Thermalization Of Jets In QCD Plasma

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Based on:

- IS arXiv:2409.04806
- Y. Mehtar-Tani, S. Schlichting, and IS JHEP 05 (2023) 091
- S. Schlichting, IS JHEP 07 (2021), 077

SoftJet 2024, Tokyo, Japan

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- Introduction
- Effective Kinetic Description
- Longitudinal Energy Loss
- Thermalization To Large Angles
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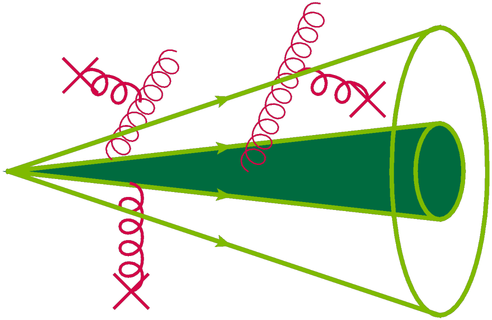
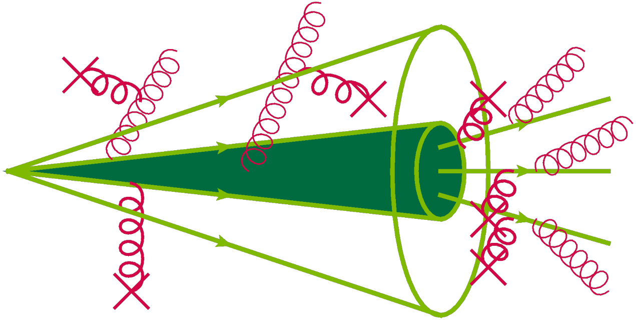
Introduction I



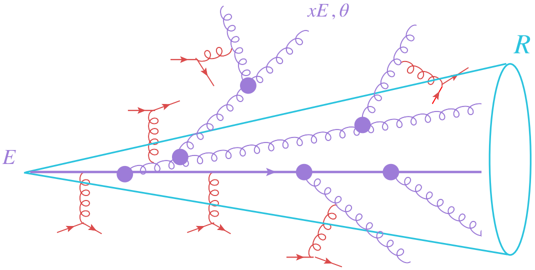
- Hard parton evolution in heavy ion collisions

⊙ Introduction I



- Hard parton evolution in heavy ion collisions
- Focus on energy loss and equilibration of hard partons ⇒ resolved by the medium

⊙ In-Medium Shower



- Main focus: Hard parton traversing a QCD plasma
- Understand: Energy cascade, out-of-cone energy loss, medium response and full thermalization of the shower ⇒ Important for low energy jets

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⊙ Effective Kinetic Description

- Leading Order Effective Kinetic Theory:

$$p^\mu \partial_\mu f_i(\vec{x}, \vec{p}, t) = C_i[\{f_i\}] \; , \tag{1}$$

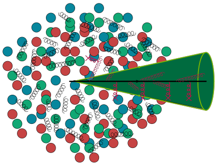
- Hard partons ⇒ Linearized Fluctuation on top of Static Equilibrium

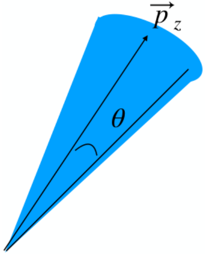
$$f(p,t) = n_{\text{eq}}(p,t) + \delta f_{\text{jet}}(p,t) \; , \tag{2}$$

- Energy distribution

$$D(x,\theta,t) \equiv x \frac{dN_a}{dx d\cos\theta} \sim \frac{\nu_a(N_f)}{E_j} \delta f_{\text{jet}}(p,\theta,t) \; . \tag{3}$$

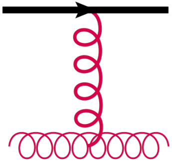
- Exact conservation of energy, momentum and valence charge → Study evolution from $\sim E$ to $\sim T$ including full thermalization



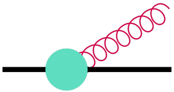


AMY EKT

- Elastic Scatterings using HTL screened $2 \leftrightarrow 2$



- Collinear Radiation effective $1 \leftrightarrow 2$



$$C[\{f_i\}] = C^{2\leftrightarrow 2}[\{f_i\}] + C^{1\leftrightarrow 2}[\{f_i\}] \,. \tag{4}$$

- Analogous to QGP thermalization in pre-Hydro phase
- Using AMY rates for in-medium radiation: $t_{\text{form}} \sim \sqrt{\frac{z(1-z)P}{\hat{q}}} \ll L$

Elastic Scattering

Elastic scattering collision integral with HTL screened matrix elements

$$C_a^{2\leftrightarrow 2}[\{f_i\}] = \frac{1}{2|p_1|\nu_a} \sum_{bcd} \int d\Omega^{2\leftrightarrow 2} |\mathcal{M}^{ab}_{cd}(p_1, p_2, p_3, p_4)|^2 \delta\mathcal{F}(p, p_1, p_2, p_3, p_4) \tag{5}$$

Full detailed balance terms ⇒ Medium response

$$\begin{aligned} \delta F(p_1, p_2, p_3, p_4) = \quad & \delta f_a(p_1)[\pm_a n_c(p_3) n_d(p_4) - n_b(p_2)(1 \pm n_c(p_3) \pm n_d(p_4))] \\ & + \delta f_b(p_2)[\pm_b n_c(p_3) n_d(p_4) - n_a(p_1)(1 \pm n_c(p_3) \pm n_d(p_4))] \\ & - \delta f_c(p_3)[\pm_c n_a(p_1) n_b(p_2) - n_d(p_4)(1 \pm n_a(p_1) \pm n_b(p_2))] \\ & - \delta f_d(p_4)[\pm_d n_a(p_1) n_b(p_2) - n_c(p_3)(1 \pm n_a(p_1) \pm n_b(p_2))] \,. \end{aligned} \tag{6}$$

⊙ Collinear Radiation

Collinear radiation (including BH & LPM effects)

$$C_g^{g\leftrightarrow gg}[\{D_i\}] = \int_0^1 dz \frac{d\Gamma^g_{gg}(\left(\frac{xE}{z}\right), z)}{dz} \Bigg[D_g\left(\frac{x}{z}\right)\left(1 + n_B(xE) + n_B\left(\frac{\bar{z}xE}{z}\right)\right)$$

$$+ \frac{D_g(x)}{z^3}\left(n_B\left(\frac{xE}{z}\right) - n_B\left(\frac{\bar{z}xE}{z}\right)\right) + \frac{D_g\left(\frac{\bar{z}xE}{z}\right)}{\bar{z}^3}\left(n_B\left(\frac{xE}{z}\right) - n_B(xE)\right)\Bigg]$$

$$- \frac{1}{2}\int_0^1 dz \frac{d\Gamma^g_{gg}(xE, z)}{dz}\Bigg[D_g(x)(1 + n_B(zxE) + n_B(\bar{z}xE))$$

$$+ \frac{D_g(zx)}{z^3}(n_B(xE) - n_B(\bar{z}xE)) + \frac{D_g(\bar{z}x)}{\bar{z}^3}(n_B(xE) - n_B(zxE))\Bigg],$$

- **AMY Radiation Rate**

$$\frac{d\Gamma^a_{bc}}{dz}(P, z, \infty) = \frac{g^2 P^a_{bc}(z)}{4\pi P^2 z^2 (1-z)^2} \mathrm{Re} \int_0^\infty dt_1 \int_{\boldsymbol{p},\boldsymbol{q}} \frac{i\boldsymbol{q}.\boldsymbol{p}}{\delta E(\boldsymbol{q})} \Gamma_3(t) \circ G(\infty, \boldsymbol{q}; t_1, \boldsymbol{p}) \,. \qquad (7)$$

- **Merging/Splitting ⇒ Thermalization of soft sector**

⊙ Outline

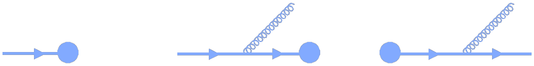
- ⊙ Introduction
- ⊙ Effective Kinetic Description
- ⊙ Longitudinal Energy Loss
- ⊙ Thermalization To Large Angles
- ⊙ Beyond AMY

Longitudinal Energy Loss

- Evolution can be divided into three regimes:
  1. Initial energy loss: mediated by single gluon radiation
  2. Energy cascade: successive emissions lead to an energy cascade
  3. Equilibration: Late time decay to soft sector

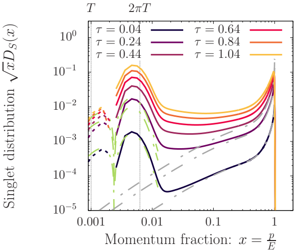
⊙ Single Emission

- Initial distribution determined from direct emissions



$$D_q(x,\tau) \simeq \delta(1-x) + \left[x\frac{d\Gamma^q_{gq}(x)}{dz} - \int_0^1 dz\; z\frac{d\Gamma^q_{gq}(x,z)}{dz}\delta(1-x)\right]\tau \tag{8}$$

- Short lived stage ⇒ Subsequent emissions lead to energy cascade



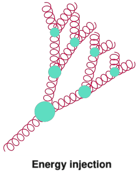
⊙ Energy Cascade

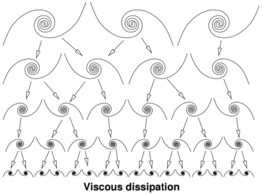
- Stationary turbulent solution in intermediate energy range $T/E \ll x \ll 1$:

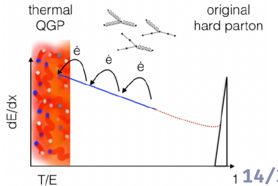
$$D_g(x) = \frac{G}{\sqrt{x}}\,, \qquad D_S(x) = \frac{S}{\sqrt{x}}\,. \tag{9}$$

- Fixed point of the differential equation

$$C_g[\{D_i\}] = \int_0^1 dz\, \frac{d\Gamma^g_{gg}\left(\frac{xE}{z}, z\right)}{dz} D_g\left(\frac{x}{z}\right) - \frac{1}{2}\frac{d\Gamma^g_{gg}(xE, z)}{dz} D_g(x) + \int_0^1 dz\, \frac{d\Gamma^g_{gq}\left(\frac{xE}{z}, z\right)}{dz} D_S\left(\frac{x}{z}\right) - N_f \int_0^1 dz\, \frac{d\Gamma^g_{q\bar{q}}(xE, z)}{dz} D_g(x)\,, \tag{10}$$





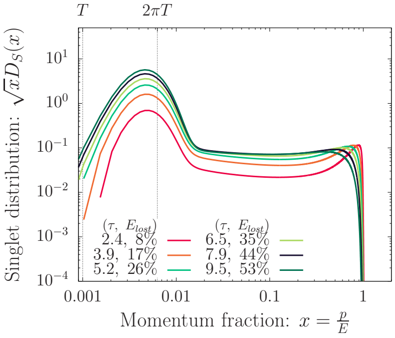


Energy Cascade

- **Stationary turbulent solution in intermediate energy range** $T/E \ll x \ll 1$**:**

$$D_g(x) = \frac{G}{\sqrt{x}}\,, \qquad D_S(x) = \frac{S}{\sqrt{x}}\,. \tag{9}$$

- **Existence of a fixed point of the differential equation comes from the fact that the rate behaves as** $1/\sqrt{xE}$



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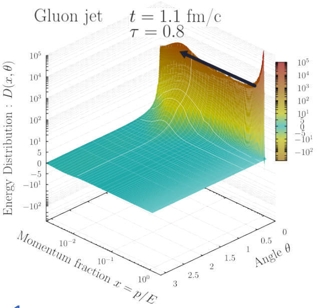
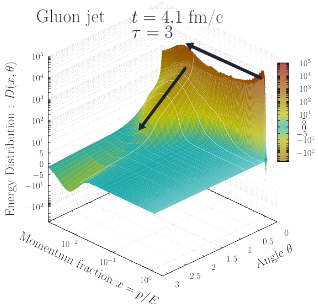
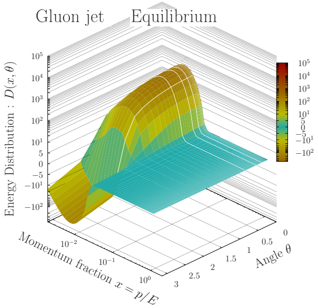
- **Thermalization To Large Angles**
- Beyond AMY

⊙ Thermalization To Large Angles

- Energy cascade to soft sector
- Broadening of soft partons to large angle
- Thermalization of soft sector





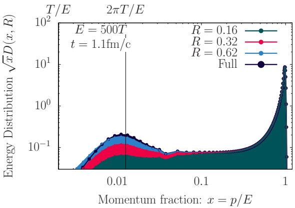


[1]

[1] $E/T = 500,\ g = 2.$

Thermalization To Large Angles

- Energy cascade to soft sector



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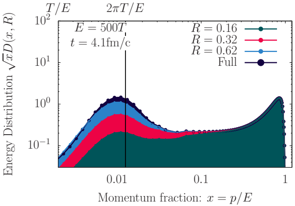
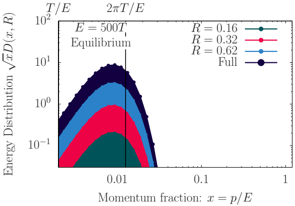
- Broadening of soft partons to large angle



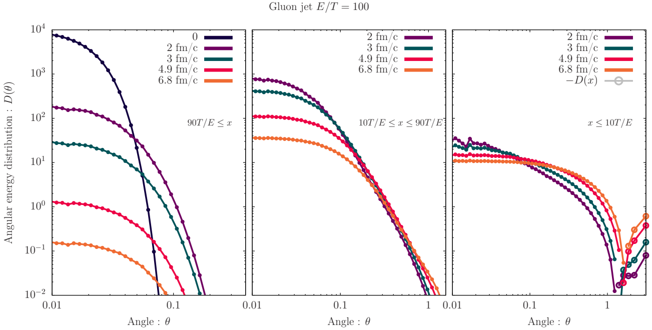
- Thermalization of soft sector



[1] $E/T = 500$, $g = 2$.

⊙ Molière Scattering

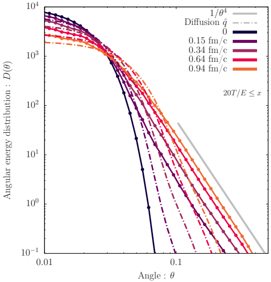
- **Angular distribution in different energy regimes**



$$\partial_t D(x, \mathbf{p}_\perp, t) \simeq \int_{\mathbf{q}_\perp} C(\mathbf{q}_\perp) \left[D(x, \mathbf{p}_\perp - \mathbf{q}_\perp, t) - D(x, \mathbf{p}_\perp, t) \right]$$

$$+ \sum_{bc} \int_0^1 dz \left[\frac{1}{z^2} \frac{d\Gamma^b_{ac}}{dz} \left(\frac{xE}{z}, z \right) D\left(\frac{x}{z}, \frac{\mathbf{p}_\perp}{z}, t \right) - \frac{1}{2} \frac{d\Gamma^a_{bc}}{dz} \left(xE, z \right) D\left(x, \mathbf{p}_\perp, t \right) \right] , \qquad (10)$$

- **Hard sector**



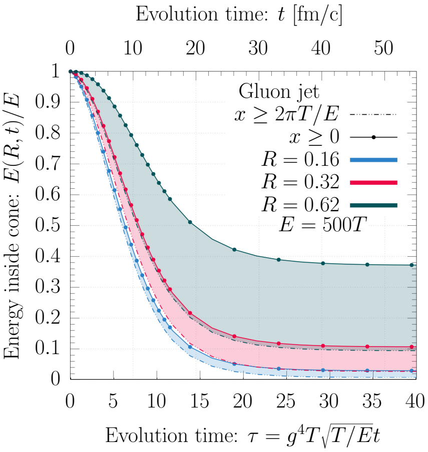
$$C(\mathbf{q}_\perp) \propto \frac{1}{\mathbf{q}_\perp^2 (\mathbf{q}_\perp^2 + \mathbf{m}_D^2)} \qquad (11)$$

⊙ Cone-Size Dependence

: $E(R,\tau) = \int_0^\infty dx \int_{\cos R}^1 d\cos\theta \;\; D(x,\cos\theta,\tau)$ (12)

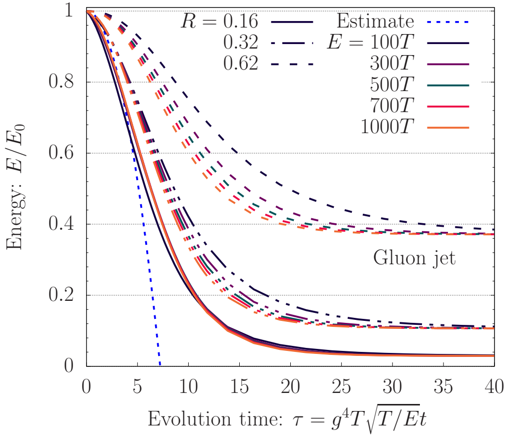
- Small cone-sizes: Soft sector does not play major role
- Large cone-sizes: Energy loss display different behavior ⇒ Dominated by thermalization

: $E_{2\pi}(R,\tau) = \int_{2\pi T/E}^\infty dx \int_{\cos R}^1 d\cos\theta \;\; D(x,\cos\theta,\tau)$ (13)



⊙ Sensitivity To Initial Parton's Energy

- Characteristic time of the turbulent cascade $t_{th} = \frac{1}{\alpha_s}\sqrt{\frac{E}{\hat{q}}}$
- Scaling between different initial parton's energy for small cone-sizes
- Broken for large cone-sizes



Modeling Jet Quenching

- In the presence of QGP, the jet spectrum factorizes

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T} = \int_0^\infty \mathrm{d}\epsilon \;\; P(\epsilon, R) \frac{\mathrm{d}\sigma_{\mathrm{vac}}}{\mathrm{d}p_T^{in}} (p_T^{in} \equiv p_T + \epsilon) \,, \tag{14}$$

- The energy loss probability $P(\epsilon, R)$ is obtained using the BDMPS rate

$$P(\epsilon, R) = \sum_{n=0}^{\infty} \frac{1}{n!} \left[\prod_{i=1}^{n} \omega_i \frac{\mathrm{d}I}{\omega_i} \mathrm{d}\omega_i \right] \delta\left(\epsilon - \sum_{i=1}^{n} \omega_i \right) \exp\left[- \int_0^\infty \mathrm{d}\omega \frac{\mathrm{d}I}{\mathrm{d}\omega} \right] \tag{15}$$

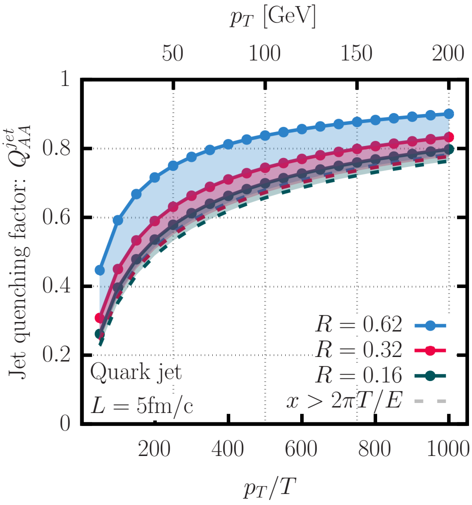
- Using a Milne transform $\Rightarrow$ Exponential
- Taking vacuum spectrum $\frac{\mathrm{d}\sigma_{\mathrm{vac}}}{\mathrm{d}p_T^{in}}(p_T^{in}) \propto (p_T^{in})^{-\alpha}$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T} = Q(p_T, R) \frac{\mathrm{d}\sigma_{\mathrm{vac}}}{\mathrm{d}p_T} \,, \quad \Rightarrow \quad Q(p_T, R) \approx \exp\left[- \int \mathrm{d}\omega \frac{\mathrm{d}I}{\mathrm{d}\omega} \left(1 - \mathrm{e}^{-n\omega/p_T} \right) \right] \tag{16}$$

⊙ Modeling Jet Quenching

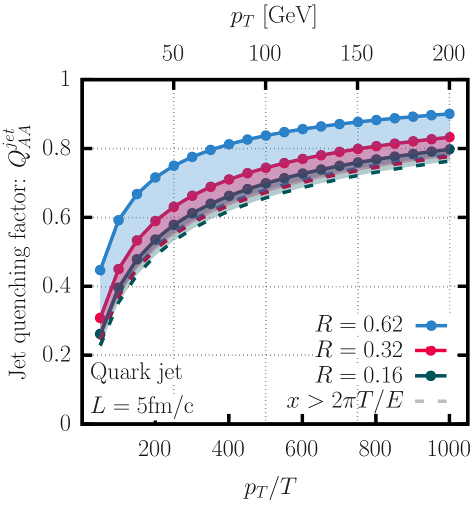
- The first emission is modeled using BDMPS finite medium rate $\frac{d\Gamma}{d\omega}(P,\omega,t)$ at time t
- Medium energy loss computed by modeling the energy remaining inside the cone $E(\omega, R, L-t)$ after a time $(L-t)$ in the medium

$$Q(p_T) = \exp\left[\int_0^L \mathrm{d}t \int \mathrm{d}\omega \times \frac{\mathrm{d}\Gamma}{\mathrm{d}\omega}\left(1 - \mathrm{e}^{-n\frac{\omega}{p_T}\left[1-\frac{E\left(\omega\,|\,R,\tau=\frac{L-t}{t_{\mathrm{th}}}\right)}{\omega}\right]}\right)\right]. \quad (17)$$



⊙ Modeling Jet Quenching

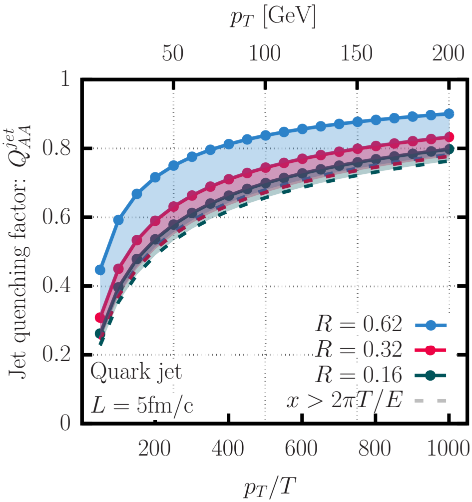
- Jets with energies $p_T \leq 25$GeV lose significant energy in length $L = 5fm$/c
  $\Rightarrow$ large suppression at low p_T, milder for $p_T \leq 200-400$GeV
- Negligible contribution of soft fragments to narrow cone
- Large cone size (≥ 0.3) $\Rightarrow$ Recover energy from the soft sector
  ▷ Medium response



⊙ Modeling Jet Quenching

⚠ Over-estimate energy loss since we neglect finite size effects, Work in progress

⚠ Requires more refined studies of near-equilibrium physics and jet recoil onto the medium



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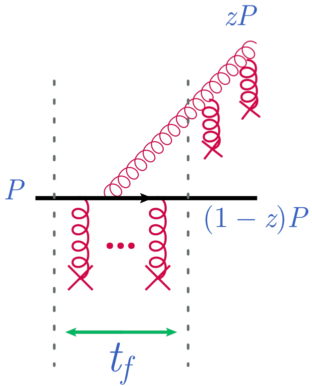
⊙ Collinear Radiation



- Multiple scatterings ⇒ induced radiation
- Emission controlled by the formation time

$$t_{\text{form}} \sim \frac{z(1-z)xE}{k_T^2} \quad , \quad k_T^2 \sim \hat{q} t_{\text{form}} \Rightarrow t_{\text{form}} \sim \sqrt{\frac{z(1-z)xE}{\hat{q}}} \,, \qquad (17)$$

- $t_{\text{form}} \ll \lambda_{\text{mfp}}$: Medium cannot resolve the quanta until it is formed
- $t_{\text{form}} \gg \lambda_{\text{mfp}}$: Multiple scatterings act coherently

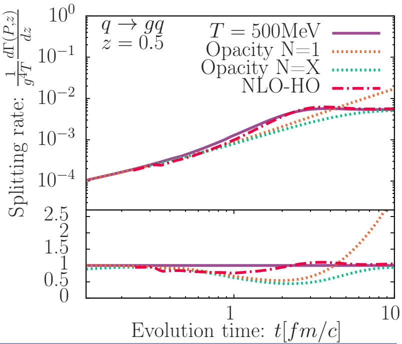
- Quantum interference ⇒ suppression of high energy radiation
  ⇒ LPM effect

[1](Baier, Dokshitzer, Mueller, Peigné, Schiff, Zakharov, Wiedemann, Arnold, Moore, Yaffe..)

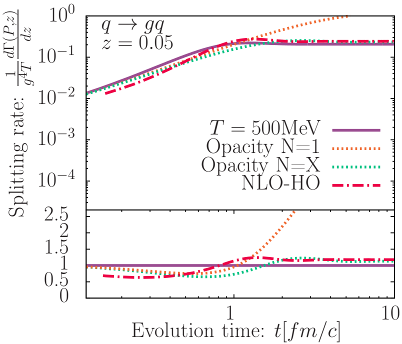
⊙ Finite Size Effects: Rates

$$\frac{d\Gamma^a_{bc}}{dz}(P, z, t) = \frac{g^2 P^a_{bc}(z)}{4\pi P^2 z^2 (1-z)^2} \mathrm{Re} \int_0^t dt_1 \int_{\boldsymbol{p},\boldsymbol{q}} \frac{i\boldsymbol{q}.\boldsymbol{p}}{\delta E(\boldsymbol{q})} \Gamma_3(t) \circ G(\infty, \boldsymbol{q}; t_1, \boldsymbol{p}) \, . \qquad (18)$$

- **Long formation time:** $z \simeq 0.5$



- **Short formation time:** $z(1-z) \simeq 1$



[1] S. Caron-Huot, C. Gale PRC(2010), S. Schlichting IS PRD(2022)

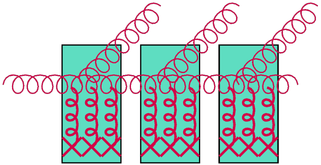
Finite Size Effects: Evolution I

- Including finite size effects in the evolution
- Global formation time:

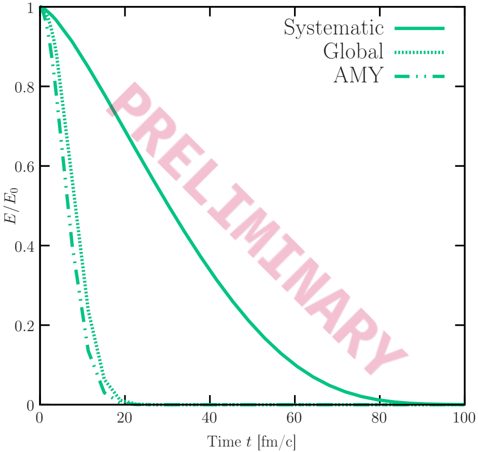
$$\partial_t D(x) = \int_0^1 dz \left[\frac{d\Gamma^a_{bc}}{dz} \left(\frac{x}{z}, z, \boldsymbol{t} \right) zD\left(\frac{x}{z}\right) - \frac{d\Gamma^a_{bc}}{dz}(x, z, \boldsymbol{t}) zD(x) \right]$$

- Systematic formation time:

$$\partial_t D(x) = \int_0^t ds \int_0^1 dz \left[\frac{d\Gamma^a_{bc}}{dz} \left(\frac{x}{z}, z, \boldsymbol{t-s} \right) zD\left(\frac{x}{z}\right) - \frac{d\Gamma^a_{bc}}{dz}(x, z, \boldsymbol{t-s}) zD(x) \right]$$

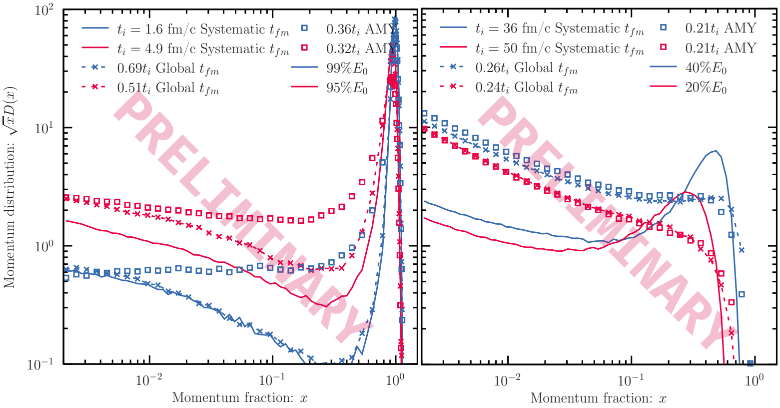


⊙ Finite Size Effects: Evolution II



- Delay of energy loss due to formation time

⊙ Finite Size Effects: Evolution III

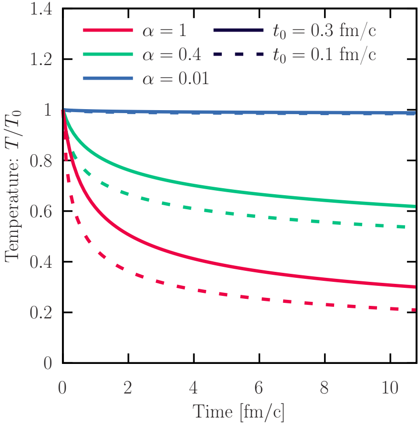


- Markedly different evolution for Systematic treatment of formation time

[1] Barata, Domínguez, Salgado, Vila JHEP (2021)

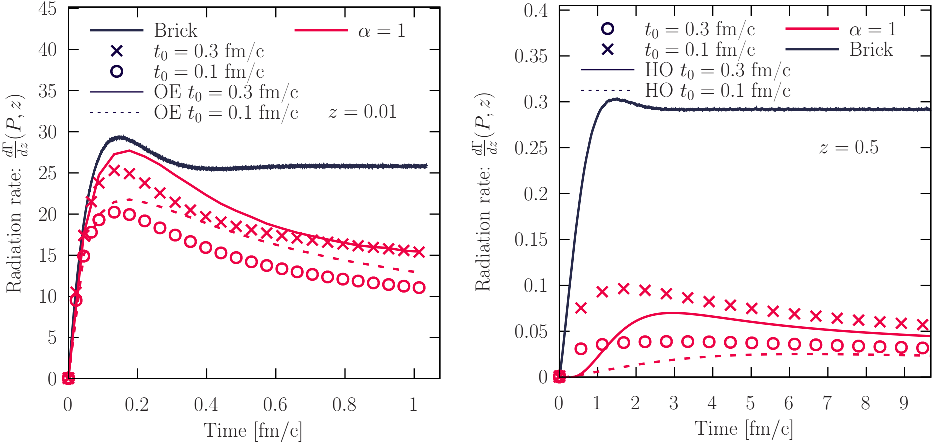
⊙ Bjorken Expansion: Medium-Induced Radiation I

- **Bjorken expansion:** $T(t) = T_0 \left(\frac{t_0}{t+t_0}\right)^{1/3}$



⊙ Bjorken Expansion: Medium-Induced Radiation II

- **Bjorken expansion:** $T(t) = T_0 \left(\frac{t_0}{t+t_0}\right)^{1/3}$



Conclusion

- Energy loss is governed by an inverse energy cascade
  $\Rightarrow$ driven by successive collinear radiation
- Energy deposited collinearly at the soft scales rapidly broadens to large angles
- Formation time lead to dramatic effect on medium-cascade $\Rightarrow$ Delay of energy loss, survival of high energy partons

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Thank you for your attention \o/
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