

BFKL NLL phenomenology of forward jets at HERA and Mueller Navelet jets at the Tevatron and the LHC

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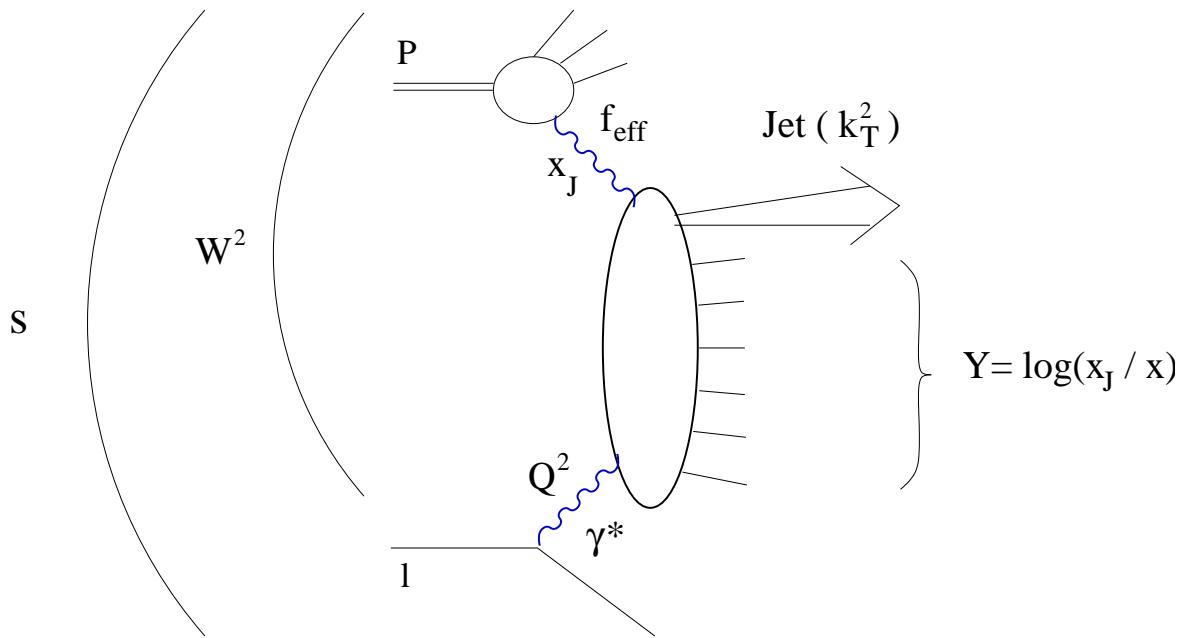
Contents:

- BFKL-NLL formalism
- Fit to H1 $d\sigma/dx$ data
- Prediction for the H1 triple differential cross section
- Prediction for Mueller Navelet jets at the Tevatron/LHC

Work done in collaboration with O. Kepka, C. Marquet, R. Peschanski

hep-ph/0609299, hep-ph/0612261

Forward jet measurement at HERA



- Typical kinematical domain where BFKL effects are supposed to appear with respect to DGLAP: $k_T^2 \sim Q^2$, and Q^2 not too large
- LO BFKL forward jet cross section: 2 parameters α_S , normalisation
- NLL BFKL cross section: one single parameter: normalisation (α_S running via RGE)

BFKL LO formalism

- BFKL LO forward jet cross section, saddle point approximation:

$$\frac{d\sigma}{dx dk_T dQ^2 dx_{jet}} = N \sqrt{\frac{Q^2}{k_T^2}} \alpha_S(k_T^2) \alpha_S(Q^2) \sqrt{A} \\ \exp\left(4\alpha(\log 2) \frac{N_C}{\pi} \log\left(\frac{x_J}{x}\right)\right) \\ \exp\left(-A \log^2\left(\sqrt{\frac{Q}{k_T}}\right)\right)$$

where

$$\frac{1}{A} = \frac{7\zeta(3)}{\pi} \alpha \log \frac{x_J}{x}$$

- 2 parameters in fits to $d\sigma/dx$: N , α

How to go to BFKL-NLL formalism?

- Simple idea: Keep the saddle point approximation, and use the BFKL NLO kernel
- Formula at NLL:

$$\frac{d\sigma}{dx} = N \left(\frac{Q^2}{k_T^2} \right)^{\text{power}} \alpha_S(k_T^2) \alpha_S(Q^2) \sqrt{A} \\ \exp \left(\alpha_S(k_T Q) \frac{N_C}{\pi} \chi(\gamma_C) \log \left(\frac{x_J}{x} \right) \right) \\ \exp \left(-A \alpha_S(k_T Q) \log^2 \left(\sqrt{\frac{Q}{k_T}} \right) \right)$$

where

$$\frac{1}{A} = \frac{3\alpha_S(k_T Q)}{4\pi} \log \frac{x_J}{x} \chi''(\gamma_C) \\ \text{power} = \gamma_C + \frac{\alpha_S(k_T Q) \chi(\gamma_C)}{2}$$

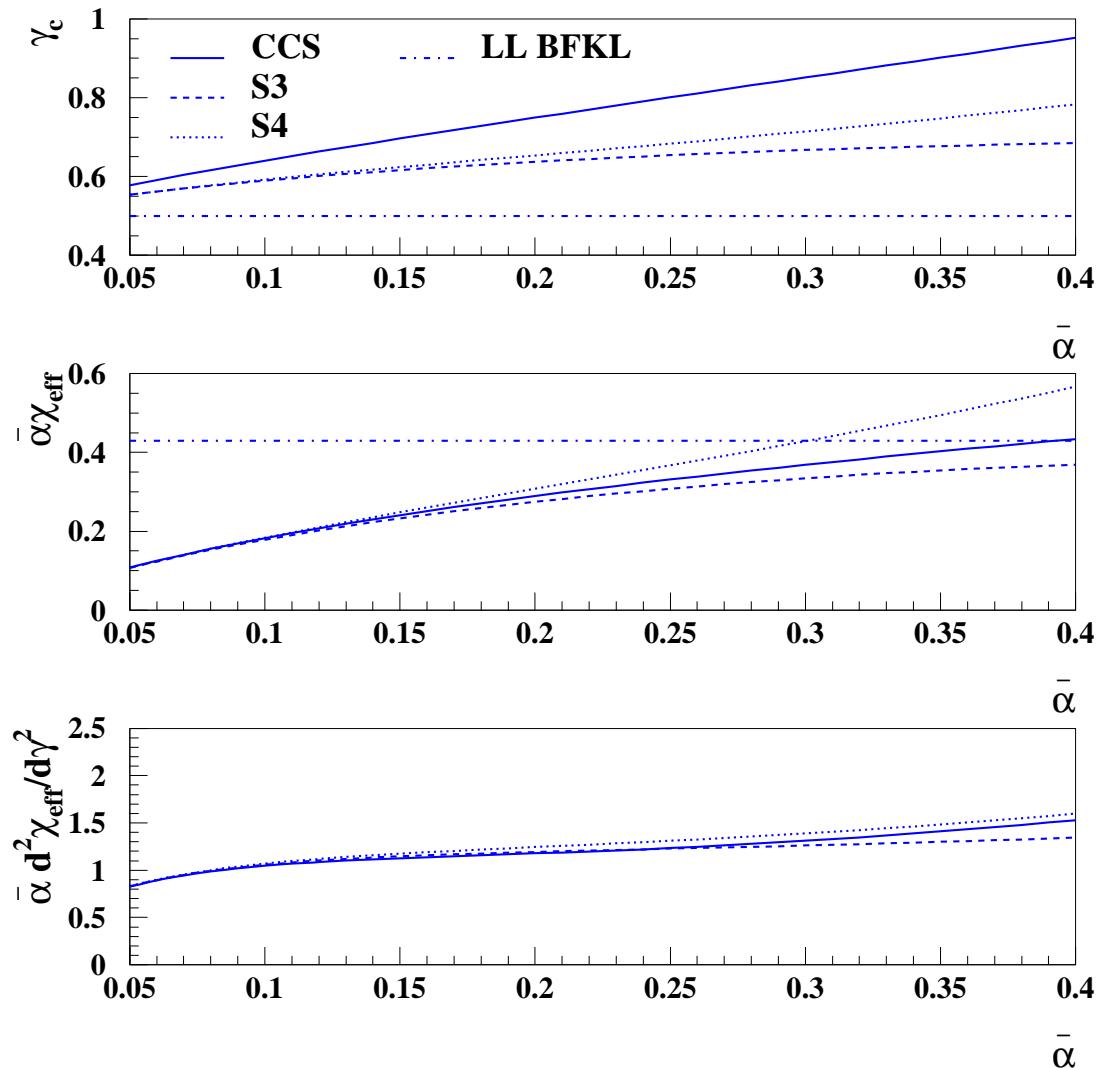
- Only free parameter in the BFKL NLL fit: absolute normalisation

How to determine γ_C , $\chi(\gamma_C)$, and $\chi''(\gamma_C)$?

- **First step:** Knowledge of $\chi_{NLO}(\gamma, \omega, \alpha)$ from BFKL equation and resummation schemes (ω is the Mellin transform of Y)
- **Second step:** Use implicit equation $\chi(\gamma, \omega) = \omega/\alpha$ to compute numerically ω as a function of γ for different schemes and values of α
- **Third step:** Numerical determination of saddle point values γ_C as a function of α as well as the values of χ and χ''
- Study performed for three different resummation schemes: S3 and S4 from Gavin Salam, and CCS from Ciafaloni et al.
- For more information and comparison to F_2 : see R. Peschanski, C. Royon, and L. Schoeffel, Nucl.Phys.B716 (2005) 401, hep-ph/0411338

γ_C , $\chi(\gamma_C)$, and $\chi''(\gamma_C)$ as a function of α

Determination of γ_C , $\chi(\gamma_C)$, and $\chi''(\gamma_C)$ as a function of α



Cross section calculation, comparison with H1 measurement

- Two difficulties: We need to integrate over the bin in Q^2 , x_{jet} , k_T to compare with the experimental measurement and we need to take into account the experimental cuts (as an example: $E_e > 10$ GeV, $k_T > 3.5$ GeV, $7 \leq \theta_J \leq 20$ degrees....)
- We perform the integration numerically: we chose the variables for which the cross section is as flat as possible to avoid numerical difficulties in precision: k_T^2/Q^2 , $1/Q^2$, $\log 1/x_{jet}$
- We take into account some of the cuts at the integration level (k_T for instance) and the other ones using a toy Monte Carlo

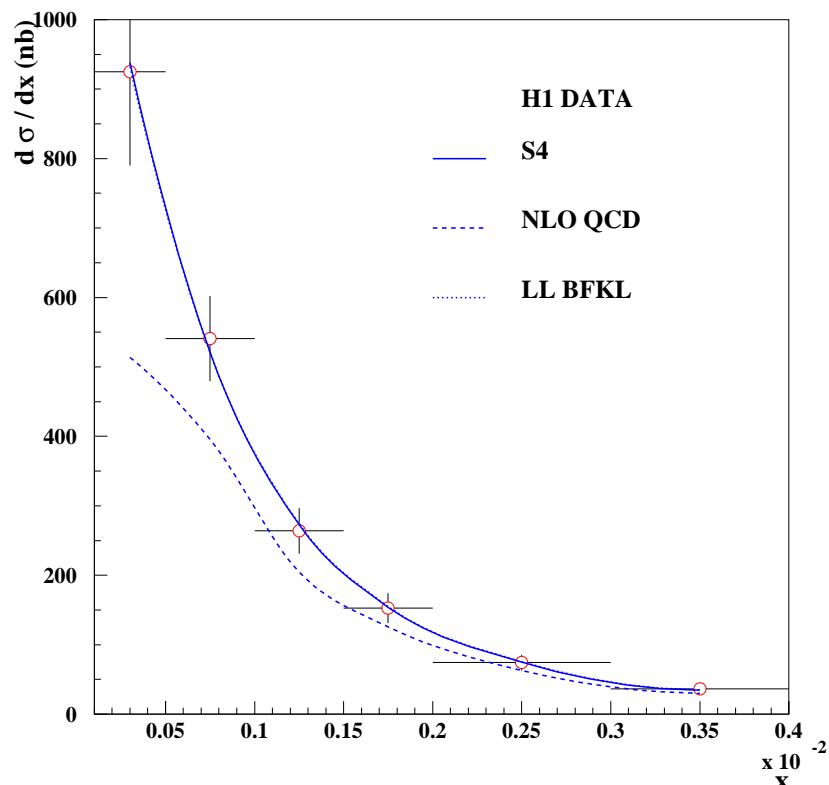
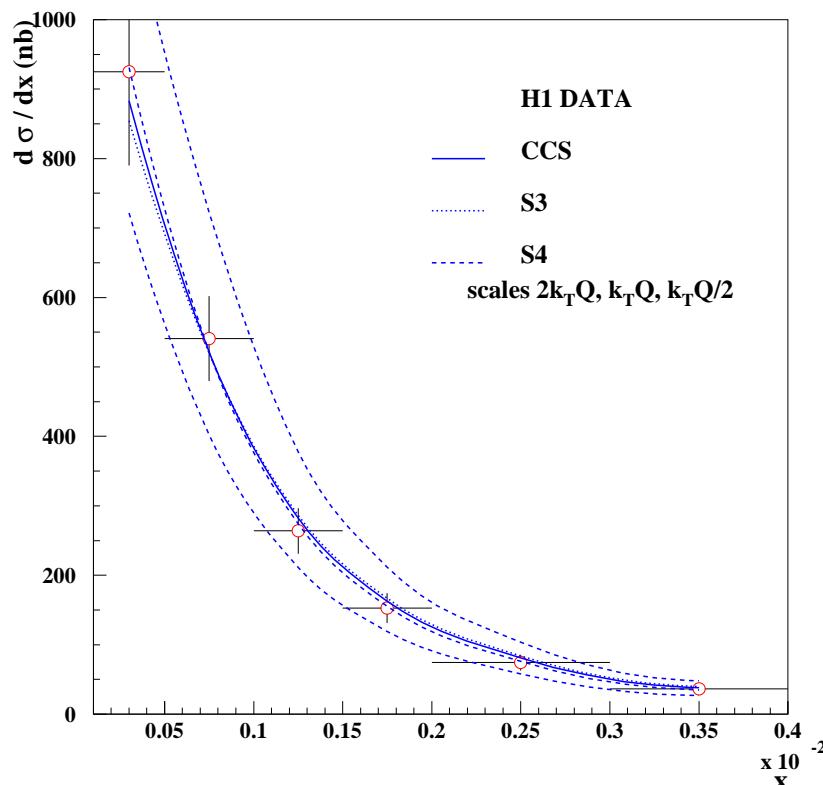
Fit procedure

- Fit to H1 $d\sigma/dx$ data only
- Fit using the 6 data points
- Results at LO: Good fit ($\chi^2 \sim 0.5/5$), but α_S small ($\alpha_S \sim 0.1$)
- $\alpha_S(k_T Q)$ is imposed using the renormalisation group equation at NLL

scheme	fit	χ^2/dof	N
CCS	stat. + syst.	0.90/5	0.1332 ± 0.0074
CCS	stat. only	22.2/4	$0.1367 \pm 0.0016 \pm 0.0170$
S3	stat. + syst.	1.74/5	0.1514 ± 0.0085
S3	stat. only	46.5/5	$0.1576 \pm 0.0018 \pm 0.0196$
S4	stat. + syst.	0.29/5	0.1094 ± 0.0061
S4	stat. only	5.4/5	$0.1096 \pm 0.0013 \pm 0.0137$

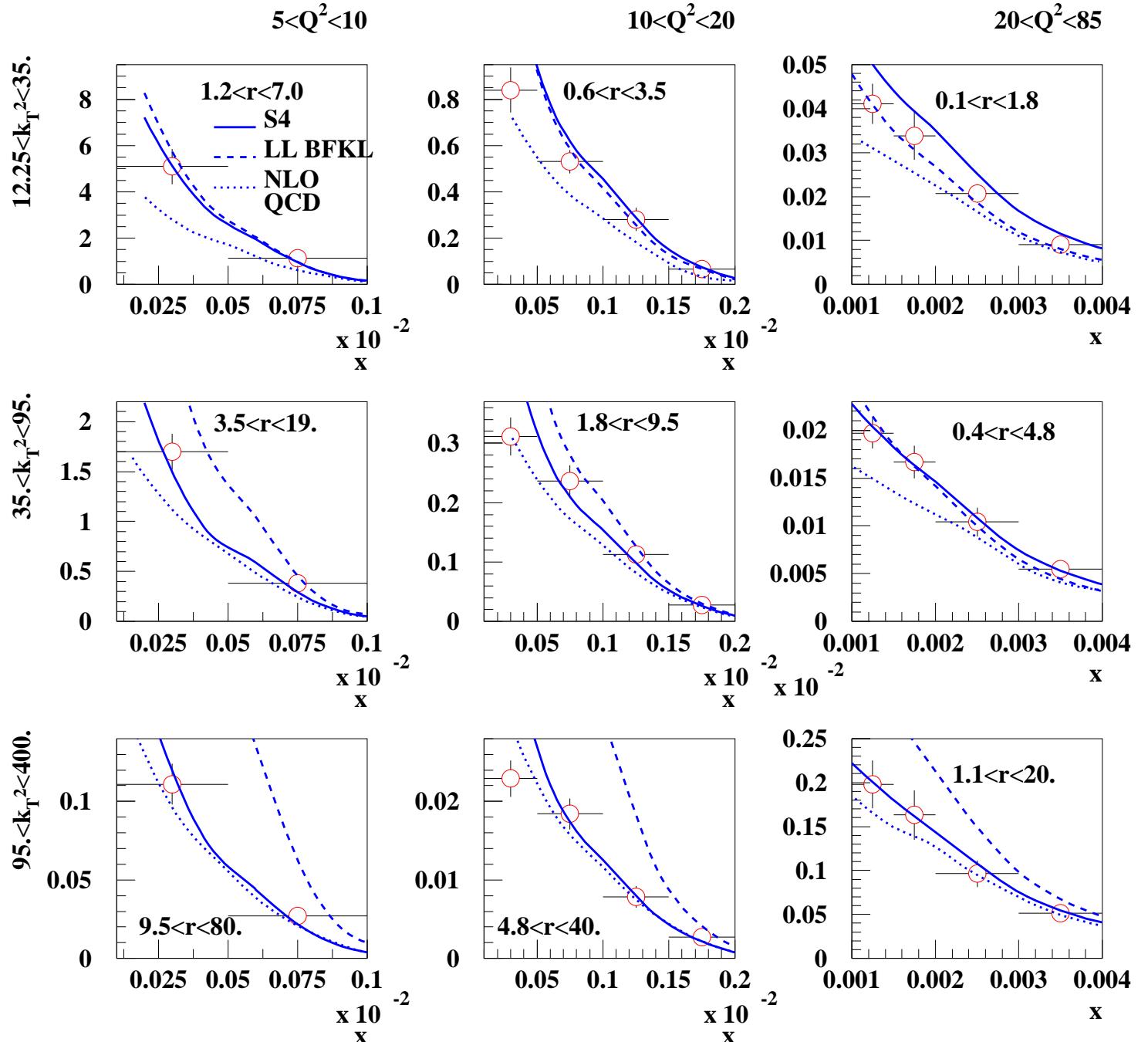
Fit results

- χ^2 for CCS: 22.2 (0.9), S3: 46.5 (1.7), S4: 5.4 (0.3)
- Good description of H1 data using BFKL LO and BFKL NLL formalism, DGLAP-NLO fails to describe the data
- BFKL higher corrections found to be small (We are in the BFKL-LO region, cut on $0.5 < kT^2/Q^2 < 5$)



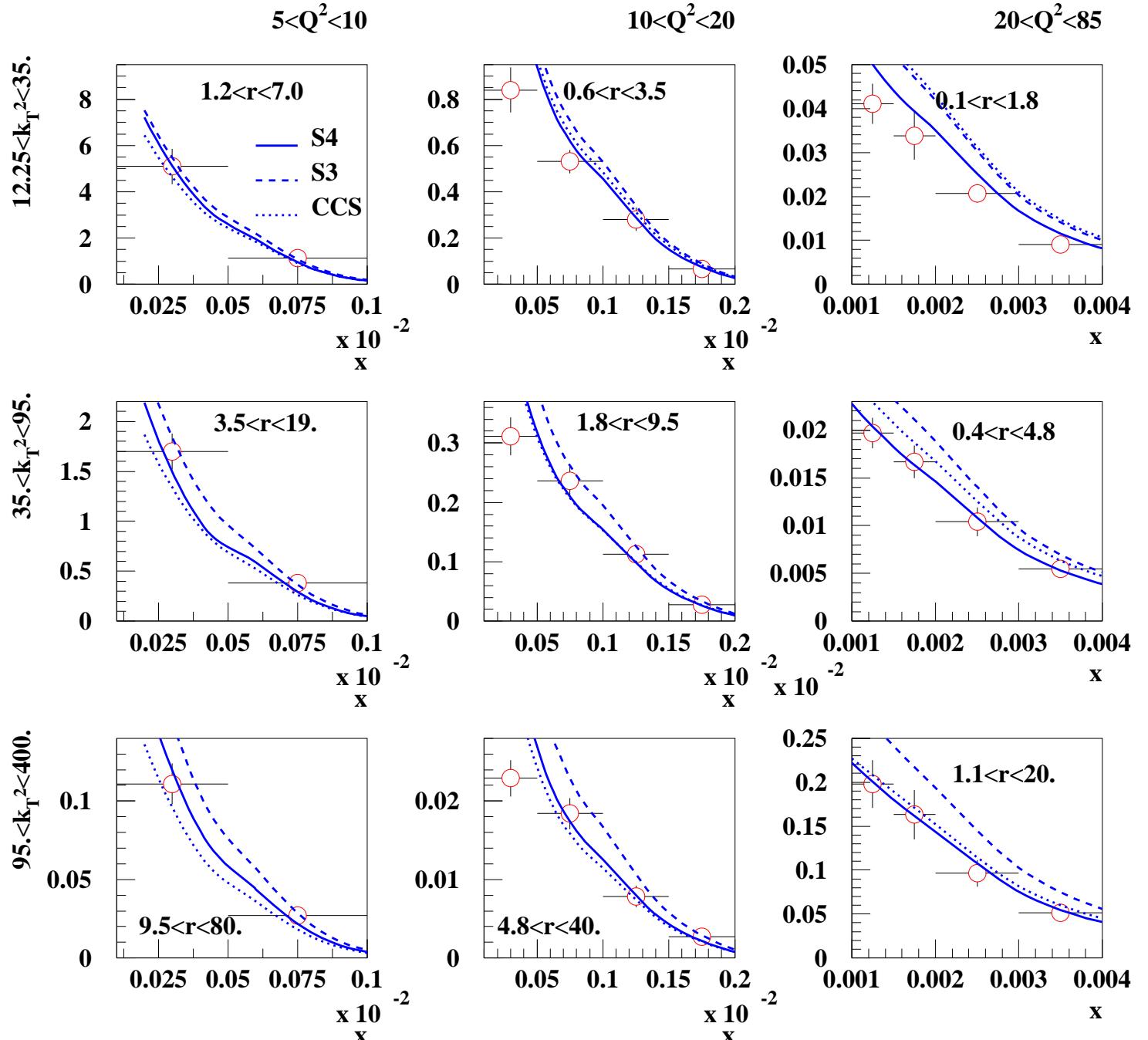
Comparison with H1 triple differential data

$d\sigma/dx dk_T^2 dQ^2$ - H1 DATA



Comparison with H1 triple differential data

$d\sigma/dx dk_T^2 dQ^2$ - H1 DATA

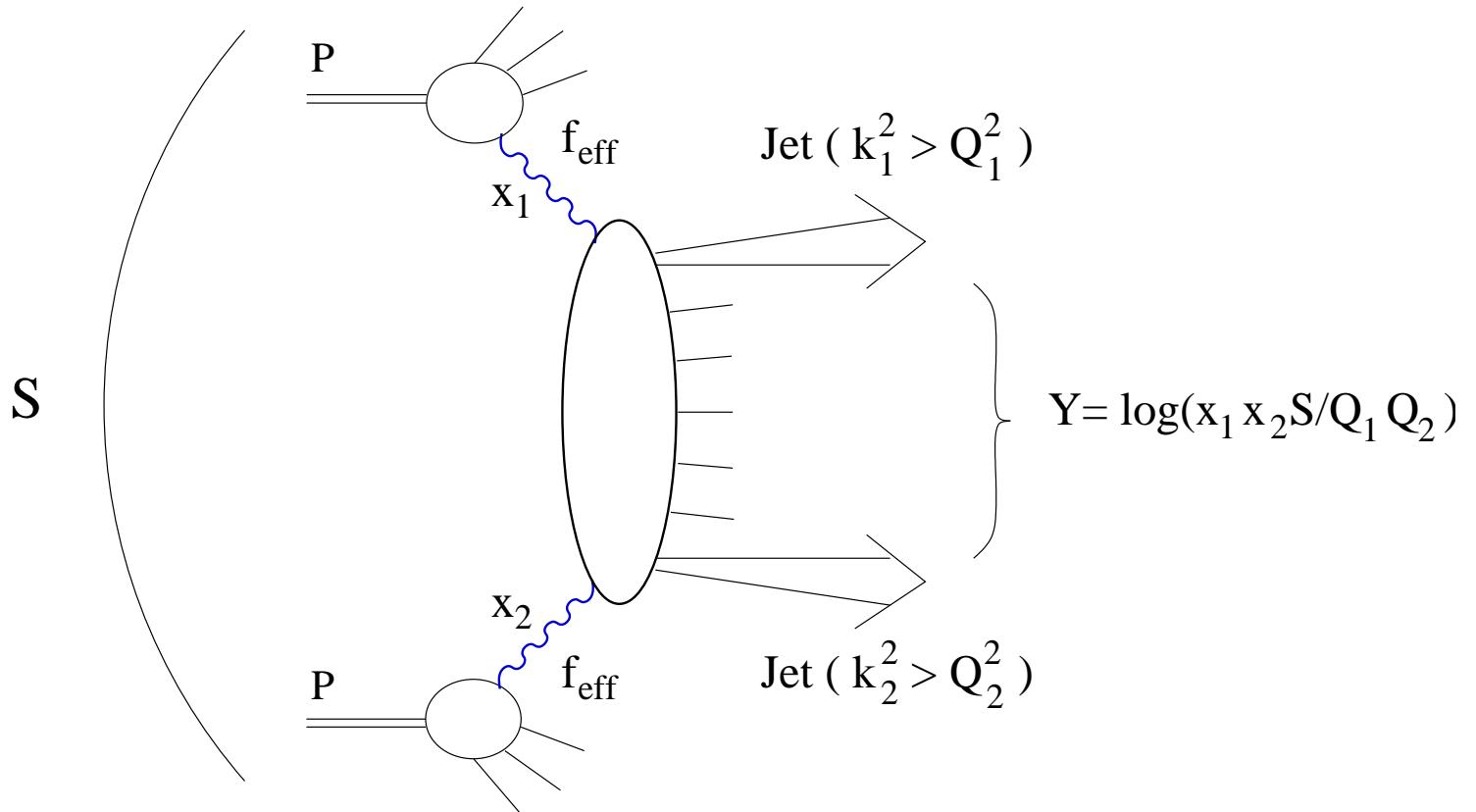


Comparison with H1 triple differential data

- DGLAP NLO predictions cannot describe H1 data in the full range, and large difference between DGLAP NLO and DGLAP LO results (DGLAP NLO includes part of the small x resummation effects)
- BFKL LO describes the H1 data when $r = k_T^2/Q^2$ is close to 1
- BFKL LO fails outside the region $r \sim 1$ specially at high Q^2
- BFKL higher order corrections found to be small (as expected) when $r \sim 1$
- Higher order BFKL corrections larger when r further away from 1, where the BFKL NLL prediction is closer to the DGLAP one (Q^2 resummation effects are starting to be large)
- BFKL NLL gives a good description of data over the full range: first success of BFKL higher order corrections, shows the need of these corrections
- Systematic additional studies: Check the effect of varying scale in α_S ($2Qk_T$, $Qk_T/2$, Q^2 , k_T^2), different assumptions for the unknown impact factors

Mueller Navelet jets

Same kind of processes at the Tevatron and the LHC



- Same kind of processes at the Tevatron and the LHC:
Mueller Navelet jets
- Study the $\Delta\Phi$ between jets dependence of the cross section:

Mueller Navelet jets: $\Delta\Phi$ dependence

- Study the $\Delta\Phi$ dependence of the relative cross section
- Relevant variables:

$$\begin{aligned}
 \Delta\eta &= y_1 - y_2 \\
 y &= (y_1 + y_2)/2 \\
 Q &= \sqrt{k_1 k_2} \\
 R &= k_2/k_1
 \end{aligned}
 \tag{1}$$

- Azimuthal correlation of dijets:

$$2\pi \frac{d\sigma}{d\Delta\eta dR d\Delta\Phi} \Bigg/ \frac{d\sigma}{d\Delta\eta dR} = 1 + \frac{2}{\sigma_0(\Delta\eta, R)} \sum_{p=1}^{\infty} \sigma_p(\Delta\eta, R) \cos(p\Delta\Phi)$$

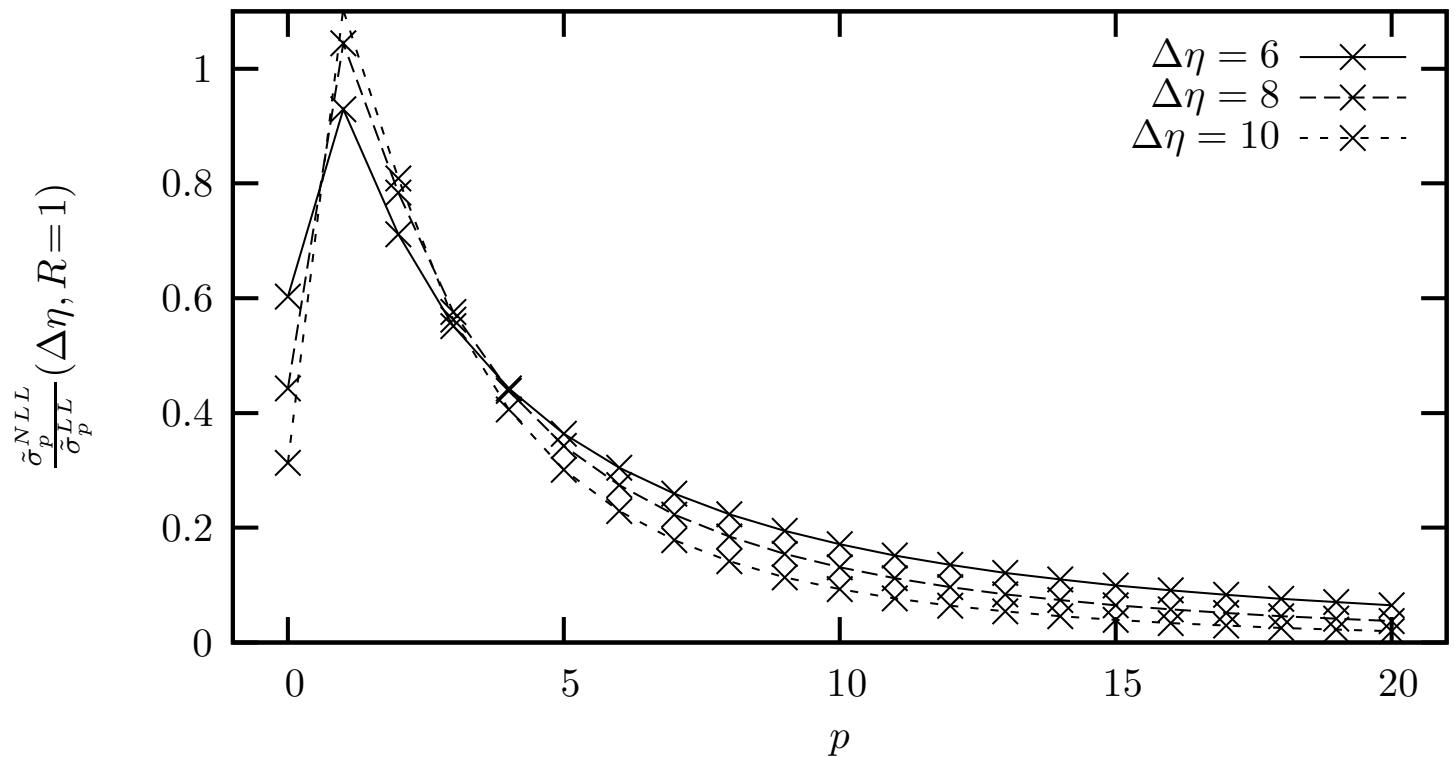
where

$$\begin{aligned}
 \sigma_p &= \int_{E_T}^{\infty} \frac{dQ}{Q^3} \alpha_s(Q^2/R) \alpha_s(Q^2 R) \\
 &\quad \left(\int_{y<}^{y>} dy x_1 f_{eff}(x_1, Q^2/R) x_2 f_{eff}(x_2, Q^2 R) \right) \\
 &\quad \int_{1/2-\infty}^{1/2+\infty} \frac{d\gamma}{2i\pi} R^{-2\gamma} e^{\bar{\alpha}(Q^2)\chi_{eff}\Delta\eta}
 \end{aligned}$$

Mueller Navelet jets: $\Delta\Phi$ dependence

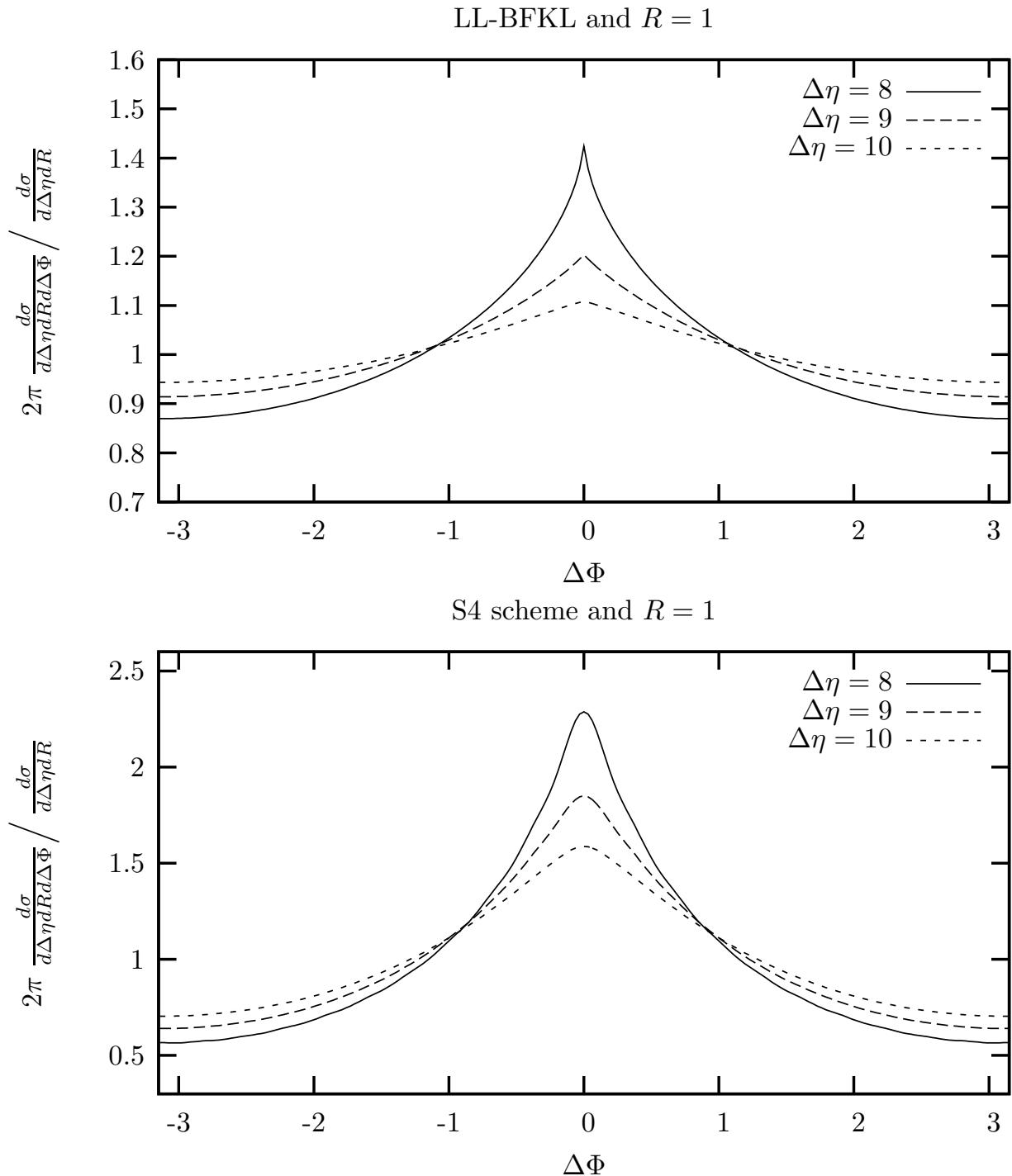
Ratio of the values of σ_i entering into the $\Delta\Phi$ spectrum between BFKL NLL and BFKL LL for different intervals in rapidity

S4/LL with $\bar{\alpha}=0.16$



Mueller Navelet jets: $\Delta\Phi$ dependence

$1/\sigma d\sigma/d\Delta\Phi$ spectrum for BFKL LL and BFKL NLL as a function of $\Delta\Phi$ for different values of $\Delta\eta$



Conclusion

- DGLAP NLO fails to describe forward jet data
- First BFKL NLL description of H1 and ZEUS forward jet data: very good description
- The BFKL scale which is used in the exponential $\alpha_S(k_T Q)$ can describe the H1 cross section measurements
- Higher order corrections small when $r = k_T^2/Q^2 \sim 1$ and larger when r is further away from 1 as expected
- BFKL NLL formalism leads to a better description than the BFKL LO one for the triple differential cross section: Resummed BFKL NLO kernels include part of the evolution in Q^2
- Mueller Navelet jets: Interesting measurement to be performed at the Tevatron/LHC to look for higher order BFKL effects, and may be saturation effects