

One gluon, two gluon: Semi inclusive observables at high energies

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based on: Alex Kovner, M.L., Heribert Weigert, Phys.Rev.D74:114023,2006

Alex Kovner and M.L., JHEP 0611:083,2006

High energy evolution of hadronic wavefunction

Hadron wave function in the gluon Fock space

$$|\Psi\rangle_{Y_0} = \Psi[a_i^{\dagger a}(x)]|0\rangle_{Y_0} \quad |\Psi\rangle = |v\rangle$$

The evolved wave function

$$|\Psi\rangle_Y = \Omega_Y(\rho, a) |v\rangle_{Y_0}; \quad |v\rangle_{Y_0} = |v\rangle \otimes |0_a\rangle$$

Gluon cloud operator in the dilute limit

$$C_Y \equiv \Omega_Y(\rho \rightarrow 0) = \text{Exp} \left\{ i \int d^2 z b_i^a(z) \int_{e^{Y_0} \Lambda}^{e^Y \Lambda} \frac{dk^+}{\pi^{1/2} |k^+|^{1/2}} \left[a_i^a(k^+, z) + a_i^{\dagger a}(k^+, z) \right] \right\}.$$

The classical WW field

$$b_i^a(z) = \frac{g}{2\pi} \int d^2 x \frac{(z - x)_i}{(z - x)^2} \rho^a(x)$$

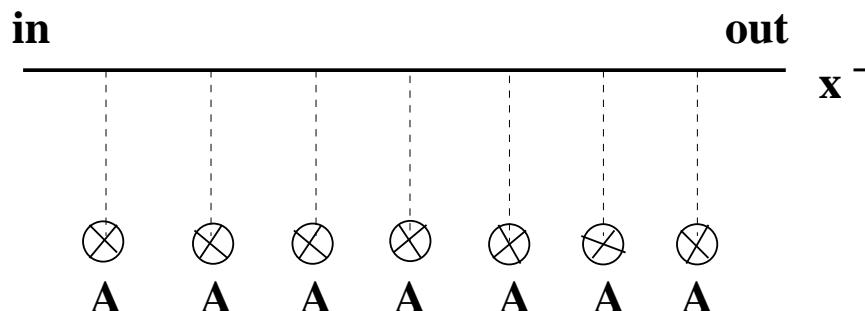
High energy scattering

We denote a S-matrix of a single gluon which scatters on a fixed configuration of chromoelectric field of the target by

$$S^{ab}(x) = \langle 0 | a_i^a(x) \hat{S} a_i^{\dagger b}(x) | 0 \rangle$$

In the light cone gauge ($A^- = 0$) the large target field component is $A^+ = \alpha_t$.

$$S(x) = \mathcal{P} \exp \left\{ i \int dx^- T^a \alpha_t^a(x, x^-) \right\} .$$



Eikonal scattering for all $k^+ \geq \Lambda$
Eikonal factor does not depend on k^+

$$|in\rangle = |z, b\rangle ; \quad |out\rangle = |z, a\rangle ; \quad |out\rangle = S |in\rangle$$

Evolution of the diagonal element of the S -matrix operator $\Sigma^P \equiv \langle P | \hat{S} | P \rangle$

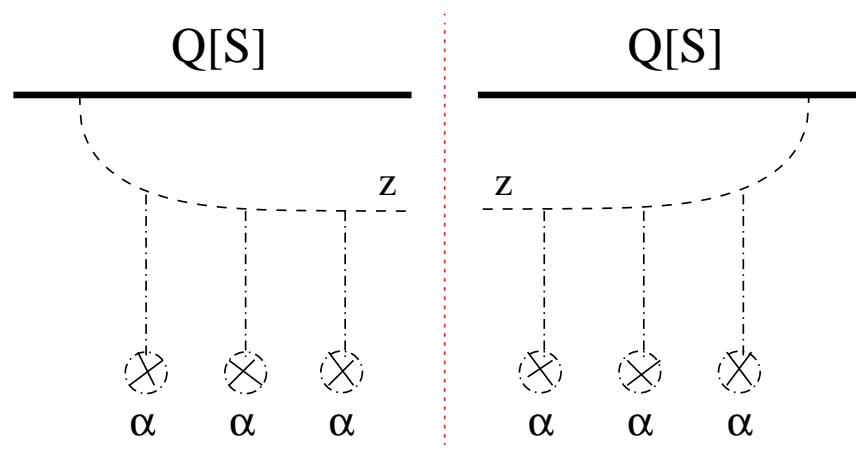
$$\partial_Y \Sigma^P = H^{JIMWLK} \Sigma^P ; \quad H^{JIMWLK} = \int_z Q_i^a(z) Q_i^a(z)$$

The gluon production amplitude

$$Q_i^a(z) = g \int_x \frac{(x-z)_i}{(x-z)^2} \left[J_L^a(z) - S^{ab}(x) J_R^b(x) \right]$$

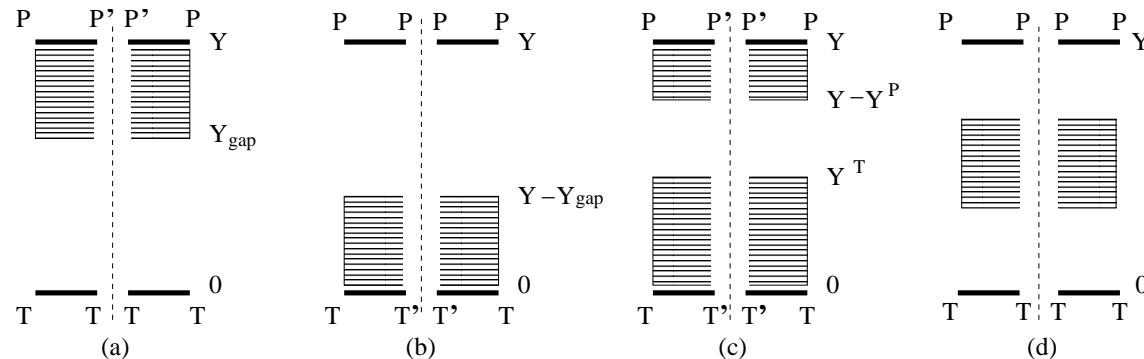
The generators of the left/right color rotations

$$J_R^a(x) = - \text{tr} \left\{ S(x) T^a \frac{\delta}{\delta S^\dagger(x)} \right\}, \quad J_L^a(x) = - \text{tr} \left\{ T^a S(x) \frac{\delta}{\delta S^\dagger(x)} \right\}$$



Semi-inclusive observables: Examples

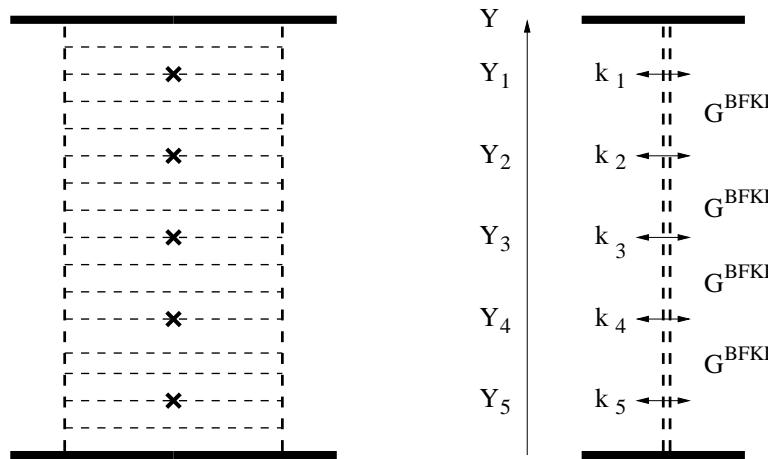
- One gluon inclusive production.
Yu. Kovchegov and K. Tuchin (2001) (dipole limit)
R. Baier, A. Kovner, M. Nardi, and U. Wiedemann (2005) (JIMWLK accuracy)
- Double gluon inclusive production (dipole limit)
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- Multi- gluon inclusive production (dipole limit).
- Diffraction (including elastic and multi gap diffraction)
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- Scattering with momentum transfer.

Multi gluon inclusive production

For the BFKL approximation:



Very schematically:

$$\frac{d\sigma}{dY_1 dk_1^2 \dots dY_n dk_n^2} \sim \Phi^T G_{Y_n - Y_0}^{BFKL} L(k_n) \dots G_{Y_1 - Y_2}^{BFKL} L(k_1) G_{Y - Y_1}^{BFKL} \Phi^P$$

What if we have multiple rescatterings?

Introduce a generating functional \mathcal{Z} :

$$\mathcal{Z}[j] \equiv \int DS W_{Y_0}^T[S] \int D\bar{S} \delta(\bar{S} - S) \mathcal{P} e^{\int_{Y_0}^Y dy [H_3 + \int_k \mathcal{O}_g(k) j(y, k)]} \Sigma^P[S^\dagger \bar{S}]$$

\mathcal{Z} generates effective Feynman rules.

The n -gluon cross section: (note analogy with the Schwinger-Keldish formalism)

$$\frac{d\sigma}{dy_1 dk_1^2 \dots dy_n dk_n^2} = \frac{\delta}{\delta j(k_1, y_1)} \dots \frac{\delta}{\delta j(k_n, y_n)} \mathcal{Z}[j] |_{j=0}$$

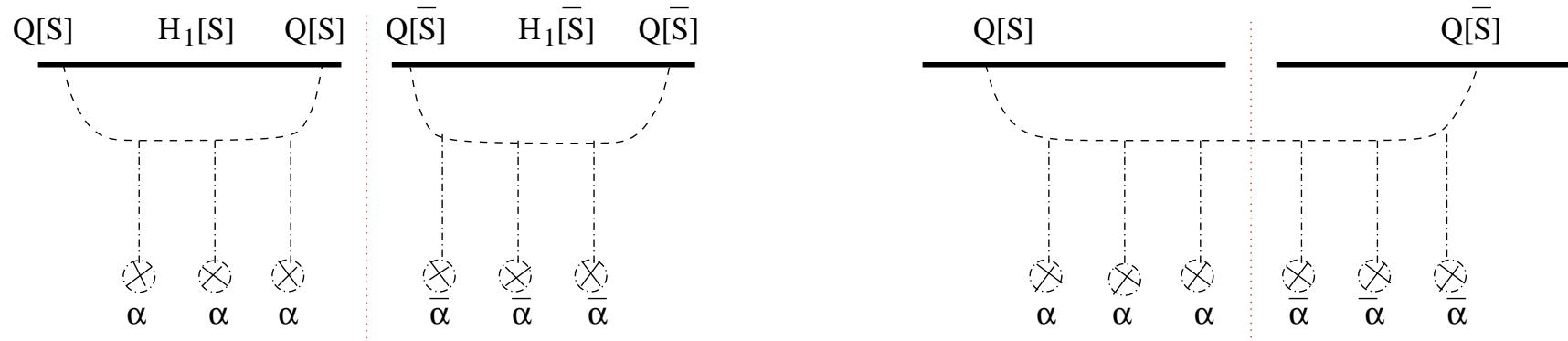
For a single dipole projectile

$$\Sigma^P = \frac{1}{N} \text{tr}[S_F(x) S_F^\dagger(y) \bar{S}_F(y) \bar{S}_F^\dagger(x)]$$

The Hamiltonian H_3 M. Hentschinski, H. Weigert and A. Schafer (2005)

$$H_3[S, \bar{S}] \equiv H_1[S] + H_1[\bar{S}] + 2 \int_z Q_i^a(z, [S]) Q_i^a(z, [\bar{S}])$$

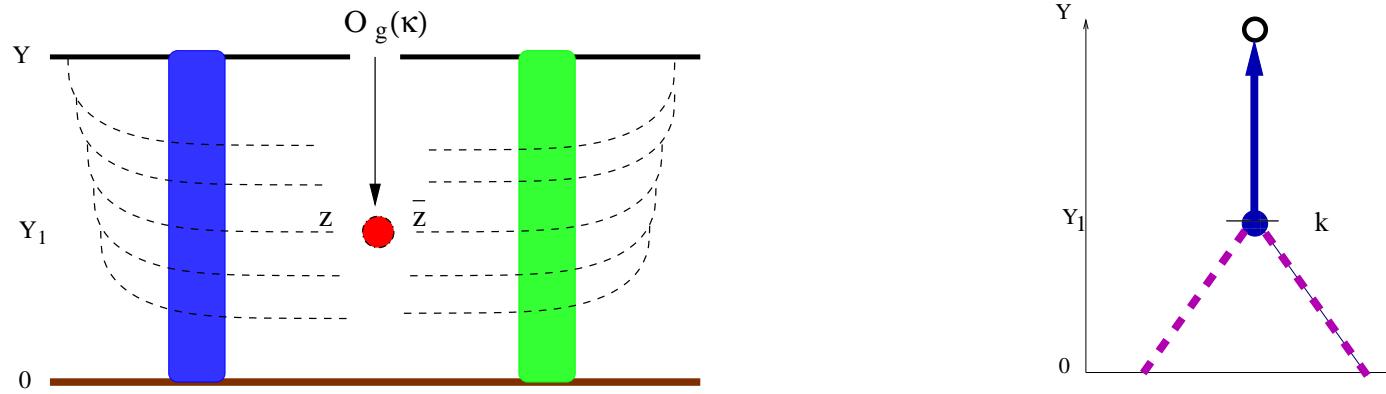
$$H_1[S] \equiv H^{JIMWLK}[S] = \int_z Q_i^a(z, [S]) Q_i^a(z, [S]), \quad H_2[S, \bar{S}] \equiv H_1[S] + H_1[\bar{S}]$$



The gluon emission operator

$$\mathcal{O}_g[S, \bar{S}] = \int \frac{d^2 z}{2\pi} \frac{d^2 \bar{z}}{2\pi} e^{i k_1 (z - \bar{z})} Q_i^a(z, [S]) Q_i^a(\bar{z}, [\bar{S}])$$

Single inclusive gluon production



$$\frac{d\sigma}{dY_1 dk_1^2} = \int DS D\bar{S} W_{Y_1}^T[S] \delta(\bar{S} - S) \mathcal{O}_g[S, \bar{S}] \Sigma_{Y-Y_1}^P[S^\dagger \bar{S}]$$

$$\begin{aligned} \frac{d\sigma}{dY_1 dk_1^2} &= \frac{\alpha_s}{\pi} \int_{z, \bar{z}} e^{i k(z - \bar{z})} \int_{x, y} \frac{(z - x)_i}{(z - x)^2} \frac{(\bar{z} - y)_i}{(\bar{z} - y)^2} n^P(x, y; Y - Y_1) \times \\ &\quad \times [\langle T_{z,y} \rangle_{Y_1} + \langle T_{x,\bar{z}} \rangle_{Y_1} - \langle T_{z,\bar{z}} \rangle_{Y_1} - \langle T_{x,y} \rangle_{Y_1}] \end{aligned}$$

$\langle T \rangle$ denoting an S -matrix of a gluonic dipole:

$$\langle T_{x,y} \rangle_{Y_1} \equiv \int DS W_{Y_1}^T[S] \text{tr}[S_x^\dagger S_y]$$

The dipole limit

Introduce new degrees of freedom.

The dipole creation operator

$$s_{x,y} = \frac{1}{N} \text{tr}[S_F(x) S_F^\dagger(y)]; \quad \bar{s}_{x,y} = \frac{1}{N} \text{tr}[\bar{S}_F(x) \bar{S}_F^\dagger(y)]$$

The quadrupole operator

$$q_{x,y,u,v} = \frac{1}{N} \text{tr}[S_F(x) S_F^\dagger(y) S_F(u) S_F^\dagger(v)]; \quad \bar{q}_{x,y,u,v} = \frac{1}{N} \text{tr}[\bar{S}_F(x) \bar{S}_F^\dagger(y) \bar{S}_F(u) \bar{S}_F^\dagger(v)].$$

The quadrupoles of the mixed type

$$q_{x,y,v,u}^{s\bar{s}} = \frac{1}{N} \text{tr}[S_F(x) S_F^\dagger(y) \bar{S}_F(u) \bar{S}_F^\dagger(v)] = q_{x,y,v,u} + t_{x,y,v,u}$$

Remember that we will set $\bar{S} = S$ in the end of our computation
→ perturbation theory in t

Re-express the Hamiltonian H_3 in new degrees of freedom

$$H_3 = H_s + H_q + H_t + V_{t \rightarrow tt}$$

H_s is the dipole Hamiltonian which generates the Balitsky Kovchegov eq. for s .

$$\partial_y s(x, y) = K^{BFKL} \otimes (s - s s)$$

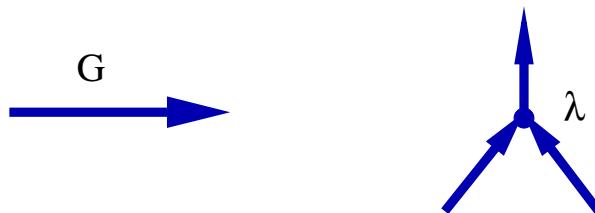
H_q generates a linear evolution of q (similar to BKP) which is also coupled to s .

$$\partial_y q(x, y, u, v) = K_1 \otimes q + K_2 \otimes q s + K_3 \otimes s s$$

H_t generates a linear evolution of t which is coupled to s and q

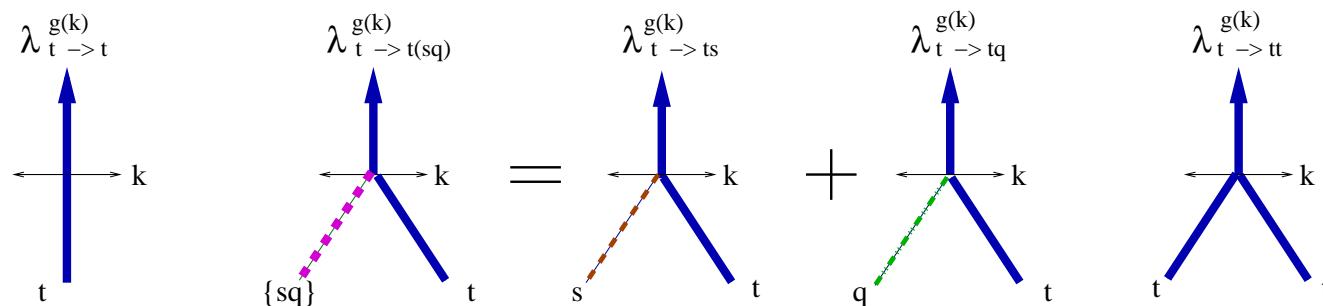
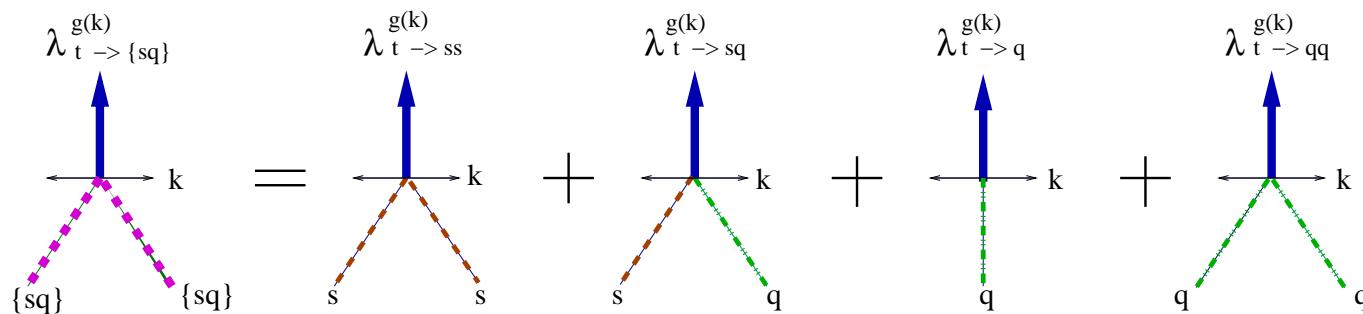
$$\partial_y t(x, y, u, v) = G^{-1}[s] \otimes t + \lambda \otimes t t$$

G is a propagator in the external “Pomeron” field s . $G \rightarrow G^{BFKL}$ for the two-point function.

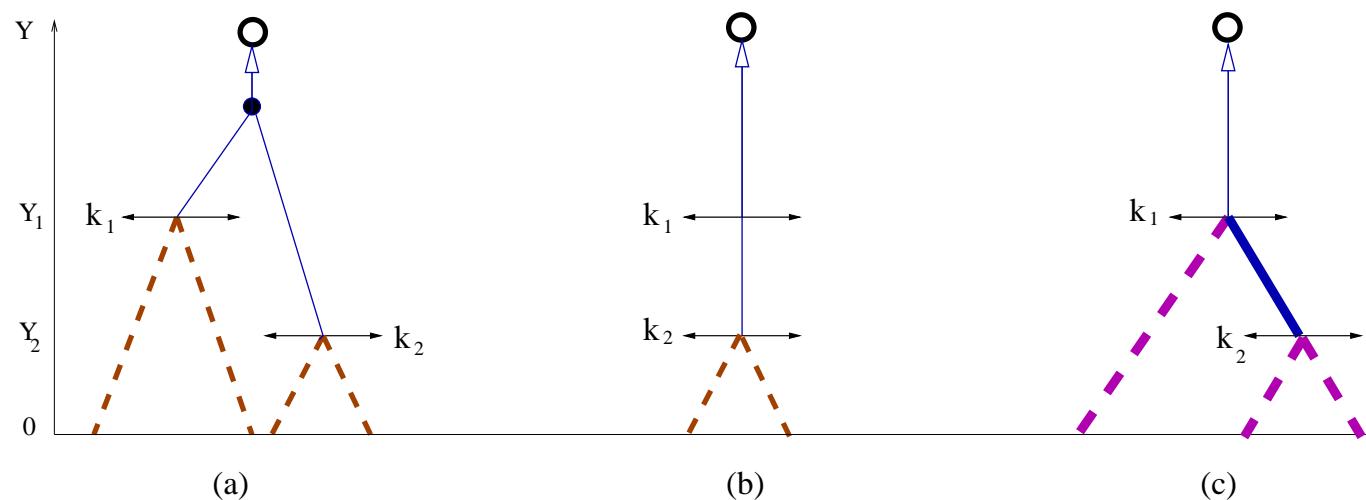


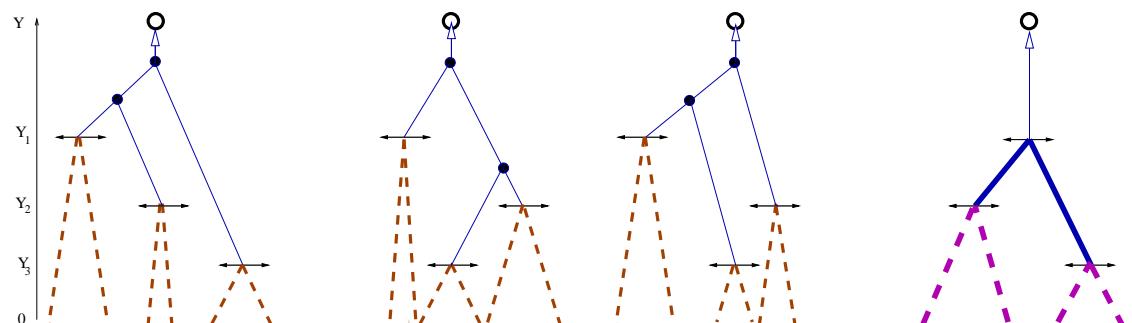
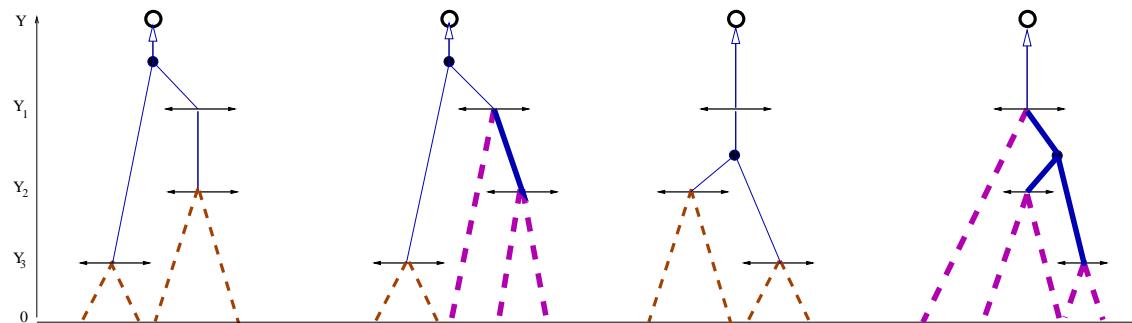
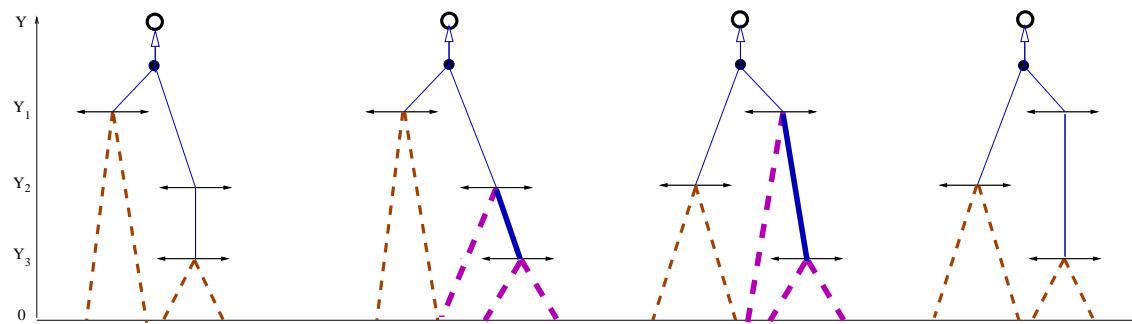
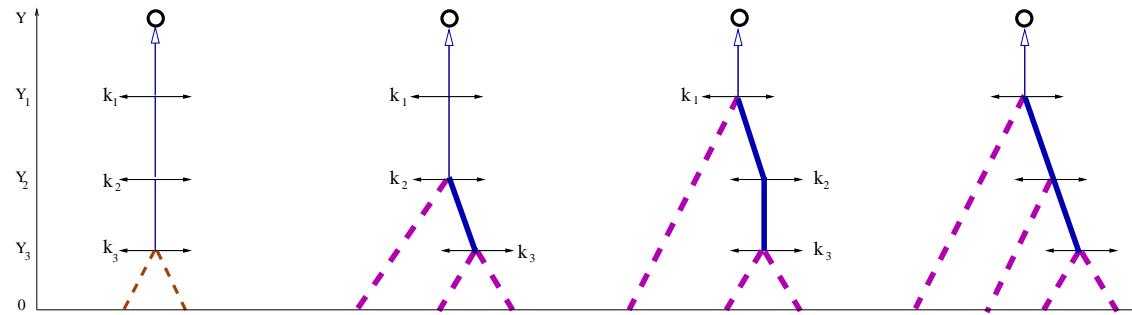
Re-express the insertion operator Q_g

$$\mathcal{O}_g(k) = A_{-1}(k) + A_0(k) + A_1(k)$$



Two gluon inclusive production





Summary and Outlook

- A systematic approach addressing semi-inclusive and exclusive processes at high energies has been developed. Many elements of our approach are independent of the explicit form of the JIMWLK/KLWMIJ Hamiltonian and those generalize to the yet unknown complete high energy evolution Hamiltonian including Pomeron loops.
- Some old results were reproduced and a few new ones were obtained.
- Phenomenology (LHC, HERA, RHIC, TeVatron) - not yet.

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The classical WW field

$$b_i^a(z) = \frac{g}{2\pi} \int d^2 x \frac{(z - x)_i}{(z - x)^2} \rho^a(x)$$

Given the evolution of the hadronic wave function one can calculate the evolution of an arbitrary observable $\hat{\mathcal{O}}(\rho)$ which depends on the color charge density

$$\langle v | \hat{\mathcal{O}}[\rho] | v \rangle = \int D\rho W[\rho] \mathcal{O}[\rho]$$

The evolution of the expectation value

$$\frac{d \langle v | \hat{\mathcal{O}} | v \rangle}{d Y} = \lim_{Y \rightarrow Y_0} \frac{\langle v | \Omega_Y^\dagger \hat{\mathcal{O}}[\rho + \delta\rho] \Omega_Y | v \rangle - \langle v | \hat{\mathcal{O}}[\rho] | v \rangle}{Y - Y_0} = - \int D\rho W[\rho] H[\rho] \mathcal{O}[\rho]$$

Charge density due to newly produced gluon

$$\delta\rho^a(x) = \int_{e^{Y_0} \Lambda}^{e^Y \Lambda} dk^+ a_i^{\dagger b}(k^+, x) T_{bc}^a a_i^c(k^+, x)$$

The dual Wilson line (charge density shift operator)

$$R(z)^{ab} = \left[\mathcal{P} \exp \int dz^- T^c \frac{\delta}{\delta \rho^c(z, z^-)} \right]^{ab}, \quad R \hat{\mathcal{O}}[\rho] = \hat{\mathcal{O}}[\rho + T]$$

KLWMIJ Hamiltonian

A. Kovner and M.L. (2005)

In the Dilute limit $\Omega \rightarrow C$

We write expand C up to quadratic order in a :

$$C_{\Delta y} = 1 + i \int d^2x b_i^a(x) \int_{e^y \Lambda}^{e^{y+\Delta y} \Lambda} \frac{dk^+}{\pi^{1/2} |k^+|^{1/2}} \left[a_i^a(k^+, x) + a_i^{\dagger a}(k^+, x) \right] - \\ - \left(\int d^2x b_i^a(x) \int_{e^y \Lambda}^{e^{y+\Delta y} \Lambda} \frac{dk^+}{\pi^{1/2} |k^+|^{1/2}} \left[a_i^a(k^+, x) + a_i^{\dagger a}(k^+, x) \right] \right)^2$$

The KLWMIJ Hamiltonian for the dilute parton system

$$H^{KLWMIJ} = \int_z b_z^a[\rho] (1 - R_z)^{ab} b_z^b[\rho]$$

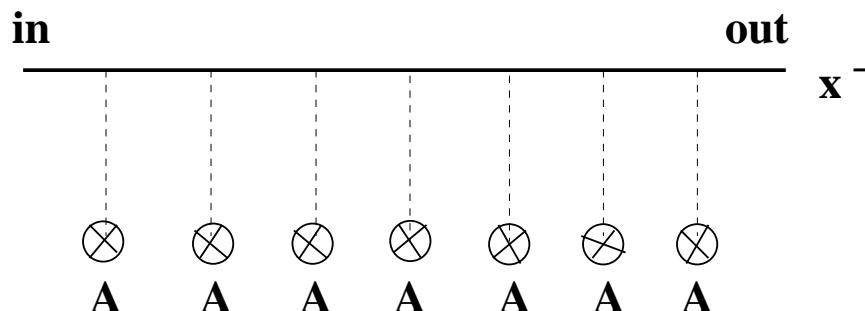
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$$S(x) = \mathcal{P} \exp \left\{ i \int dx^- T^a \alpha_t^a(x, x^-) \right\} .$$



Eikonal scattering for all $k^- \geq \Lambda$
Eikonal factor does not depend on k^-

$$|in\rangle = |z, b\rangle ; \quad |out\rangle = |z, a\rangle ; \quad |out\rangle = S |in\rangle$$

For a composite projectile which has some distribution ρ_p of gluons in its wave function

$$\Sigma_Y^{PP}[\alpha_t] \equiv \langle P | \hat{S} | P \rangle = \int d\rho_p W_Y^P[\rho_p] e^{i \int_x \rho_p \alpha_t}$$

To obtain the total S -matrix of the scattering process at a given rapidity Y one has to average Σ over the distribution of the color fields in the target.

$$\mathcal{S}(Y) = \langle T | \Sigma_{Y-Y_0}^{PP}[\alpha_t] | T \rangle.$$

The high energy evolution of the S -matrix

$$\frac{d}{dY} \mathcal{S} = - \langle T | H^{JIMWLK} \left[\alpha_t, \frac{\delta}{\delta \alpha_t} \right] \Sigma_{Y-Y_0}^{PP}[\alpha_t] | T \rangle.$$

Dense Dilute Duality (DDD): $H^{JIMWLK} \leftrightarrow H^{KLWMIJ}$

A. Kovner and M.L. (2005)

The JIMWLK Hamiltonian

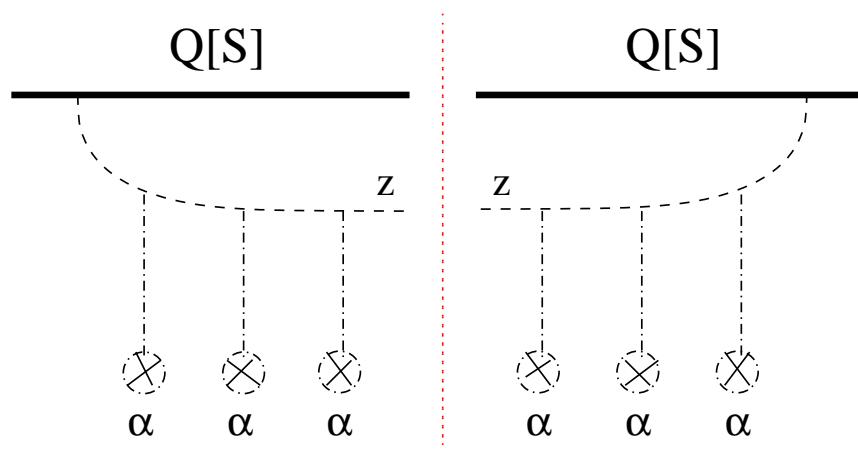
$$H^{JIMWLK} = \int_z Q_i^a(z) Q_i^a(z)$$

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The generators of the left/right color rotations

$$J_R^a(x) = -\text{tr} \left\{ S(x) T^a \frac{\delta}{\delta S^\dagger(x)} \right\}, \quad J_L^a(x) = -\text{tr} \left\{ T^a S(x) \frac{\delta}{\delta S^\dagger(x)} \right\}$$



Semi-inclusive reactions

The wave function coming into the collision region at time $t = 0$

$$|\Psi_{in}\rangle = \Omega_Y |P_v\rangle.$$

The system emerges from the collision region with the wave function

$$|\Psi_{out}\rangle = \hat{S} \Omega_Y |P_v\rangle.$$

The system keeps evolving after the collision to the asymptotic time $t \rightarrow +\infty$, at which point the measurement of an observable $\hat{\mathcal{O}}$ is made

$$\langle \hat{\mathcal{O}} \rangle = \langle P_v | \Omega_Y^\dagger (1 - \hat{S}^\dagger) \Omega_Y \hat{\mathcal{O}} \Omega_Y^\dagger (1 - \hat{S}) \Omega_Y | P_v \rangle$$

$$\langle \hat{\mathcal{O}} \rangle_Y = \mathcal{O}_Y[S, \bar{S}]|_{\bar{S}=S}$$

$$\mathcal{O}_Y[S, \bar{S}] = \langle P_v | \Omega_Y^\dagger (1 - \hat{S}^\dagger) \Omega_Y \hat{\mathcal{O}} \Omega_Y^\dagger (1 - \hat{\bar{S}}) \Omega_Y | P_v \rangle$$

High energy evolution of the observable

$$\frac{d\mathcal{O}_Y[S, \bar{S}]}{dY} = \lim_{\Delta Y \rightarrow 0} \frac{\mathcal{O}_{Y+\Delta Y}[S, \bar{S}] - \mathcal{O}_Y[S, \bar{S}]}{\Delta Y}$$

In the dilute limit $\Omega \rightarrow C$

$$\partial_Y \mathcal{O}[S, \bar{S}] = -H_3[S, \bar{S}] \mathcal{O}[S, \bar{S}]$$

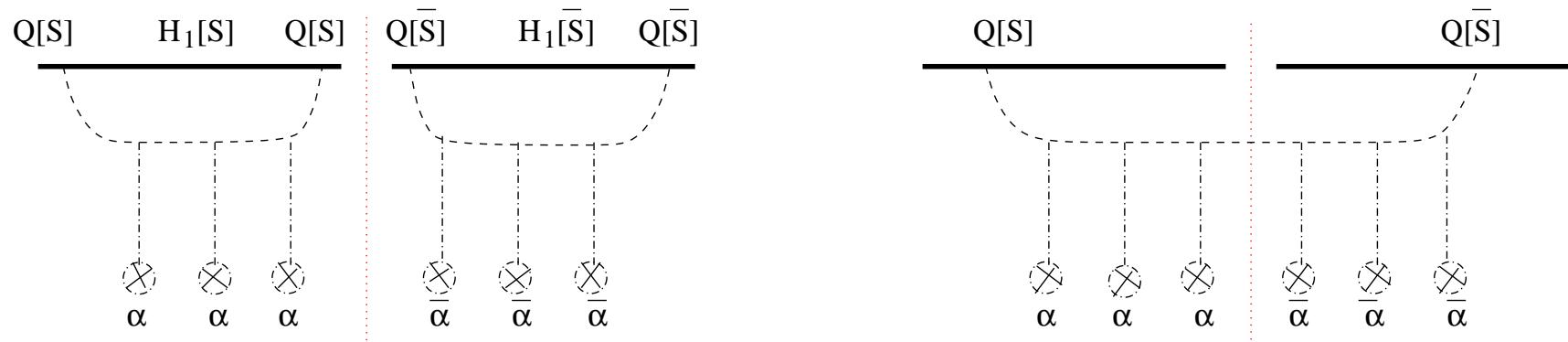
The Hamiltonian H_3

$$H_3[S, \bar{S}] \equiv H_1[S] + H_1[\bar{S}] + 2 \int_z Q_i^a(z, [S]) Q_i^a(z, [\bar{S}])$$

$$H_1[S] \equiv H^{JIMWLK}[S] = \int_z Q_i^a(z, [S]) Q_i^a(z, [S]), \quad H_2[S, \bar{S}] \equiv H_1[S] + H_1[\bar{S}]$$

For any observable the evolution operator between rapidity y_1 and y_2 is

$$U_3(y_1, y_2) = \text{Exp}[-H_3(y_2 - y_1)]$$



It is easy to generalize by considering a product of $n + 1$ local in rapidity operators $\hat{\mathcal{O}}_i$ each one depending only on degrees of freedom at fixed rapidity $Y_0 + Y_i$.

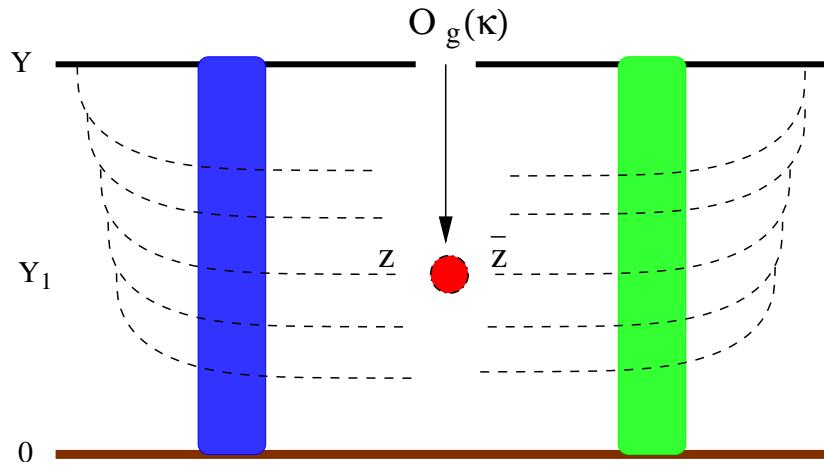
$$\langle \hat{\mathcal{O}}_n \dots \hat{\mathcal{O}}_0 \rangle = \int DS D\bar{S} W_{Y_0}^T[S] \delta(S - \bar{S}) U_3(Y_0, Y_0 + Y_n) \mathcal{O}_n[S, \bar{S}] U_3(Y_n, Y_{n-1}) \times \\ \times \mathcal{O}_{n-1}[S, \bar{S}] \dots U_3(Y_0 + Y_1, Y) \mathcal{O}_0[S, \bar{S}]$$

The operator $\hat{\mathcal{O}}_0$ determines what kind of observation is made on the valence degrees of freedom of the projectile.

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Single inclusive gluon production



The observable

$$\hat{\mathcal{O}}_g = \hat{n}(k_1, y) = \int_{e^{Y_1} \Lambda}^{e^y \Lambda} dk^+ a_i^{\dagger a}(k_1, k^+) a_i^a(k_1, k^+)$$

Introducing two external fields

$$\mathcal{O}_g[S, \bar{S}] = \frac{d}{dy} \langle 0_a | C_y (1 - \hat{S}^\dagger) C_y^\dagger \hat{\mathcal{O}}_g C_y (1 - \hat{\bar{S}}) C_y^\dagger | 0_a \rangle |_{y=Y_1}$$

The operator

$$\mathcal{O}_g[S, \bar{S}] = \int \frac{d^2 z}{2\pi} \frac{d^2 \bar{z}}{2\pi} e^{i k_1 (z - \bar{z})} Q_i^a(z, [S]) Q_i^a(\bar{z}, [\bar{S}])$$

The differential cross section

$$\frac{d\sigma}{dY_1 dk_1^2} = \langle \hat{n}(k_1, Y_1) \hat{\mathcal{O}}_0 \rangle = \int DS D\bar{S} W_{Y_1}^T[S] \delta(\bar{S} - S) \mathcal{O}_g[S, \bar{S}] \Sigma_{Y-Y_1}^{PP}[S^\dagger \bar{S}]$$

The final result for single gluon inclusive production reads

$$\begin{aligned} \frac{d\sigma}{dY_1 dk_1^2} &= \frac{\alpha_s}{\pi} \int_{z, \bar{z}} e^{i k(z - \bar{z})} \int_{x, y} \frac{(z - x)_i}{(z - x)^2} \frac{(\bar{z} - y)_i}{(\bar{z} - y)^2} n^P(x, y; Y - Y_1) \times \\ &\quad \times [\langle T_{z,y} \rangle_{Y_1} + \langle T_{x,\bar{z}} \rangle_{Y_1} - \langle T_{z,\bar{z}} \rangle_{Y_1} - \langle T_{x,y} \rangle_{Y_1}] \end{aligned}$$

$\langle T \rangle$ denoting an S -matrix of a gluonic dipole:

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