



Contribution ID: 104

Type: Poster

## Non-standard boundary behaviour arising in binary mixture problems

Consider a binary mixture model of the form  $\pi_1 \phi_1 + \pi_2 \phi_2$ , where  $\phi_1$  is standard normal and  $\phi_2$  is a completely specified heavy-tailed distribution with the same support. Gaussianity of  $\phi_1$  reflects a reduction of the raw data to a set of pivotal test statistics at each site (e.g. an energy level in a particle physics context). For a sample of independent and identically distributed values  $X_1, \dots, X_n$ , the maximum likelihood estimator  $\hat{\theta}_n$  is asymptotically normal provided that  $\theta_0$  is an interior point. This paper investigates the large-sample behaviour for boundary points, which is entirely different and strikingly asymmetric for  $\theta_0$  and  $\theta_1$ . On the right boundary, well known results on boundary parameter problems are recovered, giving  $\sqrt{n}(\hat{\theta}_n - \theta_1) \Rightarrow N(0, \Sigma)$ . On the left boundary (which corresponds to no new physics)  $\sqrt{n}(\hat{\theta}_n - \theta_0) \Rightarrow N(0, \Sigma)$ , where  $\Sigma$  indexes the domain of attraction of the density ratio when  $\theta_0 = \theta_1$ . For  $\theta_0 = \theta_1$ , which is the most important case in practice, the tail behaviour of  $\phi_2$  governs the properties of the maximum likelihood estimator and related statistics. Most notably, conditional on the event  $\{\hat{\theta}_n = \theta_0\}$ , the likelihood ratio statistic has a conditional null limit distribution that is not the usual  $N(0, 1)$ . In the talk I will omit technical details and focus on the conceptual points with a view to ascertaining whether the formulation is reasonable in a particle physics context. This is joint work with Peter McCullagh and Daniel Xiang at the University of Chicago.

**Presenter:** BATTEY, Heather (Imperial College London)

**Session Classification:** Social