BUBBLE, BUBBLE, PERTURB & TROUBLE

A NONPERTURBATIVE TEST OF NUCLEATION CALCULATIONS FOR STRONG PHASE TRANSITIONS

With Oliver Gould & David J. Weir
Based on arXiv:2404.01876

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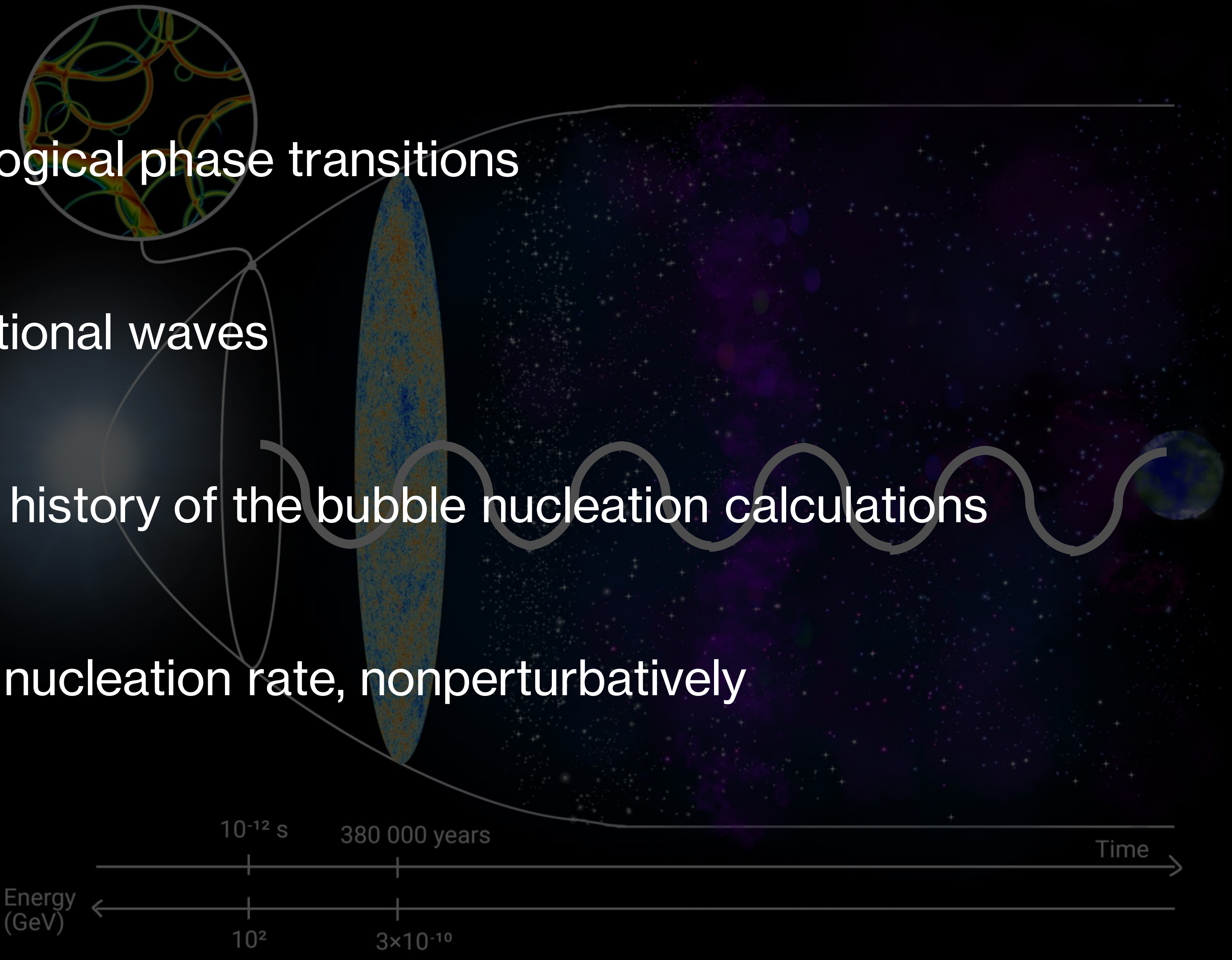
CERN Cosmo Coffee
May 8th 2024
University of Helsinki & Helsinki Institute of Physics

1. Cosmological phase transitions

2. Gravitational waves

3. A short history of the bubble nucleation calculations

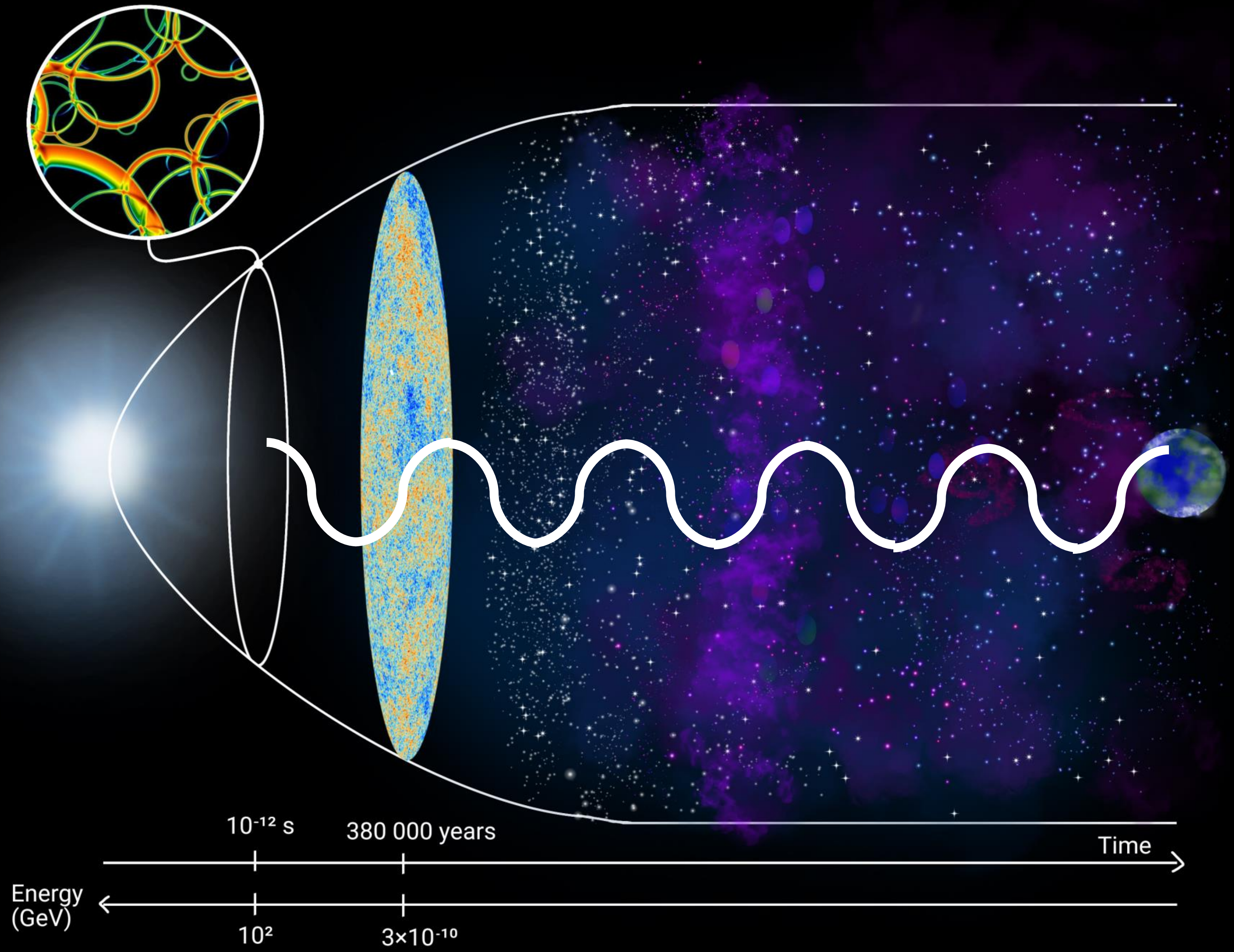
4. Bubble nucleation rate, nonperturbatively



Phase transitions

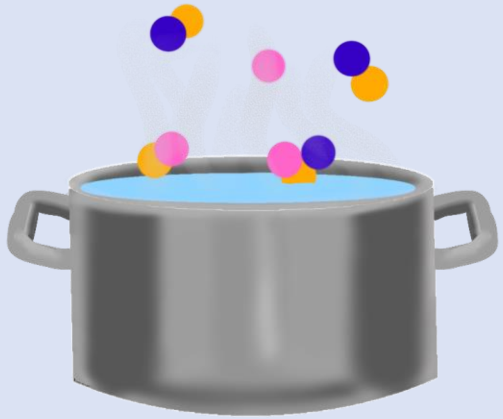
- Grand Unified Theories, Electroweak, QCD…
- In the Standard Model (SM) the electroweak PT is a crossover
- SM is incomplete → beyond SM (BSM) physics
- Things to look for: topological defects, bubbles from EWPT, … ?



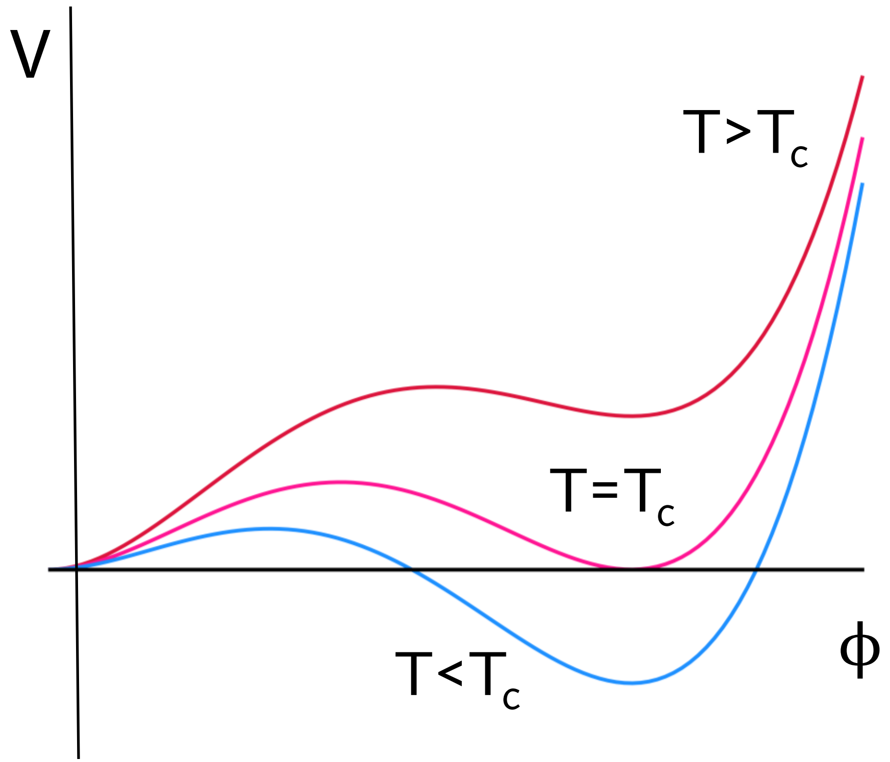


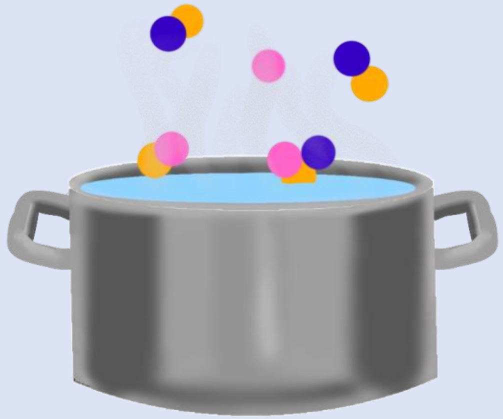
10-12 s
380 000 years
Time
Energy
(GeV)
10²
3×10-10

Electroweak Phase Transition



Electroweak Phase Transition





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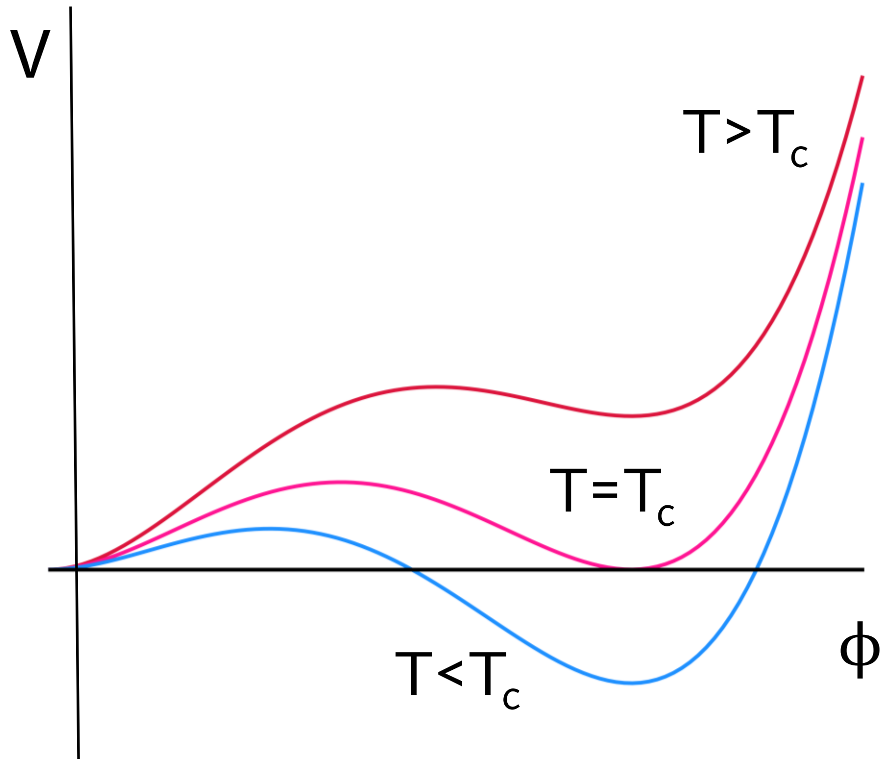

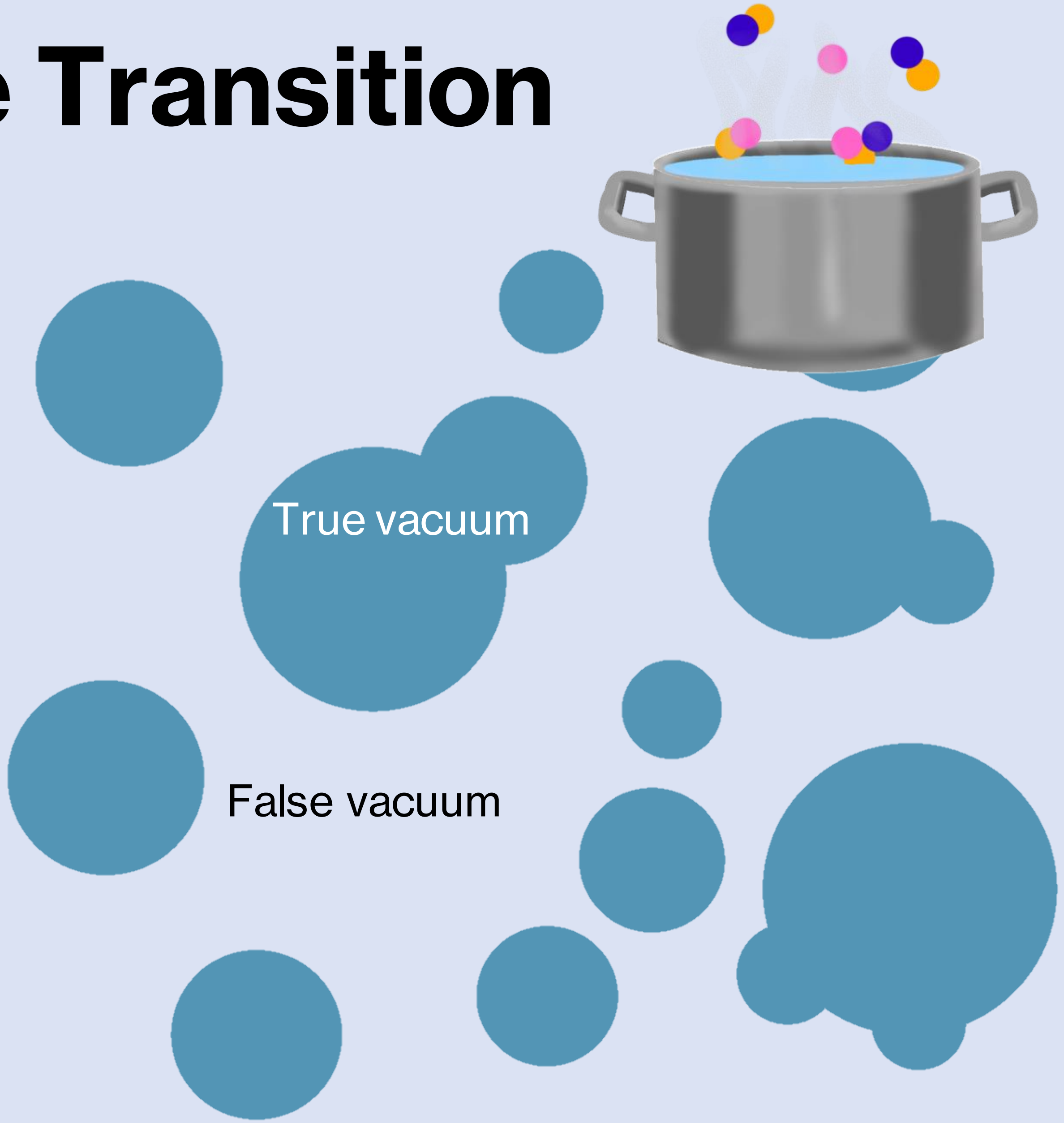

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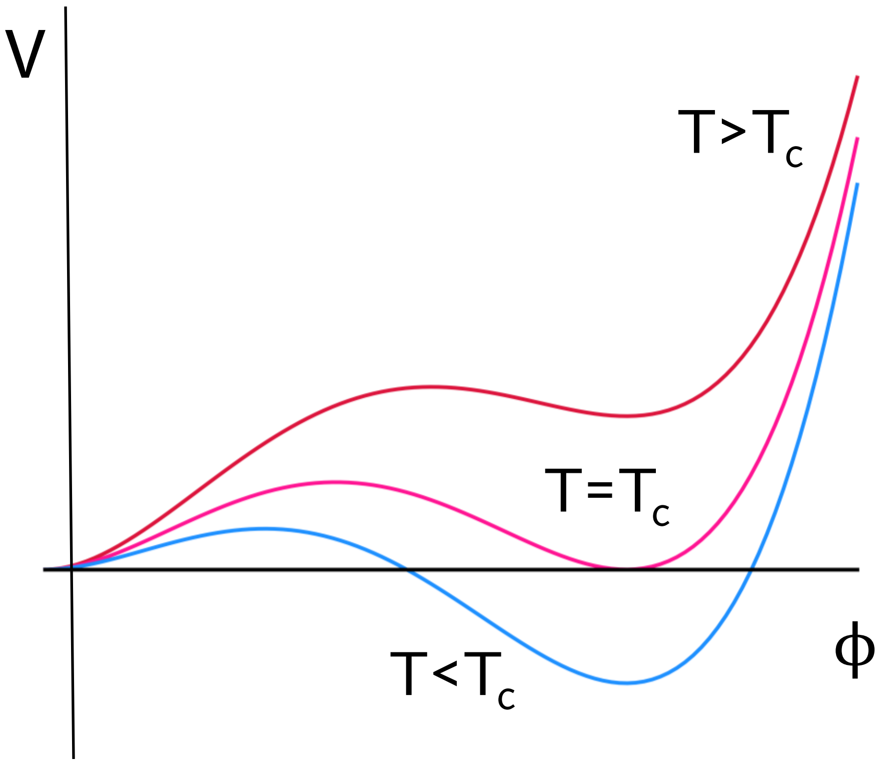
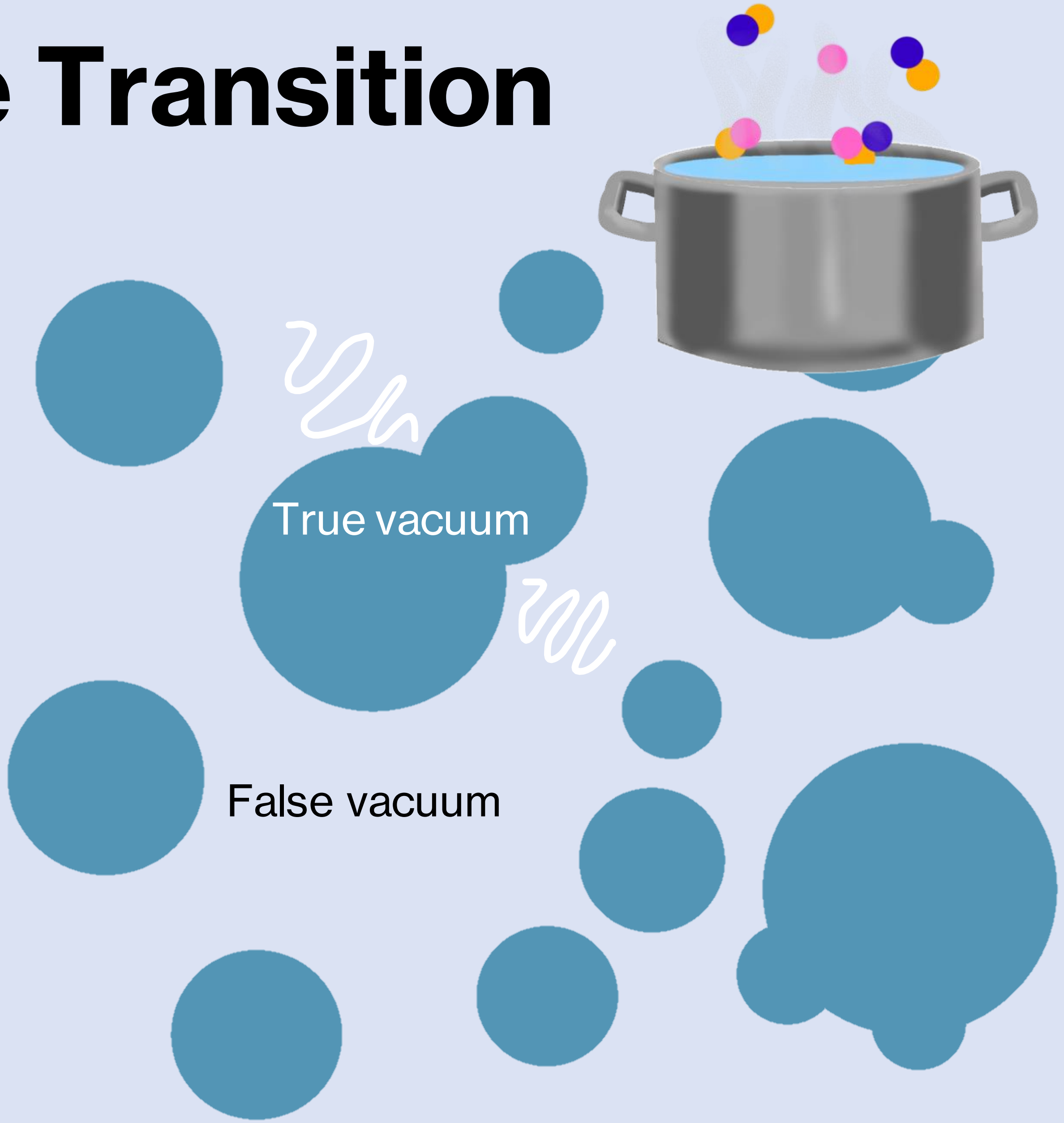
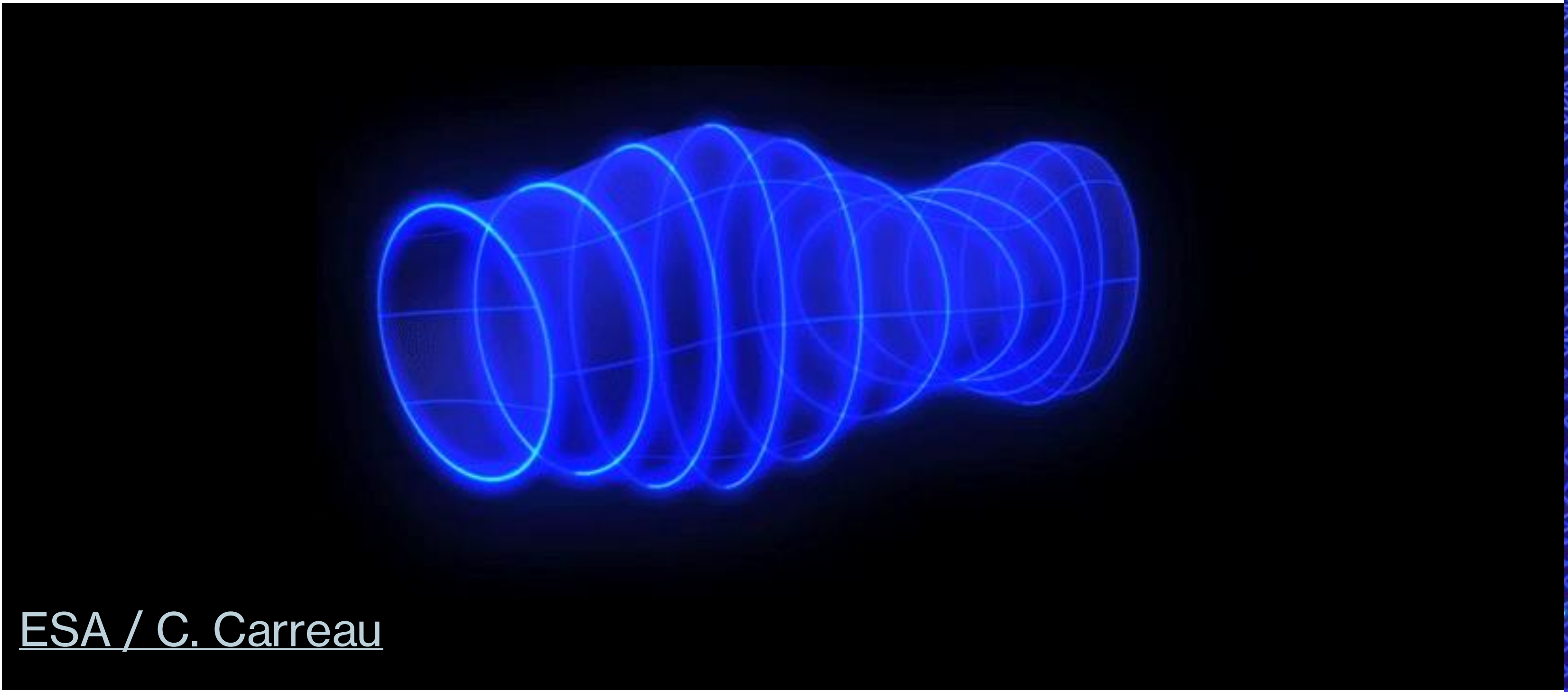




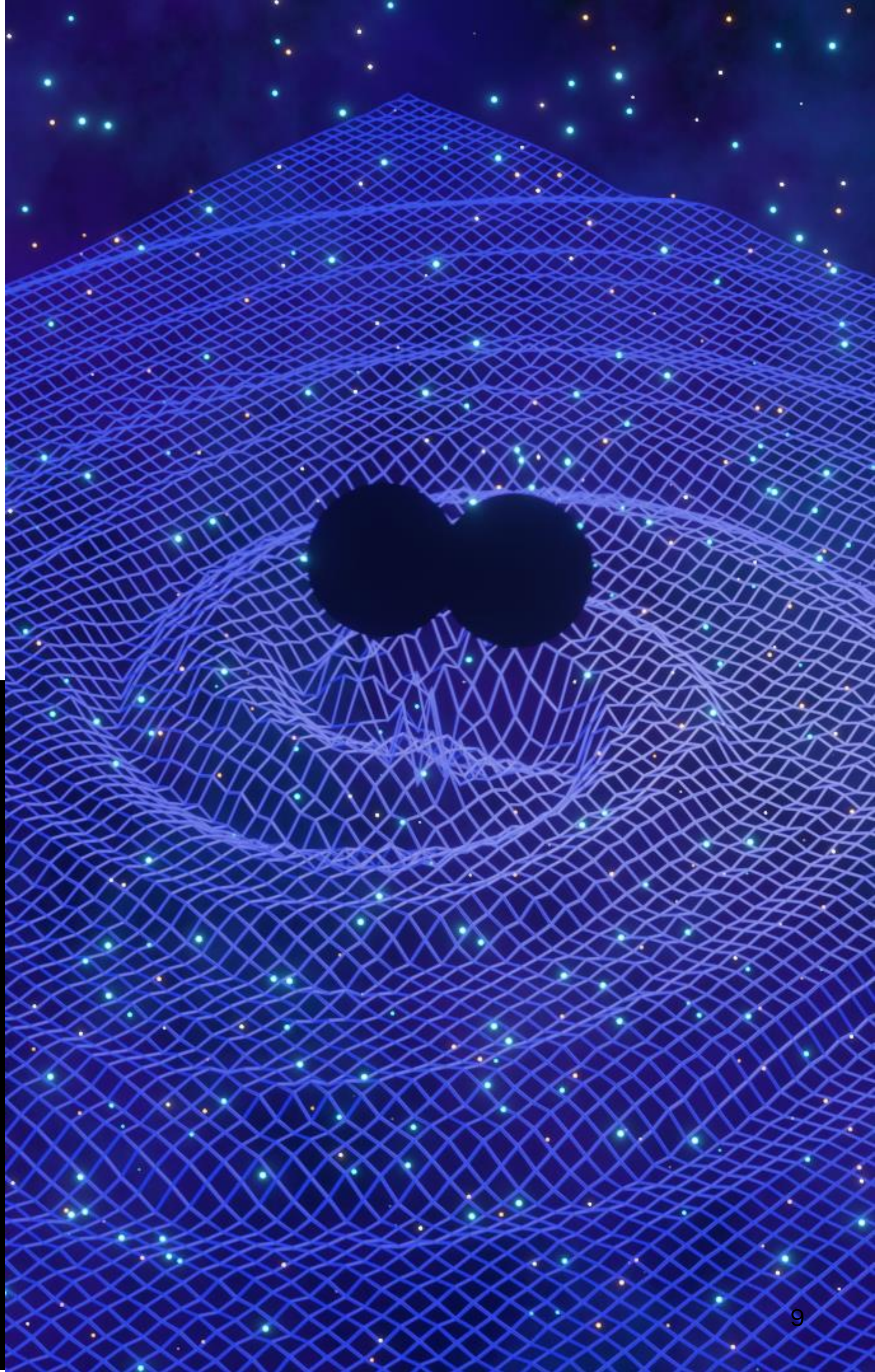
Gravitational waves

- Perturbations in the space-time geometry

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$



ESA / C. Carreau

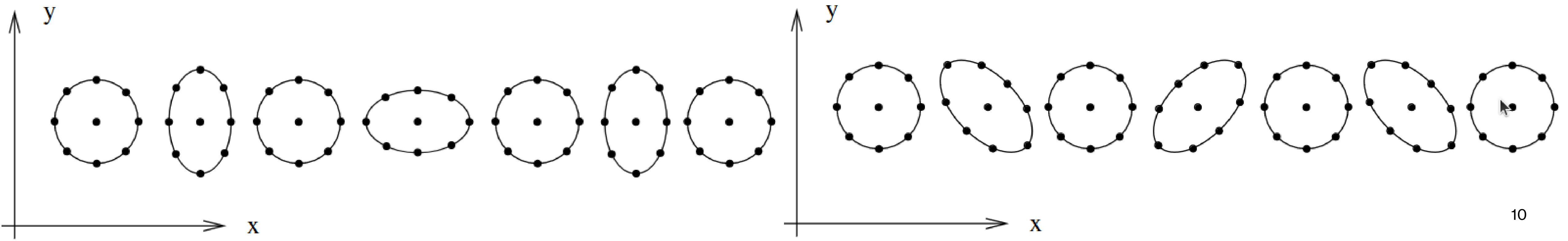


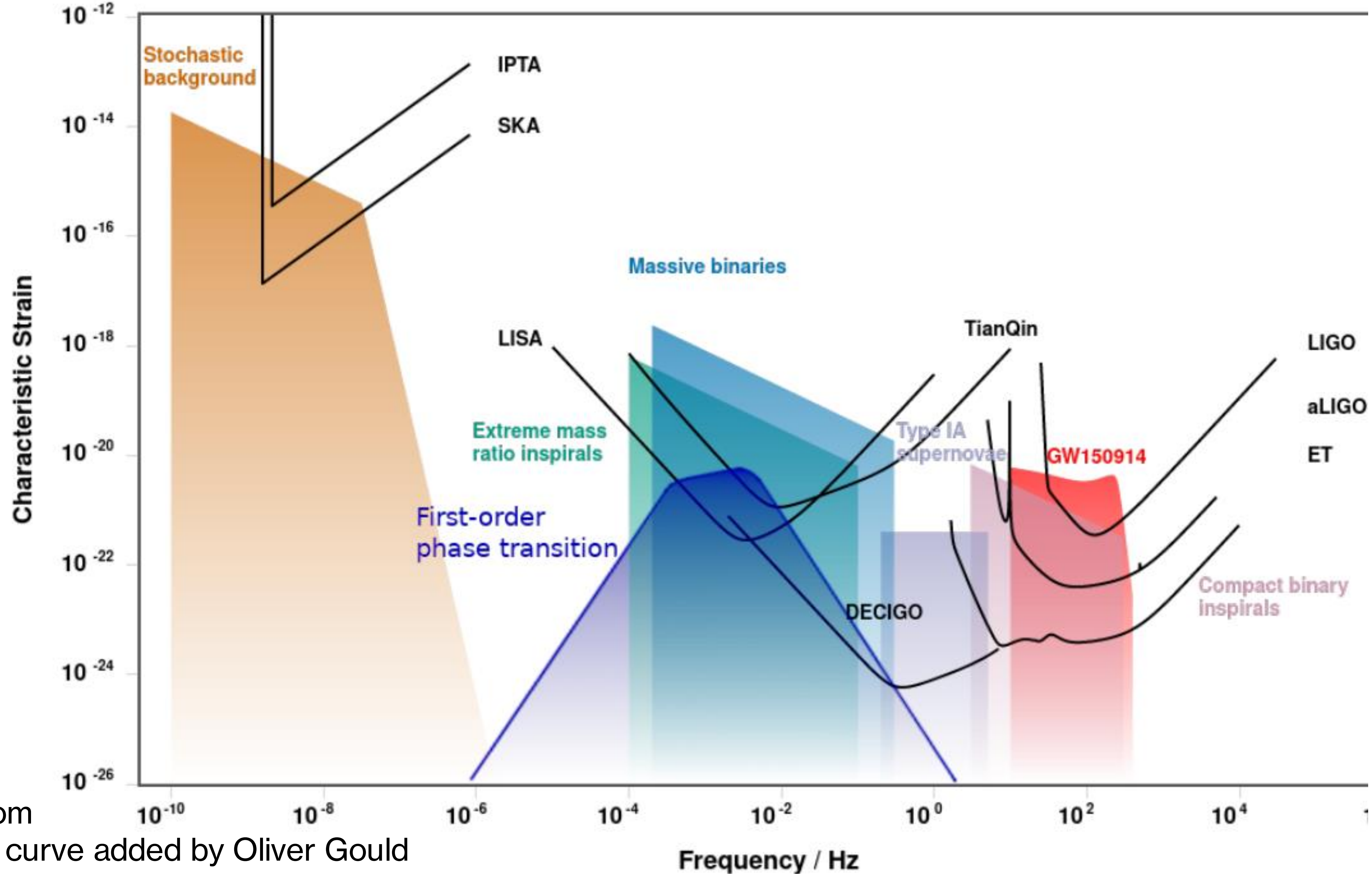
Gravitational waves

- Stochastic gravitational wave background

$$\sum \text{planar wave} \times \text{amplitude} \times \text{polarisation}$$

- GW spectrum usually modelled by broken power laws
- Main contributions: bubble wall collisions, sound waves in the surrounding plasma and turbulence





Gwplotter.com
1st order PT curve added by Oliver Gould

Gravitational waves

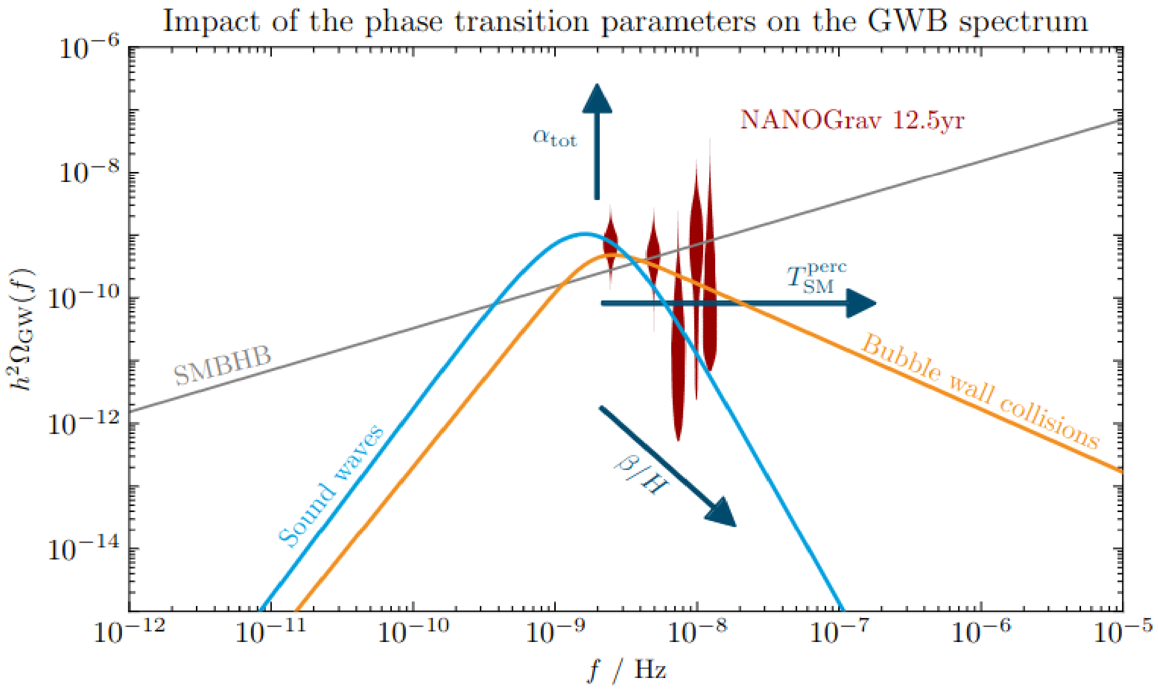
$$\Omega_{\rm gw} = F(T_*, R_*, \alpha_*, v_w)$$

T_* : transition temperature

β_* : inverse duration of the transition

α_* : transition strenght

v_w : bubble wall speed

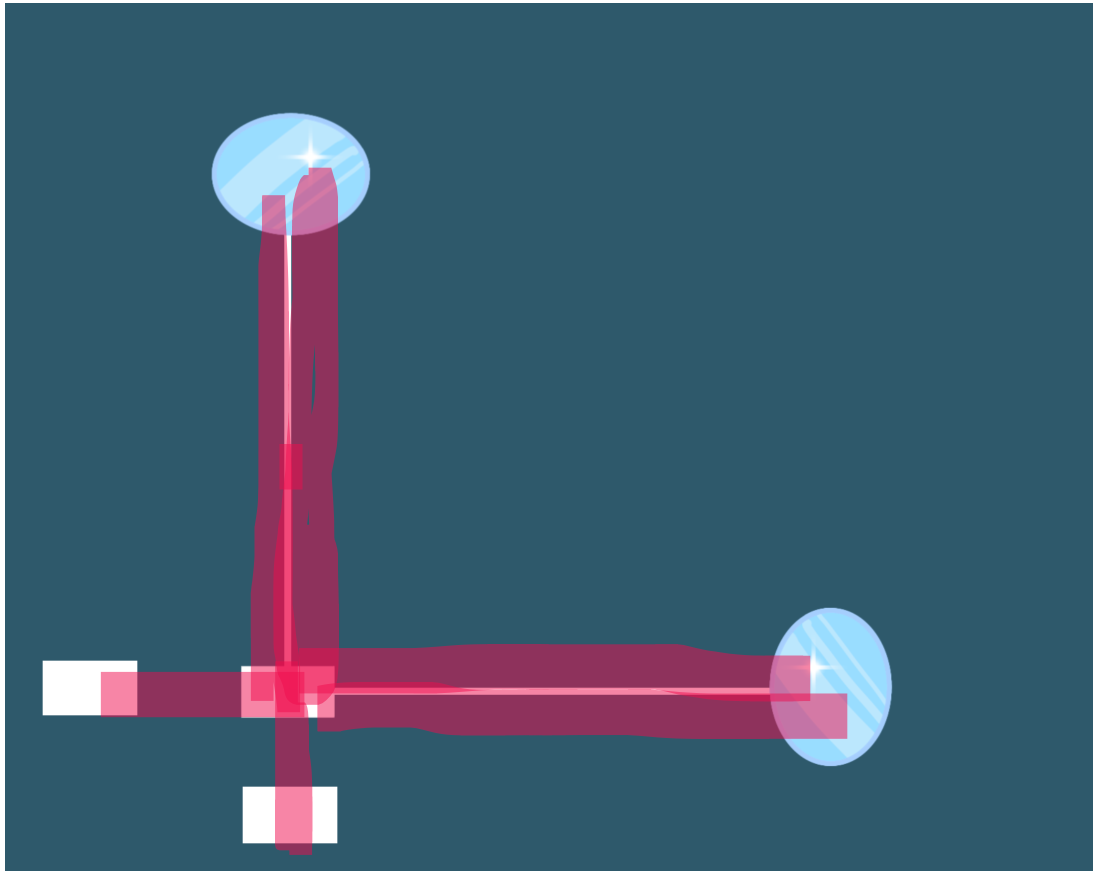


Bringmann et al. arXiv: 2306.09411

Depend on the nucleation rate!

GW DETECTION

LIGO



Caltech/MIT/LIGO Lab)

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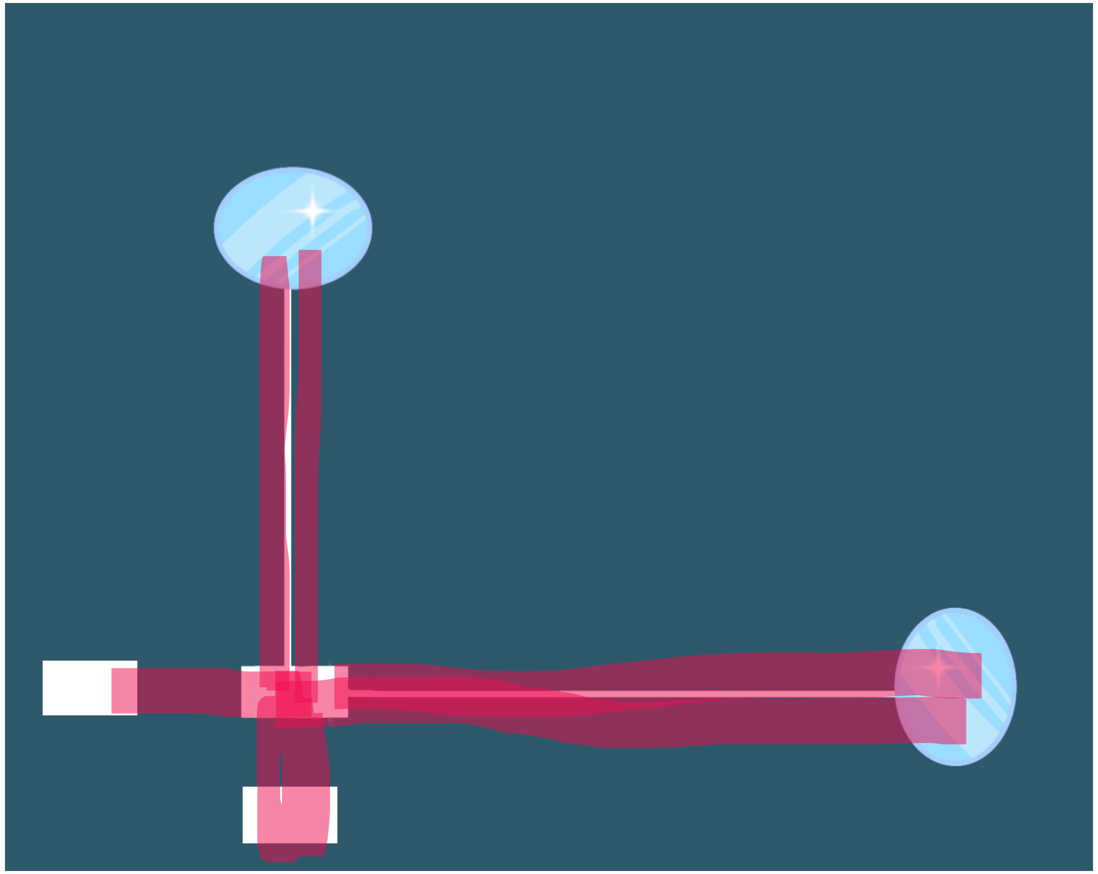

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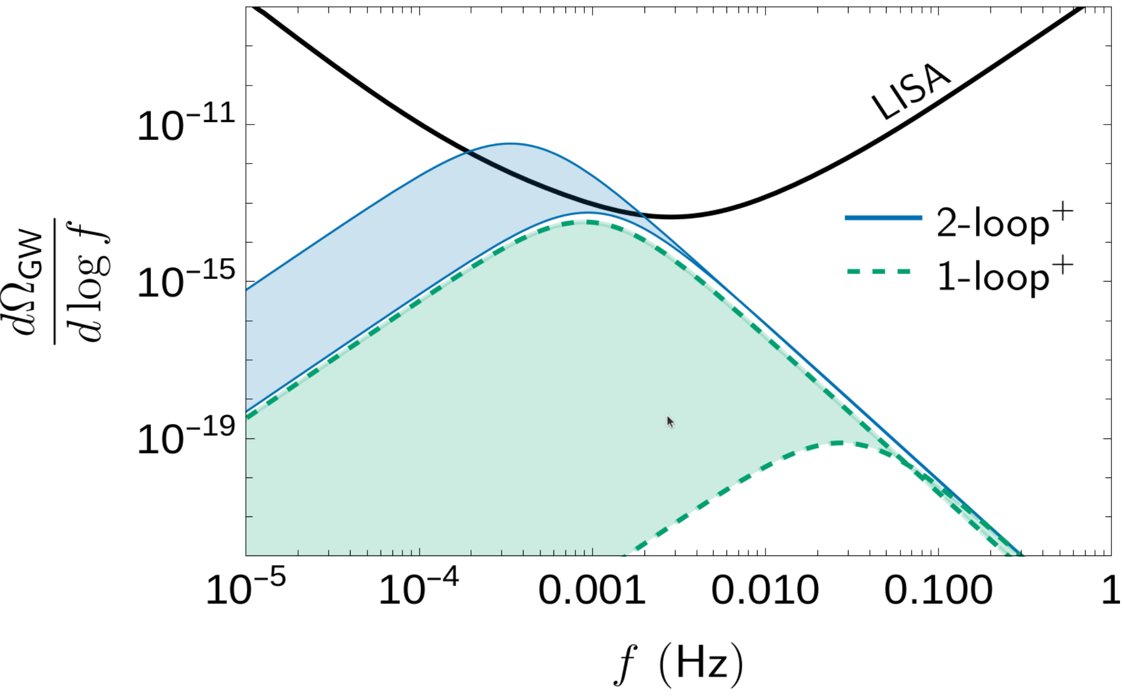
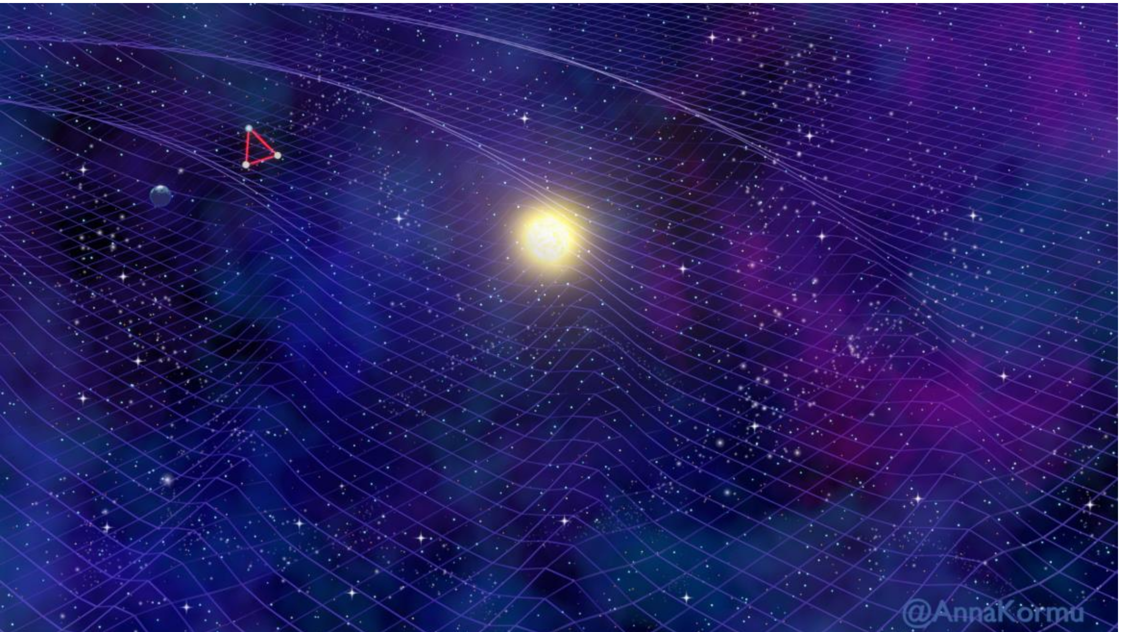


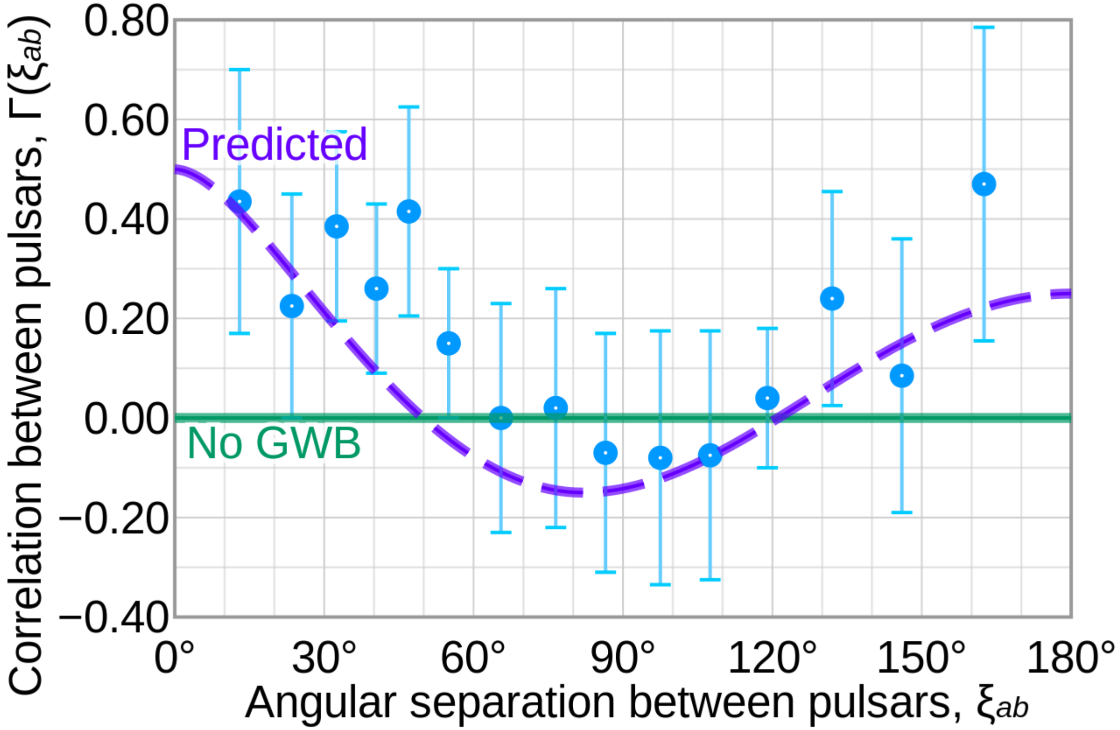
Gould, Tenkanen arXiv:2104.04399

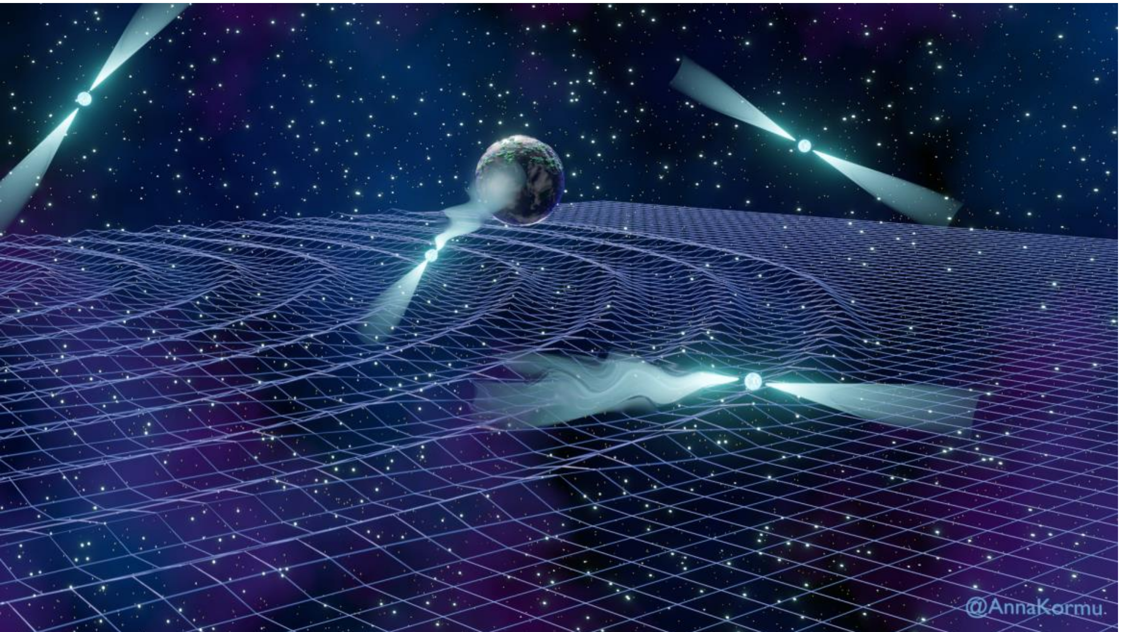


GW detection

LISA (AND OTHER SPACE INTERFEROMETERS)

- Launch in 2036, mission adoption 27.1.2024
- Three spacecraft, laser arms 2.5 million km
- Measure changes in path length between spacecraft
- Taiji & TianQin launch in 2030s





GW detection

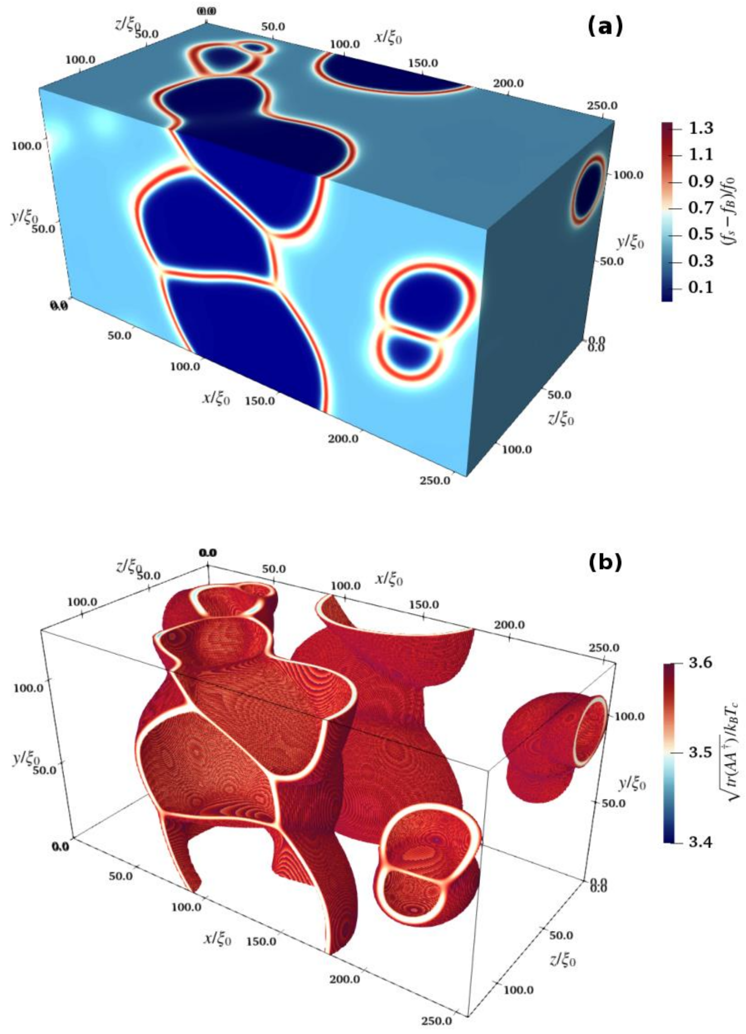
PULSAR TIMING ARRAYS

- Hints of stochastic GW background - June 2023
  (European PTA, Indian PTA, NANOGrav, Parkes PTA ’23)

- Mostly likely supermassive black holes, but new physics cannot be ruled out yet

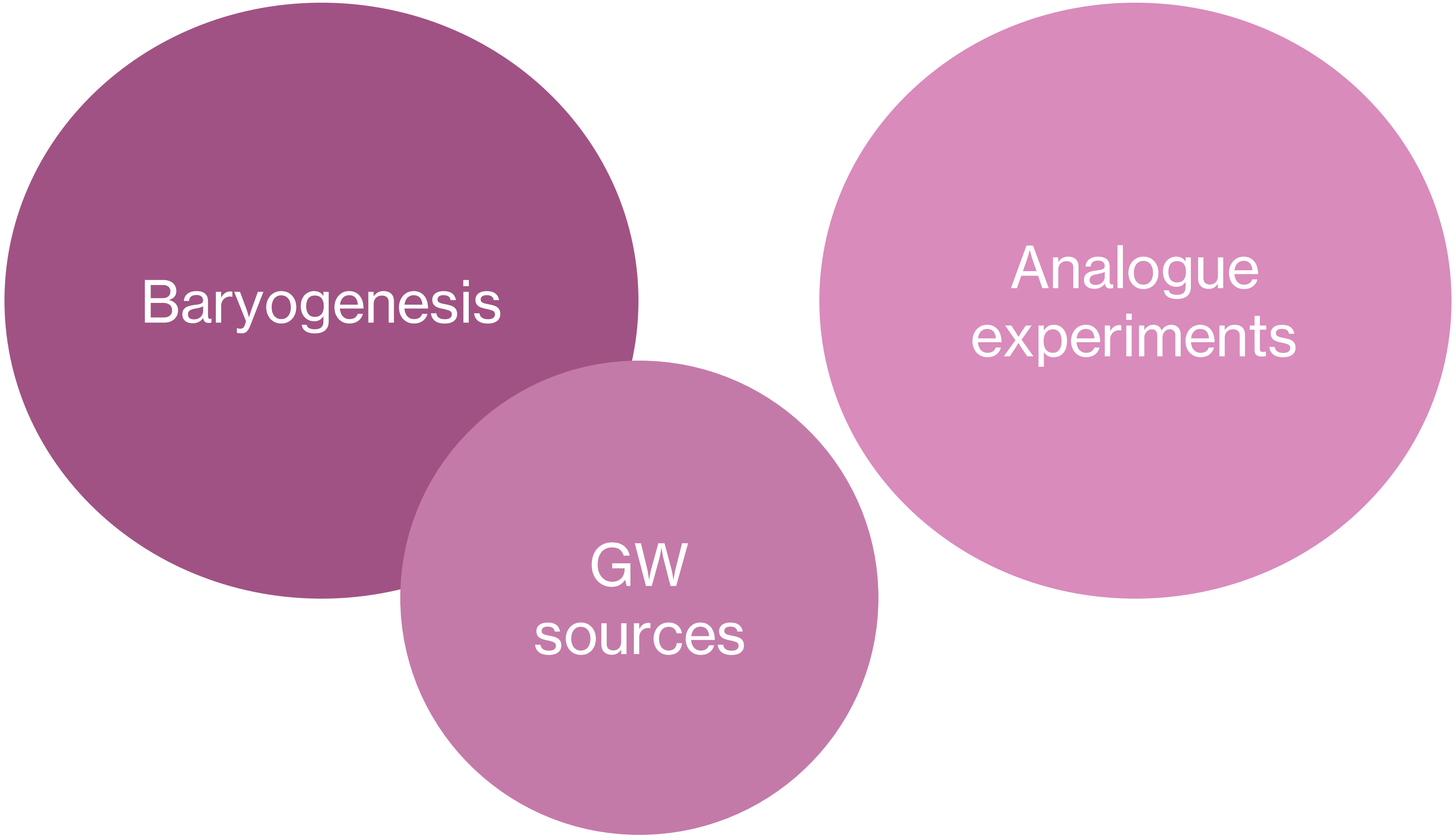
Analogue experiments

- Testing cosmology in laboratory: nucleation theory essentially the same in laboratory and in cosmology
- Superfluid Helium-3 Hindmarsh et al. arXiv:2401.07878
- Ferromagnetic superfluids Zenesini et al. arXiv:2305.05225
- Proposals to test nucleation in (other) ultracold atomic gases
  arXiv:1408.1163 arXiv:2212.03621 arXiv:2307.02549



Hindmarsh et al. arXiv:2401.07878

Key points – why the accurate estimation of bubble nucleation rate is important



Fate of the False Vacuum

- Relativistic field theory generalisation Callan & Coleman (Phys. Rev. D 16, 1762 (1977))
- Finite temperature approach introduced later by Affleck & Linde (Phys. Rev. Lett. 46, 388 (1981), Phys. Lett. B 100, 37 (1981))

$$\Gamma = A_{\mathrm{dyn}} \times \sqrt{\left|\frac{\det(S''[\phi_0]/2\pi)}{\det'(S''[\phi_{\mathrm{b}}]/2\pi)}\right|} \left(\frac{\Delta S[\phi_{\mathrm{b}}]}{2\pi}\right)^{3/2} e^{-\Delta S[\phi_{\mathrm{b}}]}$$

$$\Gamma = A_{\mathrm{dyn}} \times A_{\mathrm{stat}}$$

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Usually not computed!

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- Perturbation theory suffers from the so-called infrared problem

$$\frac{g^2}{e^{E/T}-1} \xrightarrow{E \ll T, \mathbf{p}=\mathbf{0}} \frac{g^2 T}{m}$$

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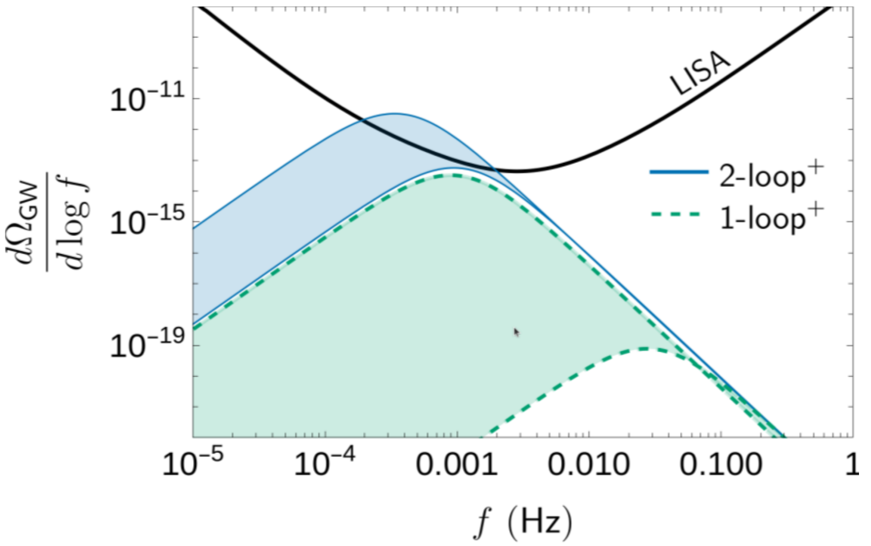

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- Introduces uncertainty! How accurate are our cosmological predictions?
- Moore, Rummukainen & Tranberg introduce a simulation method (hep-lat/0103036, hep-ph/0009132)

Key points – why the accurate estimation of bubble nucleation rate is important

Baryogenesis

GW sources

Analogue experiments

Uncertainty in perturbative calculations

Real scalar theory

For equilibrium dynamics, see Gould, arXiv:2101.05528

- Toy model possessing key features of BSM models
  - Potential has a tree-level barrier
  - Strong phase transition
  - Perturbative expansion simpler (we understand the dynamics)
- Dimensional reduction (imaginary time, high temp)

$$\mathscr{L} = -\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi) - J_1\varphi - J_2\varphi^2$$

Interaction terms

$$V(\varphi) = \sigma\varphi + \frac{1}{2}m^2\varphi^2 + \frac{1}{3!}g\varphi^3 + \frac{1}{4!}\lambda\varphi^4,$$

Model parameters

Dimensional reduction

Kajantie et al. hep-ph/9508379

- At high temperatures system looks 3d
- Dimensional reduction 4d cont → 3d cont
- Integrate out heavy modes, match correlation functions
- (3d cont → 3d lattice)

$$\mathcal{L}_{4d}$$

$$\downarrow$$

$$\mathcal{L}_{3d}$$

$$\downarrow$$

$$\mathcal{L}_{\mathrm{3d,lattice}}$$

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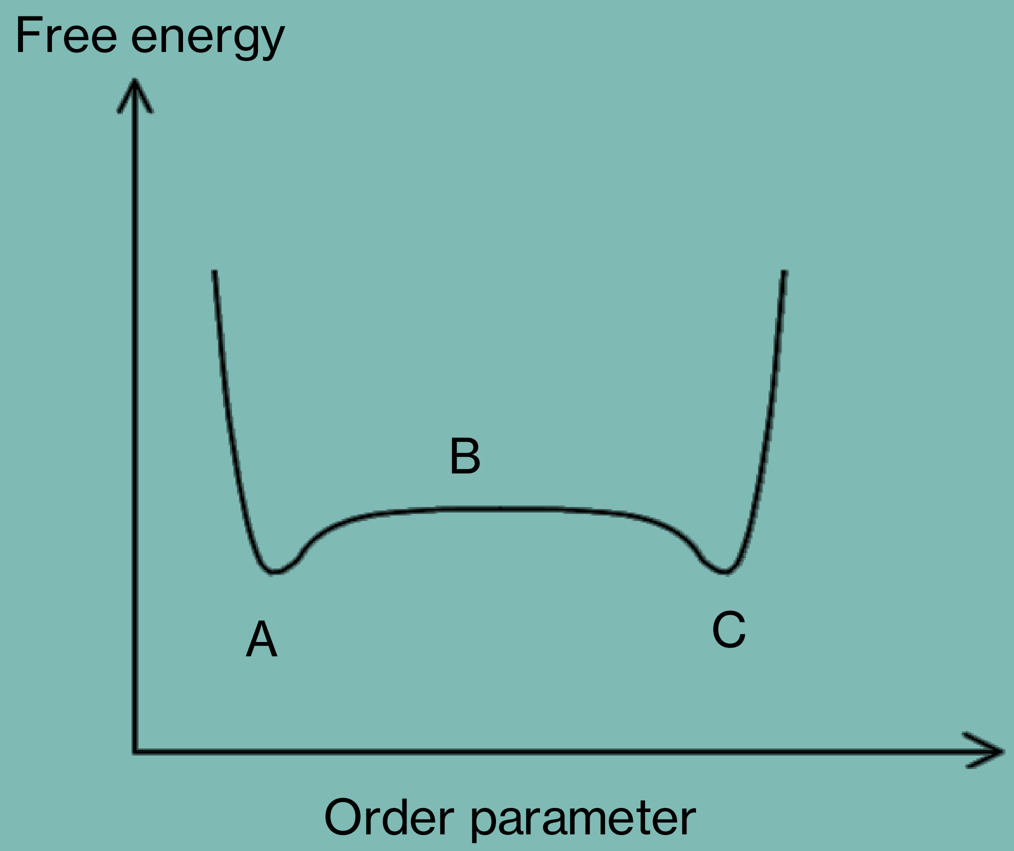
Lattice spacing — Renormalisation counterterms — Lattice parameters

$$S_{\text{lat}} = \sum_x a^3 \left[-\frac{1}{2} Z_\phi \phi_x (\nabla^2_{\text{lat}} \phi)_x + \sigma_{\text{lat}} \phi_x + \frac{1}{2} Z_\phi Z_m m^2_{\text{lat}} \phi_x^2 + \frac{1}{3!} g_{\text{lat}} \phi_x^3 + \frac{1}{4!} Z_\phi^2 \lambda_{\text{lat}} \phi_x^4 \right]$$



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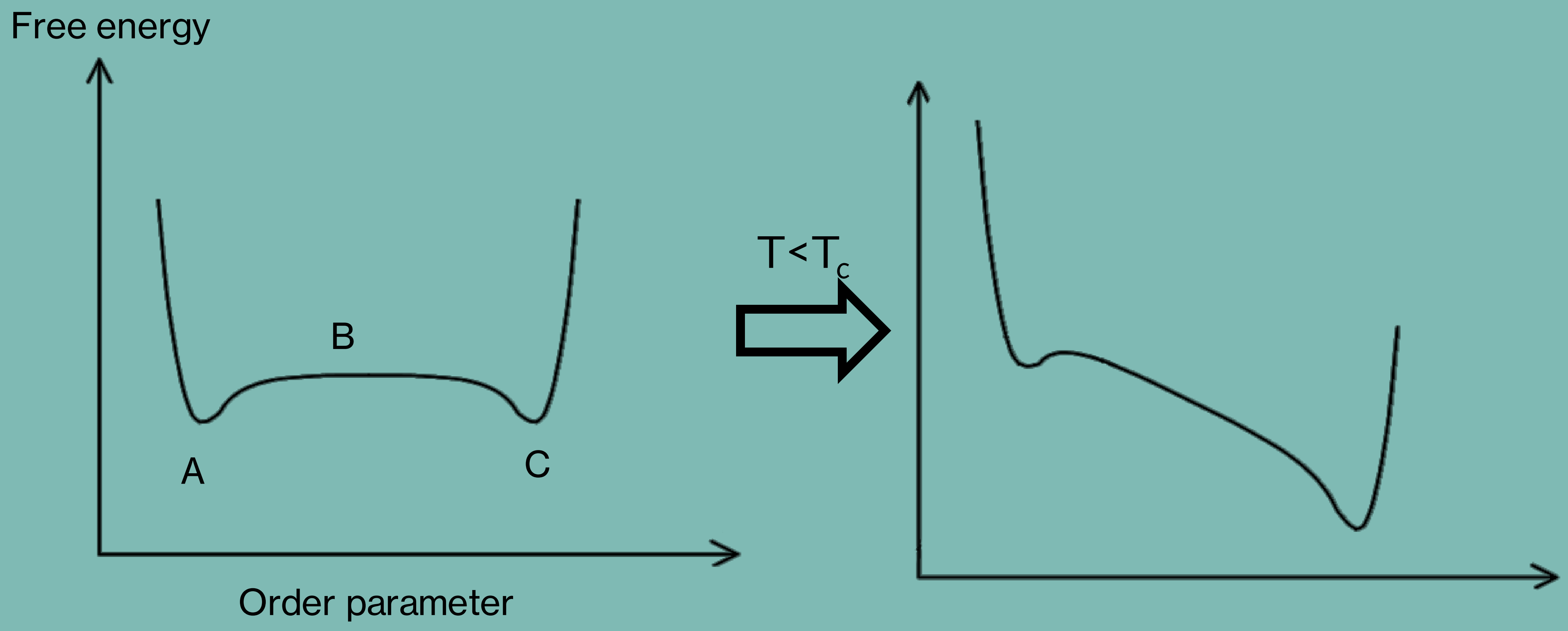

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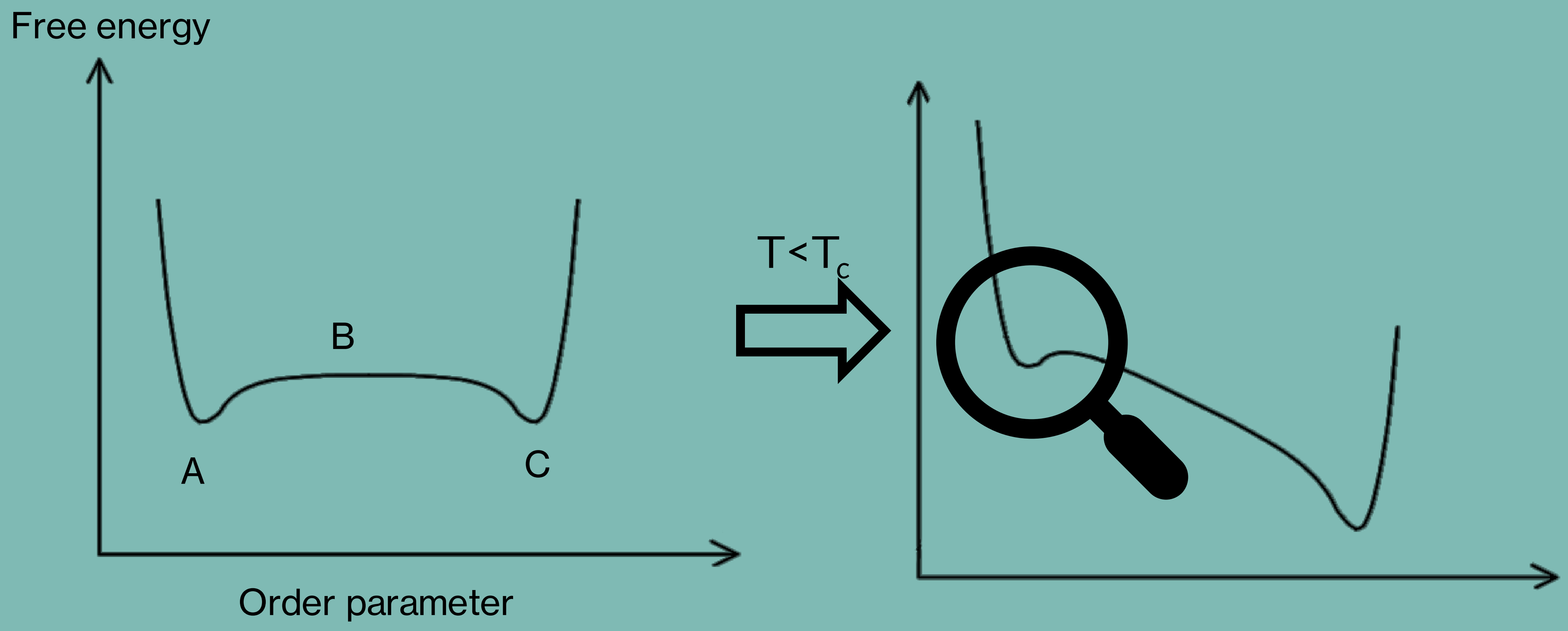

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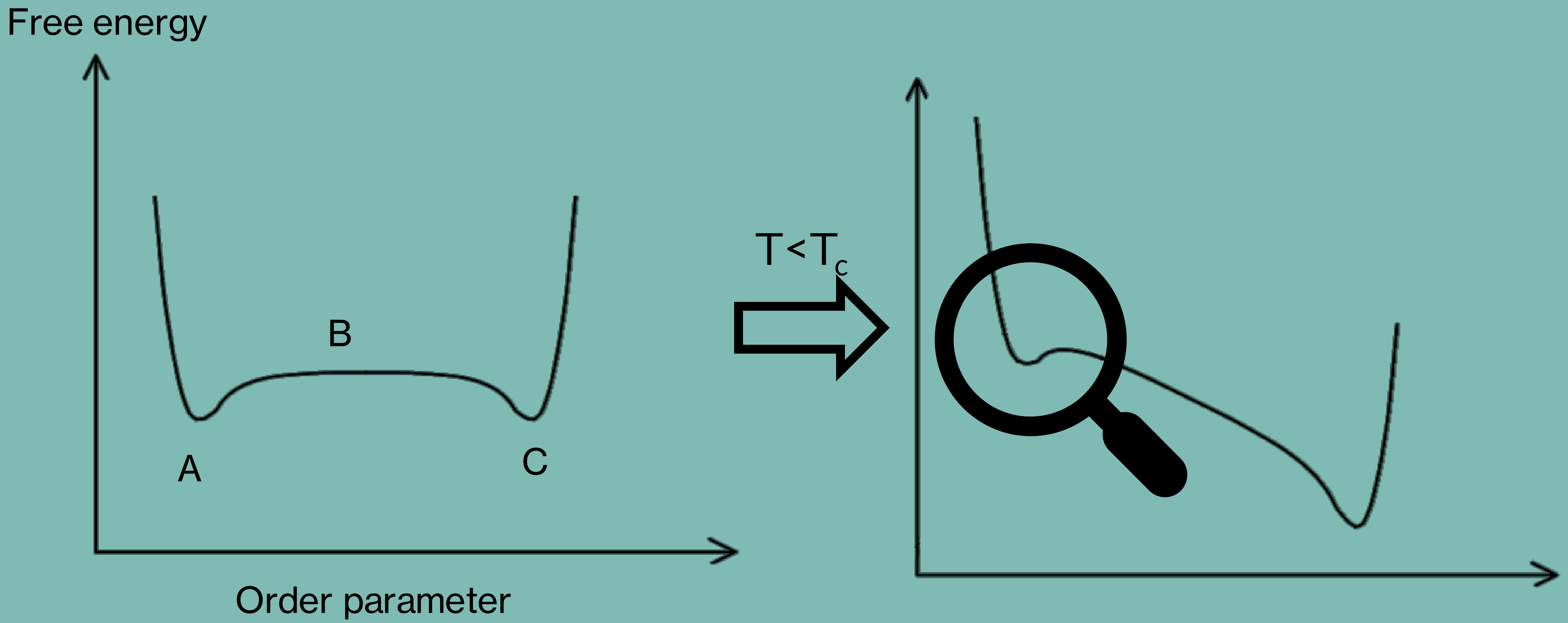

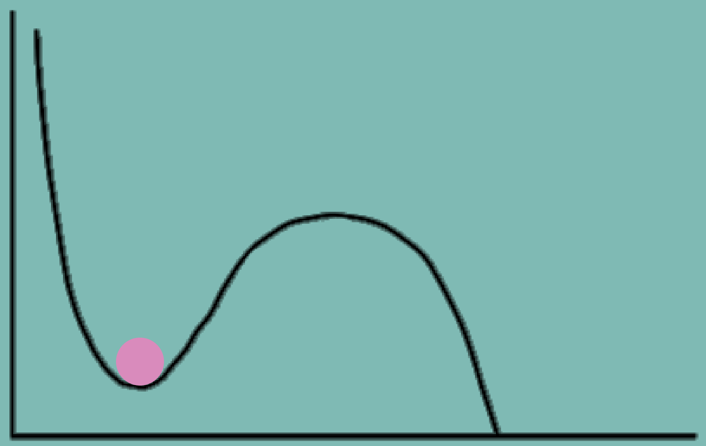

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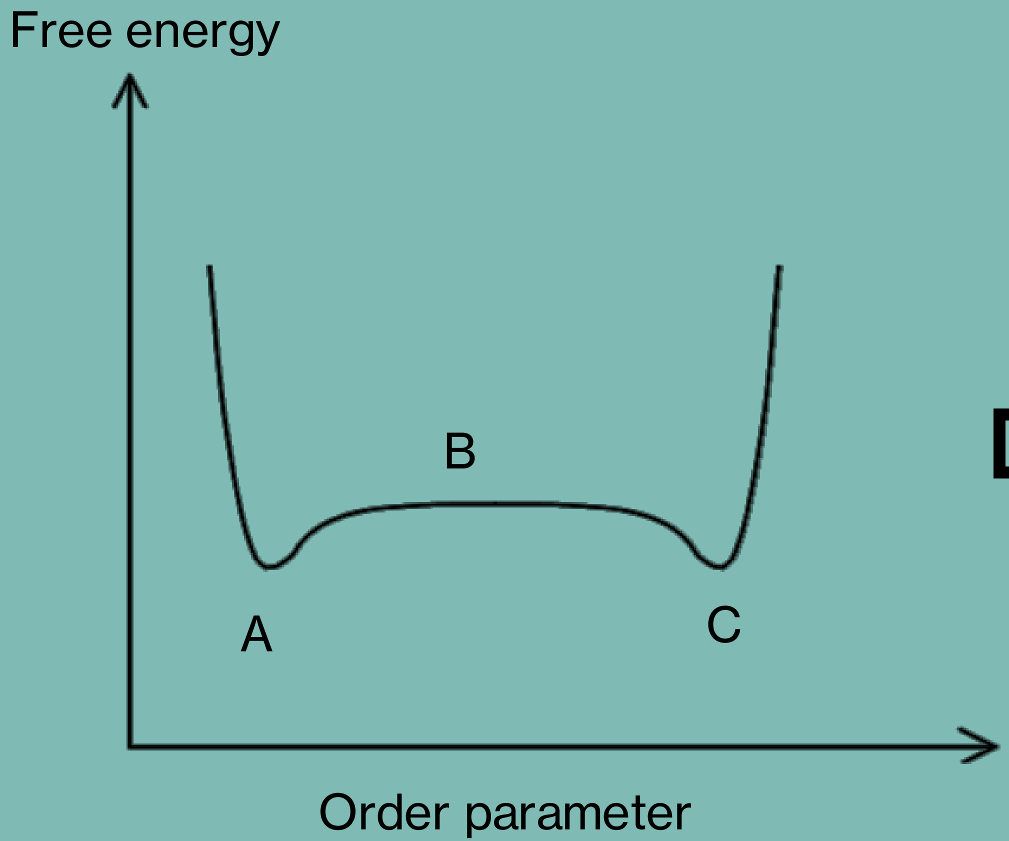

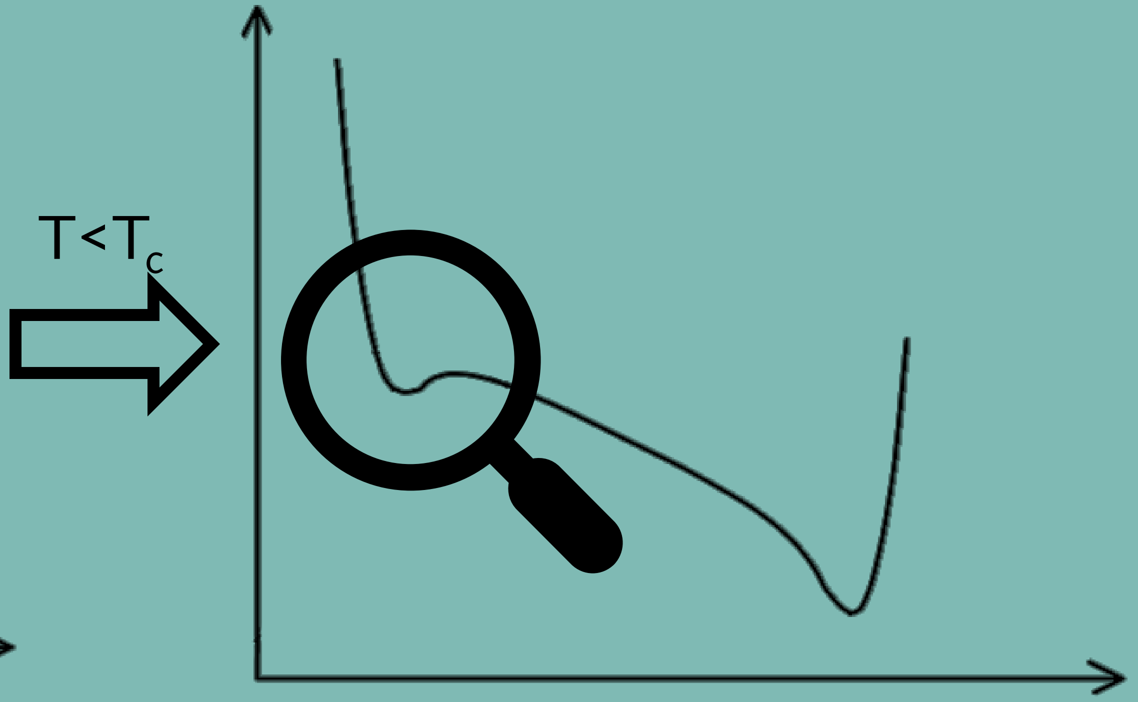

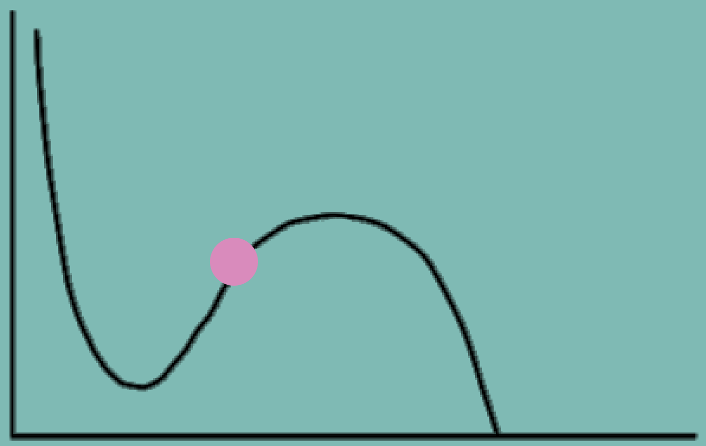

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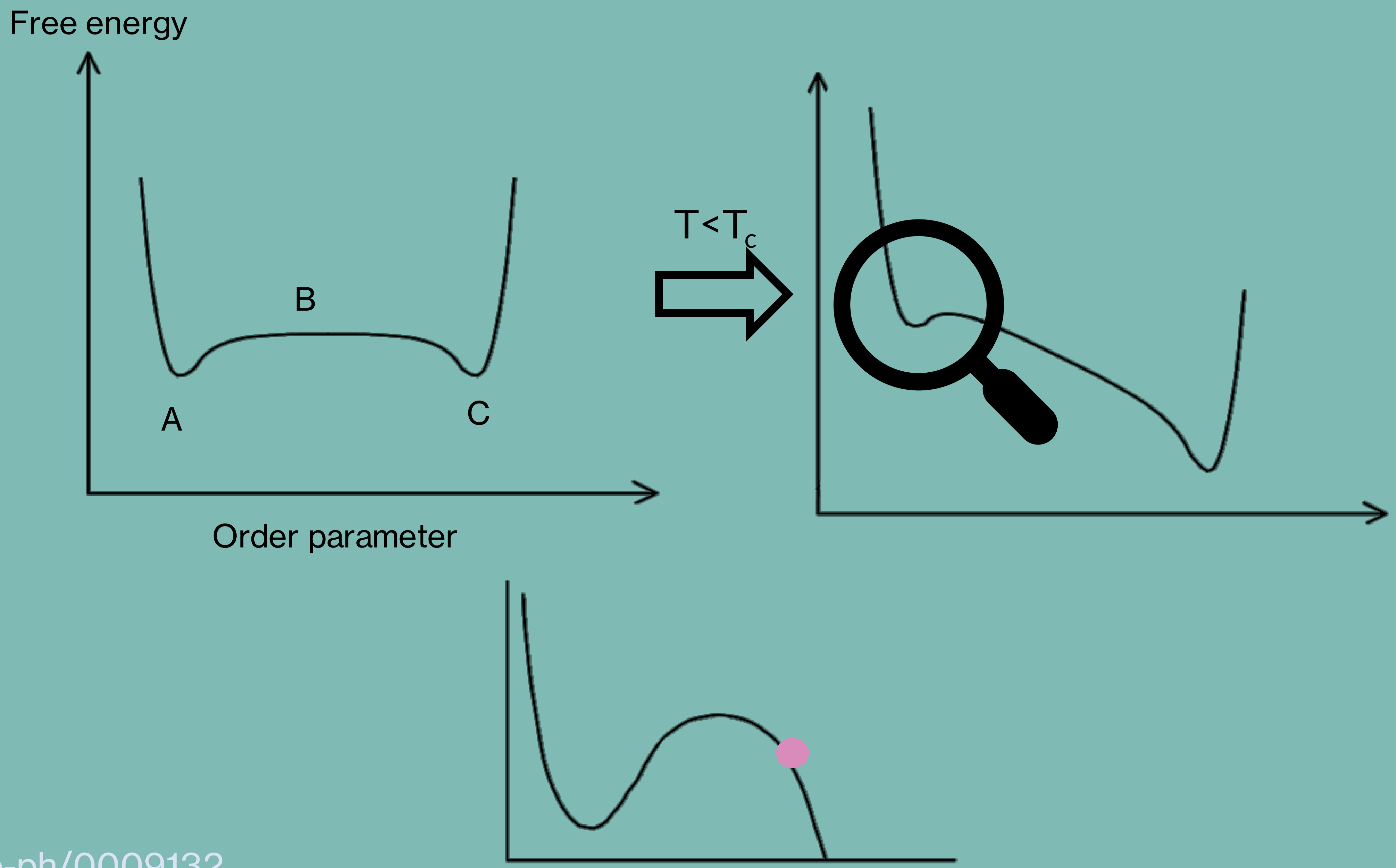
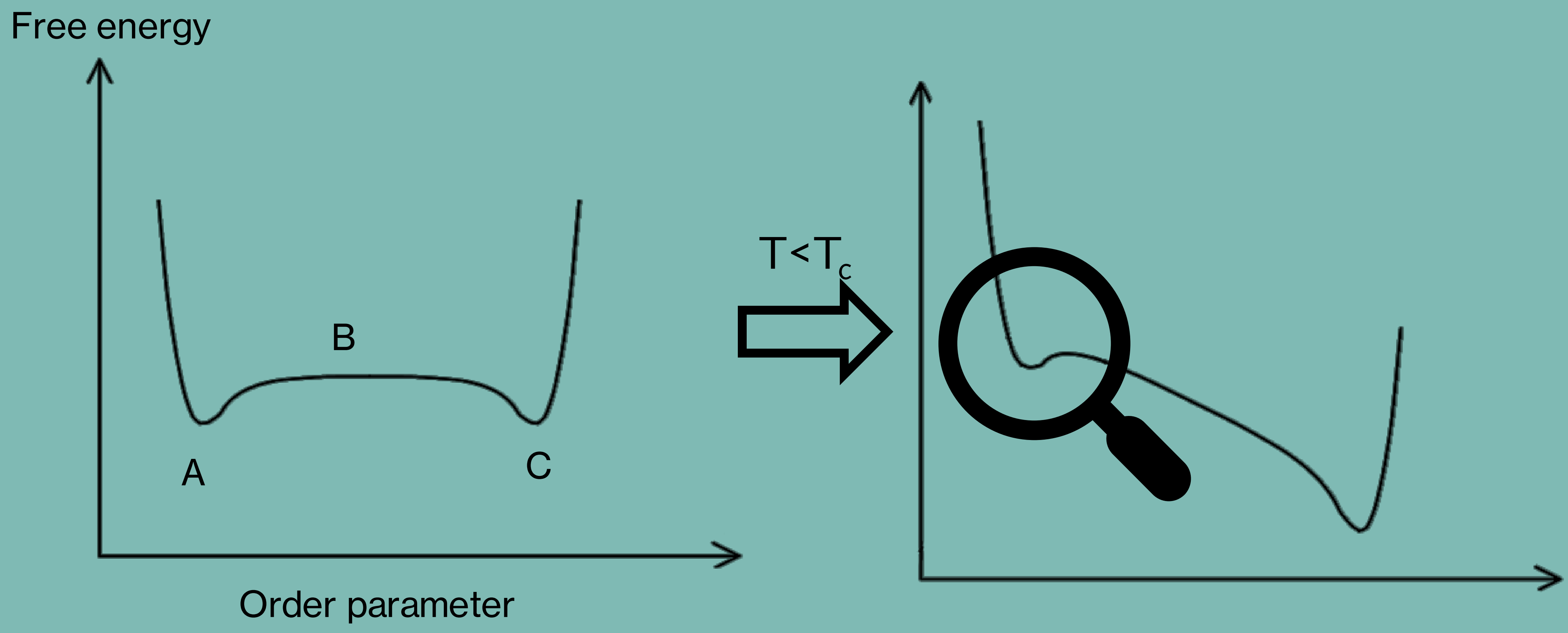
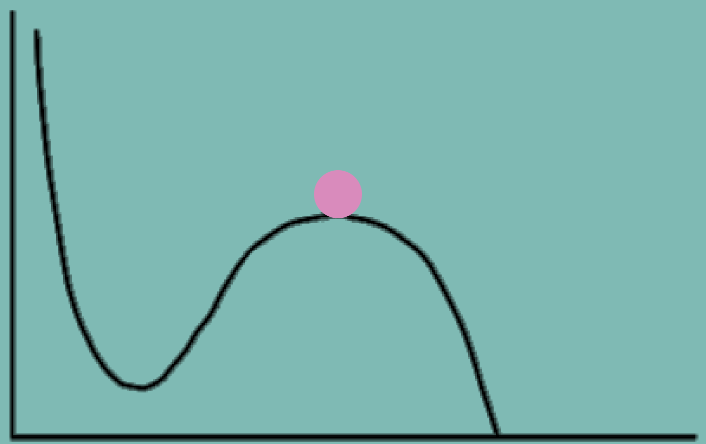


[hep-ph/0009132](hep-ph/0009132)





hep-ph/0009132

Bubble nucleation, nonperturbatively

- Langevin equation

Damping

Gaussian noise term

$$\partial_t \phi(t, \mathbf{x}) = \pi(t, \mathbf{x}),$$

$$\partial_t \pi(t, \mathbf{x}) = -\frac{\delta H_{\text{eff}}}{\delta \phi} - \gamma \pi(t, \mathbf{x}) + \xi(t, \mathbf{x})$$

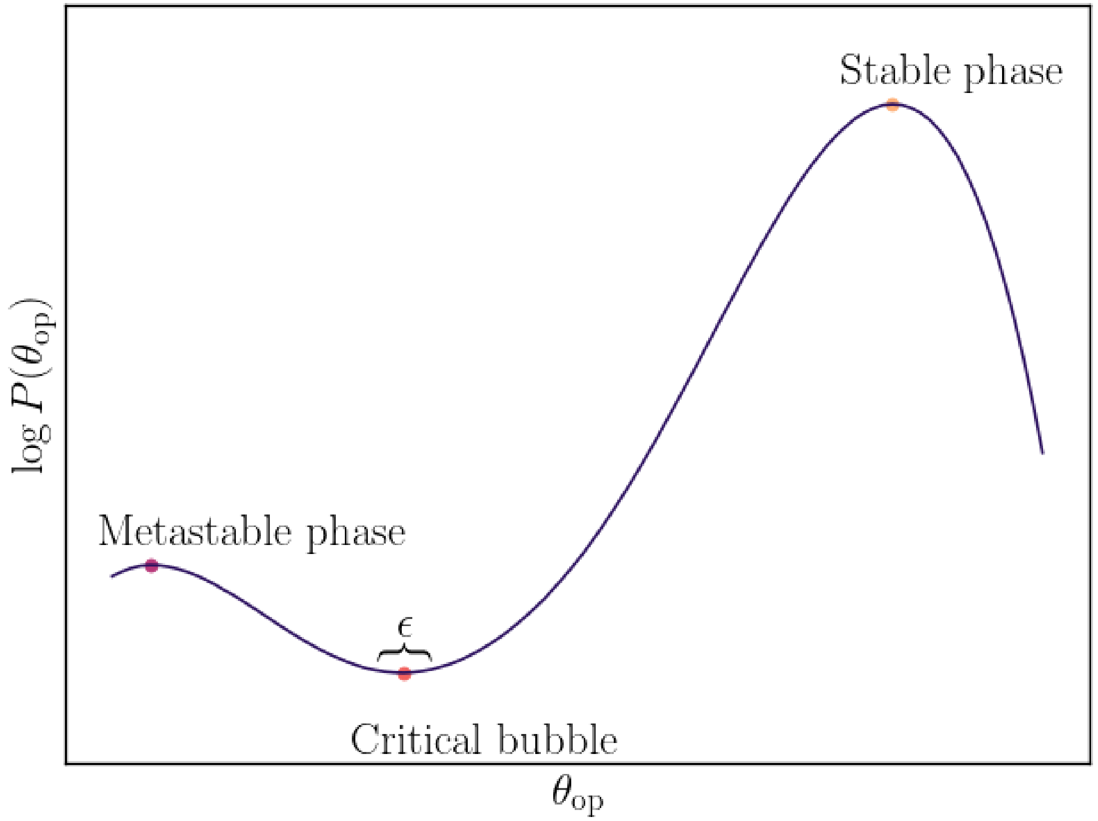
1. 2. 3. 4. ➡ $\Gamma = A_{\text{dyn}} \times A_{\text{stat}}$

Bubble nucleation, nonperturbatively

1. Pick an order parameter, simulate probability distribution

- Distinguishable behavior in the two phases

$$\theta'_{\rm op} = \bar{\phi}$$



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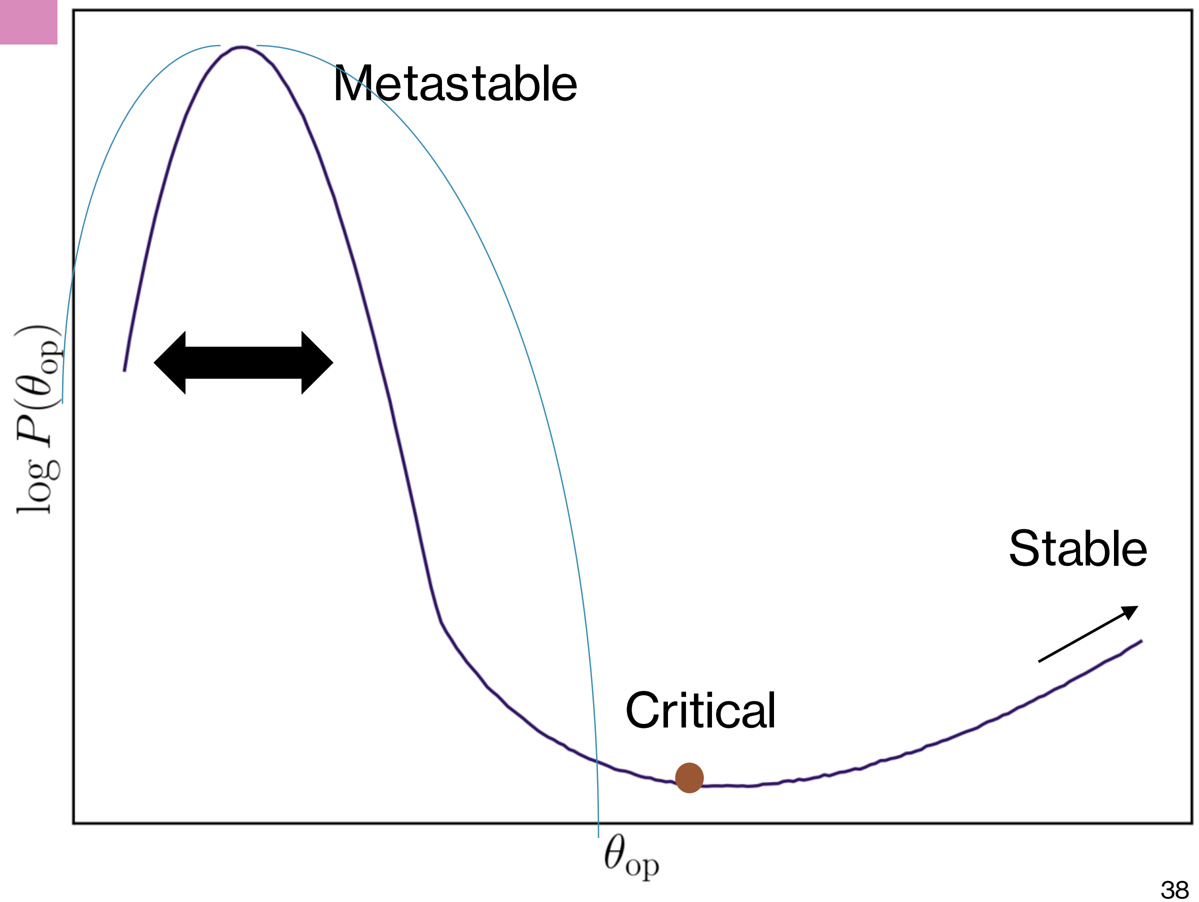

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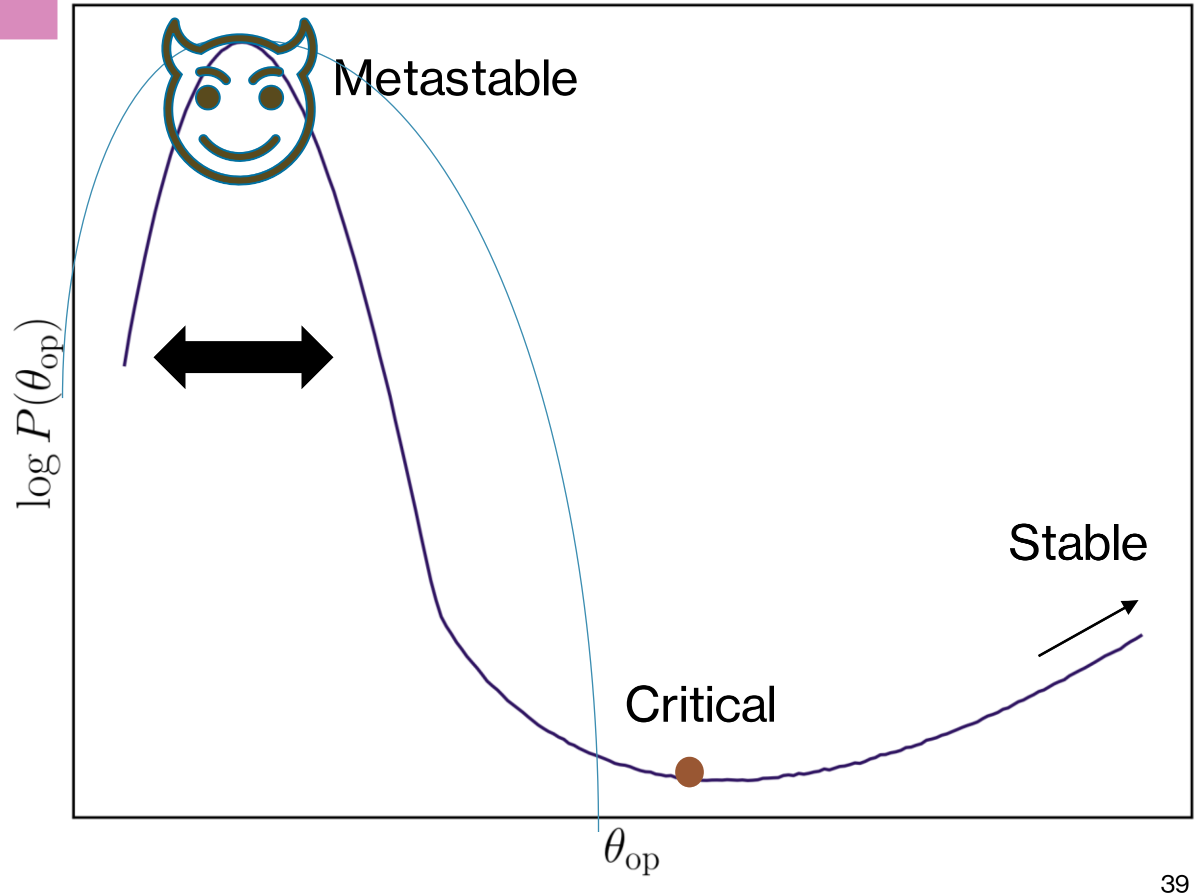
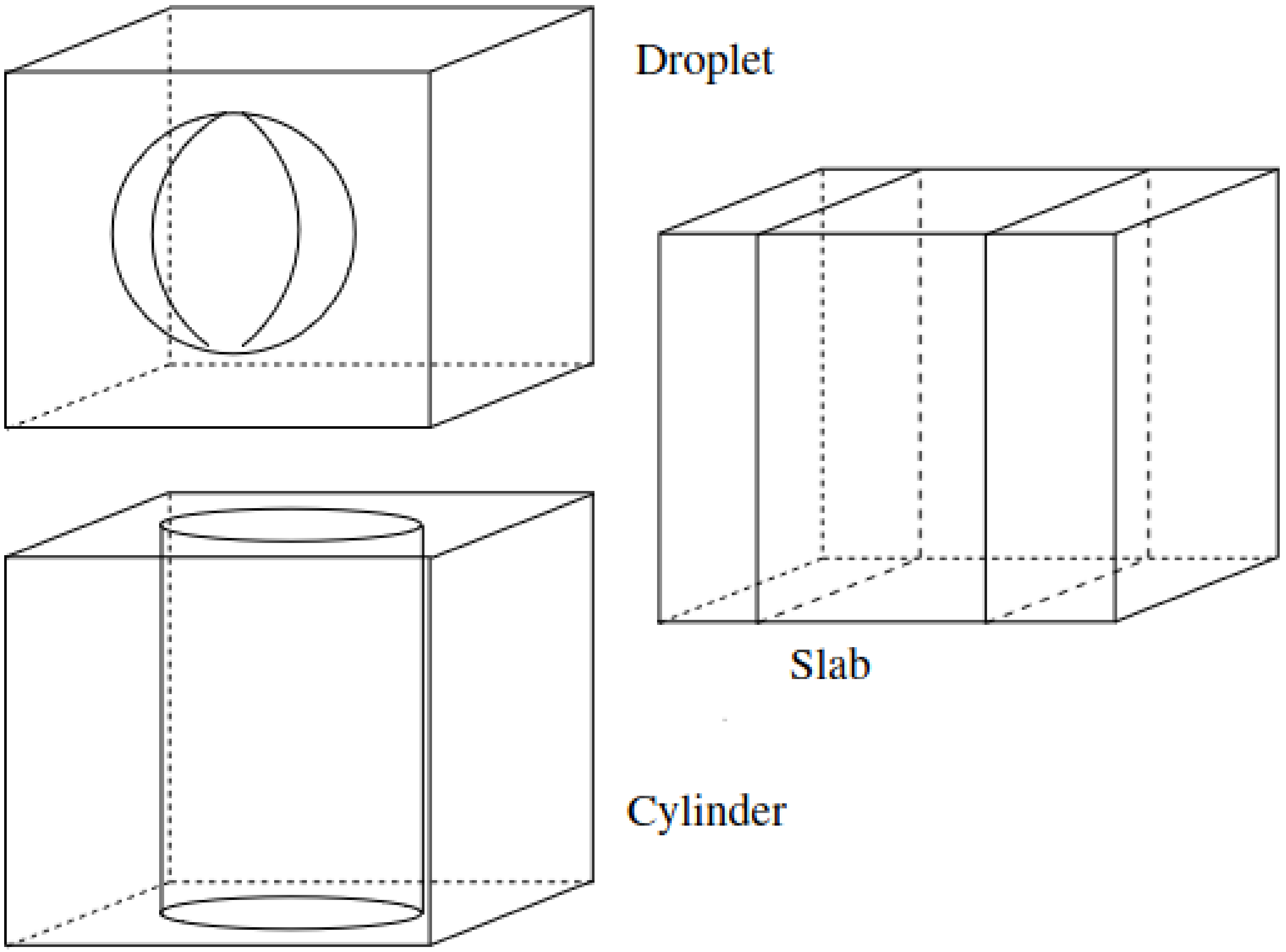




hep-lat/0103036

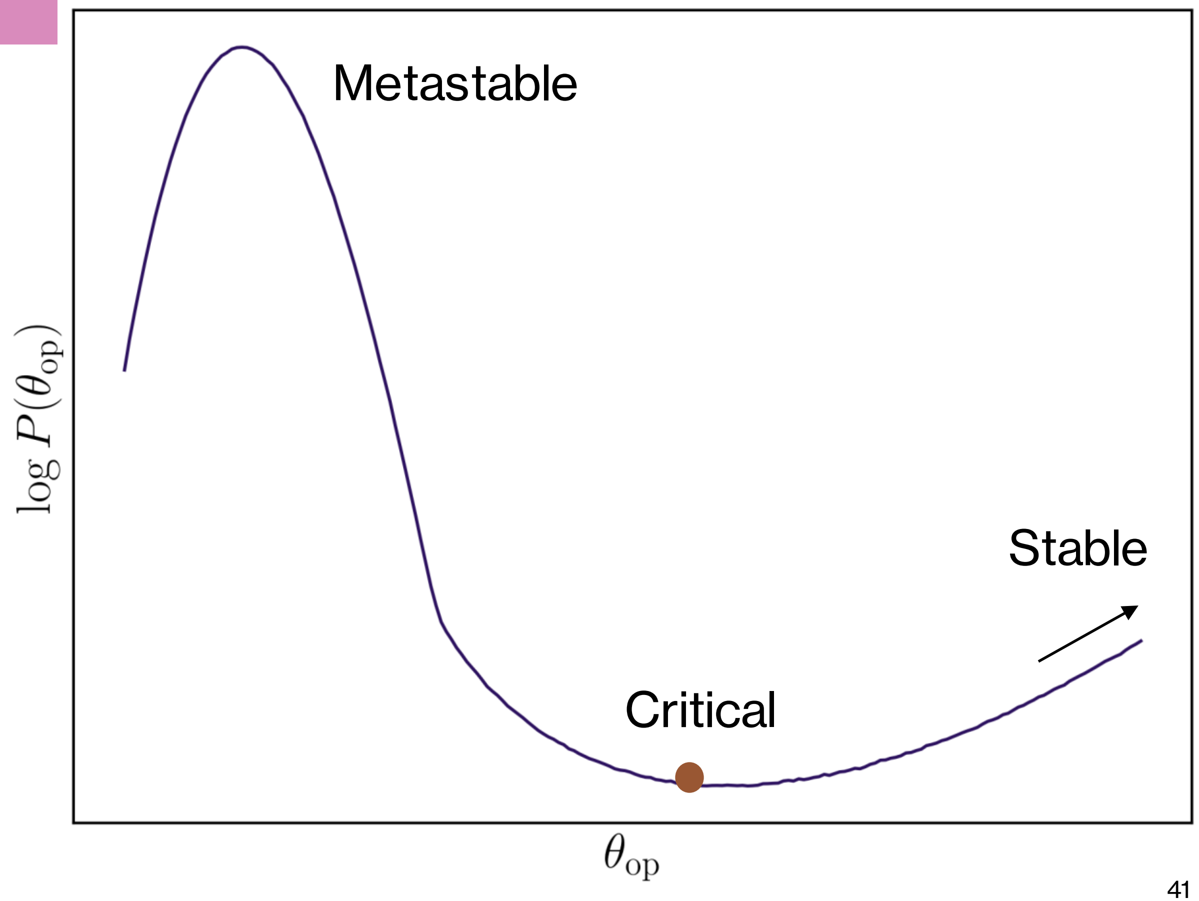
Bubble nucleation, nonperturbatively

1. Pick an order parameter, simulate probability distribution

- Distinguishable behavior in the two phases

$$\theta'_{\rm op} = \bar{\phi}$$ ✗

$$\theta_{\rm op} = \bar{\phi}^2 - 2A\bar{\phi}$$ ✓



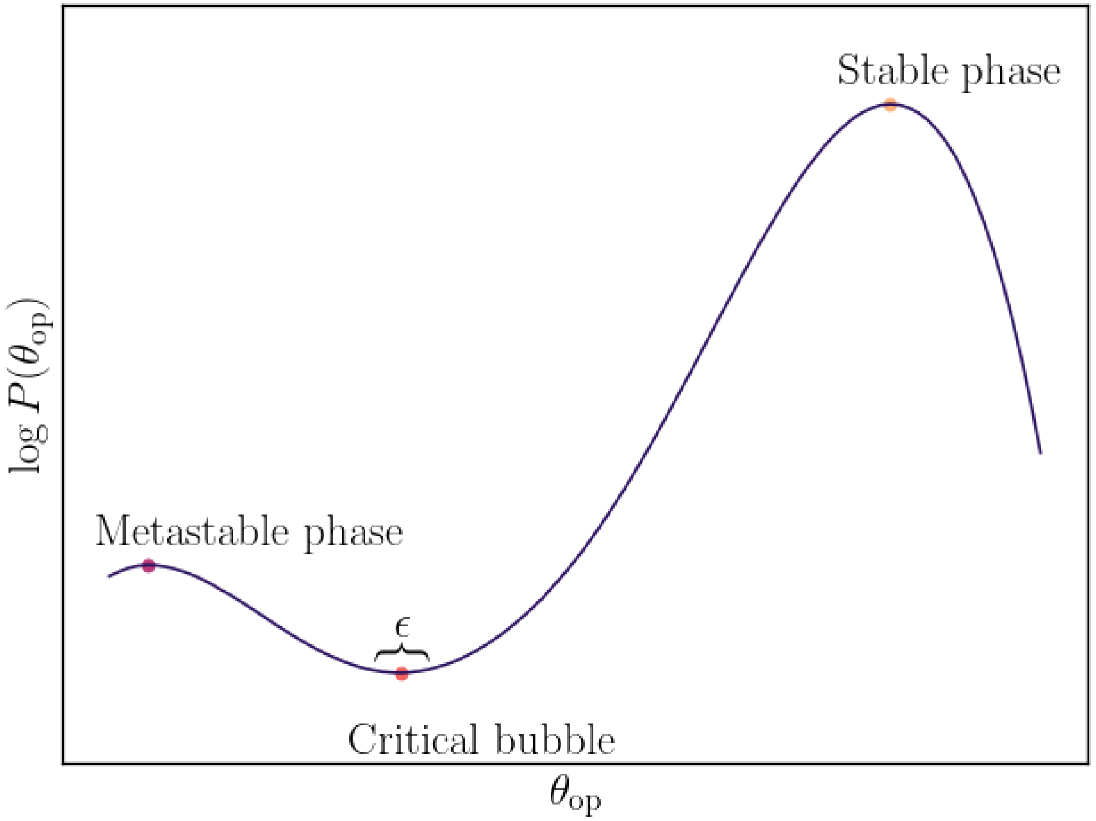
Bubble nucleation, nonperturbatively

1. Pick an order parameter, simulate probability distribution

- Distinguishable behavior in the two phases

$$\theta_{\mathrm{op}} = \bar{\phi}^2 - 2A\bar{\phi}$$

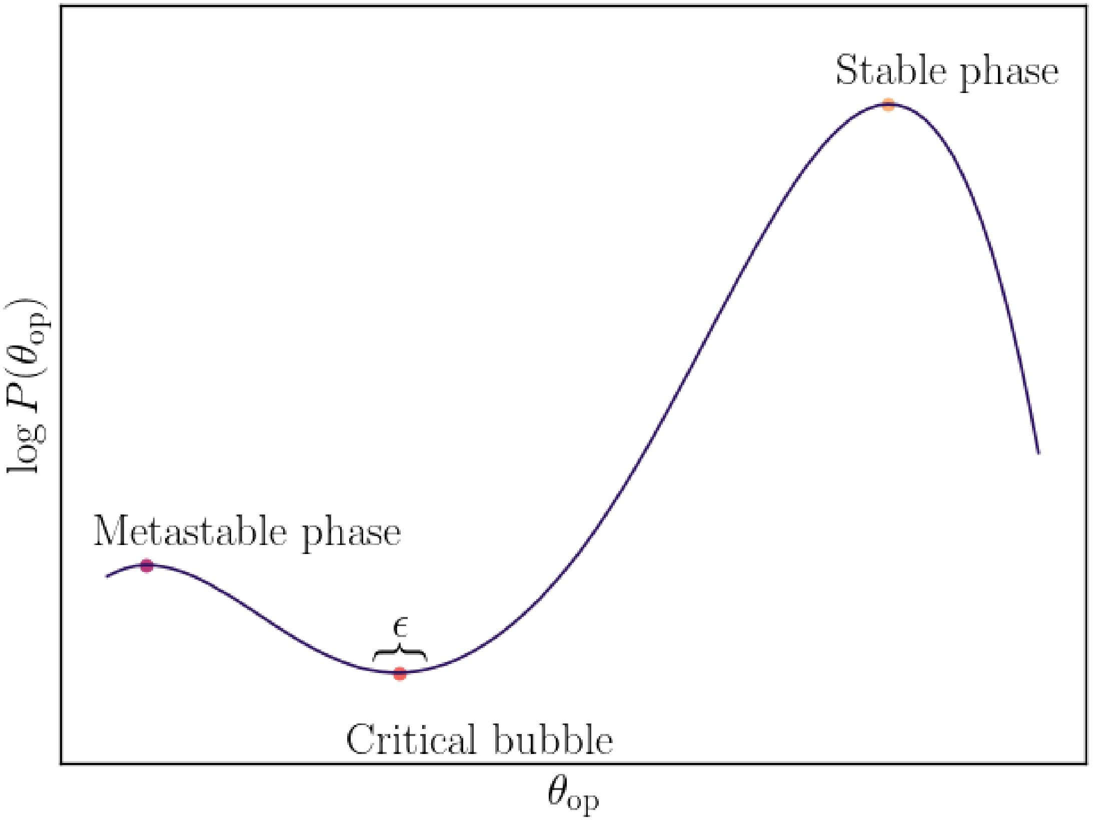
- Separatrix configurations suppressed by e^{-65}
  → Multicanonical Monte Carlo



Bubble nucleation, nonperturbatively

2. Draw separatrix configurations, calculate the probability

- Draw field configurations from a narrow range $\theta_{\mathrm{op}} \in [\theta_{\mathrm{c}} - \frac{\epsilon}{2}, \theta_{\mathrm{c}} + \frac{\epsilon}{2}]$
- These are initial conditions for the time evolution
- Calculate probability of the critical bubble and normalise it to the probability of being in the metastable phase

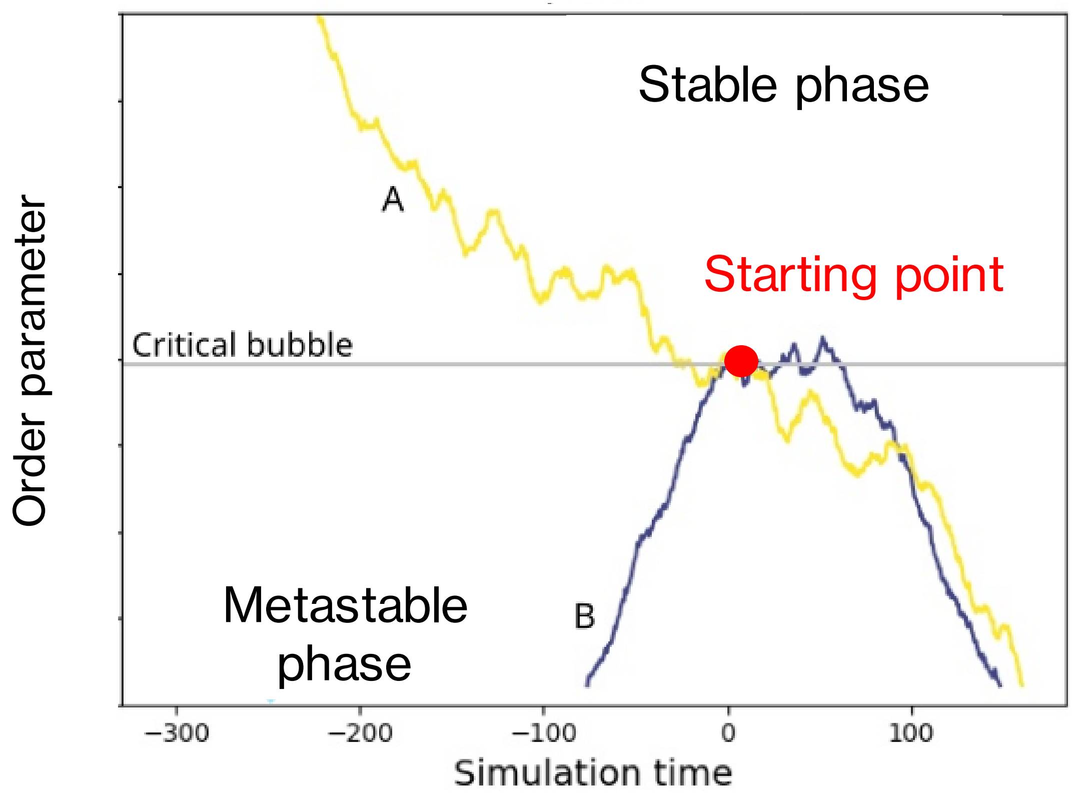


Bubble nucleation, nonperturbatively

3. Determine the tunnelling fraction

- Time evolve field configurations
- Determine if a time evolved trajectory has tunneled
- If metastable → stable (or vice versa) then $\delta_{\mathrm{tunnel}} = 1$

$$\mathbf{d} = \frac{\delta_{\mathrm{tunnel}}}{N_{\mathrm{crossings}}}$$



Bubble nucleation, nonperturbatively

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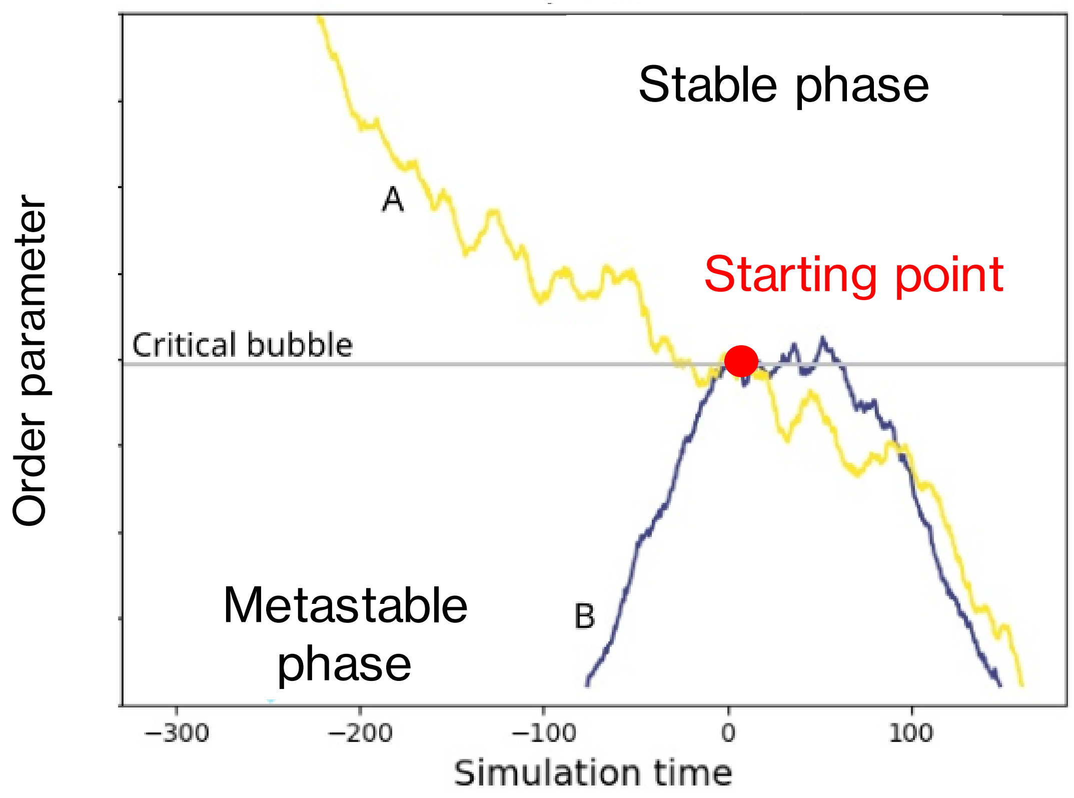


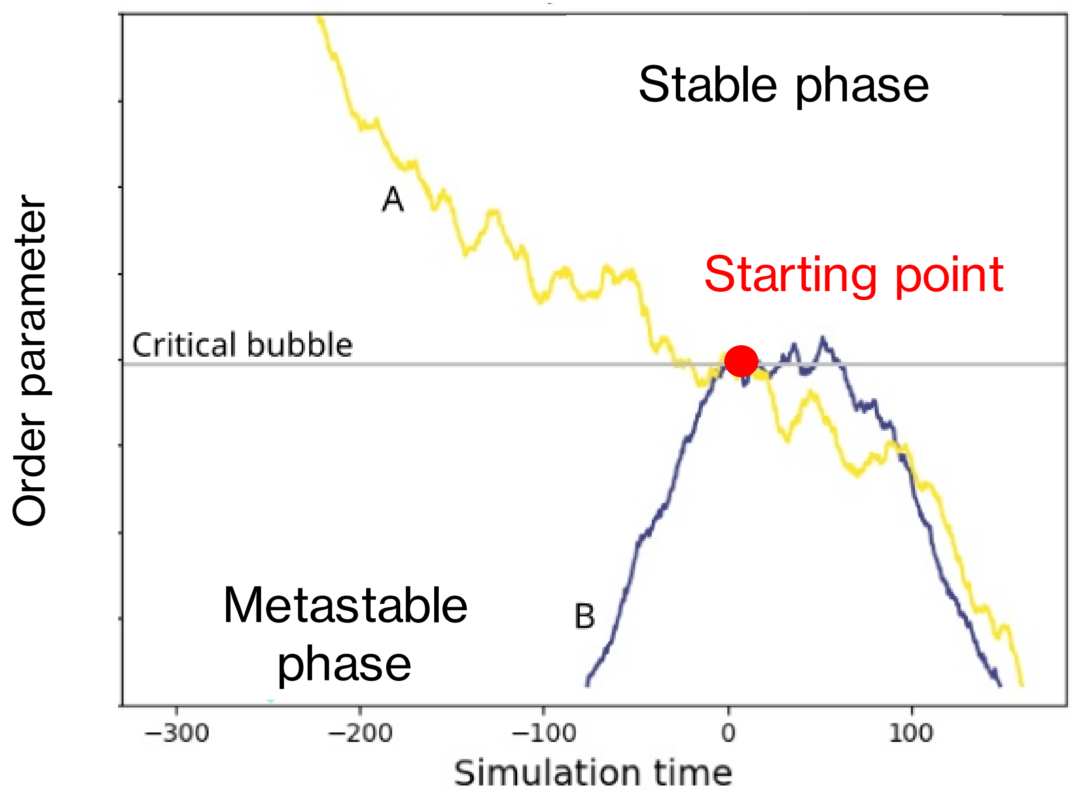
Bubble nucleation, nonperturbatively

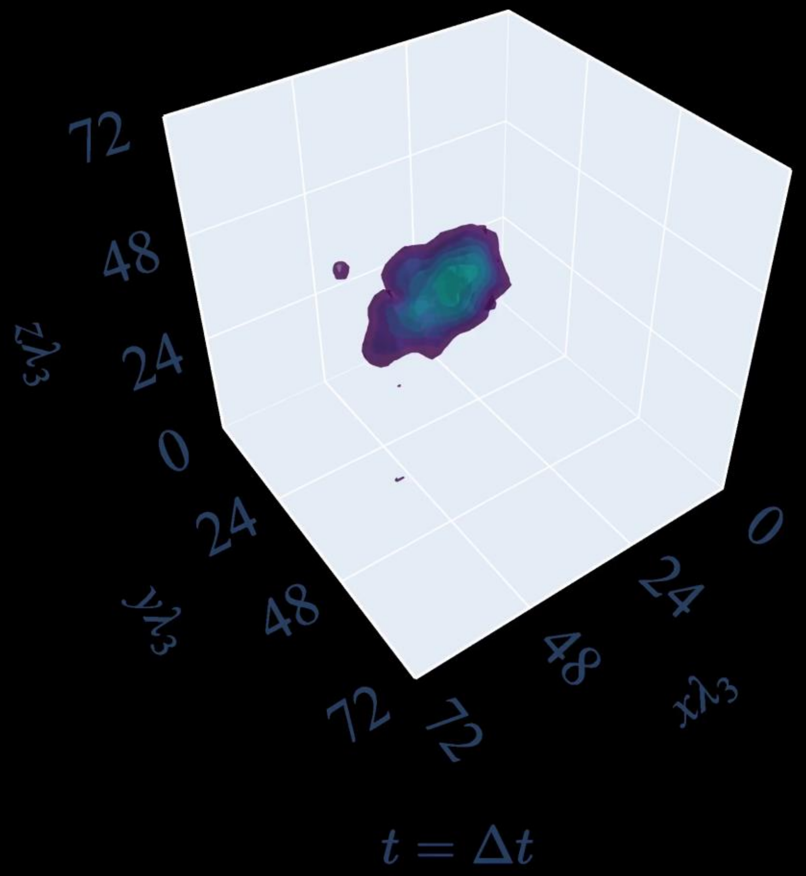
3. Determine the tunnelling fraction

- Forest-Ruth, 4th order accurate symplectic integrator built from Leapfrog

$$\pi_{t+\frac{1}{2},x} = \pi_{t,x} - \frac{\partial H_{\text{eff}}}{\partial \phi_{t,x}} \frac{\Delta t}{2},$$
$$\phi_{t+1,x} = \phi_{t,x} + a^3 \pi_{t+\frac{1}{2},x} \Delta t,$$
$$\pi_{t+1,x} = \pi_{t+\frac{1}{2},x} - \frac{\partial H_{\text{eff}}}{\partial \phi_{t+1,x}} \frac{\Delta t}{2}$$

- + noise and damping in momentum refresh





72
48
24
0
$z\lambda_3$
24
48
72
$y\lambda_3$
72
48
24
0
$x\lambda_3$
$t = \Delta t$

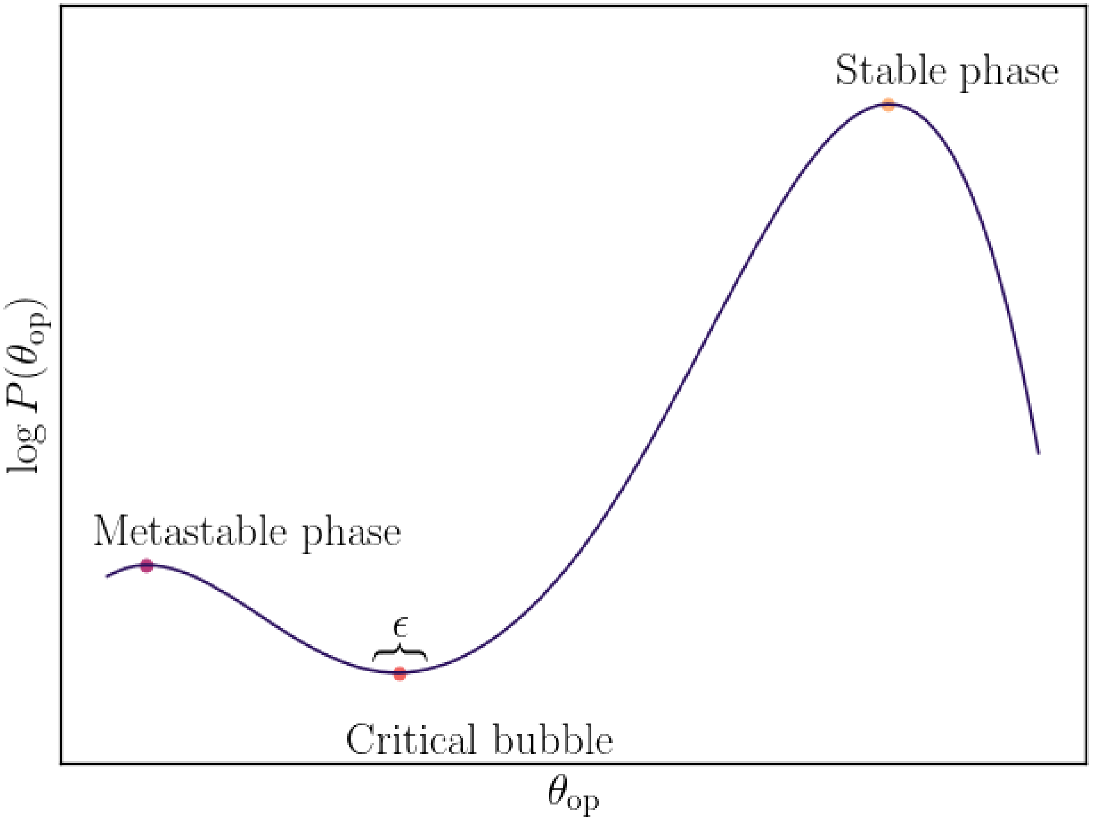
Bubble nucleation, nonperturbatively

4. Rate of change of the order parameter

- Flux - describes the rate of change of the order parameter a it crosses the separatrix

$$\langle \text{flux} \rangle = \left\langle \left| \frac{\Delta \theta_{\text{op}}}{\Delta t} \right|_{\theta_c} \right\rangle$$

- Can be solved analytically!
- Order parameter dependent!



Nucleation rate, non-perturbatively

1. Pick an order parameter, simulate probability distribution

$$P_c^{\mathrm{normalised}}$$

2. Draw separatrix configurations, calculate the probability

$$\frac{1}{2}\langle \mathbf{d} \rangle$$

3. Determine the tunnelling fraction

$$\langle \mathrm{flux} \rangle$$

4. Rate of change of the order parameter

Nucleation rate, non-perturbatively

1. Pick an order parameter, simulate probability distribution

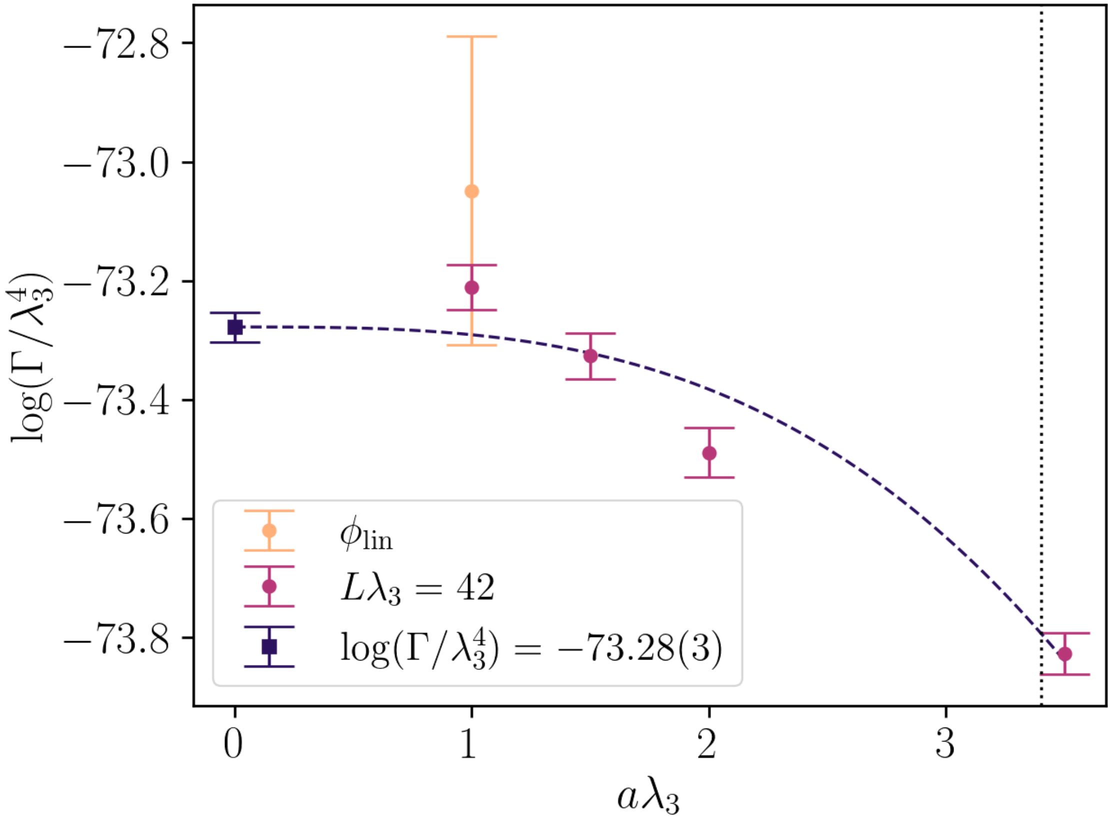
2. Draw separatrix configurations, calculate the probability

3. Determine the tunnelling fraction

4. Rate of change of the order parameter

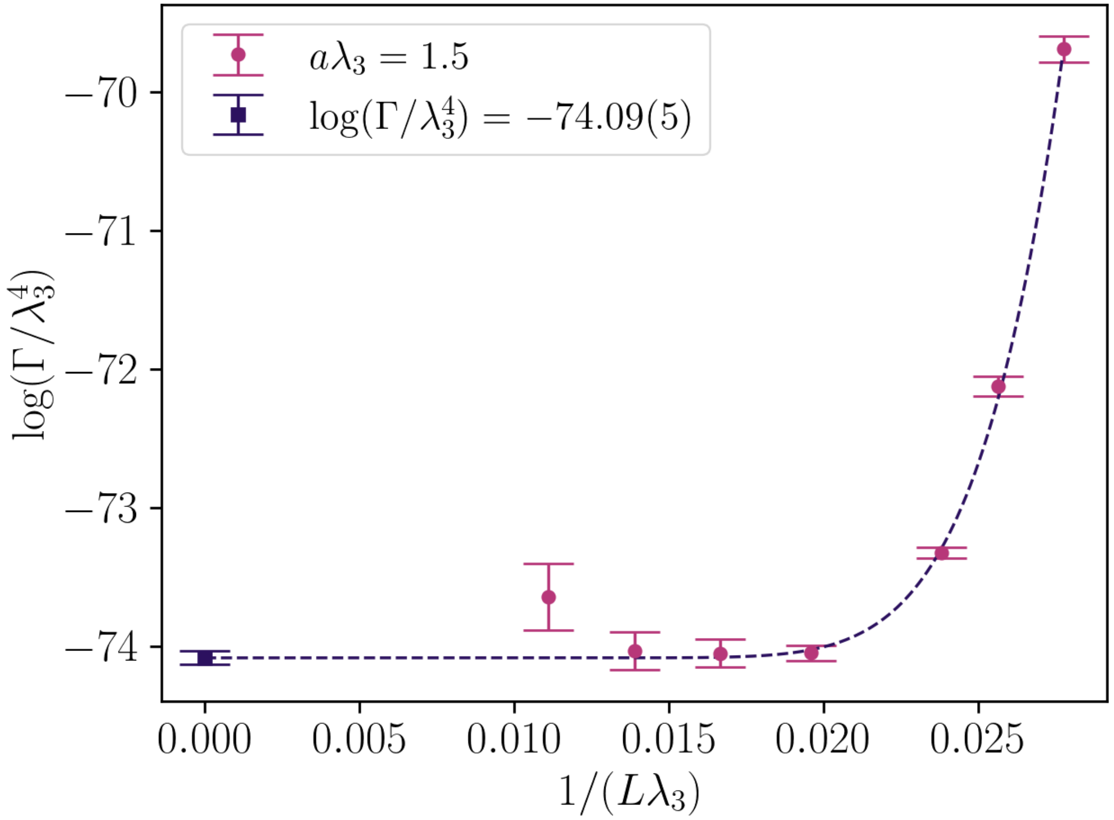
$$\Gamma \mathcal{V} \approx P_c^{\text{normalised}} \frac{1}{2} \langle \text{flux} \rangle \langle \mathbf{d} \rangle$$

Results – Continuum limit



- Zero lattice spacing extrapolation
- We use O(a^2) improved lattice discretisation
- One-loop approx for screening mass is 0.294
- Linear order parameter for reference, reduced errors in quadratic

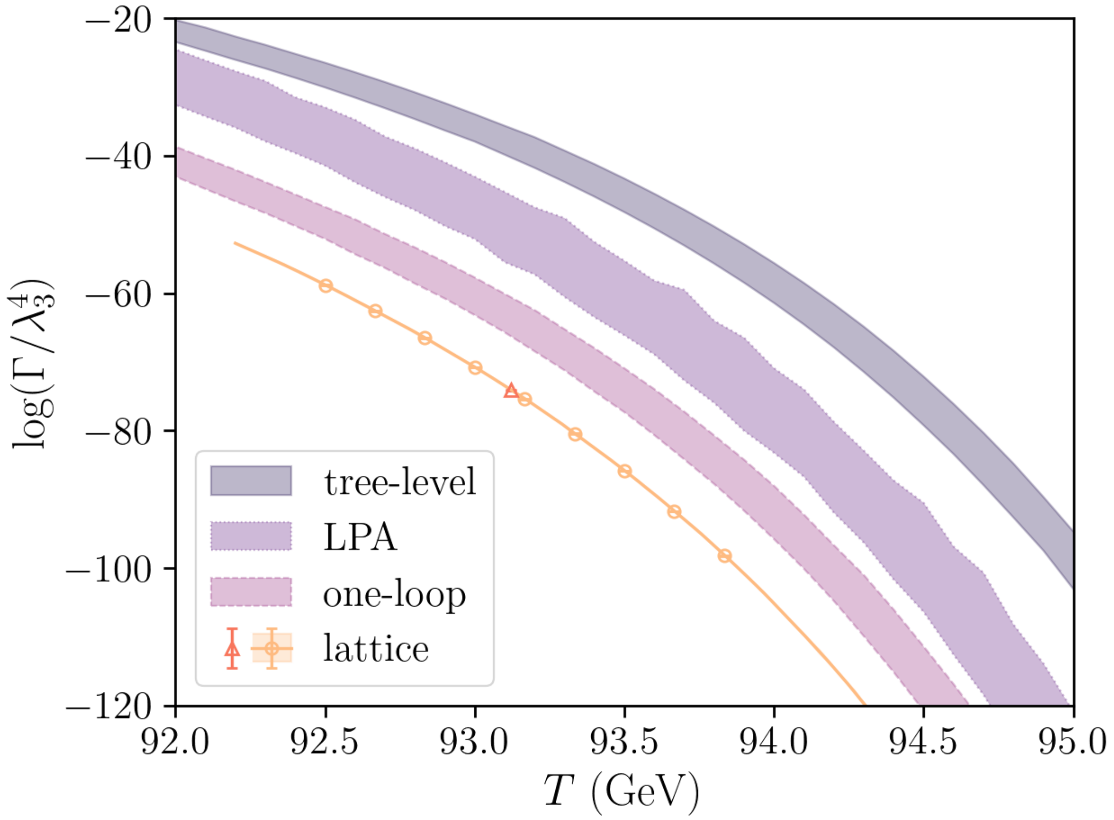
Results – Volume limit



- Long range correlations die off exponentially with distance (3d model has no massles modes)
- → we fit an exponential
- We find agreement with the perturbative screening mass

$$m_s/\lambda_3 = 0.287(12)$$

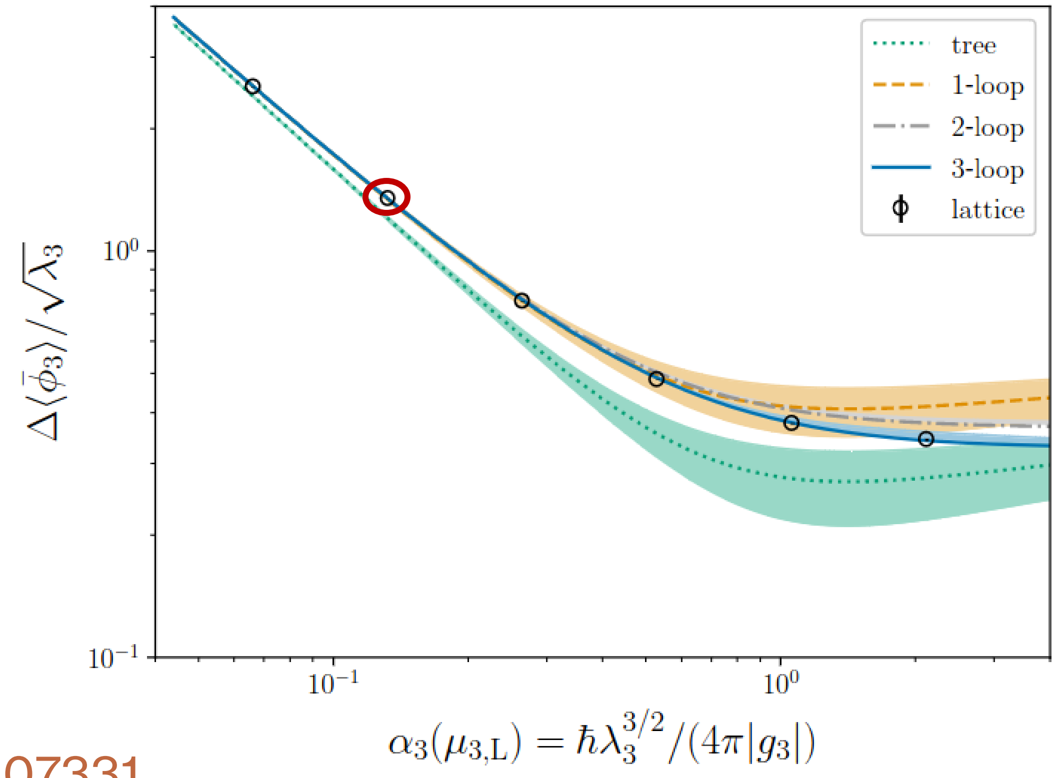
Results – Nucleation rate



- Tree-level = functional det and dynamic prefactor approx as T^4
- LPA = Local potential approx
- Lattice = one simulated param point, reweighted to other temps

Key points – do we fully understand bubble nucleation?

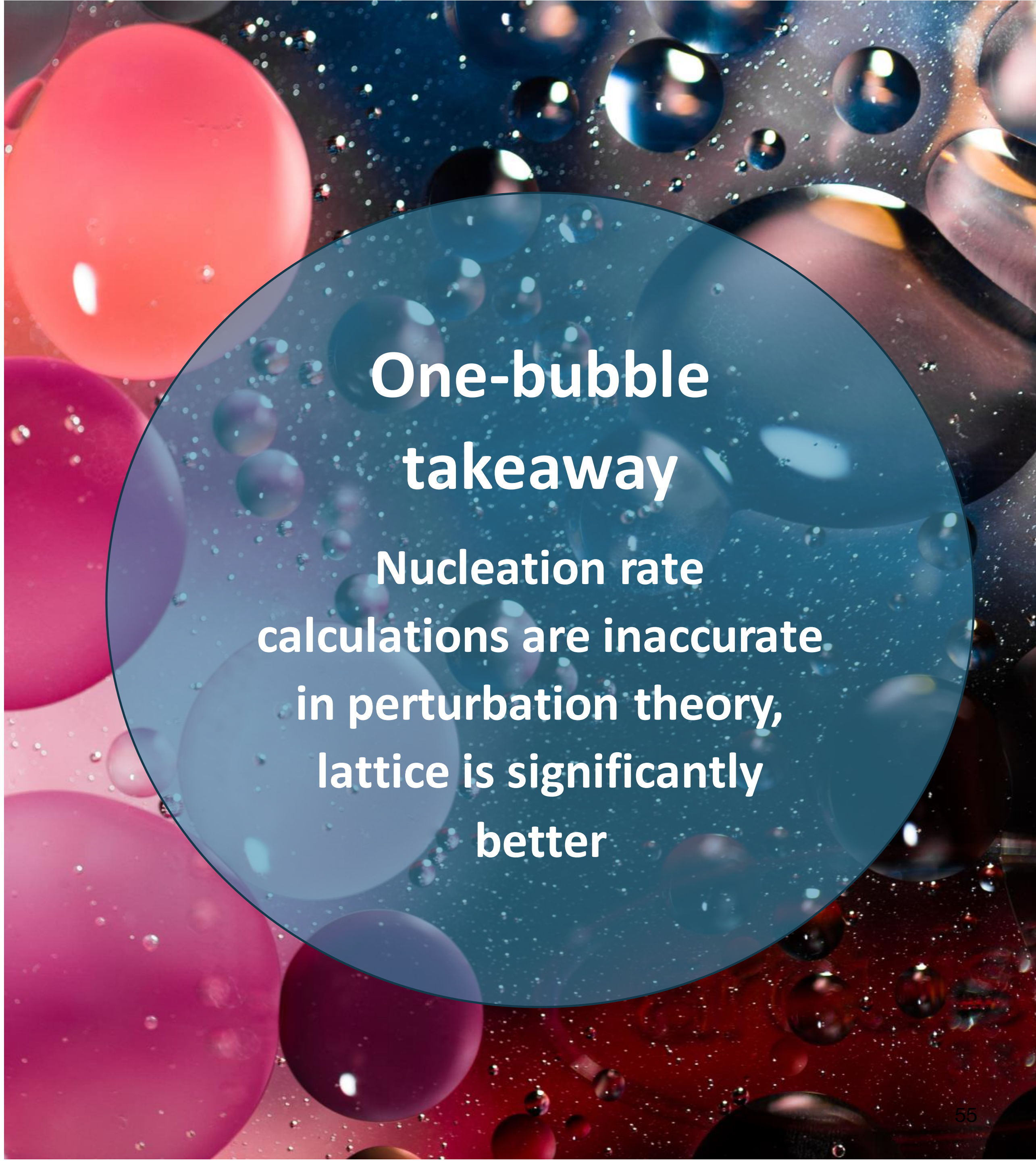
Gould, arXiv:2101.05528



- Tree-level is 100% away, one-loop is 20% away in log Γ, system should be well described by one-loop (in linear space 10^{17}% and 10^6%)
  - Do we understand the discrepancy between lattice and perturbative results?
- Latent heat on the lattice vs. one-loop agree to 1% (in linear space)
  - Semiclassical expansion breaks down? arXiv:2201.07331
    Other saddle points? arXiv:1604.06090, arXiv:1806.06069
    Something else?

Conclusions

- Allows us to calibrate the uncertainty in PT parameters when obtained from perturbative results
- Accurate computations of the nucleation rate are crucial for calculating e.g. the GW power spectrum
- Our simulations show us a suppression of the nucleation rate by a factor of 20 compared to the one loop estimate
- Method and results can be applied to other theories



BACKUP – DIMENSIONAL REDUCTION

- Fourier expansion

$$\int_\tau \int_x \left[\frac{1}{2}\psi(\tau,x)(-\nabla^2 - \partial_\tau^2 + m^2)\psi(\tau,x)\right] = \frac{1}{T}\sum_n \int_x \left[\frac{2}{2}\phi_n(x)(-\nabla^2 + (n\pi T)^2 + m^2)\phi_n(x)\right]$$

- Masses of the Fourier (Matsubara) modes are now

$$m_n^2 = (n\pi T)^2 + m^2$$

BACKUP – DIMENSIONAL REDUCTION

Hard: $E \sim \pi T$

Soft: $E \sim gT$

Supersoft: $E \sim g^{3/2}T$

Ultrasoft: $E \sim g^2T$

Backup - Reweighting

- Simulations are computationally expensive → use reweighting the order parameter histogram at different parameter points
- In our case we reweight in two parameters