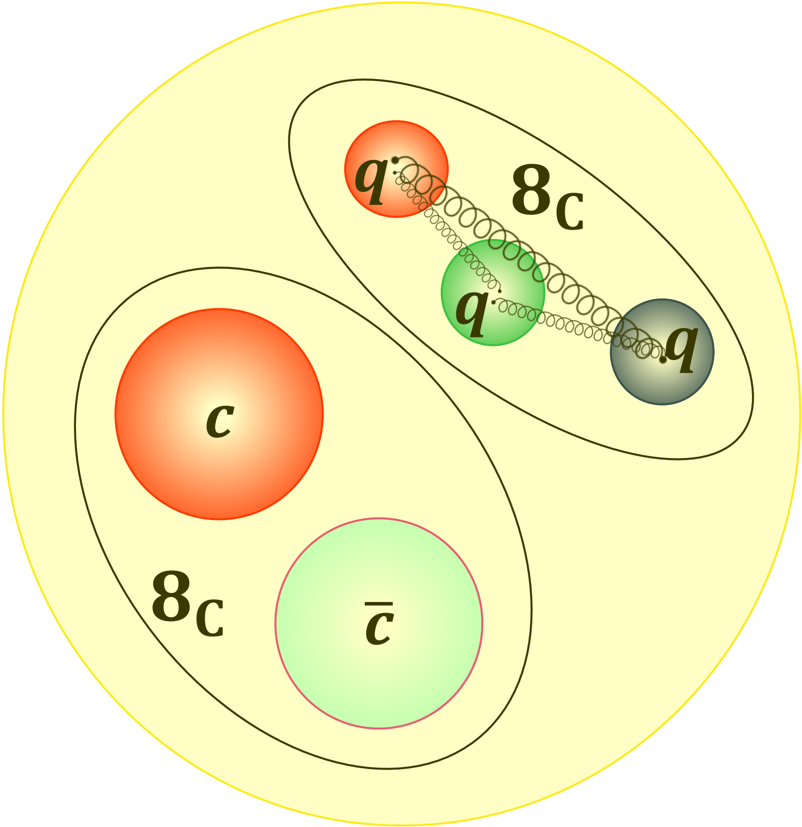
An 'Hadro-Charmonia' Model

- The starting point is more or less the same of the Born-Oppenheimer approach. We consider the $c\bar{c}$ system in a color octet as well as the three light quarks sysyem;

- Fermi statistics for ligth quarks;

- **Exchange Interaction**. But to use to use the exchange interaction, we need to have a precise symmetry on the spin-orbital part which means a complete symmetry in the **color-flavor** sector

$$\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{1} \oplus \mathbf{8} \oplus \mathbf{8'} \oplus \mathbf{10}$$ (flavor)



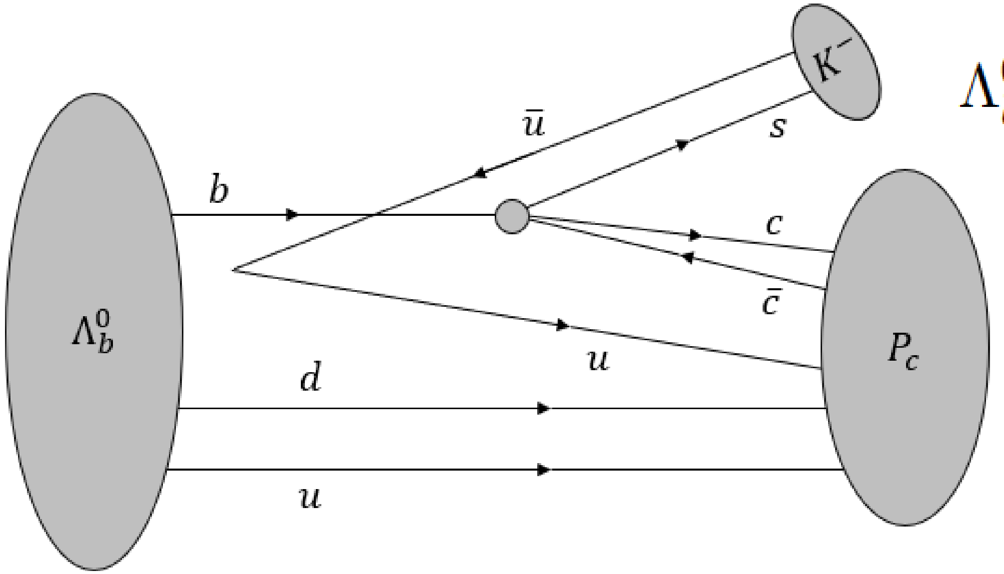
Symmetric and Antisymmetric Combination

1. $$\mathbf{S}^{abc}_{ijk} = 6\left(\eta_i^{[a}\eta_j^{b]}\eta_k^c - \eta_j^{[a}\eta_k^{b]}\eta_i^c\right) = 6\left(\eta^a_{[i}\eta^b_{j]}\eta^c_k - \eta^a_{[j}\eta^b_{k]}\eta^c_i\right)$$ P

$$\eta^{a,i}\eta^{b,j} = \eta^{b,j}\eta^{a,i}$$

2. $$\mathbf{A}^{abc}_{ijk} = 6\left(\psi_i^{[a}\psi_j^{b]}\psi_k^c - \psi_j^{[a}\psi_k^{b]}\psi_i^c\right) = 6\left(\psi^a_{(i}\psi^b_{j)}\psi^c_k - \psi^a_{(j}\psi^b_{k)}\psi^c_i\right)$$ $\tilde{P}$

$$\psi^{a,i}\psi^{b,j} = -\psi^{b,j}\psi^{a,i}$$



$$\Lambda_b^0 \to (J/\psi\, p)\, K^-$$

The diquark ud has initially $[ud]^{\bar{3}_c}_{\bar{3}_F\, 0_S}$ (good diquark) so it means that is in a symmetric configuration wrt color-flavor. We assume that the color-flavor symmetry is preserved in the process so we choose the S tensor for the pentaquarks produced in association with K^-.

Our 'Hadro-Charmonia' model

- **Fermi statistics for ligth quarks**

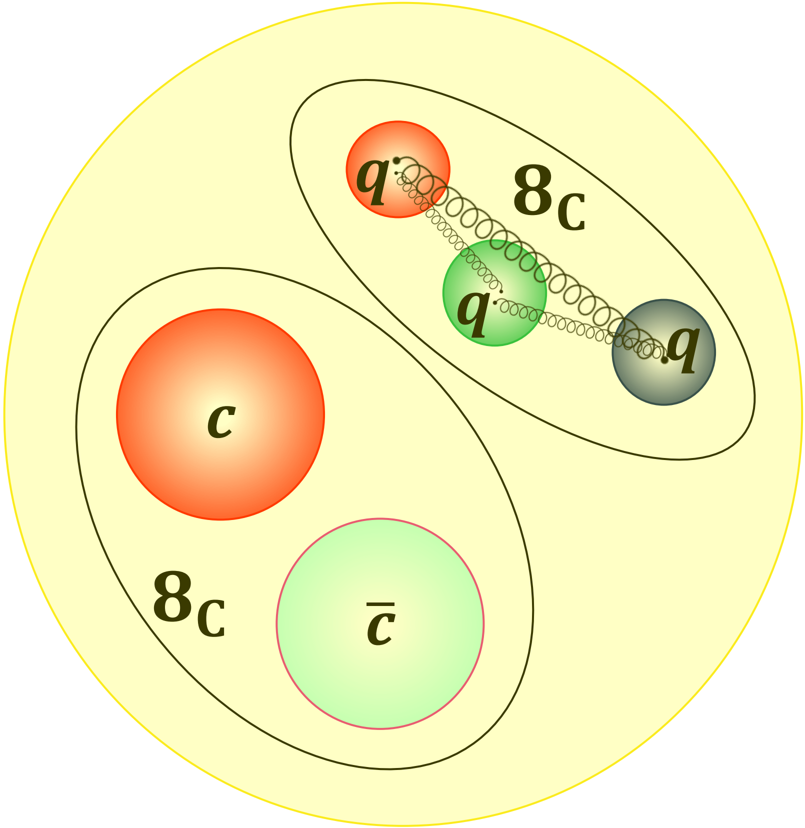
Color-Flavour	+	Spin-Orbital
S		A
A		S

- **Exchange Interaction**

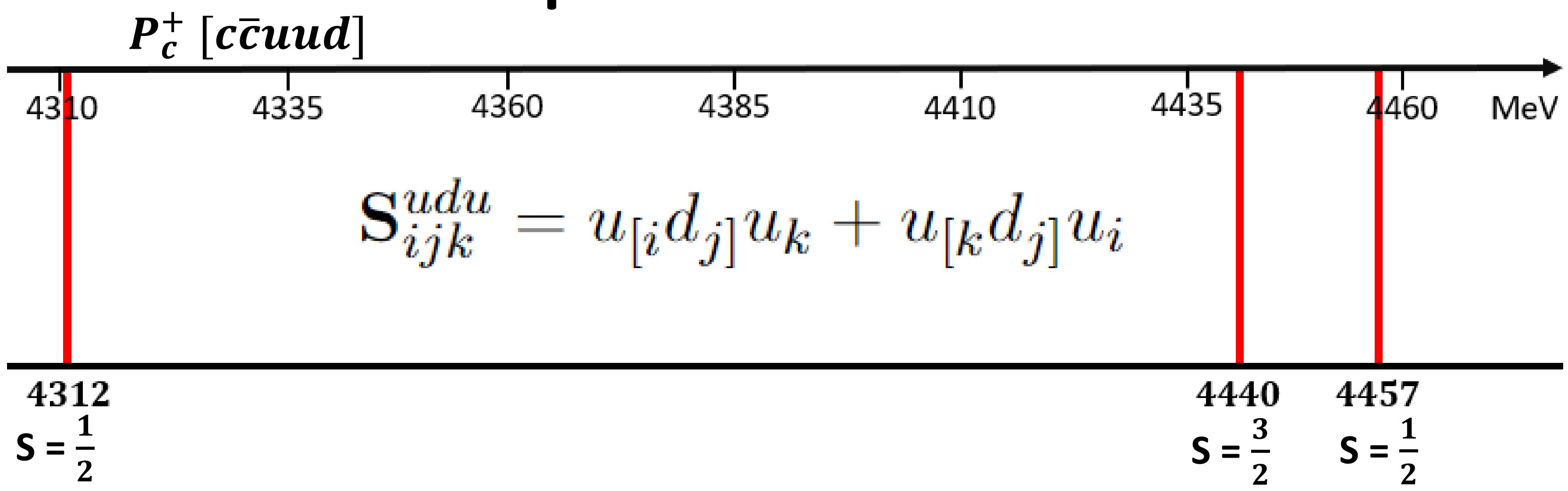
$$V = -\sum_{\text{pairs}} J_{ab} \left(\frac{1}{2} + 2\mathbf{S}_a \cdot \mathbf{S}_b \right)$$

- **J couplings:**

Experimental Data + qq interactions in one gluon exchange approximation + Spin assignment



J parameters: how do we fit?



1. $J_S^{uu} = J_S^{qq} > 0, \qquad J_A^{ud} = J_A^{qq} < 0$
2. $J_S = -\frac{1}{2} J_A \longrightarrow 2(C(\mathbf{6}) - 2C(\mathbf{3})) = -(C(\bar{\mathbf{3}}) - 2C(3))$
3. $$\begin{cases} M_{P_c^+}(4457) - M_{P_c^+}(4312) = 2|J_S^{qq} - J_A^{qq}| \\ M_{P_c^+}(4440) - \frac{M_{P_c^+}(4457)+M_{P_c^+}(4312)}{2} = -2J_A^{qq} - J_S^{qq} \end{cases}$$

We search for a solution of the sistem that respects the constraints on the sign an ratio of the couplings.

Spin assignment and Mass prediction

	Symmetric [MeV]	Antysimmetric [MeV]	No symmetry [MeV]
J^{qq}	$29.9^{+2.5}_{-2.8}$	$-42.8^{+2.4}_{-1.6}$	-
J^{qs}	17.9 ± 2	-25.7 ± 2	-3.9 ± 2

We have the spin prediction for these particles. Each triplet is ordered from top to bottom with $S = 1/2, 3/2, 1/2$.

$$\Delta E_{1/2} = \pm\sqrt{J_{12}^2 + J_{13}^2 + J_{23}^2 - J_{12}J_{13} - J_{12}J_{23} - J_{13}J_{23}}$$

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	Mass [MeV]		Mass [MeV]
$P_c(4312)$	$(4311.9^{+7}_{-0.9})$	$P_{cs}(4459)$	$(4458.8^{+6}_{-3.1})$
$P_c(4440)$	(4440.0^{+4}_{-5})	$\mathbf{P}'_{cs}$	4541 ± 6
$P_c(4457)$	$(4457.3^{+7}_{-1.8})$	$\mathbf{P}''_{cs}$	4565 ± 6
$\widetilde{\mathbf{P}}''_{\mathbf{c}}$	4187 ± 7	$\widetilde{P}_{cs}(4338)$	(4338.2 ± 0.8)
$\widetilde{\mathbf{P}}'_{\mathbf{c}}$	4276 ± 12	$\widetilde{\mathbf{P}}'_{\mathbf{cs}}$	4387 ± 4
$\widetilde{P}_c(4337)$	$4332 \pm 7\ (4337^{+7}_{-4}{}^{+2}_{-2})$	$\widetilde{\mathbf{P}}''_{\mathbf{cs}}$	4435 ± 4

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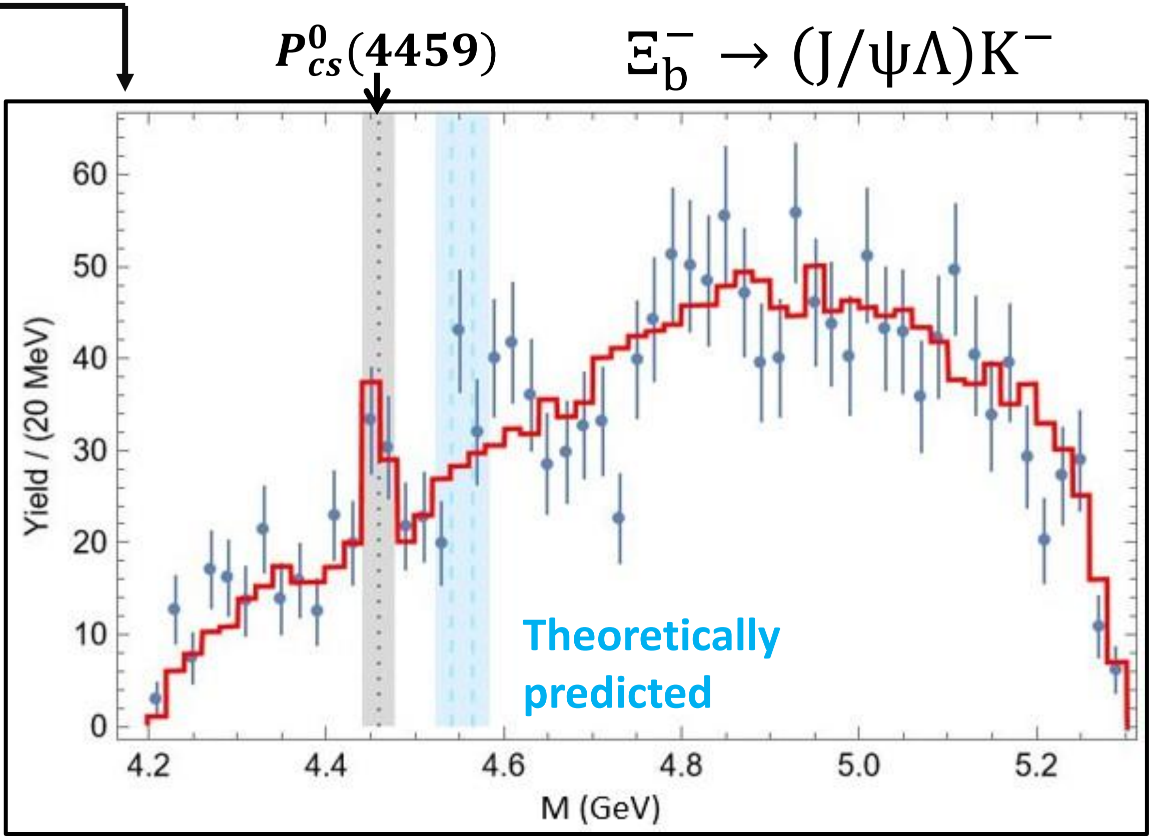

R. Aaij et al. arXiv:2012.10380v2 [hep−ex]

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→ **Not allowed in B^- decay**

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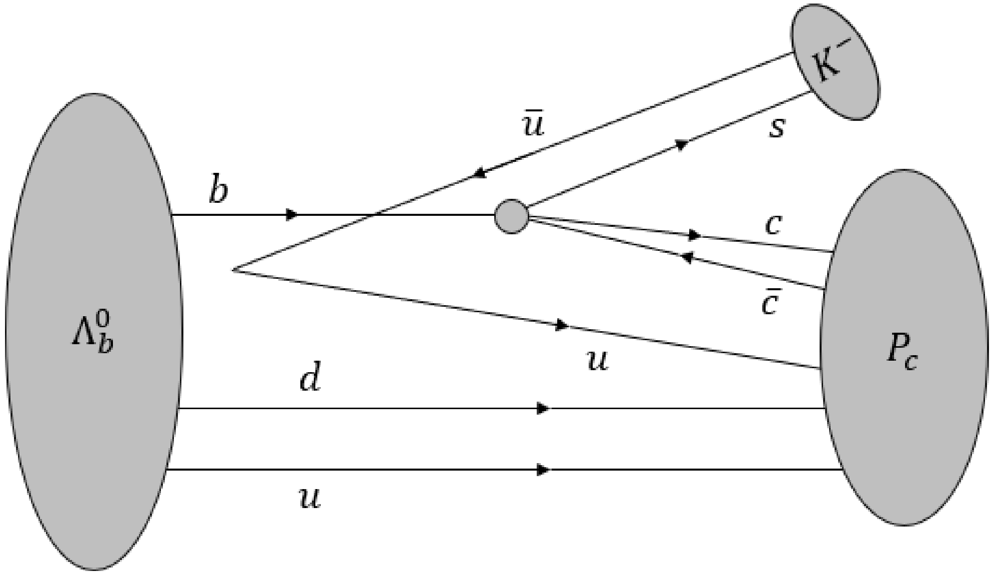
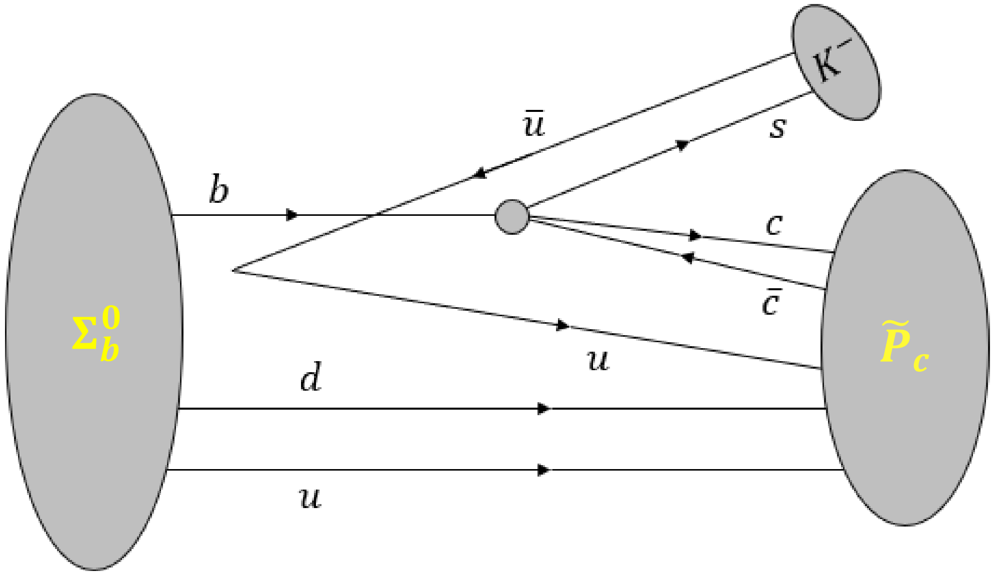
Good vs Bad diquarks



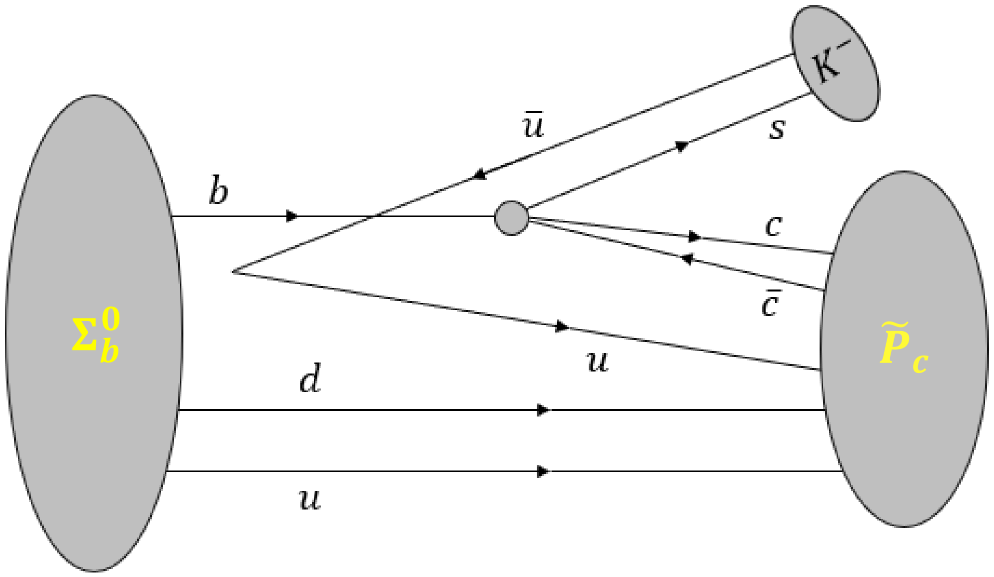
Triplet produces P pentaquarks



Sextet produces $\widetilde{\boldsymbol{P}}$ pentaquarks!

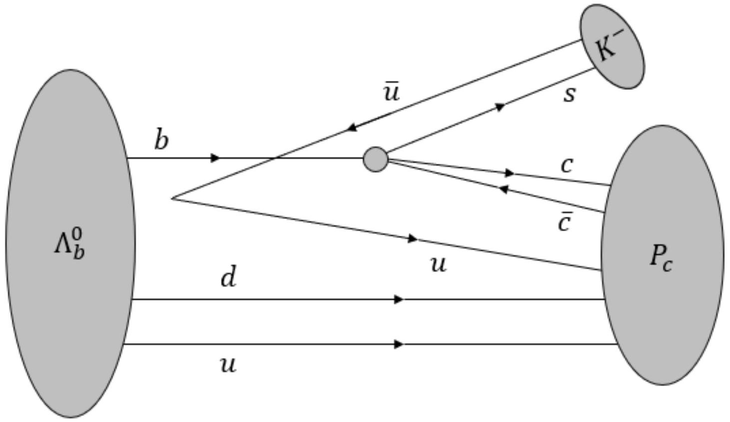
Good vs Bad diquarks

Dominant Channel	Cabibbo suppressed Channel
$\Sigma_b^+ \to \widetilde{P}_{cs}\pi^+$	$\Sigma_b^+ \to \widetilde{P}_{cs}K^+$
$\Sigma_b^0 \to \widetilde{P}_{cs}\pi^0$	$\Sigma_b^0 \to \widetilde{P}_{cs}K^0$
$\Sigma_b^- \to \widetilde{P}_{cs}\pi^-$	$\Sigma_b^0 \to \widetilde{P}_{cs}K^-$
$\Xi_b^{\prime 0} \to \widetilde{P}_{cs}\bar{K}^0$	$\Xi_b^{\prime 0} \to \widetilde{P}_{cs}\pi^0$
$\Xi_b^{\prime -} \to \widetilde{P}_{cs}\bar{K}^-$	$\Xi_b^{\prime -} \to \widetilde{P}_{cs}\pi^-$
-	$\Omega_b^- \to \widetilde{P}_{cs}K^-$



Sextet produces $\widetilde{P}$ pentaquarks!

Suppression Mechanism in Baryons



$$\Lambda_\mathrm{b}^0 \to P_\mathrm{c} K^- \to (\mathrm{J}/\psi p) K^-$$

Suppression of the antisimmetric color-flavor part

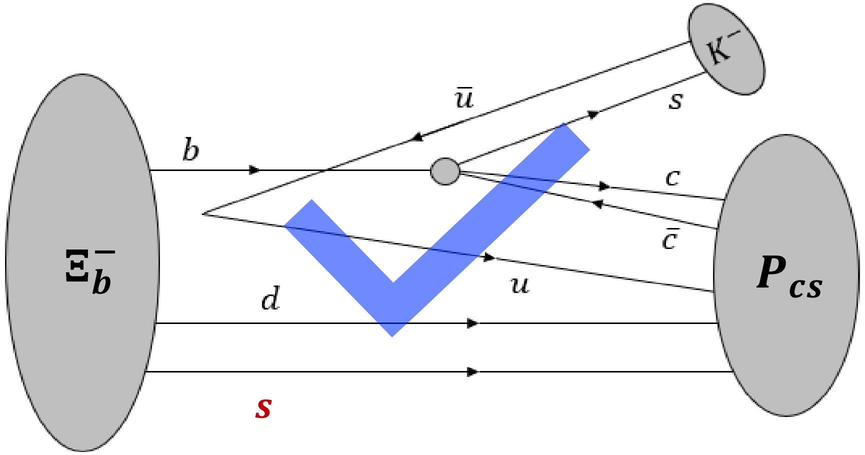
⬇

We do not observe

$\Lambda_\mathrm{b}^0 \to \tilde{P}_c\, K^-$

This is the only possible diagram in this case, **but** there are processes with **other diagrams**

$$\Xi_\mathrm{b}^- \to P_\mathrm{cs} K^- \to (\mathrm{J}/\psi \Lambda) K^-$$



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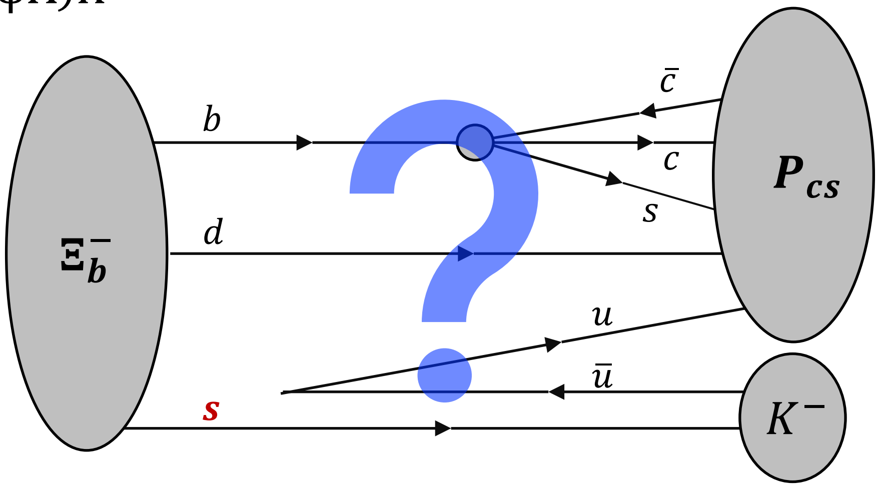
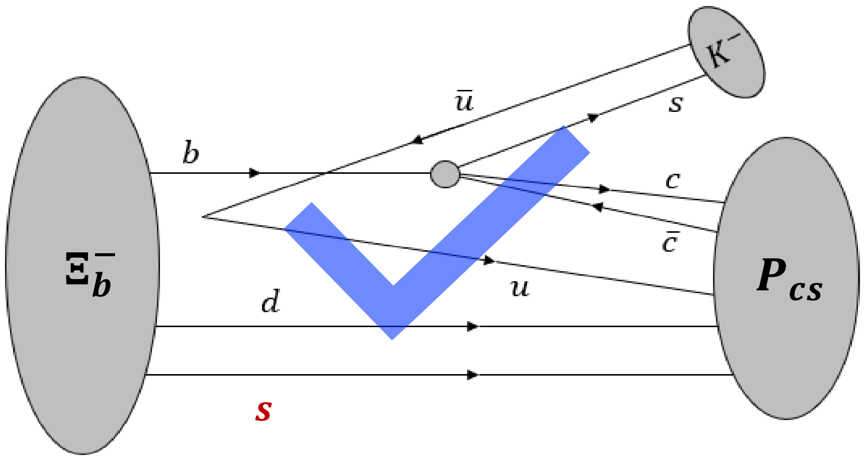


Suppression Mechanism in Baryons

$\Xi_b^- \to P_{cs} K^- \to (J/\psi\Lambda) K^-$



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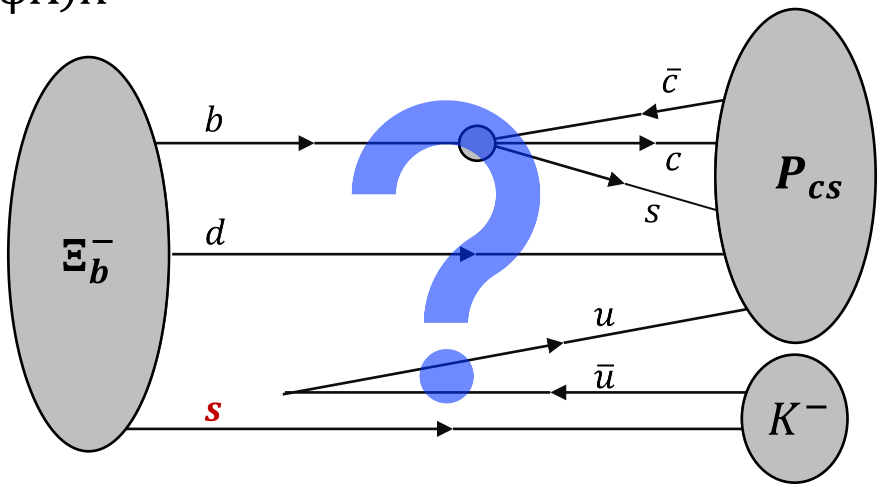
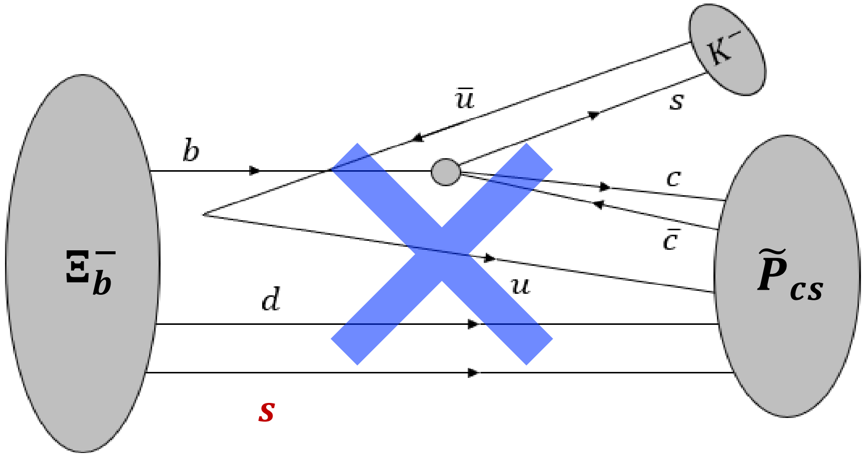


$\Xi_b^- \to \tilde{P}_{cs} K^- \to (J/\psi\Lambda) K^-$



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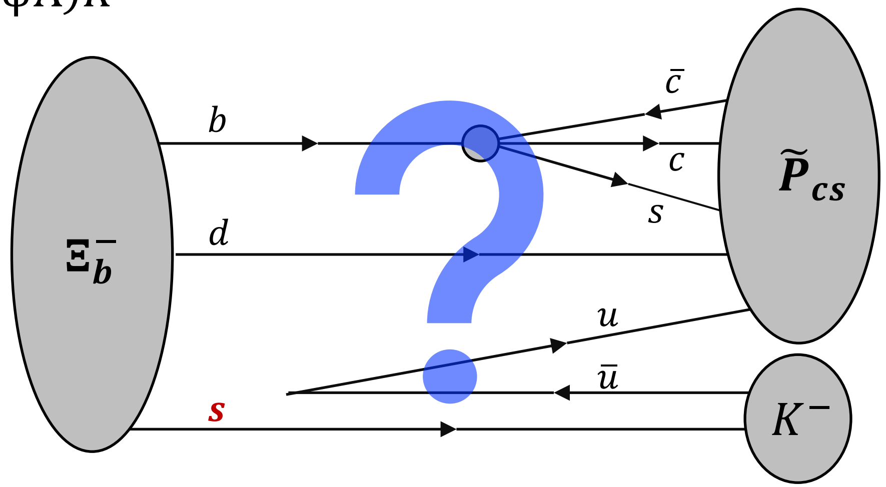
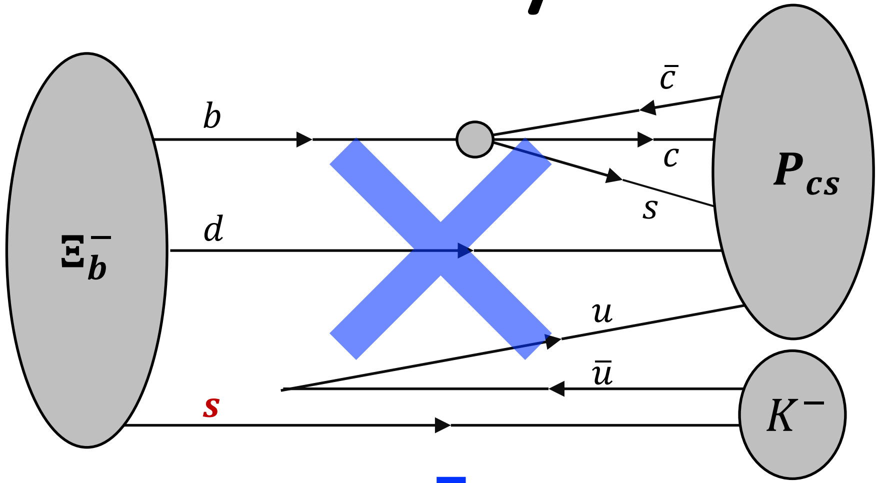


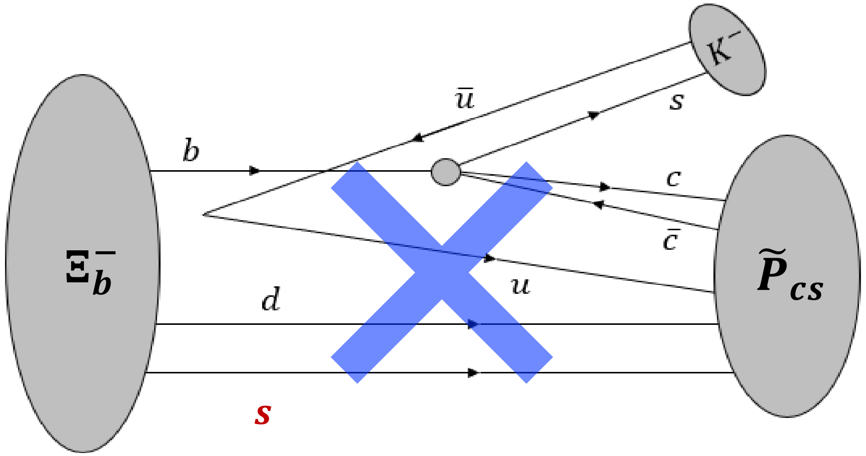
Suppression of the antisimmetric color-flavor part

Suppression Mechanism in Baryons

$$\mathcal{R} = \frac{|\mathcal{A}(\Lambda_b^0 \to P_c(4312)\, K^-)|^2}{|\mathcal{A}(\Xi_b^- \to P_{cs}(4459)\, K^-)|^2} \approx 6$$



$\Xi_b^- \to \tilde{P}_{cs} K^- \to (J/\psi\Lambda) K^-$ **Suppressed**



Suppression of the antisimmetric color-flavor part

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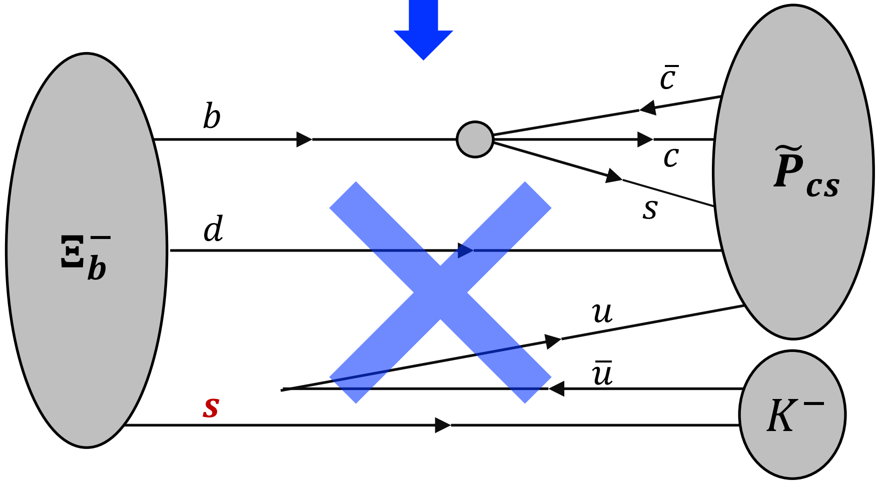

Conclusions

- By exploiting Fermi statistics and exchange interaction, we are able to provide a prediction for the spin of the observed pentaquarks and make predictions for the masses of the remaining pentaquarks to complete the triplets;
- The data suggests two different types of production for pentaquarks. Our model, based on the existence of two tensors in color-flavor space, could provide a way to account for this experimental fact. Further studies are necessary, particularly to determine the value of the ratio

$$\mathscr{R} = \frac{|\mathscr{A}(\Lambda_b^0 \to P_c(4312)\, K^-)|^2}{|\mathscr{A}(\Xi_b^- \to P_{cs}(4459)\, K^-)|^2}\ ;$$

- The decay of heavy baryons containing the bottom quark and belonging to the baryonic sextet can be used to study the pentaquarks P_{cs}, which are not visible in the decay of B^-.


THANK
YOU!

Backup

Young Tableau

$$M^{abc} \equiv \left(T^{abc}, \begin{array}{|c|c|}\hline a & b \\ \hline c & \\ \cline{1-1} \end{array} \right) = T^{abc} + T^{bac} - T^{cba} - T^{bca}$$

$$\widetilde{M}^{abc} \equiv \left(T^{abc}, \begin{array}{|c|c|}\hline a & c \\ \hline b & \\ \cline{1-1} \end{array} \right) = T^{abc} + T^{cba} - T^{bac} - T^{cab}$$

$$\overline{M}^{abc} \equiv \left(T^{abc}, \overline{\begin{array}{|c|c|}\hline a & b \\ \hline c & \\ \cline{1-1} \end{array}} \right) = T^{abc} - T^{bac} + T^{cba} - T^{bca}$$

$$S_{ijk}^{abc} \equiv \overline{M}_2^{ijk} \widetilde{M}_1^{abc} + \overline{M}_2^{ikj} M_1^{acb}$$

$$A_{ijk}^{abc} \equiv \widetilde{M}_2^{ijk} M_1^{abc} - M_2^{ikj} \widetilde{M}_1^{acb}$$

(Anti-) Symmetric under the exchange of any pair
e.g. $(\boldsymbol{i}, \boldsymbol{a}) \leftrightarrow (\boldsymbol{b}, \boldsymbol{j})$

$$\mathbf{S}^{uds}_{ijk} = u_{[i}d_{j]}s_k + u_{[k}d_{j]}s_i \; \left[\begin{array}{l} J_A^{qs} = k\, J_S^{qs} \\ J^{ds} = J^{qs} = \dfrac{J_S^{qs} + J_A^{qs}}{2} = \dfrac{1+k}{2} J_S^{qs} \\ k_\kappa = \dfrac{\kappa_A^{qs}}{\kappa_A^{qq}} \approx 0.60 \longrightarrow J_A^{qs} = k_\kappa\, J_A^{qq} \end{array} \right.$$

	Symmetric [MeV]	Antysimmetric [MeV]	No symmetry [MeV]
J^{qq}	$29.9^{+2.5}_{-2.8}$	$-42.8^{+2.4}_{-1.6}$	-
J^{qs}	17.9 ± 2	-25.7 ± 2	-3.9 ± 2

Baryon Octet Matrix

We consider the greek letters to be spin indices and the latin letters to be flavor ones

$$\mathbf{B}^{abc}_{\alpha\beta\gamma} = \epsilon_{ijk}\left(\mathbf{S}^{abc}_{\alpha\beta\gamma}\right)^{ijk} \Longrightarrow \mathbf{B}^{abc}_{\alpha\beta\gamma} = \epsilon_{ijk}\left(\psi^a_{[\alpha}\psi^b_{\beta]}\psi^c_{\gamma} - \psi^a_{[\beta}\psi^b_{\gamma]}\psi^c_{\alpha}\right)^{ijk}$$

Baryon Operator

$$\mathcal{B}^a_b \equiv \frac{1}{2}\epsilon_{bcd}\mathbf{B}^{cda} \Longrightarrow \mathcal{B}^a_b = \begin{pmatrix} \frac{\Lambda^0}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ & p \\ \Sigma^- & \frac{\Lambda^0}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda^0 \end{pmatrix}$$

$$|p, +\frac{1}{2}\rangle \propto u^{\uparrow}u^{\uparrow}d^{\downarrow} - u^{\uparrow}u^{\downarrow}d^{\uparrow} \qquad \left[\mathbf{B}^{duu}_{\downarrow\uparrow\uparrow}\right]_{SF} = d_{[\downarrow}u_{\uparrow]}u_{\uparrow} = u_{\uparrow}u_{\uparrow}d_{\downarrow} - u_{\uparrow}u_{\downarrow}d_{\uparrow}$$

H. Georgi, Lie algebras in particle physics. From isospin to unified theories