Large-signal Code TESLA and its Application to the Modeling of Single-Beam and Multiple-Beam Klystrons and IOTs*

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* This work was supported by the U.S. Office of Naval Research.

Outline:

- Current state of the art in the klystrons and MBKs modeling
- Overview of the TESLA algorithms, its main assumptions and features
- Examples of application of the code TESLA to the modeling of single-beam and multiple-beam klystrons and IOTs
- Importance of 2D effects for accurate modeling of klystrons
- Parallel TESLA modeling of MBKs and IOTs
- Introduction of the reduced-order 1.5D TESLA algorithm and its comparisons with the original 2.5D TESLA
- Summary/Conclusion

Current State-of-the-Art of the Vacuum Electronics Device Modeling: *application to klystrons and MBKs*

3D PIC Codes

(CST Particles Studio)

First principles

- ***Direct solution of Maxwell's equations + Lorentz equation***
- ***Geometry driven approach***
- ***Fully time domain***

Main advantage:

General purpose tool

Disadvantages:

- **High demand on computer resources**
- **Frequently uses simplified geometry to reduce mesh**

- ***Wide range of applicability***
- ***Allows to explore new concepts***

2D/2.5D Large-signal codes

(*TESLA, TESLA-Z*)

TESLA algorithm relies on approximations

- ***Frequency & slow-time domain***
- ***RF period averaging***
- ***Employs axial-symmetry inside beam-tunnel***

Main advantage:

Computationally efficient: fast and runs on a single core of laptop or desktop PC

Disadvantages:

Device-specific

- ***Became industry standard design tool:***
  - ***TESLA -> klystrons, MBKs, IOTs, EIKs***

Recently developed large-signal code **TESLA-Z** *enabled geometry-driven modeling of wide-range of linear-beam VEDs by relying* on *generalized, frequency-dependent impedance matrix; it was successfully applied to the modeling of advanced MBK with two-gap cavities and filter-loading.*

Comparisons of approximations, models and settings used in different codes: *from 1D to 1.5D and to 2D/2.5D simulations*

Code type and name / feature	**1D large-signal algorithms (AJDISK, KlyC)**	**1.5D large-signal algorithms (TESLA-1.5D, KlyC-1.5D)**	**2D/2.5D large-signal algorithms (TESLA-2.5D, MAGIC-2D)**	**3D PIC codes (CST Particles Studio)**
particles model	disks (1d particles)	rings (2d particles)	rings (2d particles)	3d particles
beam-model	----	immersed	internal beam model or imported beam (axisymmetric 2D profile)	internal beam model or imported beam
beam-tunnel profile	constant	constant or variable	arbitrary axisymmetric 2D profile	arbitrary 2D/3D profile
magnetic-field value or profile	**infinite**	**infinite**	arbitrary axisymmetric 1D or 2D profile	arbitrary 2D/3D profile
RF Ez-field across the beam/beam-tunnel	averaged	variable	variable	variable
RF Er-field across the beam/beam-tunnel	absent	absent	variable	variable
RF B_θ -field across beam/beam-tunnel	absent	absent	variable	variable

Large-signal 2.5D klystron code TESLA: main features of the algorithm

TESLA - Telegraphist's Equations Solution for Linear-beam Amplifiers

• **Simulations code for:**	**Single Beam Klystrons (SBK)** **Multiple Beam Klystrons (MBK)** **Extended Interaction Klystrons (EIK)** **Inductive Output Tubes (IOT)**
• **Self Consistent:**	**Mutual influence of EM fields and electron trajectories** **Complete solution of Maxwell's equations inside beam-tunnel** **Space charge effects (1D/quasi-2D solver)**
• **Effective:**	**Compact Representation of Fields-modes** **Multiple time scale approach –average over wave period** **Can treat transients, high Q structures** **Runs on a PC –10s seconds to 10s minutes**
• **Original Code:**	**Used Fortran-66/77 language constructs** **Static memory management (run-time memory ~ 1-2 GB)** **Command-line version only** **Approximate treatment of slow-particles; couldn't treat reflected**

TESLA evolved from MAGY: ***M. Botton, T.M. Antonsen, Jr., B. Levush, K.T. Nguyen and, A.N. Vlasov, "MAGY: A time-dependent code for simulation of slow and fast microwave sources," IEEE Trans. On Plasma Science, Vol. 26, No. 3, pp. 882-892, June. 1998.***

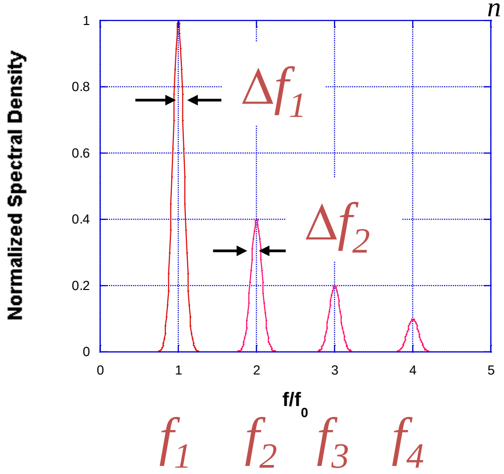
Original TESLA algorithm: ***A.N. Vlasov, T.M. Antonsen, Jr., D.P. Chernin, B.Levush and E.L.Wright ,"Simulation of Microwave Devices With External Cavities Using MAGY", IEEE Trans. On Plasma Science, 30, 3, pp. 1277-1291, 2002.***

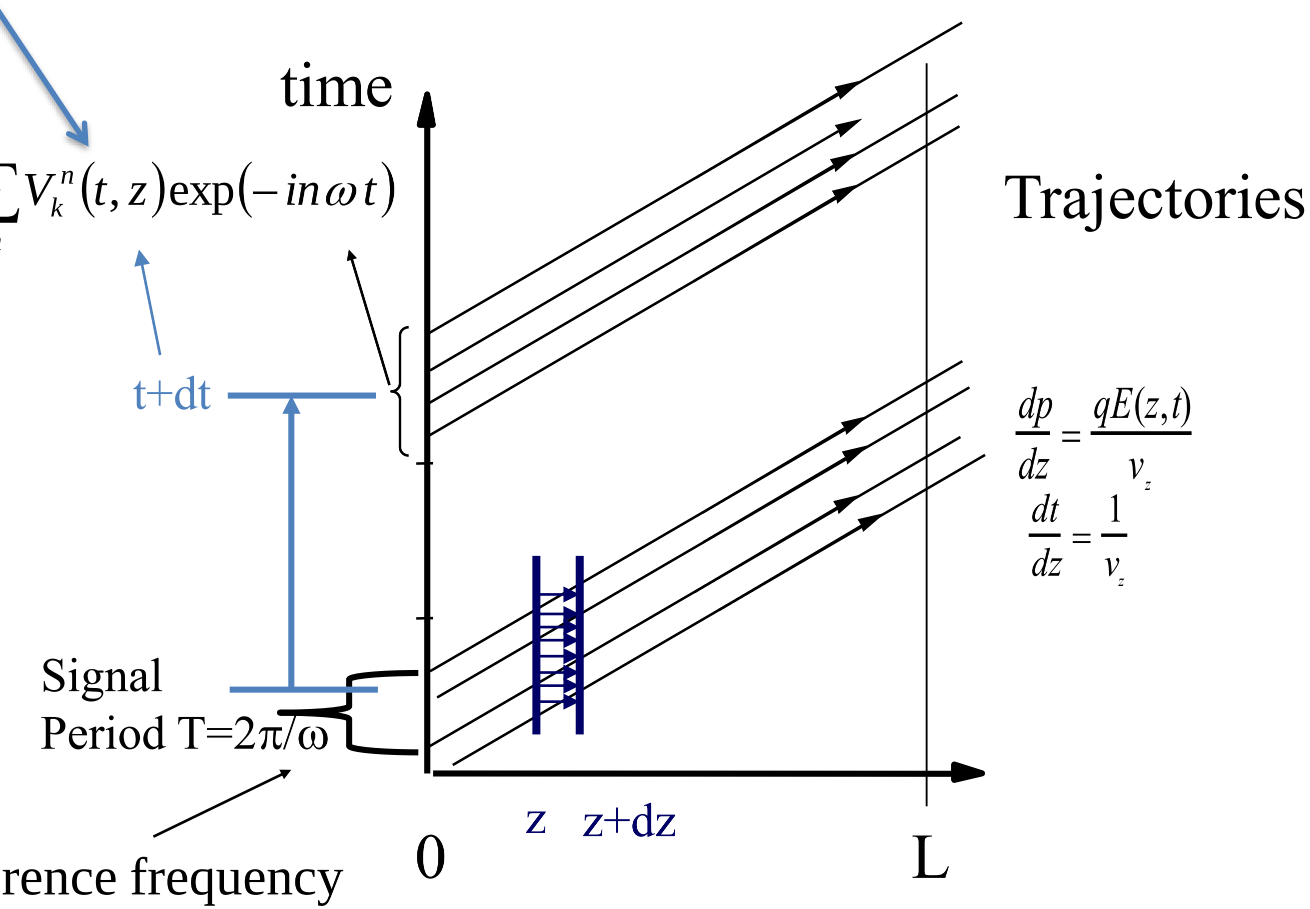
Code TESLA validation: S.J. Cooke, K.T. Nguyen, A.N. Vlasov, T.M. Antonsen, Jr., B. Levush .T.A. Hargreaves and M.F. Kirshner, "Validation of the Large-signal Klystron Simulation Code TESLA", ***IEEE Trans. Plasma Science*****, 32 (3), pp. 1136-1146, June 2004.**

TESLA - Frequency & Time Domain Hybrid Simulation*

Slow-time (envelope) approximation

$\Delta f_n << f_n$





$$\sum_n V_k^n(t,z)\exp(-in\omega t)$$

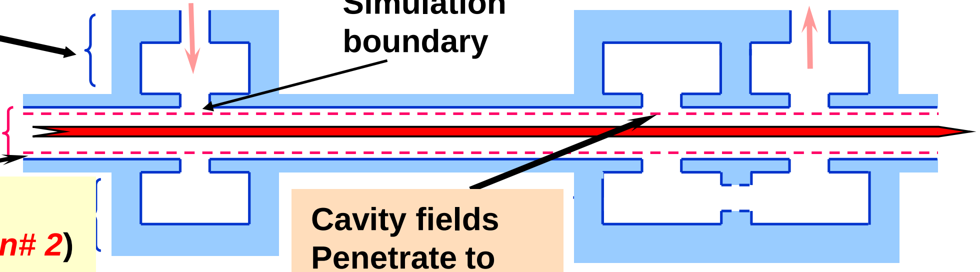
$$\frac{dp}{dz}=\frac{qE(z,t)}{v_z}$$

$$\frac{dt}{dz}=\frac{1}{v_z}$$

*T.M. Antonsen, Jr., I.A. Chernyavskiy, D. Chernin, A.N. Vlasov, and B. Levush, “Advances in the Theory and Modeling of Linear Beam VE Amplifiers,” published in a Special Issue of IEEE Transactions on Electron Devices, vol. 70, no. 6, pp. 2680- 2692, June 2023; DOI: 10.1109/TED.2023.323452

Key Elements of TESLA Model: *domain separation and modeling inside the beam-tunnel*

*2D Large-signal code **TESLA**[1] – **T**elegraphist's **E**quations **S**olution for **L**inear-beam **A**mplifiers*



External Structure region (*domain# 1*)

Input signal

Simulation boundary

Output signal

Beam tunnel Region (*domain# 2*)

Cavity fields Penetrate to Beam tunnel through gaps

- **Separation of external structure region from beam tunnel**;
- **Full Solution of Maxwell's Equations inside beam tunnel**
- Slow-time (envelope) approximation; time advance algorithm with large time steps
- Coupled Evolutionary equations for fields in external region (Equations for Modes of Resonance Cavity) and inside beam tunnel (Matrix Telegraphist's Equations)
- Relativistic 3D Equations of Electron Motion in *d/dz* (and *d/dt* [3]) representations

Beam tunnel fields: Sum over local cross-section eigenmodes[2]

$$\mathbf{E}(\mathbf{r},t) = \mathrm{Re}\left\{\sum\left[(\mathbf{E}_{\mathbf{T}}^{n}(\mathbf{r},t) + E_z^n(\mathbf{r},t)\mathbf{i}_{\mathbf{z}})e^{-i\omega_n t}\right]\right\},$$

$$\mathbf{B}(\mathbf{r},t) = \mathrm{Re}\left\{\sum_n\left[(\mathbf{B}_{\mathbf{T}}^{n}(\mathbf{r},t))e^{-i\omega_n t}\right]\right\},$$

$$\mathbf{E}_{\mathbf{T}}^{n} = \sum_k V_k'^n(z,t)\mathbf{e}_{\mathbf{k}}'(\mathbf{r}_{\mathbf{T}},z),$$

$$\mathbf{B}_{\mathbf{T}}^{n} = \sum_k I_k'^n(z,t)\mathbf{b}_{\mathbf{k}}'(\mathbf{r}_{\mathbf{T}},z).$$

$$i\frac{\omega_n}{c}E_z^n = \frac{4\pi}{c}j_z^n - \nabla_\perp\cdot\left(\mathbf{B}_{\mathbf{T}}^{n}\times\mathbf{i}_{\mathbf{z}}\right)$$

Sum over temporal harmonics

Eigen-modes of the local cross-section

External Structure fields: Sum over external structure eigenmodes

$$\mathbf{E} = \mathrm{Re}\left(\sum_s V_s(t)\,\mathbf{e}_{\mathbf{s}}\,e^{-i\omega_1 t}\right),$$

$$\mathbf{B} = \mathrm{Re}\left(\sum_s I_s(t)\,\mathbf{b}_{\mathbf{s}}\,e^{-i\omega_1 t}\right)$$

$$\begin{cases} \dfrac{d\mathbf{p}_j}{dz} = \dfrac{q}{v_{zj}}\left[\mathbf{E} + \dfrac{\mathbf{v}_j\times\mathbf{B}}{c}\right] \\ \dfrac{d\mathbf{r}_{\perp j}}{dz} = \dfrac{\mathbf{v}_{\perp j}}{v_{zj}} \\ \dfrac{d(\omega t_j)}{dz} = \dfrac{\omega}{v_{zj}} \end{cases}$$

$$\mathrm{E} = \mathrm{E}_{\mathrm{RF}} + \mathrm{E}_{\mathrm{DC}}$$

$$\mathrm{B} = \mathrm{B}_{\mathrm{RF}} + \mathrm{B}_{\mathrm{DC}} + \mathrm{B}_0$$

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[1] A.N. Vlasov et al., "Simulation of microwave devices with external cavities using MAGY," IEEE TPS, vol. 30, no. 3, pp. 1277–1291, Jun. 2002.

[2] M. Botton et al., "MAGY: A Time-Dependent Code for Simulation of Slow and Fast Microwave Sources", IEEE TPS, vol. 26, no.3, pp.882-892, June 1998.

[3] I.A. Chernyavskiy et al., "Simulation of Klystrons with Slow and Reflected Electrons Using Large-Signal Code TESLA", IEEE TED, vol.54, no. 6, pp.1555-1561, June 2007.

Particle equations, used in TESLA: accurate description for slow and reflected particles

division on v_z

Equations of Motion

$$\begin{cases} \dfrac{d\mathbf{p}_j}{dz} = \dfrac{q}{v_{zj}}\left[\mathbf{E} + \dfrac{\mathbf{v}_j \times \mathbf{B}}{c}\right] \\ \dfrac{d\mathbf{r}_{\perp j}}{dz} = \dfrac{\mathbf{v}_{\perp j}}{v_{zj}} \\ \dfrac{d(\omega t_j)}{dz} = \dfrac{\omega}{v_{zj}} \end{cases}$$

$\mathbf{E} = \mathbf{E}_{\mathbf{RF}} + \mathbf{E}_{\mathbf{DC}}$

$\mathbf{B} = \mathbf{B}_{\mathbf{RF}} + \mathbf{B}_{\mathbf{DC}} + \mathbf{B}_{\mathbf{0}}$

"Partial" treatment

$$\begin{cases} \dfrac{d\mathbf{p}_j}{dz} = \dfrac{q}{v_{zj}}\left[\dfrac{\mathbf{v}_j \times \mathbf{B}_0}{c}\right] \\ \dfrac{d\mathbf{r}_{\perp j}}{dz} = \dfrac{\mathbf{v}_{\perp j}}{v_{zj}} \\ \dfrac{d(\omega t_j)}{dz} = \dfrac{\omega}{v_{zj}} \end{cases}$$

Fast, but limited approach: works very well for electrons with longitudinal velocity above threshold ($v_z > v_{z\text{-th}}$), but does not work for slow, stopped and return particles ($v_z \le v_{z\text{-th}}$)

"Full" treatment

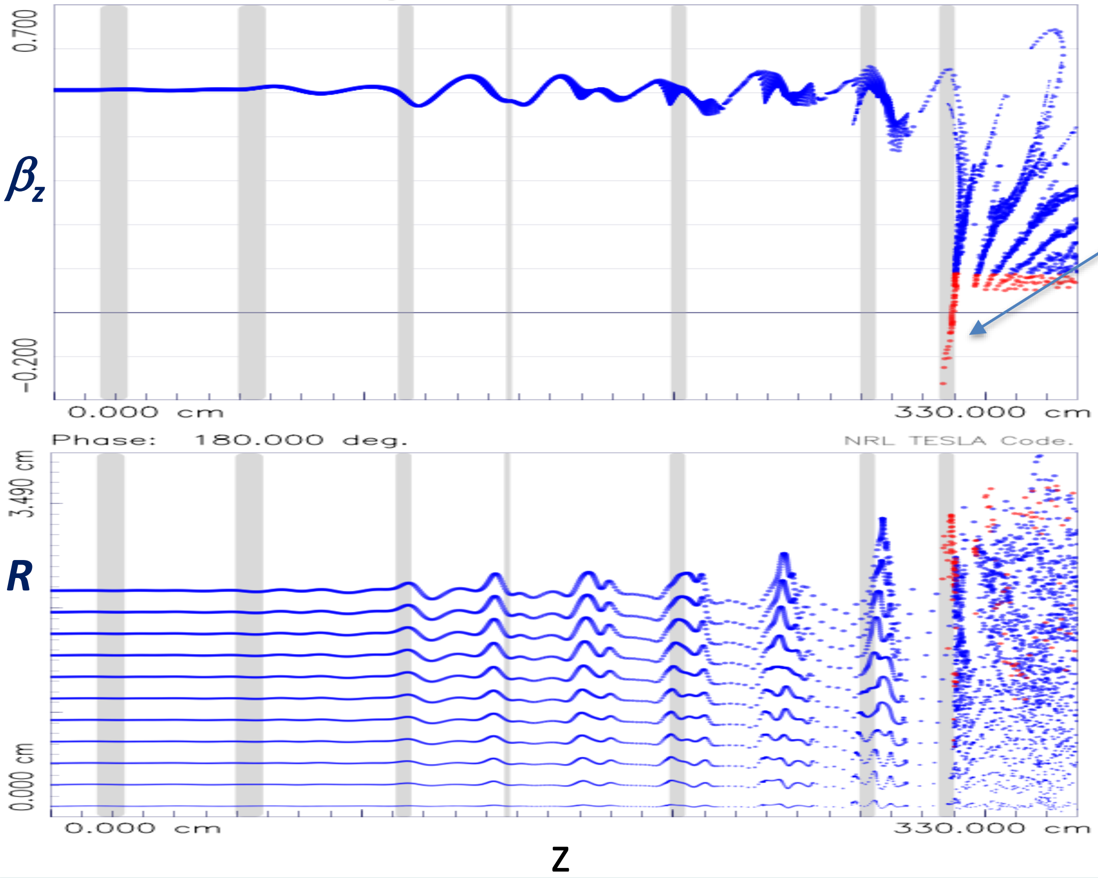
$$\begin{cases} \dfrac{d\mathbf{p}_j}{dt} = q\left[\mathbf{E} + \dfrac{\mathbf{v}_j \times \mathbf{B}}{c}\right] \\ \dfrac{d\mathbf{r}_j}{dt} = \mathbf{v}_j \\ \dfrac{d(\omega t_j)}{dt} = \omega \end{cases}$$

More general approach: works for all kind of electrons, including stopped and returned particles.

***I.A. Chernyavskiy, A. N. Vlasov, T.M. Antonsen, Jr., S.J. Cooke, B. Levush, K.T. Nguyen – *"Simulation of Klystrons with Slow and Reflected Electrons Using Large-Signal Code TESLA"*, *IEEE-TED*, Vol.54, No.6, June 2007**

Key features of the current, Fortran-95/2000 implementation of the 2.5D large-signal code TESLA[1]:

Particles phase-space (β_z vs z): example, based on the modeling of the SLAC 7-cavity klystron:



Current implementation of the code[1-5]:

- ➢ uses Fortran-95/2000 language and dynamical memory management; highly structured and optimized implementation[1];
- ➢ has user friendly interface (GUI) and advanced pre-/post-processing;
- ➢ accurate treatment for slow, returned and reflected particles[2] ;
- ➢ has fully 2D DC space-charge solver[3] ;
- ➢ has parallel extension for accurate modeling of multiple-beam devices with non-identical beams/beam-tunnels[4] ;
- ➢ supports beam-importing from the gun-code MICHELLE and advanced options for particle's depopulation[5] ;
- ➢ And more …

Run-time: ~240 s on a single core of 2.6 GHz laptop
Used memory: ~210 MB

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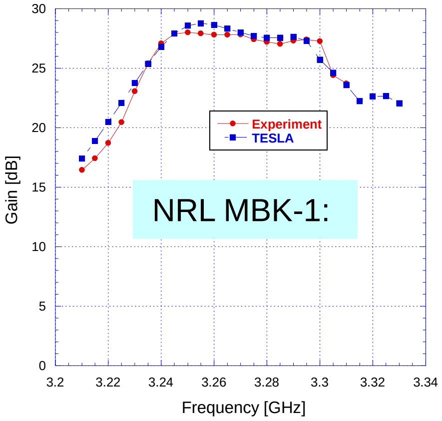
[1] I.A. Chernyavskiy et al., "Large-signal code TESLA: improvements in the implementation and in the model," *2006 IEEE International Vacuum Electronics Conference (IVEC 2006)*, Monterey, CA, 2006.
[2] I.A. Chernyavskiy et al., *"Simulation of Klystrons with Slow and Reflected Electrons Using Large-Signal Code TESLA"*, IEEE Trans. on Electron Dev., Vol.54, No.6, June 2007
[3] I.A. Chernyavskiy et al. "Full 2D model for DC space charge fields in the large-signal code TESLA," presented on *2007 IEEE Pulsed Power & Plasma Science Conference*, Albuquerque, NM (2007).
[4] I.A. Chernyavskiy et al. *"Parallel Simulation of Independent Beam-tunnels in Multiple Beam Klystrons Using TESLA"*, IEEE *Trans. on Plasma Science*, Vol. 36, No. 3, pp.670-681, June 2008
[5] I.A. Chernyavskiy et al., "End-to-End Analysis Using MICHELLE and TESLA Codes", presented on ICOPs 2009, San Diego, CA, 2009

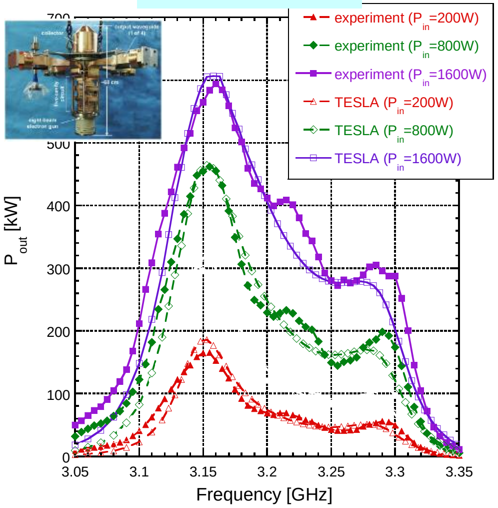
Application of the klystron code TESLA to different types of linear-beam amplifiers:

TESLA modeling of NRL S-band 8-beams MBKs

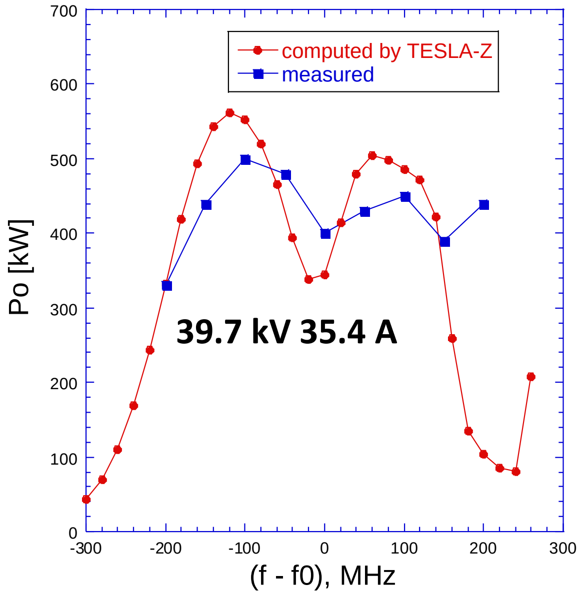


NRL MBK-1:

NRL MBK-2:

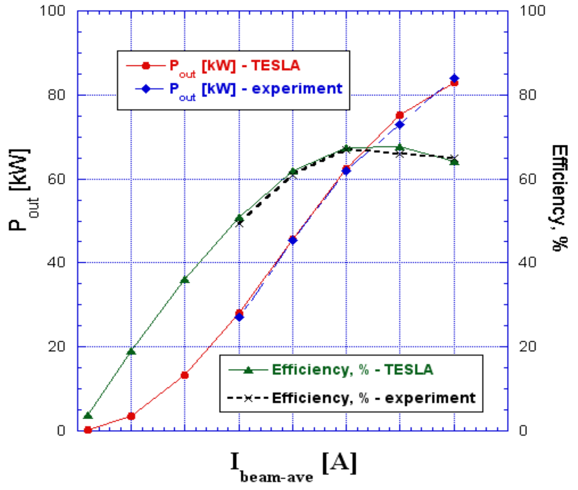


Comparisons of TESLA-Z predictions with the measured data for NRL MBK-3 (IVEC 2018)



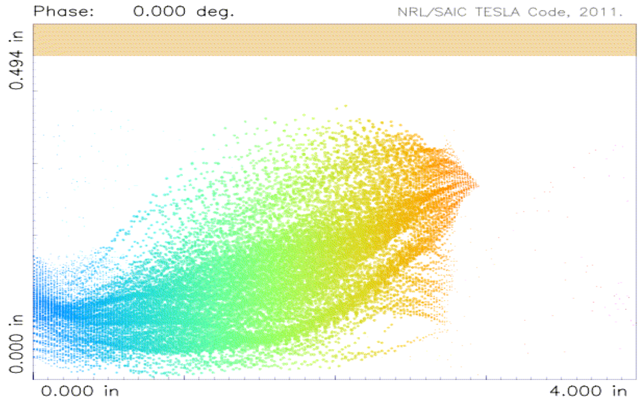
39.7 kV 35.4 A

TESLA modeling of a L-band IOT (*Courtesy of CPI, Palo Alto*): IVEC 2012



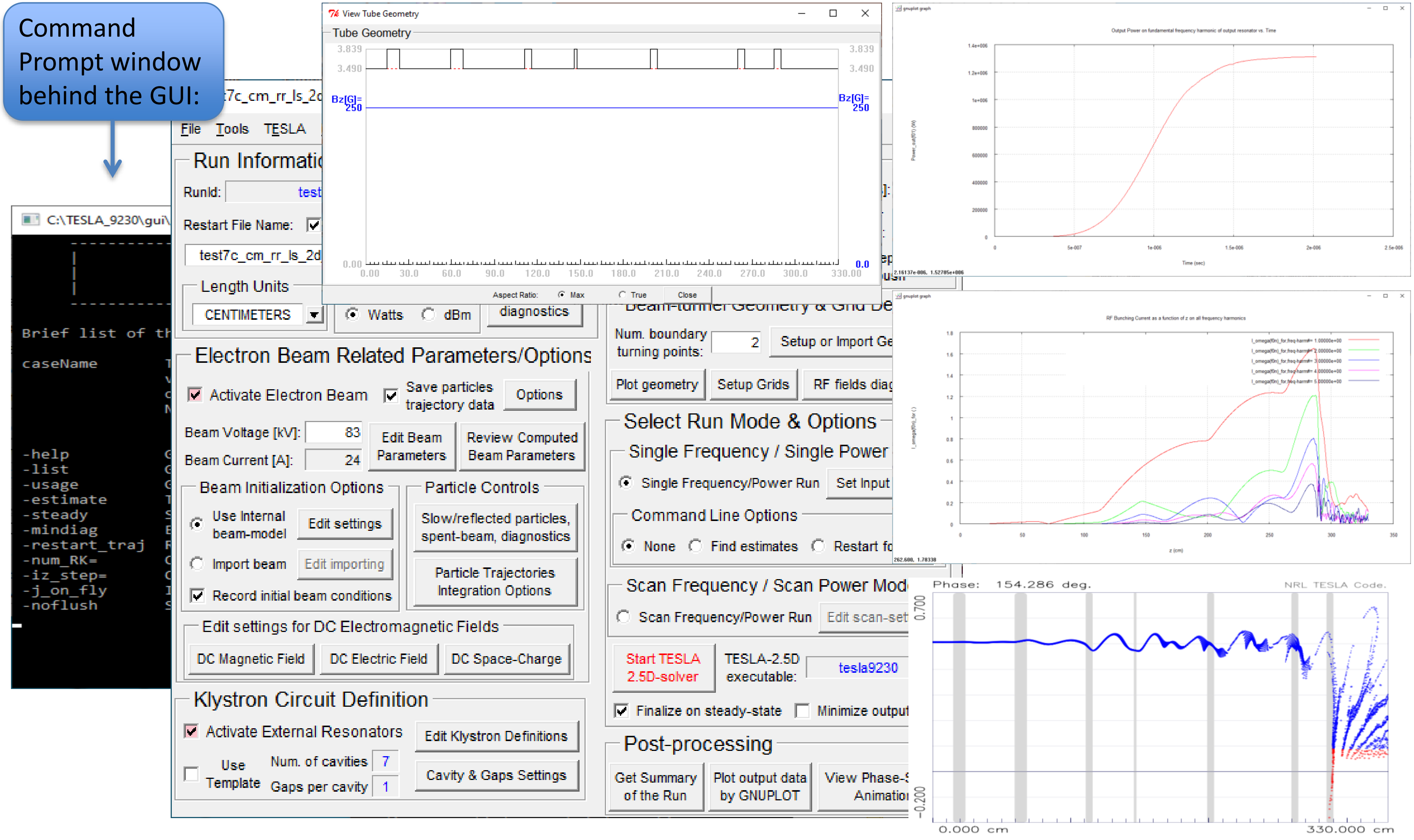
TESLA modeling of MB-IOT: IVEC 2012

(*E. Wright, et.al.*, "High-Power Multiple-Beam IOT Design")



TESLA Python-based GUI: wide functionality, pre- and post-processing options

Command Prompt window behind the GUI:



```
C:\TESLA_9230\gui\
        ----------
        |
        |
        |
        ----------

Brief list of th

caseName


-help
-list
-usage
-estimate
-steady
-mindiag
-restart_traj
-num_RK=
-iz_step=
-j_on_fly
-noflush
```

RunId: test

Restart File Name: ☑ test7c_cm_rr_ls_2d

Length Units: CENTIMETERS

● Watts ○ dBm

☑ Activate Electron Beam

☑ Save particles trajectory data

Beam Voltage [kV]: 83

Beam Current [A]: 24

Beam Initialization Options: ● Use Internal beam-model ○ Import beam

☑ Record initial beam conditions

☑ Activate External Resonators

☐ Use Template

Num. of cavities: 7

Gaps per cavity: 1

Num. boundary turning points: 2

● Single Frequency/Power Run

Command Line Options: ● None ○ Find estimates ○ Restart fo

○ Scan Frequency/Power Run

TESLA-2.5D executable: tesla9230

☑ Finalize on steady-state

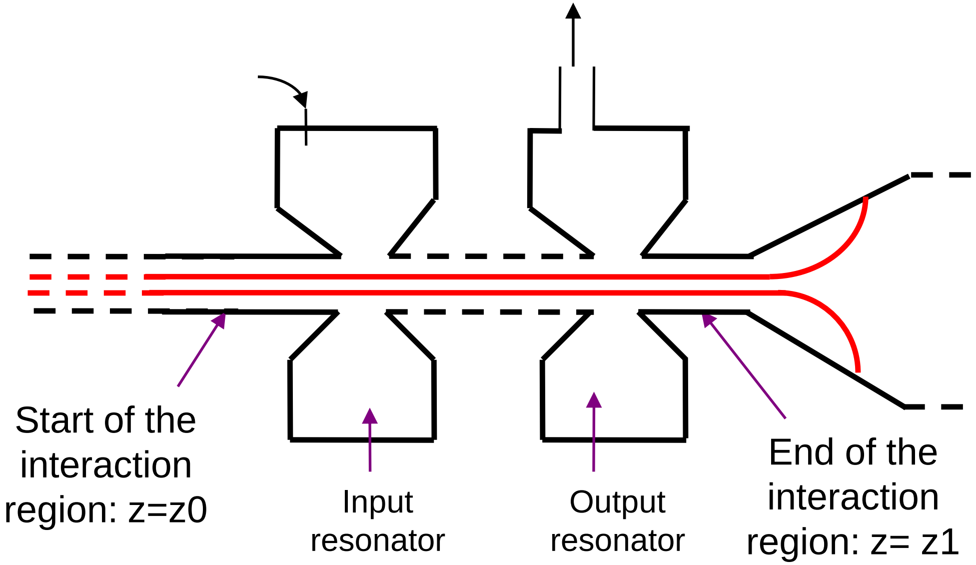
☐ Minimize output

Aspect Ratio: ● Max ○ True

Modeling with the internal beam models in TESLA

Available in TESLA set of the pre-defined models for the electron beam's initialization:

- Immersed cathode beam;
- Rigid-Rotor model;
- Shielded cathode model;
- Partially-shielded cathode;



Main assumption: it is a completely formed beam given far away from the formation region(s);

Effectiveness: in general it is a very good approximation and it provides very reasonable results for simple designs

Interaction region: part, simulated by the code TESLA

Interaction region's extend: *dz = z1 – z0*

Limitations pre-defined beam model:

- becomes more approximate & inaccurate when the beam doesn't satisfy the mentioned above assumptions;

- Intended to be applied to the simple designs or to be used as a first estimate for the real designs;

Factors, which make this limitations more pronounced:

- Complex design with the non-uniform beam-tunnel's profile;

- Compact device, whose formation region overlap with the interaction region;

- Non-uniform or complex magnetic field's profile;

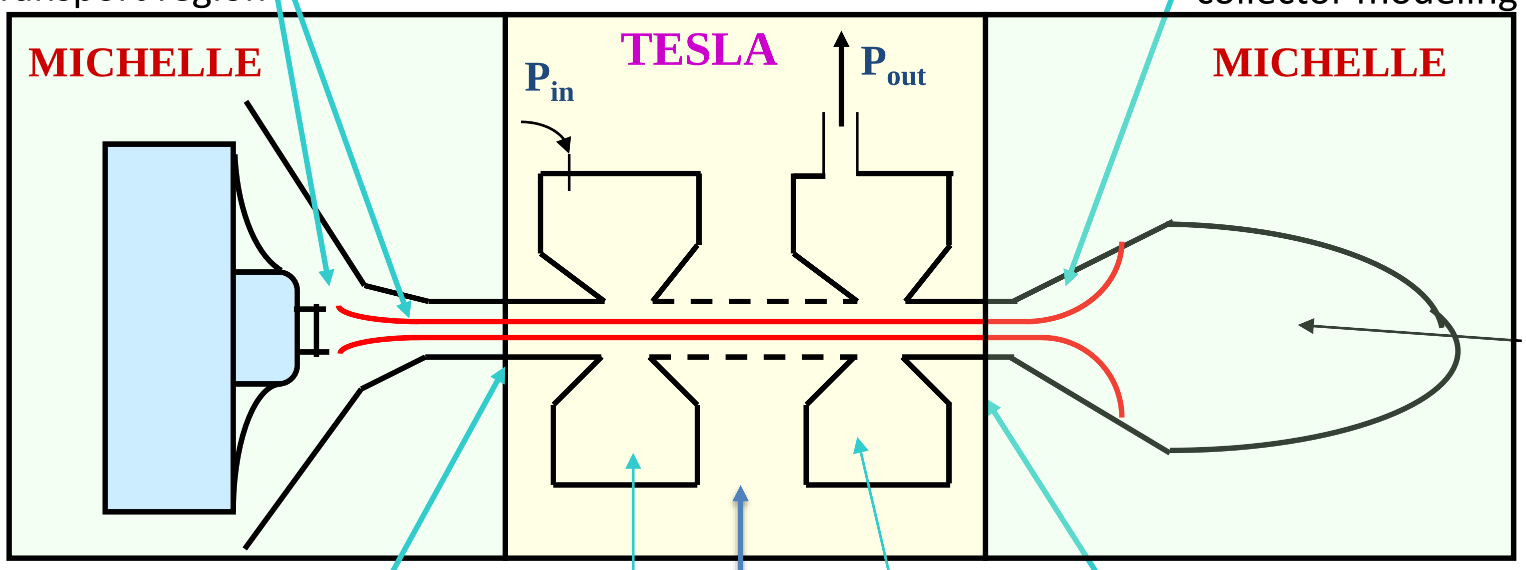
Integration of the 2.5D TESLA with the gun-code MICHELLE for an end-to-end analysis*:

"Seamless" transition between the codes for the full end-to-end analysis of the device:

Gun and collector's regions simulated by the gun-code MICHELLE**

Electron beam's formation and transport region

Spent beam and collector modeling



Collector

Start of the interaction region

Input resonator

Output resonator

End of the interaction region

Interaction region simulated by the large-signal code TESLA*

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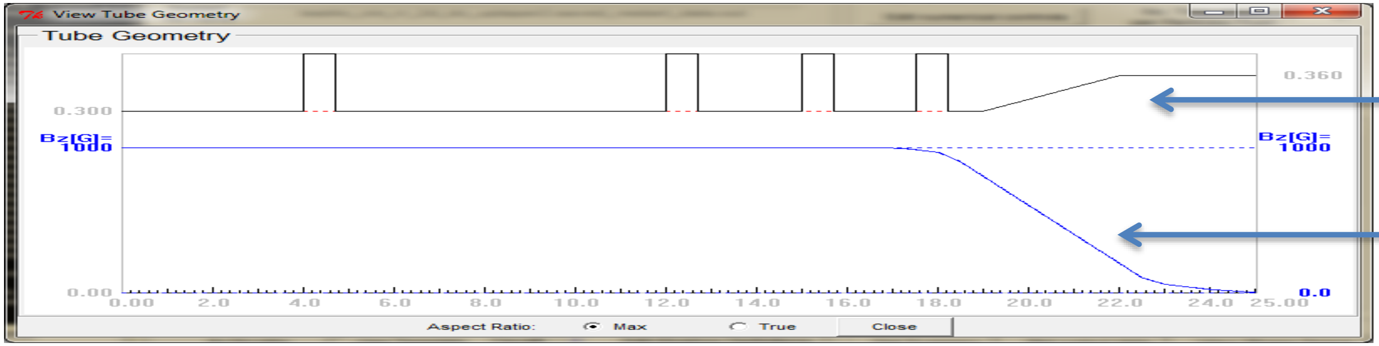
*I.A. Chernyavskiy, John J Petillo, Alexander T. Burke, Alexander N. Vlasov, Baruch Levush, Edward L Wright, "End-to-end Analysis Using MICHELLE and TESLA Codes", presented on International Conference on Plasma Science, ICOPS & SOFE, San Diego, June 2009.

**Petillo J.J., E.M. Nelson, J.F. DeFord, N.J. Dionne, B. Levush, "Recent Developments to the MICHELLE 2D/3D Electron Gun and Collector Modeling Code", IEEE Trans. Electron Devices, vol. 52, no. 5, pp.742-748, May 2005.

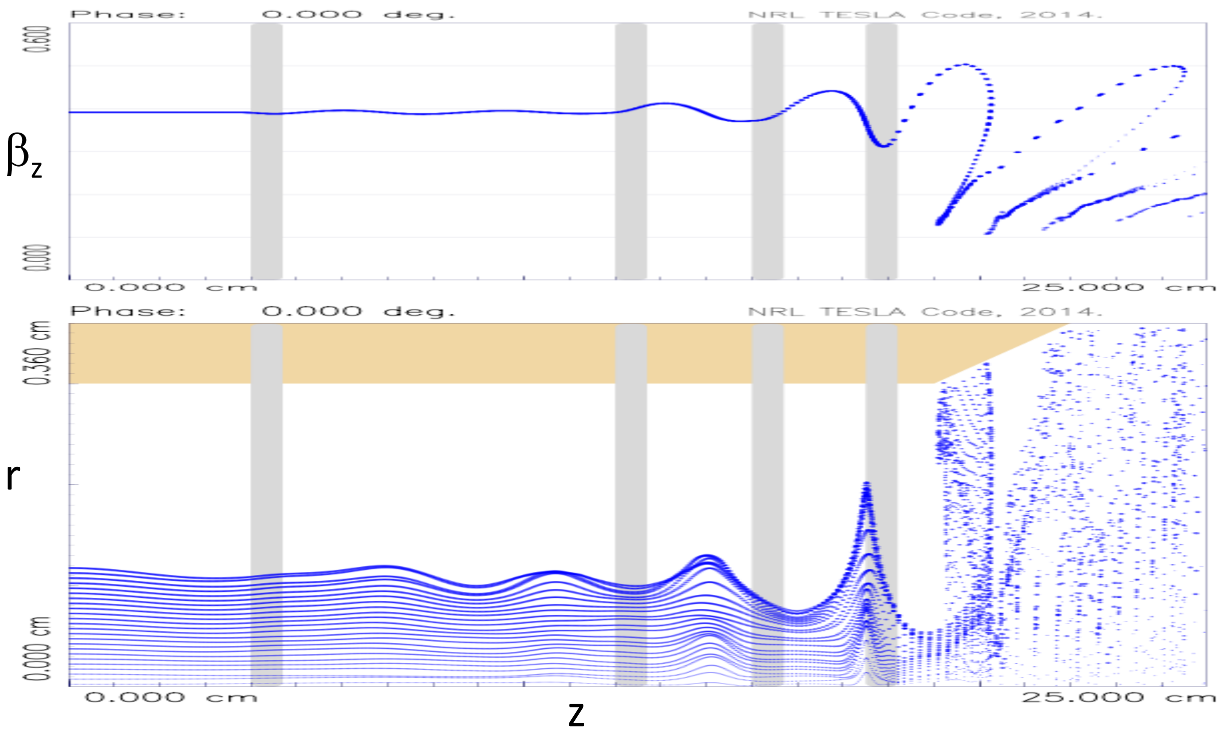
Example to illustrate 2D effects when using more realistic geometry and magnetic field profile:



Aspect Ratio: ● Max ○ True

Tapered beam-tunnel's profile

Tapered B_z field

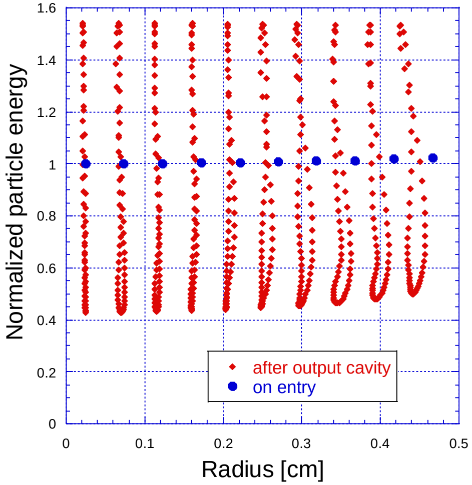


Increased danger and potential for body current and slow particles

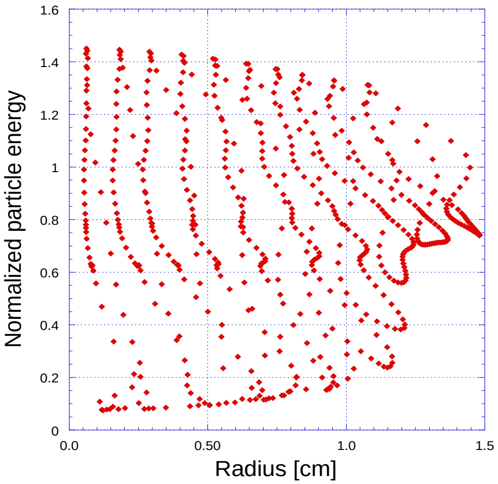
Particles’ reflection from the after-cavity region*

Widening in the distribution of particles in energy on exit right after the output cavity’s gap

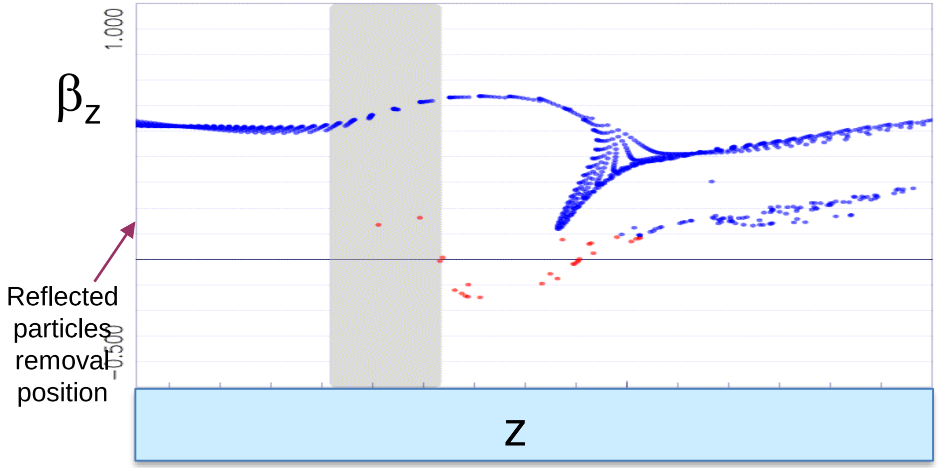
Initial energy on entry



Dramatic increase in the fraction of “slow” particles due to the effects in the after-cavity region, especially in the presence of the up-taper (additional DCSC potential depression inside the up-taper)

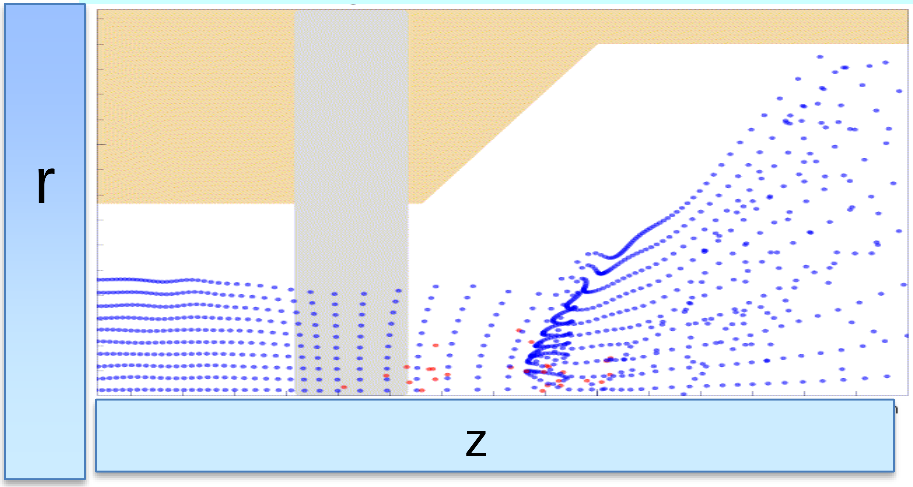


Particles phase-space in the vicinity of the output cavity’s gap



Reflected particles removal position

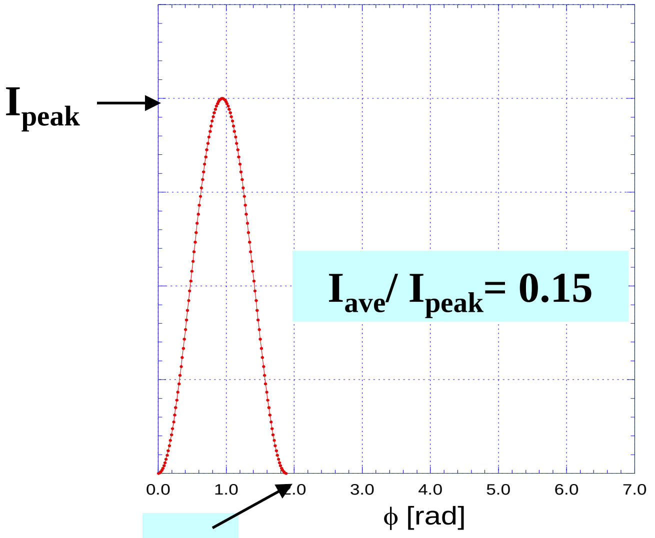
Particles trajectories in the vicinity of the output cavity’s gap and up-taper



I.A. Chernyavskiy et al., “TESLA Modeling of After-Cavity Interaction in High-Efficiency Klystrons”, abstract and presentation on IVEC 2010, Monterey, CA, 2010.

Applying TESLA to the modeling of the IOT-type devices: pre-modulated electron beam model

Used by the code an initial profile of the pre-modulated electron beam current:



$I_{ave}/I_{peak} = 0.15$

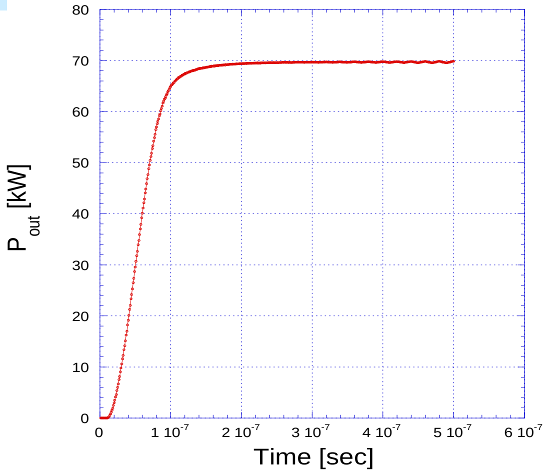
φ_w

$$\varphi_w / 2\pi = 2\, I_{ave}/I_{peak} = 0.3$$

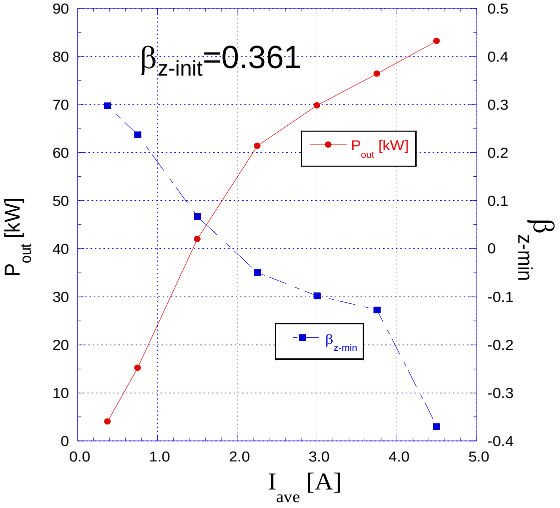
Pre-modulated beam model

$$I_b(\varphi) = I_{peak} \begin{cases} \sin^2(\pi\, \varphi/\varphi_w), & 0 < \varphi \le \varphi_w; \\ 0, & \varphi_w < \varphi < 2\pi; \end{cases}$$

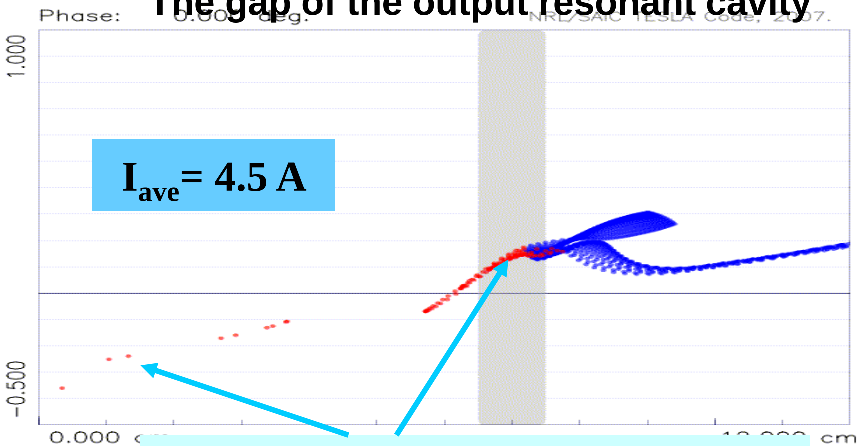
Convergence of output power in time: $I_{ave} = 3.0$ A



Dependences of the Pout and min(betaz) vs average beam current:



The gap of the output resonant cavity

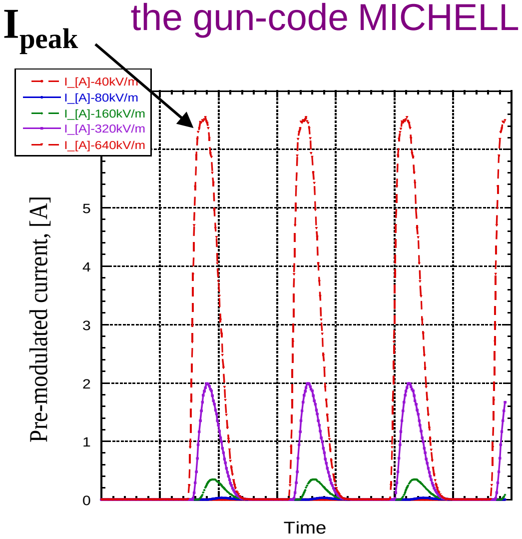


$I_{ave} = 4.5$ A

Fraction of slow and reflected particles

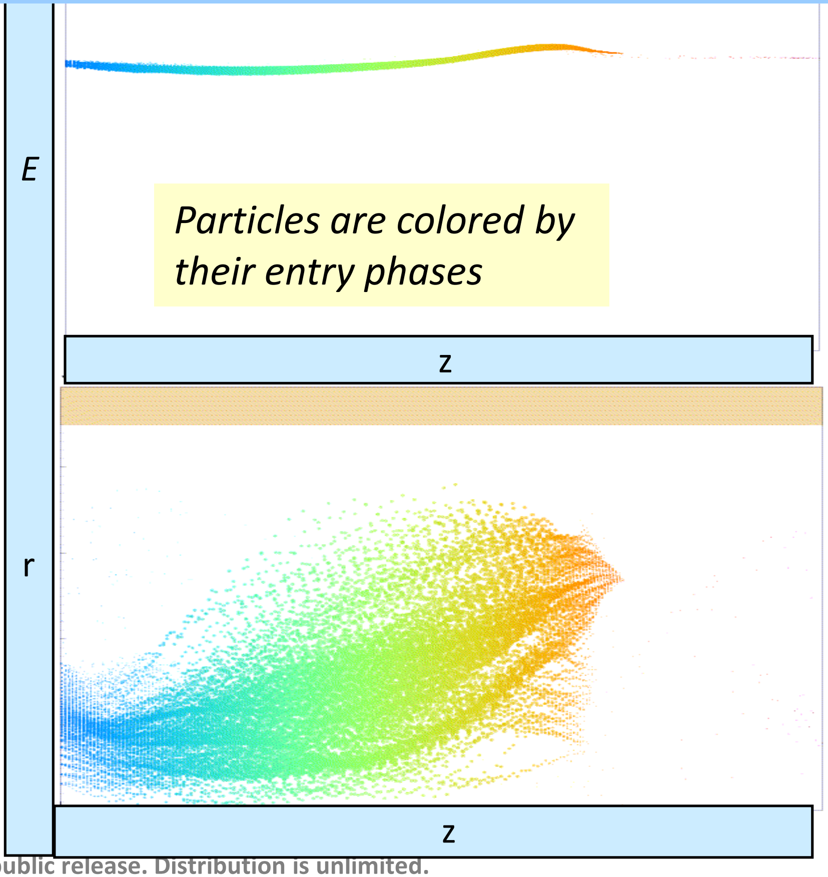
Applying TESLA to the modeling of the IOT-type devices: importing pre-modulated beam's profile

Profile of the pre-modulated electron beam current imported into TESLA (produced by the gun-code MICHELLE):



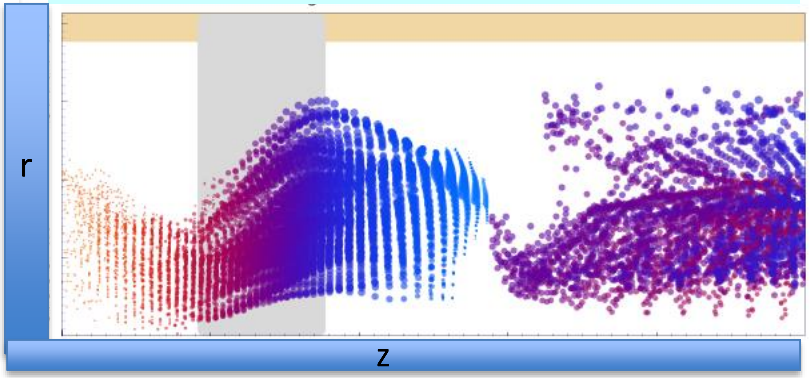
Beam current as a function of the field's strength on the gap of the gridded gun (courtesy of E. Wright)

Example with the TESLA beam-transport of a pre-modulated beam, imported from the gun-code MICHELLE: the original time-dependent beam with ~3.8M "*MICHELLE*" particles have been depopulated down to ~32K "*TESLA*" particles (*E. Wright, et.al.,* **"High-Power Multiple-Beam IOT Design"**: IOT and 500 kW to 1 MW average power UHF MB IOT modeling, simulation, code validation and design efforts using the state-of-the-art codes MICHELLE, TESLA and Analyst.)



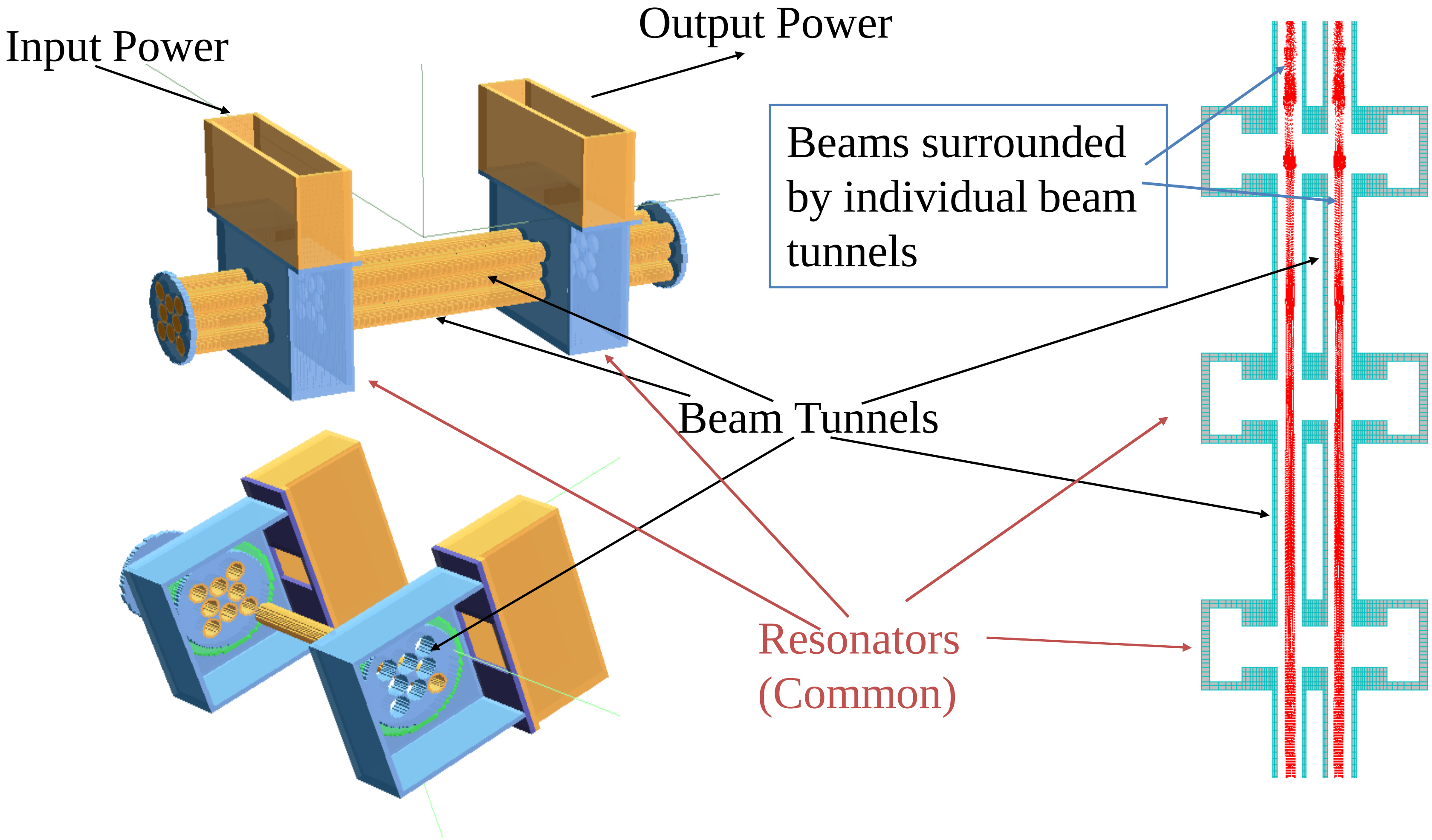
Particles are colored by their entry phases

TESLA simulation of the output cavity beam-wave interaction.



E.Wright et al., "High-Power Multiple-Beam IOT Design", IVEC 2012

Multiple Beam Klystrons (MBKs) based on fundamental mode interaction*:



Input Power

Output Power

Beams surrounded by individual beam tunnels

Beam Tunnels

Resonators (Common)

*These plots are borrowed from the work of Dr. Khanh Nguyen (Beam-Wave Research) related to NRL MBK-1 , MBK-2 and MBK-3 designs developed by using MAGIC-3D modeling.

From emulation to accurate modeling of multiple beams/beam-tunnels*:

One code - Two compilation & running modes:

Serial or *Non-parallel* version of TESLA-MB:

Uses approximation of identical multiple beams / beam-tunnels

\+

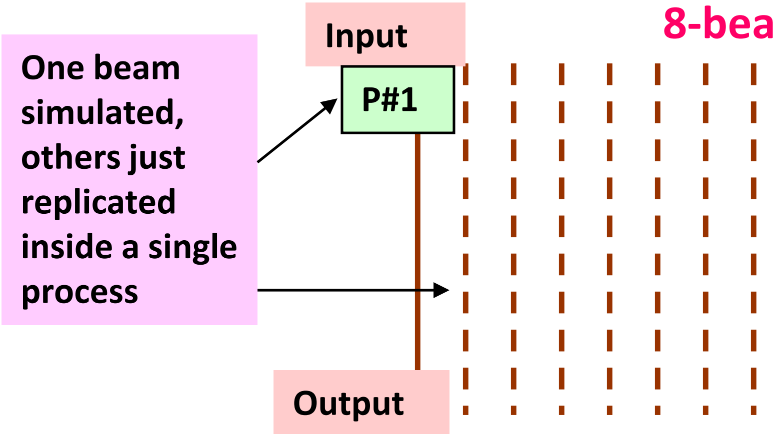
Procedure of averaging of R/Q's over all gaps for every cavity

Parallel version of TESLA-MB* executable:

Capable to model mostly general case with all beams / beam-tunnels being non-identical

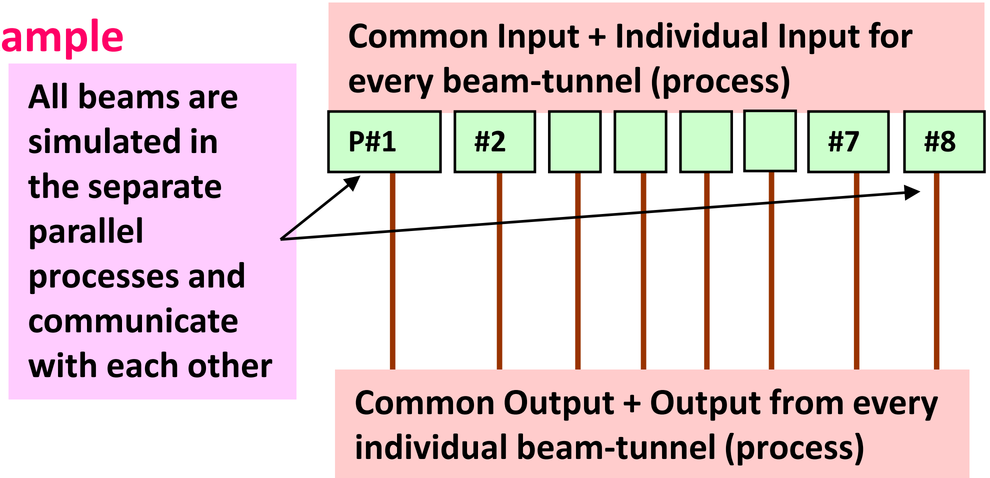
\+

Measured values of R/Q for all gaps and for all cavities (*other parameters can vary as well*)



One beam simulated, others just replicated inside a single process

8-beams example



All beams are simulated in the separate parallel processes and communicate with each other

Common Input + Individual Input for every beam-tunnel (process)

Common Output + Output from every individual beam-tunnel (process)

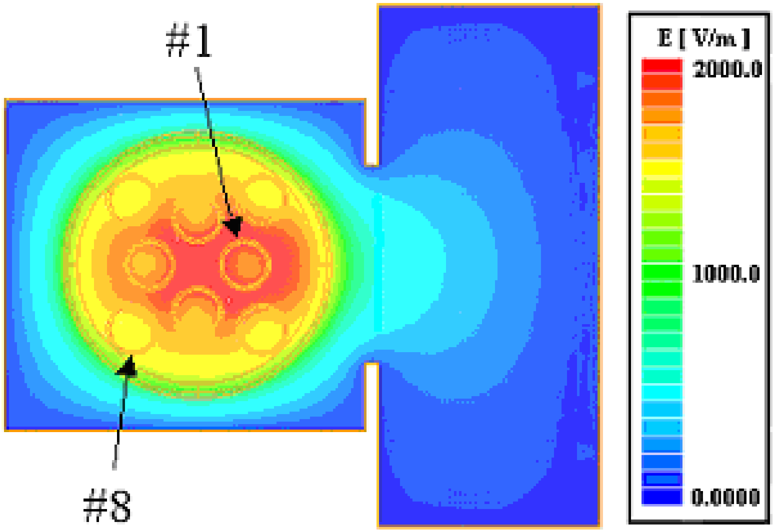
Code TESLA-MB uses MPI protocol to support runs in a parallel mode and provide communication between multiple parallel processes. It can be executed on a multi-core desktop PC or on Unix/Linux clusters (using single process per core/processor).

***Igor A. Chernyavskiy, Simon J. Cooke, Alexander N. Vlasov, Thomas M. Antonsen, Jr., David K. Abe, Baruch Levush and Khanh T. Nguyen – "*Parallel Simulation of Independent Beam-tunnels in Multiple Beam Klystrons Using TESLA*", IEEE *Trans. on Plasma Science*, Vol. 36, No. 3, pp.670-681, June 2008.**

Limitations of the approximation of identical beams/ beam-tunnels and modeling of experimental MBK*

Output cavity of the 4-cavity 8-beams MBK: *low Q cavity with single output waveguide, gives a large spread in R/Q*

Run 8 processes with the use of measured values of $(R/Q)^k_j$ for every k-th gap of every j-th cavity inside 8 separate beam-tunnels: 4×8 = 32 R/Q values



Alignment of 8 beam-tunnels with various field's intensity and R/Q

Parameters for the output cavity of the 4-cavity 8-beam MBK

Beam-tunnel#		$(R/Q)_{out}$, Ω
1	Group of inner beam-tunnels (higher R/Q)	46
2		42
3		40
4		39
5	Group of outer beam-tunnels (lower R/Q)	33
6		34
7		29
8		28

Inner **beam-tunnel with *highest* $(R/Q)_{out}$**

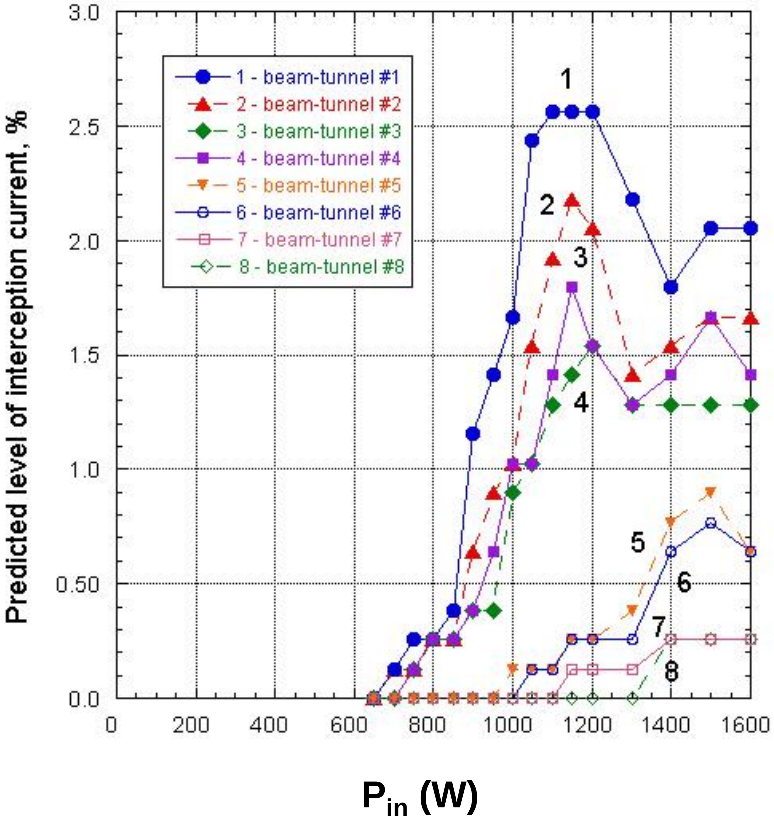
~65%

Outer **beam-tunnel with *lowest* $(R/Q)_{out}$**

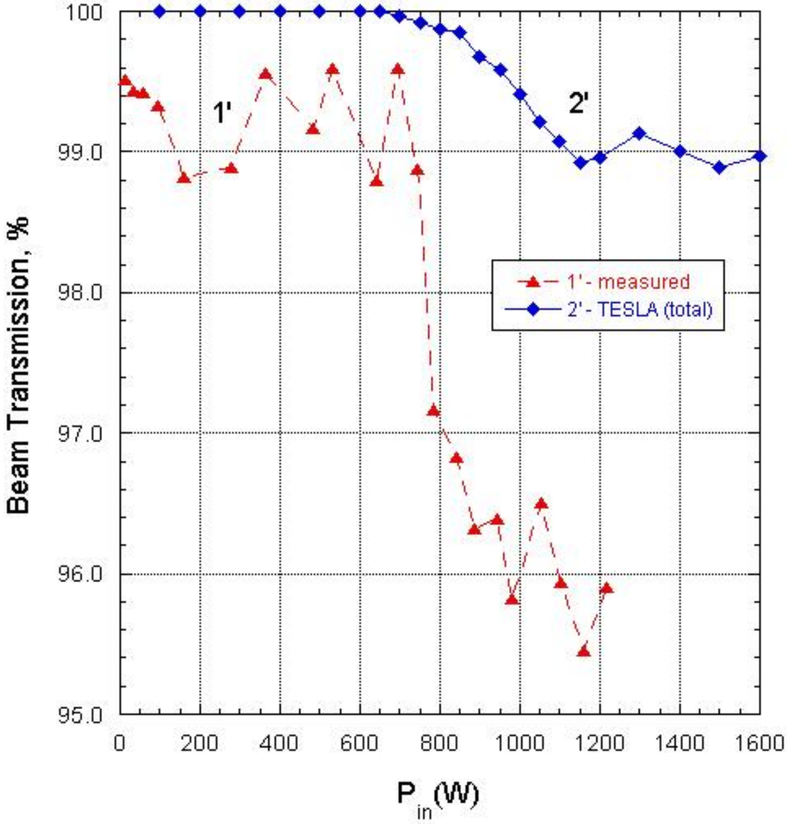
***Abe D.K., Pershing D.E., Nguyen K.T., Wood F.N., Myers R.E., Eisen E.L., Cusick M., Levush B., "Demonstration of an S-band, 600-kW Fundamental-Mode Multiple-Beam Klystron", *Electron Device Letters*, vol. 26, no. 8, pp. 590-592, August 2005.**

Detailed analysis of an interception current in the MBK1*

Predicted interception current per beam-tunnel:

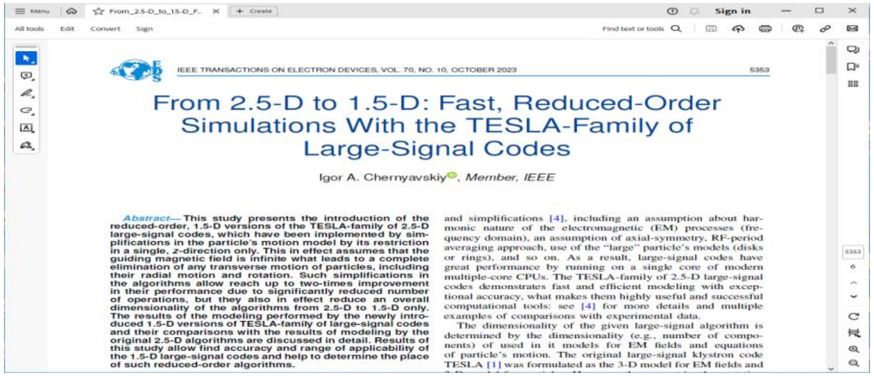


Experimental and theoretically predicted beam transmission:



***I.A.Chernyavskiy, D.K. Abe and B. Levush, "Detailed Analysis of the Interception Current Predicted by the Large-Signal Code TESLA for an Experimental Multiple Beam Klystron", IEEE-TED 2009 Special Issue (May 2009).**

From 2.5D to 1.5D: fast modeling with reduced order TESLA algorithm



IEEE TRANSACTIONS ON ELECTRON DEVICES, VOL. 70, NO. 10, OCTOBER 2023 5353

From 2.5-D to 1.5-D: Fast, Reduced-Order Simulations With the TESLA-Family of Large-Signal Codes

Igor A. Chernyavskiy, *Member, IEEE*

Abstract—This study presents the introduction of the reduced-order, 1.5-D versions of the TESLA-family of 2.5-D large-signal codes, which have been implemented by simplifications in the particle's motion model by its restriction in a single, *z*-direction only. This in effect assumes that the guiding magnetic field is infinite what leads to a complete elimination of any transverse motion of particles, including their radial motion and rotation. Such simplifications in the algorithms allow reach up to two-times improvement in their performance due to significantly reduced number of operations, but they also in effect reduce an overall dimensionality of the algorithms from 2.5-D to 1.5-D only. The results of the modeling performed by the newly introduced 1.5-D versions of TESLA-family of large-signal codes and their comparisons with the results of modeling by the original 2.5-D algorithms are discussed in detail. Results of this study allow find accuracy and range of applicability of the 1.5-D large-signal codes and help to determine the place of such reduced-order algorithms.

and simplifications [4], including an assumption about harmonic nature of the electromagnetic (EM) processes (frequency domain), an assumption of axial-symmetry, RF-period averaging approach, use of the "large" particle's models (disks or rings), and so on. As a result, large-signal codes have great performance by running on a single core of modern multiple-core CPUs. The TESLA-family of 2.5-D large-signal codes demonstrates fast and efficient modeling with exceptional accuracy, what makes them highly useful and successful computational tools: see [4] for more details and multiple examples of comparisons with experimental data.

The dimensionality of the given large-signal algorithm is determined by the dimensionality (e.g., number of components) of used in it models for EM fields and equations of particle's motion. The original large-signal klystron code TESLA [1] was formulated as the 3-D model for EM fields and

Introducing reduced order particle's push into TESLA - from 3D to 1D particle trajectories[1]: dramatic simplification in the algorithm and its additional wide optimization allowed to speed-up particle's push in ~ 3-times.

Additional equations of motion[2]: following in d/dt instead

$$\begin{cases} \dfrac{d\mathbf{p}_j}{dz} = \dfrac{q}{v_{zj}}\left[\mathbf{E} + \dfrac{\mathbf{v}_j \times \mathbf{B}}{c}\right] \\ \dfrac{d\mathbf{r}_{\perp j}}{dz} = \dfrac{\mathbf{v}_{\perp j}}{v_{zj}} \\ \dfrac{d(\omega t_j)}{dz} = \dfrac{\omega}{v_{zj}} \end{cases}$$

Fast and accurate approach, but breaks in cases when $v_z \rightarrow 0$ or becomes < 0

division on v_z

$$\mathbf{E} = \mathbf{E}_{RF} + \mathbf{E}_{DC}$$

$$\mathbf{B} = \mathbf{B}_{RF} + \mathbf{B}_{DC} + \mathbf{B}_0$$

$$\frac{dp_{zj}}{dz} = \frac{q}{v_{zj}} E_z$$

$$\frac{d(\omega t_j)}{dz} = \frac{\omega}{v_{zj}}$$

$$\begin{cases} \dfrac{d\mathbf{p}_j}{dt} = q\left[\mathbf{E} + \dfrac{\mathbf{v}_j \times \mathbf{B}}{c}\right] \\ \dfrac{d\mathbf{r}_j}{dt} = \mathbf{v}_j \\ \dfrac{d(\omega t_j)}{dt} = \omega \end{cases}$$

More general approach: works for all kind of electrons, including stopped and returned particles.

Reduce the number of particle's degrees of freedom from 3 to 1

$$E_z = E_{z,RF} + E_{z,DC}$$

$$\frac{dp_{zj}}{dt} = qE_z$$

$$\frac{d(\omega t_j)}{dt} = \omega$$

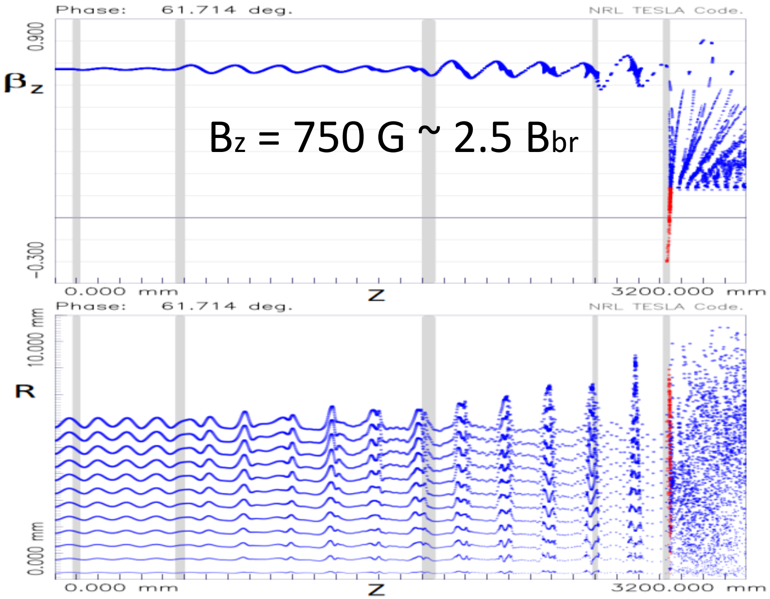
---

[1] I.A. Chernyavskiy, *"From 2.5-D to 1.5-D: Fast, Reduced-Order Simulations With the TESLA-Family of Large-Signal Codes,"* IEEE TED, v. 70, no. 10, pp. 5353-5358, Oct. 2023

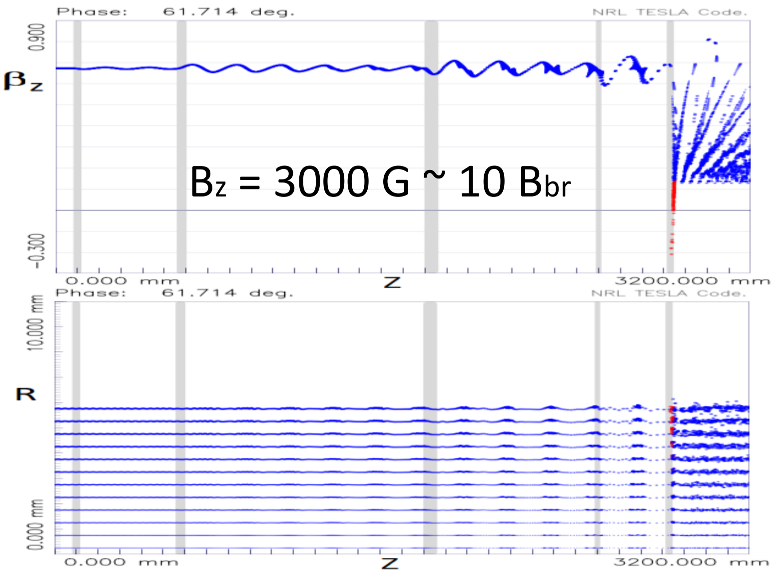
[2] I.A. Chernyavskiy et al., *"Simulation of Klystrons with Slow and Reflected Electrons Using Large-Signal Code TESLA"*, IEEE TED., Vol.54, No.6, June 2007.

From 2.5D to 1.5D: evolution of particles' β_z and their trajectories along the device

2.5D TESLA modeling

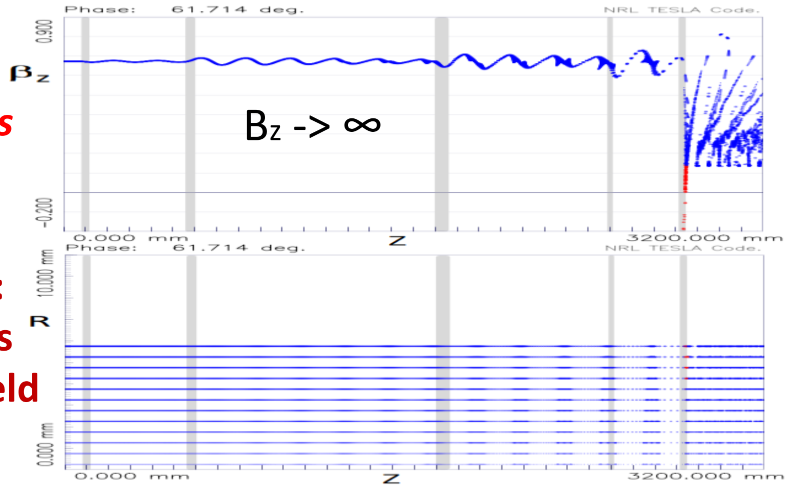


B_z = 750 G ~ 2.5 B_{br}



B_z = 3000 G ~ 10 B_{br}

1.5D TESLA modeling



B_z -> ∞

Example based on the CERN 5-cavity klystron with 2.1 MW output power and 73% efficiency, which originally was simulated by 1.5D code KlyC:

1. J. Cai and I. Syratchev, "Advanced techniques for high efficeincy klystron simulation," FCC Week 2018, CERN. Available online: https://indico.cern.ch/event/656491/contributions/2932265/

2. J. Cai and I. Syratchev, "KlyC: 1.5-D Large-signal Simulation Code for Klystrons," *IEEE Trans. on Plasma Sci.*, vol. 47, no. 4, pp.1734-1741, Apr.2019. doi: 10.1109/TPS.2019.2904125.

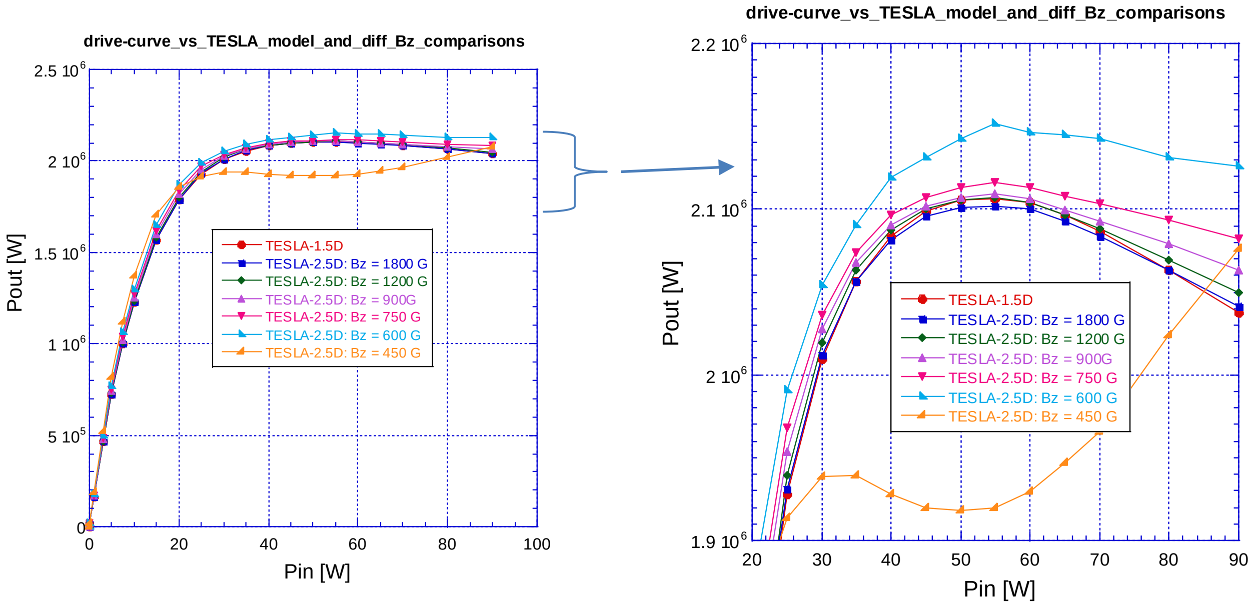
Up to ~2 times faster runs !

multiple rings: every ring sees different E_z-field

Comparisons between predictions of the 2.5D and 1.5D TESLA algorithms:

Effect of the focusing magnetic field's value on predicted drive-curve and body current in the 5-cavity klystron

B_{br} ~ 300 G



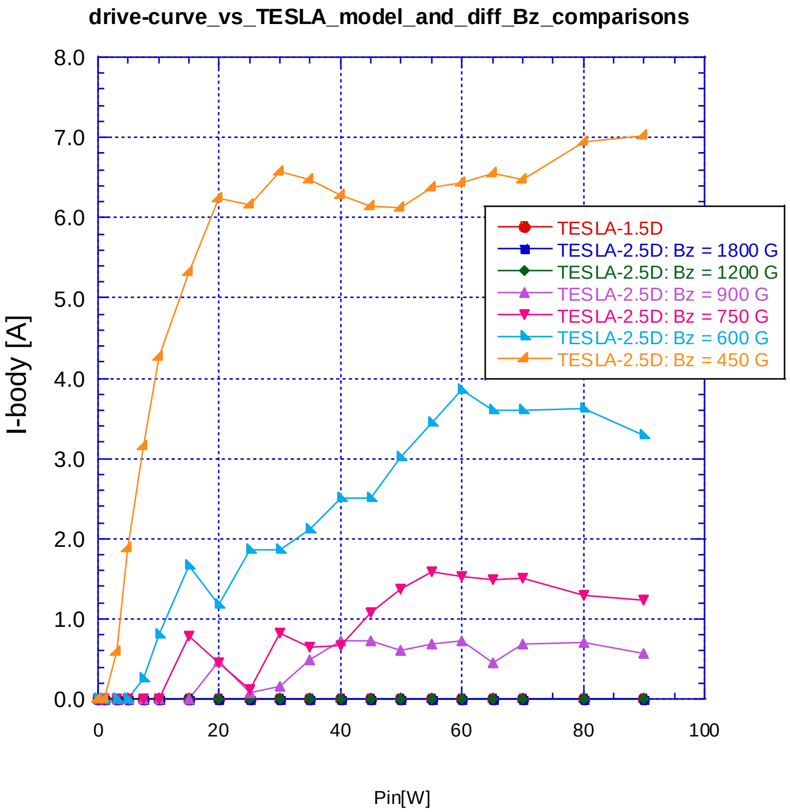
2.5D TESLA modeling: reduction in the value of the focusing magnetic field below ~6 x B_{br} resulted in increasingly different predictions for the drive-curve and increased level of predicted body current

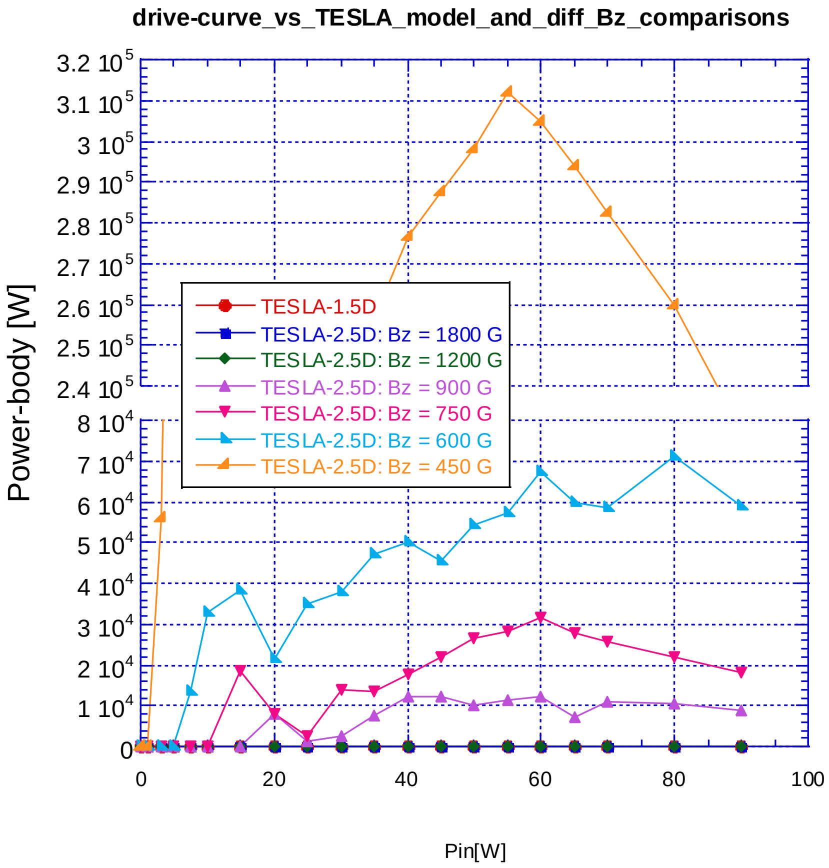
- All 1D and 1.5D models in fact assumes that magnetic field is infinite and particles are moving along one (Z) direction only.
- Capability to take into account and accurately model 2D effects are the main advantages of the fully 2D models and codes.

---

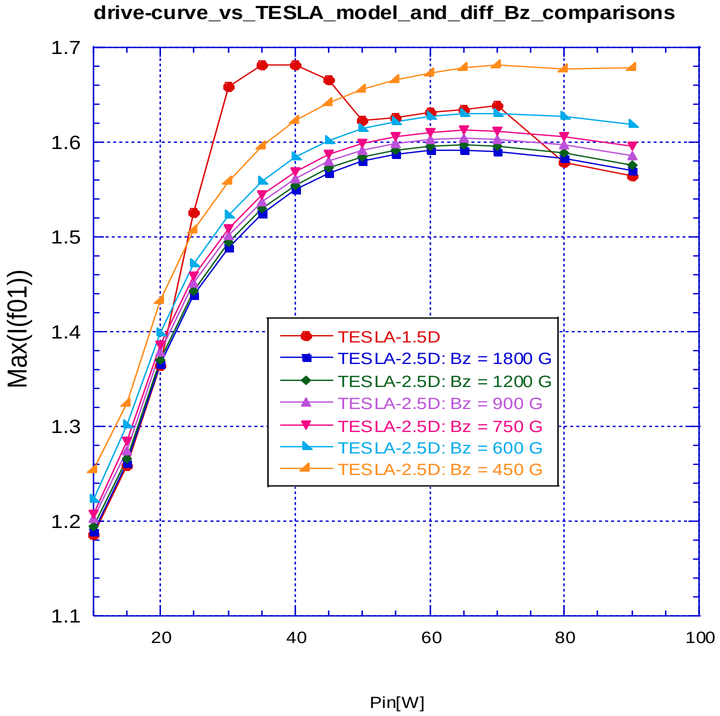
[1] I.A. Chernyavskiy, *"From 2.5-D to 1.5-D: Fast, Reduced-Order Simulations With the TESLA-Family of Large-Signal Codes,"* IEEE TED, v. 70, no. 10, pp. 5353-5358, Oct. 2023

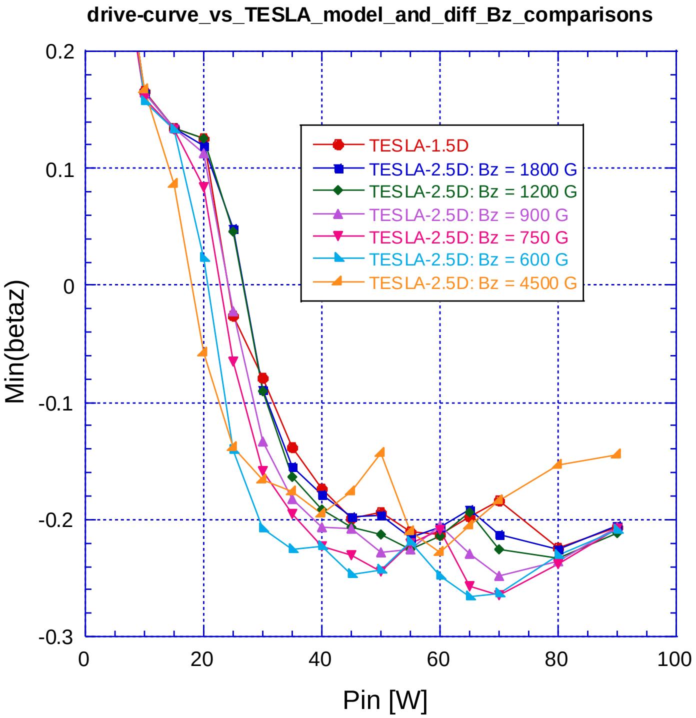
Comparisons for predicted body-current and deposited beam power


drive-curve_vs_TESLA_model_and_diff_Bz_comparisons


drive-curve_vs_TESLA_model_and_diff_Bz_comparisons

Comparisons for the maximum bunched current and minimum βz:


drive-curve_vs_TESLA_model_and_diff_Bz_comparisons

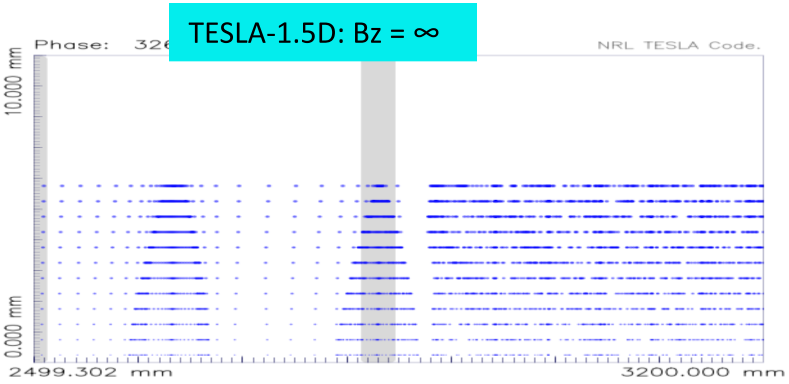

drive-curve_vs_TESLA_model_and_diff_Bz_comparisons

Brief list of 2D effects and additional options, which are not available for modeling by the reduced order codes:

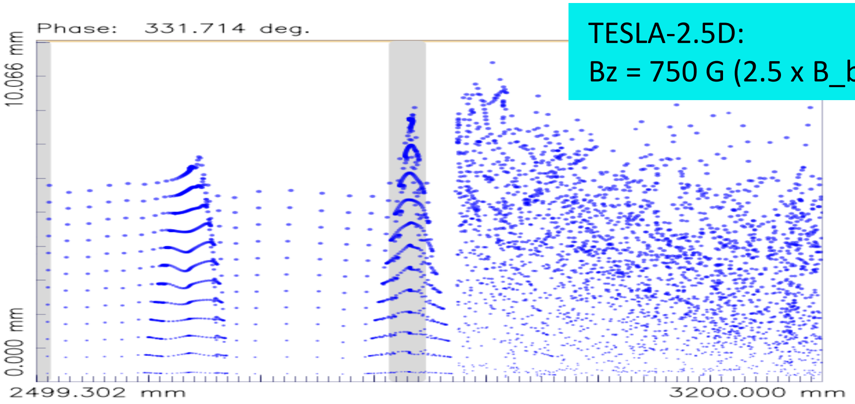
1) **Effects of the finite magnetic field**, including particle's rotation, **electron beam's expansion and scalloping**;
2) Importing and use of more **realistic beam-configurations, obtained by a gun-code modeling**;
3) Capability to calculate **realistic spent-beam**, which can be used for collector modeling;
4) Capability to model beam-tunnel's **wall interception by particles** (**body-current**);
5) Capability to model accurately the **non-uniform geometry of the beam-tunnel's profile**;
6) Capability to model with **non-uniform magnetic field-profile** to improve beam-transport and its interaction and eliminate possible reflection of "slow" particles;

TESLA-1.5D vs TESLA-2.5D Modeling*:

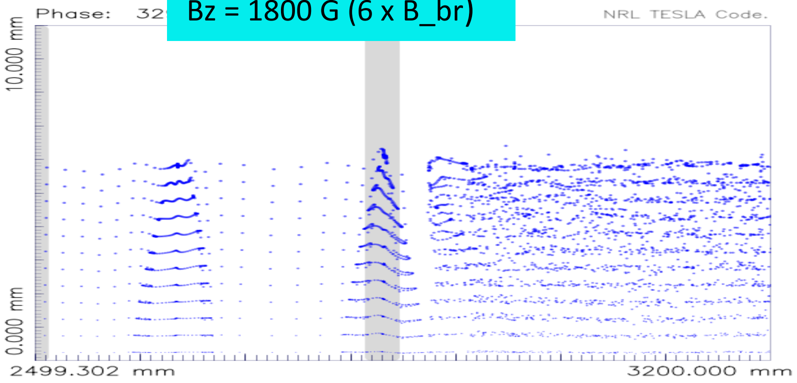
Changes in the predicted shape of the bunch in the vicinity of the output cavity gap depending on the used beam-model (1.5D or 2.5D) and value of the focusing magnetic field:



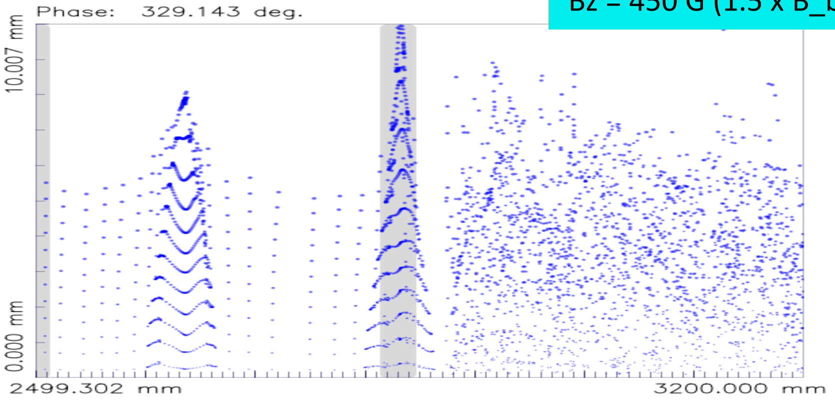
TESLA-1.5D: Bz = ∞



TESLA-2.5D:
Bz = 750 G (2.5 x B_br)



TESLA-2.5D:
Bz = 1800 G (6 x B_br)



TESLA-2.5D:
Bz = 450 G (1.5 x B_br)

*I.A. Chernyavskiy et al., "**Critical Analysis of Simulations With Large-signal Codes: Gains and Losses of Modeling with Reduced-order Algorithms**", Conference Abstract of the IVEC 2024.

Summary:

- We have presented an overview of the large-signal code TESLA algorithm and its application to the modeling of the linear-beam Vacuum Electronics (VE) amplifiers based on resonance structures, including klystrons and IOTs;
- TESLA is a fully 2D algorithm, which allows to take into account effects of the finite magnetic field focusing, including beam's radial motion and its expansion, wall interception current, deposited on a wall beam-power and more;
- TESLA algorithm is a light-weight application, which allows fast, accurate and efficient modeling of VE amplifiers on a single core of the laptop or desktop computer;
- Utilization of the modern computers with multiple-core CPUs allows to run simultaneously many instances of the code TESLA;
- Parallel extension of the code TESLA enables accurate modeling of multiple-beam VE amplifiers and allows take into account effects of non-identical beams and beam-tunnels;
- Recently introduced reduced-order 1.5D TESLA algorithm enabled even faster approximate large-signal modeling to get preliminary predictions for the given device;
- Recently made corrections in the TESLA algorithm enabled accurate modeling of relativistic devices; code was validated by comparisons with the predictions of the code MAGIC-2D and available measurements;