Jet modification in the QGP and the hadronic phases with SUBA-Jet framework

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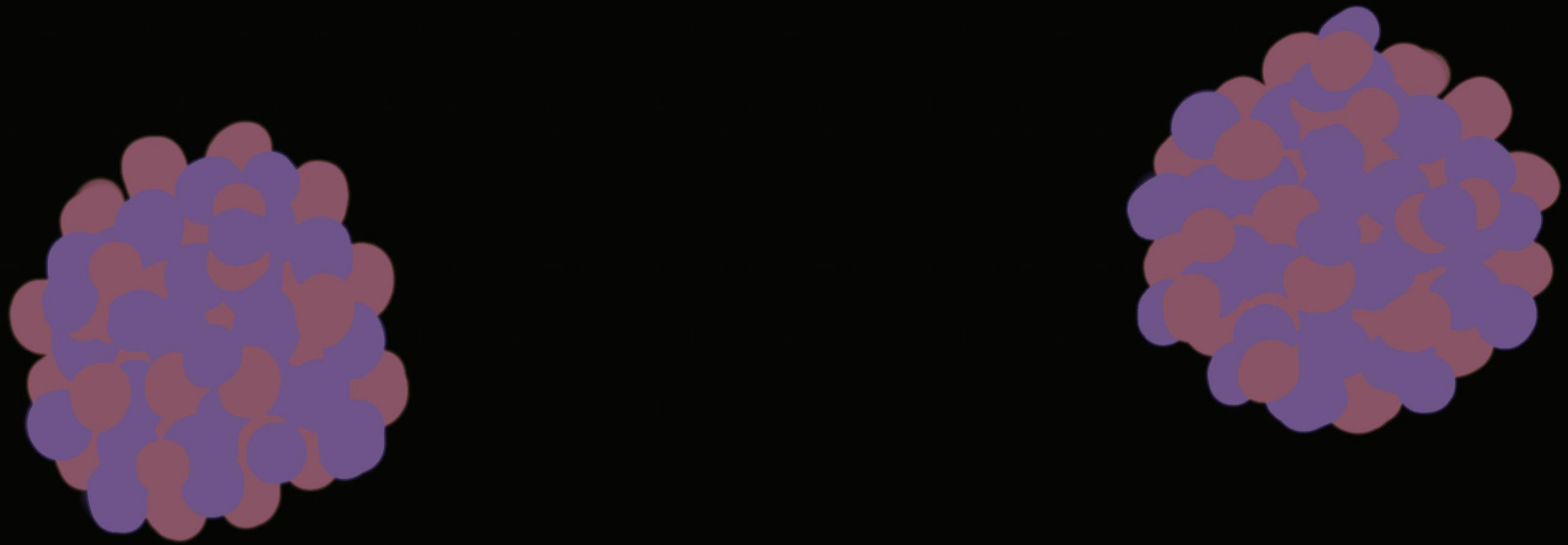
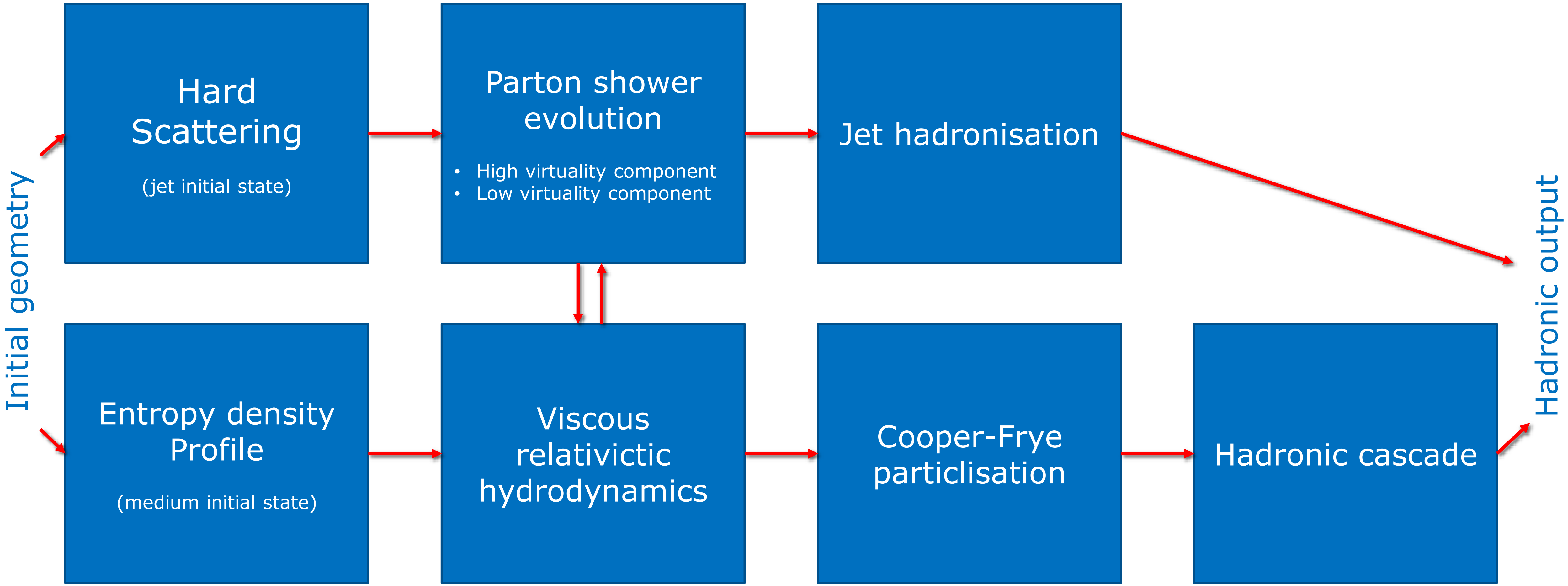
SoftJet2024



Heavy-Ion Collision

Two heavy-ion nuclei collide and create hot medium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(jet initial state)
Parton shower evolution
High virtuality component
Low virtuality component
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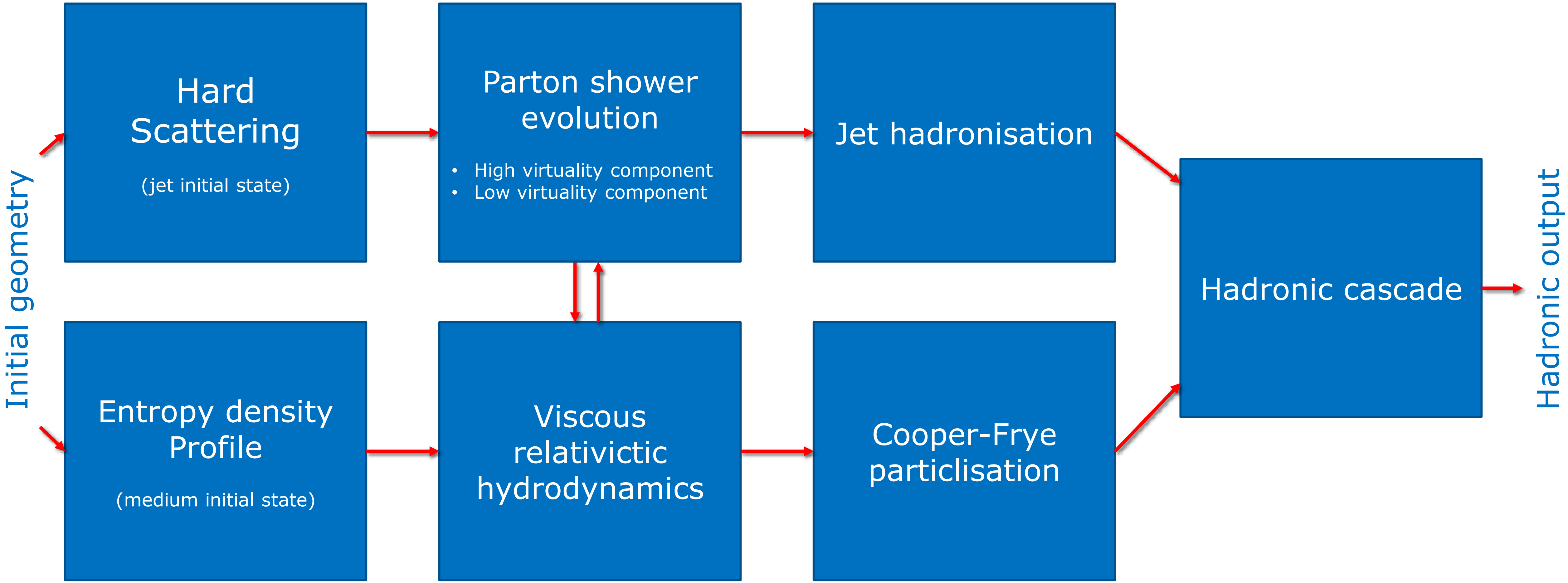
Entropy density Profile
(medium initial state)
Viscous relativictic hydrodynamics
Cooper-Frye particlisation
Hadronic cascade
Hadronic output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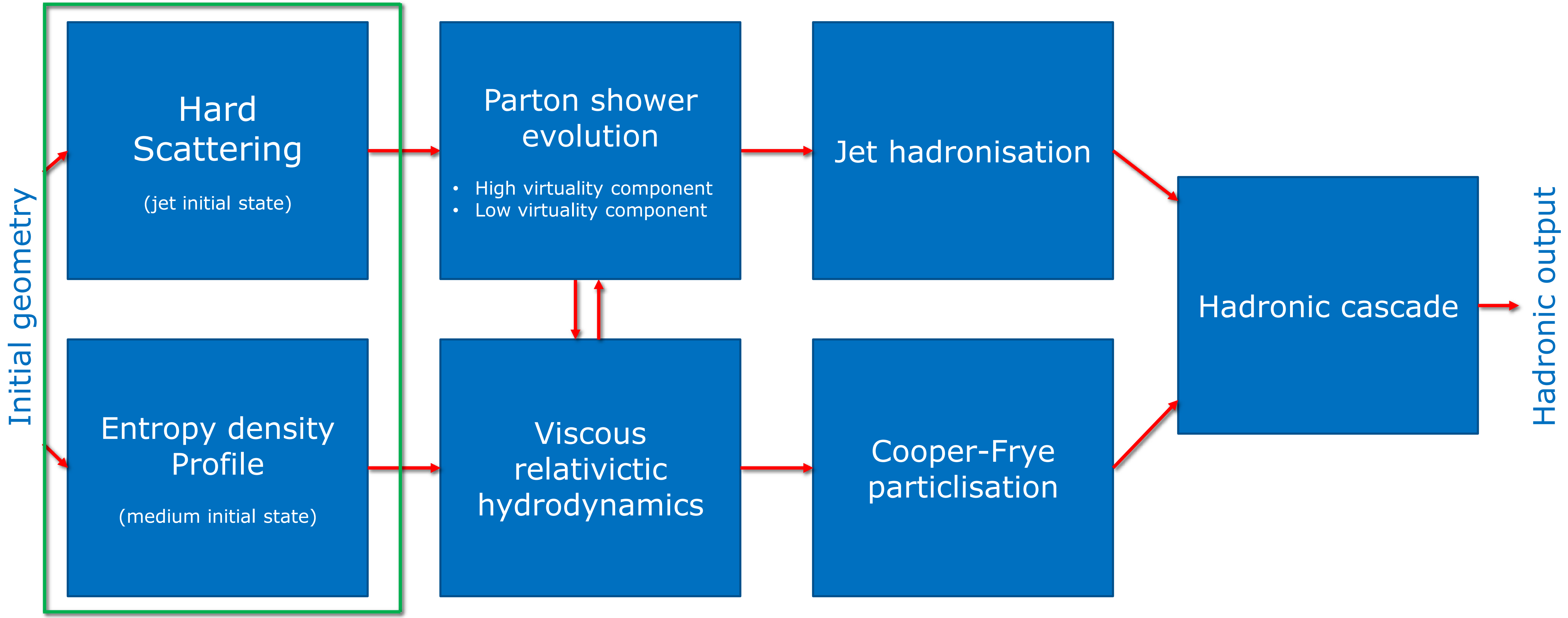
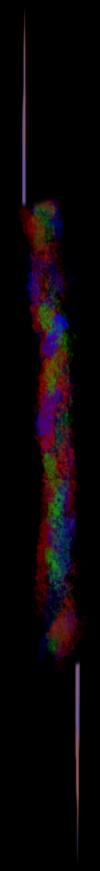
Initial state



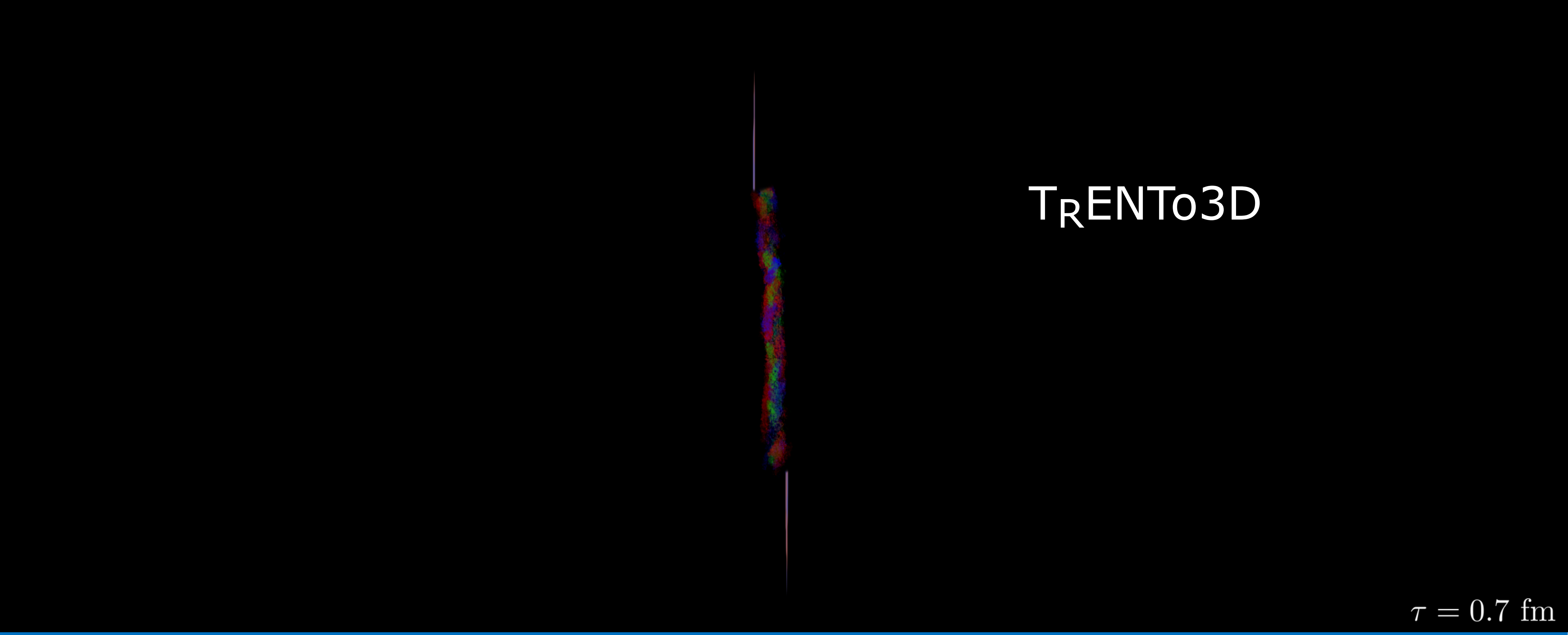
Initial Geometry of the Event

Two nuclei before collision



Medium Initial State

3D initial entropy density at the equilibrium time



Medium Initial State

TRENTo3D is a non-dynamical initial state model

TRENTo Initial State

Participant thickness function:

[arXiv:1412.4708, arXiv:1610.08490]

$$T_A(x,y) \equiv \int \mathrm{d}z \rho_A^{\mathrm{part}}(x,y,z)$$

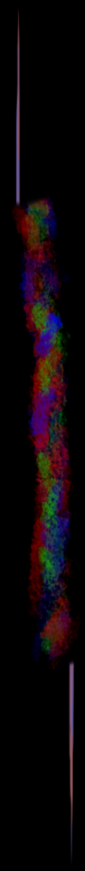
Reduced thickness function (proportional to entropy density):

$$T_{\mathrm{R}}\left(p; T_A, T_B\right) \equiv \left(\frac{T_A^p + T_B^p}{2}\right)^{1/p}$$

$$T_{\mathrm{R}}\left(p; T_A, T_B\right) \propto \left.\frac{\mathrm{d}s}{\mathrm{d}\eta_s}\right|_{\substack{\tau=\tau_0 \\ \eta_s=0}}$$

Prescription preferred by multiple Bayesian analyses:

$$T_{\mathrm{R}}\left(0; T_A, T_B\right) = \sqrt{T_A T_B}$$

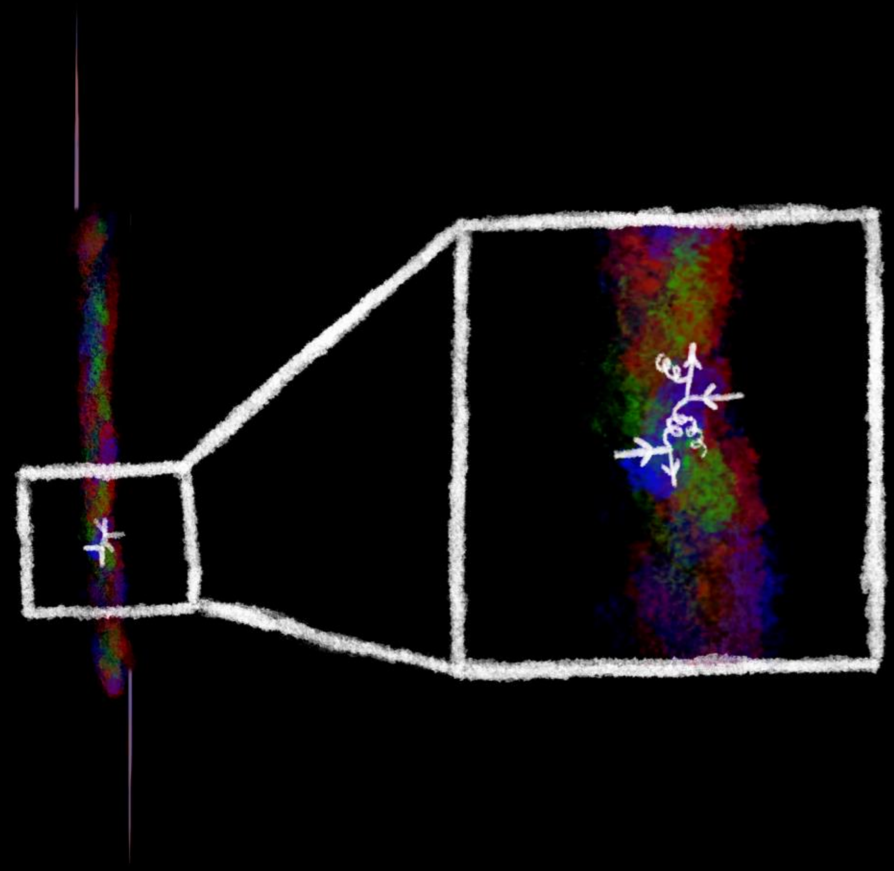


TRENTo3D

$\tau = 0.7\ \mathrm{fm}$

Medium Initial State

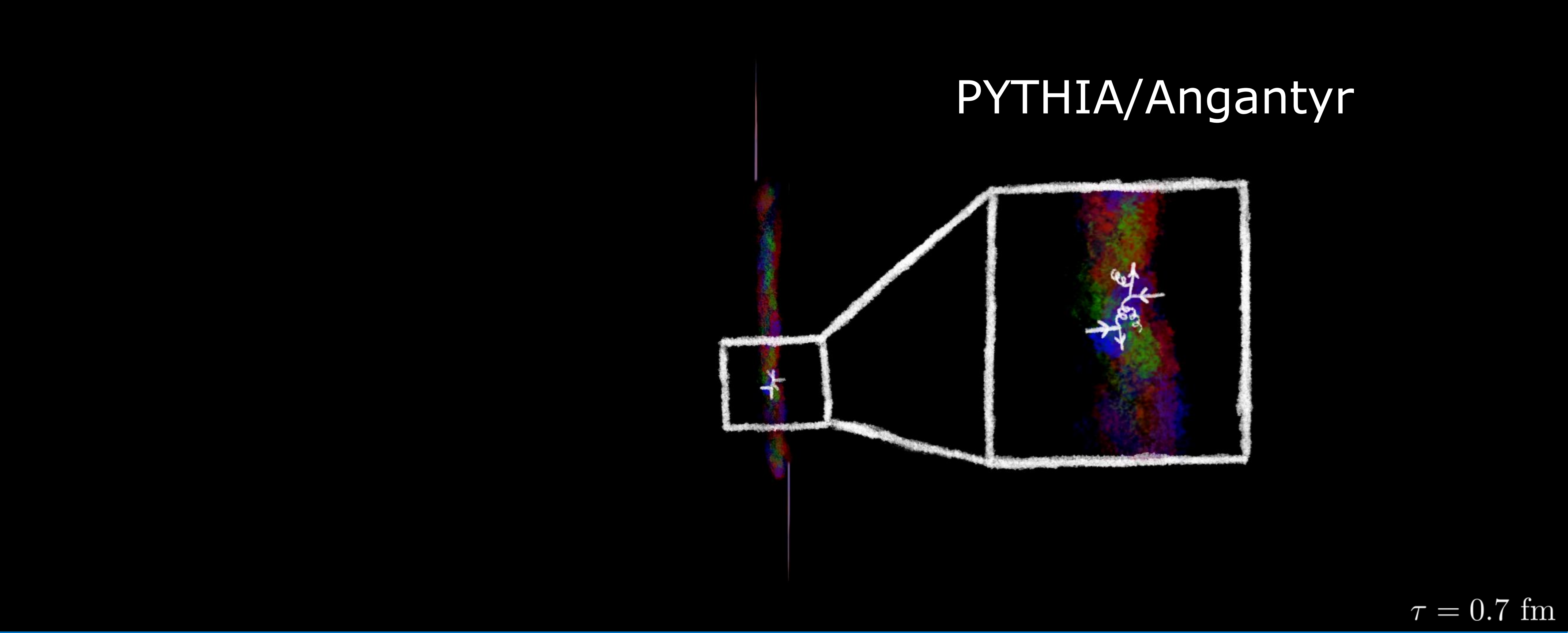
TRENTo is a non-dynamical initial state model



$\tau = 0.7 \text{ fm}$

Hard Parton Initial State

Hard scattering at the beginning of the collision



Hard Parton Initial State

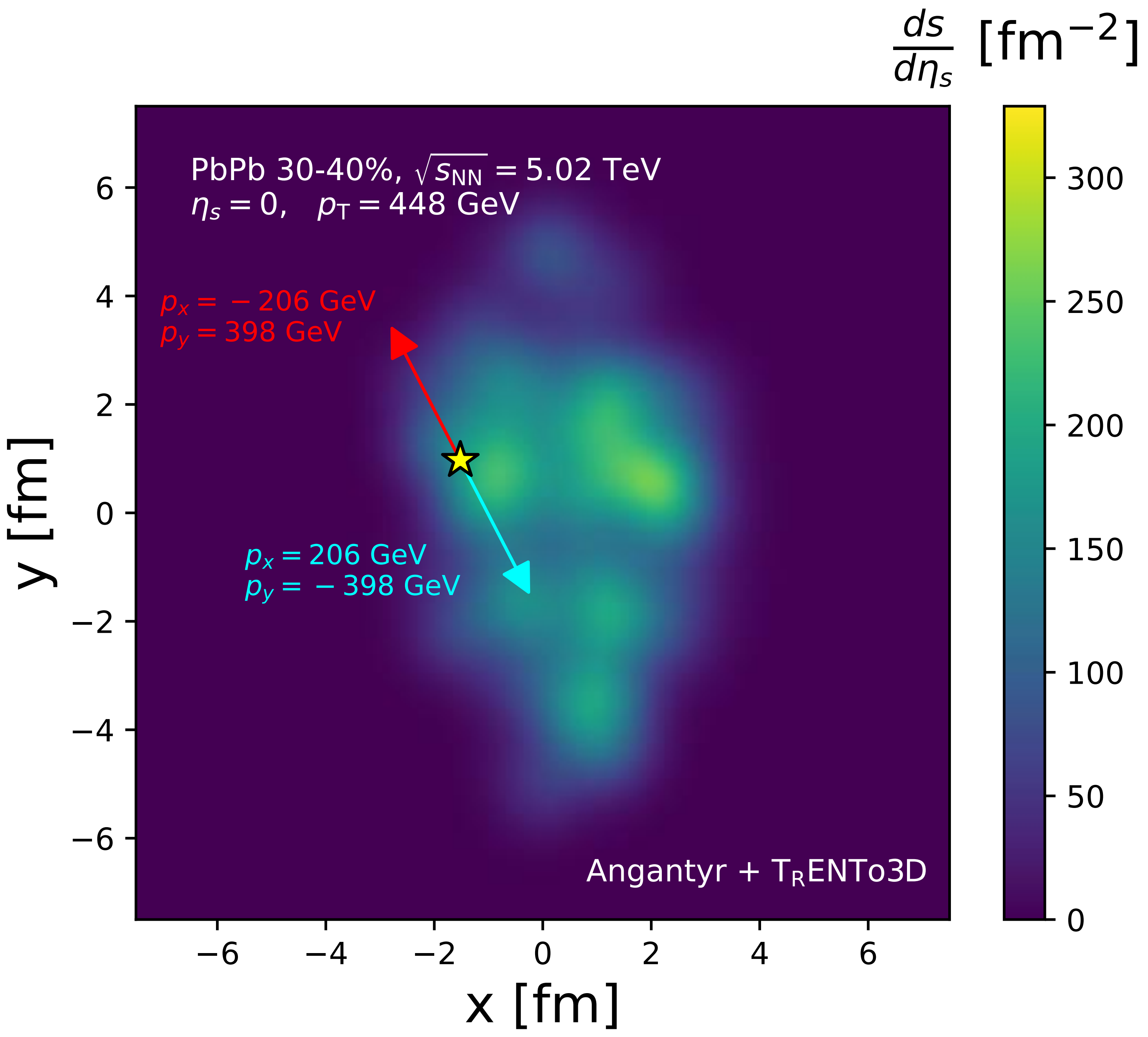
PYTHIA is p_{T}-ordered Monte Carlo framework

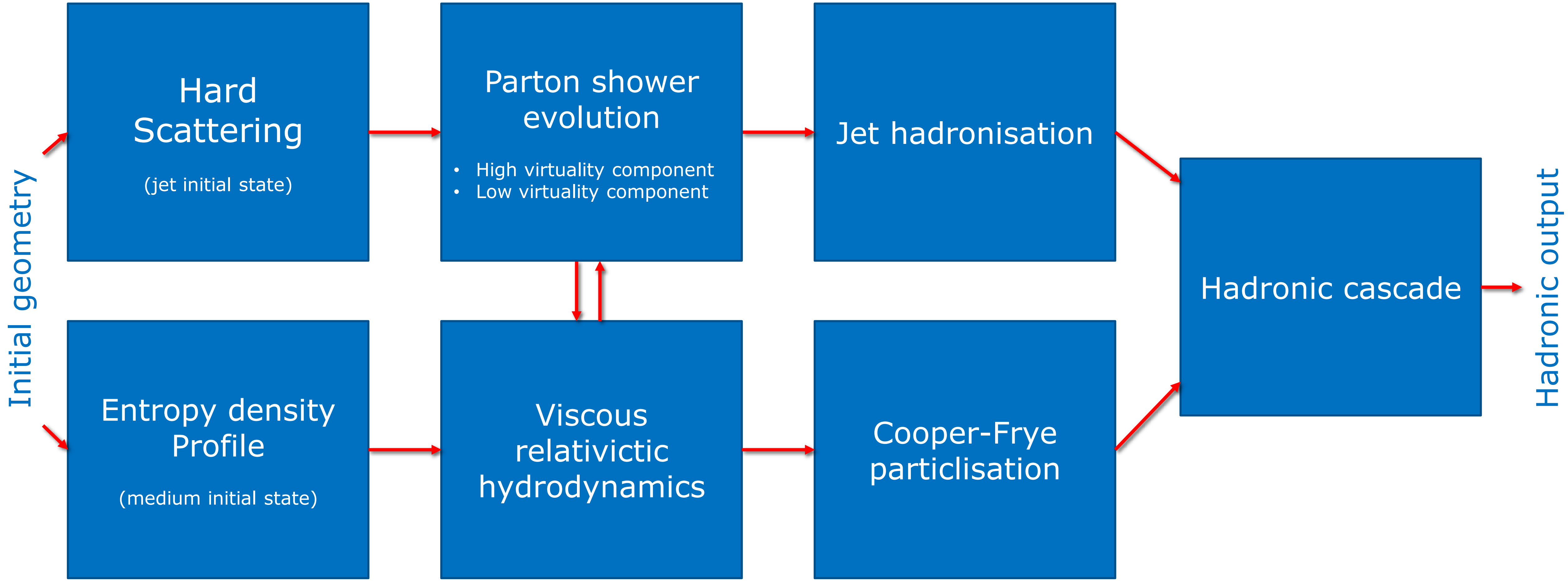
Angantyr is a model for heavy-ion collisions as extrapolation of pp collisions (PYTHIA)

Initial state

T$_R$ENTo initial entropy density

Angantyr initial hard partons

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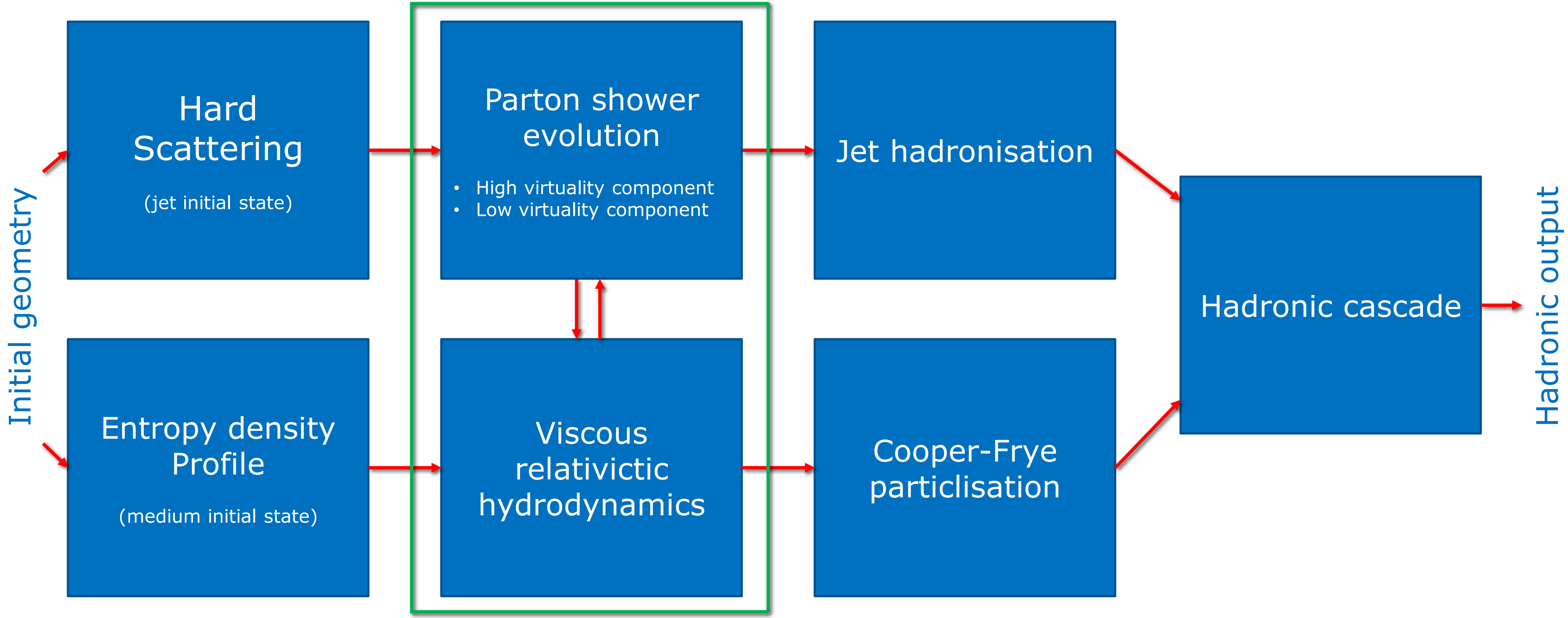
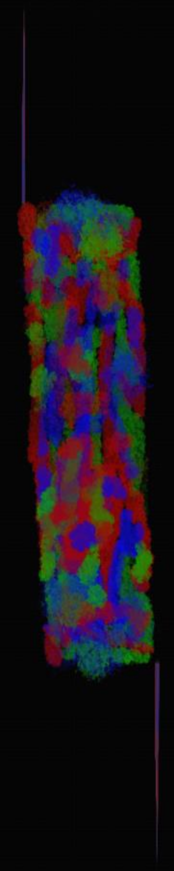
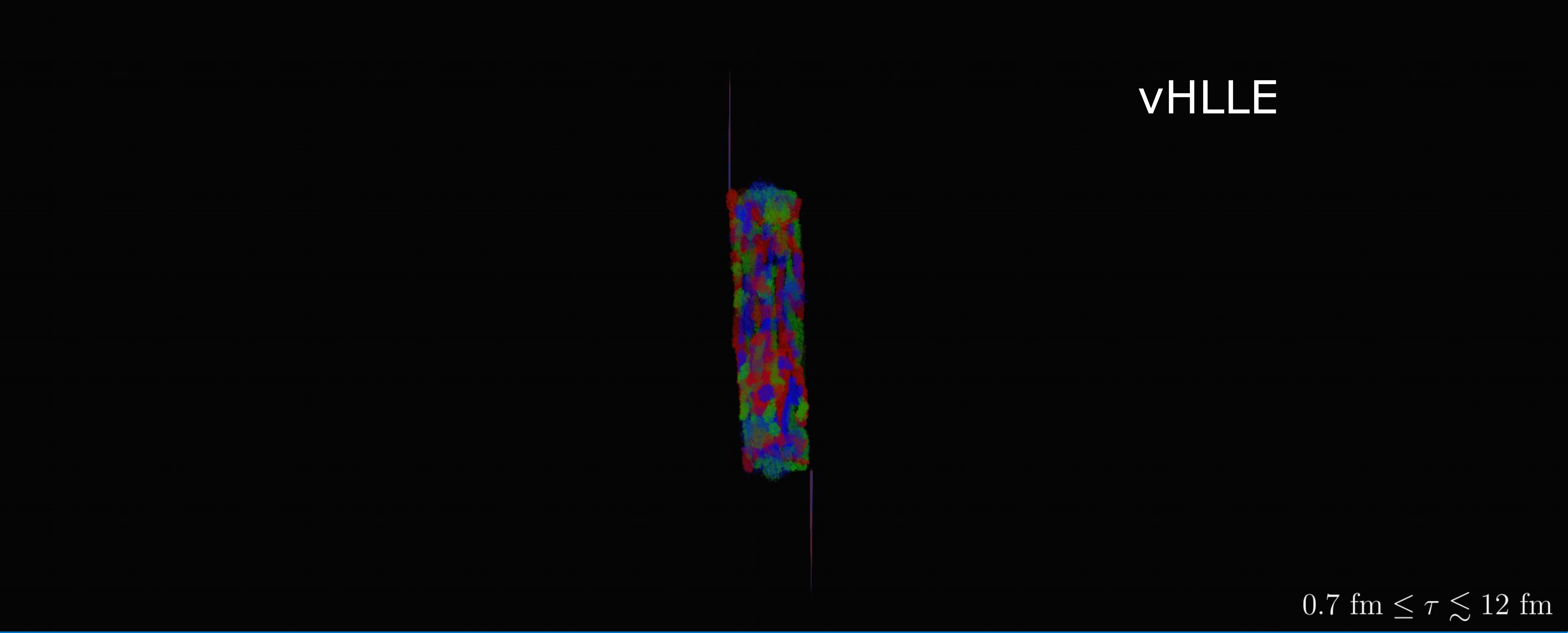
Medium and Jet Evolution





Medium Evolution

Second-order hydrodynamics with temperature-dependant bulk and shear viscosity



Medium Evolution

vHLLE is based on Israel-Steward formalism

Second-Order Hydrodynamics

Israel-Stewart equations in the 14-momentum approximation:

[arXiv:1312.4160]

① $$u^\mu \partial_\mu \Pi = \frac{-\zeta \partial_\mu u^\mu - \Pi}{\tau_\Pi} - \frac{\delta_{\Pi\Pi}}{\tau_\Pi} \Pi \partial_\mu u^\mu + \frac{\lambda_{\Pi\pi}}{\tau_\Pi} \pi^{\mu\nu} \sigma_{\mu\nu}$$

② $$u^\alpha \partial_\alpha \pi^{\langle\mu\nu\rangle} = \frac{2\eta\sigma^{\mu\nu} - \pi^{\mu\nu}}{\tau_\pi} - \frac{\delta_{\pi\pi}}{\tau_\pi} \pi^{\mu\nu} \partial_\mu u^\mu + \frac{\phi_7}{\tau_\pi} \pi_\alpha^{\langle\mu} \pi^{\nu\rangle\alpha} - \frac{\tau_{\pi\pi}}{\tau_\pi} \pi_\alpha^{\langle\mu} \sigma^{\nu\rangle\alpha} + \frac{\lambda_{\pi\Pi}}{\tau_\pi} \Pi \sigma^{\mu\nu}$$

$$\sigma^{\mu\nu} \equiv \eta \partial^{\langle\mu} u^{\nu\rangle} = \eta \left[\frac{1}{2} \left(\Delta^{\alpha\mu} \Delta^{\beta\nu} + \Delta^{\beta\mu} \Delta^{\alpha\nu} \right) - \frac{1}{3} \Delta^{\mu\nu} \Delta^{\alpha\beta} \right] \partial_\alpha u_\beta$$

Transport coefficients: η, ζ

$$\frac{\delta_{\Pi\Pi}}{\tau_\Pi} = \frac{2}{3}, \quad \frac{\lambda_{\Pi\pi}}{\tau_\Pi} = \frac{8}{5}\left(\frac{1}{3} - c_s^2\right), \quad \frac{\delta_{\pi\pi}}{\tau_\pi} = \frac{4}{3}, \quad \phi_7 = \frac{9}{70p}, \quad \frac{\tau_{\pi\pi}}{\tau_\pi} = \frac{10}{7}, \quad \frac{\lambda_{\pi\Pi}}{\tau_\pi} = \frac{6}{5}$$

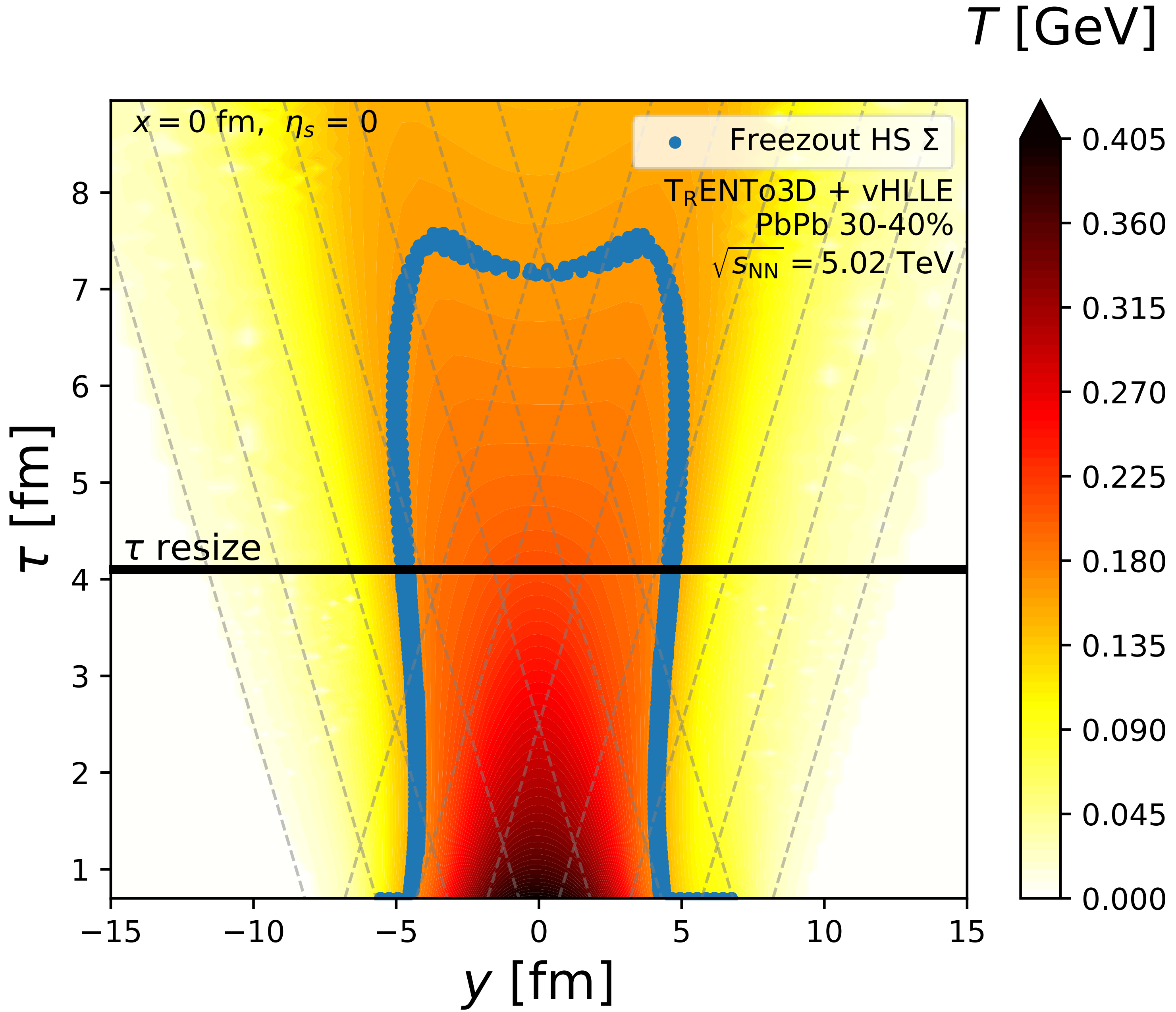
Relaxation times:

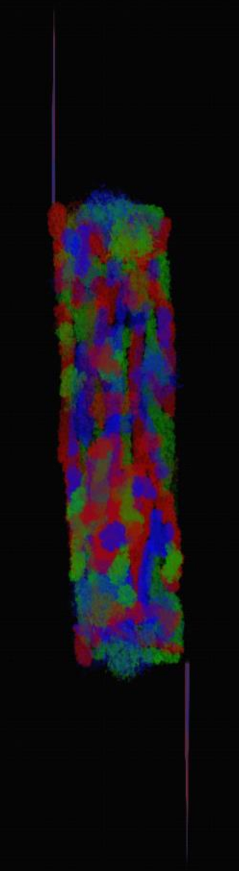
$$\tau_\pi = \frac{5\eta}{sT}, \quad \tau_\Pi = \frac{\zeta}{15\left(\frac{1}{3} - c_s^2\right)^2 sT}$$

Temperature evolution in the medium

T_RENTo initial entropy density

vHLLE hydrodynamics





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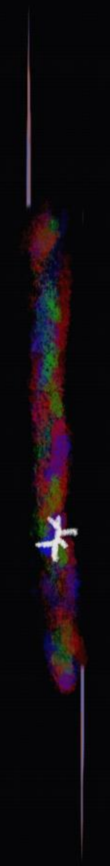


$0.7\ \mathrm{fm} \leq \tau \lesssim 12\ \mathrm{fm}$

Parton Shower Evolution Inside the Medium

Parton cascade evolves inside the medium

Parton Shower Evolution Inside the Medium

SUBA-Jet is recently developed parton shower with coherent gluon radiation

SUBA-Jet

High-virtuality evolution in a vacuum is governed by the DGLAP equations:

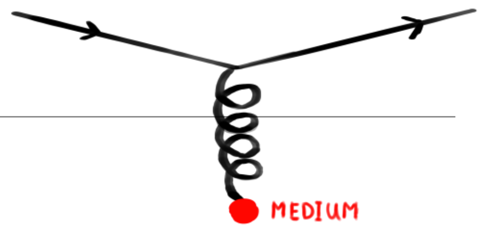
$$S_a\left(Q_{a\uparrow}, Q_a\right) = \exp\left(-\sum_{a\to b,c}\int_{Q_a^2}^{Q_{a\uparrow}^2}\frac{\mathrm{d}Q^2}{Q^2}\int_{x^-}^{x^+}\mathrm{d}x\,\frac{\alpha_s\left(x(1-x)Q^2\right)}{2\pi}P_{a\to b,c}(x)\right)$$

A local, continuous increase in virtuality modifies high-virtuality evolution:

$$\frac{\mathrm{d}Q^2}{\mathrm{d}t} = \hat{q}(T,p)$$

[arXiv:2404.14579]

SUBA-Jet



Low-virtuality component involves both **elastic collision**:

$$\frac{\mathrm{d}^2\Gamma^{\mathrm{q}}_{\mathrm{el}}}{\mathrm{d}^2 q_{\mathrm{T}}} = n_{\mathrm{q}}(T)\frac{\mathrm{d}^2\sigma^{\mathrm{qq}}_{\mathrm{el}}}{\mathrm{d}^2 q_{\mathrm{T}}} + n_{\mathrm{g}}(T)\frac{\mathrm{d}^2\sigma^{\mathrm{qg}}_{\mathrm{el}}}{\mathrm{d}^2 q_{\mathrm{T}}} \qquad \frac{\mathrm{d}^2\sigma^{\mathrm{qq}}_{\mathrm{el}}}{\mathrm{d}^2 q_{\mathrm{T}}} = \frac{2C_{\mathrm{F}}}{N_{\mathrm{c}}}\frac{\alpha_s^2}{\left(q_{\mathrm{T}}^2+\mu^2\right)^2} \qquad \frac{\mathrm{d}^2\sigma^{\mathrm{qg}}_{\mathrm{el}}}{\mathrm{d}^2 q_{\mathrm{T}}} = \frac{2C_{\mathrm{A}}}{N_{\mathrm{c}}}\frac{\alpha_s^2}{\left(q_{\mathrm{T}}^2+\mu^2\right)^2}$$

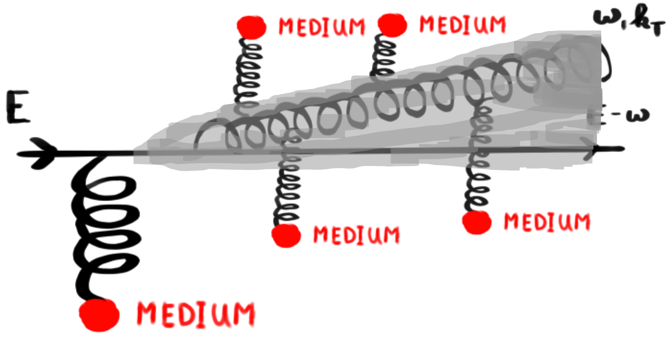
Inelastic collisions in SUBA-Jet follow the Gunion-Bertsch cross-section

$$\frac{\mathrm{d}^5\sigma_{\mathrm{rad}}}{\left(\mathrm{d}x\ \mathrm{d}^2 l_\perp \mathrm{d}^2 k_\perp\right)} \sim \left|\mathcal{M}_{\mathrm{el}}\right|^2 \times P_g \times \Theta$$

[arXiv:2404.14579]

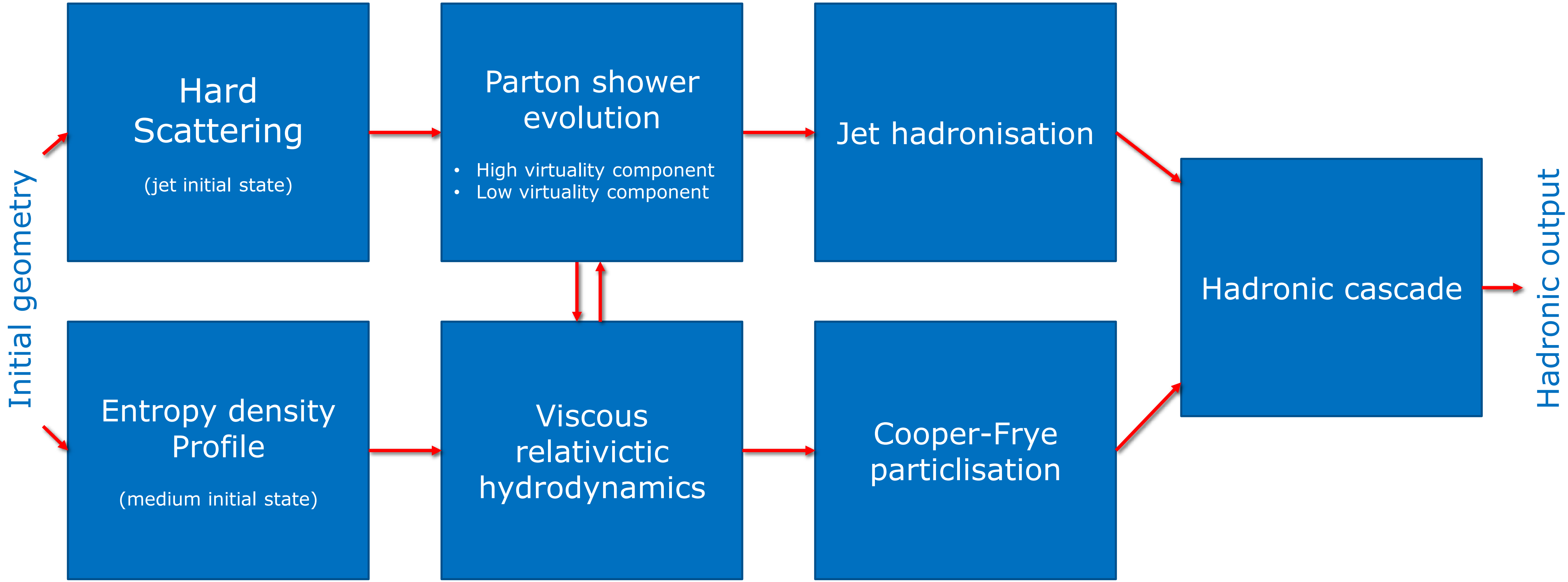
Coherent gluon radiation (QCD analog to LPM)

- Formation time of trial-radiated gluon



Parton Shower Evolution Inside the Medium

SUBA-Jet is newly developed parton shower with coherent gluon radiation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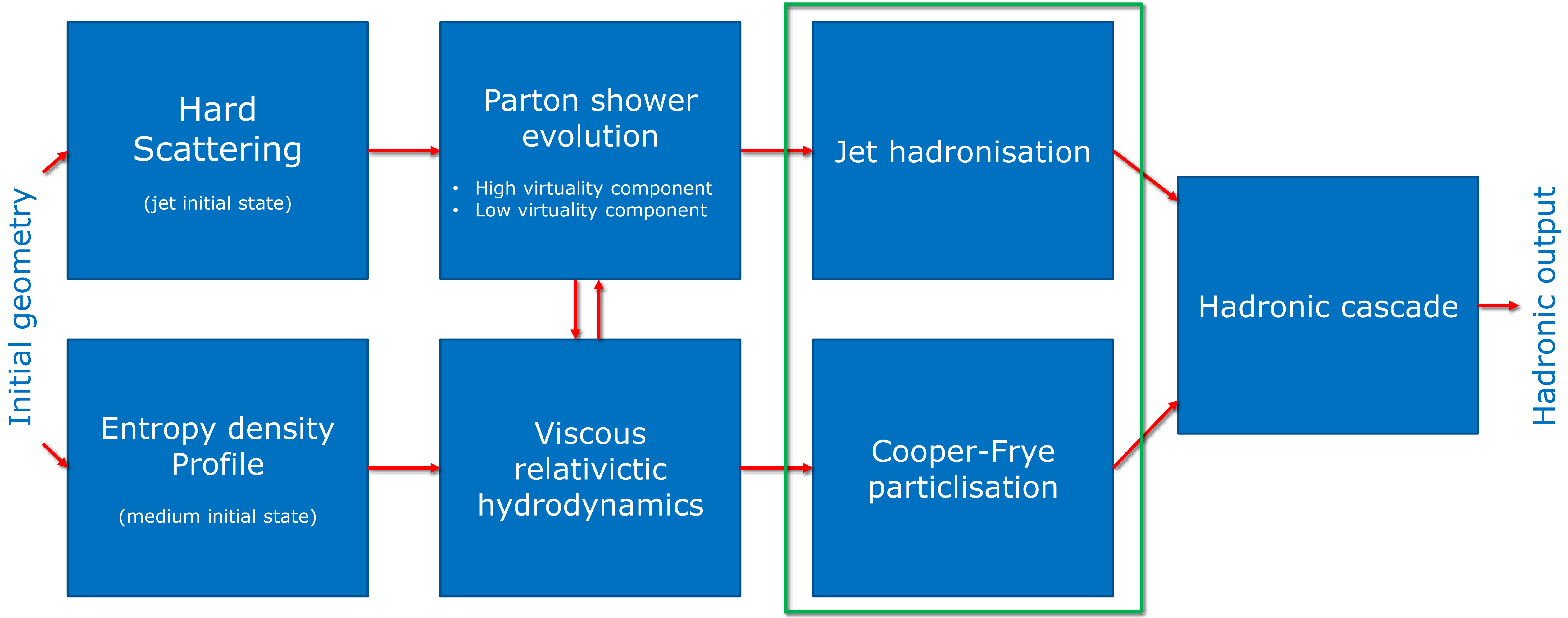
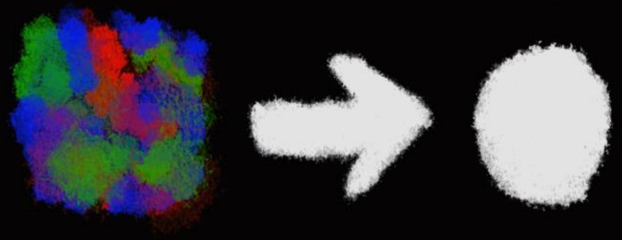
Hadronisation and Particlisation

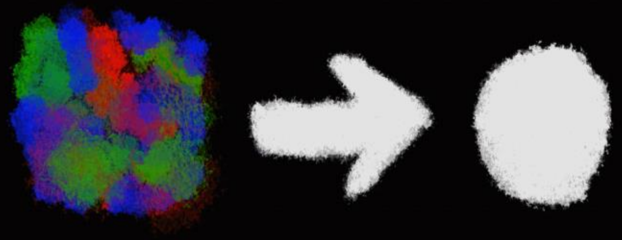




Particlisation

Transition from fluid to hadronic degrees of freedom happens at freeze-out hypersurface isotherm

SMASH-hadron-sampler



$0.7\ \mathrm{fm} \leq \tau \lesssim 12\ \mathrm{fm}$

Particlisation

SMASH-hadron-sampler provides particlisation according to the properties of the SMASH hadron resonance gas via grand-canonical ensemble

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SMASH-hadron-sampler

Partcilisation

Cooper-Frye formula

[arXiv:1502.01978, arXiv:2112.08724]

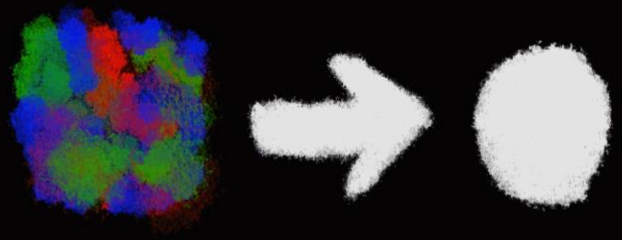
$$\frac{\mathrm{d}N^i}{p_\mathrm{T}\mathrm{d}y\mathrm{d}p_\mathrm{T}\mathrm{d}\phi_p} = \frac{g}{(2\pi)^3}\int_\Sigma \mathrm{d}^3\sigma_\mu p^\mu \left(f_0^i(x,p) + \delta f^i(x,p)\right)$$

only shear viscous corrections are considered

Isothermal freezout hypersurface Σ with the volume element

$$\mathrm{d}^3\sigma_\mu = \varepsilon_{\mu\upsilon\rho\sigma}\frac{\mathrm{d}\Sigma^\mu}{\mathrm{d}r_x}\frac{\mathrm{d}\Sigma^\rho}{\mathrm{d}r_y}\frac{\mathrm{d}\Sigma^\sigma}{\mathrm{d}\eta_s}\cdot\mathrm{d}r_x\mathrm{d}r_y\mathrm{d}\eta_s$$

SMASH-hadron-sampler



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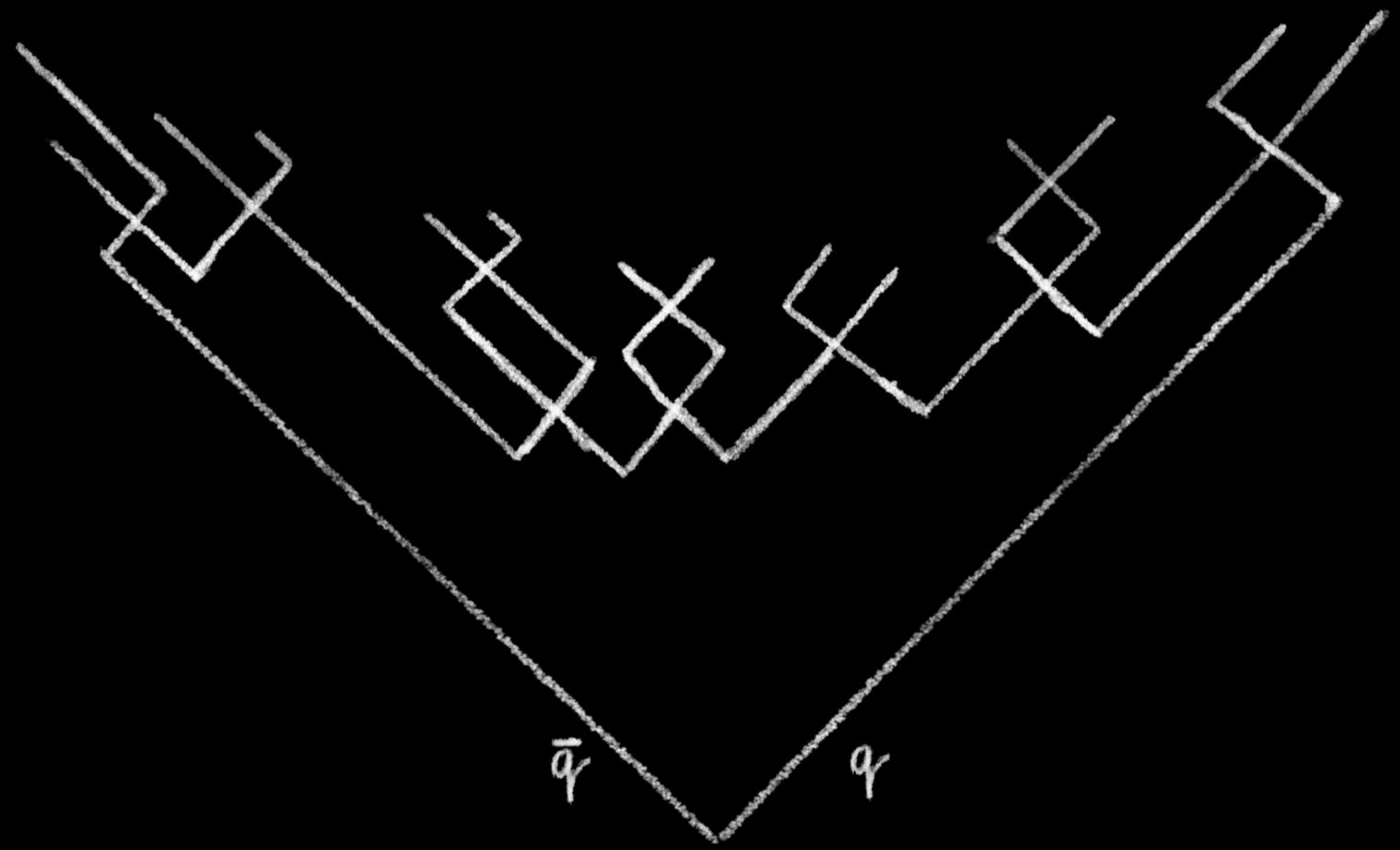
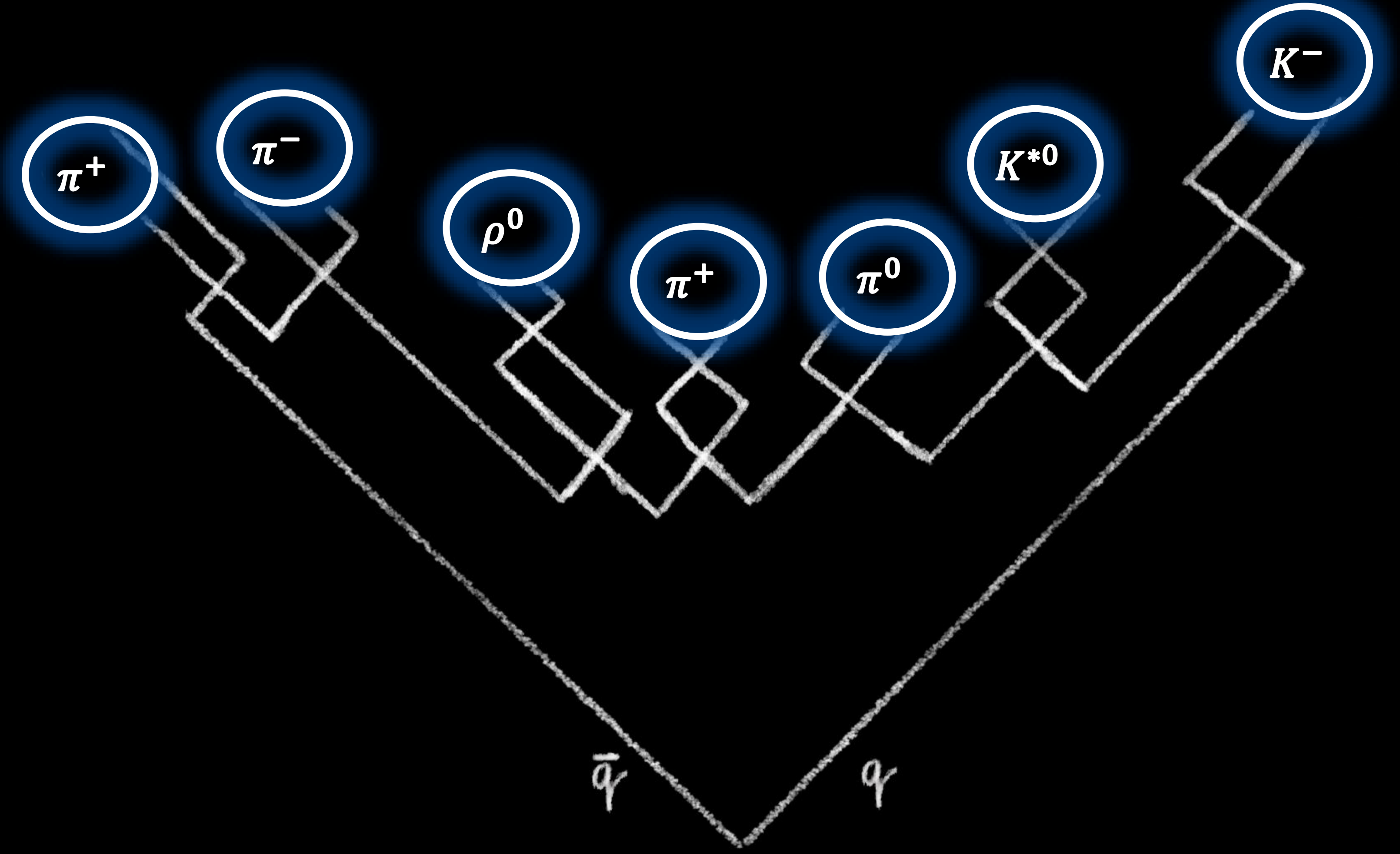


Parton Hadronisation

Lund string model with the space–time hadronisation structure



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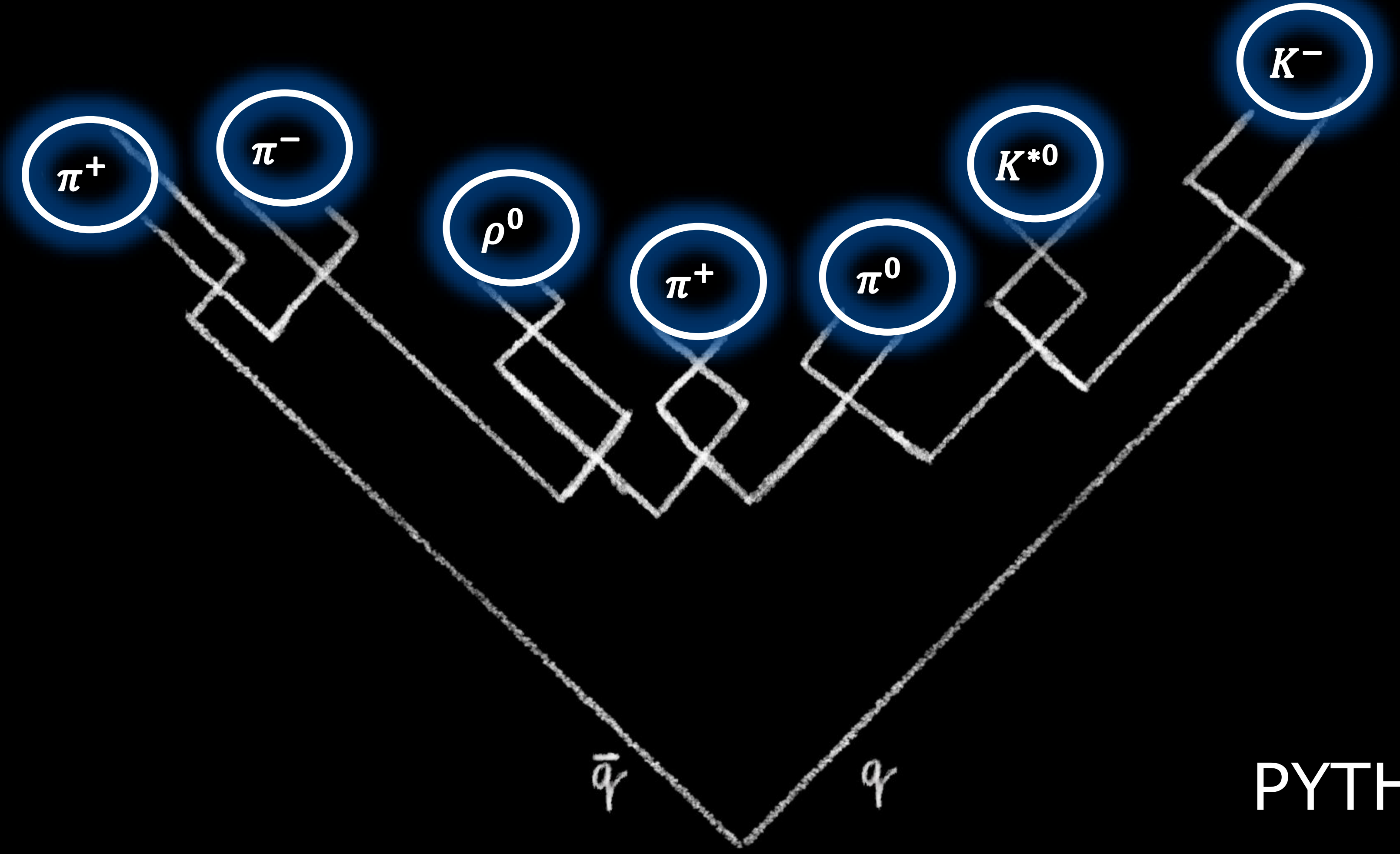
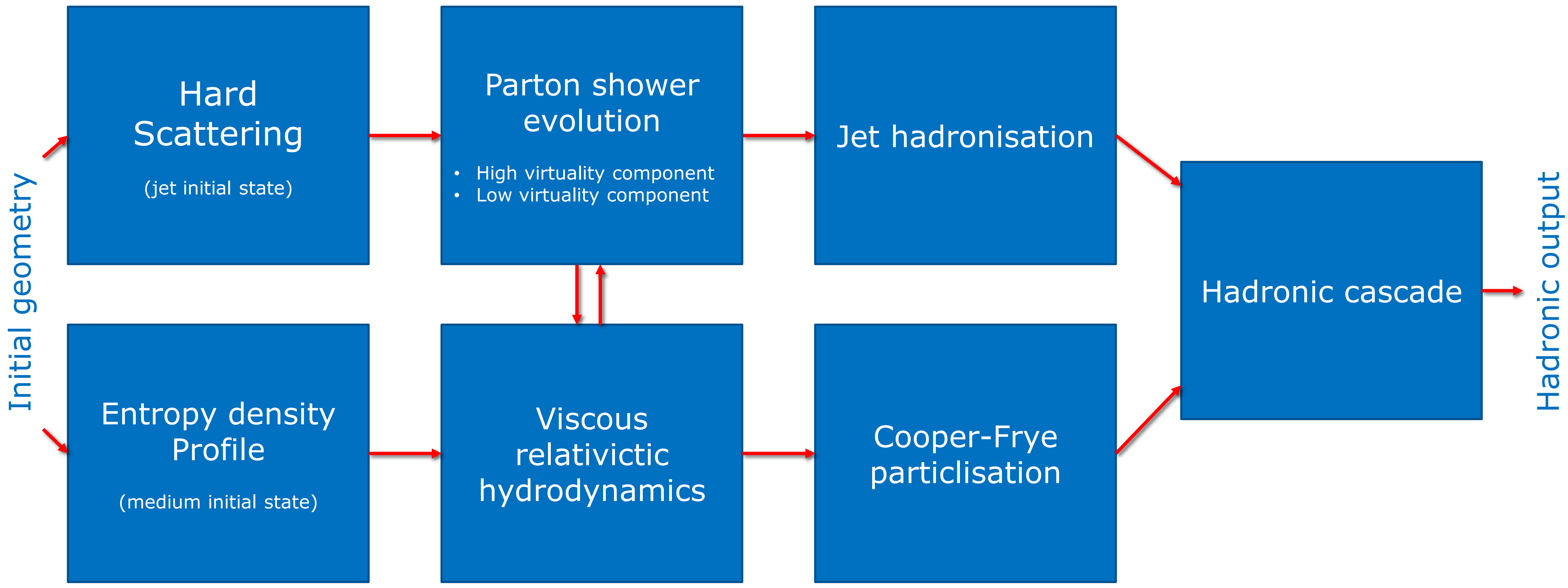


PYTHIA

Parton Hadronisation

PYTHIA can hadronise partons into hadrons with the Lund string model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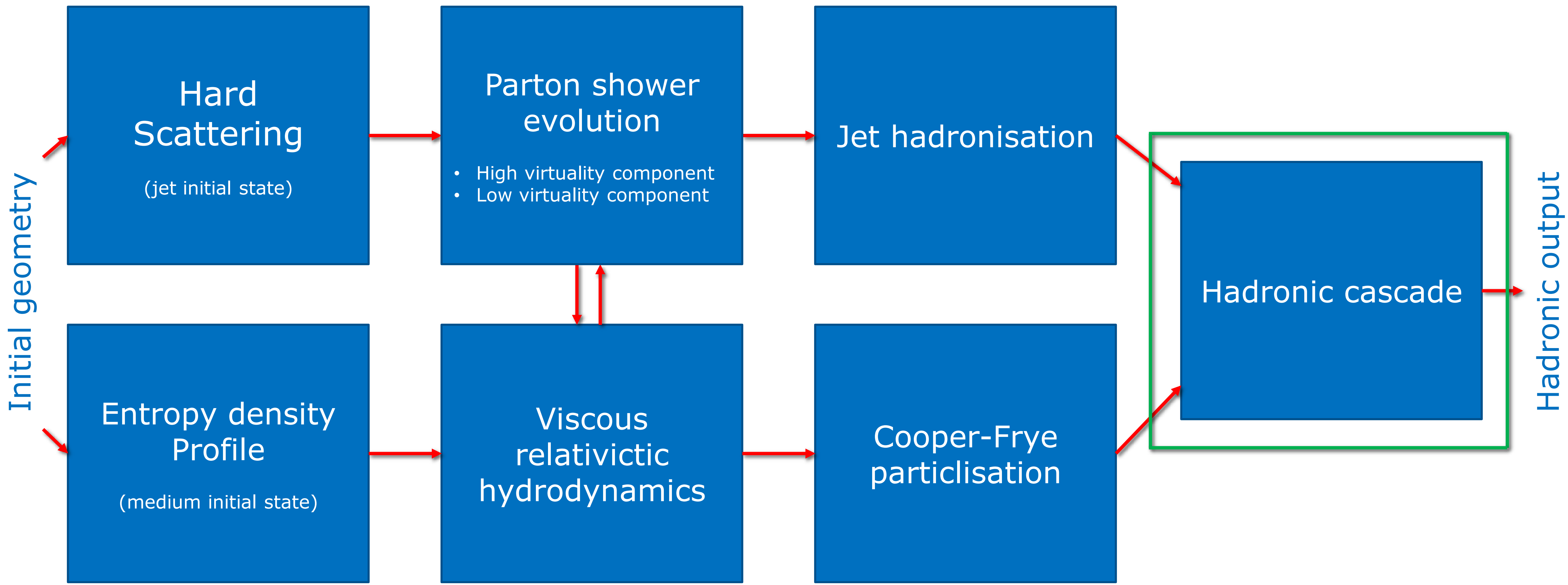
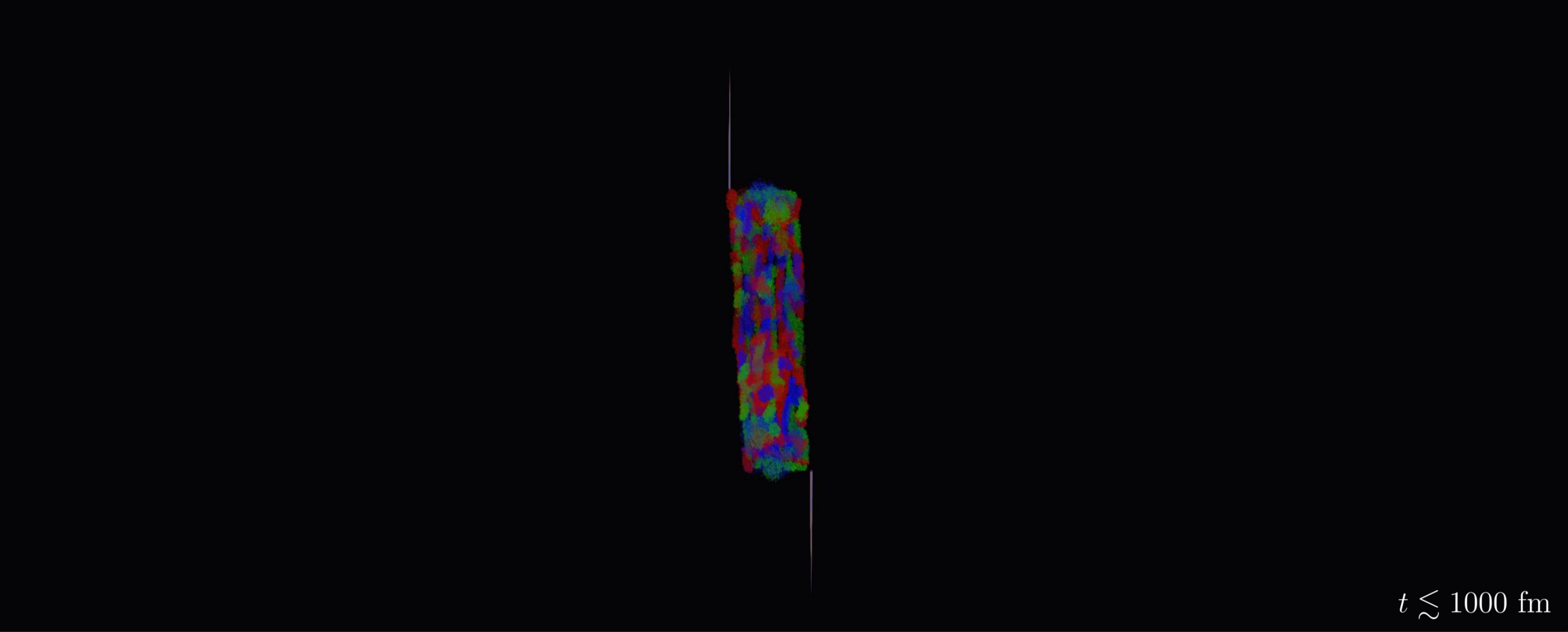
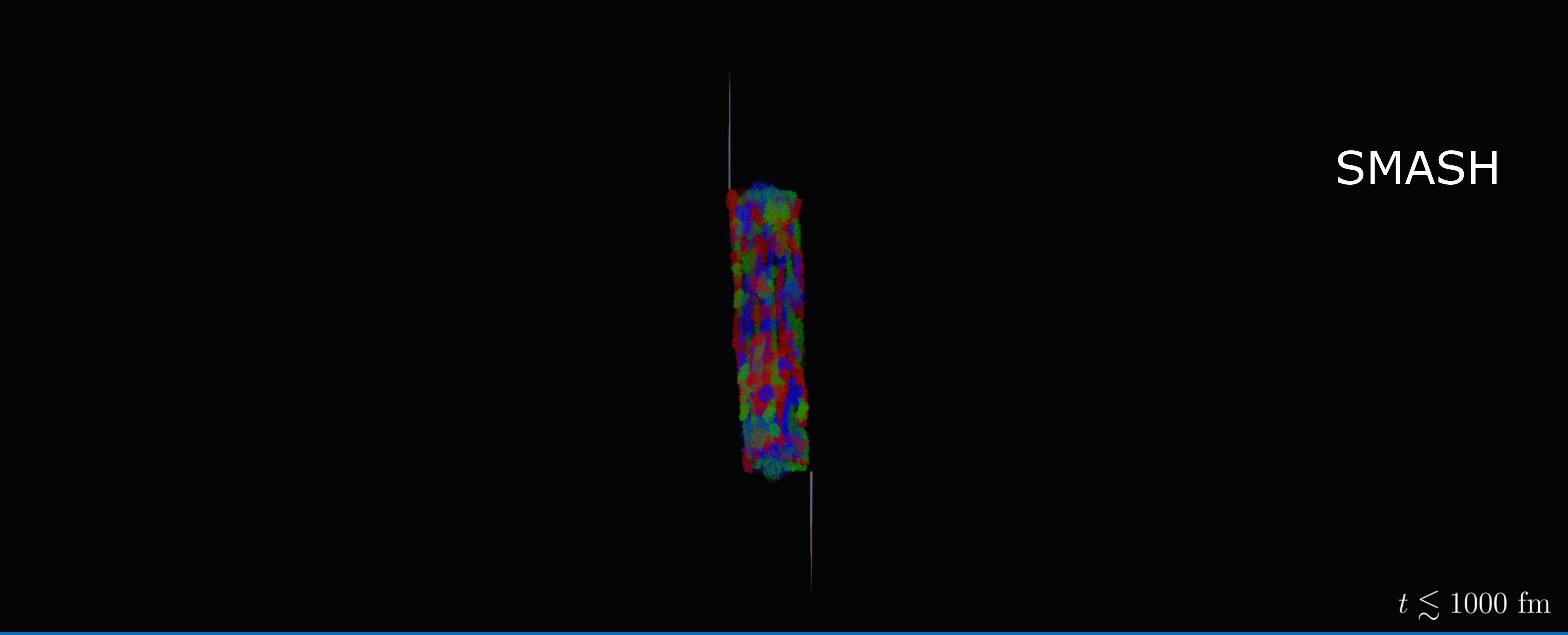
Afterburner





Hadronic Non-equilibrium Stage

Medium hadrons (from particlisation) and jet hadrons (from hadronisation) are evolved together in hadronic afterburner



Hadronic Non-equilibrium Stage

SMASH is a relativistic hadronic transport model designed to simulate non-equilibrium hadronic dynamics

SMASH

Boltzmann equation

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m} \cdot \nabla_{\vec{x}} f + \vec{F} \cdot \frac{\partial f}{\partial \vec{p}} = \mathcal{C}\left[f\right]$$

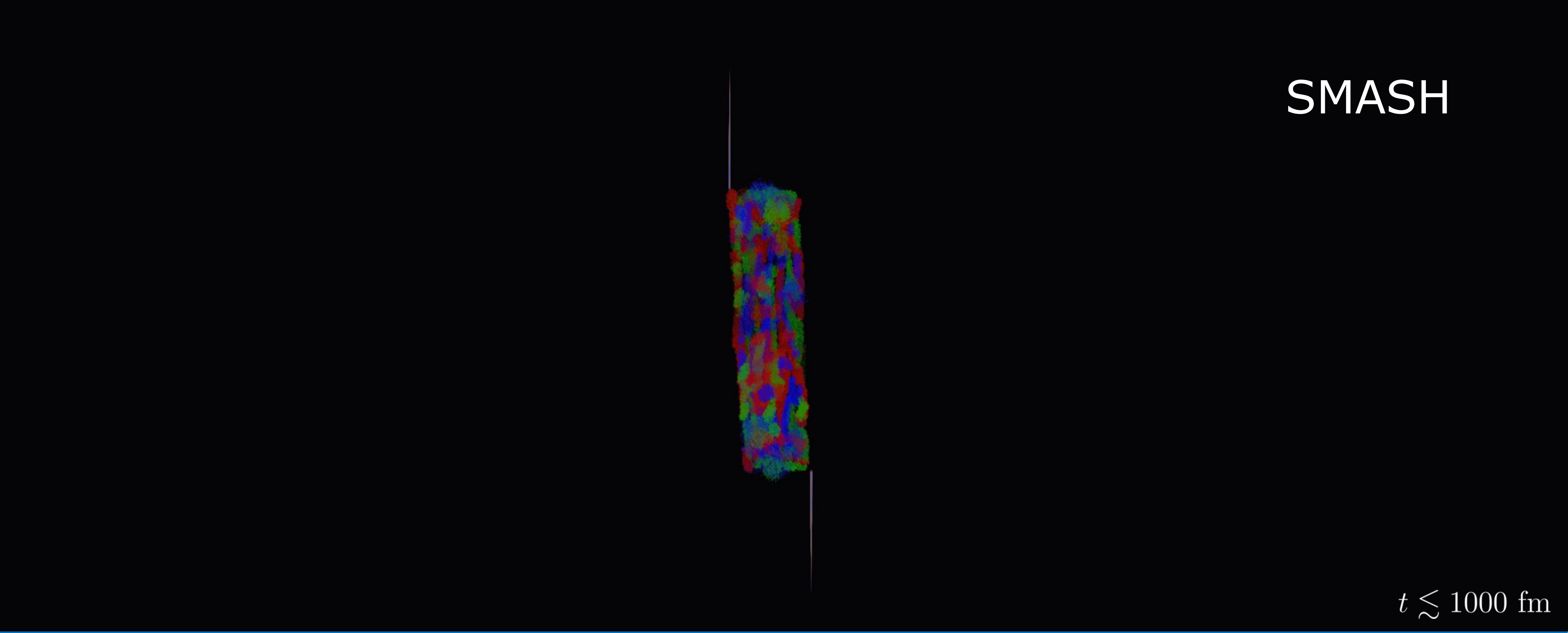
arXiv:1606.06642

Right side (collision kernel)

- Elastic $2 \to 2$
- Inelastic $2 \to 2$
- Inelastic $2 \to 1$
- Decay $1 \to 2$
- String (high-energy) $2 \to n$

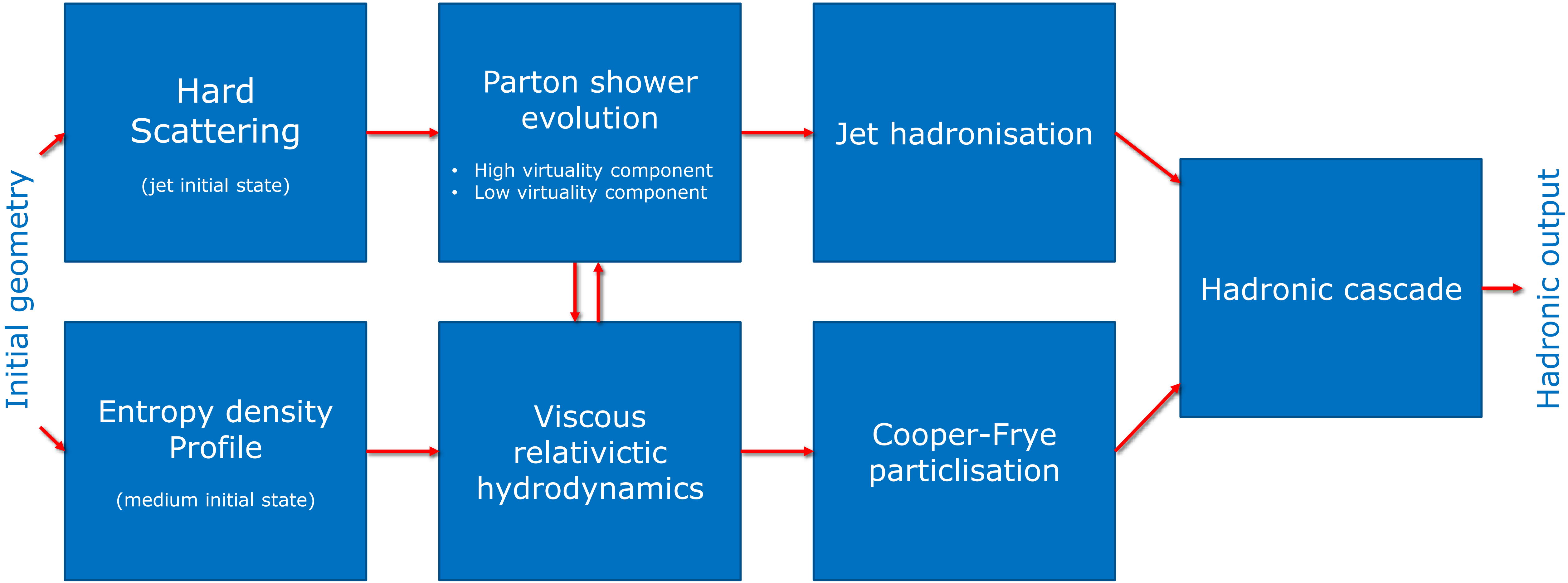
Test particle effective solution

$$N \mapsto N N_{\text{test}} \qquad \sigma \mapsto \sigma N_{\text{test}}^{-1}$$



Hadronic Afterburner

SMASH is a relativistic hadronic transport model designed to simulate non-equilibrium hadronic dynamics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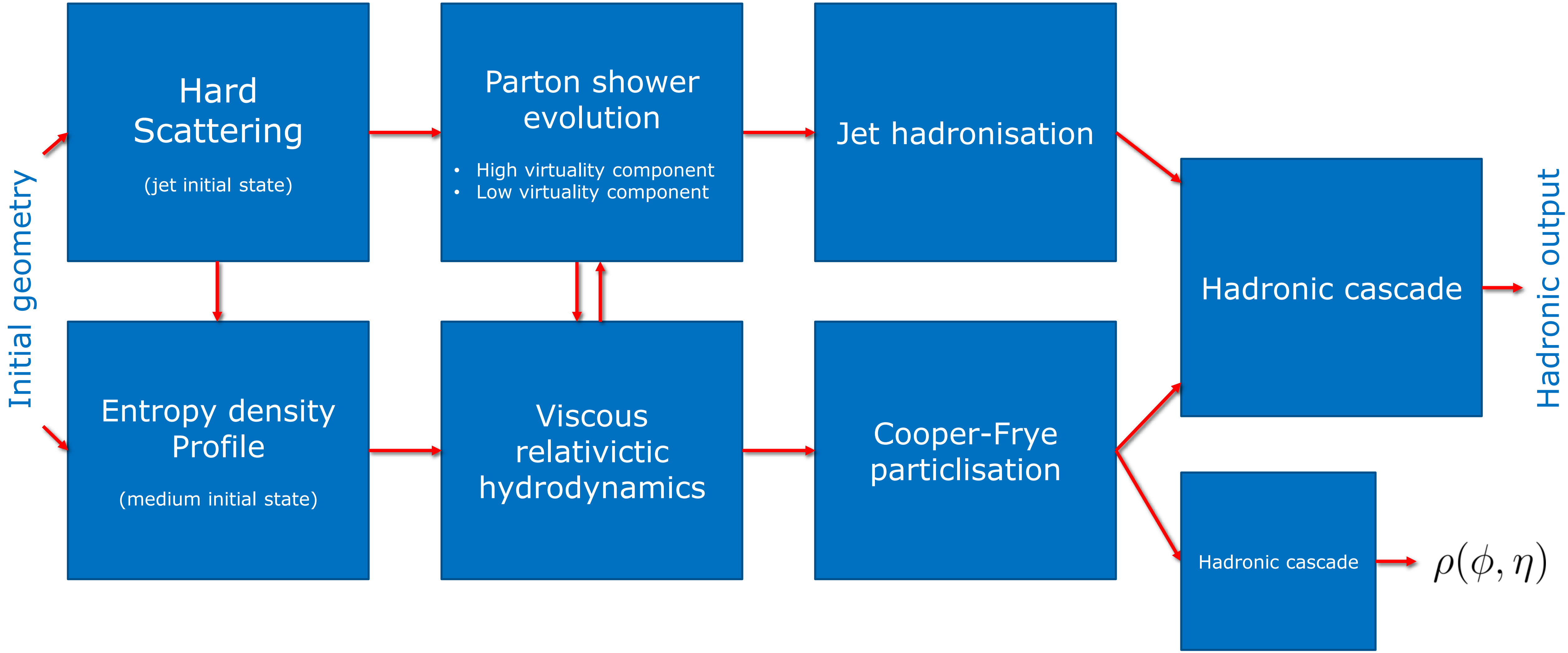
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$\rho(\phi, \eta)$

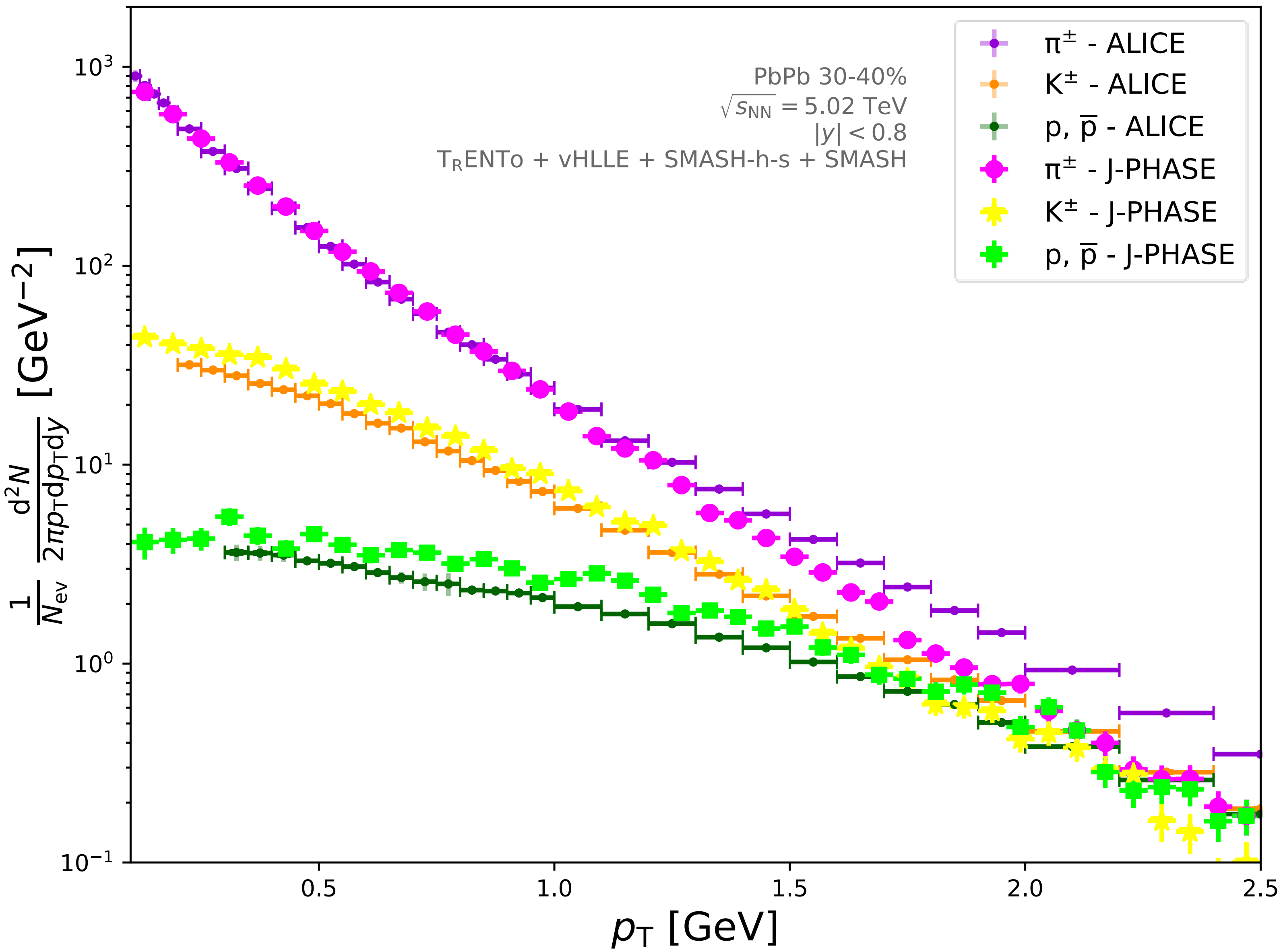
Transverse momenta spectra

T_RENTo initial entropy density

vHLLE hydrodynamics

SMASH-hadron-sampler particlisation

SMASH hadronic afterburner



Pseudorapidity distribution

T$_R$ENTo initial entropy density

vHLLE hydrodynamics

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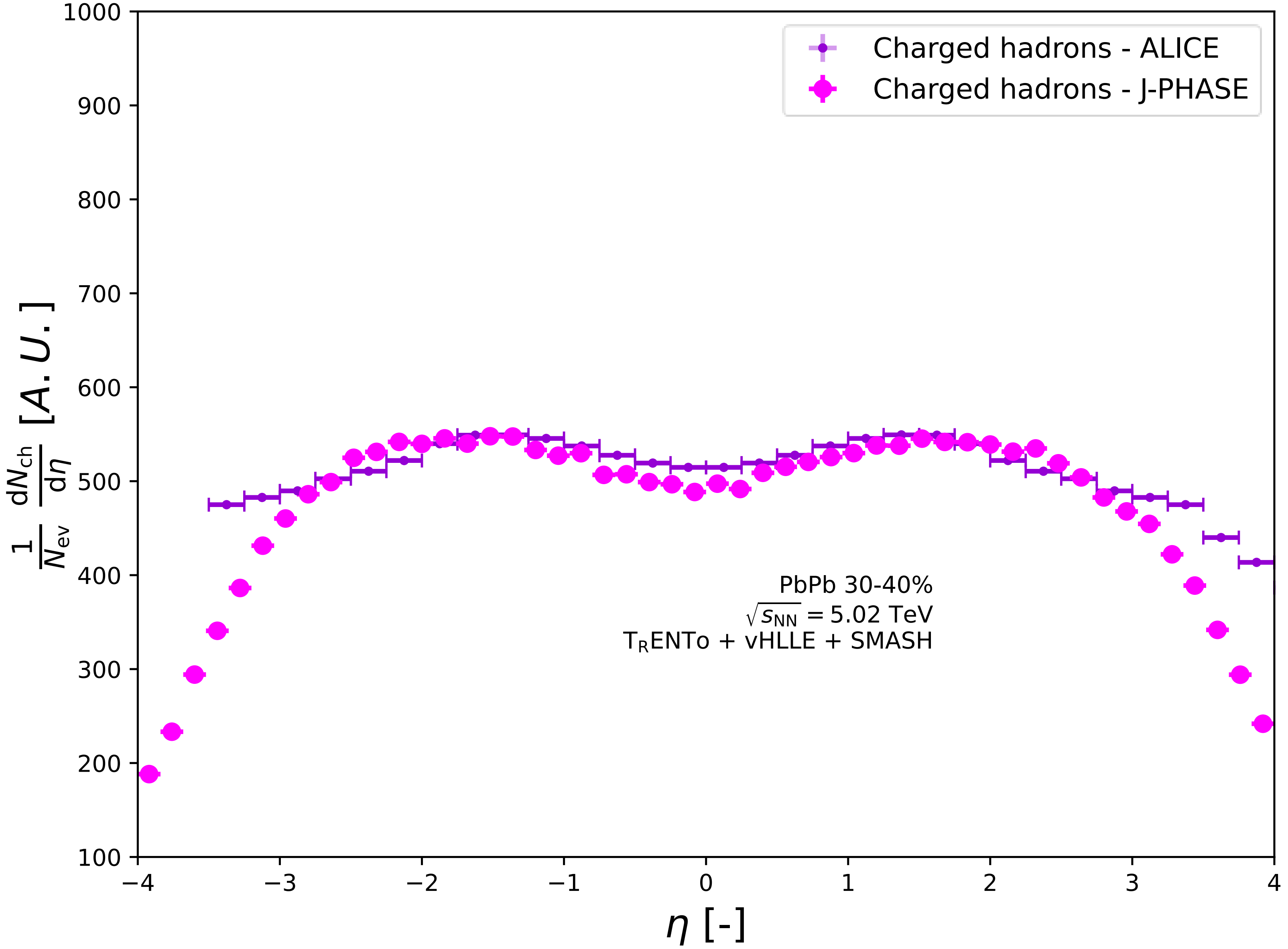
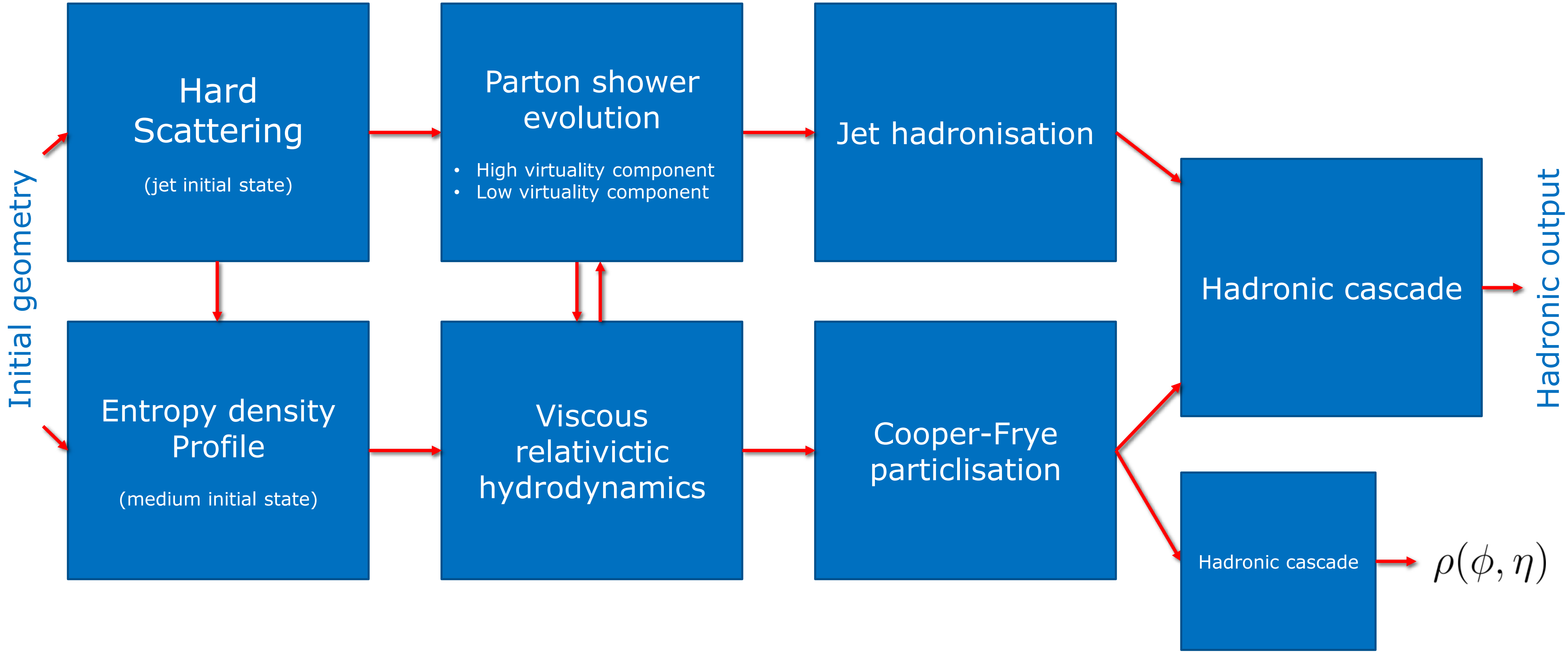
SMASH hadronic afterburner

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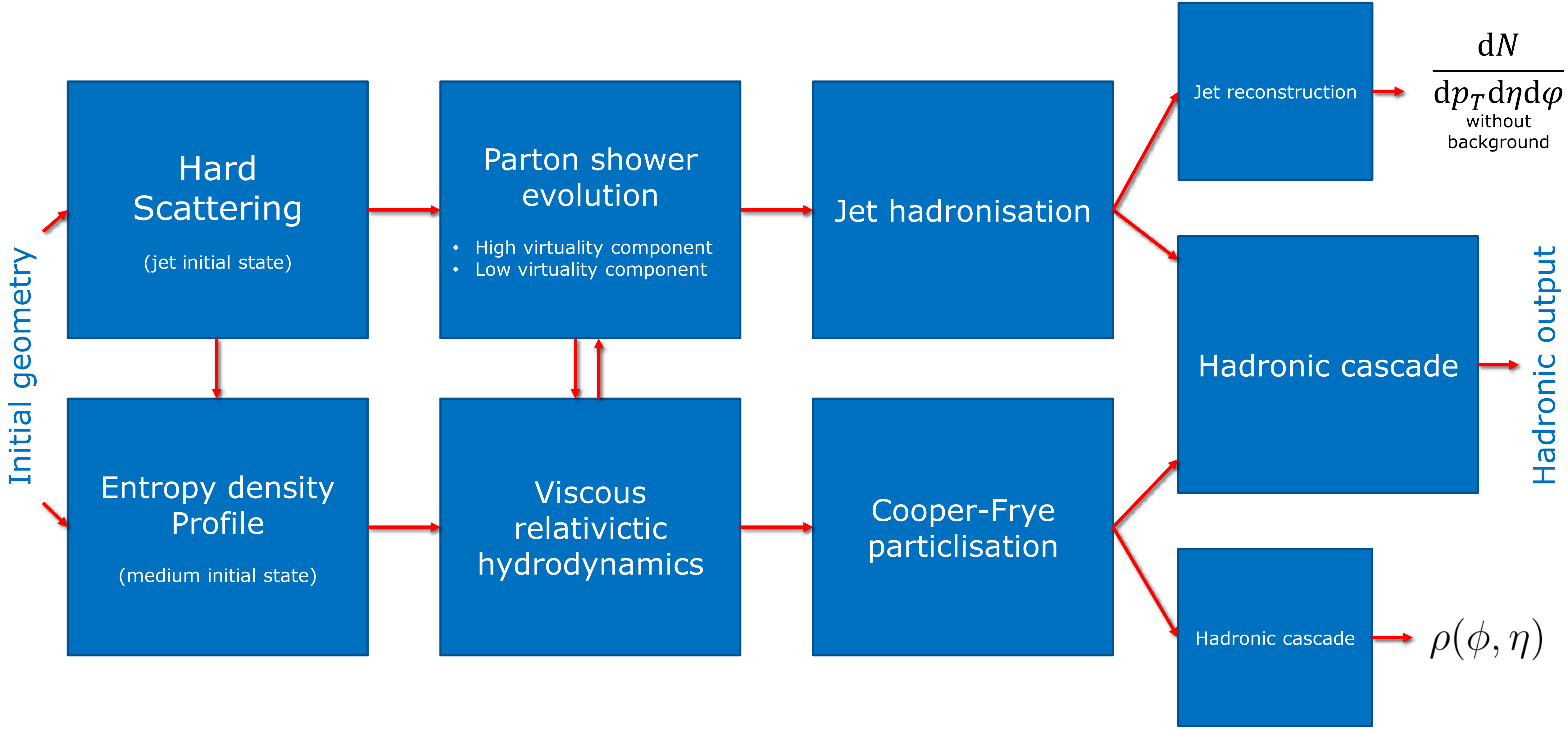
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Jet reconstruction
$\frac{\mathrm{d}N}{\mathrm{d}p_T \mathrm{d}\eta \mathrm{d}\varphi}$
without background
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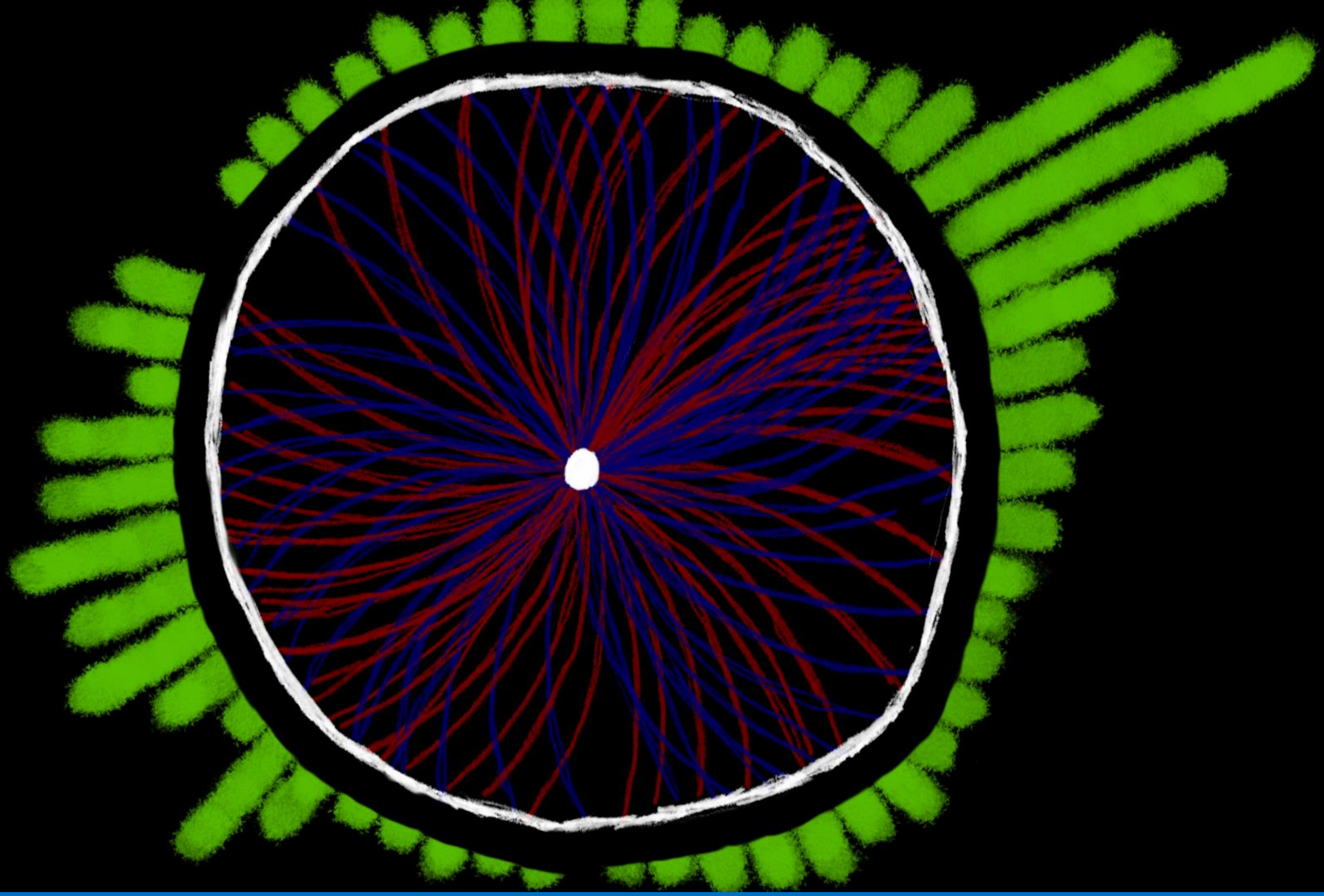
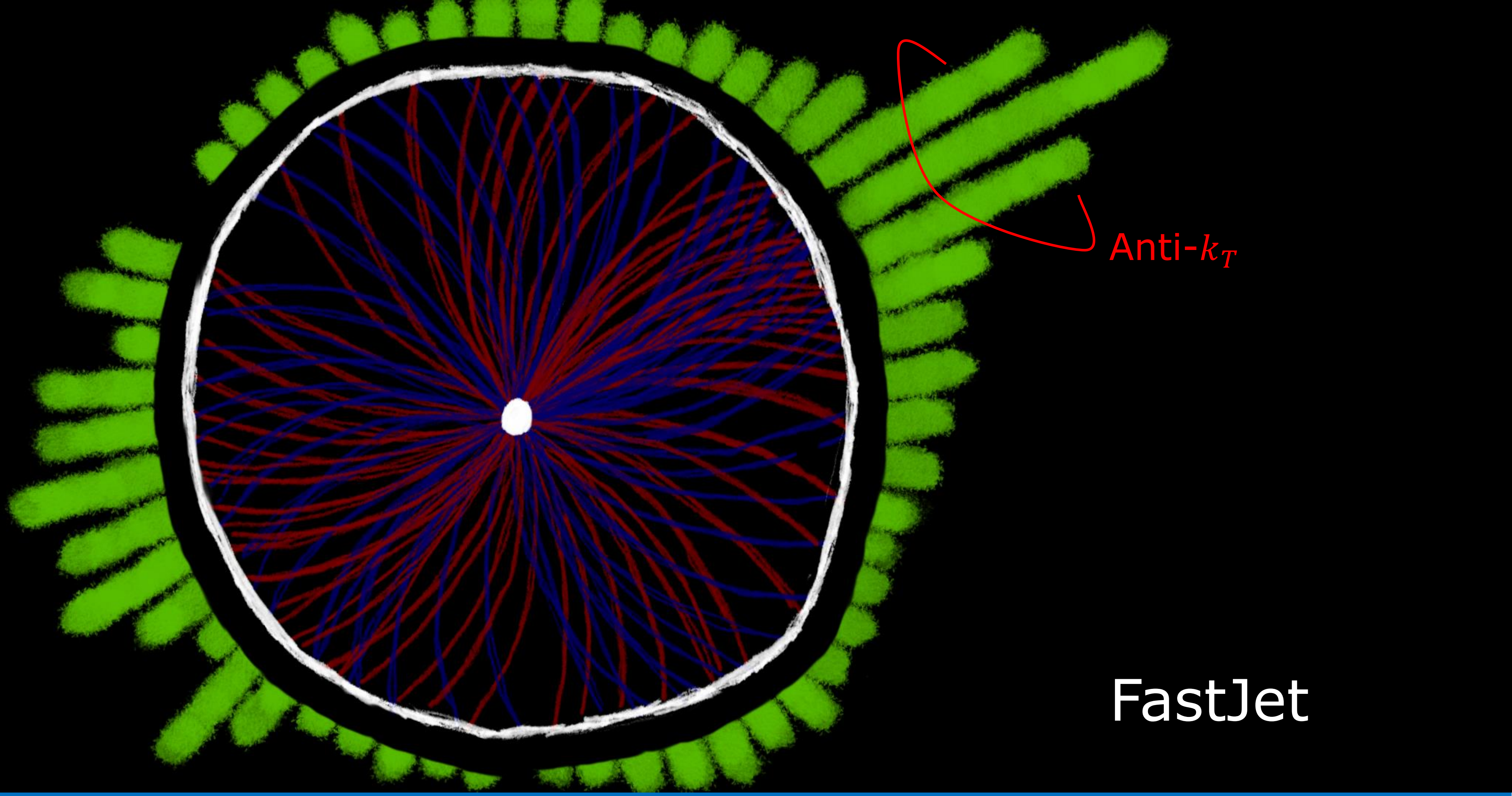


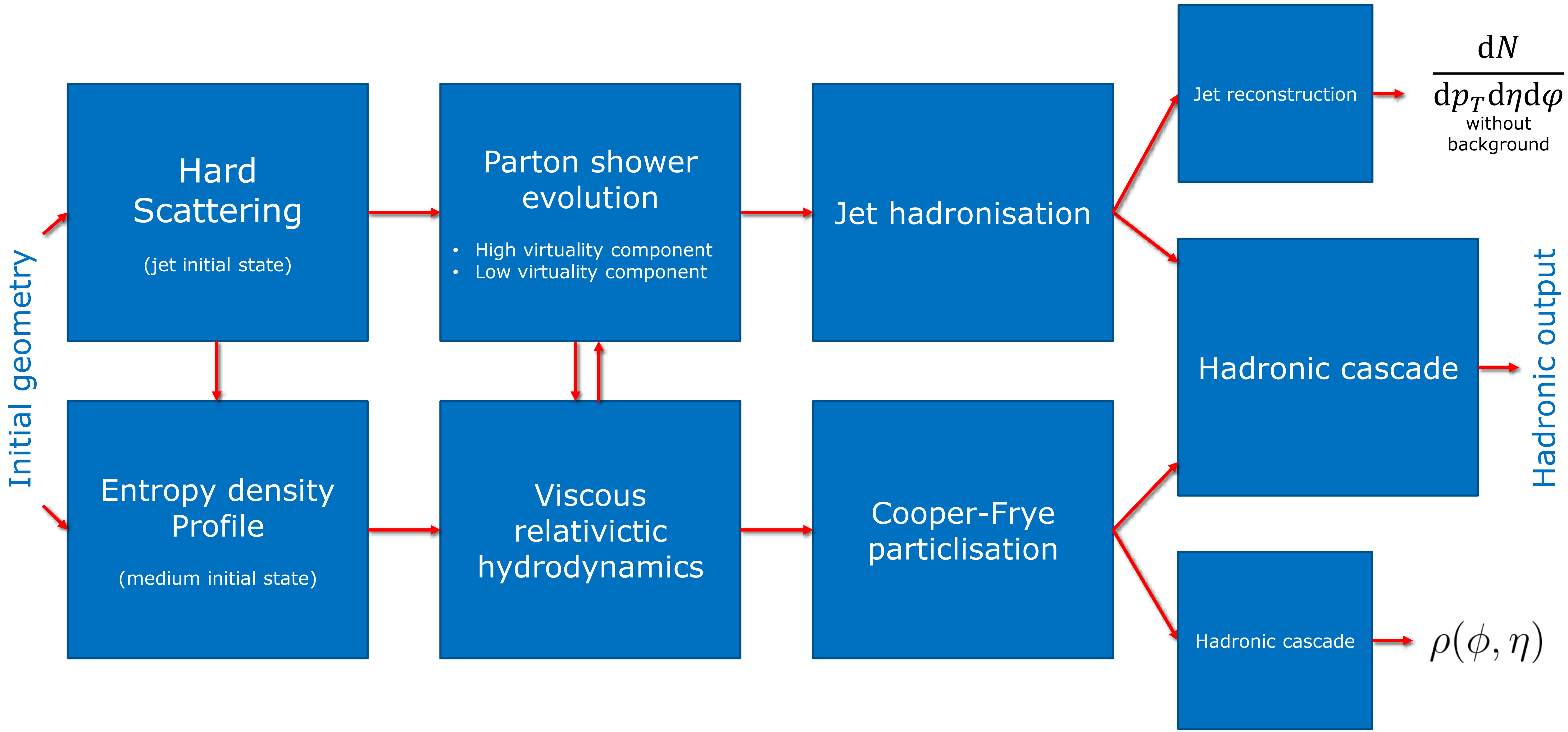
Jet Reconstruction

Hadrons are reconstructed with sequential clustering algorithm into jets



Jet Reconstruction

Anti-k_T algorithm implemented in FastJet can reconstruct jets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$\frac{dN}{dp_T d\eta d\varphi}$
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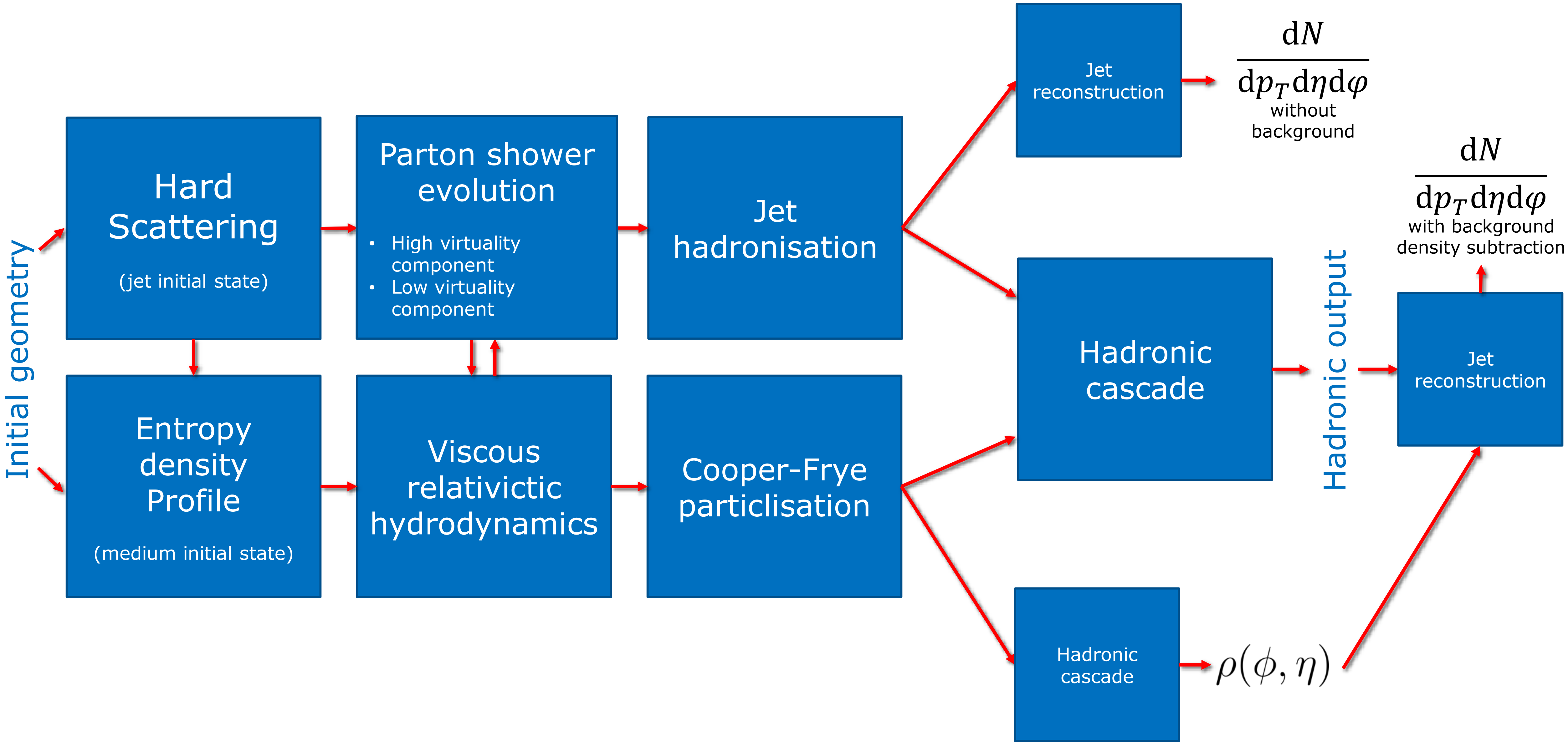
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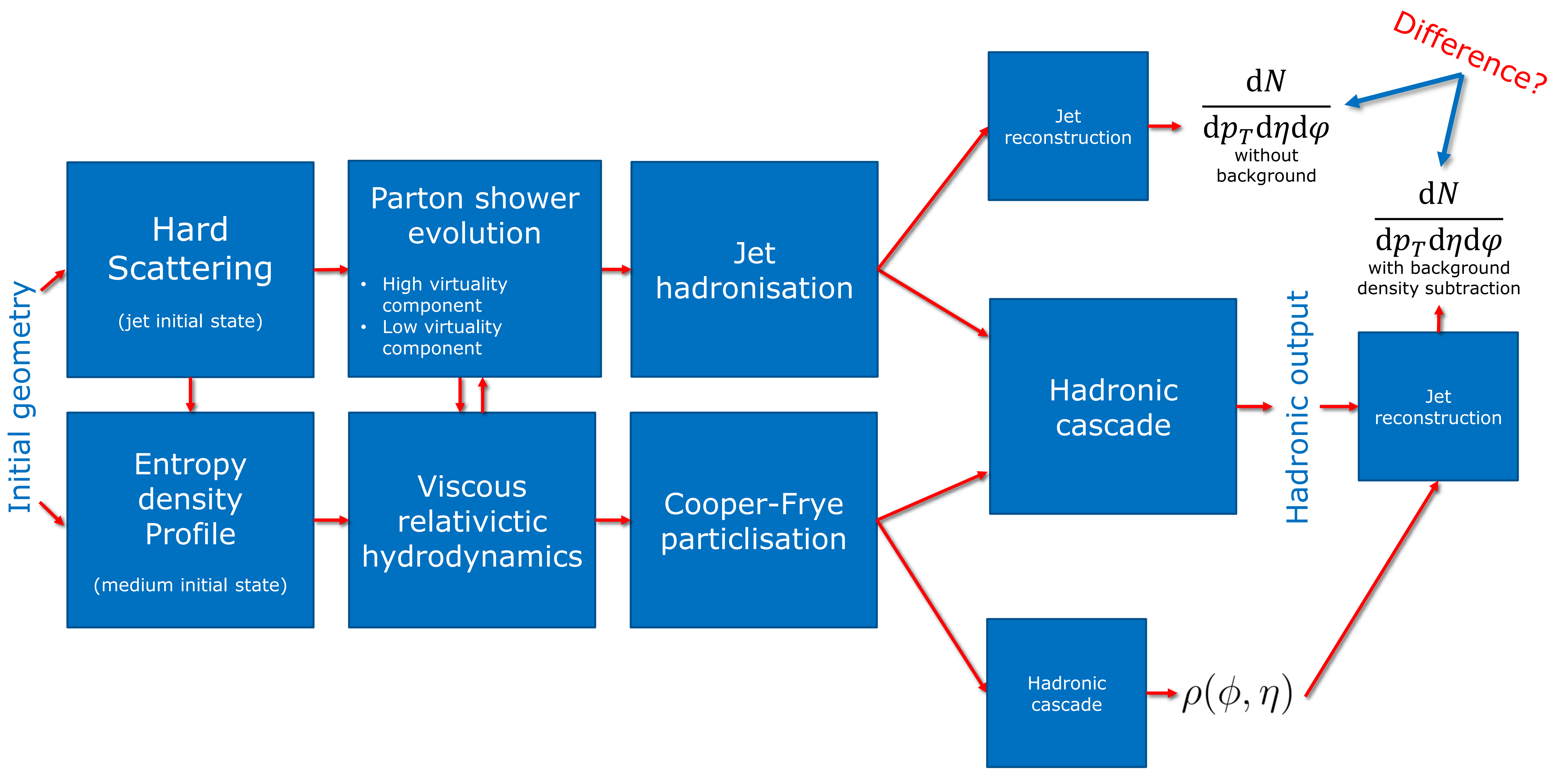
(jet initial state)
Parton shower evolution
High virtuality component
Low virtuality component
Jet hadronisation
Entropy density Profile
(medium initial state)
Viscous relativictic hydrodynamics
Cooper-Frye particlisation
Jet reconstruction
$\frac{dN}{dp_T d\eta d\varphi}$
without background
Hadronic cascade
Hadronic output
Jet reconstruction
$\frac{dN}{dp_T d\eta d\varphi}$
with background density subtraction
Hadronic cascade
$\rho(\phi, \eta)$


Difference?
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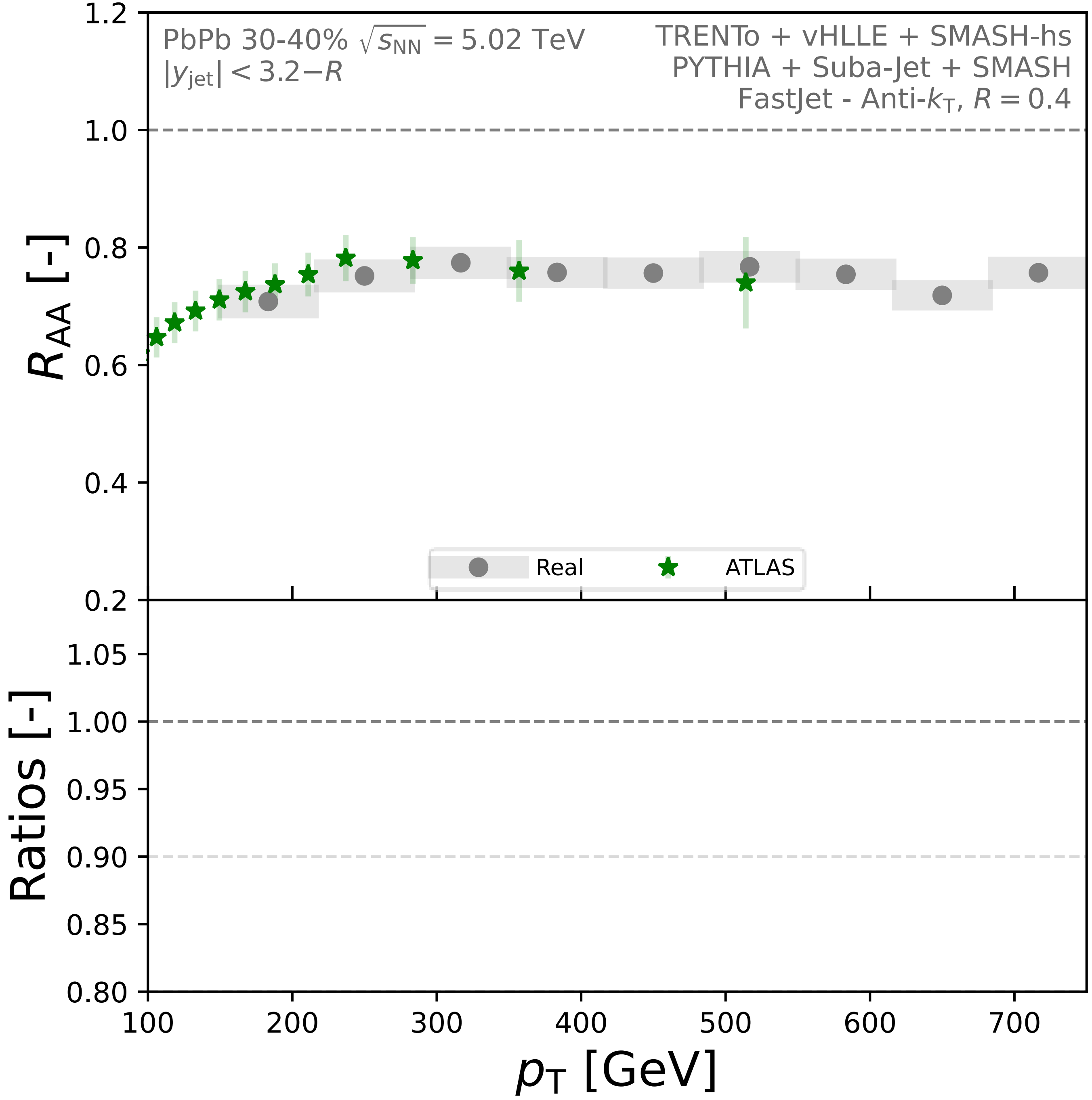
Nuclear modification factor

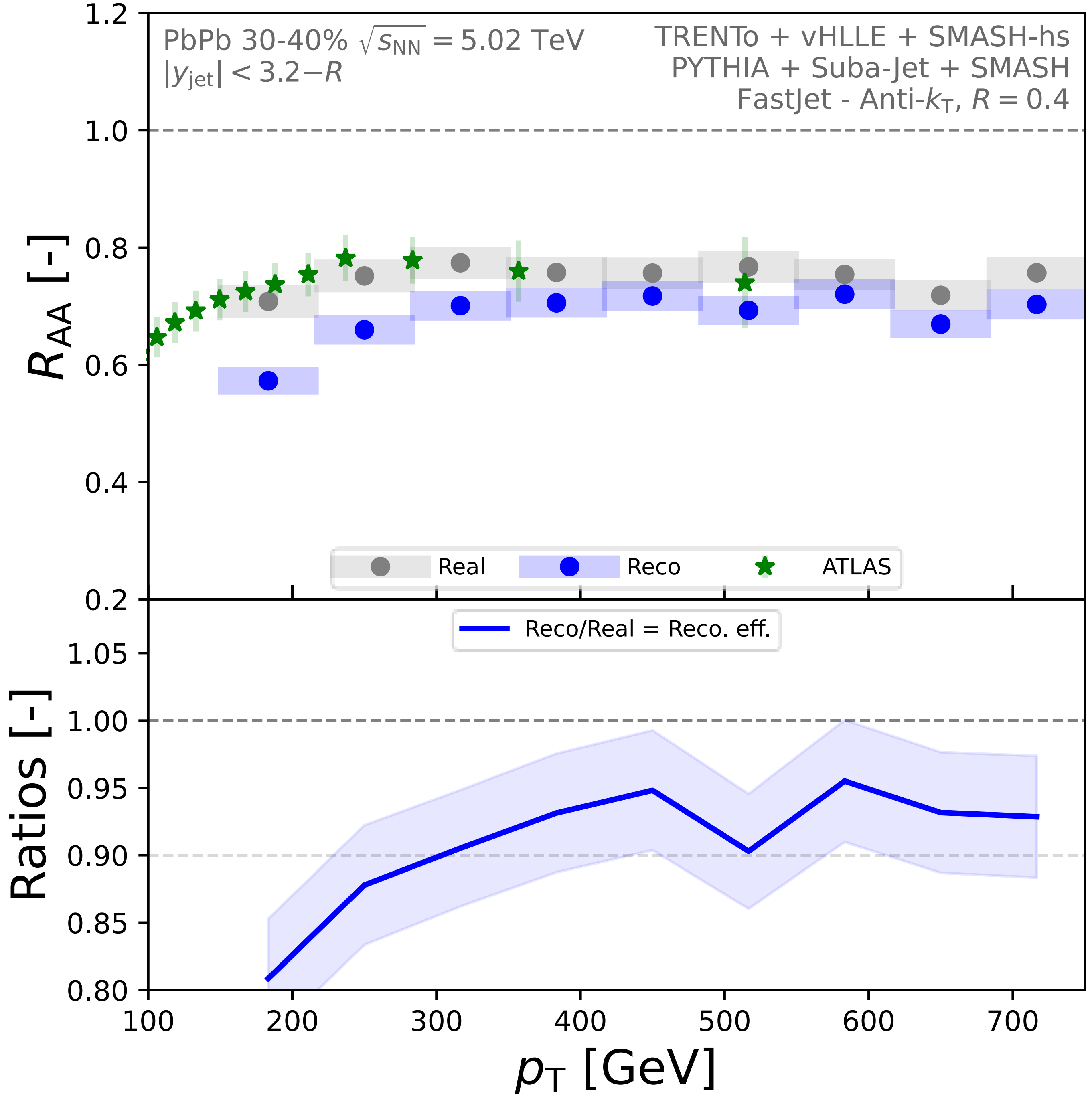
"Real" - Hadronised parton shower without background, followed by simple reconstruction.



Nuclear modification factor

"Real" - Hadronised parton shower without background, followed by simple reconstruction.

"Reco" - Jets and soft hadrons are combined (without interaction), followed by jet reconstruction with background subtraction



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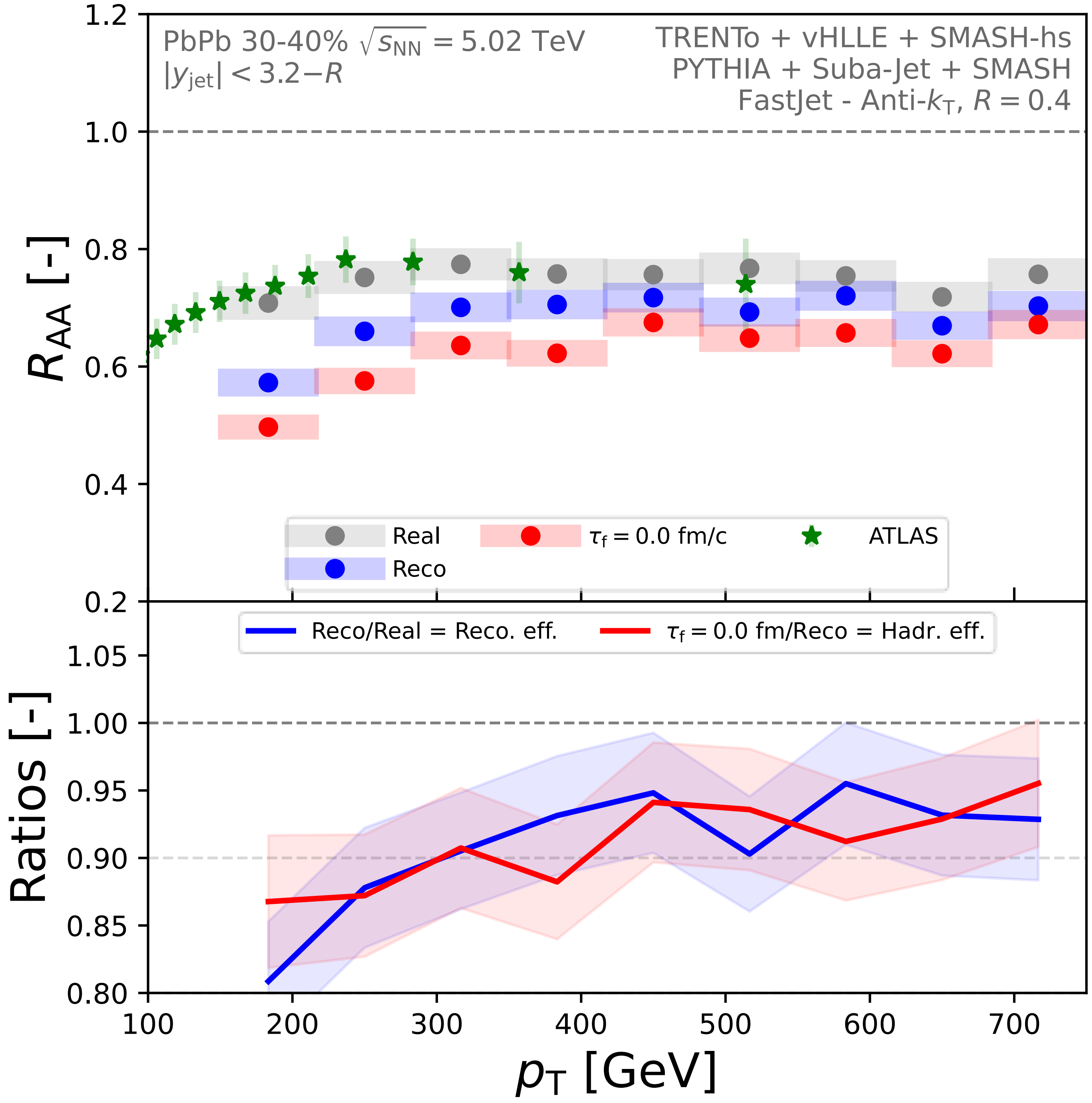
"$\tau_F = x$" - Jet hadrons interact with soft hadrons after a formation proper time, followed by the same reconstruction as in "Reco"



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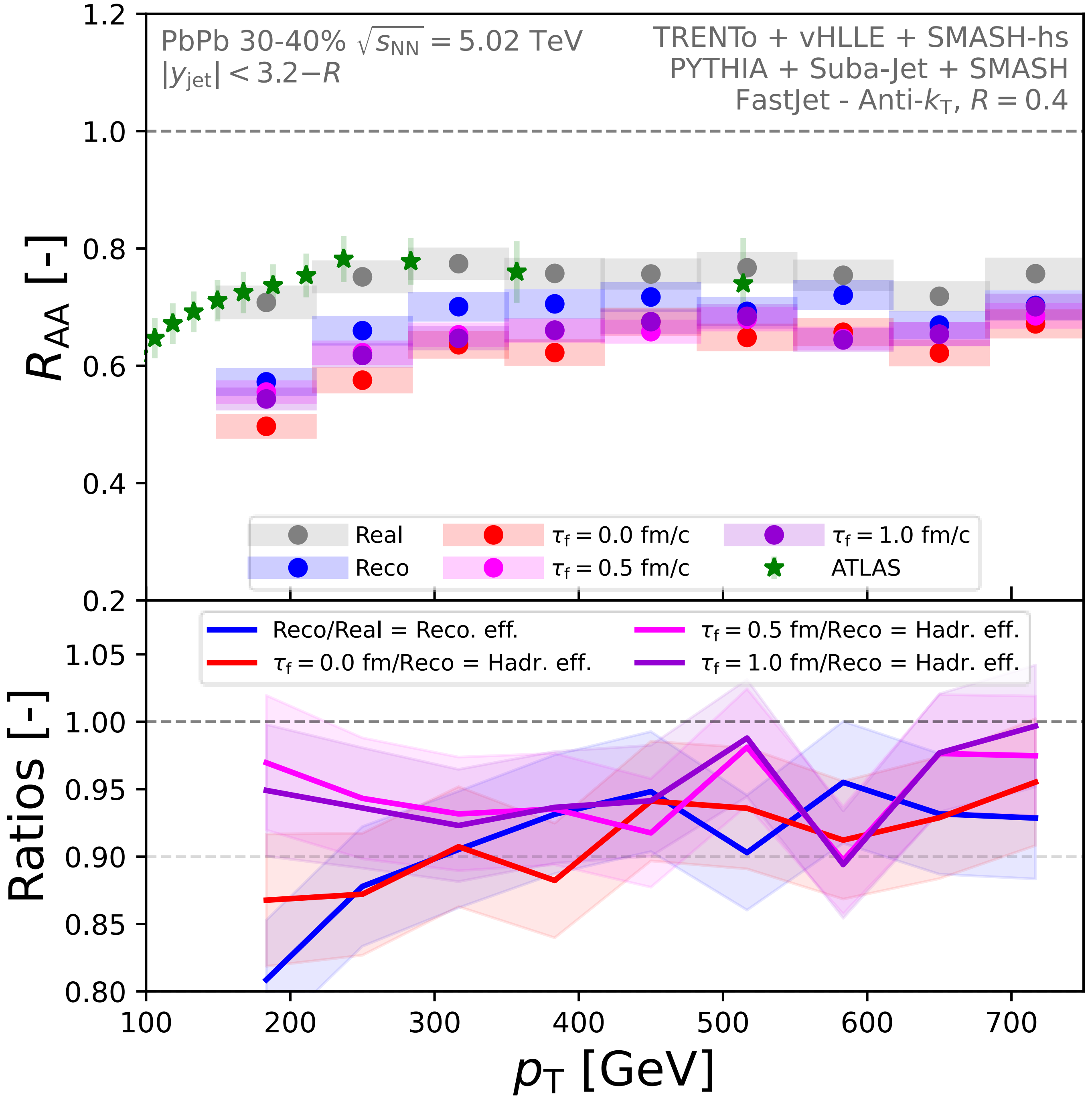

Jet shape

"Real" - Hadronised parton shower without background, followed by simple reconstruction.

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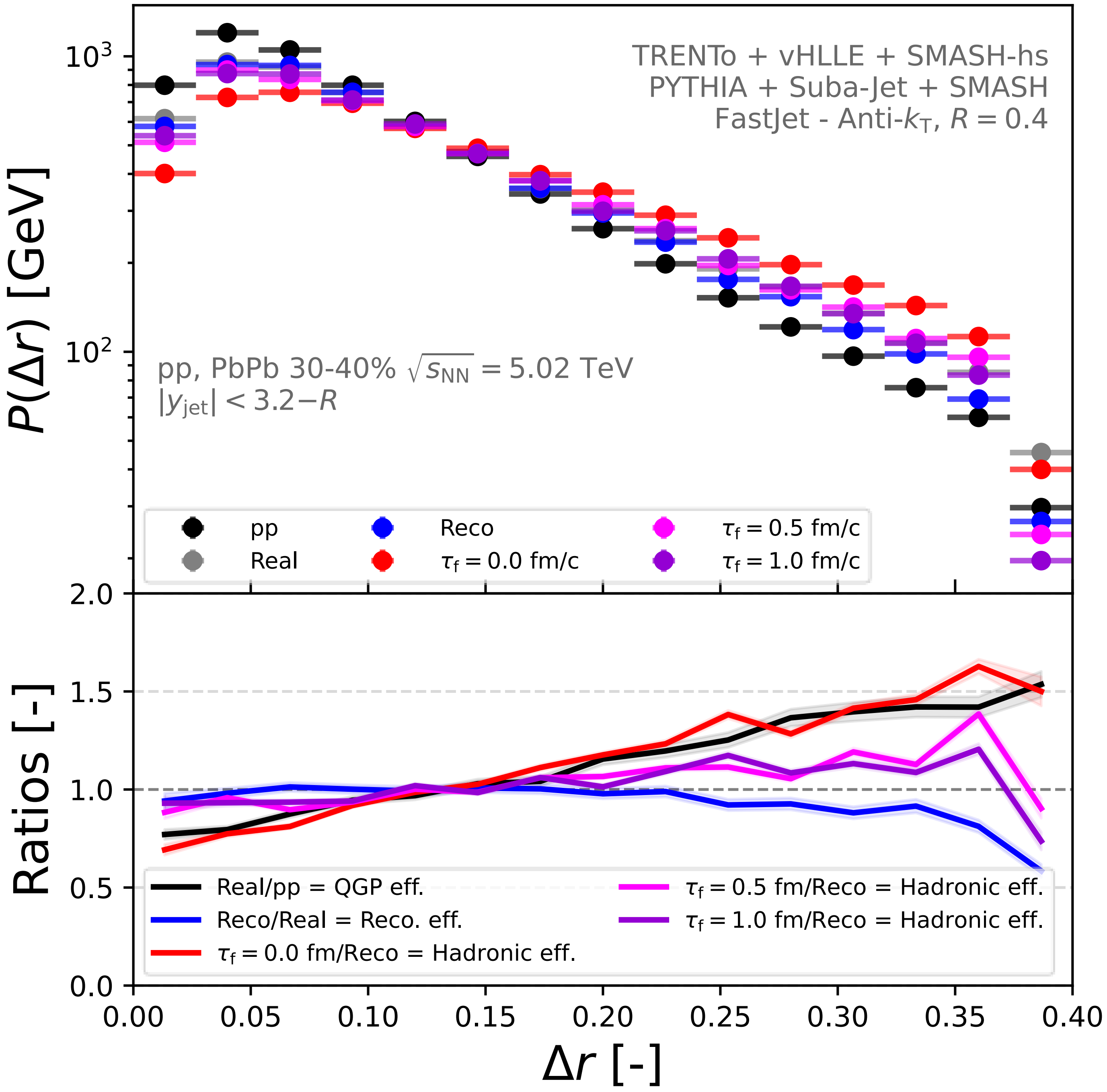

Jet fragmentation function

"Real" - Hadronised parton shower without background, followed by simple reconstruction.

"Reco" - Jets and soft hadrons are combined (without interaction), followed by jet reconstruction with background subtraction

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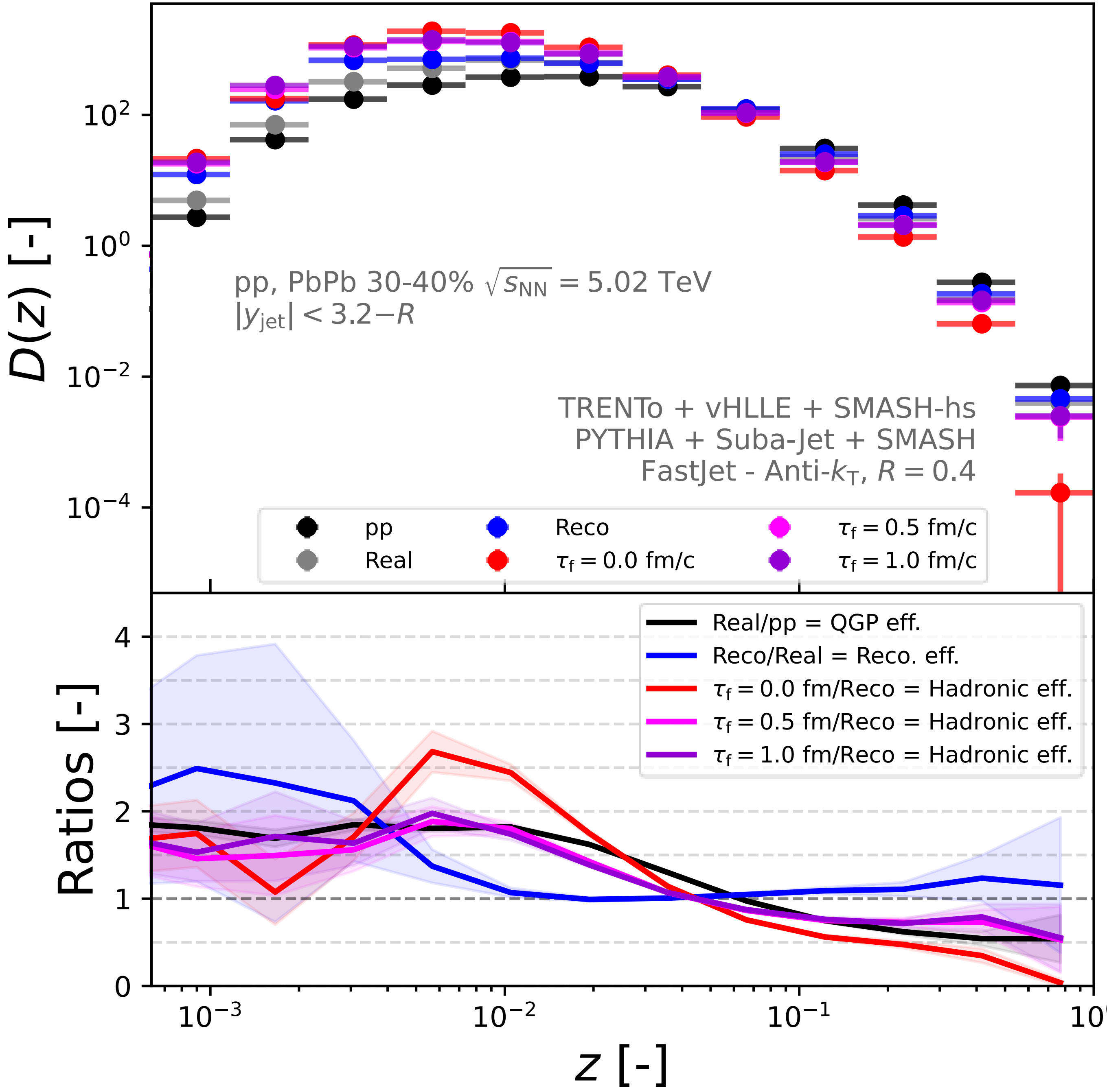


Conclusion

- A comprehensive framework for heavy-ion collisions **J-PHASE-Generator** (**J**et **P**articles evolved in **H**ydrodynamic and **A**fterburner **S**tages **E**vent **Generator**), was constructed
  - Low-transverse momenta observables - **T_RENTo** + **vHLLE** + **SMASH-hadron-sampler** + **SMASH**
  - Jet observables
    - QGP effects - **PYTHIA/Angantyr** (initial seed) + **SUBA-jet** + **PYTHIA** (hadronisation) [+ medium simulation]
    - Hadronic effects - complete framework incorporating **SMASH** hadronic rescattering
- We studied jet hadronic phase in **PbPb 30-40% at 5.02 TeV**
  - Three scenarios for the **formation proper time** of jet hadrons (1.0, 0.5, and 0.0 fm/c)
  - Visible effect on the **jet nuclear modification factor** is observed for all formation proper time values
  - Large enhancement of the **jet shape** at large distances from the jet axis
  - In the extreme scenario of **zero formation proper time** modification of the **jet shape** in the hadronic phase becomes **comparable to** that in the **QGP phase**
- Existing paradigm of **neglecting interactions in the hadronic phase** based on formation proper time argument **may not be entirely accurate**

Outlook

- Analysis of **hadronic effects** on the jet observables:
  - With different **centrality bins** (~multiplicity)
  - Dependent on **intrajet multiplicity** is planned
- Improving **reconstruction**
- Add non-primary **resonance decays** after hadronisation for "Real"
- Add **medium response** in the form of **wake** and **recoiled partons**
- Explore **parameter space** of this framework and fit experimental results
  - **Baysian analysis**
- Add **coalescence** hadronisation
- Add **pre-equilibrium** stage for **medium** and **jet** evolution
- Explore **heavy flavour** and **substructure** observables

Thank you for your attention!

Acknowledgement to the Czech Science Foundation (GAČR) for supporting this research

TRENTo3D Initial State

Rapidity-dependent entropy profile

arXiv:1610.08490

$$s\left(\mathbf{x}, \eta_s\right)\big|_{\tau=\tau_0} \propto f(\mathbf{x}) g(\mathbf{x}, y) \frac{dy}{d\eta}$$

$$g(\mathbf{x}, \eta) d\eta = g(\mathbf{x}, y) dy$$

$$\frac{dy}{d\eta} = \frac{J \cosh \eta}{\sqrt{1 + J^2 \sinh^2 \eta}}$$

$$g(\mathbf{x}, y) = \mathcal{F}^{-1}\{\tilde{g}(\mathbf{x}, k)\}$$

$$\log \tilde{g} = i\mu k - \frac{1}{2}\sigma^2 k^2 - \frac{1}{6} i\gamma\sigma^3 k^3 + \cdots$$

Ideal Hydrodynamics

Ideal fluid dynamics:

$$T_0^{\mu\nu} = \varepsilon u^\mu u^\nu - p\Delta^{\mu\nu}, \qquad \text{were } \Delta^{\mu\nu} = \eta^{\mu\nu} - u^\mu u^\nu$$

$$N_{0,i}^\mu = n_i u^\mu$$

Equations of motion from orthogonal projection:

$$u_\mu \partial_\nu T_0^{\mu\nu} = 0 \longrightarrow u^\mu \partial_\mu \varepsilon + (\varepsilon + p)\partial_\nu u^\nu = 0 \qquad \text{(Continuity eq.)}$$

$$\Delta_{\sigma\mu} \partial_\nu T_0^{\mu\nu} = 0 \longrightarrow (\varepsilon + p) u^\mu \partial_\mu u_\sigma - \Delta_\sigma^\nu \partial_\nu p = 0 \qquad \text{(Euler eq.)}$$

Navier-Stokes Formalism (first order)

$$\begin{aligned} T^{\mu\nu} &= T_0^{\mu\nu} + \Pi^{\mu\nu} \\ &= T_0^{\mu\nu} + \pi^{\mu\nu} - \Pi\Delta^{\mu\nu} \\ &= \varepsilon u^\mu u^\nu - (p+\Pi)\Delta^{\mu\nu} + \pi^{\mu\nu} \end{aligned}$$

$$\pi^{\mu\nu} = \eta\partial^{<\mu}u^{\nu>} = \eta\left[\frac{1}{2}\left(\Delta^{\alpha\mu}\Delta^{\beta\nu} + \Delta^{\beta\mu}\Delta^{\alpha\nu}\right) - \frac{1}{3}\Delta^{\mu\nu}\Delta^{\alpha\beta}\right]\partial_\alpha u_\beta$$

$$\Pi = -\zeta\partial_\mu u^\mu$$

Equation of motion with orthogonal projection:

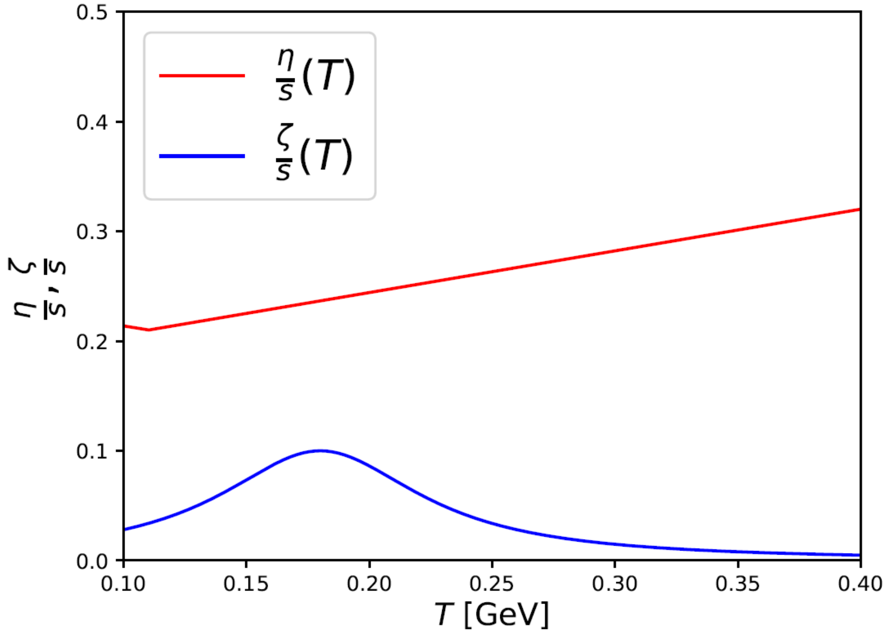
$$u^\mu\partial_\mu\varepsilon + (\varepsilon + p + \Pi)\partial_\nu u^\nu + \pi_{\mu\nu}\partial^{<\mu}u^{\nu>} = 0 \qquad \text{(Continuity eq.)}$$

$$(\varepsilon + p + \Pi)u^\mu\partial_\mu u^\sigma - \partial^\sigma(p+\Pi) + \Delta^{\sigma\mu}\partial^\nu\pi_{\mu\nu} - \pi^{\sigma\nu}u^\mu\partial_\mu u_\nu = 0 \qquad \text{(N-S eq.)}$$

Bulk and Shear Viscosity

- η/s can only be computed for simplified scenarios
  - pQCD (leading log) $\frac{\eta}{s} \sim \frac{1}{\alpha_s^2 \ln(\alpha_s^{-1})}$
    - Small α_s
  - AdS/CFT limit $\frac{\eta}{s} \geq \frac{1}{4\pi} \approx 0.08$
    - Large α_s

- η/s cannot be computed for realistic QGP
  - Comparison of different η/s to the data (i.e., Bayesian analysis)
- ζ/s cannot be computed even for simplified scenarios
  - Comparison of different ζ/s to the data (i.e., Bayesian analysis)
  - Must be carefully tested for numerical stability



String Hadronisation

Yoichiro Nambu nQCD potential → Lund string model

$$V_{\mathrm{QCD}} = -\frac{4}{3}\frac{\alpha_s}{r} + \kappa r + \ldots$$

$$\left|\frac{\mathrm{d}E}{\mathrm{d}z}\right| = \left|\frac{\mathrm{d}p_z}{\mathrm{d}z}\right| = \left|\frac{\mathrm{d}E}{\mathrm{d}t}\right| = \left|\frac{\mathrm{d}p_z}{\mathrm{d}t}\right| = \kappa$$

String breaking mechanism

$$\frac{1}{\kappa}\frac{\mathrm{d}\mathcal{P}_q}{\mathrm{d}^2 p_\perp} \propto \exp\left(-\frac{\pi m_{\perp q}^2}{\kappa}\right) = \exp\left(-\frac{\pi p_\perp^2}{\kappa}\right)\exp\left(-\frac{\pi m_q^2}{\kappa}\right)$$

$$u\bar{u} : d\bar{d} : s\bar{s} : c\bar{c} \approx 1 : 1 : 0.3 : 10^{-11}$$

Bayesian Analysis of TRENTo Parameters

- Principal component analysis
  - Linear transformation of observables $\left(\frac{\mathrm{d}N_{\mathrm{ch}}}{\mathrm{d}\eta}, \langle p_{\mathrm{T}}\rangle, v_2\{2\}, v_3\{2\}, v_4\{2\}\right)$
  - Reduces the number of variables that must be evaluated

arXiv:1804.06469

- Gaussian process
  - Estimates the principal components
  - Evaluates likelihood
  - No running model (hydro)
- The Bayes theorem sampled by MCMC
  - Posterior ∝ Likelihood×Prior

$$P(\mathbf{x} \mid \mathbf{y}) \propto P(\mathbf{y} \mid \mathbf{x}) P(\mathbf{x})$$

Parameter	Value
$n_{2.76}$	13.94
$n_{5.02}$	18.50
p	0.0
k	1.044
w	0.956 fm
d	1.27 fm

($p = 0 \Rightarrow$ geometric mean)

T_RENTo Initial State

PHYSICAL INPUT

- Impact parameter $\boldsymbol{b}$
- Inelastic nucleon-nucleon cross section $\sigma_{\mathrm{NN}}^{\mathrm{inel}}$
- Nuclear density ρ_A
- Normalization n

MODEL PARAMETERS

- Reduced thickness parameter p
- Fluctuation k
- Nucleon width w
- Nucleon minimum distance d

Jet Reconstruction by Sequential Clustering Algorithms

Particle i-beam distance

$$d_{iB} = p_{\mathrm{T}i}^{a}$$

arXiv:1111.6097

i-j particles distance

$$d_{ij} = \min(p_{\mathrm{T}i}^{a}, p_{\mathrm{T}j}^{a}) \frac{R_{ij}^2}{R}$$

$a = -2 \Leftrightarrow$ anti-k_{T}

Euclidian distance in $y - \phi$ plane

$$R_{ij} = \sqrt{(y_i - y_j)^2 + (\phi_i - \phi_j)^2}$$

If d_{ij} is minimum → combine i and j

If d_{iB} is minimum → i is jet

Repeat