Insight into the electrical conductivity of quark gluon plasma through photon production

Nicholas J. Benoit, PostDoc at Hiroshima University

Collaborators:

T. Miyoshi, C. Nonaka, A. Sakai, and H. R. Takahashi

Phenomenological model used in this work:

[1] K. Nakamura, T. Miyoshi, C. Nonaka, and H. R. Takahashi, Eur. Phys. J. C 83, 229 (2023)
[2] K. Nakamura, T. Miyoshi, C. Nonaka, and H. R. Takahashi, Phys. Rev. C 107, 014901 (2023)
[3] K. Nakamura, T. Miyoshi, C. Nonaka, and H. R. Takahashi, Phys. Rev. C 107, 034912 (2023)

What is our work?

New model using relativistic resistive magneto-hydrodynamics (RRMHD) for quark-gluon plasma (QGP)

$$\nabla_\mu N^\mu = 0 \qquad \nabla_\mu F^{\mu\nu} = -J^\nu$$
$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu {}^* F^{\mu\nu} = 0$$
$$\nabla_\mu S^\mu \geq 0 \qquad J^\mu = q u^\mu + \sigma e^\mu$$

+

Electromagnetic dissipative corrections to photon yield from QGP

$$E_k \frac{d\mathcal{R}^{\mathrm{EM}}}{d^3\vec{k}} \simeq C \alpha_s \alpha_{\mathrm{EM}} \mathcal{I} \mathcal{L}_c \sum_a \delta f^{(1)}_{a,\mathrm{EM}}(X,k)$$

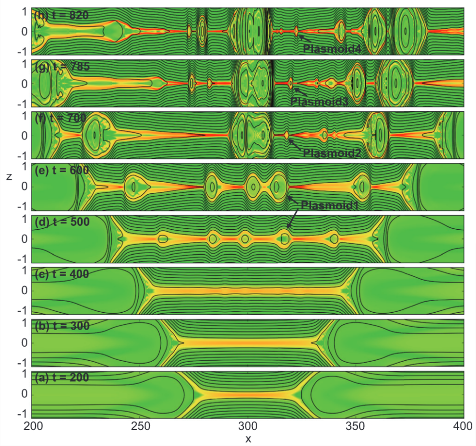
Goal: Estimate the electric conductivity (σ) of QGP independent of lattice and direct pQCD calculations.

Why study QGPs electric conductivity?

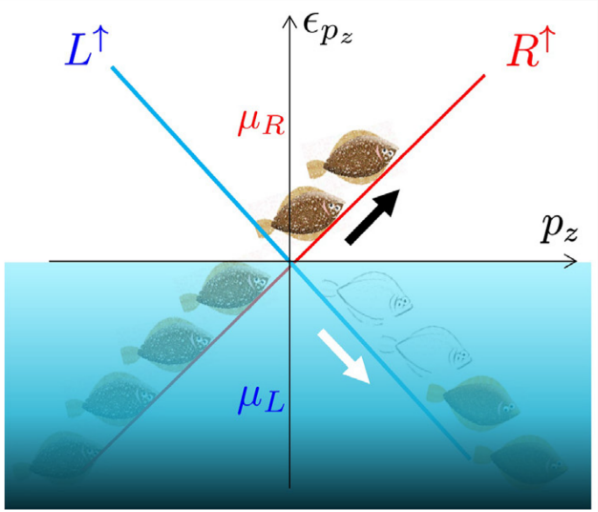
Multiple motivations

Important for interesting phenomena

1) Chiral Magnetic Effect (CME)
2) Magnetic reconnection
3) EM probe diffusion



T. Shibayama, K. Kusano, T. Miyoshi, T. Nakabou, and G. Vekstein, Physics of Plasmas 22, 100706 (2015).



CME from K. Hattori, K. Itakura, and S. Ozaki, Progress in Particle and Nuclear Physics 133, 104068 (2023).

Quark-gluon plasma (QGP) electric conductivity

What is QGP electric conductivity (σ)?

- Studied by lattice calculations (~10 papers), pQCD, and kinetic transport theories

Review: G. Aarts and A. Nikolaev, Eur. Phys. J. A 57, 118 (2021); 2008.12326 [hep-lat]

Electric Conductivity on the Lattice

$$\sigma = \frac{1}{6}\frac{\partial}{\omega}\left(\int d^4x e^{i\omega t}\langle[j_\mu^{\text{em}}(t,x), j_\mu^{\text{em}}(0,0)]\rangle\right)|_{\omega=0}$$

Uses linear-response theory (Kubo formula)

Low energy limit of the electromagnetic spectral function

where the EM current is,

$$j_\mu^{\text{em}}(x) = \sum_{f=1}^{N_f}(eq_f)\overline{\psi}^f(x)\gamma_\mu\psi^f(x)$$

depends on the quark flavors

Quark-gluon plasma (QGP) electric conductivity

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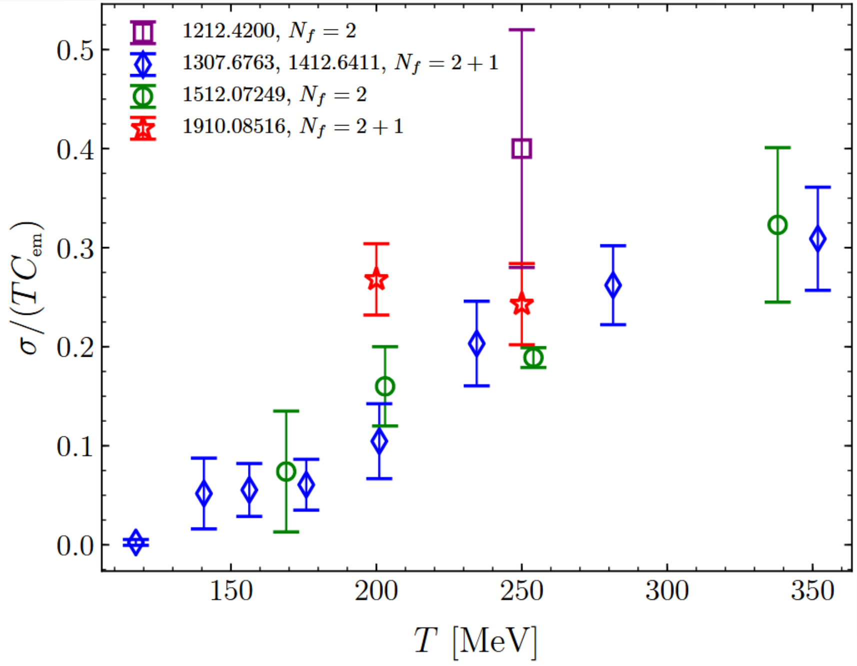

- Does not include external magnetic field effects
- Uses approximately realistic pion mass
- General agreement among results using a variety of methods and parameters (see backup or paper)

Review: G. Aarts and A. Nikolaev, Eur. Phys. J. A 57, 118 (2021); 2008.12326 [hep-lat]

What about heavy-ion collisions (HIC)?

Is it possible to measure electric conductivity in HICs?

- Yes! Collision environment has QGP + EM fields

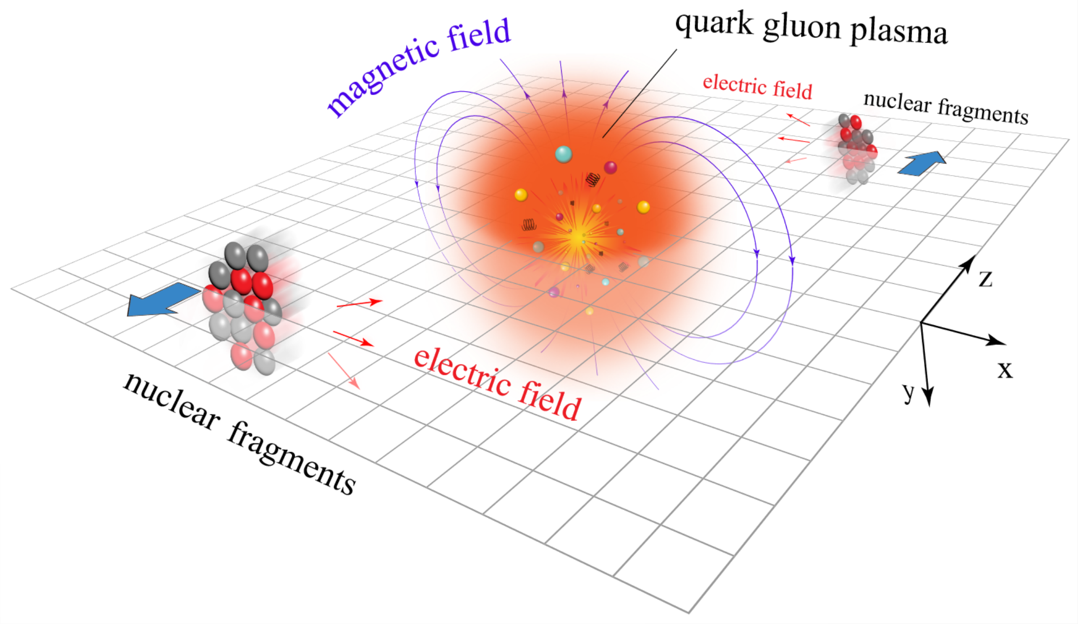

Image: STAR Collaboration (2024)

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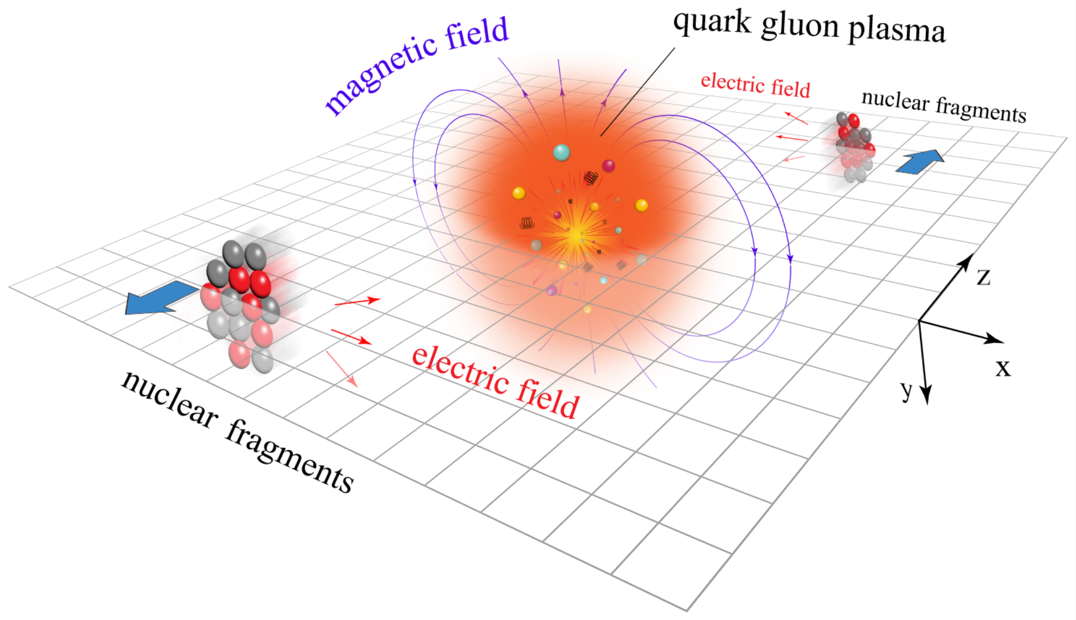
Proposed EM observables

- Dileptons → i.e., Y. Akamatsu, H. Hamagaki, T. Hatsuda, and T. Hirano, Phys. Rev. C 85, 054903 (2012).
- Photons → i.e., J.-A. Sun and L. Yan, Phys. Rev. C 109, 034917 (2024).

Proposed charge dependent directed flow

- Asymmetic collisions → i.e., Y. Hirono, M. Hongo, and T. Hirano, Phys. Rev. C 90, 021903 (2014).
- Symmetric collisions



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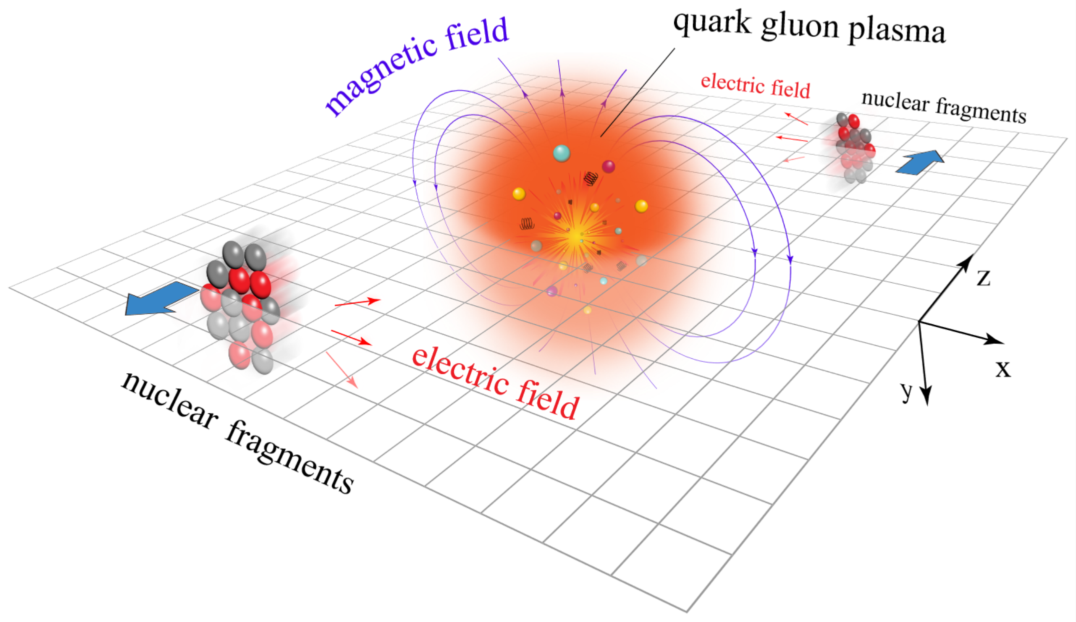

Image: STAR Collaboration (2024)

Using QGP photons as a probe of electric conductivity

Electromagnetic dissipation for QGP photons

Electromagnetic fields inside QGP

- EM fields penetrating QGP drive charge carriers out-of-equilibrium

$$J^\mu = q u^\mu + \sigma F^{\mu\nu} u_\nu$$

EM current in the QGP medium

First order dissipation from the EM fields

Electromagnetic dissipation for QGP photons

Electromagnetic fields inside QGP

- EM fields penetrating QGP drive charge carriers out-of-equilibrium

$$J^\mu = qu^\mu + \sigma F^{\mu\nu} u_\nu$$

EM current in the QGP medium

First order dissipation from the EM fields

- Taking the Boltzmann equation in the relaxation time approximation focus on 2→2 processes,

$$k^\mu \partial_\mu f_a + eQ_a F^{\mu\nu} k_\mu \frac{\partial f_a}{\partial k^\nu} = -\frac{k^\mu u_\mu}{\tau_R} \delta f^{(n)}_{a,EM}$$

J. A. Sun and L. Yan
Phys. Rev. C 109, 034917 (2024)

Vlasov term for the external EM fields

Order "n" corrections to the quark distribution function because of dissipation from the EM fields

Electromagnetic dissipation for QGP photons

Electromagnetic fields inside QGP

- For this calculation we focus on 1st order corrections,

$$k^\mu \partial_\mu f_a + eQ_a F^{\mu\nu} k_\mu \frac{\partial f_a}{\partial k^\nu} = -\frac{k^\mu u_\mu}{\tau_R} \delta f^{(n)}_{a,EM}$$

J. A. Sun and L. Yan
Phys. Rev. C 109, 034917 (2024)

$$f_a = f_{a,eq} + \delta f^{(1)}_{a,EM} + \delta f^{(2)}_{a,EM} + \delta f^{(3)}_{a,EM} + \cdots$$

Ordered by the EM field strength

$$\delta f^{(1)}_{a,\mathrm{EM}}(X,k) = -\frac{-f_{a,eq}(1-f_{a,eq})}{T\chi_{el} k^\mu u_\mu} e\sigma Q_a e^\mu k_\mu$$

Electric conductivity of QGP from Landau matching with the current

$$J^\mu = qu^\mu + \sigma F^{\mu\nu} u_\nu$$

EM fields in the fluid rest frame

$$e^\mu = \left(\gamma v_k E^k, \quad \gamma E^i + \gamma \epsilon^{ijk} v_j B_k\right)$$

Electromagnetic dissipation for QGP photons

Electromagnetic fields inside QGP

- What we do is calculate the fluid + EM field contributions using hydrodynamics

Temperature and four-velocity

Electric conductivity value

Values that come from a hydrodynamic calculation

$$\delta f^{(1)}_{a,\mathrm{EM}}(X,k) = -\frac{-f_{a,eq}(1-f_{a,eq})}{T\chi_{el}k^{\mu}u_{\mu}} e\sigma Q_a e^{\mu} k_{\mu}$$

$$e^{\mu} = \left(\gamma v_k E^k, \quad \gamma E^i + \gamma \epsilon^{ijk} v_j B_k\right)$$

Spacetime dependent EM fields in QGP medium

Electric susceptibility of QGP

$$\chi_{a,el} = -\frac{1}{3}\int \frac{d\vec{p}}{(2\pi)^3 E_p}(p^{\sigma}p^{\nu}\Delta_{\sigma\nu})\frac{-f_{a,eq}(1-f_{a,eq})}{p^{\mu}u_{\mu}}$$

Electromagnetic dissipation for QGP photons

Electromagnetic fields inside QGP

- What we do is calculate the fluid + EM field contributions using hydrodynamics
- All of those values can be calculated self-consistently using relativistic resistive magneto-hydrodynamics (RRMHD)

Temperature and four-velocity

Electric conductivity value

Values that come from a hydrodynamic calculation

$$\delta f^{(1)}_{a,\mathrm{EM}}(X,k) = -\frac{-f_{a,eq}(1-f_{a,eq})}{T\chi_{el}k^\mu u_\mu} e\sigma Q_a e^\mu k_\mu$$

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Introduction to our model for QGP

Relativistic Resistive Magneto-Hydrodynamics (RRMHD) Model

- Ideal relativistic hydrodynamics + Maxwell's equations

$$\nabla_\mu N^\mu = 0 \qquad \nabla_\mu F^{\mu\nu} = -J^\nu$$

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu {}^*F^{\mu\nu} = 0$$

$$\nabla_\mu S^\mu \geq 0$$

$$T^{\mu\nu} = T^{\mu\nu}_{\mathrm{m}} + T^{\mu\nu}_{\mathrm{f}}$$

$$T^{\mu\nu}_{\mathrm{m}} = (\epsilon + p_{\mathrm{gas}})\, u^\mu u^\nu + p_{\mathrm{gas}} g^{\mu\nu}$$

$$T^{\mu\nu}_{\mathrm{f}} = F^{\mu\lambda} F^\nu_\lambda - \frac{1}{4} g^{\mu\nu} F^{\lambda\delta} F_{\lambda\delta}$$

This is the Lorentz force acting on the plasma

- In the QGP medium the EM field lifetimes are extended

$$J^\mu = q u^\mu + \sigma e^\mu$$

We assume a scalar electrical conductivity (no temperature dependence)

We do not include any other kinds of dissipation, i.e., viscosity

Photon elliptic flow using RRMHD + EM dissipation

QGP photon elliptic flow + conductivity

Why elliptic flow?

- Rate of QGP photon production should be increased by the EM fields

$$E_k \frac{d\mathcal{R}}{d^3\vec{k}} = E_k \frac{d\mathcal{R}^{\mathrm{QGP}}}{d^3\vec{k}} + E_k \frac{d\mathcal{R}^{\mathrm{EM}}}{d^3\vec{k}}$$

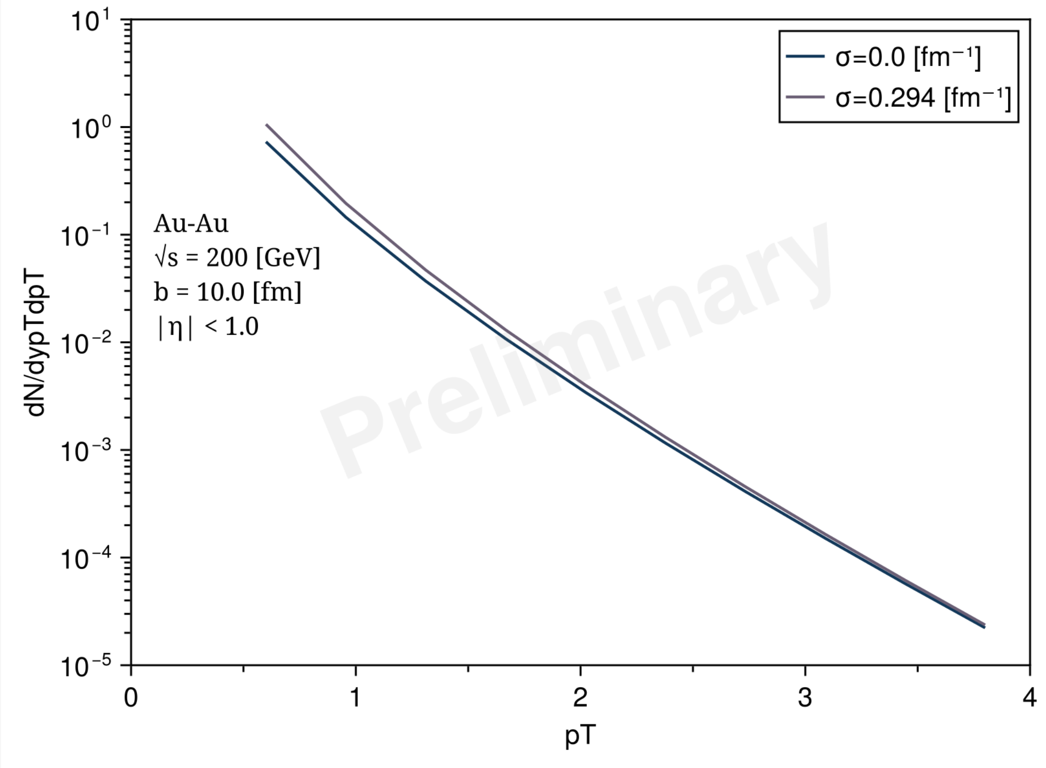
$$E_k \frac{d\mathcal{R}^{\mathrm{EM}}}{d^3\vec{k}} \simeq C\alpha_s \alpha_{\mathrm{EM}} \mathcal{I}\mathcal{L}_c \sum_a \delta f^{(1)}_{a,\mathrm{EM}}(X,k)$$

However, if the electric conductivity value is smaller than 1, and the EM fields are short lived. This correction is much smaller than the bulk QGP.

QGP photon elliptic flow + conductivity

Why elliptic flow?

- Rate of QGP photon production should be increased by the EM fields



$$E_k \frac{d\mathcal{R}^{\mathrm{EM}}}{d^3\vec{k}} \simeq C\alpha_s\alpha_{\mathrm{EM}}\mathcal{I}\mathcal{L}_c \sum_a \delta f^{(1)}_{a,\mathrm{EM}}(X,k)$$

Small contribution to the transverse momentum spectrum

$$\sigma_{\mathrm{lattice}} \simeq 0.0294\ [\mathrm{fm}]^{-1}$$

QGP photon elliptic flow + conductivity

Why elliptic flow?

- Rate of QGP photon production should be increased by the EM fields

$$E_k \frac{d\mathcal{R}}{d^3\vec{k}} = E_k \frac{d\mathcal{R}^{\mathrm{QGP}}}{d^3\vec{k}} + E_k \frac{d\mathcal{R}^{\mathrm{EM}}}{d^3\vec{k}}$$

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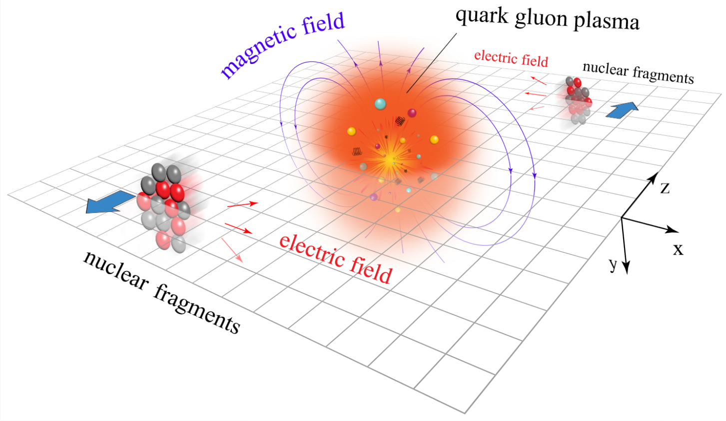


Image: STAR Collaboration (2024)

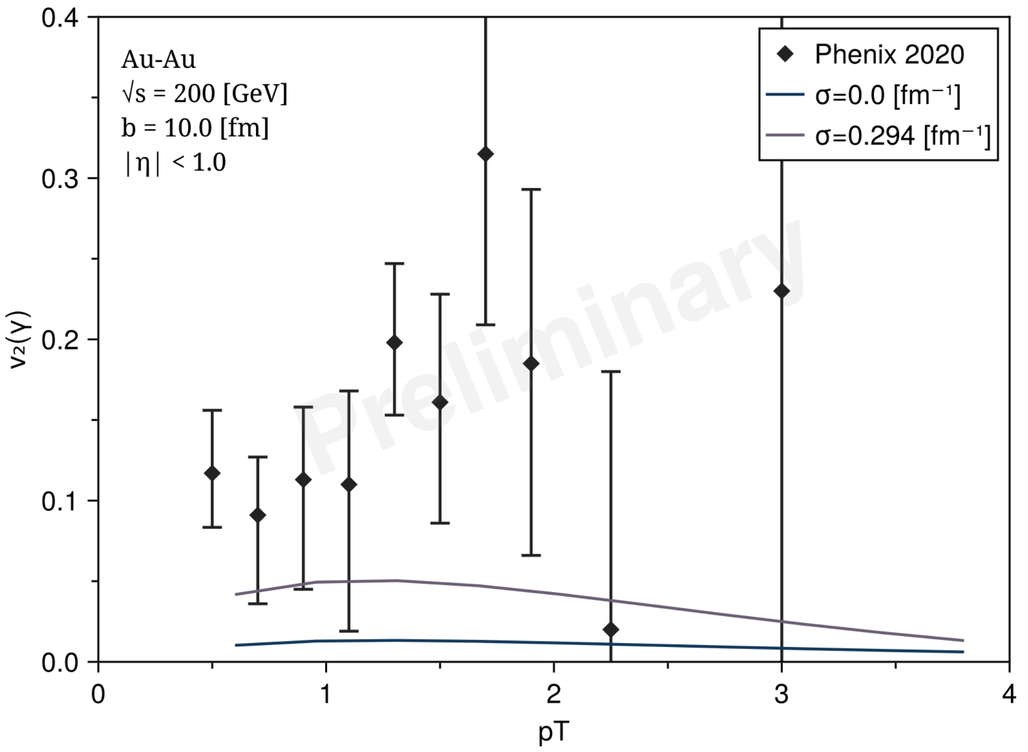
Largest magnetic field has an elliptic orientation, so we can expect a larger impact from the EM corrections on elliptic flow?

QGP photon elliptic flow + conductivity

Large enhancement for non-zero conductivity

- Symmetric collision @ RHIC
- Tilted optical Glauber model initial energy density + smooth initial EM fields
- Enhancement is similar to J. A. Sun and
- L. Yan, whom only included static By-field

$$v_2(\gamma) \equiv \frac{v_0 v_2 + v_0^{\rm EM} v_2^{\rm EM}}{v_0 + v_0^{\rm EM}}$$



- **Photon elliptic flow could constrain QGP conductivity, but experimental error is large**

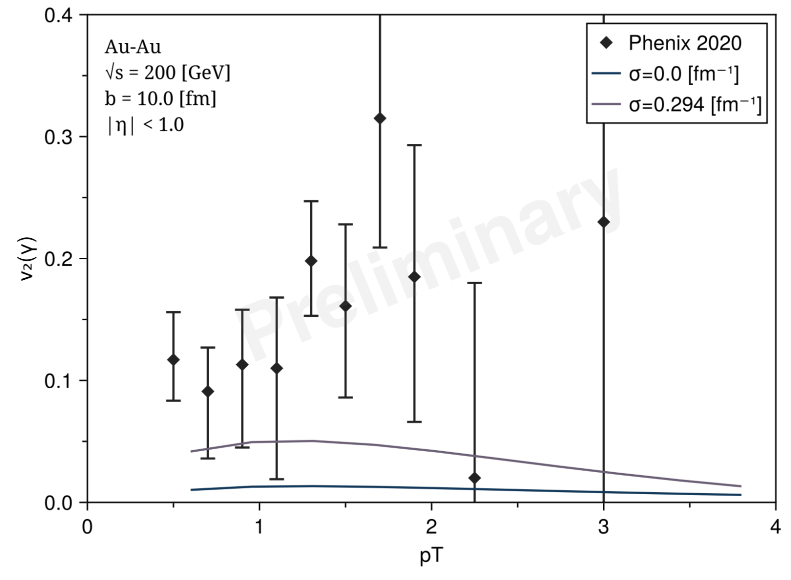
Summary

Demonstrated how this new calculation:

Relativistic resistive magneto-hydrodynamics (RRMHD) for quark-gluon plasma (QGP)

+

EM dissipative corrections to QGP photon elliptic flow



Can constrain QGP electric conductivity (σ) using experimental data

In future we aim to include viscosity and Hadron photon production

Backups

What is QGP conductivity?

Conductivity is a QGP transport parameter

- They characterize the space-time evolution of QGP

Shear viscosity	η	Energy-momentum
Bulk viscosity	ζ	
Charm-diffusion coefficient	D	Heavy-flavor quantum numbers
Thermal conductivity	κ	Heat via baryon current
Electric conductivity	σ	Electrical charges via the electric current

Initial Conditions for the QGP RRMHD

	RHIC
Ion	197 Au
$\sqrt{s_{\mathrm{NN}}}$	200 GeV^2
τ_0	0.4 fm
ϵ_0	55.0 $\mathrm{GeV/fm}^3$
σ_{NN}	41.3 mb
η_s	5.9 fm
ω_η	0.4
ϵ_{vac}	0.10 $\mathrm{GeV/fm}^3$

TABLE I. Initial parameters used for this work.

Quark-gluon plasma (QGP) electric conductivity

What is QGP electric conductivity (σ)?

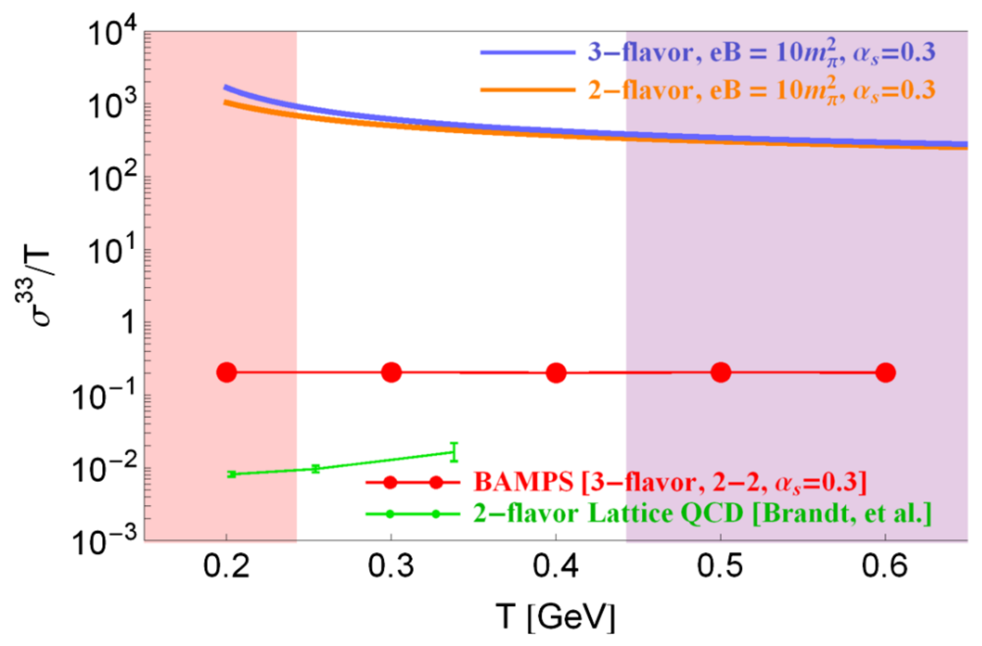
- Studied by lattice calculations (~10 papers), pQCD, and kinetic transport theories

Unfortunately lattice calculations, pQCD, and kinetic transport calculations can disagree

Includes an external magnetic field

- Several orders of magnitude difference
- Blue+Orange is Strong Field calculation
- Red is Boltzman Transport
- Green is Lattice

Fig. 8 from K. Hattori and D. Satow, Phys. Rev. D 94, 114032 (2016)



Results also vary depending on the details of the calculation

Quark-gluon plasma (QGP) electric conductivity

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Table 1 Details of the lattice QCD ensembles to compute the electrical conductivity. Fermion properties refer to sea quarks. Here a_τ and a_s denote the temporal and spatial lattice spacing respectively

Ref.	arXiv number	N_f (sea)	Fermion type	m_π [MeV]	a_τ [fm]	a_s/a_τ	Discretisation
[15]	hep-lat/0301006	0	Quenched	–	$a_\tau \to 0$	1	Continuum limit
[16]	hep-lat/0703008	0	Quenched	–	0.0488, 0.0203	1	Fixed cutoff
[17]	1012.4963	0	Quenched	–	$a_\tau \to 0$	1	Continuum limit
[18]	1112.4802	0	Quenched	–	0.015	1	Fixed scale
[19]	1212.4200	2	Wilson-clover	270	0.0486(4)(5)	1	Fixed cutoff
[20]	1307.6763	2 + 1	Wilson-clover	384(4)	0.0350(2)	3.5	Fixed scale
[21]	1412.6411	2 + 1	Wilson-clover	384(4)	0.0350(2)	3.5	Fixed scale
[22]	1512.07249	2	Wilson-clover	270	0.0486(4)(5)	1	Fixed scale
[23]	1604.06712	0	Quenched	–	$a_\tau \to 0$	1	Continuum limit
[24]	1910.08516	2 + 1	Staggered	134.2(6)	0.0618, 0.0493	1	Fixed cutoff

Review: G. Aarts and A. Nikolaev, Eur. Phys. J. A 57, 118 (2021); 2008.12326 [hep-lat]

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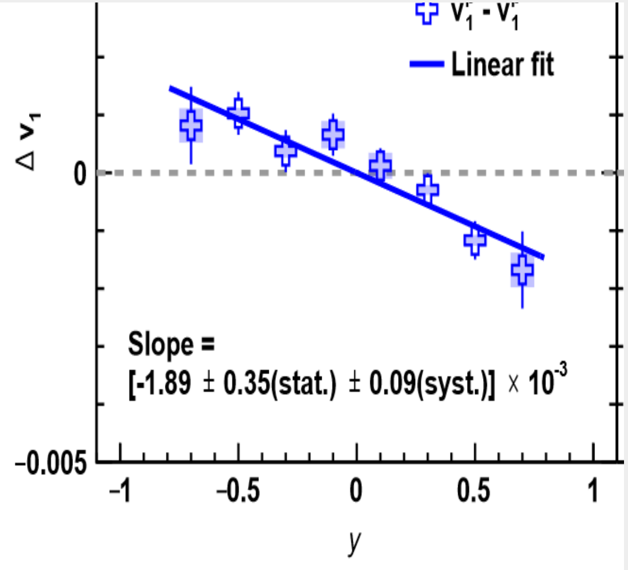
$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu {}^* F^{\mu\nu} = 0$$
$$\nabla_\mu S^\mu \geq 0 \qquad J^\mu = q u^\mu + \sigma e^\mu$$

+

New 2024 results from STAR
Phys. Rev. X 14, 011028 (2024)
arXiv:2304.03430 [hep-ex]



Estimation the electric conductivity of QGP independent of lattice and pQCD calculations.

Charge dependent directed flow

Electromagnetic fields inside QGP

- Faraday: the decay of the fragments magnetic field
- Lorentz Force or Hall Effect: from the longitudinal expansion of the fluid
- Coulomb: produced by spectators and fluid

Size of the effects depend upon the collision

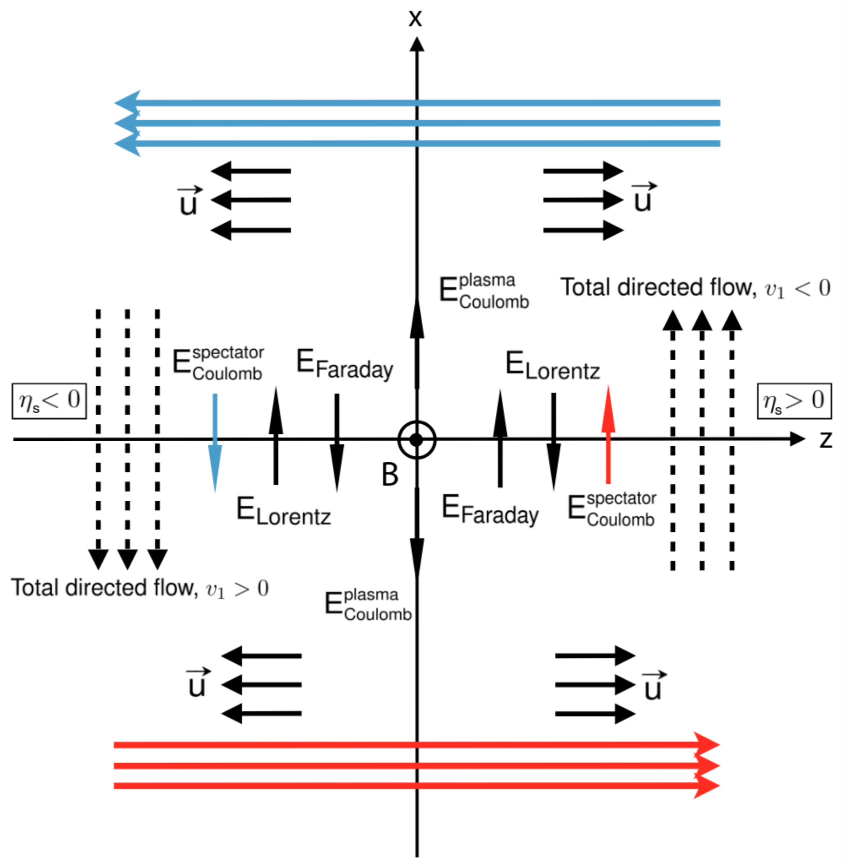


Image: U. Gürsoy, D. Kharzeev, E. Marcus, K. Rajagopal, and C. Shen, Phys. Rev. C 98, 055201 (2018).

Magnetic fields in heavy-ion collisions

Collision system with the same two nuclei

- Electromagnetic fields are a superposition (summation) from all the protons in the nuclear fragments

$$E_{x,y} = \int d^3xq \frac{\vec{x}\sigma}{4(t-z/v)^2} e^{\frac{-\vec{x}^2\sigma}{4(t-z/v)}},$$

$$E_z = -\int d^3xq \frac{(t-z/v) - \vec{x}^2\sigma/4}{\gamma^2(t-z/v)^3} e^{\frac{-\vec{x}^2\sigma}{4(t-z/v)}}$$

$$B_{\text{fragment}} = v \times E$$

$$\mathcal{B}_{\text{total}} = B_{\text{fragment 1}} + B_{\text{fragment 2}}$$

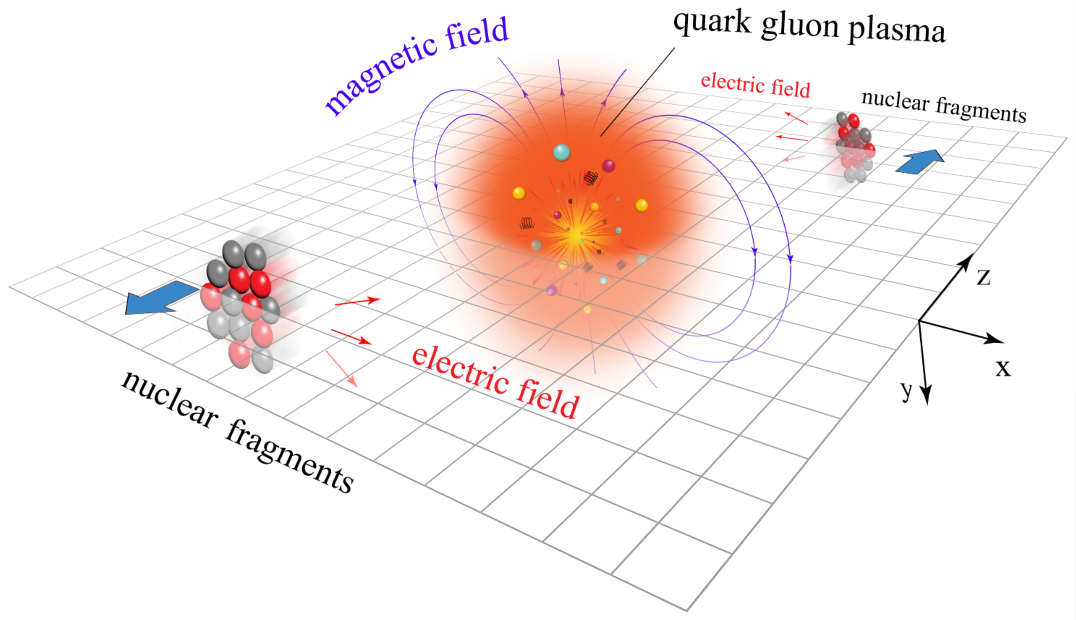
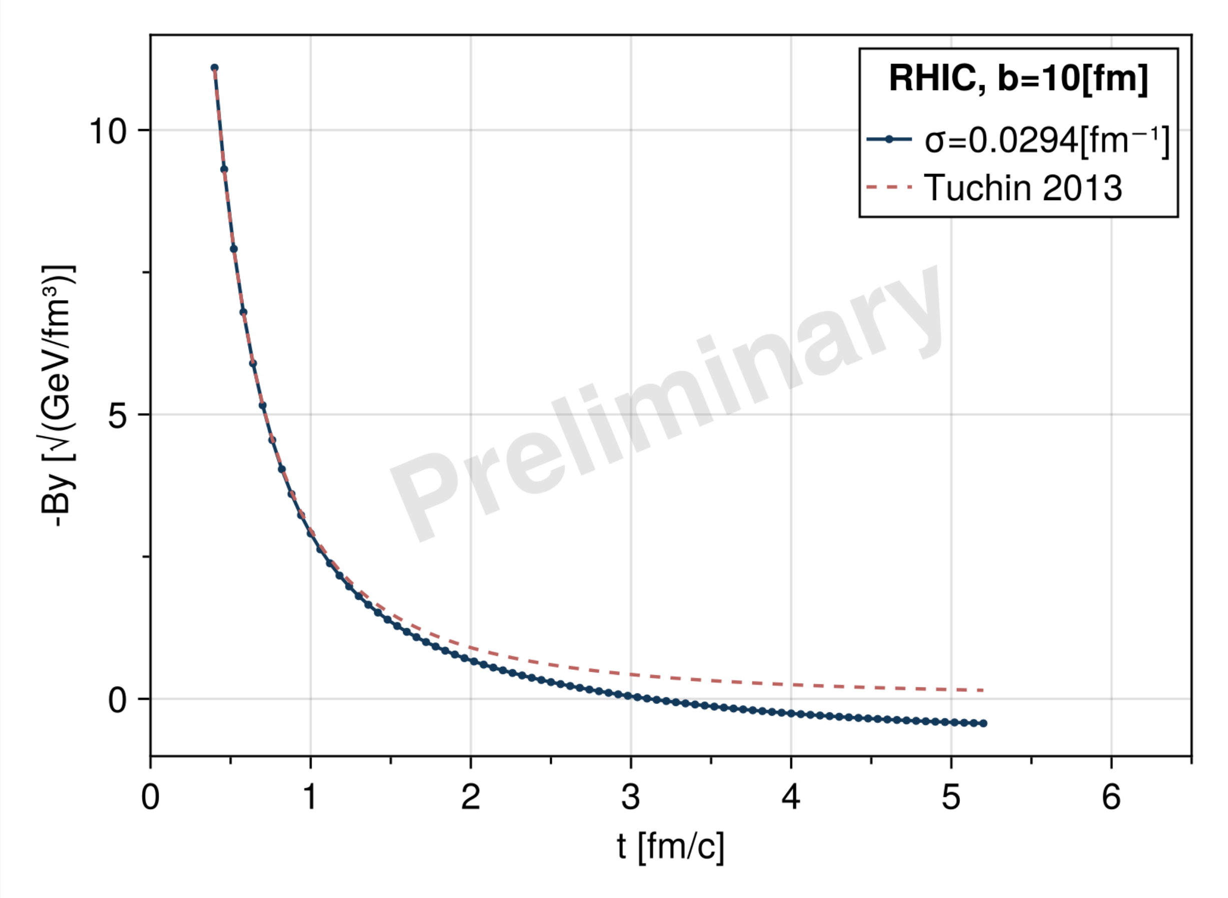


Image: STAR Collaboration (2024)

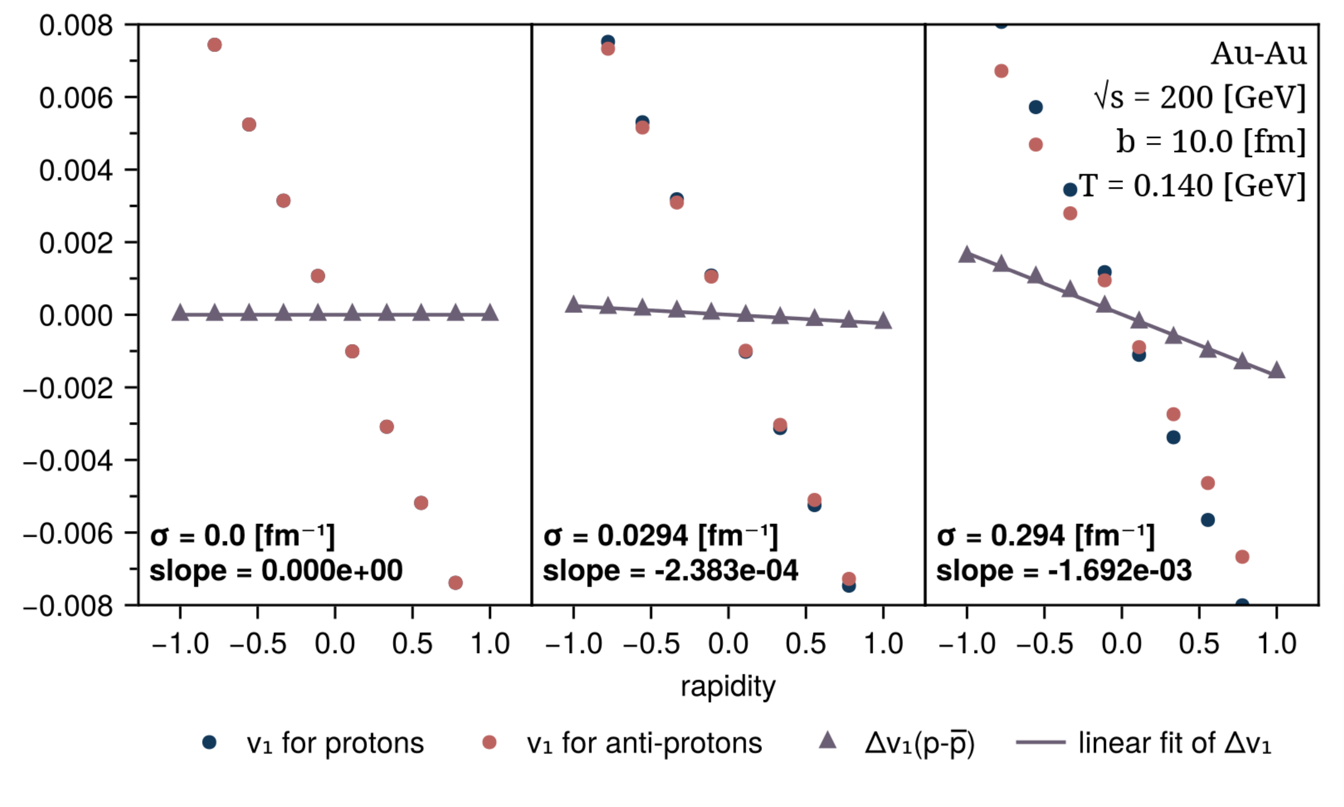
Time evolution of the magnetic field RRMHD

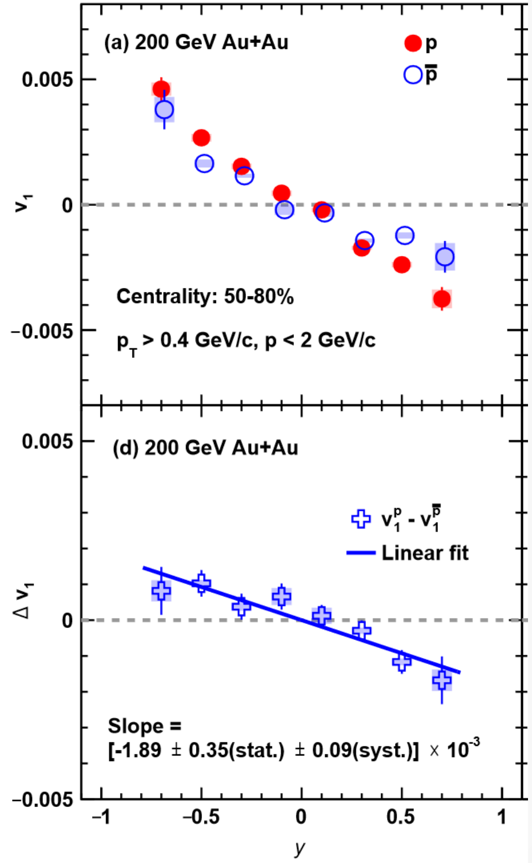


- Time evolution of the magnetic field at the center of the grid (x=0, y=0, \eta = 0) comparing our RRMHD model to an analytic calculation of the field generated by the collision fragments.
- The first point is equal, because the initial condition of the RRMHD model is the analytic solution.
- We attribute the differences between the models at later times to be due to the EM sources. The analytic solution only includes the ions as a source, whereas the RRMHD model only includes QGP charges as a source.

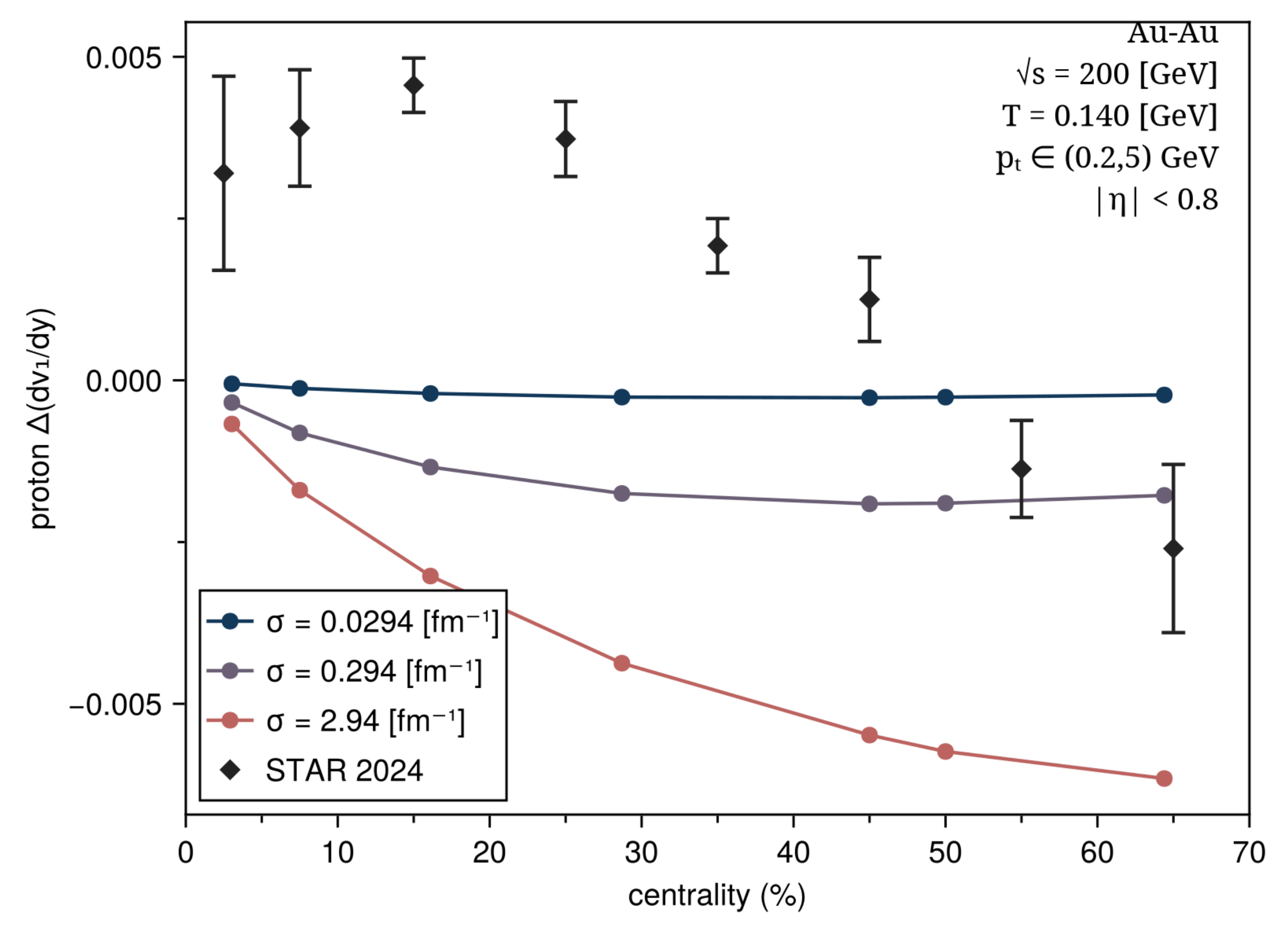
Charged directed flow results

Our RRMHD model vs the STAR 2024 experimental results





Centrality dependence of charged directed flow



Introduction to our model for QGP

Relativistic Resistive Magneto-Hydrodynamics (RRMHD) Model

- Start with ideal relativistic hydrodynamics
- Assume the plasma has a main fluid-current

$$\nabla_\mu N^\mu = 0$$ Continuity equation (i.e. net-baryon current)

$$\nabla_\mu T^{\mu\nu} = 0$$ Total energy-momentum tensor

$$\nabla_\mu S^\mu \geq 0$$ 2nd law of thermodynamics

- Assume a secondary current in the plasma creates the EM fields
- Then, assume the EM fields are too short-lived for nonlinear effects
- Also the plasma is highly ionized, so magnetization/polarization are ignored

$$\alpha = \frac{n_i}{n_i + n_n} \approx 1$$

Ionization ratio captures how much of the plasma number density is neutral

$$\nabla_\mu F^{\mu\nu} = -J^\nu$$
$$\nabla_\mu {}^*F^{\mu\nu} = 0$$

Maxwell's equations

Introduction to our model

Model created by K. Nakamura, T. Miyoshi, C. Nonaka, and H. R. Takahashi

- To summarize, we use ideal relativistic hydrodynamics + Maxwell's equations

$$\nabla_\mu N^\mu = 0 \qquad \nabla_\mu F^{\mu\nu} = -J^\nu$$
$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu {}^*F^{\mu\nu} = 0$$
$$\nabla_\mu S^\mu \geq 0 \qquad J^\mu = q u^\mu + \sigma e^\mu$$

→ system of semi-hyperbolic differential equations

$$\partial_0[\sqrt{-g}\mathbf{U}] + \partial_i[\sqrt{-g}\mathbf{F}^i] = \sqrt{-g}(\mathbf{S}_e + \mathbf{S}_s)$$

conserved variables, fluxes, explicit source terms, stiff source terms

$$\mathrm{U} = \begin{pmatrix} D \\ \Pi_j \\ \epsilon \\ B^j \\ E^j \\ q \\ \vdots \end{pmatrix} \quad \mathrm{F}^i = \begin{pmatrix} Dv^i \\ T^i_j \\ \Pi^i \\ \epsilon^{jik}E_k + g^{ij}\psi \\ -\epsilon^{jik}B_k + g^{ij}\phi \\ J^i \\ \vdots \end{pmatrix} \quad \mathrm{S}_e = \begin{pmatrix} 0 \\ \frac{1}{2}T^{ik}\partial_j g_{ik} \\ -\frac{1}{2}T^{ik}\partial_0 g_{ik} \\ 0 \\ -qv^i \\ 0 \\ \vdots \end{pmatrix} \quad \mathrm{S}_s = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -J^i_c \\ 0 \\ \vdots \end{pmatrix}$$

Solving Relativistic Resistive Magneto-Hydrodynamics

These are difficult differential equations

- Require non-trivial initial conditions to solve (non-equilibrium)
- General analytic solutions are not known (semi-hyperbolic equations)
- We will use a curvilinear coordinate system (Milne coordinates)

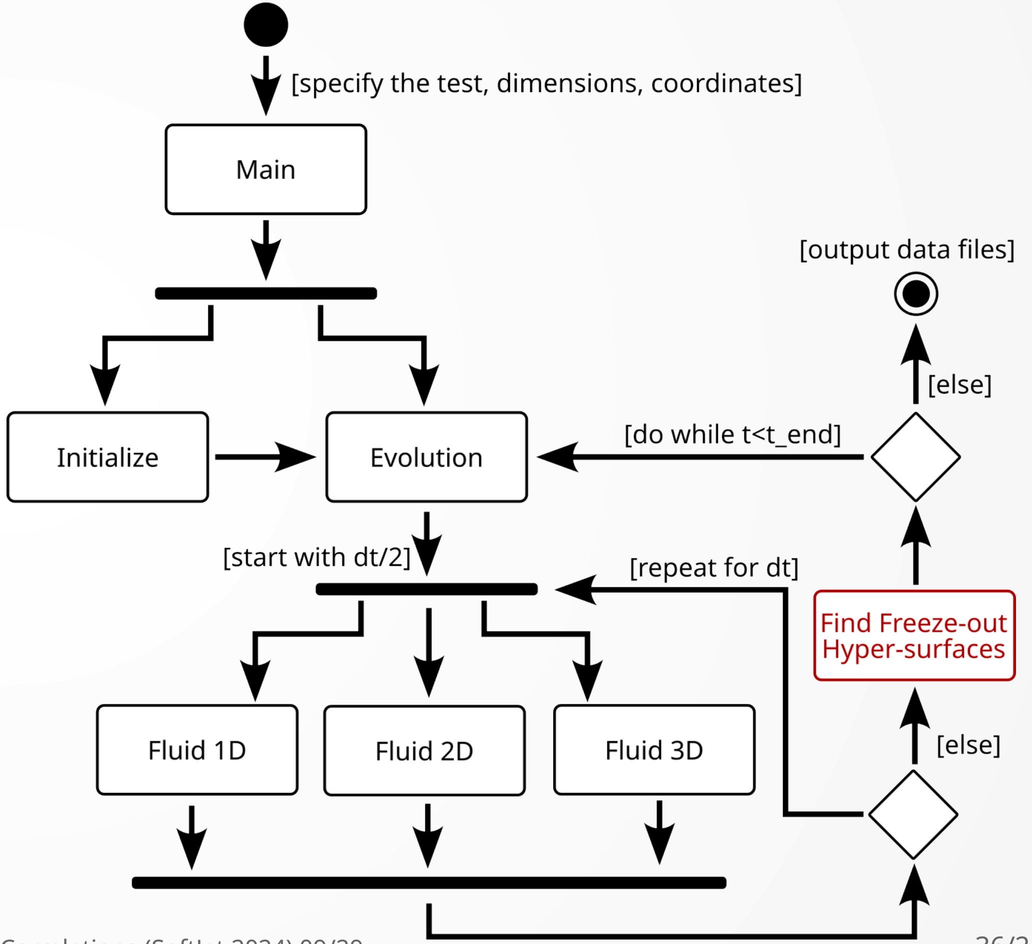
Solution

Use numerical methods from computational physics

Solving Relativistic Resistive Magneto-Hydrodynamics

Overview of the numerical model

- Create initial conditions
- Solve differential equations
  - Strang Splitting method
  - Runge-Kutta 2
  - primitive variable second order interpolation
- At freezeout, stop time evolution and output data
- Process data using Cooper-Frye method



Heavy-Ion Collisions

Relativistic charges in heavy-ion collisions

- Heavy nuclei are accelerated to relativistic velocities to study QCD
  - High-energy could be 200 GeV, ~2.56 TeV, ~6.78 TeV
  - Heavy nuclei = e.g., Au, Pb, Cu, U, Ru, Zr, …

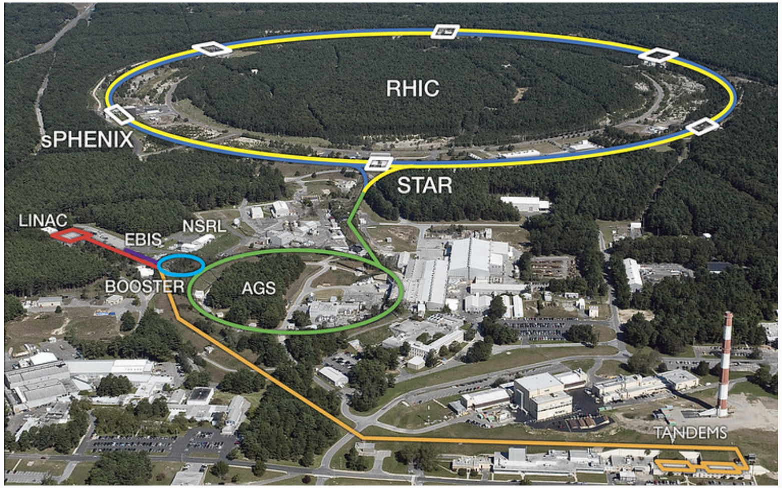


Image: Brookhaven National Lab (BNL)
https://www.bnl.gov/rhic/

Why study quark-gluon plasma (QGP)?

Quantum Chromodynamics (QCD)

- At high temperatures hadrons are melted into a strongly interacting quark-gluon plasma (QGP)
- In QGP quarks and gluons are a many-body system that acts with cohernet motion
- The motion is described using various transport parameters