Want to accelerate your Track Fitting?

Try the General Triplet Track Fit

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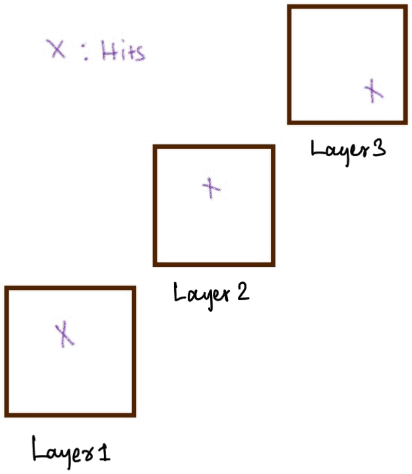
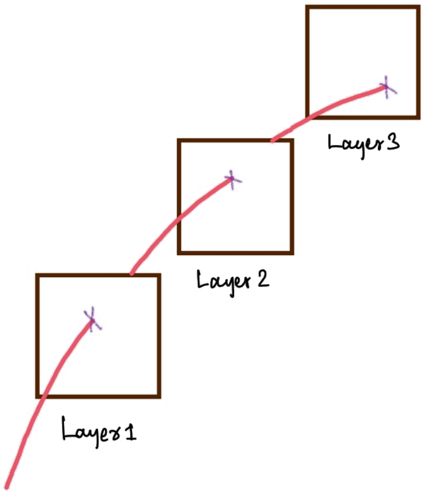
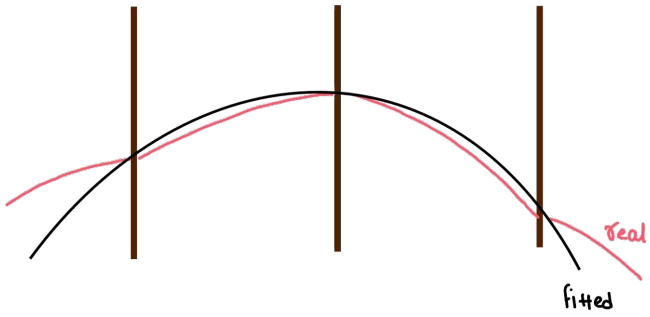
Track Fitting

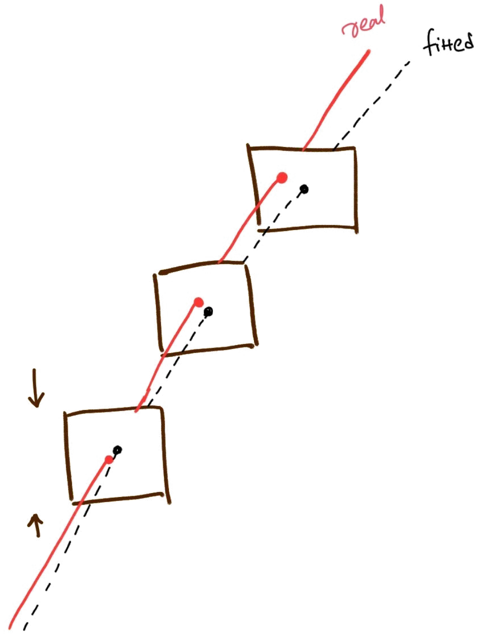
- Track fitting - adapting parameters of a track model to measurements
  - Models describe trajectory in magnetic field, materials, energy loss.
  - Fitting also takes uncertainties into account.





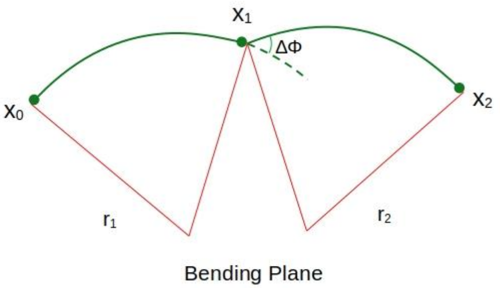
Materials, Sensor resolutions?

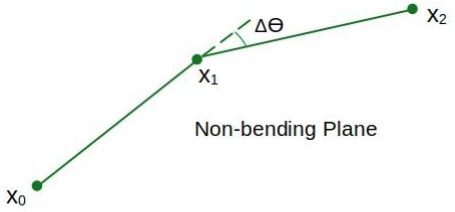




Triplets…

- Using the *triplet* representation
    - At least three hits required to measure track parameters.
    - Less complex than ‘quadruplets’.
    - Kinematics overconstrained ➔ filter bad combinations with fit quality.
    - Can be used to seed conventional track finding algos.
- There exists an analytical solution for fitting triplets with Multiple Coulomb scattering (MS) and hit position uncertainties ✅





General Triplet Track Fit

- Generalized from MS-only triplet fit [1] to include hit uncertainties [2].
- Fit quality for a triplet:

$$\chi^2(p, \vec{\delta}_k) = \frac{\Delta\Theta_{\mathrm{MS}}(p,\vec{\delta}_k)^2}{\sigma^2_{\Theta_{\mathrm{MS}}}} + \frac{\Delta\Phi_{\mathrm{MS}}(p,\vec{\delta}_k)^2}{\sigma^2_{\Phi_{\mathrm{MS}}}} + \sum_{hit\ k=0}^{2} \vec{\delta}_k^{\top} V_k^{-1} \vec{\delta}_k$$

$$\vec{\delta}_k = \vec{r}_{\mathrm{fit},k} - \vec{r}_{\mathrm{meas},k}$$

- Total momentum p & hit position shifts $\vec{\delta}_k$ are fitted by minimizing the χ^2.

[1]: Berger, Kozlinskiy, Kiehn, & Schöning, *A new three-dimensional track fit with multiple scattering* 2017
[2]: Schöning, *A General Track Fit Based on Triplets* 2024

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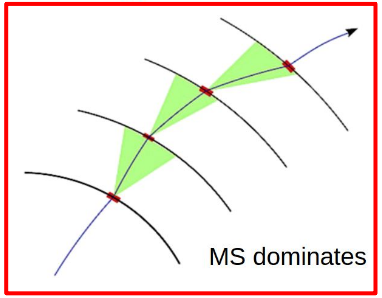

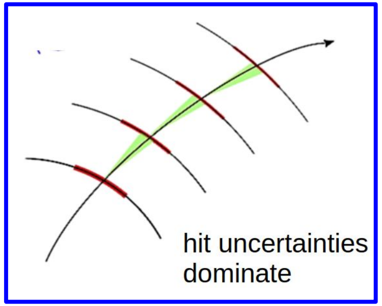

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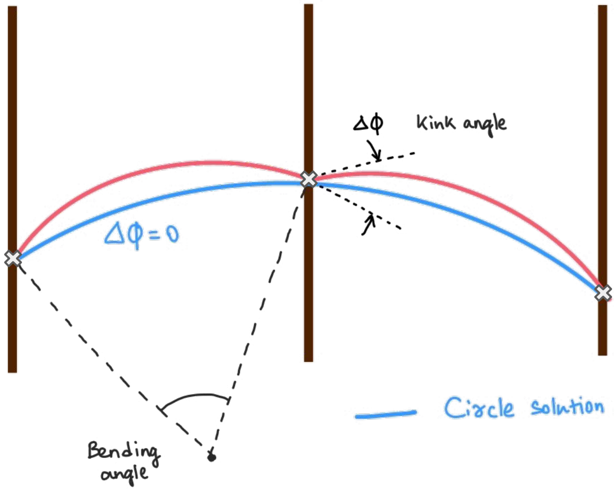
Triplet Parameters

- Kink angles can be obtained from bending angles.
- Bending angles are given by transcendental equations ➞ no algebraic solution…
- Use approximate solution connecting three hits with a circle in the transverse plane.



Linearized kink angle:

$$\Delta\Phi = \widetilde{\Phi} + \rho_{\Phi} c_{3D} \qquad c_{3D} = R_{3D}{}^{-1} = \frac{|q|B}{p}$$

Kink angle for vanishing curvature (c_{3D}= 0)

Dependance of kink angle on curvature

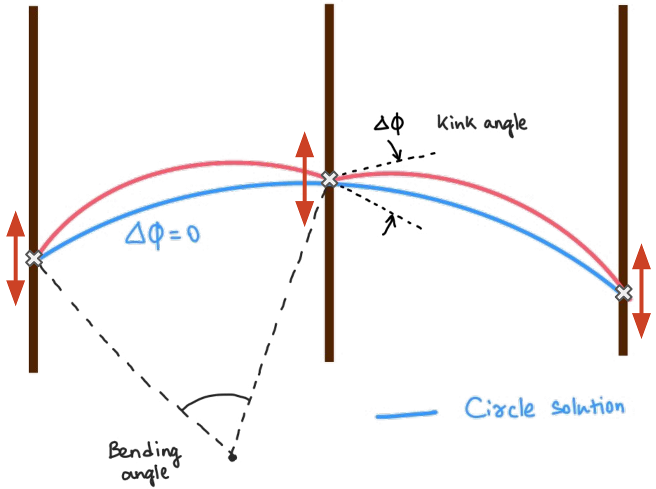
…similarly for the kink angle in non-bending plane

Triplet Parameters

Contributions to kink angles from position uncertainties taken into account using directional derivatives of kink angles w.r.t hit position shifts.

$$\Delta\Theta_{\text{MS}} = \Delta\Theta - \Delta\Theta_{\text{hit}};\ \Delta\Phi_{\text{MS}} = \Delta\Phi - \Delta\Phi_{\text{hit}}$$

with apparent kinks $\Delta\Theta,\ \Delta\Phi$



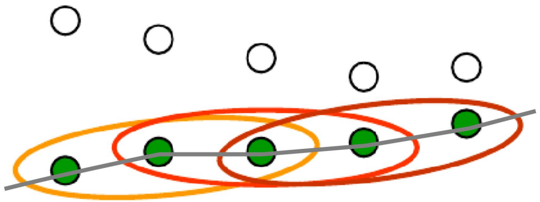
…similarly for the kink angle in non-bending plane

General Triplet Track Fit

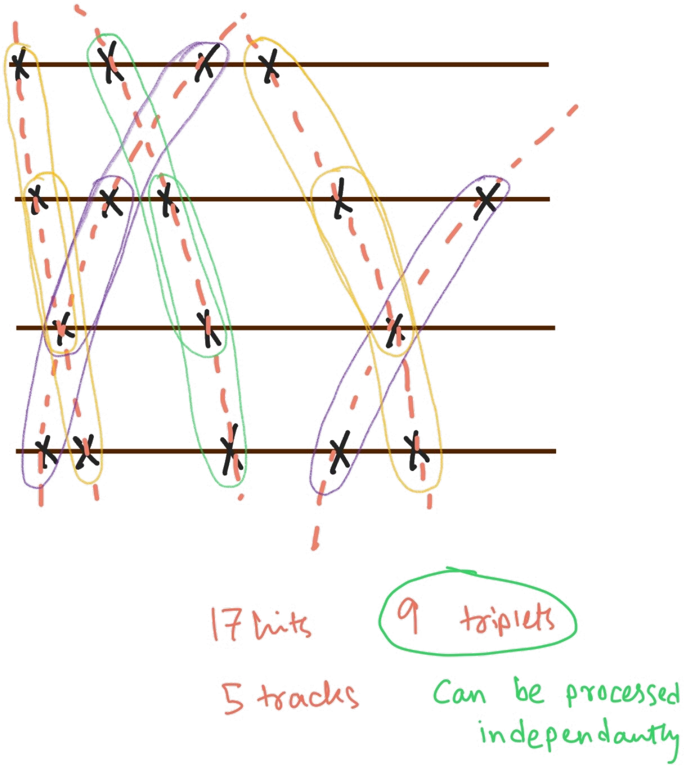
- Linearized expressions of kink angles & contribution of position uncertainties are plugged back into χ^2 - and minimized.

- The fit is factorized into two steps:
  - Local calculation of triplet parameters.
  - Global fit of all connected triplets in a track - requires a matrix inversion.

- Several advantages of the two step procedure:
  - **Parallelization** of the problem ➝ **next slide**.
  - Global fit is completely detector independent.
    - Triplet parameters absorb detector dependencies (geometry, magnetic field).
  - Use triplet compatibility (fitted p, q) before combination.



Parallelization from Triplets



Track Finding using triplets
- Sachin's talk next

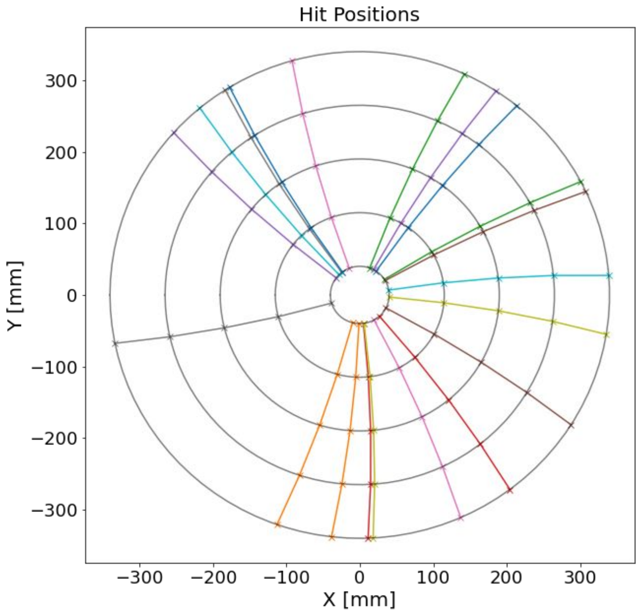
MS-only fit implemented on GPUs for Mu3e [1]

[1]: Mu3e Collaboration, *Technical design of the phase-I Mu3e experiment* 2021

Toy Simulation

- Developed for standalone testing of the fits
  - Minimal setup: provides tight control over results.
- Particles propagated through uniform, solenoidal B-field.
- Detector with 5 equally spaced barrel layers (axis parallel to B-field).

Generated particles	Position	(0,0,0) - beamspot of size 45 mm along z
	Azimuthal angle (Φ)	Uniform in [0, 2π]
	Polar angle (ϴ)	70° (η= 0.36)
	Charge	+1
Simulated uncertainties	Multiple scattering	x/X_0 = 2% (per layer)
	Hit uncertainties	12 µm in sensor transverse directions



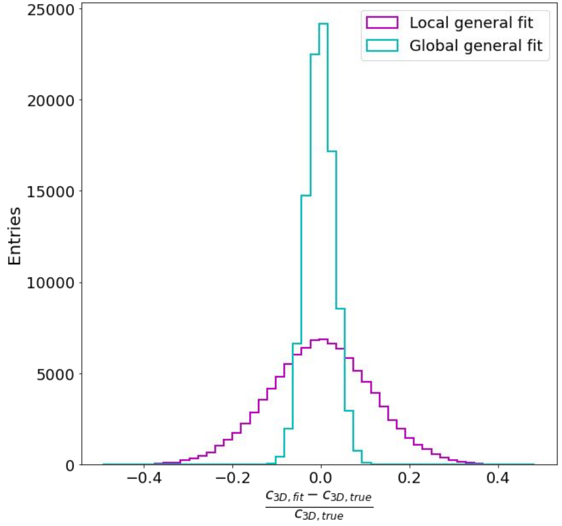
In bending plane

Implementation on CPU

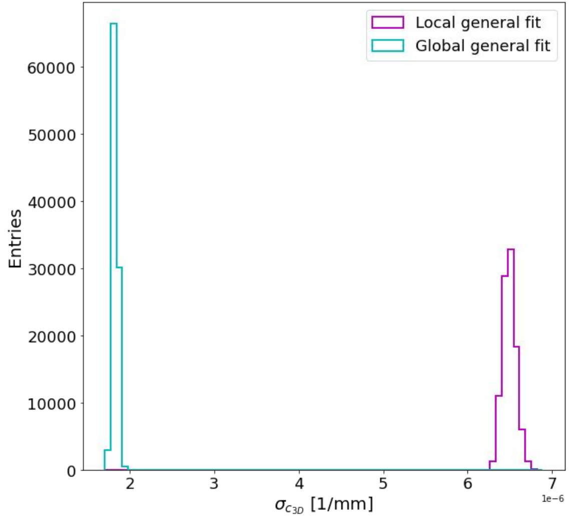
Curvature: $c_{3D} = {R_{3D}}^{-1} = \frac{|q|B}{p}$ to avoid numerical instabilities at high momenta

p_T = 10 GeV

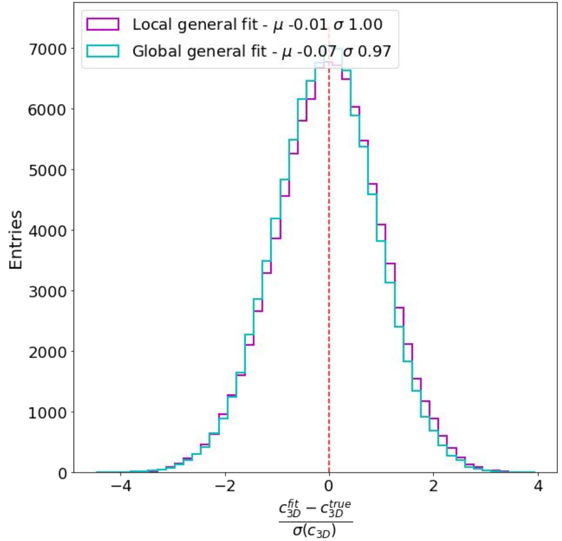
Relative resolution



Fit uncertainty



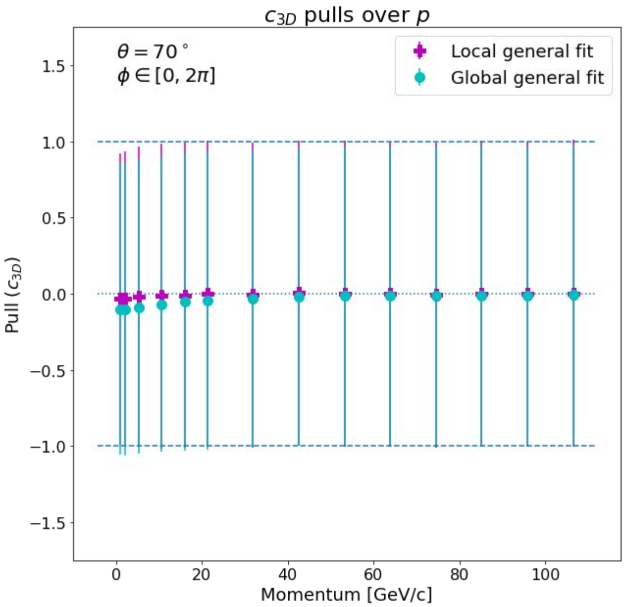
Pull distribution



Plots for local fit everywhere are for one triplet; the global fit combines all (three) triplets on track.

Pull distribution with 0 mean and 1 sigma implies correct estimation of curvatures and uncertainties.

Curvature Pulls

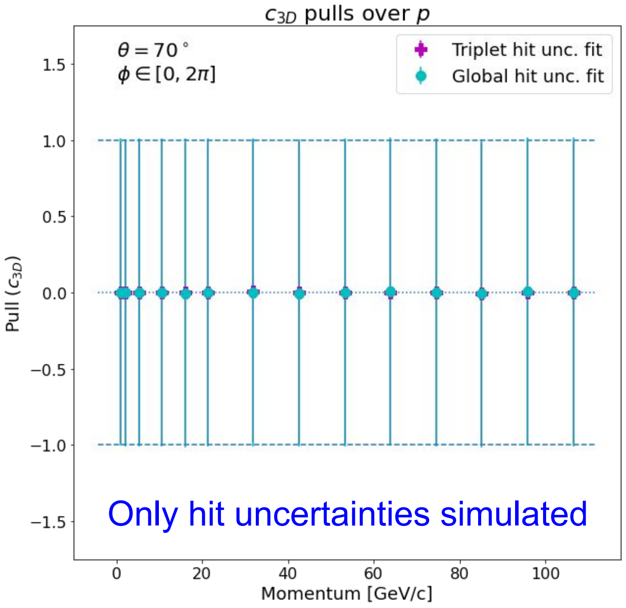


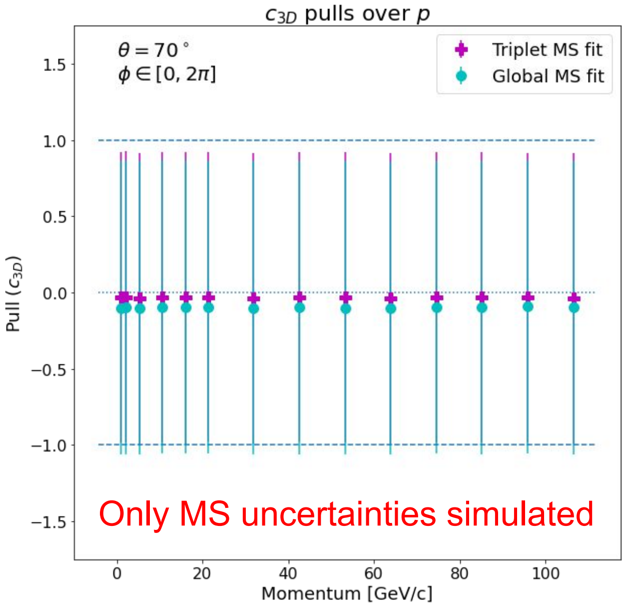
Correct estimation of curvatures and uncertainties over a large momentum range.

Dominating MS effects at low momenta → correlation between curvatures and uncertainties → bias in pull distributions.

➤ Well understood and can be corrected.

Curvature Pulls





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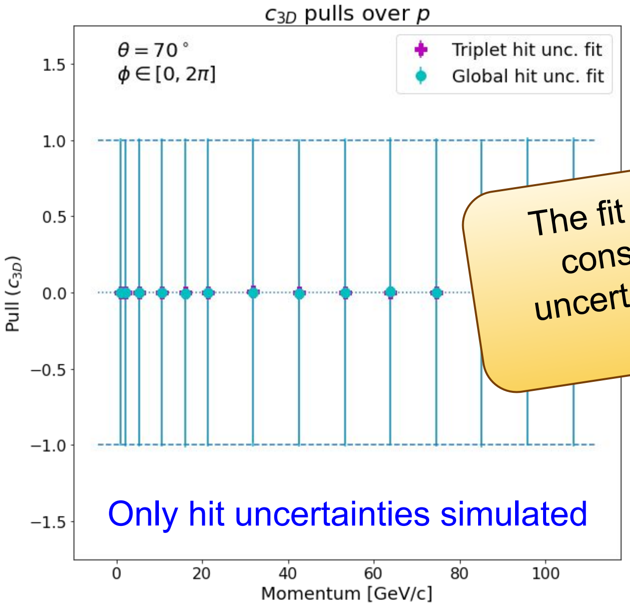
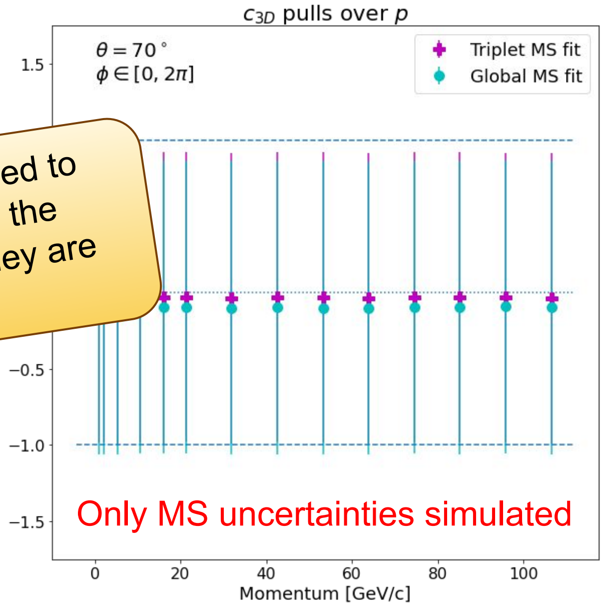


The fit can be configured to consider only one of the uncertainties where they are dominating



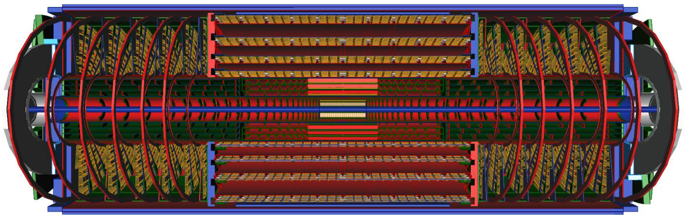
Summary & Outlook

- General triplet track fit with Multiple scattering and hit position uncertainties.
  - Plan to exploit massive parallelism by using GPUs ➔ for fast track fitting in ATLAS EF.
- The triplet fit was tested using standalone simulation.
- Correct estimation of curvatures and uncertainties observed over a large momentum range.
- Gain from using MS-only triplet fits, wherever possible.
- Next steps: Integrating in `traccc`; study performance in ITk.

Backup

ATLAS Event Filter Tracking

- Series of upgrades underway to prepare for Hi-Lumi LHC
  - Inner Detector (ID) to be replaced with all-Si Inner Tracker (ITk).
  - Trigger and DAQ systems have to cope with increased data rate and event complexity.
- Event Filter (EF) to reduce 1 MHz Level-0 rate to 10 kHz.
  - Running offline-like algorithms on commercial hardware.
  - Possibly using accelerators (GPUs, FPGAs) to gain performance.
- Tracking in Event Filter (EF Tracking)
  - Crucial to maintain sensitivity while keeping trigger rates under control.
  - Regional and full-scan tracking at 1 MHz and 150 kHz respectively.
  - Most expensive computationally.



ATLAS Collaboration, *Expected tracking and related performance with the updated ATLAS Inner Tracker layout at the High-Luminosity LHC* - ATL-PHYS-PUB-2021-024

Triplet Parameters

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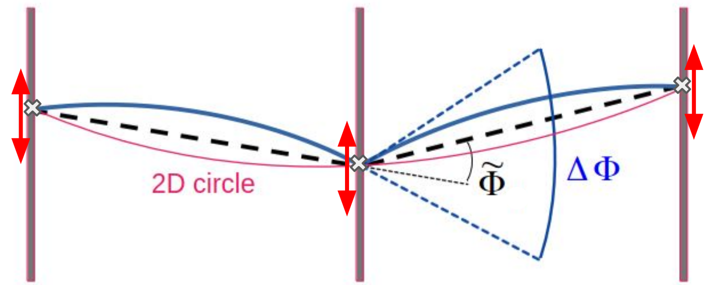
Use approximate solution connecting three hits with a circle in the transverse plane.

$\odot \mathbf{B}$

Bending plane:



$$\Delta\Phi_{\text{hit}} = \sum_{k=1}^{3} \overrightarrow{h_{\Phi\,k}} \cdot \vec{\delta}_k$$
with $\overrightarrow{h_{\Phi\,k}} = \text{grad}\,\Delta\Phi(\vec{x_k})$

Contributions to kink angles from position uncertainties taken into account using directional derivatives of kink angles w.r.t hit position shifts.

Linearized kink angle: $\Delta\Phi = \tilde{\Phi} + \rho_\Phi c_{3D}$ $\qquad c_{3D} = R_{3D}{}^{-1} = \frac{|q|B}{p}$

Kink angle for vanishing curvature (c_{3D}= 0)

Dependance of kink angle on curvature

$$\Delta\Theta_{\text{MS}} = \Delta\Theta - \Delta\Theta_{\text{hit}}; \; \Delta\Phi_{\text{MS}} = \Delta\Phi - \Delta\Phi_{\text{hit}}$$
with apparent kinks $\Delta\Theta, \; \Delta\Phi$

…similarly for the scattering angle in non-bending plane

χ^2 Minimization

$$\chi^2(c_{3D};\vec{\boldsymbol{\delta}}) = \sum_{\text{triplet } j}^{n_{\text{hit}}-2} \frac{1}{\sigma^2_{\text{MS},j}} \left(\tilde{\Theta}_j + \rho_\Theta\, c_{3D} - \Delta\Theta_{\text{hit},j}(\vec{\boldsymbol{\delta}})\right)^2 + \sum_{\text{triplet } j}^{n_{\text{hit}}-2} \frac{\sin^2\hat{\vartheta}_j}{\sigma^2_{\text{MS},j}} \left(\tilde{\Phi}_j + \rho_\Phi\, c_{3D} - \Delta\Phi_{\text{hit},j}(\vec{\boldsymbol{\delta}})\right)^2 + \sum_{\text{hit } k=0}^{n_{\text{hit}}-1} \frac{{\delta_{k_i}}^2}{{\sigma_{k_i}}^2}$$

Global fit χ^2 :

$$\chi^2(c_{3D};\vec{\boldsymbol{\delta}}) = \left(\boldsymbol{\Psi} + \boldsymbol{\rho}\, c_{3D} - \vec{\boldsymbol{H}}\vec{\boldsymbol{\delta}}\right)^T \boldsymbol{D}_{\text{MS}} \left(\boldsymbol{\Psi} + \boldsymbol{\rho}\, c_{3D} - \vec{\boldsymbol{H}}\vec{\boldsymbol{\delta}}\right) + \vec{\boldsymbol{\delta}}^T\, \vec{\vec{\boldsymbol{D}}}_{\text{hit}}\, \vec{\boldsymbol{\delta}}$$

Triplet parameters from linearization ($\boldsymbol{\Psi}$, $\boldsymbol{\rho}$)

Hit position derivatives of kink angles ($\vec{\boldsymbol{H}}$)

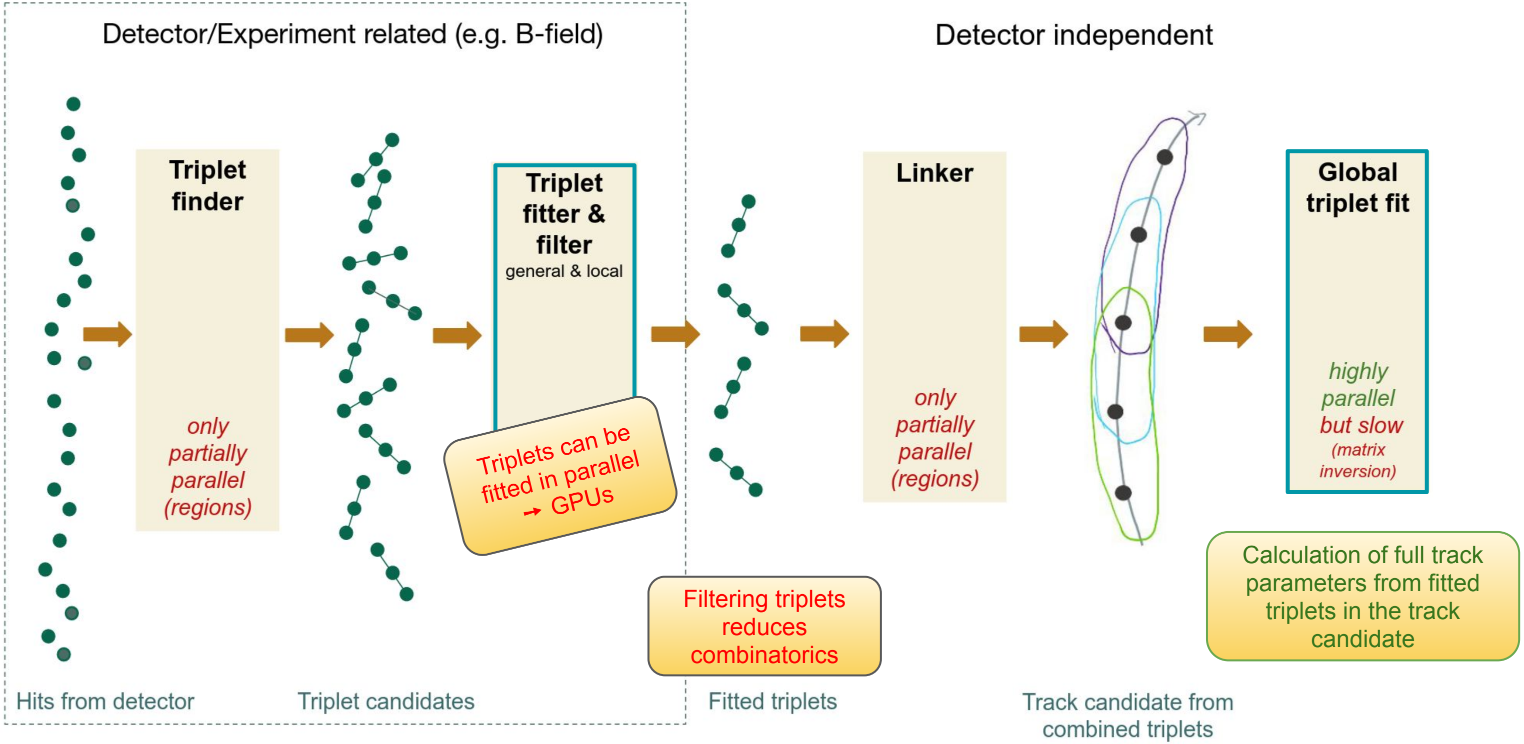
Multiple scattering uncertainty matrix ($\boldsymbol{D}_{\text{MS}}$)

Hit position uncertainty matrix ($\vec{\vec{\boldsymbol{D}}}_{\text{hit}}$)

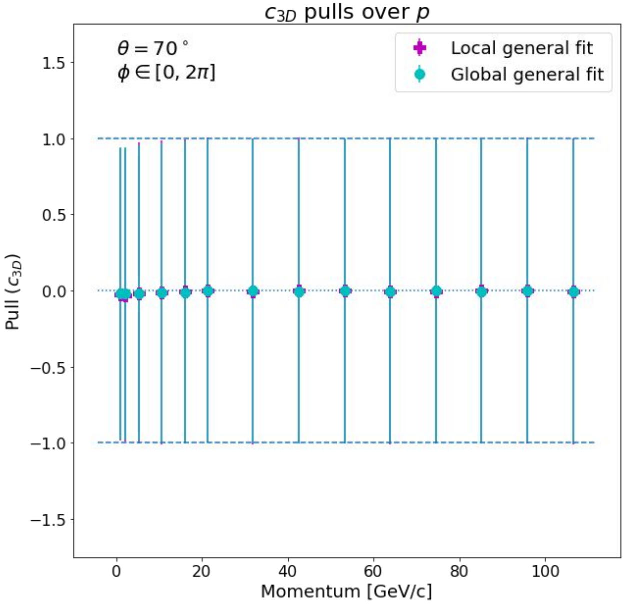
Hit position shifts ($\vec{\boldsymbol{\delta}}$)

Minimization involves a matrix inversion.

Triplet Fit: Hits to Tracks



Curvature Pulls



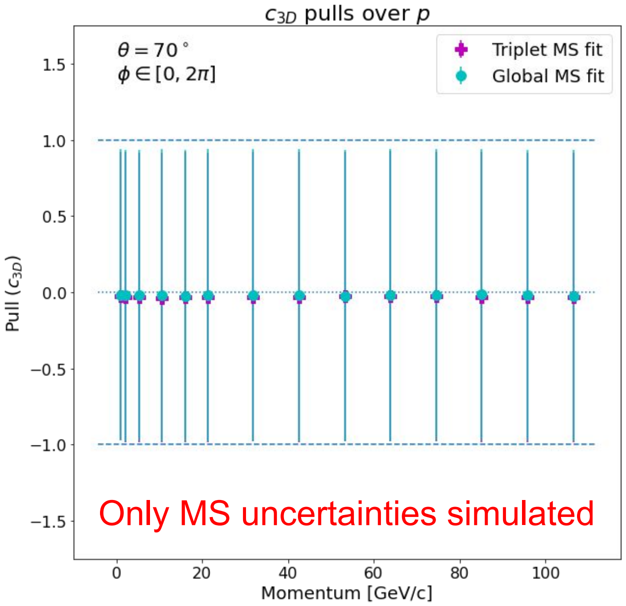
Correct estimation of curvatures and uncertainties over a large momentum range.

Bias goes away if the 'correct' MS uncertainties - estimated using the curvature from the global fit - are used, instead of those estimated using the linearization/local fits.

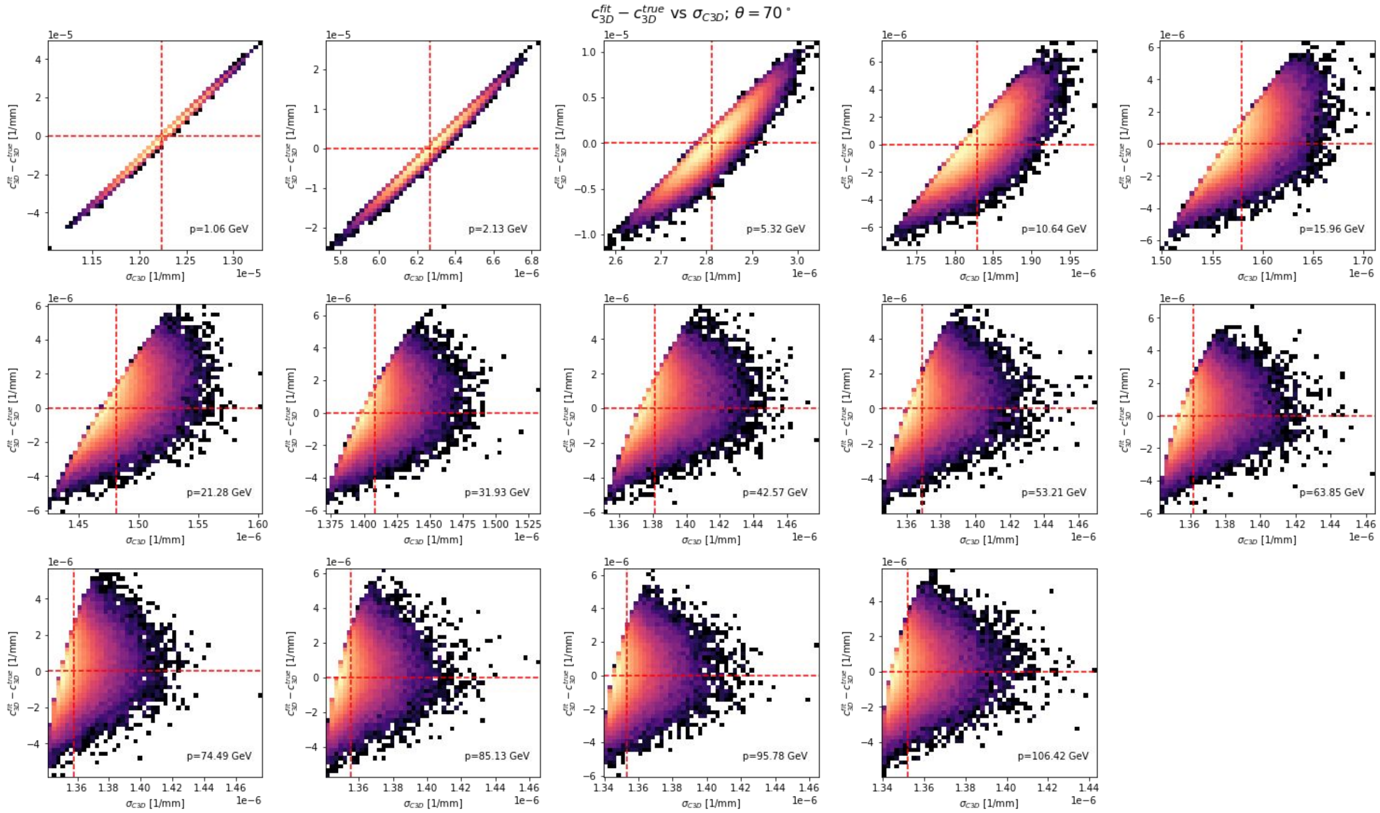
➞ iterative!

Curvature Pulls

With MS uncertainties estimated from global fit curvatures.



Pull Bias



c_{3D} and $\sigma(c_{3D})$ strongly correlated at low momenta (dominating MS uncertainties)

Two effects:

- pulls for a single triplet have $\mu < 0$ (negative residuals divided by smaller σ-s)

- statistical effect: combining triplets make the biases larger