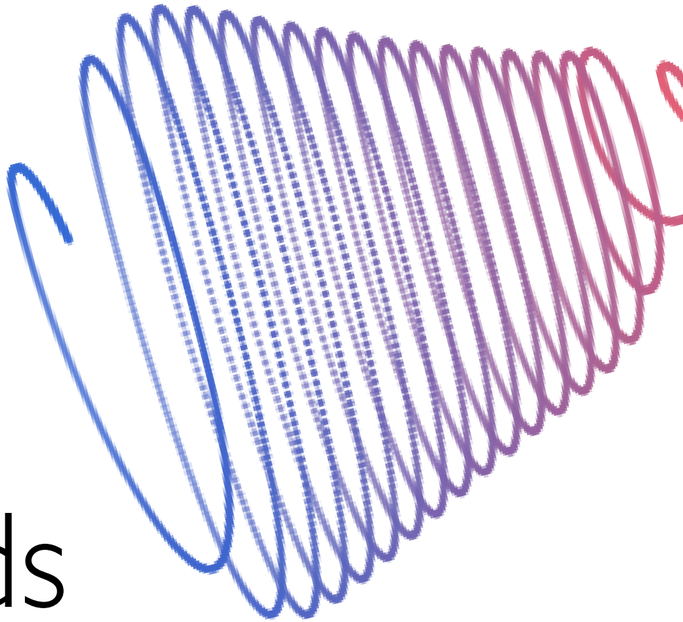


Final Cooling:

Tracking within solenoids and absorbers in RF-Track

R. Taylor

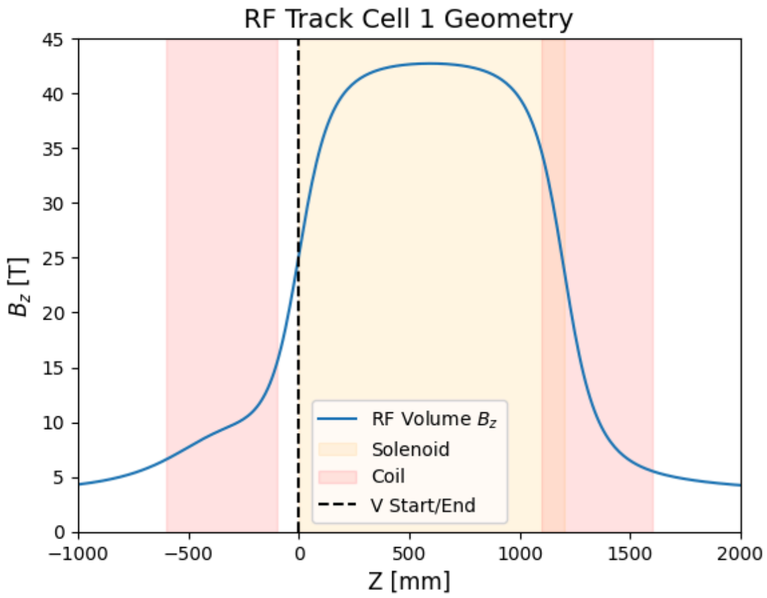
Muon Cooling WG

25th April 2024

1. **A Solenoid:**
    a. Comparing single-particle integration step & algorithm
2. **Solenoid with Absorber:**
    a. Track 1000 particles with same initial conditions
    b. Observe randomness due to scattering
    c. Calculate single-particle scattered transverse emittance
    d. Observe difference with integration step & algorithm

A Solenoid:

Comparing single-particle integration step & algorithm



- 40 T solenoid from Cell 1 of E. Fol's Final Cooling
- Tracked a particle of arb. initial conditions
  - Applied frequent watch points (wp) throughout the cell
- **Four** RF-Track algorithms track particle throughout cell
  - Integrate in steps of t [mm/c] = βrel * S [mm]

Runge-Kutta Order 2

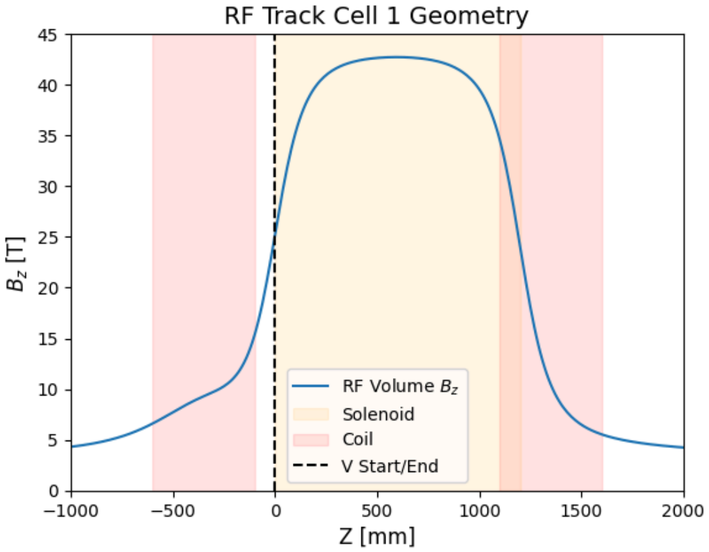
Leapfrog

Runge-Kutta-Fehlberg Ord 4,5

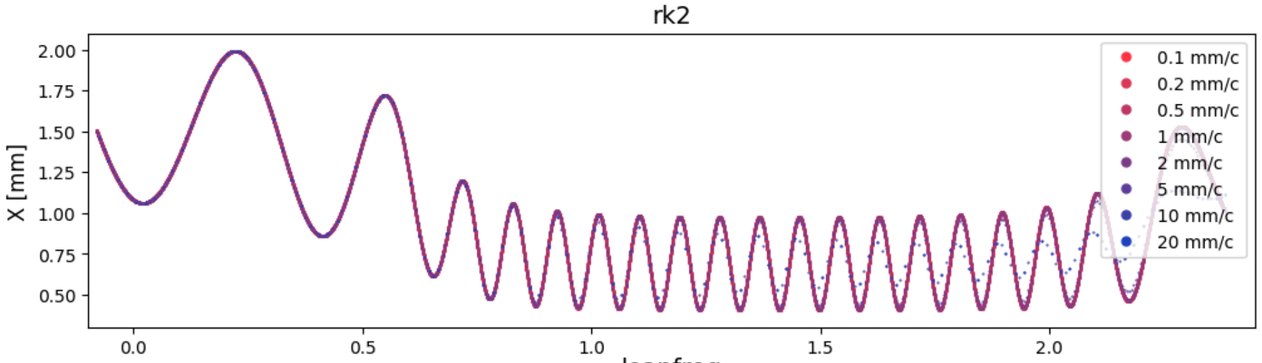
Analytic (*A. Latina*)

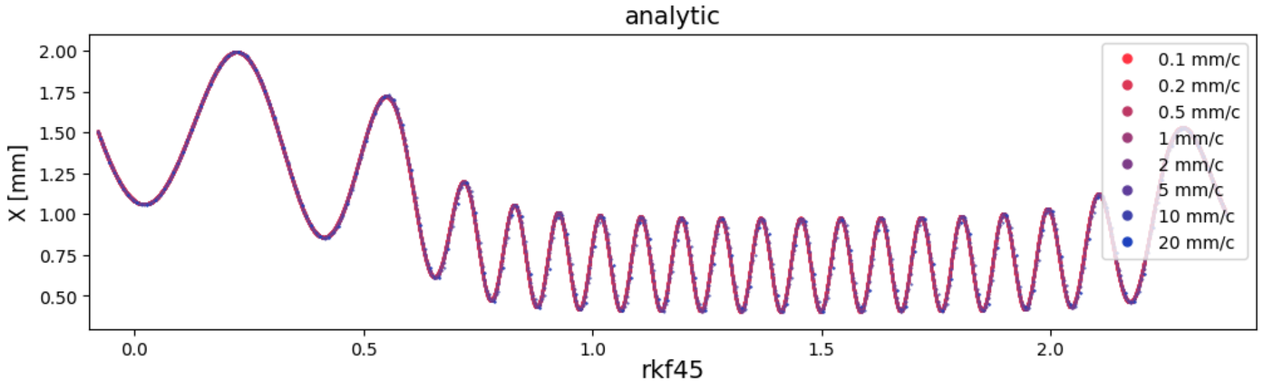
A Solenoid:

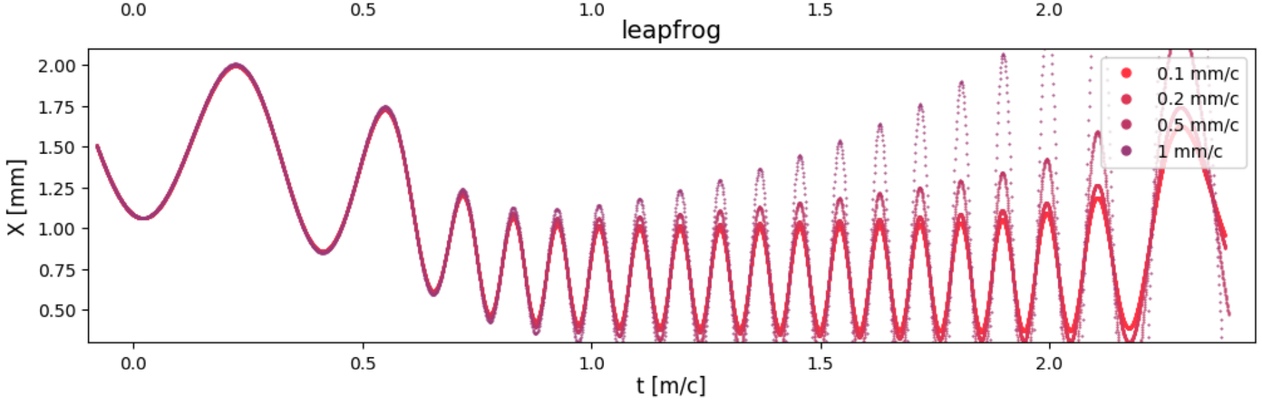
Comparing single-particle integration step & algorithm

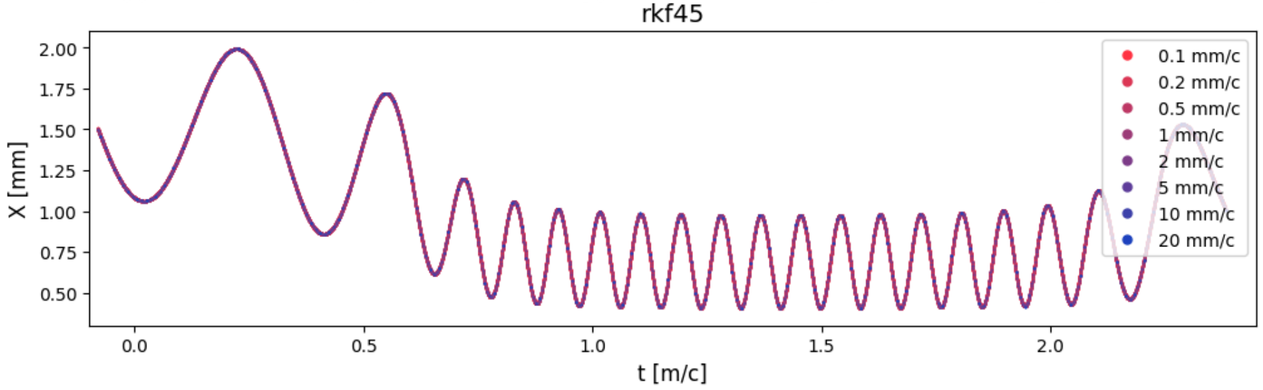


X-coordinate oscillation in steps of time (mm/c)



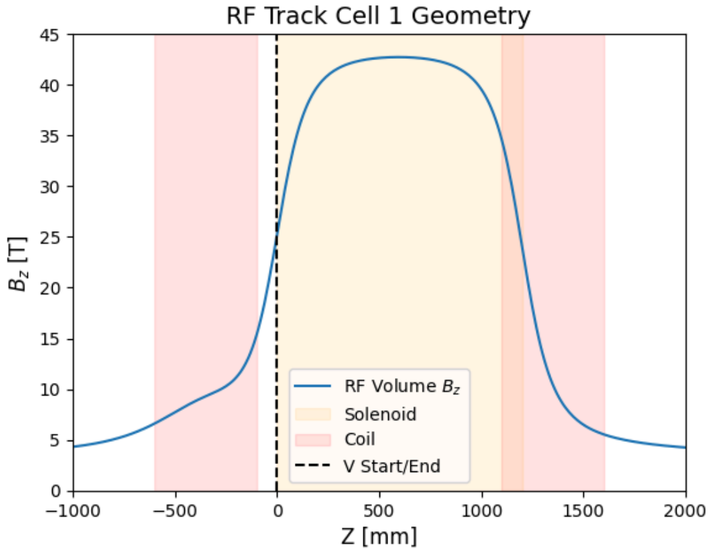




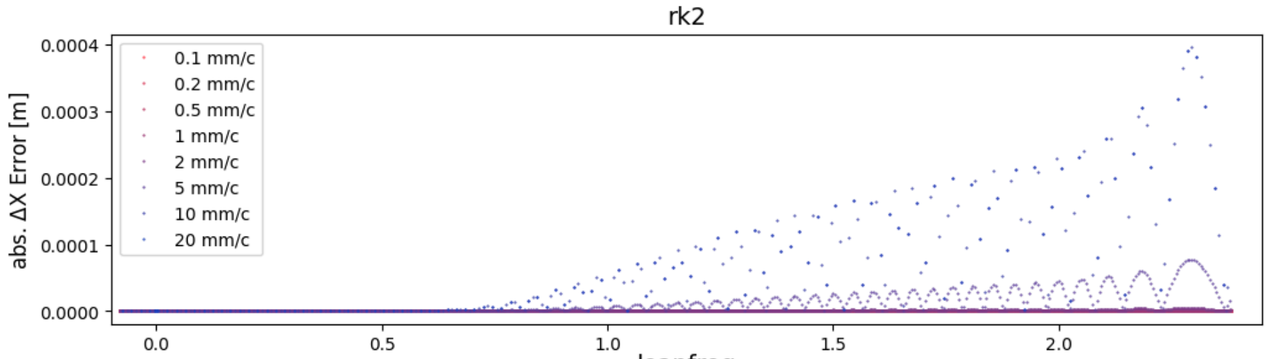


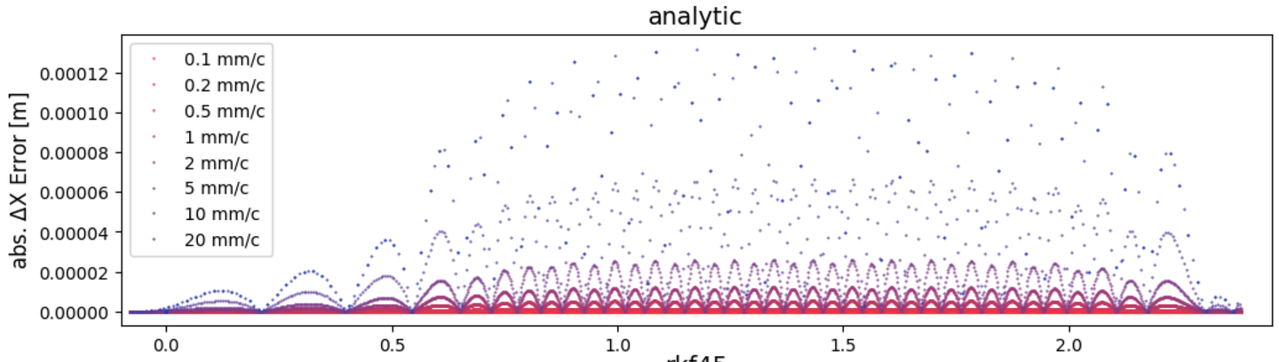
A Solenoid:

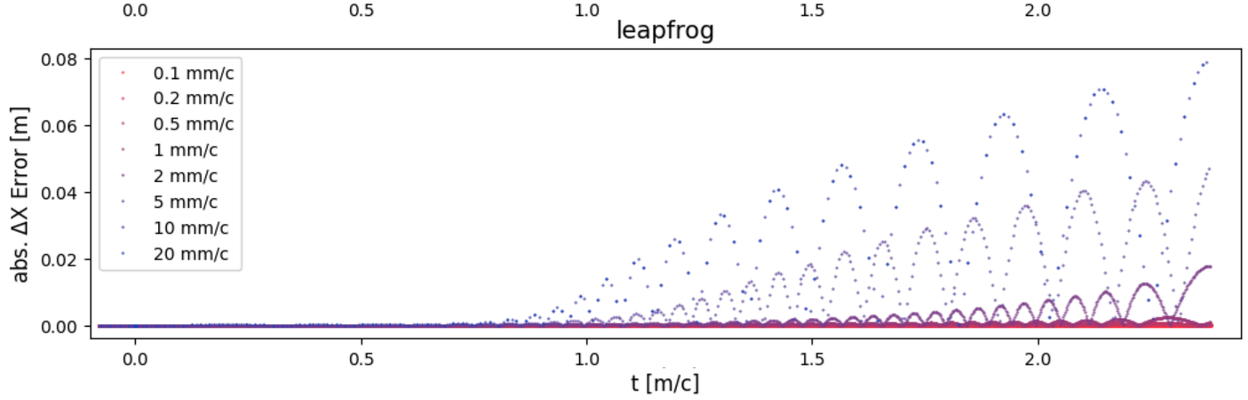
Comparing single-particle integration step & algorithm

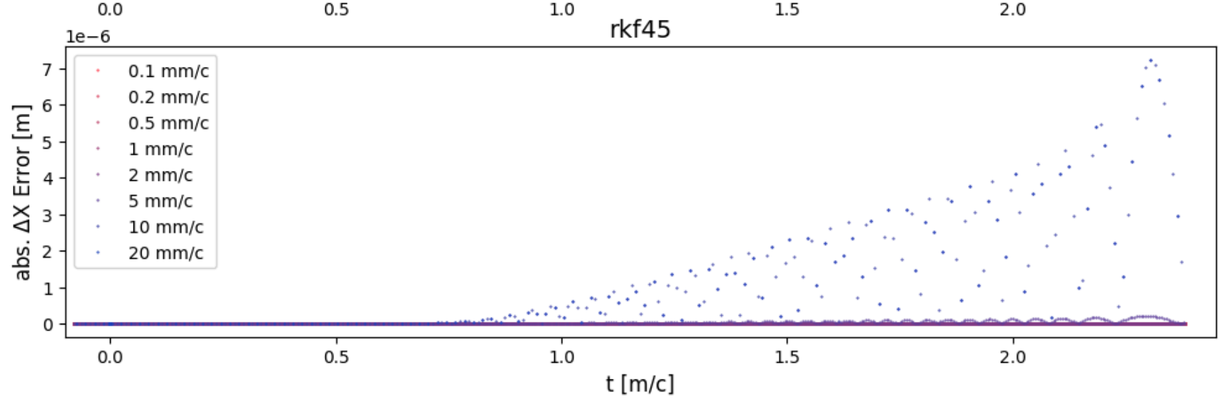


Error on X-coordinate oscillation (relative to 0.1 mm/c)





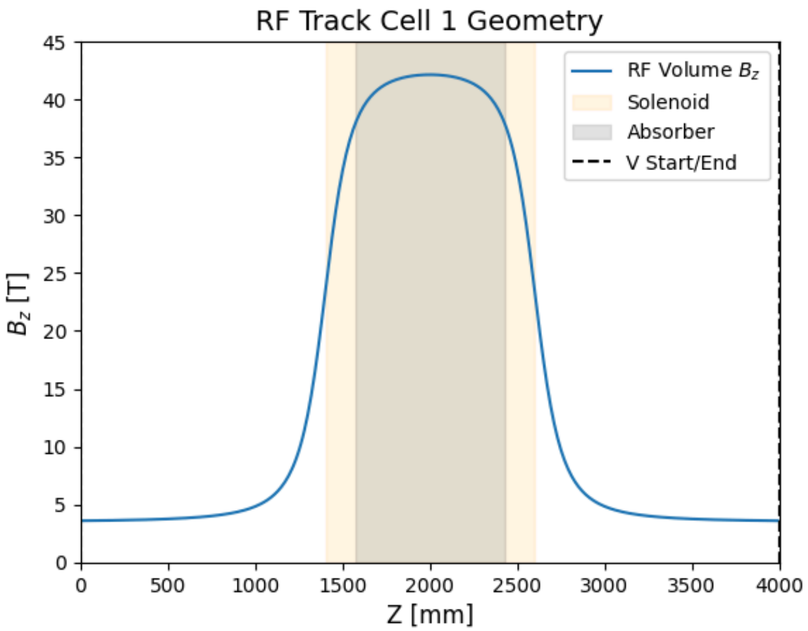


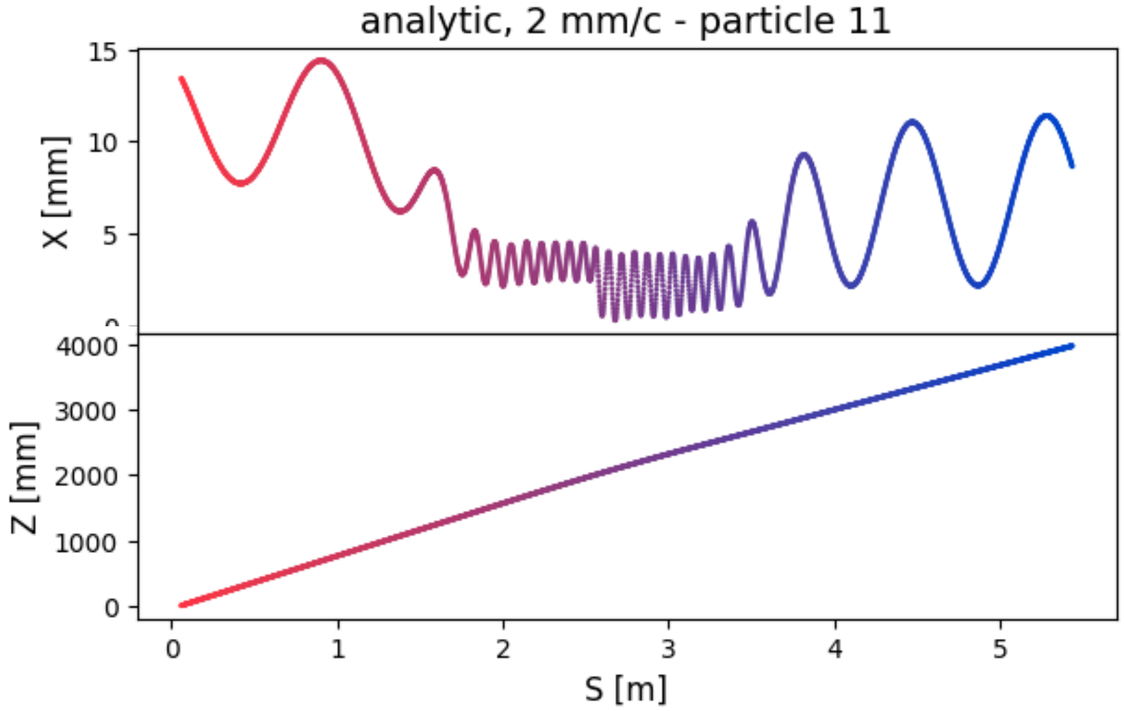


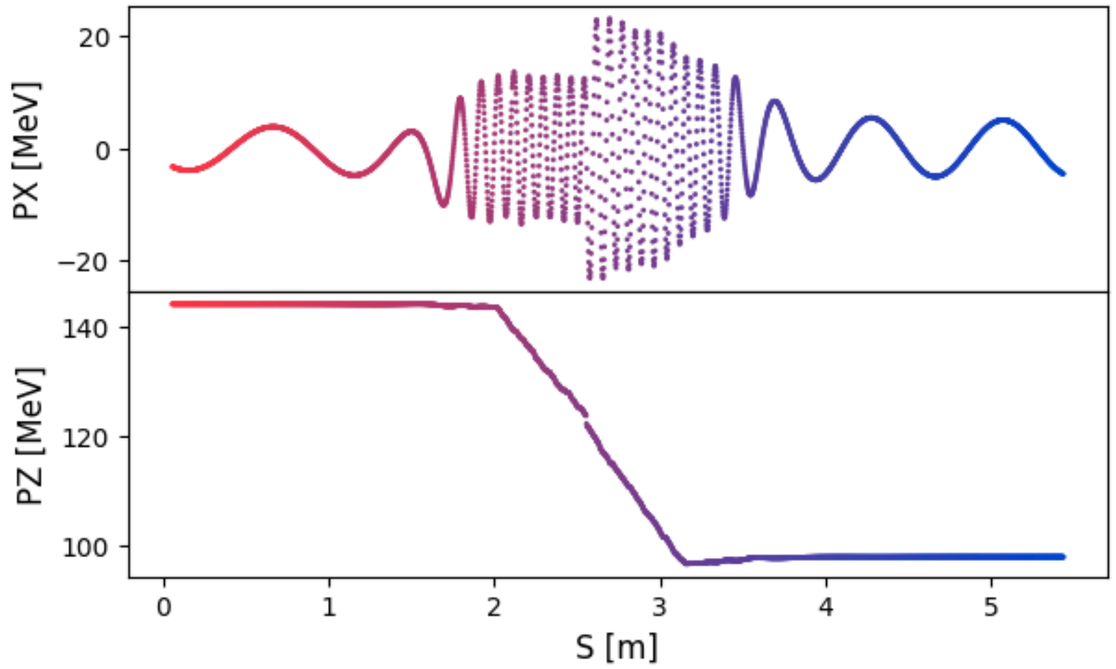
Solenoids + Absorber:

Comparing single-particle integration step & algorithm

- 40 T solenoid, no coils, 0.85 m LH absorber
- Tracked a particle of arb. initial conditions



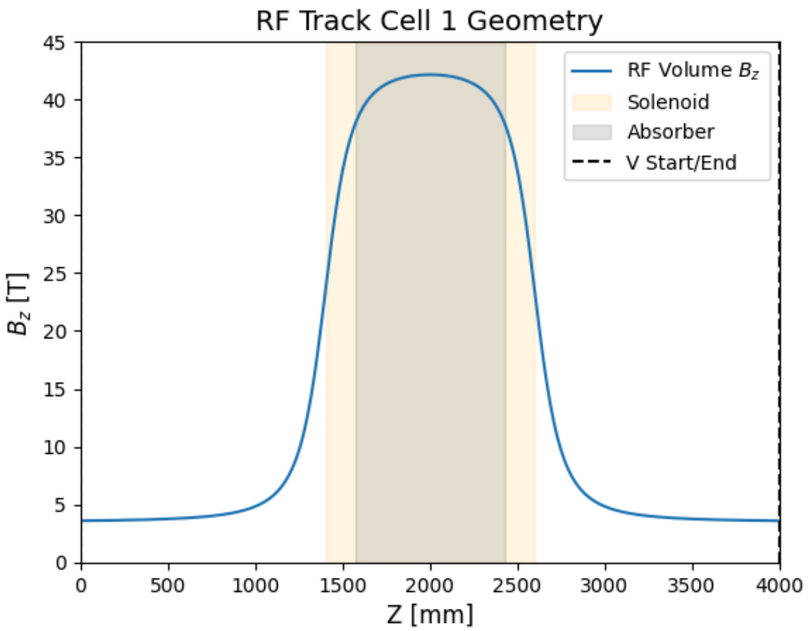


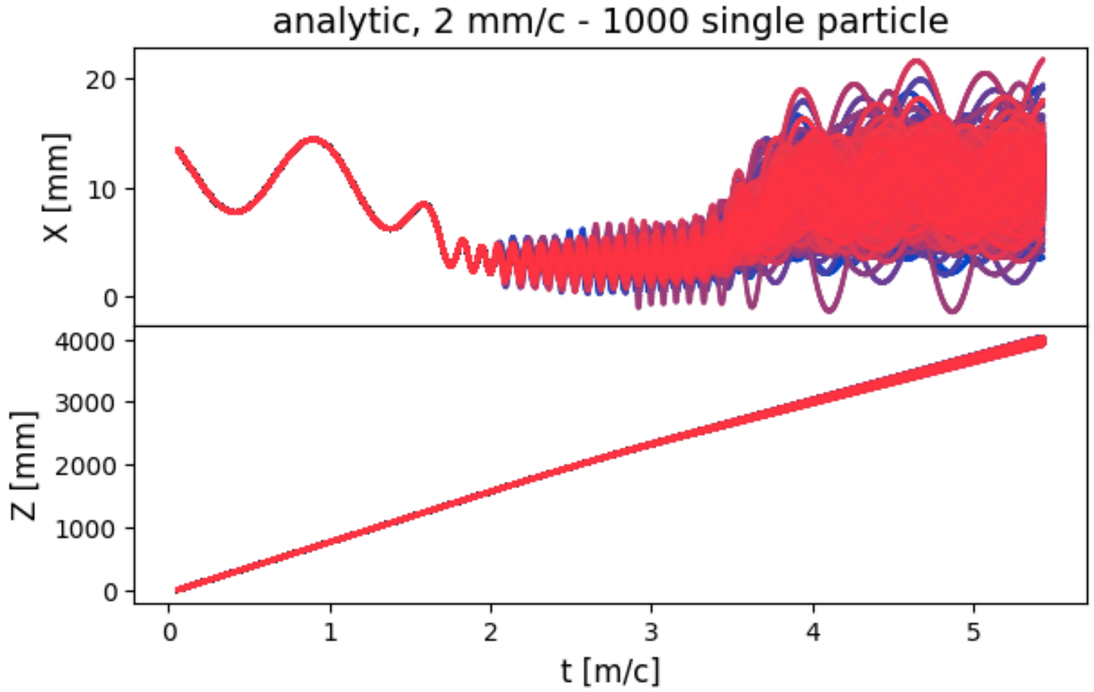


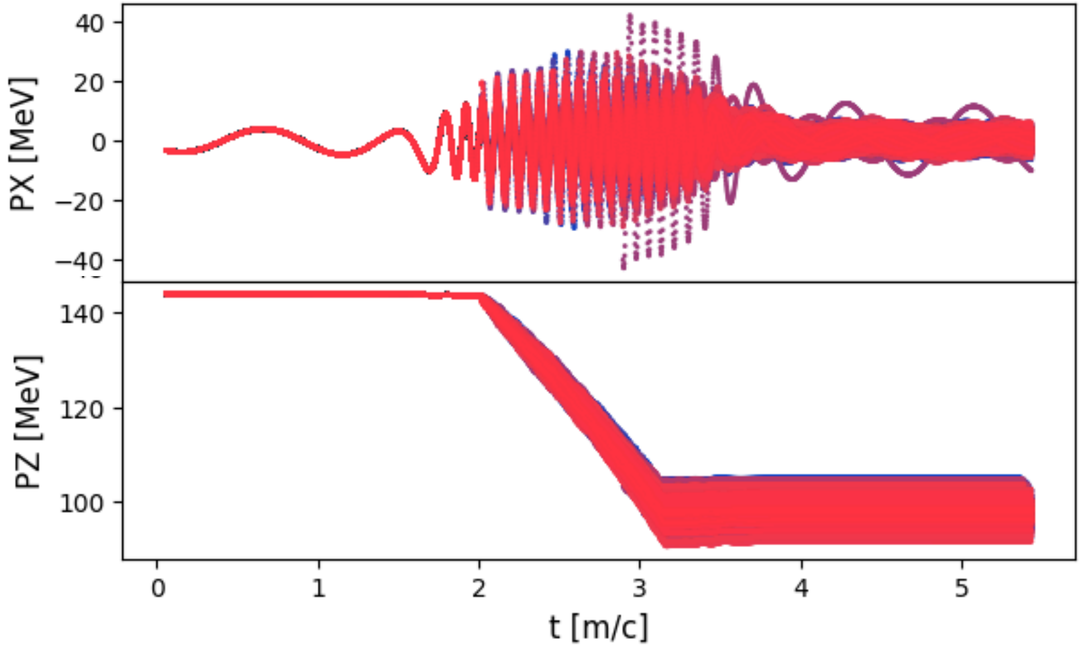
Solenoids + Absorber:

Observe randomness due to scattering

- 40 T solenoid, no coils, 0.85 m LH absorber
- Tracked 1000 particle of same initial conditions





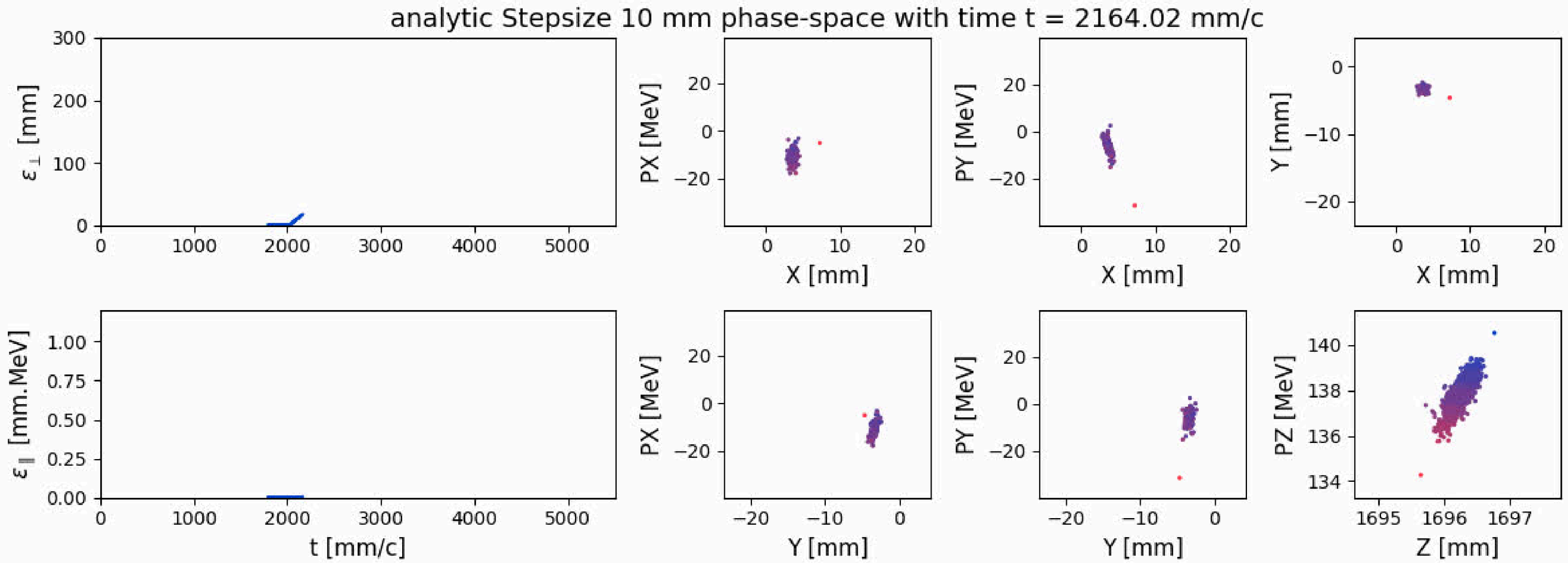


Solenoids + Absorber:

Calculate single-particle scattered emittance

$$\varepsilon_\perp = \frac{\sqrt[4]{\det(\mathrm{Cov}([x, x', y, y']))}}{m_\mu}$$

$$\varepsilon_\parallel = \frac{\sqrt{\det(\mathrm{Cov}([z, P_z]))}}{m_\mu}$$



Solenoids + Absorber:

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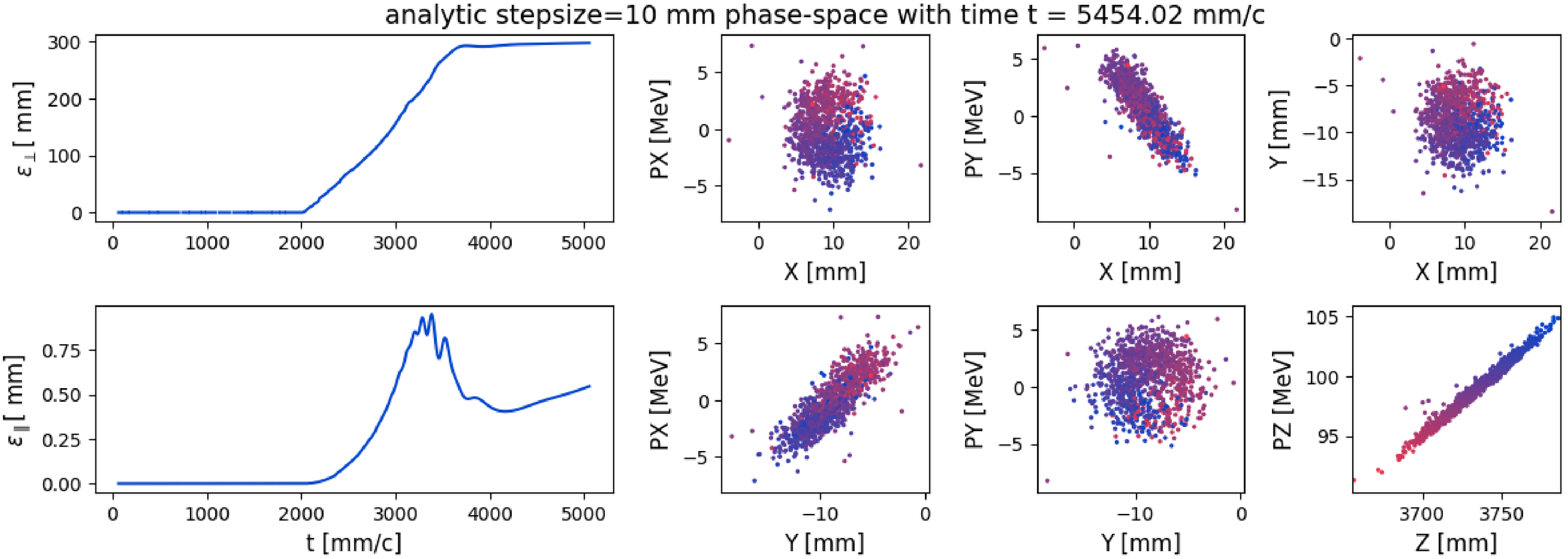

Solenoids + Absorber:

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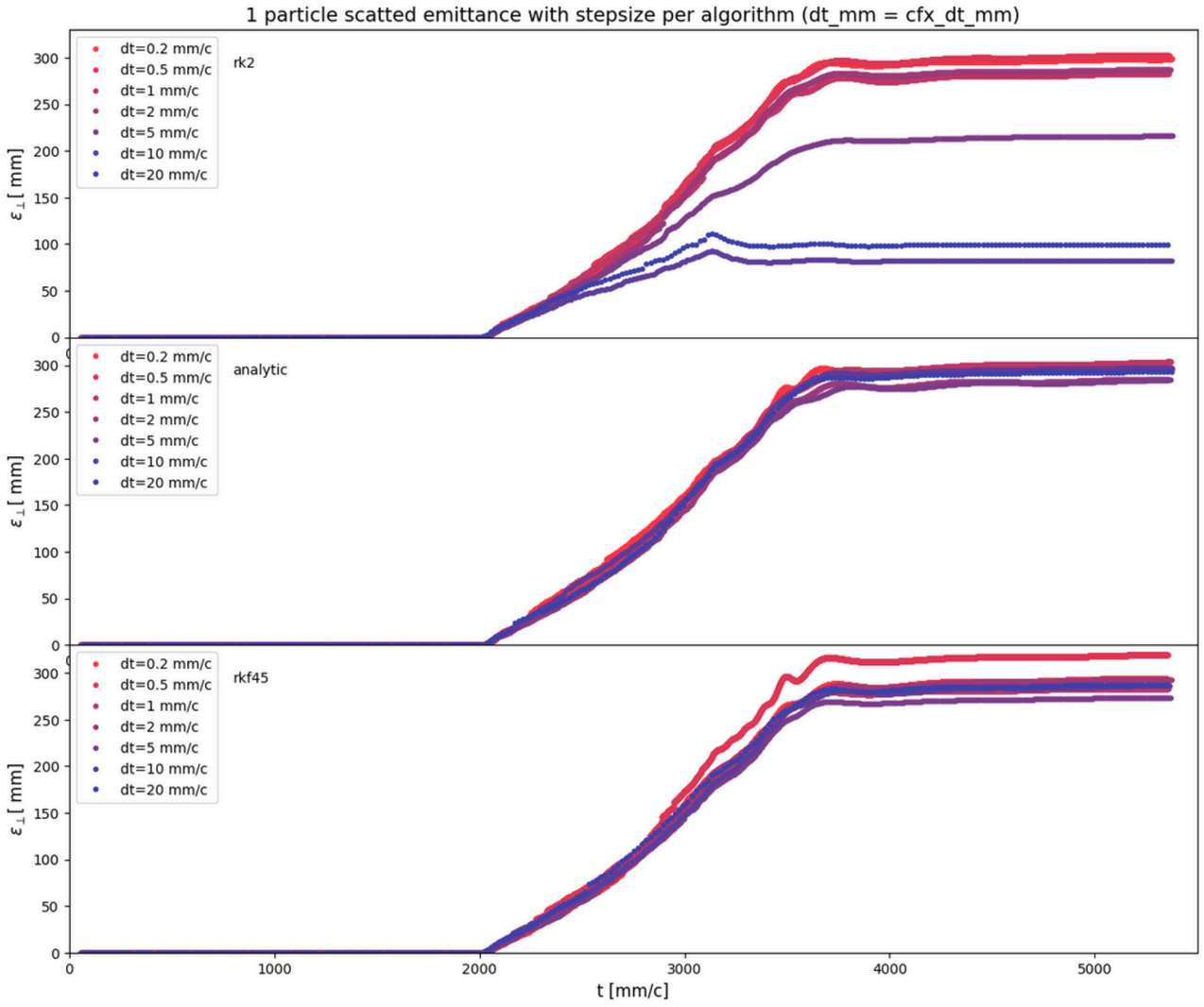
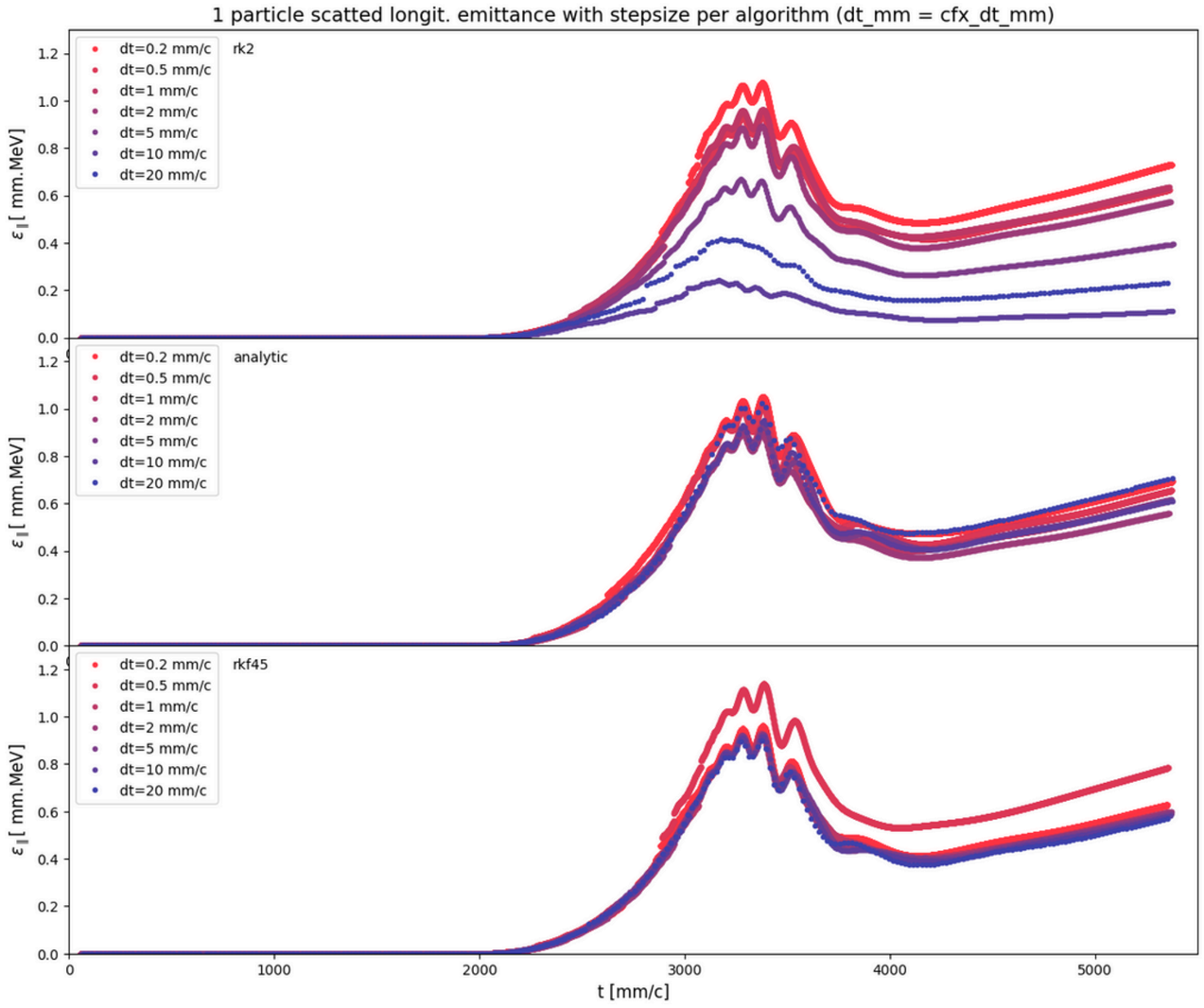




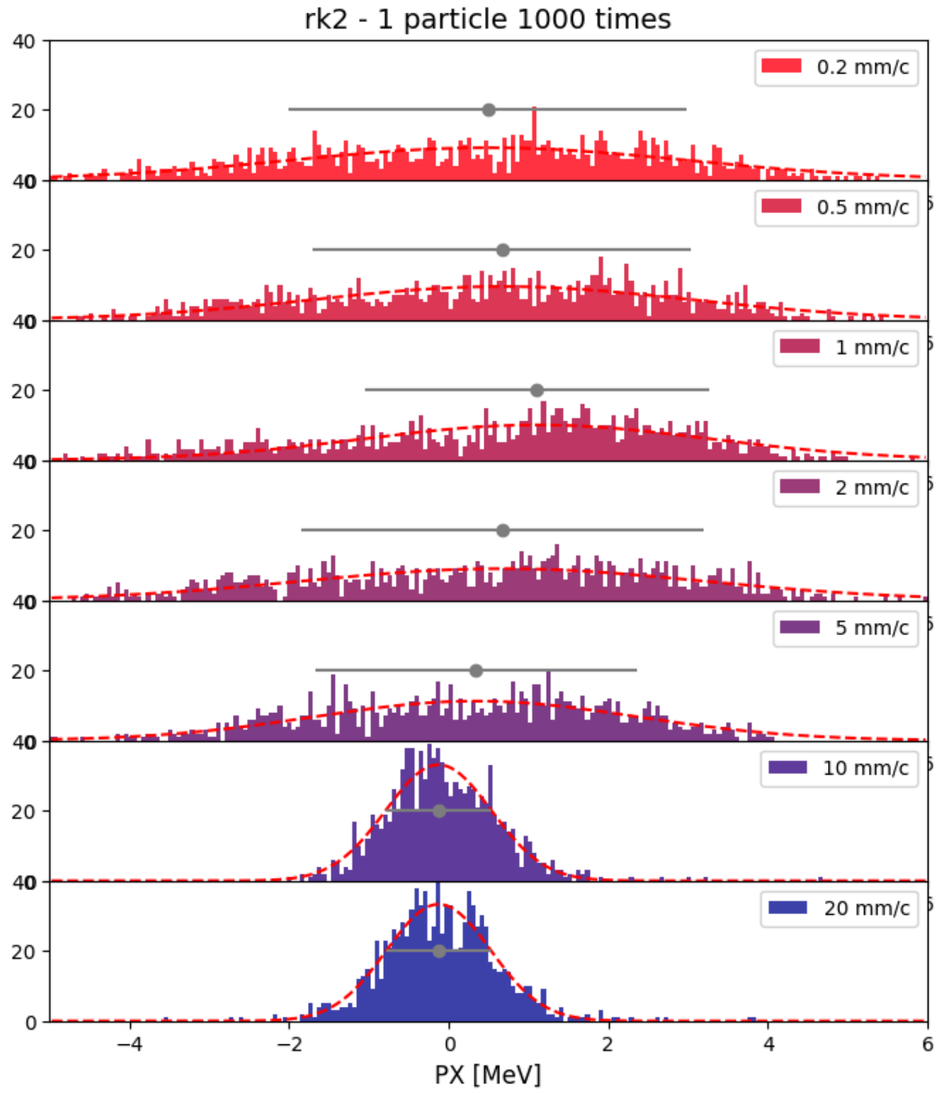
Conclusions

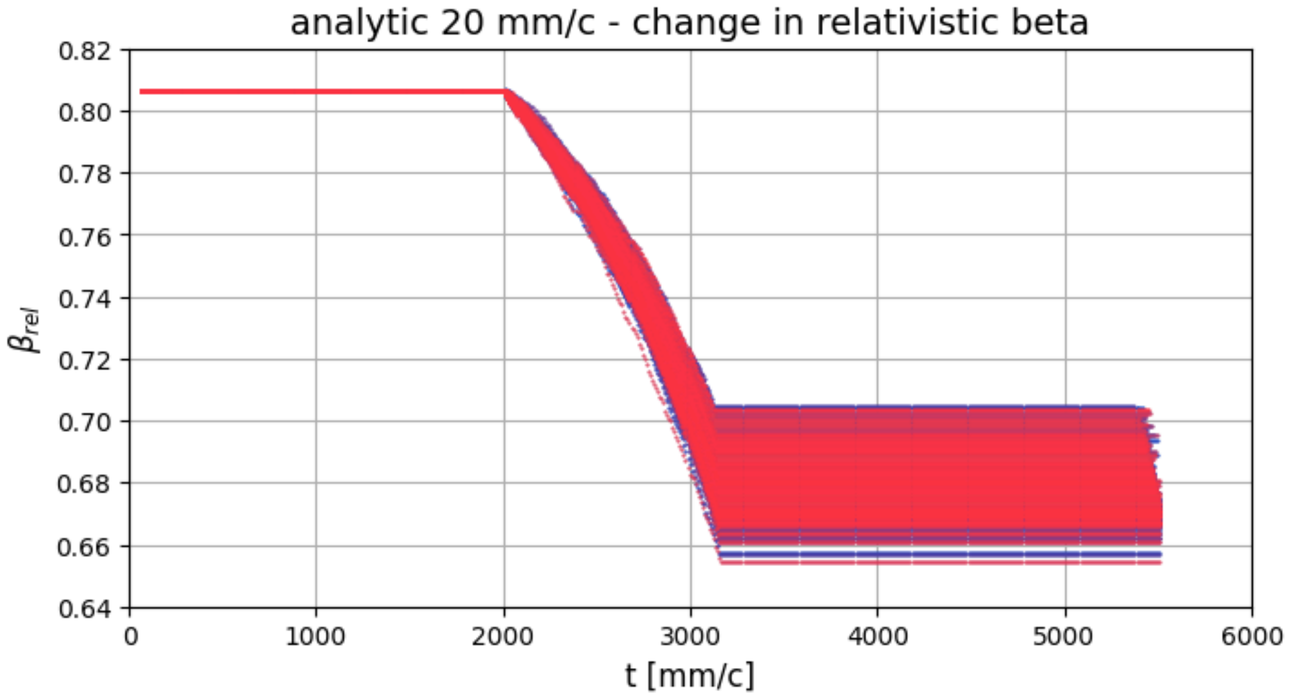
- RKF45 has lowest convergence error, but analytical has error $\propto$ Bz.
- Scattered emittance growth depends on integration step size
  - Analytical converges the fastest. **Recommend for High T fields.**

Next Steps

- Check on-axis particle for change in emittance growth.
- Show cooling in simple solenoid/absorber example.
- Slice in space rather than in time.

Extra Plots


rk2 - 1 particle 1000 times
0.2 mm/c
0.5 mm/c
1 mm/c
2 mm/c
5 mm/c
10 mm/c
20 mm/c
PX [MeV]


analytic 20 mm/c - change in relativistic beta
β_{rel}
t [mm/c]