# Accounting for Selection Effects in Supernova Cosmology with Simulation-Based Inference & Hierarchical Bayesian Modelling

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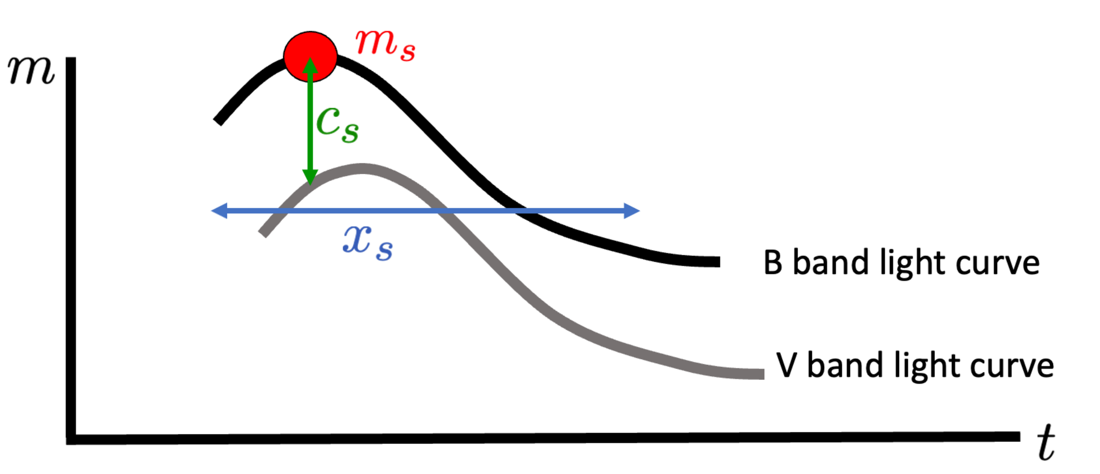
**Type Ia supernovae** (SNe Ia) are **exploding stars** that can be used to put constrains on the **nature of our universe**. SNe **selection effects** can cause **bias** to propagate through to our posteriors on **cosmological parameters**. We develop a novel technique of using a **normalising flow** to learn the **non-analytical likelihood** of observing a SN for a given survey from simulations. The learnt likelihood is then used in a **hierarchical Bayesian model** with **Hamiltonian Monte Carlo** sampling to put constraints on **different cosmological models.**

## How do we standardise SNe Ia?

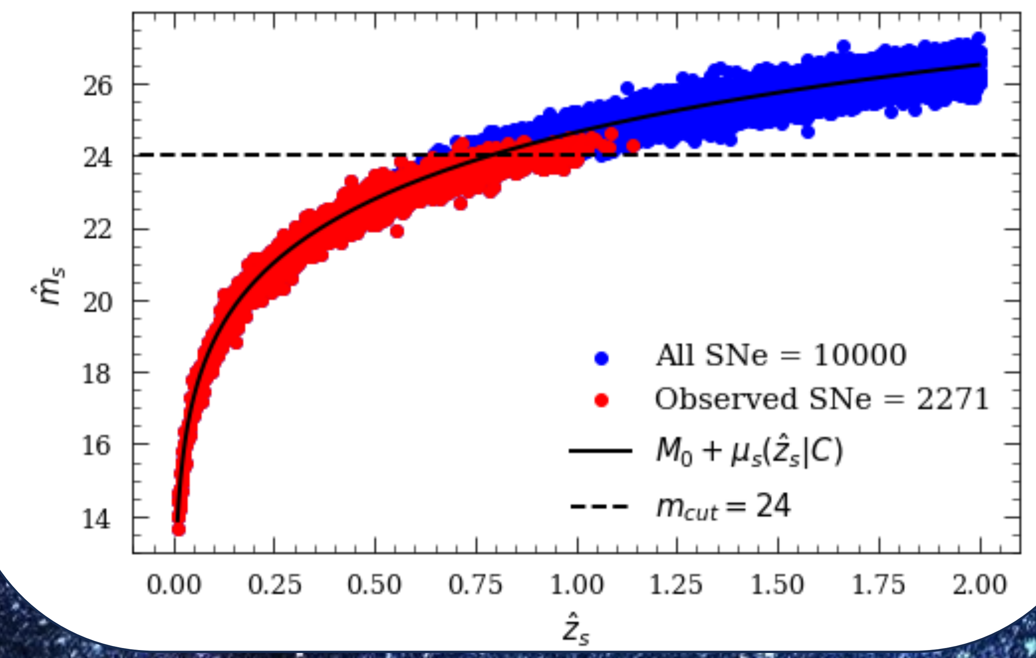



Tripp Formula [1]: $m_s = \mu(z_s|\boldsymbol{C}) + \underline{M_0} + \underline{\alpha}\, x_s + \underline{\beta}\, c_s$

- SNe Ia are exploding stars coming from similar physical processes.
- We can standardise them to model their absolute brightness and distance.
- Population analysis of SN Ia distances and redshifts allows us to constrain cosmology $\boldsymbol{C}$.

## Malmquist Bias

- One challenge with population analyses of SNe Ia is Malmquist bias, where we preferentially observe the brighter SNe due to limitations of our telescopes.
- This bias can propagate through to our constraints on cosmology.
- Traditional methods of accounting for this include bias corrections with simulations [2] and assuming the selection is analytic [3].

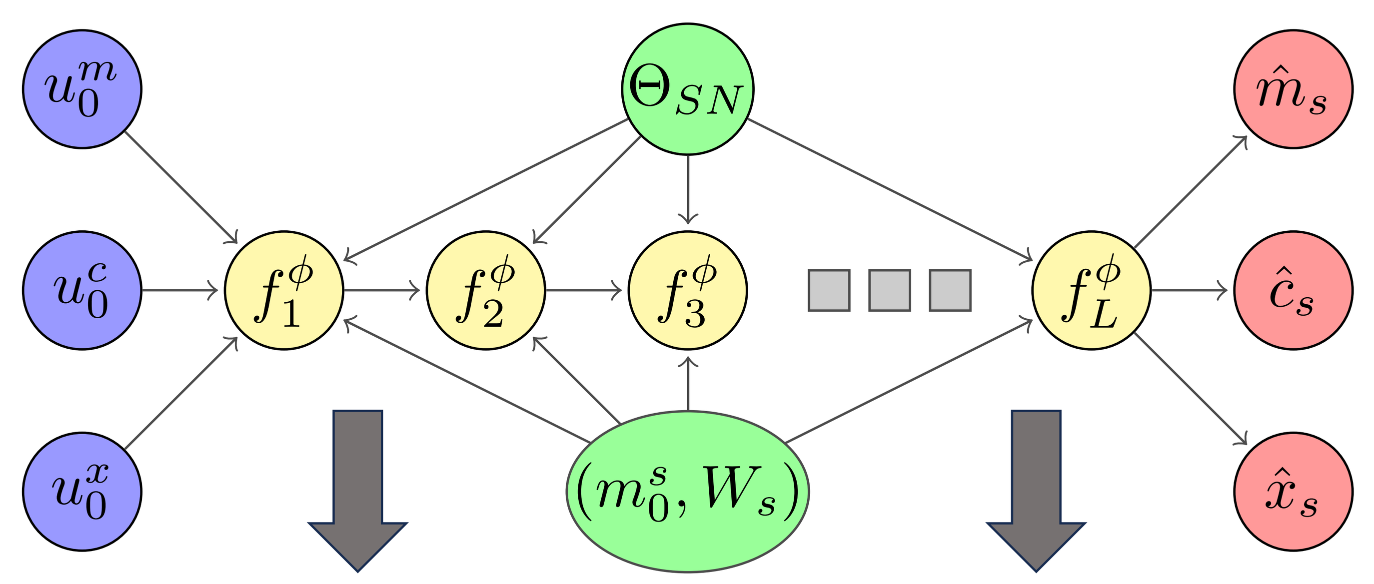


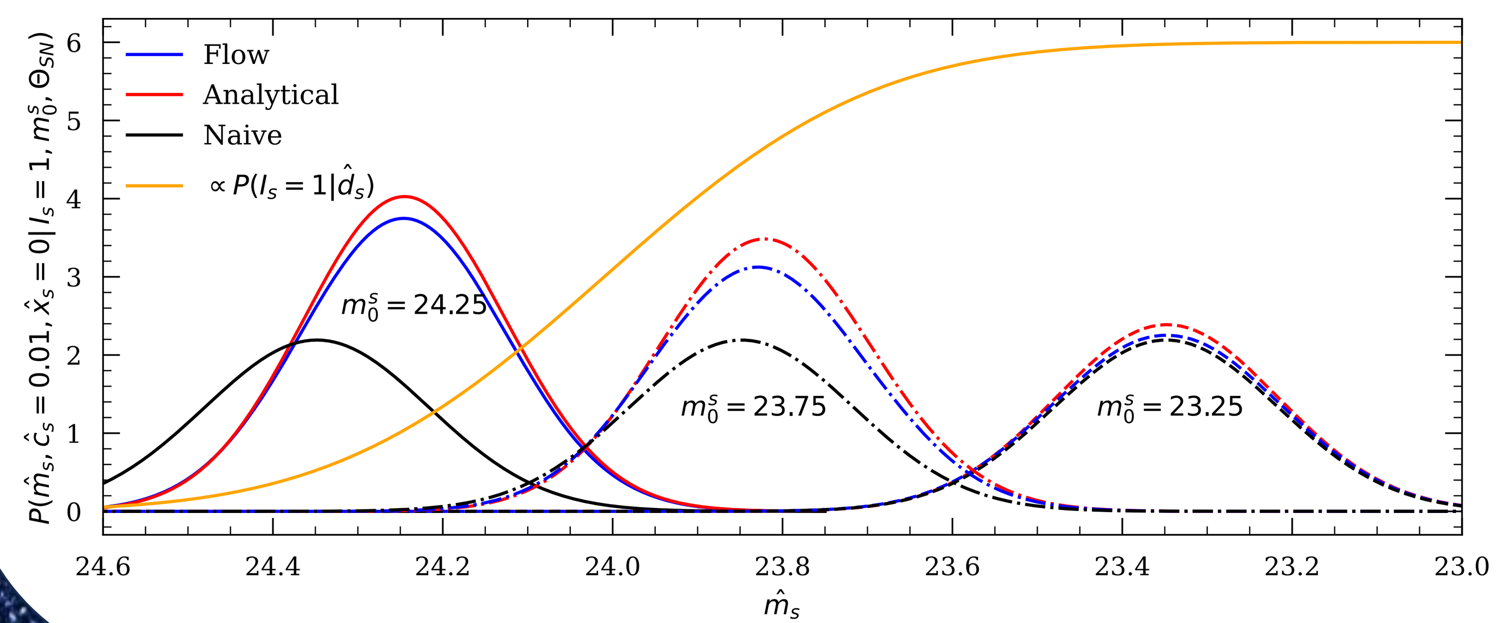

## Toy Simulations

$$\hat{\boldsymbol{d}}_{\boldsymbol{s}} = (\hat{m}_s, \hat{c}_s, \hat{x}_s)$$
$$\boldsymbol{\theta} = (\boldsymbol{C}, M_0, \alpha, \beta, \ldots)$$

$$P(\text{Selection}|\hat{\boldsymbol{d}}_{\boldsymbol{s}}) = \Phi\left(\frac{m_{\text{cut}} - (\hat{m}_s + a\hat{x}_s + b\hat{c}_s)}{\sigma_{\text{cut}}}\right) \quad [3]$$

- To validate our method we make toy simulations where the analytical likelihood is tractable.
- This allows the approximated posteriors on the cosmology and supernova parameters to be compared with the analytical solutions.

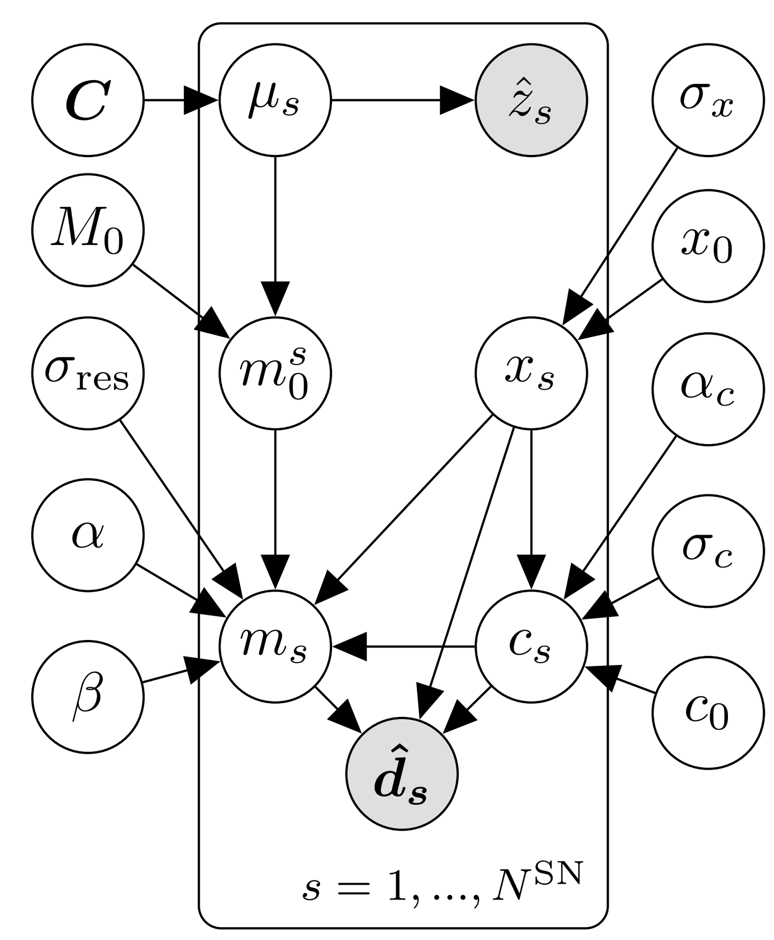



## Learning the Likelihood with a Normalising Flow

- Normalising flow [4] required as more realistic simulations have skewed likelihoods.
- Learn three-dimensional likelihood from simulations.
- Transform unit Gaussians into observed data by maximizing log likelihood.
- Defining $m_0^s = M_0 + \mu(\hat{z}_s|\boldsymbol{C})$ allows us to constrain different cosmological models.
- Learnt likelihoods agree well with analytical solutions.

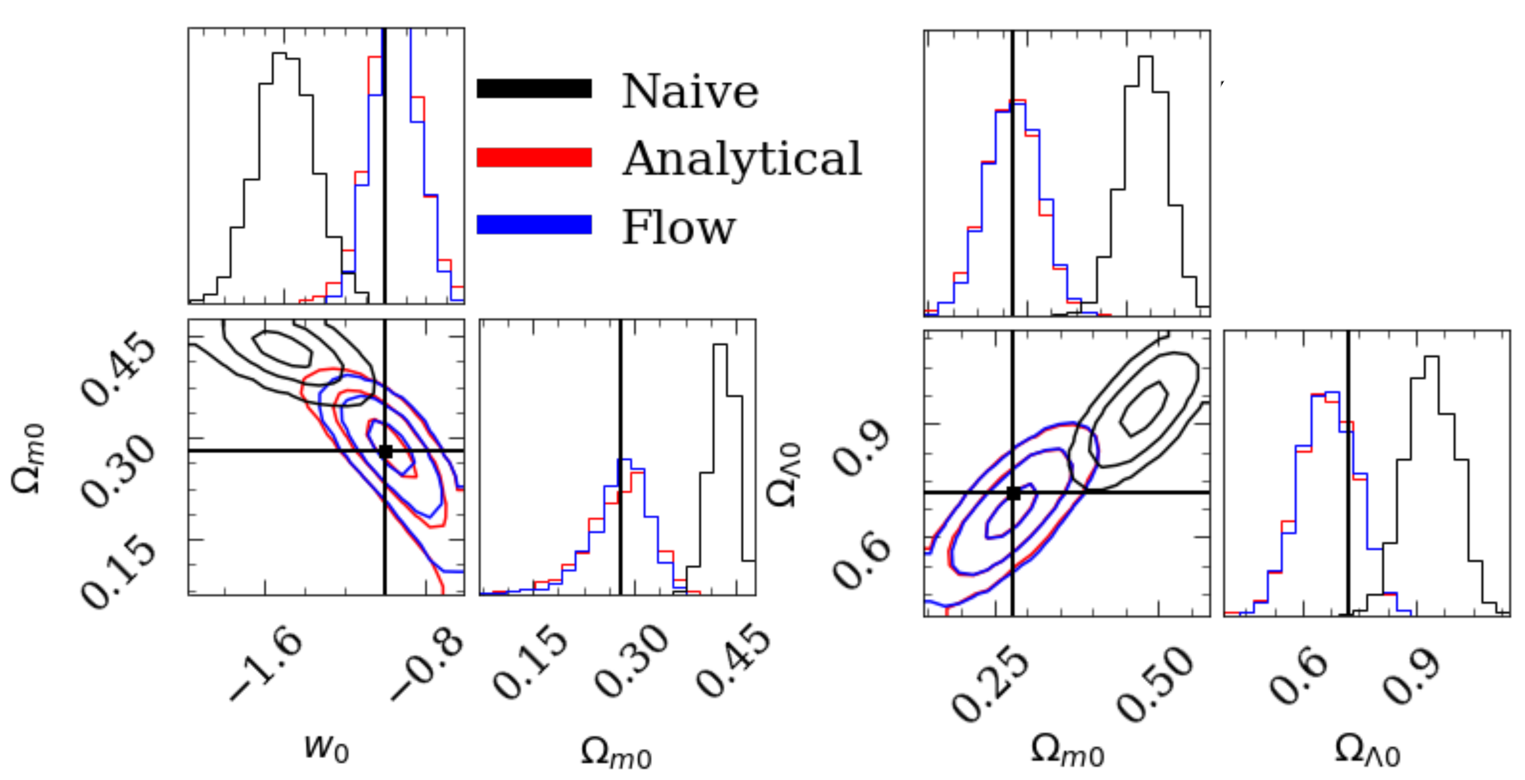
## Hierarchical Bayesian Model



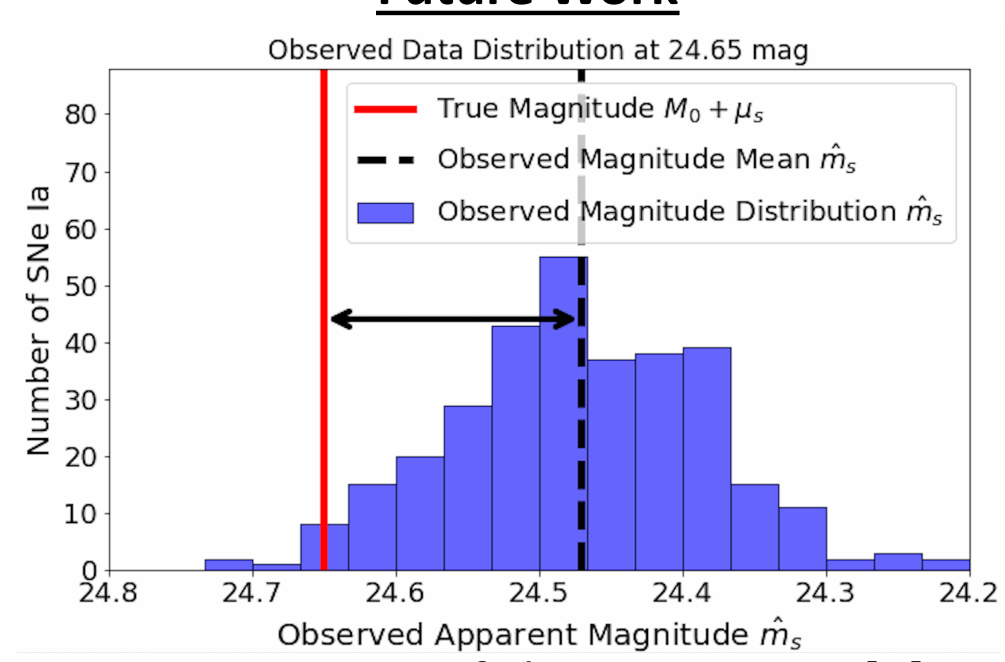

- Place normalising flow into a hierarchical Bayesian model [3,5,6].
- Sample with Hamiltonian Monte Carlo [7].

## Results

- Agreement with analytical posteriors across all 11 hyperparameters to within $1\sigma$.
- Same normalising flow successfully reused to constrain different cosmological models.




## Future Work




- Train on state-of-the-art SNANA [8] survey simulations for non-analytical cosmology posteriors on real data.

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