IMPERIAL

Modern Cosmology

Opportunities & Challenges

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Outline

- Current physical model of the Universe
- Open questions
- Observations
- Numerical methods
- Statistical methods
- Summary

Cosmic chronology

inflation

~ Gaussian primordial density perturbations

recombination (decoupling)

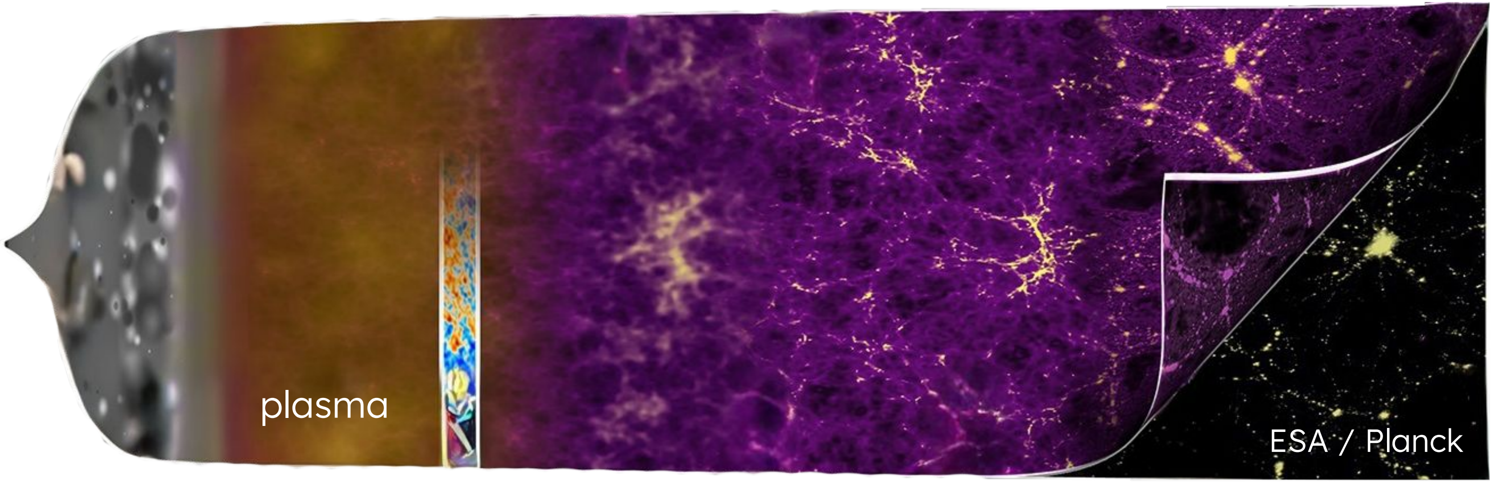
reionization

gravity (GR)

cosmic expansion

non-Gaussian cosmic large-scale structure

Big Bang

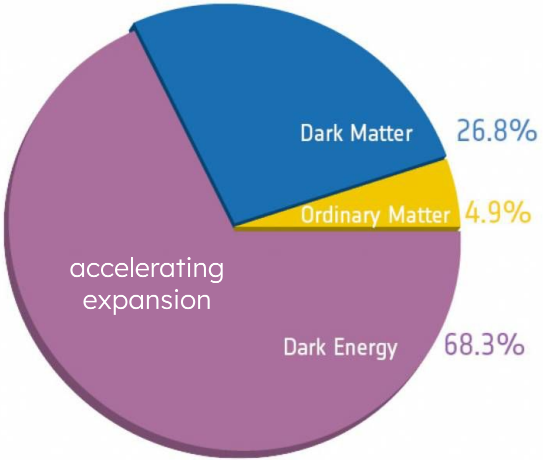


ΛCDM predicts this evolution after postulating specific energy densities

Cosmological Principle: homogeneity & isotropy

Open questions in ΛCDM

cosmological parameters



ESA / Planck

- Amplitude / tilt of primordial power spectrum
- Optical depth during reionization
- Size of Hubble horizon at decoupling

- Initial conditions statistics
- Theory of gravity
- Accelerated expansion
- Neutrino masses

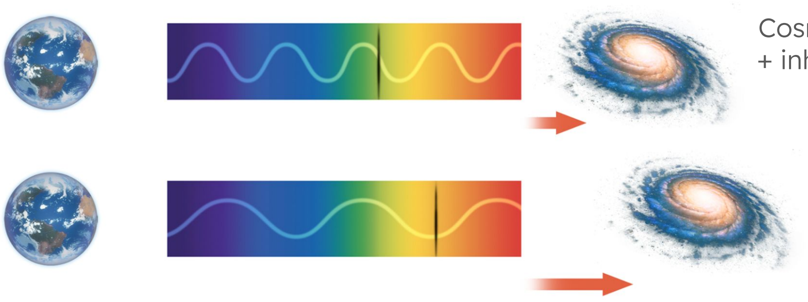
and more.

We answer these questions through **observations**.

What do the observations consist of?

redshifts

$$z_{\text{obs}} = \frac{\lambda_{\text{obs}}}{\lambda_{\text{em}}} - 1$$

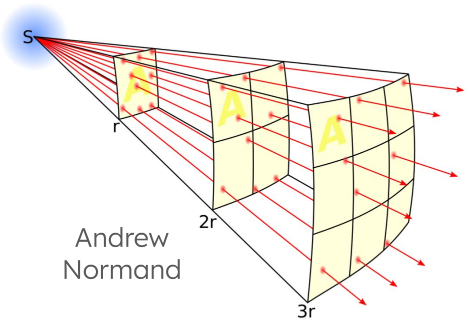


Cosmic expansion + inhomogeneities

Mark Garlick

magnitudes

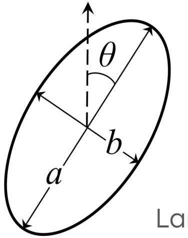
$$m - M = 5\log(d_L) - 5$$



Andrew Normand

galaxy shapes

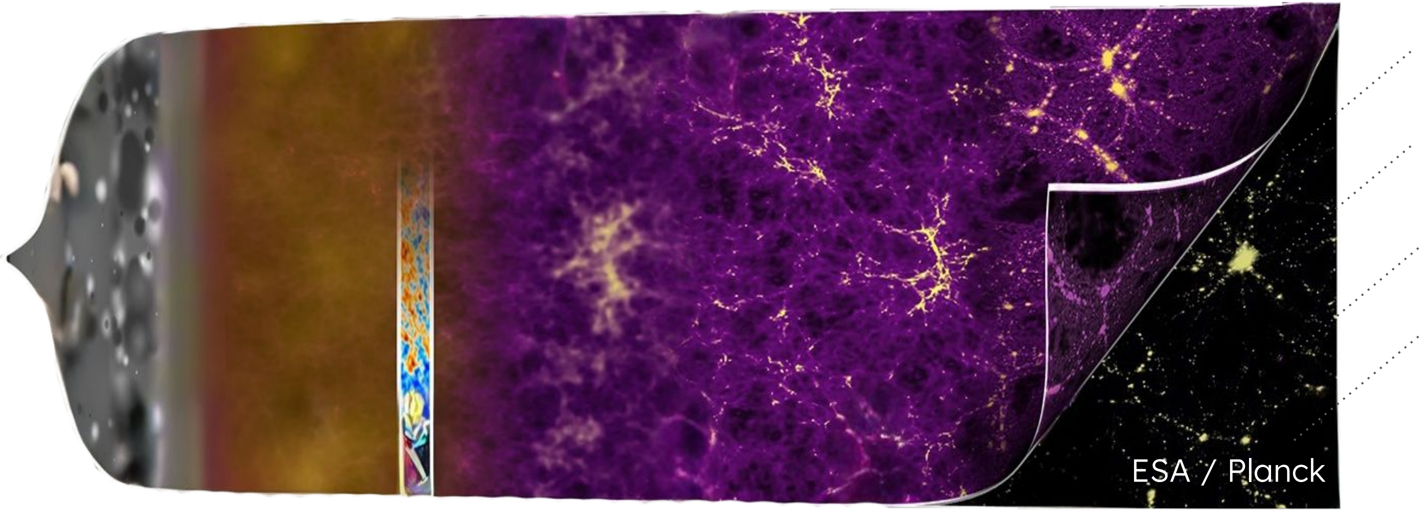
$$\varepsilon = \frac{a-b}{a+b}\exp(2i\theta)$$



Lamman+24

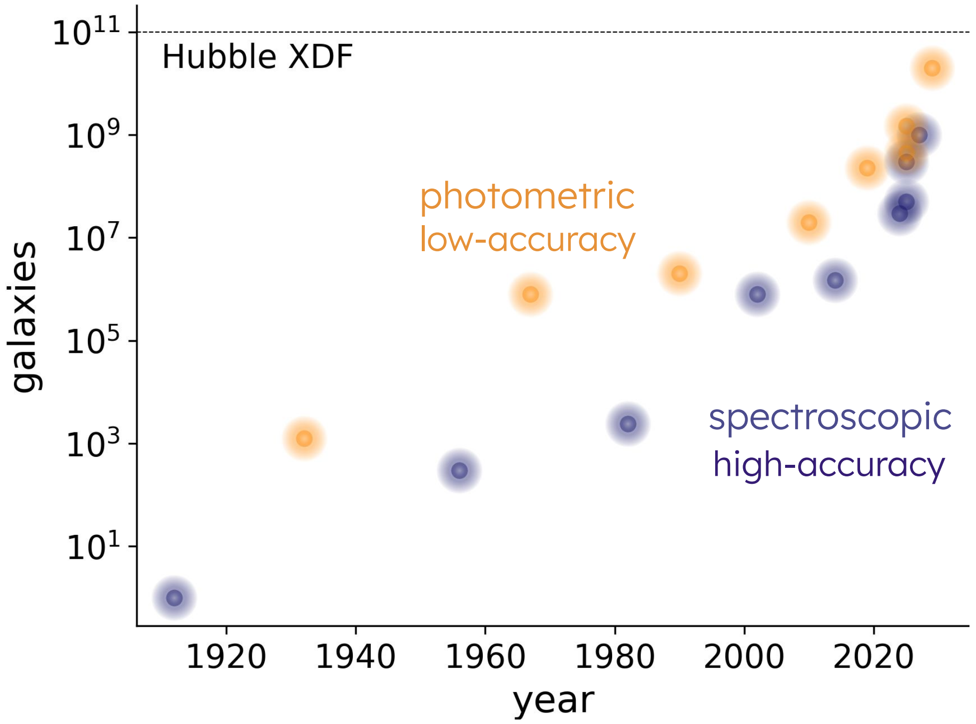
What physics do we utilise to interpret these data?

How do we obtain a physical interpretation of the data?



Since the 20th century but 2020-: era of stage-IV surveys

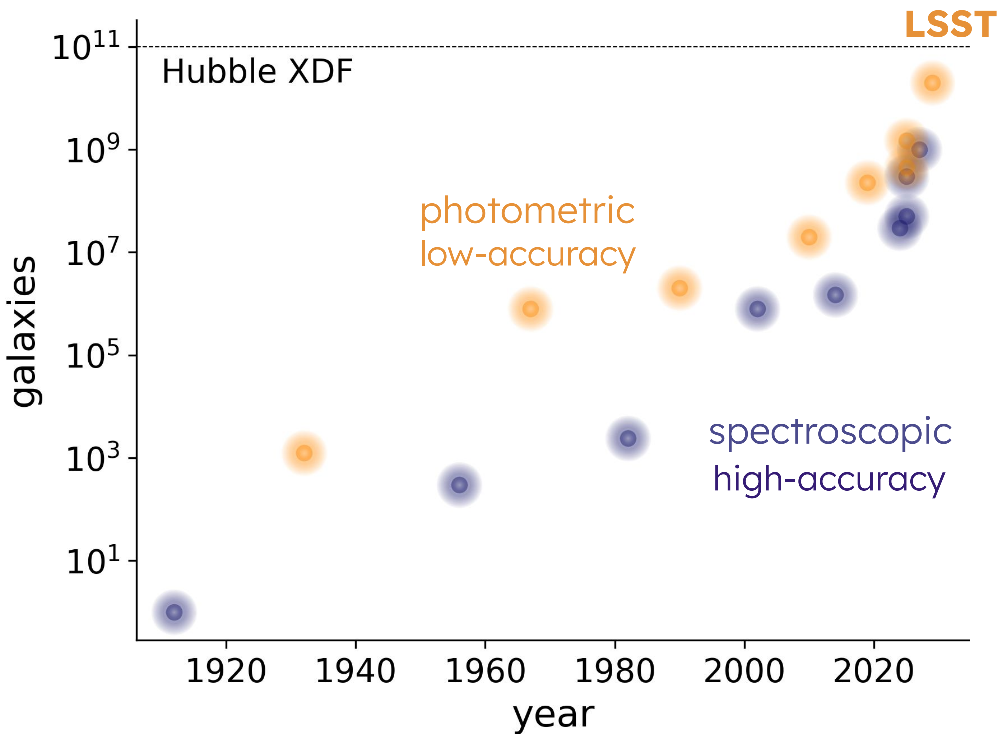
An unprecedented amount of data



NASA / ESA

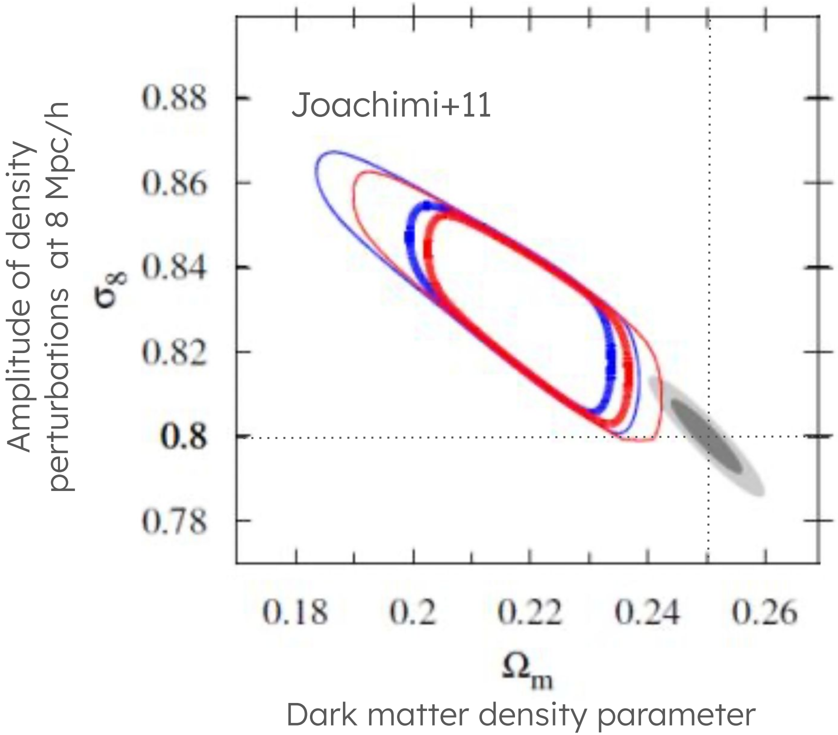
Reaching the limit of observable galaxies

We are limited by systematic effects rather than noise



- 37 billion astronomical sources
- 500 PB imaging
- 50 PB catalogue
- ~10^6 real-time alerts / night (large samples of rare events)

Systematics can bias our cosmological conclusions

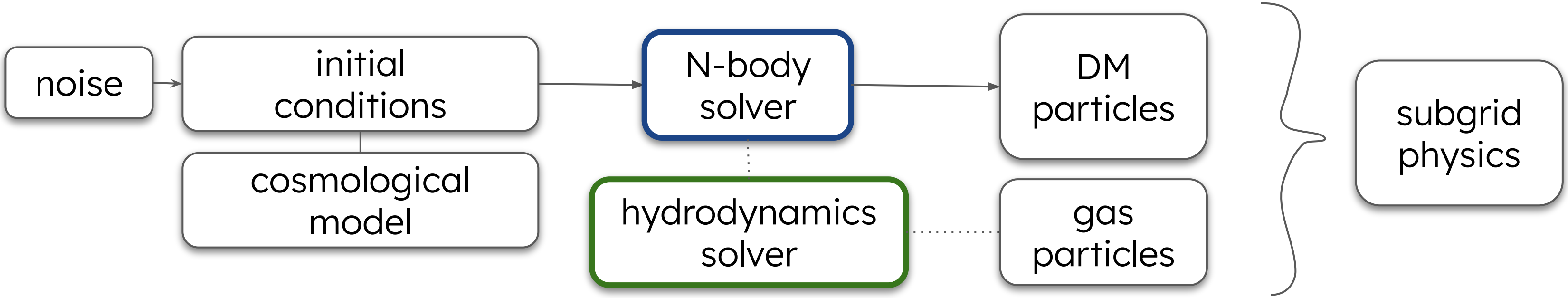


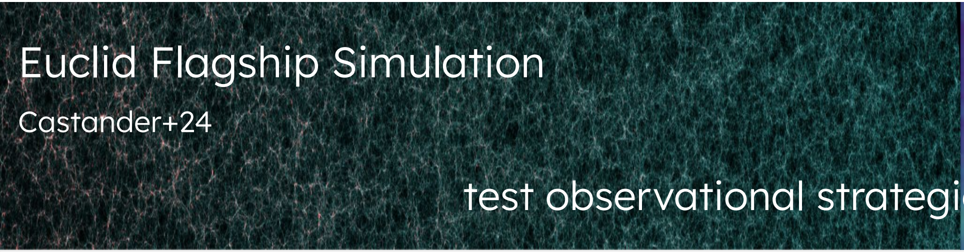
Probe	Systematics (indicatively)
Lensing	galaxy shape, redshift accuracy
Clustering	baryonic physics
Supernovae	astrophysics

Systematics come both from instrumental effects and our physical understanding

Our cosmological toolkit

- Cosmological observations: opportunities & challenges
- Cosmological simulations: N-body / hydrodynamical simulations





test observational strategies / impact of systematics

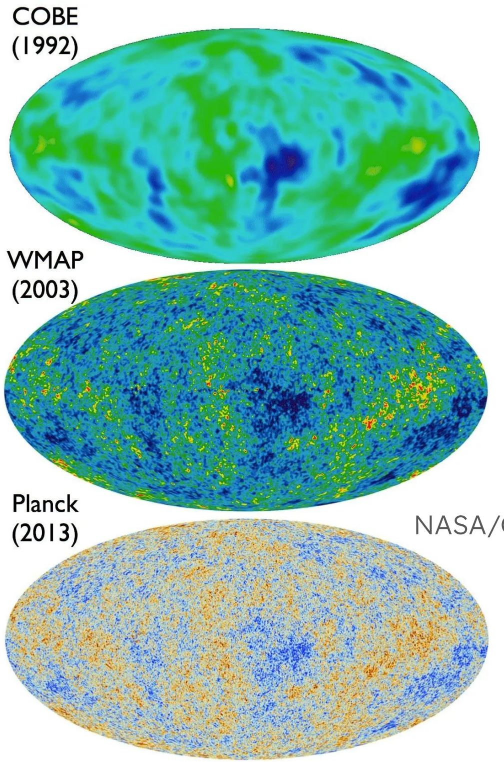
Our cosmological toolkit

- Cosmological observations
- Cosmological simulations
    - N-body dark matter / hydrodynamical simulations
    - Calibrate errors / test observational strategies / impact of systematics
- Statistical methods
    - Inverse: Hypothesis testing / parameter estimation / model selection
    - Forward: given known parameters, what is the data distribution?
    - Frequentist / Bayesian
    - Compressed or full dataset

Cosmological experiments present some peculiarities.

Cosmological statistics is peculiar

- We can look back in time
- The experiments are not controlled
- Cannot be repeated
- We observe only one sky (cosmic variance)
- Observations suffer from selection effects
- We assume the **Cosmological Principle**
  - homogeneity & isotropy
  - $\langle q \rangle_{sky} = \langle q \rangle_{stat}$



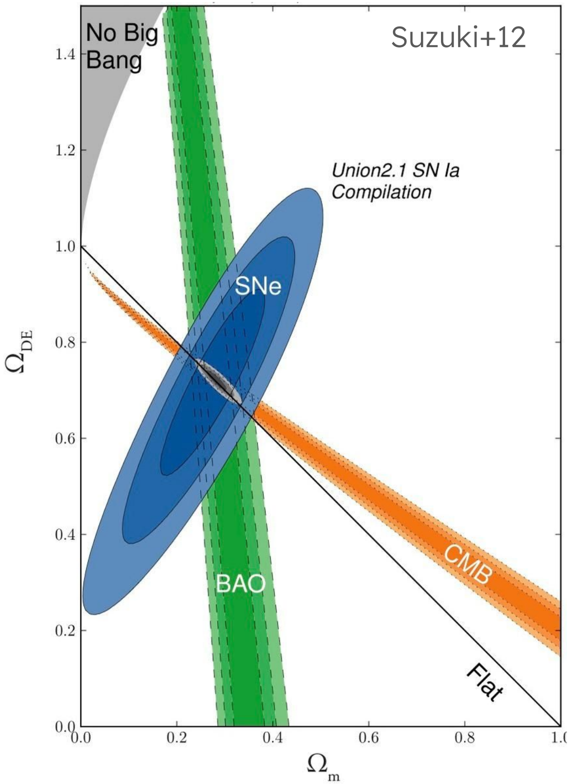
NASA/COBE/DMR

Bayesian approaches are preferred

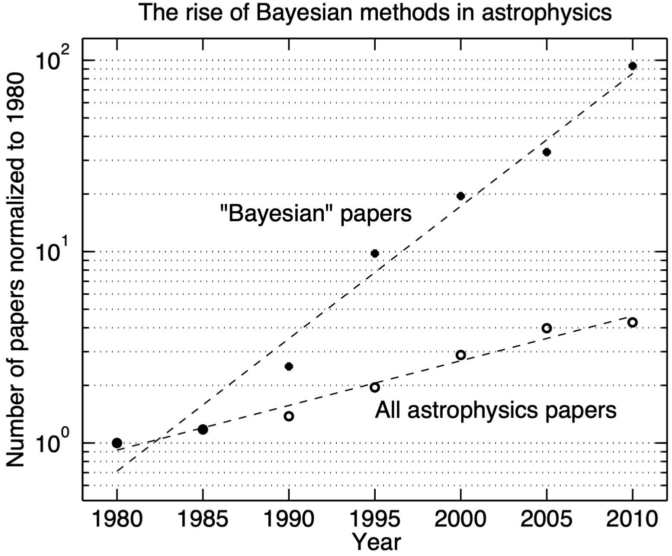
What is the probability of a hypothesis given the data?

$$p(\boldsymbol{\theta}|\mathbf{x}) = \frac{p(\mathbf{x}|\boldsymbol{\theta})p(\boldsymbol{\theta})}{p(\mathbf{x})}$$

Most powerful constraints from probe combination



- Explicit assumptions
- Nuisance parameters
- Prior information



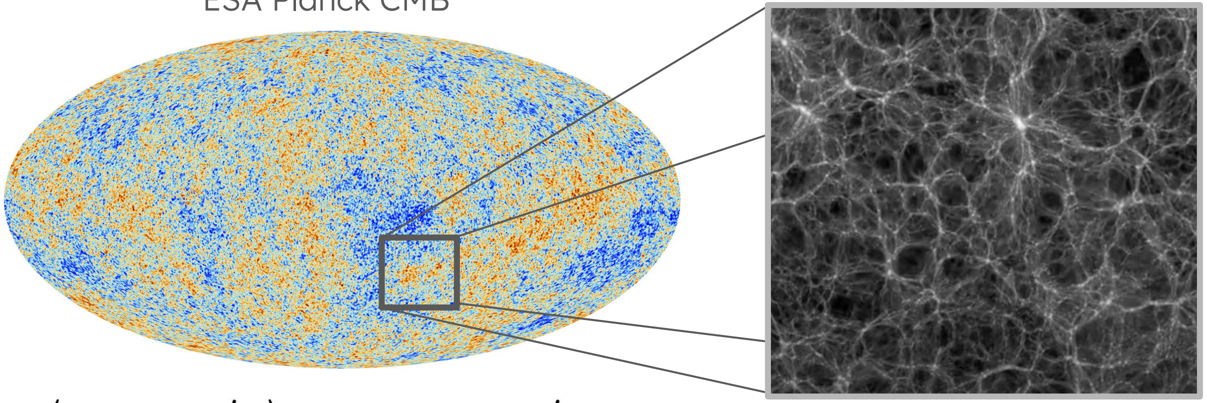
R. Trotta

Two regimes of cosmological statistics

Gaussian
temperature anisotropies

ESA Planck CMB

Millennium Simulation



non-Gaussian
cosmic large-scale structure (small scales)

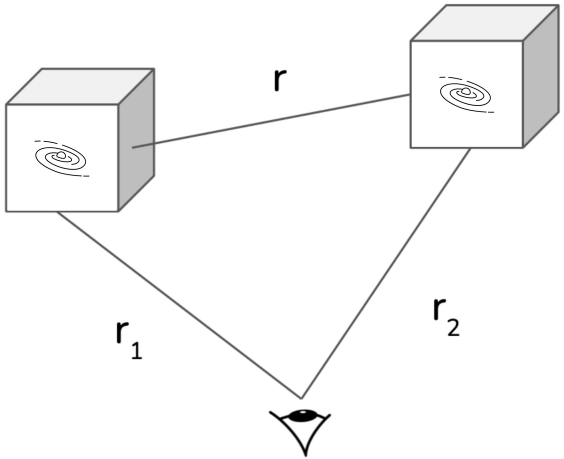
$$\left\langle \mathcal{R}(\vec{k})\mathcal{R}(\vec{k}')^* \right\rangle = (2\pi)^3 \delta(\vec{k} - \vec{k}') P(k)$$

$$P(k) = \frac{2\pi^2}{k^3}\mathcal{P}_{\mathcal{R}}(k) \quad \mathcal{P}_{\mathcal{R}}(k) = A_s (k/k_{\mathrm{pivot}})^{n_s - 1}$$

Summary statistics are sufficient for Gaussian and isotropic fields

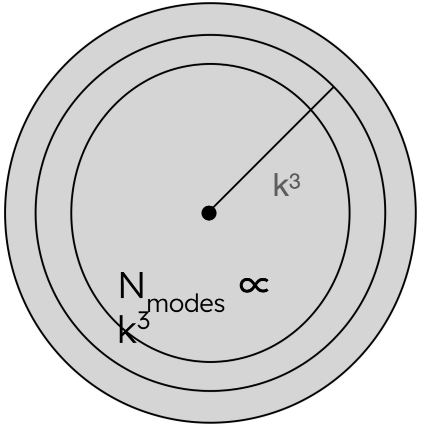
The case of 2-point statistics

2-point correlation function



$$\xi(r) = \langle \delta(\mathbf{r_1})\delta(\mathbf{r_2}) \rangle$$

power spectrum



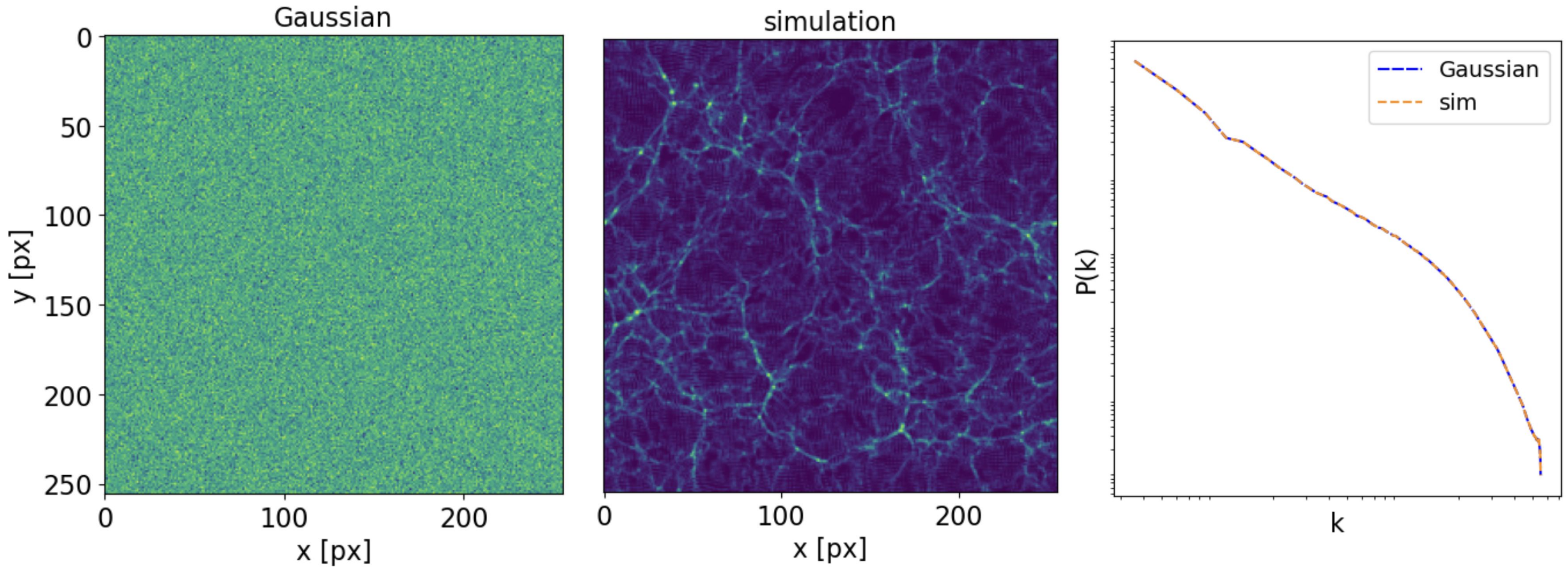
$$P(k) = \frac{1}{(2\pi)^{3/2}} \int \xi(r) e^{i\mathbf{kr}} d^3\mathbf{r}$$

Non-Gaussian and anisotropic fields have non-zero higher-order statistics

Information beyond 2-pt statistics

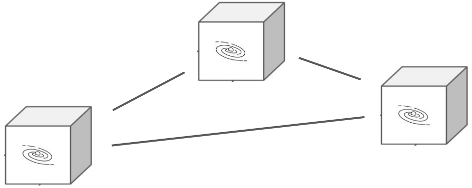
CAMELS

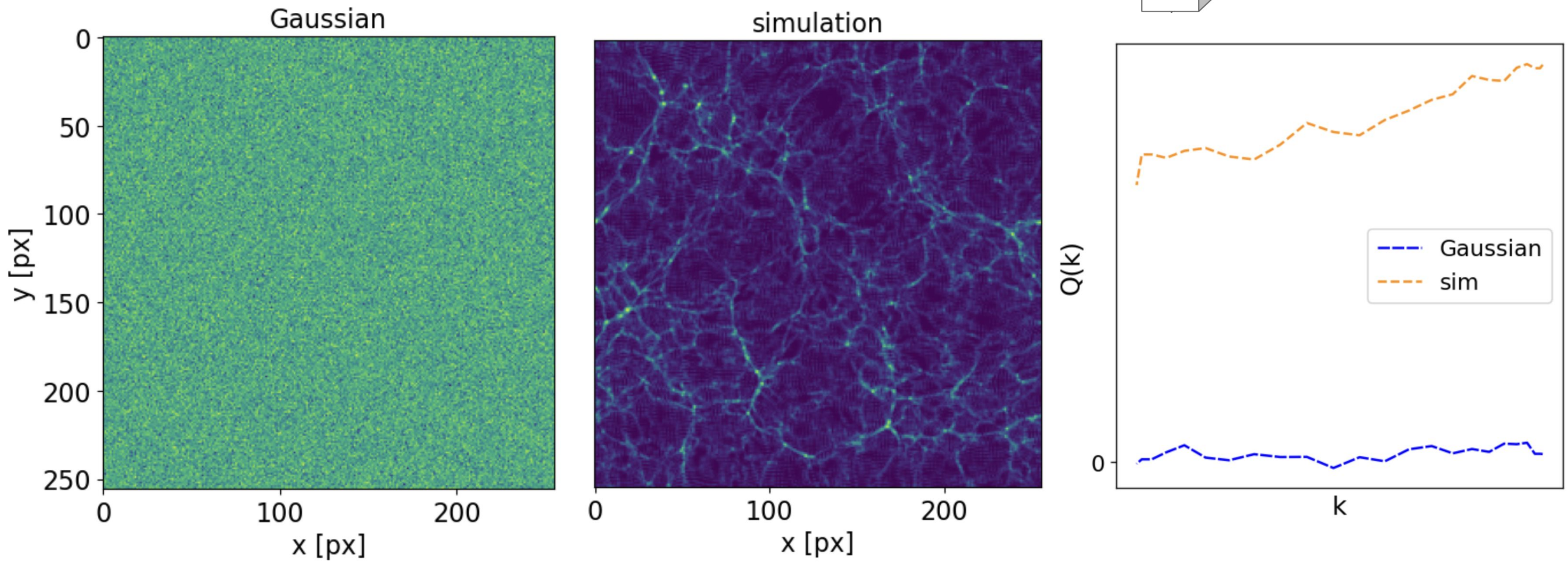
Villaescusa-Navarro+20



Can be distinguished through higher-order statistics

Higher-order statistics

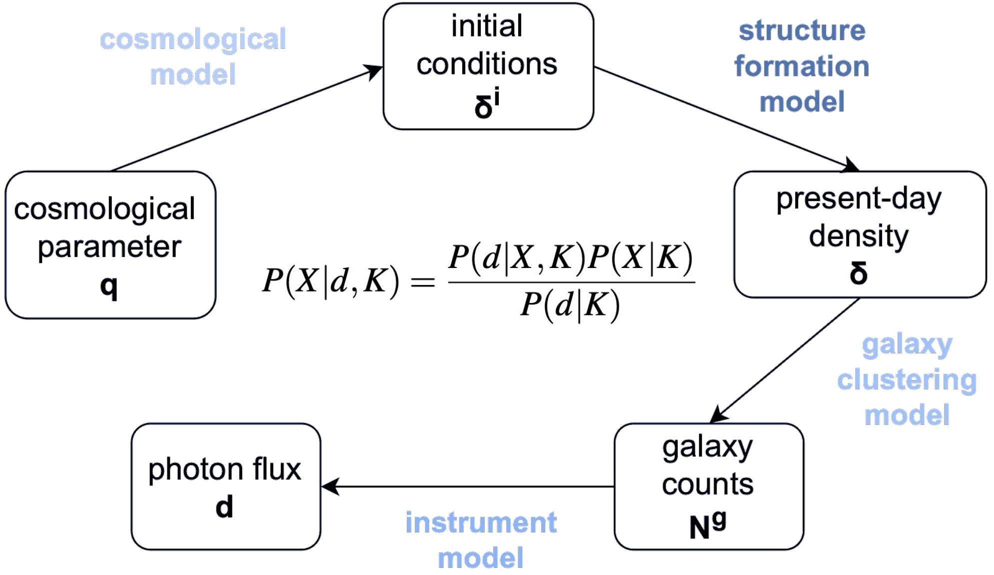




How can we access higher-order statistics?

Beyond higher-order statistics

- Field-level inference: each pixel is a random variable to infer
- Forward modelling: accounting for systematic effects self-consistently

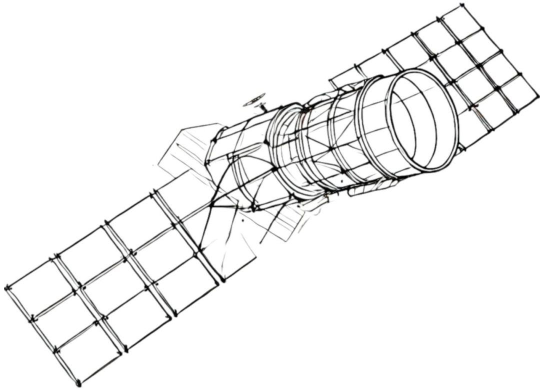


$$P(X|d,K) = \frac{P(d|X,K)P(X|K)}{P(d|K)}$$

- Likelihood-based
- Simulation-based

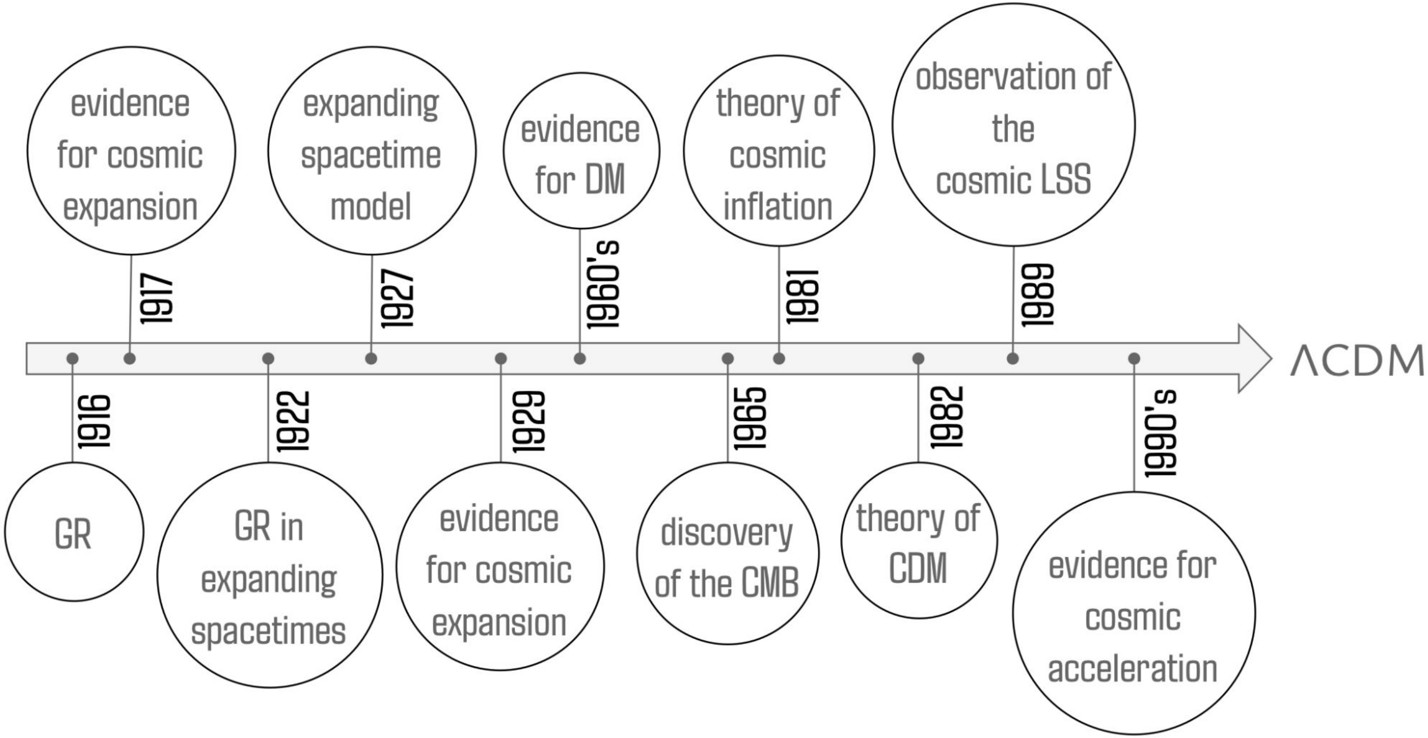
Summary

- How did the Universe evolve from the Big Bang until today?
- Unprecedented amount of observed and simulated data
- Limited by systematic effects
- Bayesian inference for 2-pt statistics and beyond
  - Likelihood-based
  - Simulation-based



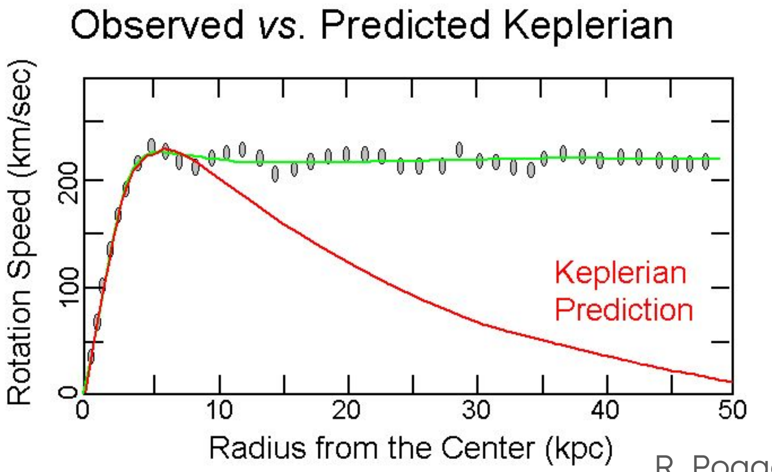
Bonus slides

Motivation for ΛCDM

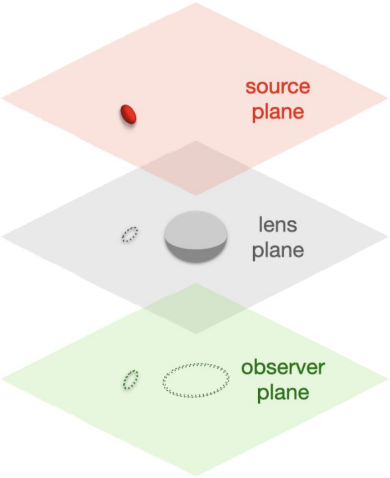


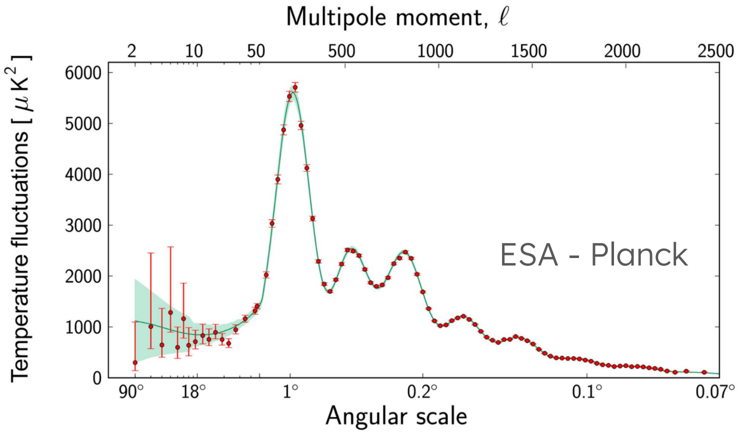
Evidence for DM

- Galaxy rotational velocities
- Gravitational lensing
- Cosmic Microwave Background
- And more.



R. Pogge



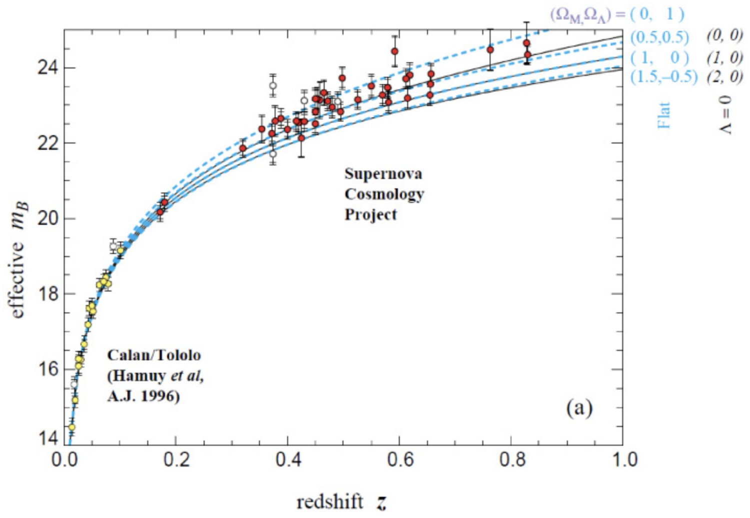


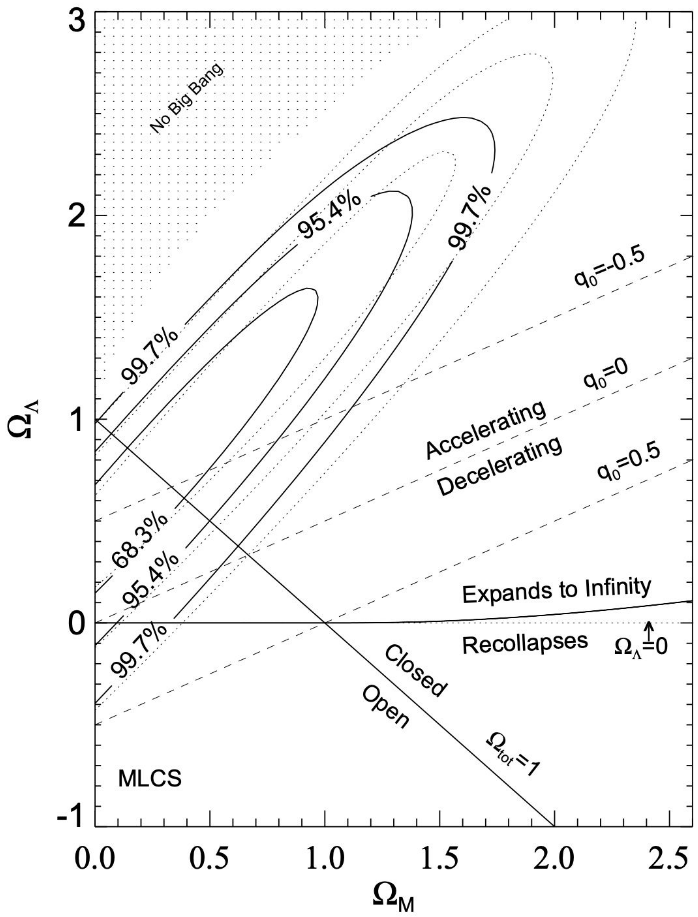
ESA - Planck

Evidence for DE

- Supernovae
- And more.

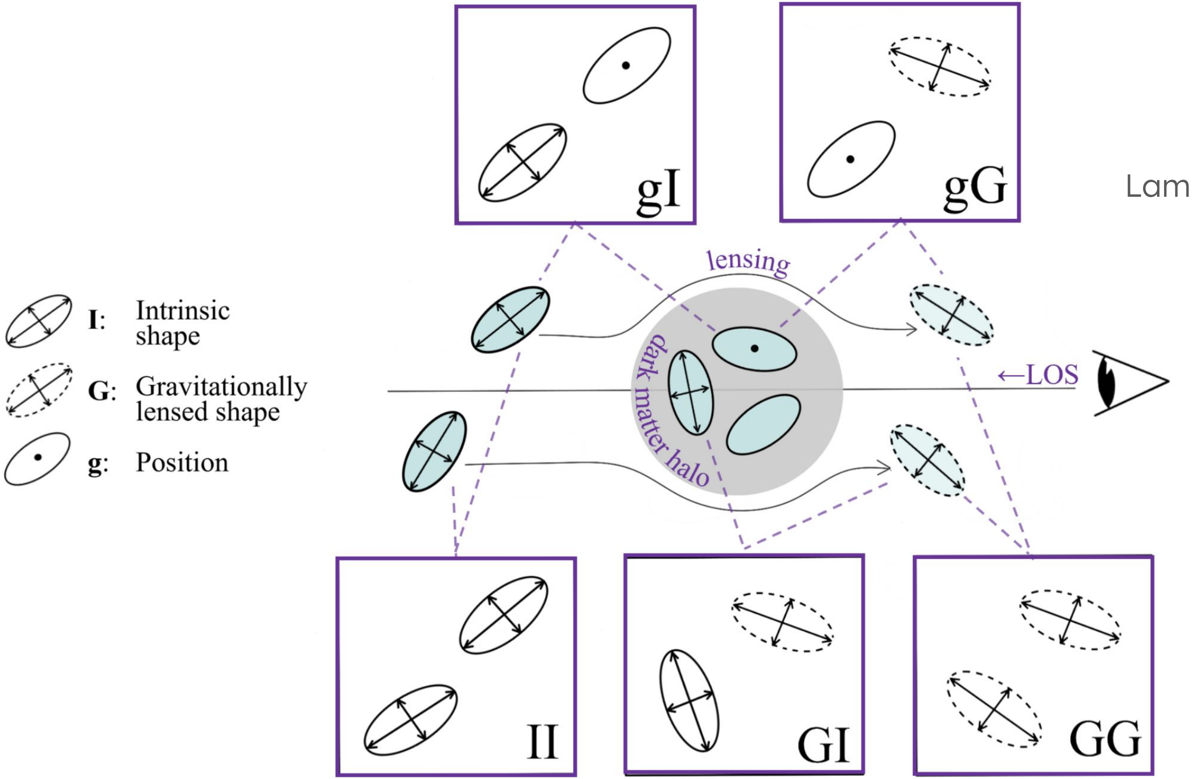
Perlmutter+99





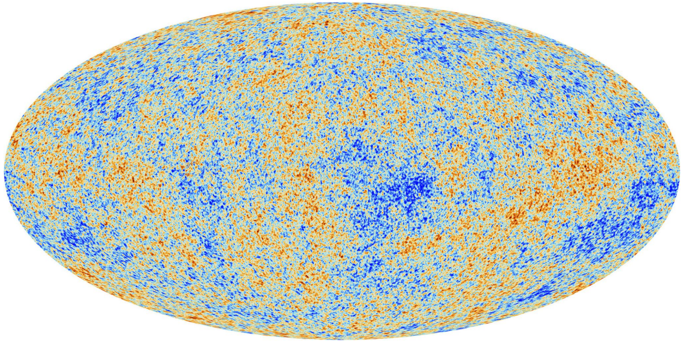
Riess+98

Weak lensing



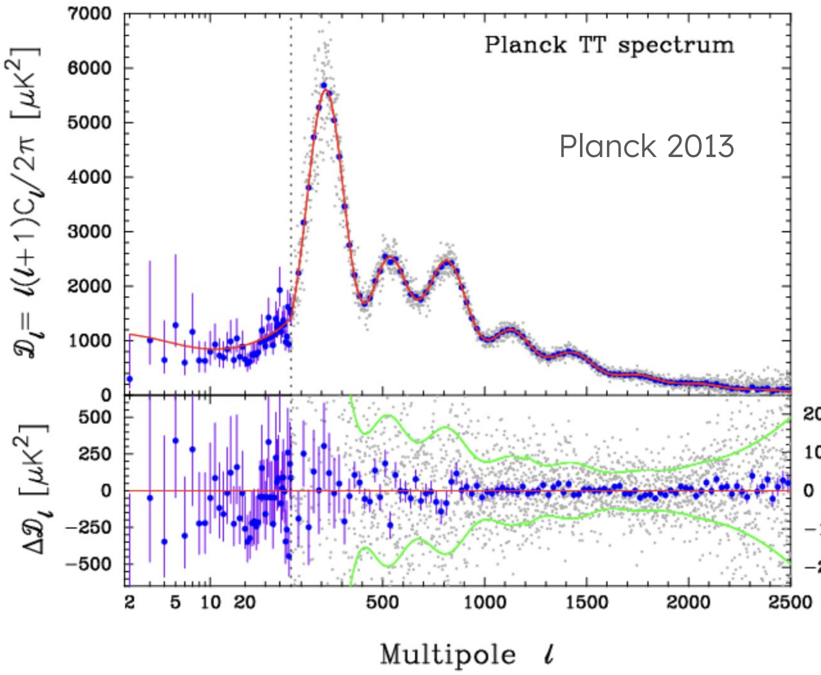
Lamman+24

Cosmic Microwave Background



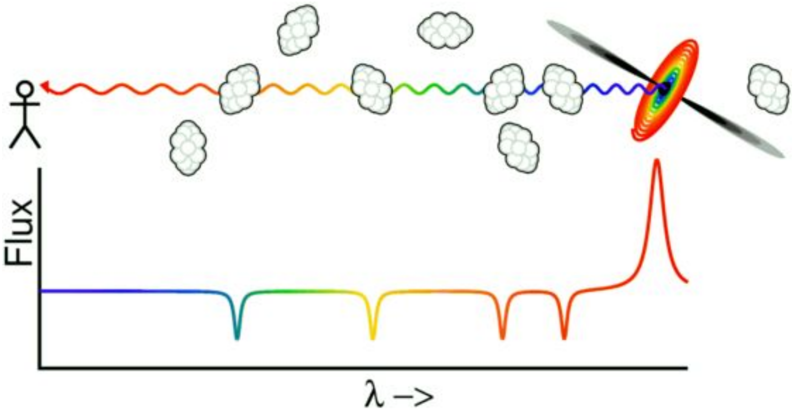
$$C(\theta) \equiv \left\langle \frac{\Delta T}{T_0}(\hat{\mathbf{m}}) \frac{\Delta T}{T_0}(\hat{\mathbf{n}}) \right\rangle, \qquad \hat{\mathbf{m}} \cdot \hat{\mathbf{n}} = \cos\theta$$

$$C(\theta) = \sum_{\ell=2}^{\infty} \frac{2\ell+1}{4\pi} C_\ell P_\ell(\cos\theta)$$

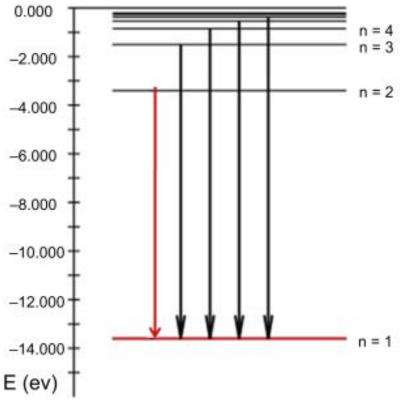


Lya forest

Edward Wright



Quasar (luminous galaxy, visible at large distances, emitting at all wavelengths)

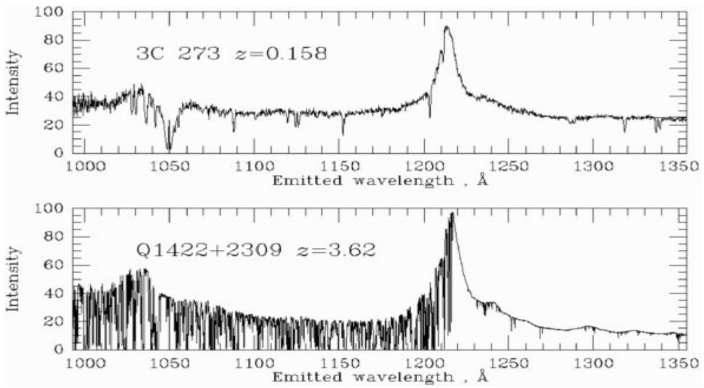


λ (1 → ∞) = 91.1 nm

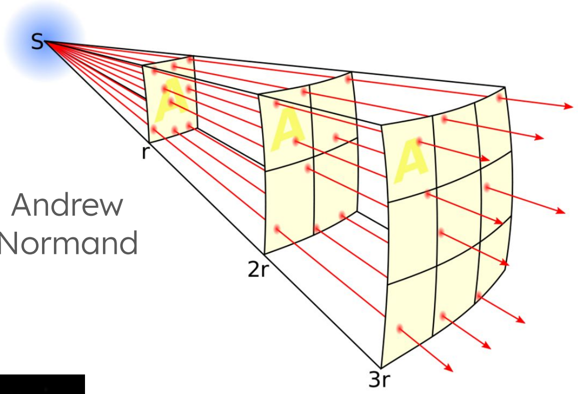
λ (1 → 6) = 93.7 nm
λ (1 → 5) = 94.9 nm
λ (1 → 4) = 97.2 nm
λ (1 → 3) = 102.5 nm
λ (1 → 2) = 121.5 nm
Lyman-α

Ghysels-Dubois+21

Bill Keel

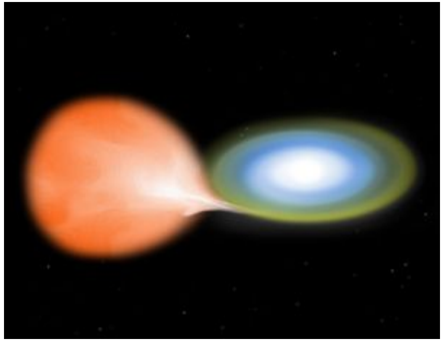


Supernovae: standard candles

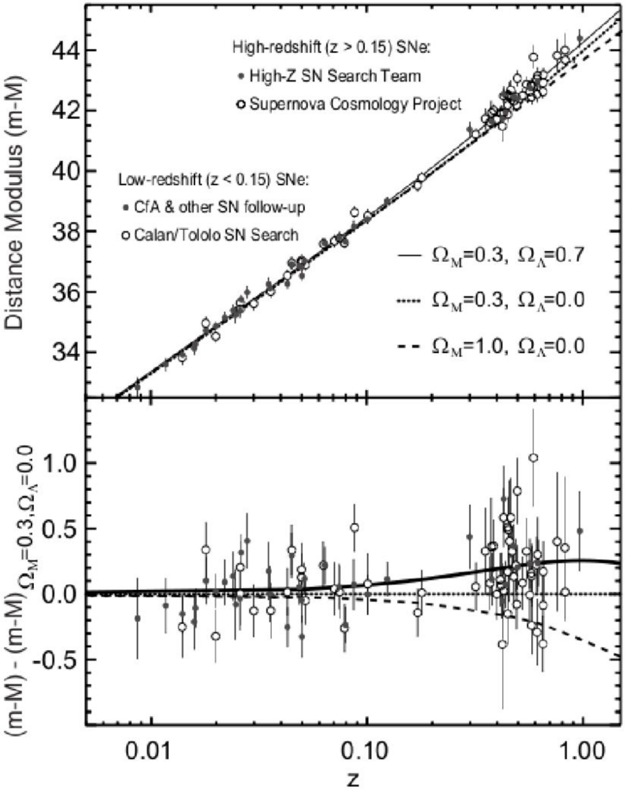


Andrew Normand

CXC/M.Weiss

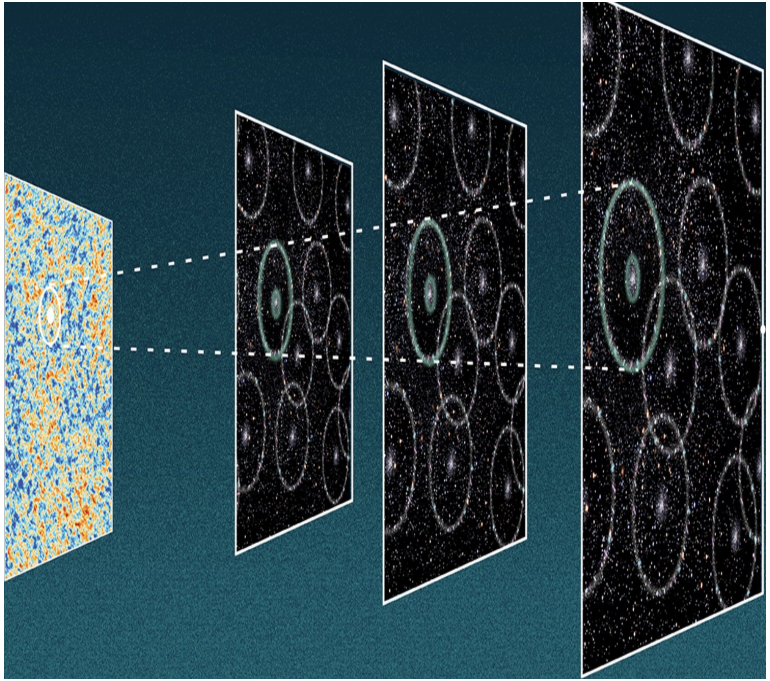


White dwarf exceeds the Chandrasekhar limit → standard candle



Perlmutter+98

Baryon Acoustic Oscillations: standard rulers



Euclid Consortium

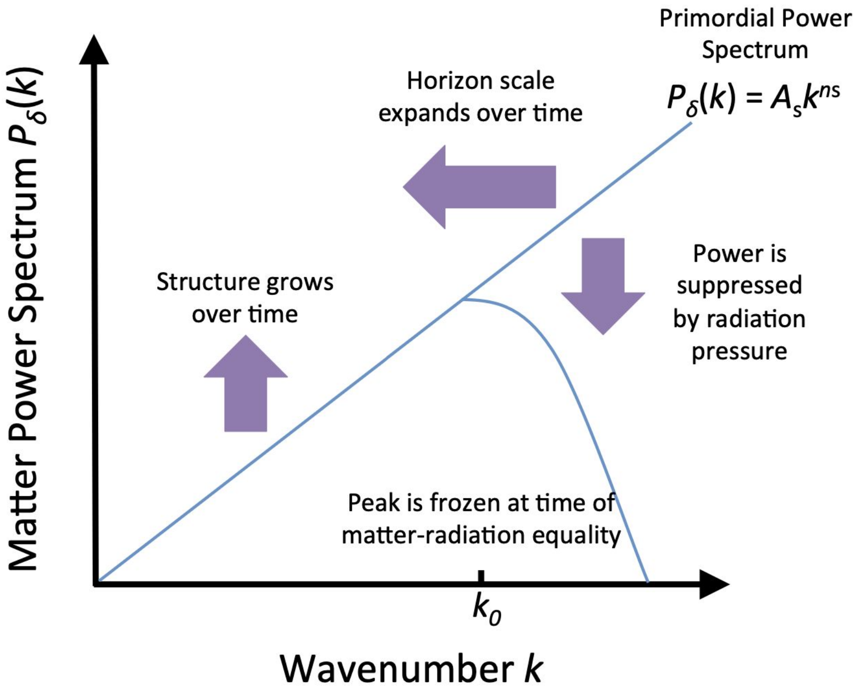
- Hubble horizon: causal contact
- Inflation stretches perturbations beyond the horizon → freeze in
- Later perturbations re-enter → seeds of structure formation

Acoustic waves in the plasma that freeze at recombination

$$\Delta\theta = \frac{\Delta\chi}{d_A(z)}$$

$$d_A(z) \propto \int_0^z \frac{dz'}{H(z')}$$

Shape of matter power spectrum

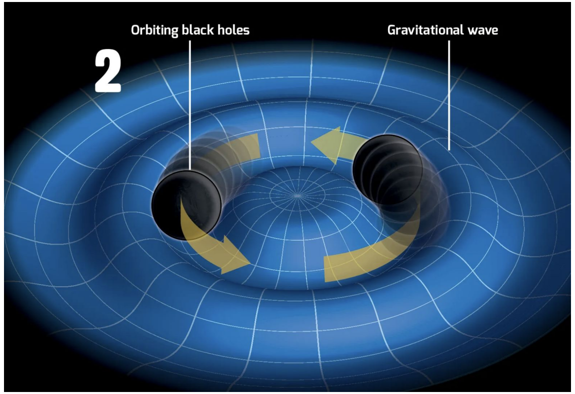


Simon Samuroff

Gravitational waves

- Provide estimates of luminosity distance, but not redshifts
- EM counterpart → redshift determination → cosmology

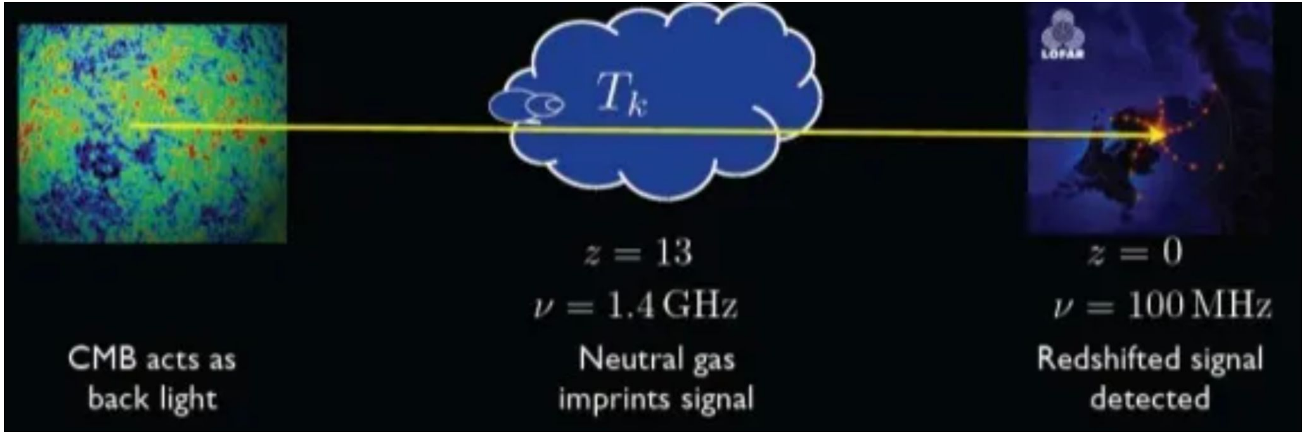
$$E(z) = \frac{H(z)}{H_0} = \sqrt{\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\Lambda}$$



Ben Gilliland/STFC

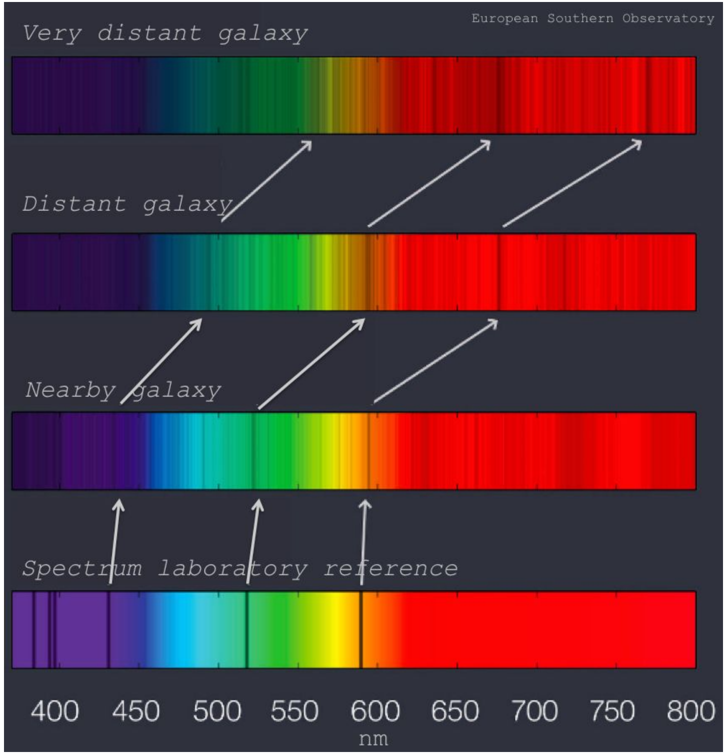
21-cm line

- Onset of recombination: transition between hyperfine energy levels in hydrogen (spin-flip transition) → 21-cm line
  - Mapping of 21-cm: 3D distribution of dark matter
  - “Holes” in 21-cm that occur due to reionization of neutral hydrogen → reionization



Gianni Bernardi

How are redshifts measured?



How to construct Gaussian field with a given power spectrum?

See Garrett Goon's tutorial!

$$\langle \varphi_{\mathbf{k}} \varphi_{-\mathbf{k}} \rangle' = P(k) \langle \phi_{\mathbf{k}} \phi_{-\mathbf{k}} \rangle' = P(k) \ .$$

Spelled out in more detail, we will perform the following steps:

1. Consider a white noise field of unit amplitude: $\varphi_{\mathbf{k}}$ with $\langle \varphi_{\mathbf{k}} \varphi_{-\mathbf{k}} \rangle' = 1$.
2. Generate a position-space realization of the white noise, denoted by $R_{\text{white}}(\mathbf{x})$. That is, $R_{\text{white}}(\mathbf{x})$ is a particular map showing the values of $\varphi(\mathbf{x})$ at various positions $\mathbf{x}$ and for which $\langle \varphi(\mathbf{x}) \varphi(\mathbf{y}) \rangle' = \delta^d(\mathbf{x} - \mathbf{y})$.
3. Fourier transform the realization: $R_{\text{white}}(\mathbf{x}) \longrightarrow R_{\text{white}}(\mathbf{k})$.
4. Multiply $R_{\text{white}}(\mathbf{k})$ by the square root of the power spectrum to create $R_P(\mathbf{k}) = P^{1/2}(k) R_{\text{white}}(\mathbf{k})$.
5. Fourier transform $R_P(\mathbf{k})$ back to position-space to get the desired realization: $R_P(\mathbf{x}) = \int \mathrm{d}^d \tilde{k} \, e^{i\mathbf{k}\cdot\mathbf{x}} R_P(\mathbf{x})$.