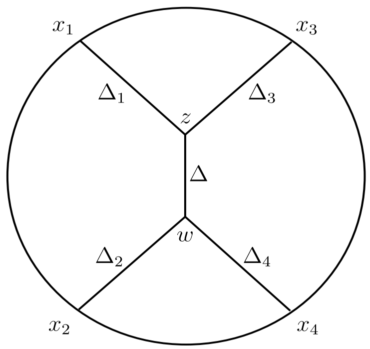
Conformal Invariance and Pomeron Interaction from AdS/CFT



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Technique: Summing generalized Witten Diagrams

Freedman et al., hep-th/9903196

Brower, Polchinski, Strassler, and Tan, hep-th/0003115

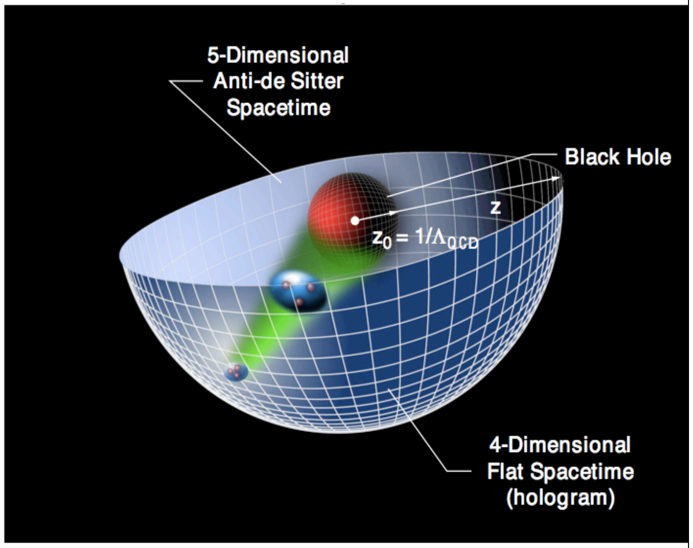
Outline

- extreme strong coupling limit, $1/(g^2N)=0$: SUPRA graphs in AdS-5
- reduction to AdS-3 at high energy
- $1/(g^2N)$ nonzero,
- higher order diagrams, e.g., eikonal, fan graphs, etc., importance of confinement

Conformal Invariance

- AdS-5 Background metric:

$$d^2z = \frac{1}{z_0^2}\left\{dx_\mu dx^\mu + d^2 z_0\right\}$$



- Scalar Propagator:

$$S = \int dz\sqrt{g}\left\{\partial_M\phi(z)g^{MN}\partial_N\phi(z) + \Delta(\Delta - d)\phi^2(z)\right\}$$

$$\langle\phi_\Delta(z)\phi_\Delta(w)\rangle = G^{(5)}_\Delta(z,w)$$

$$\left\{-\frac{1}{\sqrt{g}}\partial_M\sqrt{g}g^{MN}\partial_N + \Delta(\Delta - d)\right\}G^{(5)}_\Delta(z,w) = \delta^5(z-w)$$

Conformal Invariance

Scalar Propagator is a function of a single variable--
chordal distance between two points, (z,w):

$$u = \frac{(x-y)^2 + (z_0 - w_0)^2}{2 z_0 w_0}$$

Best to use momentum representation

$$G(u) = \int \frac{d^4 q}{(2\pi)^4} e^{iq\cdot(x-y)} (z_0 w_0)^2 I_2(q z_<) K_2\,(q z_>)$$

Momentum Representations

$$(-\partial_{z_0} z_0^{-(d-1-2S)} \partial_{z_0} + q^2 \; z_0^{-(d+1-2S)} + z_0^{-(d-1-2S)} \; m^2)\phi(u) = 0$$

- Direct calculation, for s=0, d=4, m=0:

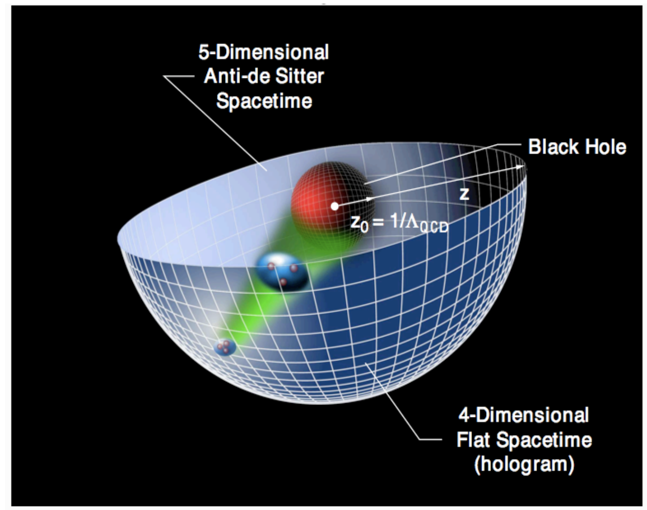
$$\tilde{G}(q, z_0, w_0) = (z_0 w_0)^2 I_2(q z_<) K_2 \; (q z_>) \; .$$

$$0 < z_0, w_0 < \infty$$

- Spetral in t= -q^2, (no mass gap):

$$\tilde{G}(q, z_0, w_0) = (z_0 w_0)^2 \int_0^\infty k \; dk \; \frac{J_2(k z_0) J_2(k w_0)}{k^2 - t} \; .$$

Confinement --- Massive Tensor Glueballs



Hard-Wall Example:

$$0 < z_0 < z_1 = 1/\Lambda$$

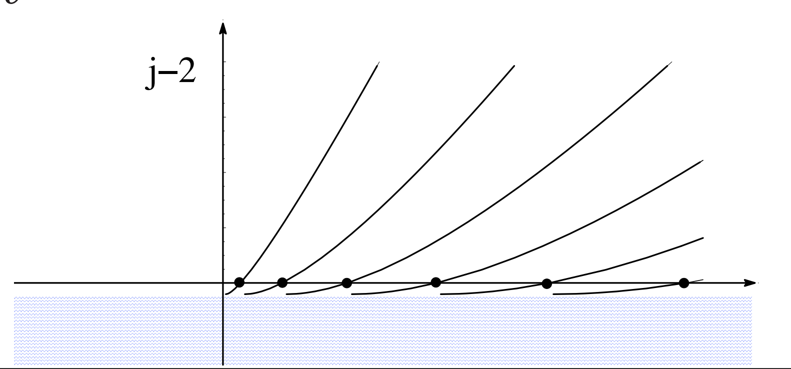
Boundary Condition:

$$\partial_{z_1}[z_1^2 J_2(m_n z_1)] = 0$$

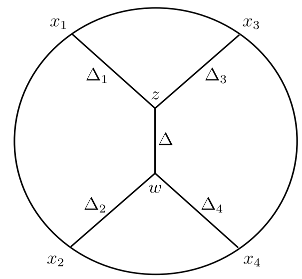
Spectral Representation, discrete spectrum (with mass gap):

$$\tilde{G}(q, z_0, w_0) = (z_0 w_0)^2 \sum_n \frac{\Phi_n(z_0)\Phi_n(w_0)}{m_n^2 - t}$$

$$t = -q^2$$



One Graviton Graph



Coordinate Space:

$$I_{grav}(x_1,x_2,x_3,x_4) = \frac{g_s^2}{4}\int dz\sqrt{g}\int dw\sqrt{g}\; T^{MN}(x_1,x_3,z)G_{MNM'N'}(z,w)T^{M'N'}(x_2,x_4,w)$$

Momentum Space:

$$p_1+p_2\to p_3+p_4$$

$$T_4^{(1)}(p_1,p_2,p_3,p_4) = \frac{1}{4}\int dz_0\sqrt{g}\int dw_0\sqrt{g}\;\tilde{T}^{MN}(p_1,p_3,z_0)\tilde{G}_{MNM'N'}(q,z_0,w_0)\tilde{T}^{M'N'}(p_2,p_4,w_0)$$

One Graviton Exchange at High Energy

- d=4, Delta=4:
- Graviton propagator:

$$\tilde{G}_{++,--}(q, z_0, w_0) = \frac{2}{(z_0 w_0)^2} \tilde{G}(q, z_0, w_0)$$

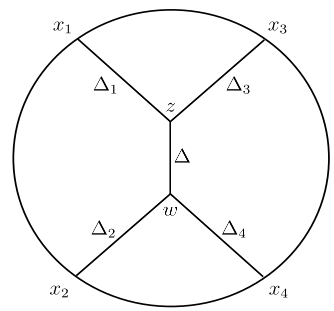
- Energy-momentum tensor:

$$\begin{aligned}
\tilde{T}_{13}^{++}(p_1, p_3, z_0) &= 2(ip_1^+)(ip_3^+)\tilde{K}_\Delta(p_1, z_0)\tilde{K}_\Delta(p_3, z_0) \\
\tilde{T}_{24}^{--}(p_2, p_4, w_0) &= 2(ip_2^-)(ip_4^-)\tilde{K}_\Delta(p_2, w_0)\tilde{K}_\Delta(p_4, w_0)
\end{aligned}$$

One Graviton in Momentum Representation at High Energy

$$T_4^{(1)}(p_1,p_2,p_3,p_4) = g_s^2 \int \frac{dz_0}{z_0^5} \int \frac{dw_0}{w_0^5}\ \tilde{K}_\Delta(p_1^2,z_0)\tilde{K}_\Delta(p_3^2,z_0)\mathcal{T}_4^{(1)}(p_i,z_0,w_0)\tilde{K}_\Delta(p_2^2,w_0)\tilde{K}_\Delta(p_4^2,w_0)$$

$$p_1 + p_2 \to p_3 + p_4$$



$$\mathcal{T}_4^{(1)}(p_i,z_0,w_0) = s^2\tilde{G}_{++,--}(q,z_0,w_0) = \frac{2s^2}{(z_0w_0)^2}\tilde{G}(q,z_0,w_0)$$

Reduction to AdS-3 at High Energy for Near Forward Scattering

* momentum transfer q is transverse:

$$\mathcal{T}_4^{(1)}(s, x_\perp - y_\perp) = (1/2\pi)^2 \int d^2 q_\perp e^{i(x_\perp - y_\perp)\cdot q_\perp} T_4^{(1)}(s, -q_\perp^2)$$

$$= g_s^2 \int \frac{dz_0}{z_0^5} \int \frac{dw_0}{w_0^5} \tilde{K}_\Delta(p_1^2, z_0)\tilde{K}_\Delta(p_3^2, z_0)\mathcal{K}(s, x_\perp - y_\perp, z_0, w_0)\tilde{K}_\Delta(p_2^2, w_0)\tilde{K}_\Delta(p_4^2, w_0)$$

* AdS-3 Propagator:

$$\mathcal{K}(s, x_\perp, z_0, w_0) = \frac{2s^2}{z_0 w_0} G^{(3)}_{\Delta_2}(x_\perp, z_0, w_0)$$

* Isometry of Euclidean AdS-3 is SL(2C) ---
the same symmetry group as BFKL kernel

Strong Coupling Pomeron Propagator--

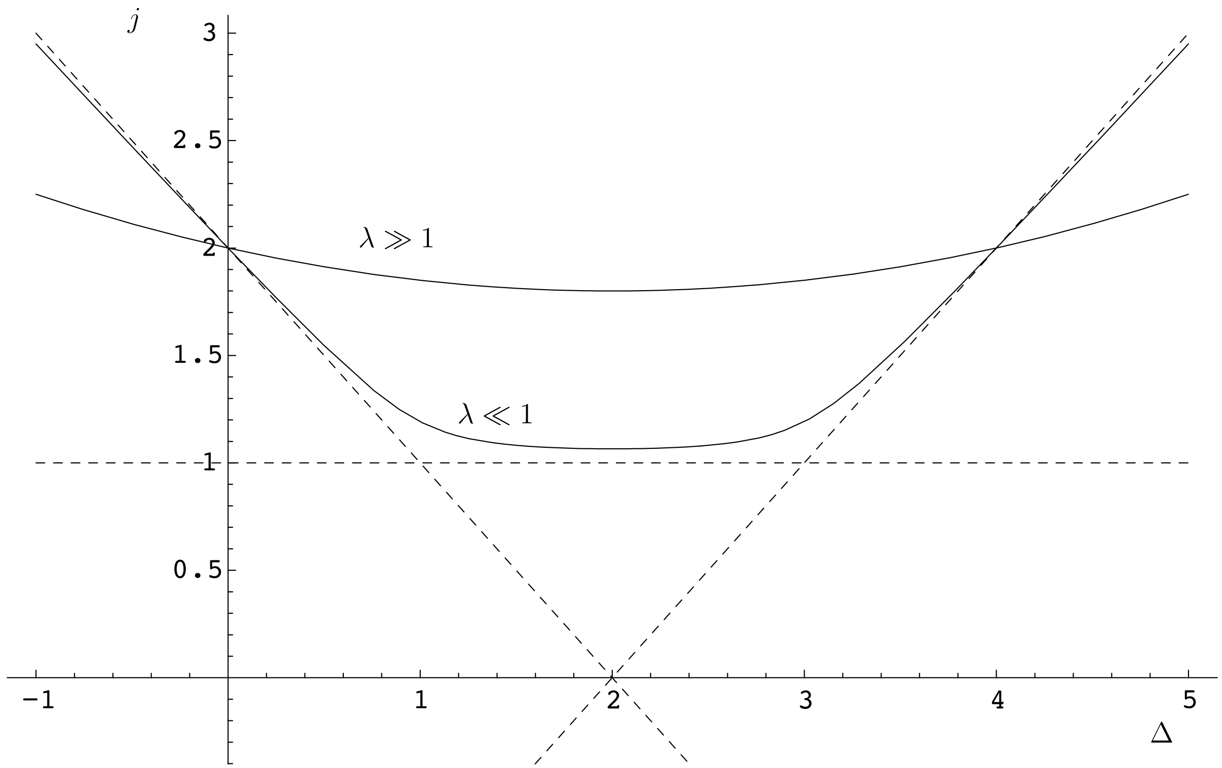
Conformal Limit

- Spin 2 -------> J
- Use Complex angular momentum reprsentation
- Use J-dependent Dimension:

$$\Delta: \quad 4 \to \Delta(J) = 2 + [2\sqrt{\lambda}(J - J_0)]^{1/2} = 2 + \sqrt{\bar{j}}$$

- BFKL-cut: $\qquad J_0 = 2 - \dfrac{2}{\sqrt{\lambda}}$

Spin-Dimension Curve



Strong Coupling Pomeron Propagator--Comparison with BFKL

- AdS-3 propagator:

$$\mathcal{K}(j, x_\perp - x'_\perp, z, z') = \frac{1}{4\pi z z'} \frac{\left[y + \sqrt{y^2 - 1}\right]^{(2-\Delta_+(j))}}{\sqrt{y^2 - 1}} \; ,$$

$$y \pm 1 = \frac{(z \mp z')^2 + (x_\perp - x'_\perp)^2}{2 z z'}$$

- BFKL kernel:

$$\Phi_{n,\nu}(b_1 - b_0, b_2 - b_0) = \left[\frac{b_1 - b_2}{(b_1 - b_0)(b_2 - b_0)}\right]^{i\nu + (1+n)/2} \left[\frac{\bar{b}_1 - \bar{b}_2}{(\bar{b}_1 - \bar{b}_0)(\bar{b}_2 - \bar{b}_0)}\right]^{i\nu + (1-n)/2}$$

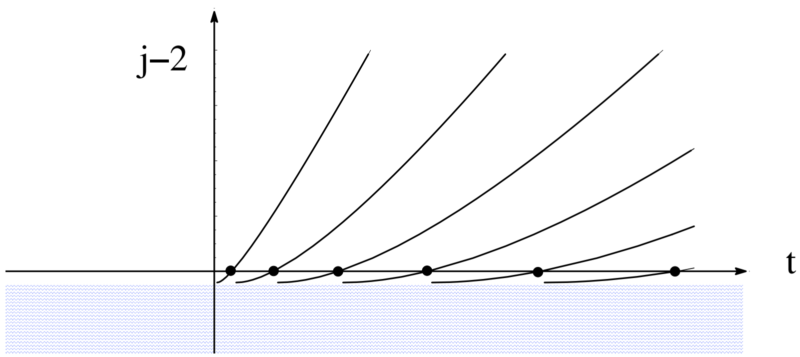
Strong Coupling Pomeron Propagator--with Confinement

Spectral Rep. in Conformal limit:

$$G(j,t,z,z') = \int_0^\infty dk\ k\ \frac{J_{\sqrt{j}}(kz)J_{\sqrt{j}}(kz')}{k^2 - t}$$

Spectral Rep. with Confinement

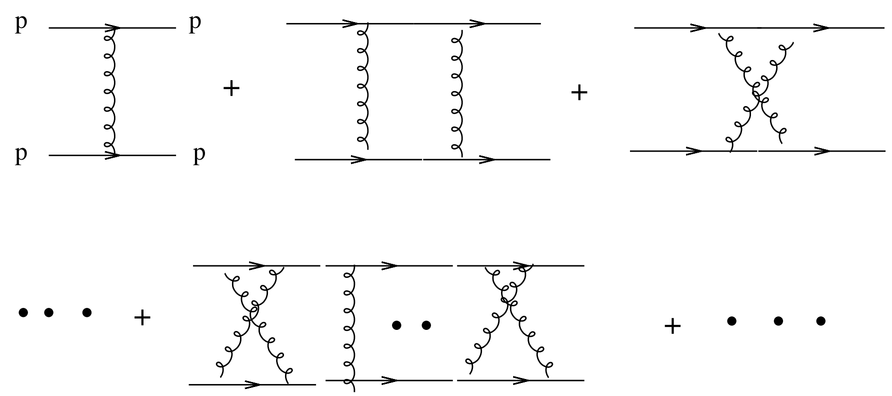
$$G(j,t,z,z') = \sum_n \frac{\psi_n(z)\psi_n^*(z')}{t_n(j) - t}$$



Ref. Brower, Polchinski, Strassler, Tan, het-0603115

Higher Order Diagrams:

Eikonal Sum:



Fan Diagrams:

AdS-3 Pomeron Calculus:

Warning: Breaking Conformal Invariance--

Physics changes when confinement is

taken into account !!

Gauge/String Duality provides a robust framework for concrete calculations and predictions!!

Ref: hep-th/0603115, and papers in preparation,
R. C. Brower, M. Strassler, J. Polchinski, CIT