IR beam losses and MDI collimators

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Outline

- **Introduction**
  - FCC-ee beam loss studies
  - FCC-ee collimation system
  - Workflow for detector background evaluation

- **FCC-ee beam losses and collimation simulations**
  - Simulation benchmark with SuperKEKB data

- **Updates on FCC-ee beam loss simulations: focus on IR beam losses**
  - Beam halo losses
  - Beam-gas losses
  - Touschek scattering losses
  - Beam-beam losses
  - Top-up injection losses
  - Losses from fast instabilities

- **Outlook and next steps**

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FCC-ee beam loss studies: overview

- **Studies and simulations of beam losses** in the FCC-ee are being performed to study:

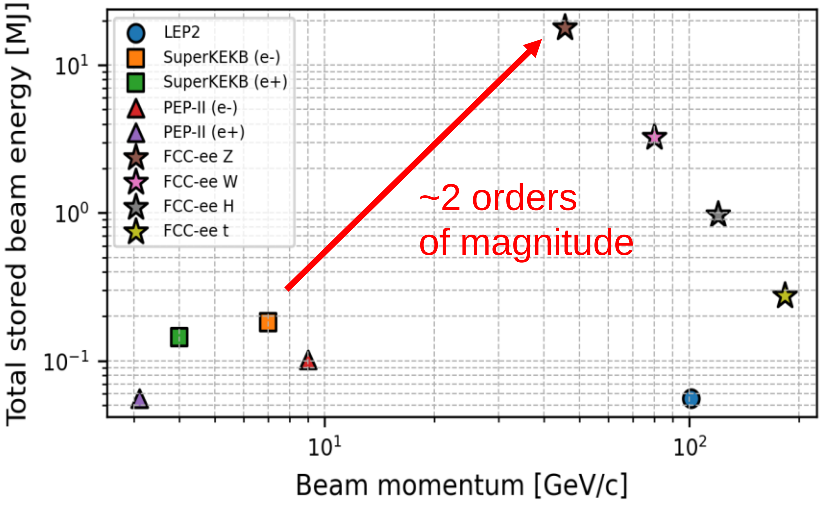
Optimization of the FCC-ee collimation system design

Minimization of beam losses on sensitive components (e.g. superconducting magnets)

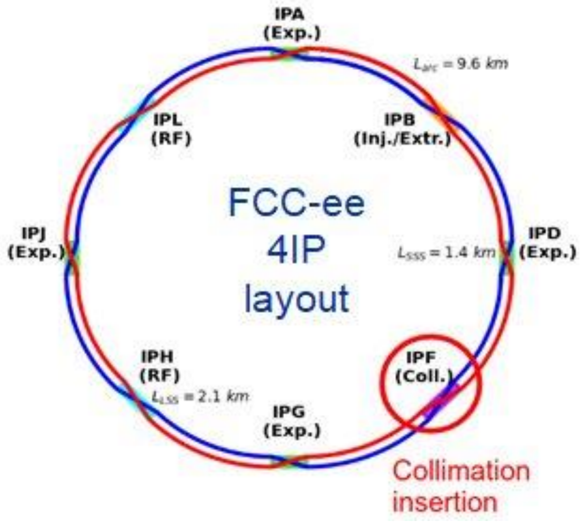
Minimization of beam losses in the experimental interaction regions (IRs): can be source of backgrounds

- **FCC-ee presents unique challenges**:
  - ➢ **17.5 MJ** stored beam energy in the ***Z* mode** (45.6 GeV)
  - ➢ New regime for collimation of e± beams (highly destructive beams)

- **Collimation strategy for the FCC-ee**
  - ○ Beam-halo (global) collimation
    - ▪ + local protection collimation (e.g., experimental IRs)
  - ○ Secondary particle shower absorbers
  - ○ Synchrotron radiation (SR) collimation – upstream of the IPs



FCC-ee collimation system



- **Dedicated long straight section in PF for global beam halo collimation**
  - Two-stage betatron and off-momentum collimation
  - Ensure protection of the aperture bottlenecks in different conditions
    - **Aperture bottleneck at Z:** 11.5σ (H plane), 81.5σ (V plane)
  - Secondary particle shower absorbers between primary and secondary betatron collimators (FLUKA team)

- **SR collimators upstream of each IP**
  - Designed to reduce detector backgrounds and power loads in the inner beampipe due to photon losses

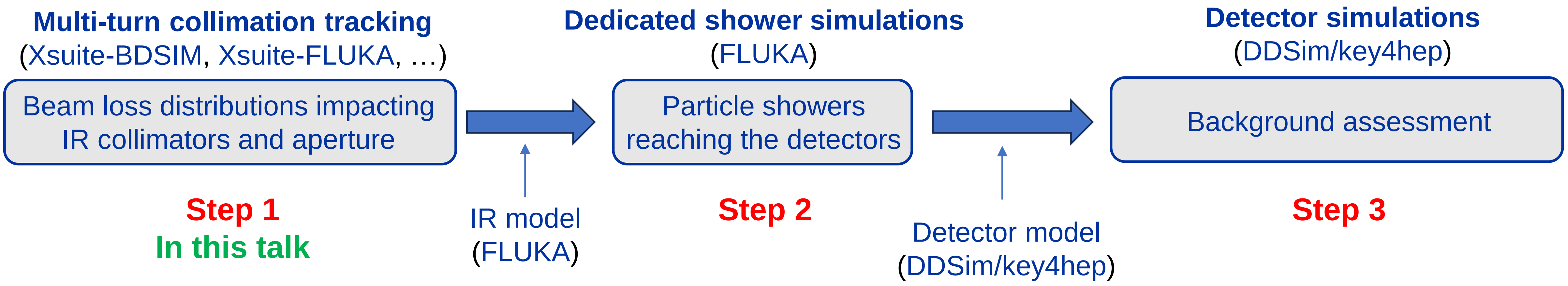
- **Tertiary local protection collimators**
  - Placed upstream of each IP: 690 m and 420 m upstream
  - s-location optimized for optimal phase-advance (multiple of π) between TCTs and { SR collimators / aperture bottlenecks

SR collimators
aperture bottlenecks

- More details in: [G. Broggi, Collimation studies for the FCC-ee, FCC week 2025]

Workflow for detector background evaluation

- **Workflow to evaluate beam-induced backgrounds**:



Multi-turn collimation tracking
(Xsuite-BDSIM, Xsuite-FLUKA, …)
Beam loss distributions impacting IR collimators and aperture
Step 1
In this talk

IR model
(FLUKA)

Dedicated shower simulations
(FLUKA)
Particle showers reaching the detectors
Step 2

Detector model
(DDSim/key4hep)

Detector simulations
(DDSim/key4hep)
Background assessment
Step 3

- **MDI collimator impacts from multi-turn collimation tracking of beam-gas scattered particles provided to the FLUKA team to move on to the «step 2»**
  - First results in A. Frasca, TID and Fluence in the detector and IR
  - Similar studies for injection background and to assess fast instabilities impact on detector
    - G. Nigrelli, First look at injection backgrounds

- Similar studies are planned for all beam loss scenarios
  - GOAL: converge on a complete beam-induced background evaluation campaign

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FCC-ee beam loss scenarios

- The FCC-ee Z mode is the current focus: has the highest stored beam energy **17.5 MJ**

- **Important to identify different beam loss scenarios and define the ones to protect against**

- Current selection of beam loss scenarios to study and simulate:
  - **Generic beam halo losses**
  - Beam losses from **interactions with residual gas**
  - Beam losses from **Touschek scattering**
  - Beam losses from **beam-beam interactions**
  - Beam losses due to **fast instabilities**

In this talk: focus on IR losses

  - Beam losses from **top-up injection** (G. Nigrelli, previous talk)

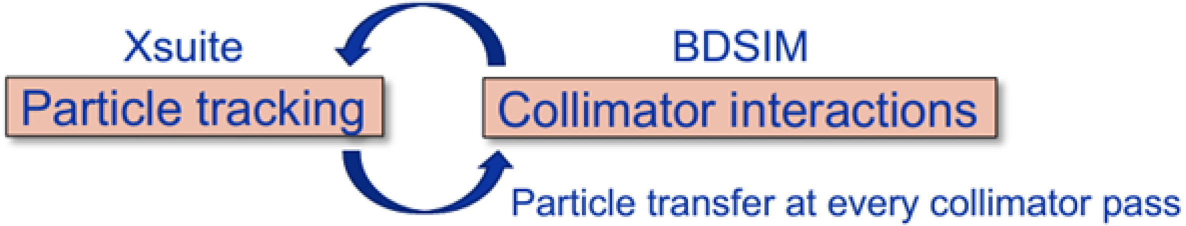
  - Beam losses from interactions with **thermal photons**: study planned for 2025-2026
    - Important effect at LEP - analytical estimates first to understand potential impact on FCC-ee

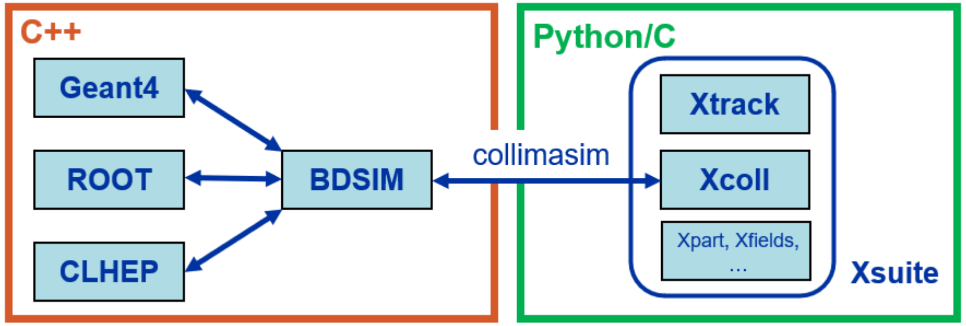
  - Accidental scenarios (asynchronous dump, others): waiting for inputs to set up models

FCC-ee collimation simulations

- **FCC-ee presents unique challenges for collimation simulations**
  - ➢ Synchrotron radiation and magnet strength adjustment (tapering) to compensate it
  - ➢ Complex beam dynamics – strong sextupoles in the lattice and strong beam-beam effects
  - ➢ Detailed aperture and collimator geometry modelling + beam particle-matter interactions
  - ➢ Electron/positron Large accelerator system – 90+ km beamline

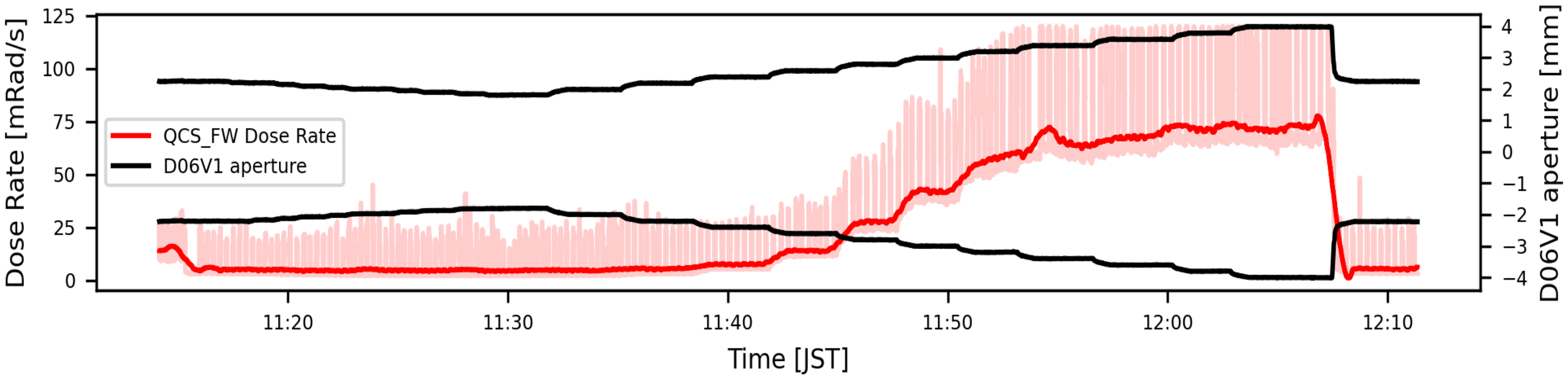
- **Xsuite + BDSIM (Geant4) coupling** (JINST paper)
  - ➢ Developed for FCC collimation simulations
  - ➢ Benchmarked against {
    - other simulation codes: MAD-X, pyAT, Sixtrack-FLUKA
    - measured data from: SPS, LHC, SuperKEKB, DAFNE is also considered
  - ➢ Other tools available: Xsuite-FLUKA coupling – already used for hadron beam collimation, ready to use for lepton beam collimation



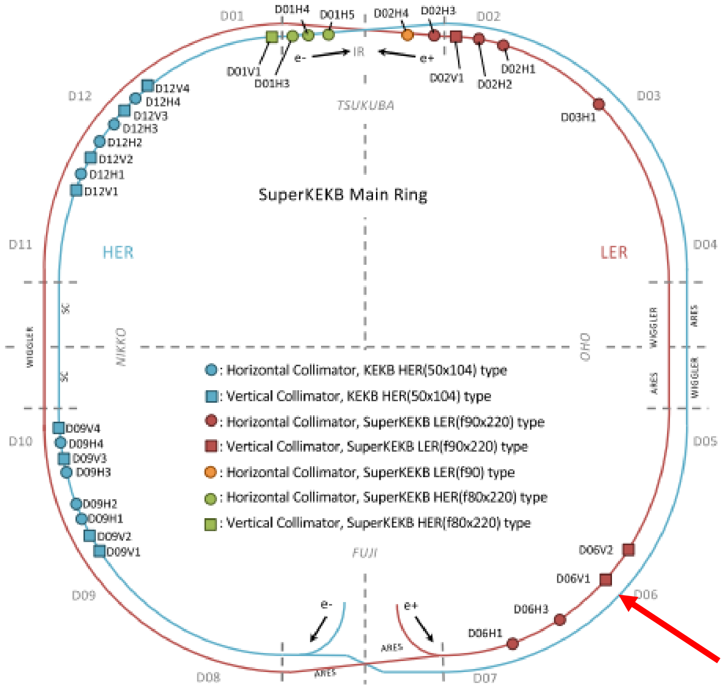


Xsuite-BDSIM benchmark at SuperKEKB

- Comparison of Xsuite–BDSIM simulations with measured backgrounds at SuperKEKB (IPAC'25 paper)
- Dedicated single-beam background study at SuperKEKB LER (June 2020, A. Natochii, PRAB)
  - D06V1 collimator aperture scan
  - Radiation dose rates measured by Belle II radiation monitors



SuperKEKB collimator layout as of June 2020

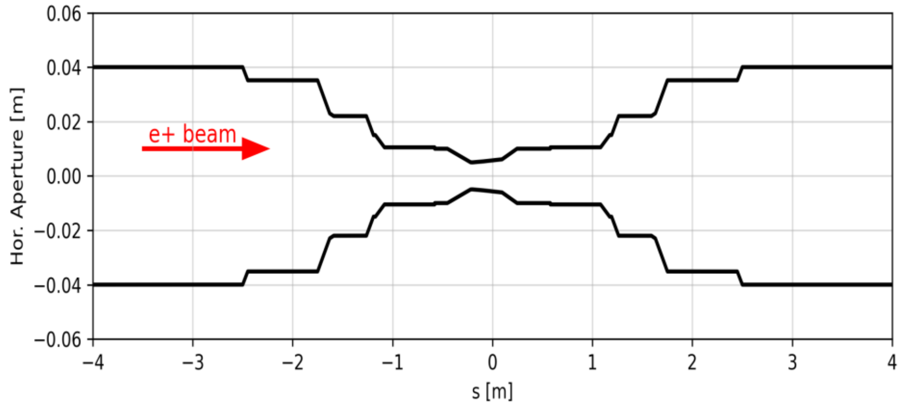


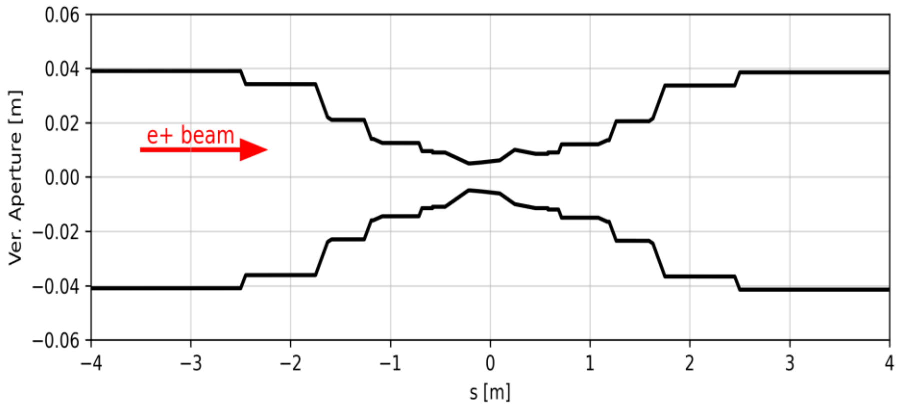
- Experimental conditions reproduced in simulations
  - Beam-gas (Brems, Coulomb) and Touschek scattering
  - IR aperture losses recorded and used as input to detector simulations (basf2*) to estimate the response of IR radiation monitors

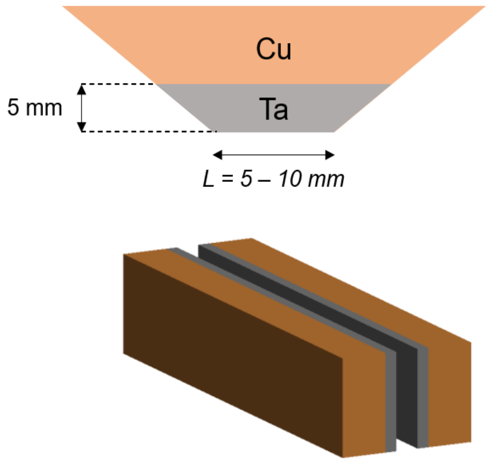
***basf2**: Belle II Analysis Software Framework – Geant4-based framework for Belle II detector simulations

Xsuite-BDSIM benchmark at SuperKEKB

- **SuperKEKB** (LER) **aperture model**

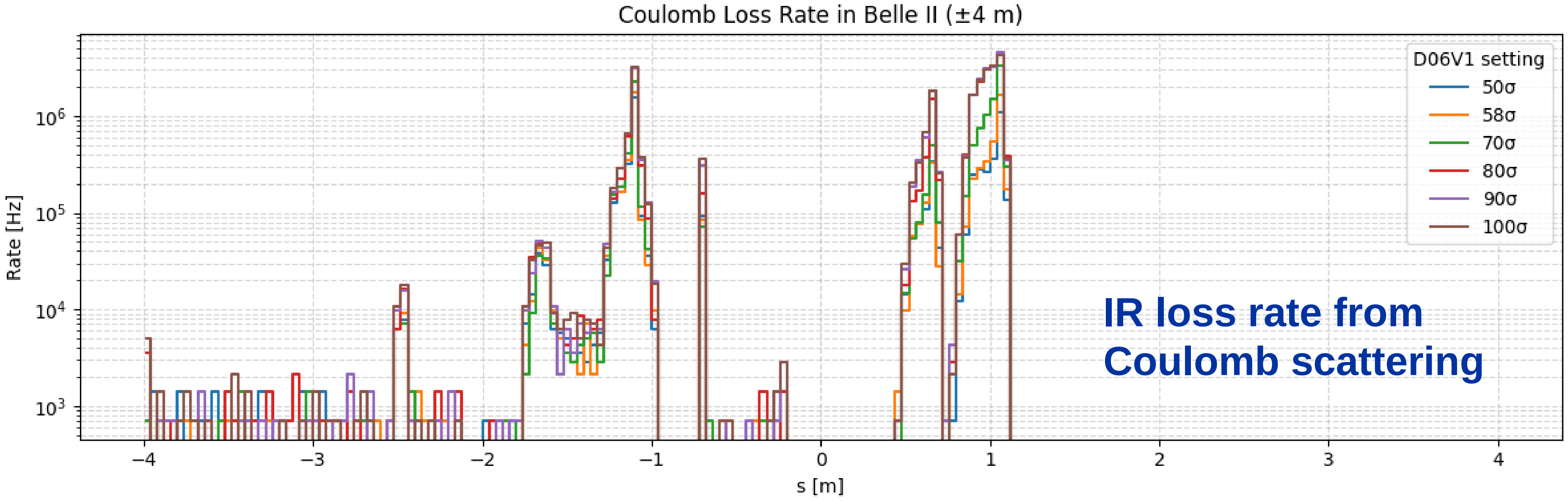






BDSIM CollimatorTipJaw

New collimator type implemented in BDSIM to reproduce the SuperKEKB-type collimators

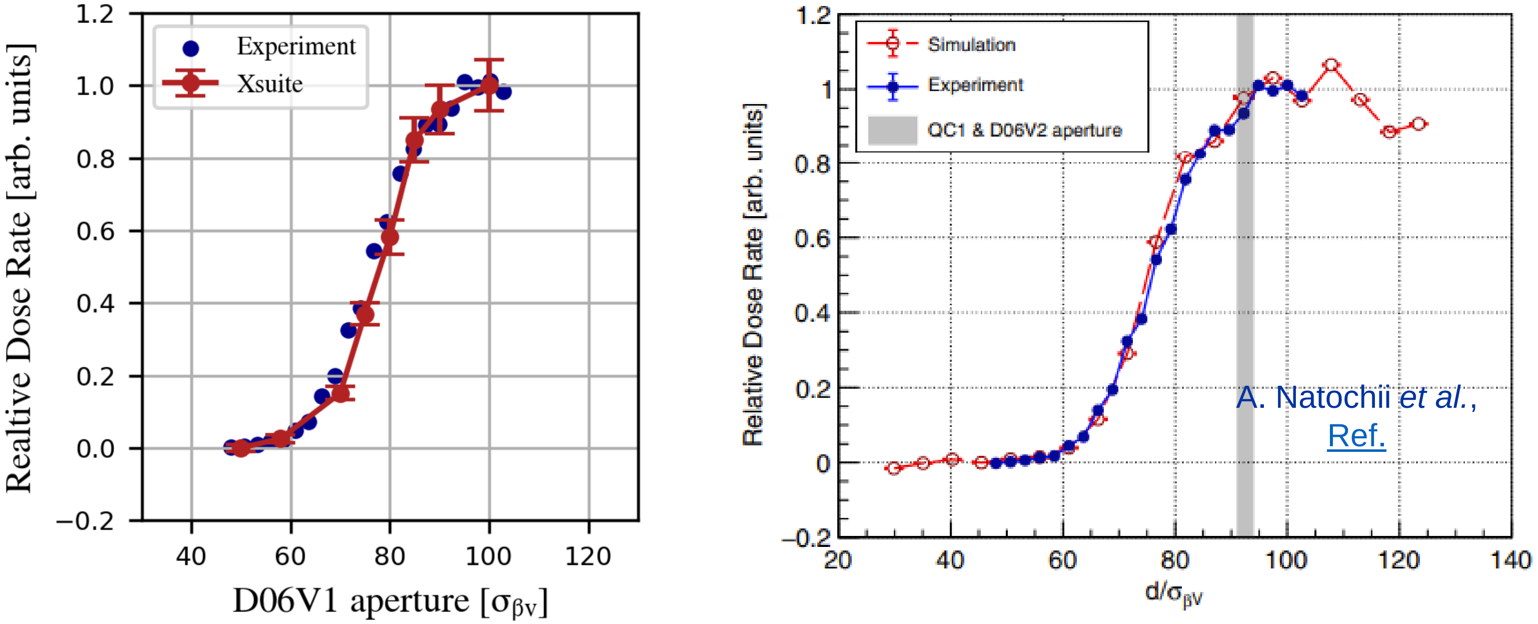

Coulomb Loss Rate in Belle II (±4 m)

IR loss rate from Coulomb scattering

Xsuite-BDSIM benchmark at SuperKEKB

- Propagating IR losses in basf2 detector simulations and comparing with the measured background trend vs. D06V1 collimator aperture:

Relative total dose rate for QCS-FW diamond detector vs. D06V1 collimator aperture in units of σβV



A. Natochii *et al.*, [Ref.](Ref.)

- Simulations accurately reproduce the measured background trend

- In absolute, Data/MC~3-4, consistently with independent SAD/Geant4 simulations
    - Excellent result given uncertainties on gas pressure and composition, ideal machine without errors, etc.

- Validates Xsuite–BDSIM for collimation and background studies in e+e− machines

Many thanks to G. Iadarola, A. Natochii, J. Salvesen, KEK colleagues and EAJADE programme for the support!

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  - Losses from fast instabilities

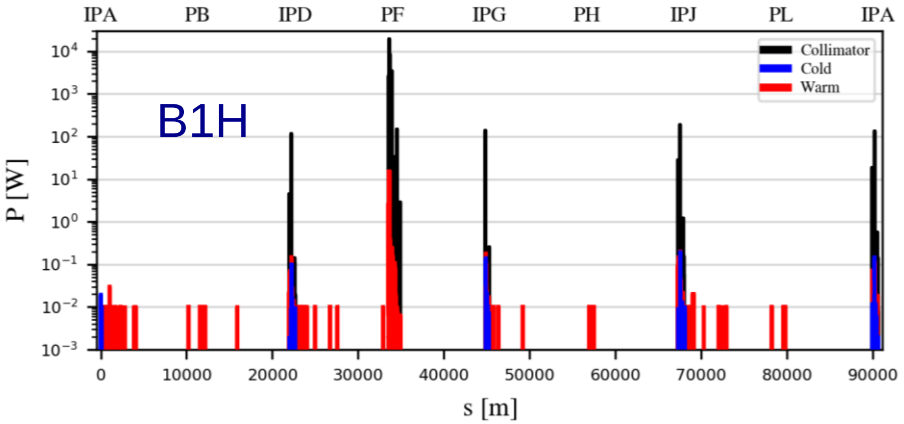
- **Outlook and next steps**

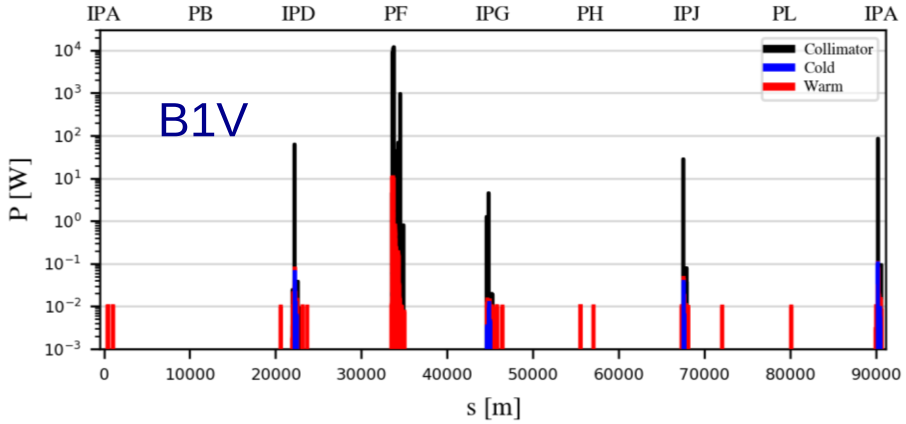
Generic halo losses for the Z mode

- **Generic halo losses**:
  - Slow loss process assumed
  - The loss process is not simulated
  - Beam halo directly impacts one of the TCPs
  - Beam lifetime drop to 5 min is assumed

- Horizontal and vertical betatron collimation losses (B1H, B1V)

- **Good halo cleaning performance overall**
  - Losses well contained within the collimation insertion PF
  - Loss suppressed by:
    - ~2 orders of magnitude on the TCTs
    - ~4-5 orders of magnitude on the SR collimators
    - >5 orders of magnitude on all the other elements outside PF

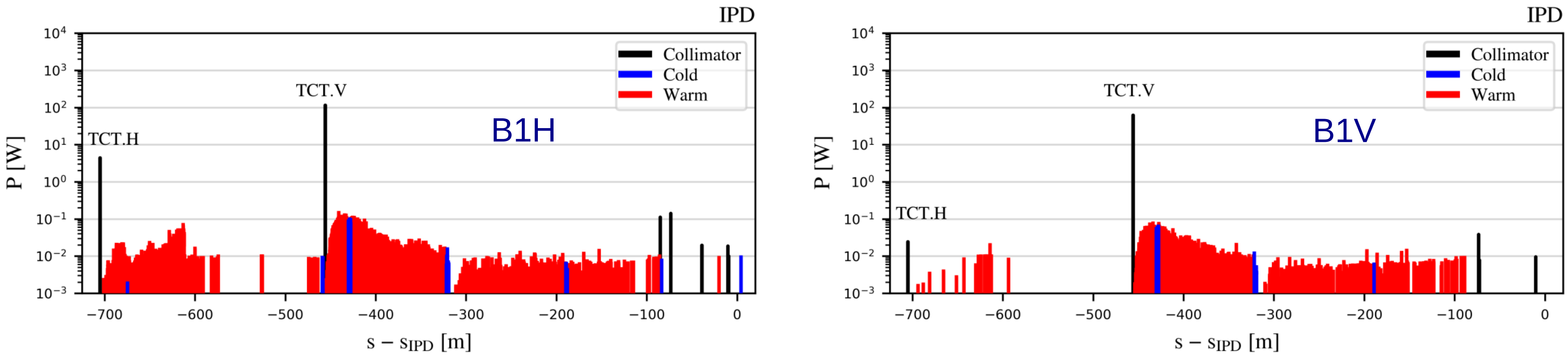




*Loss maps are obtained by binning aperture losses in 10 cm bins, except for the collimators, for which the binning is equal to the collimator length

IR generic halo losses for the Z mode

- **IR losses from generic beam halo losses**: focus on IPD (farther from collimation insertion in PF)

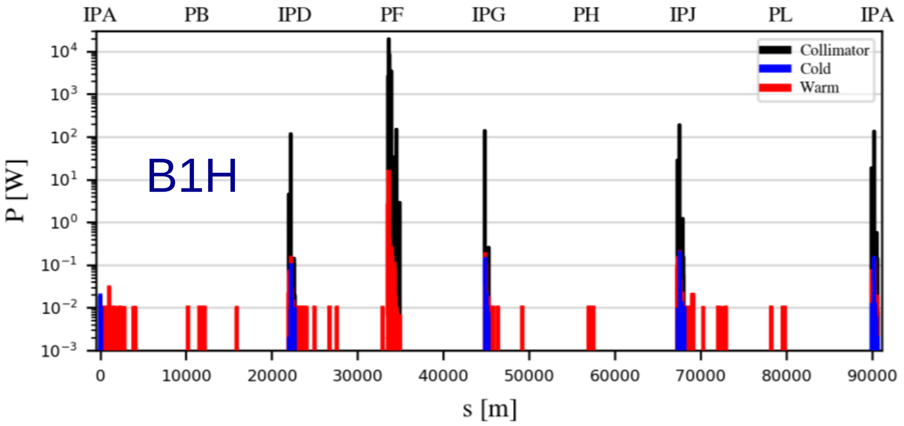


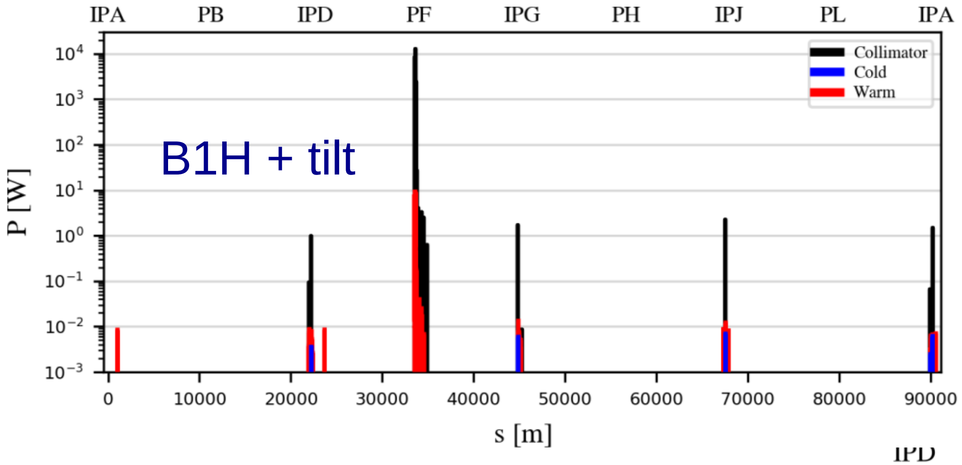
- **Vast majority of losses intecepted by the local protection tertiary collimators** (**TCTs**)
  - Max. recorded power loads ~100 W
  - Residual losses on SR collimators with max. recorded power loads ~100 mW
  - Near absence of losses beyond the last SR collimator

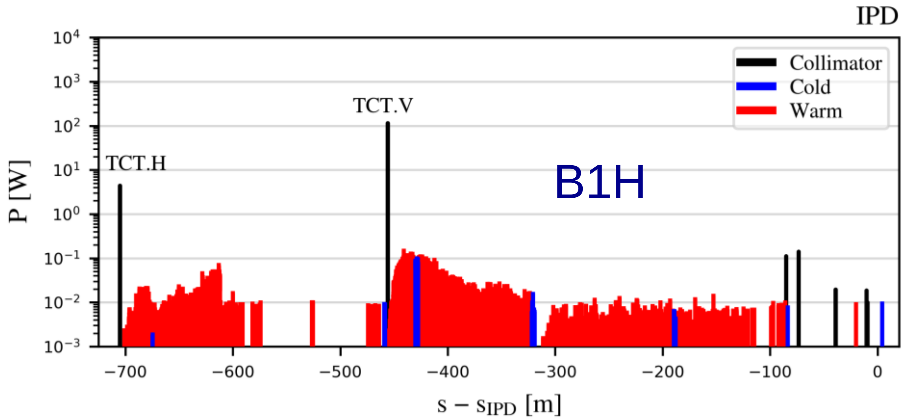
- Showers from TCTs may determine radiation to tunnel/environment and may generate backgrounds
  - Shower absorbers downstream of the TCTs will be studied by the FLUKA team (similarly to PF)
  - Impact on background to be studied

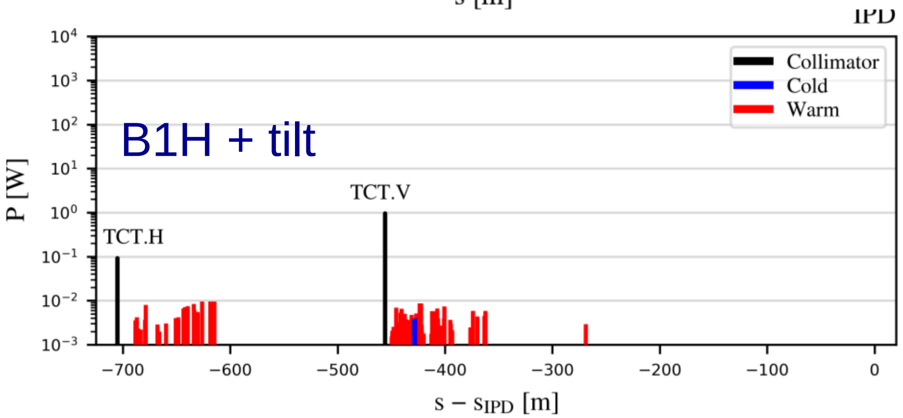
Collimator angular alignment

- As found in previous studies, collimator angular alignment can significantly improve the collimation performance







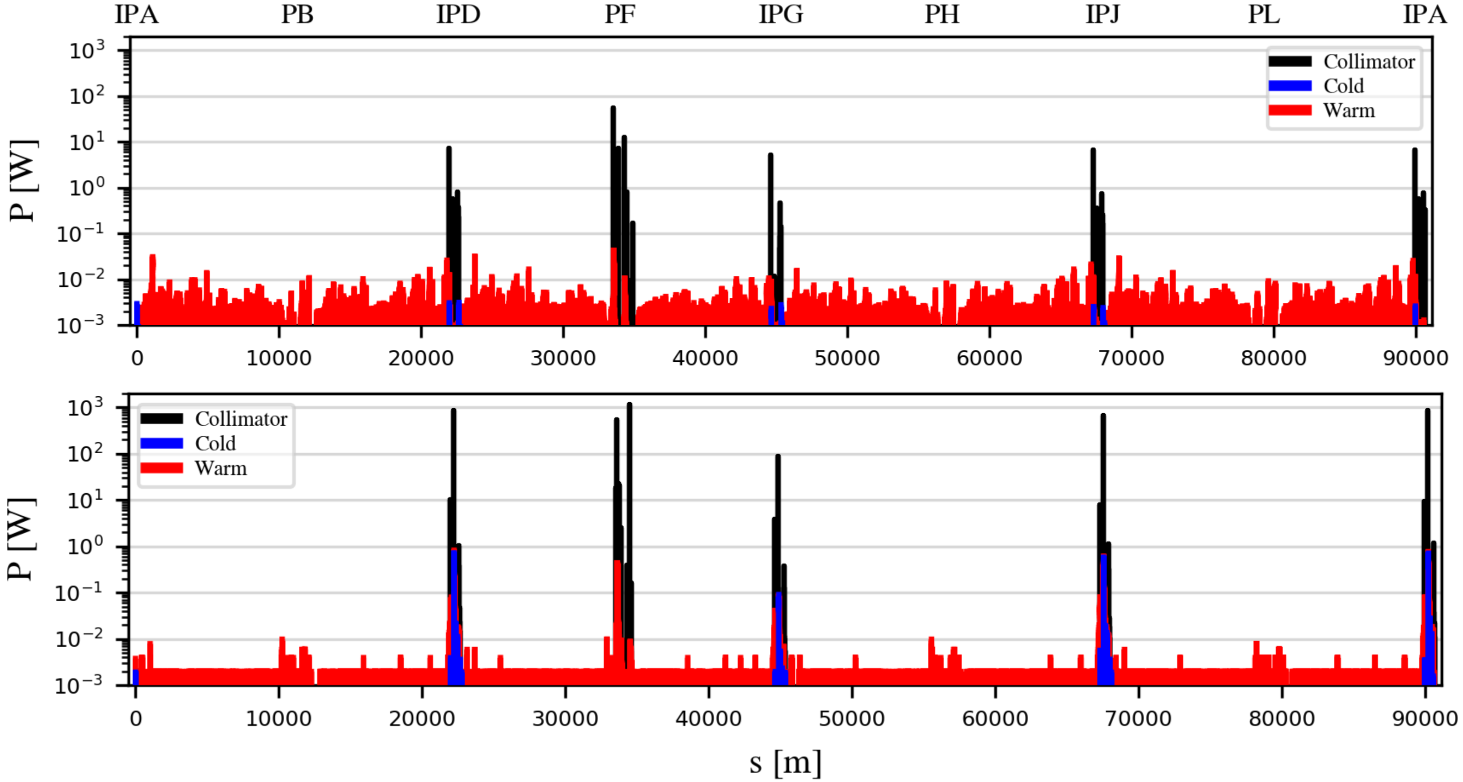


- **Important input for the hardware design** (A. Perillo-Marcone, FCC week 2025 talk)
  - ➢ Collaboration with the engineering team
  - ➢ Tolerances on the collimator angular alignment are being evaluated

Beam-gas losses for the *Z* mode

*1h beam conditioning at full nominal current (1.27 A): pressure is expected to condition down further (up to a factor ~100) over time

- A scattering routine to simulate beam-gas interactions while tracking in Xsuite has been developed (IPAC'25 paper)
  - Based on realistic pressure and gas composition profile provided as input



Beam-gas bremsstrahlung

- Estimated lifetime*: 274 min
- Expected to increase to > 100 h in a fully conditioned machine

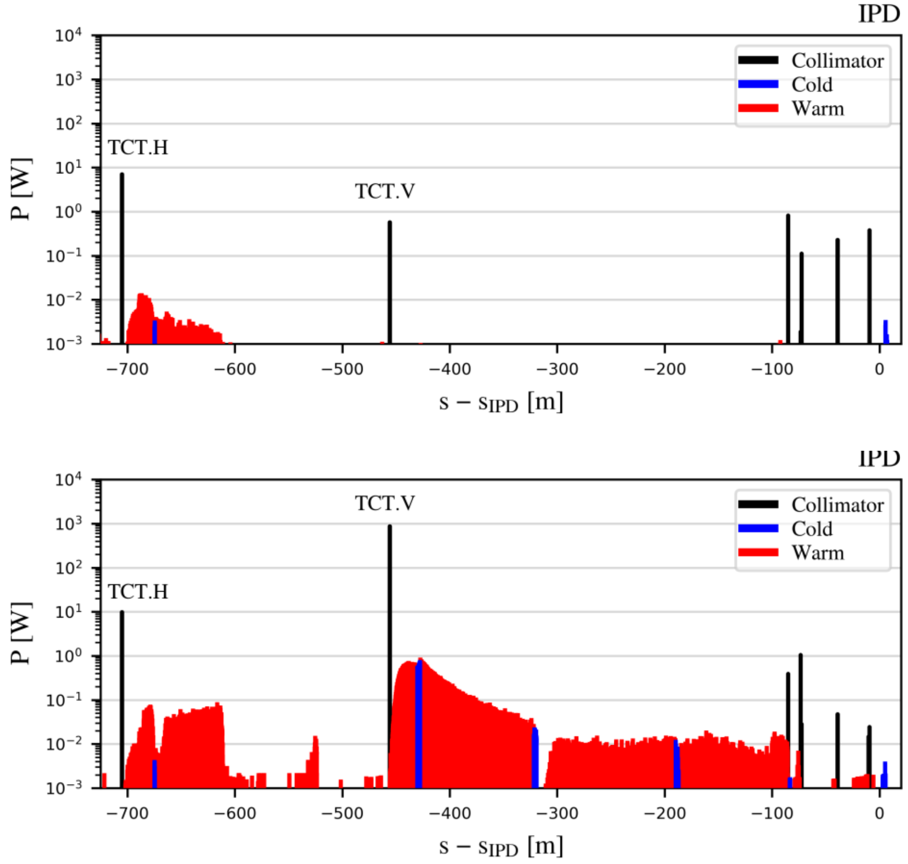
Beam-gas Coulomb scattering

- Estimated lifetime*: 41 min
- Expected to increase to > 10 h in a fully conditioned machine

- At the FCC-ee beam energies, bremsstrahlung is expected to dominate beam-gas-induced lifetime degradation
  - BUT, under particular machine conditions, such as limited DA, even small angular deflections from a single Coulomb scattering event can be sufficient to drive particles beyond the DA limit

IR beam-gas losses for the *Z* mode

*1h beam conditioning at full nominal current (1.27 A): pressure is expected to condition down further (up to a factor ~100) over time



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Beam-gas Coulomb scattering

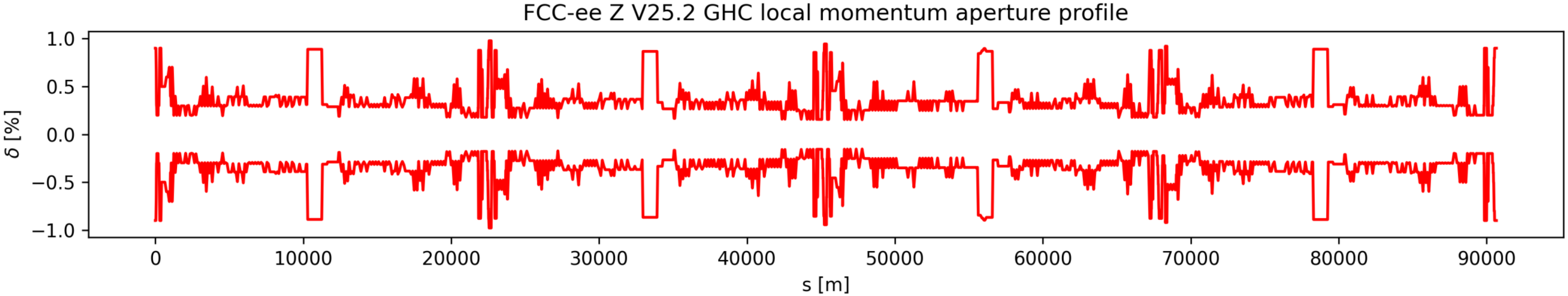
- Estimated lifetime*: 41 min
- Expected to increase to > 10 h in a fully conditioned machine

- **Vast majority of losses intecepted by the TCTs**
  - Max. recorded power loads ~kW (Coulomb)
  - Residual losses on SR collimators with max. recorded power loads ~1 W
  - Near absence of losses beyond the last SR collimator

- Showers from TCTs may cause sizeable radiation to tunnel/environment and may generate backgrounds
  - Shower absorbers downstream of the TCTs will be studied by the FLUKA team (similarly to PF)
  - Impact on backgrounds is being evaluated ([A. Frasca, TID and Fluence in the detector and IR]())

Touschek scattering losses for the Z mode

- **A Monte Carlo routine to simulate Touschek scattering has recently developed**
  - The routine follows the approach by A. Xiao and M. Borland (PRSTAB paper) implemented in ELEGANT
  - Monte Carlo method + Piwinski formula
  - Local momentum aperture profile provided as input (retrieved from tracking):

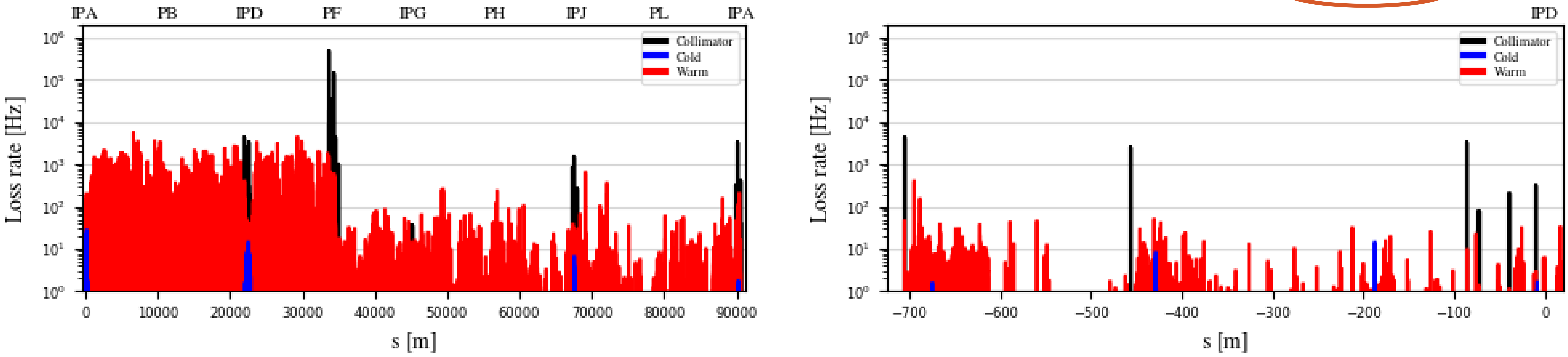


- Touschek lifetime estimates in SuperKEKB/DAFNE in agreement with expectations
- More details on the implementation in backup slides (or G. Broggi, Xsuite-based simulations of Touschek effect in DAFNE, FCC-ee MDI meeting #65)
- **First application of the routine to the FCC-ee** (*Z* mode)
  - The Z mode is the one at which Touschek scattering is most important (lowest energy, highest intensity)

Touschek scattering losses for the Z mode

- **First tracking-estimated Touschek lifetime in the FCC-ee (*Z*): 2069 min** (~35 h)
  - Consistent with previous MAD-X/SAD analytical estimates ([M. Boscolo, FCC week 2019](#))
  - Touschek scattering is not a lifetime-limiting effect in the FCC-ee
    - Lifetime from radiative Bhabha scattering: 22 min
    - Lattice lifetime (q + BS + lattice): 83 min
    - Beam-gas lifetime: 36 min (1h conditioning), >500 min (conditioned machine)

PRELIMINARY
detailed checks ongoing



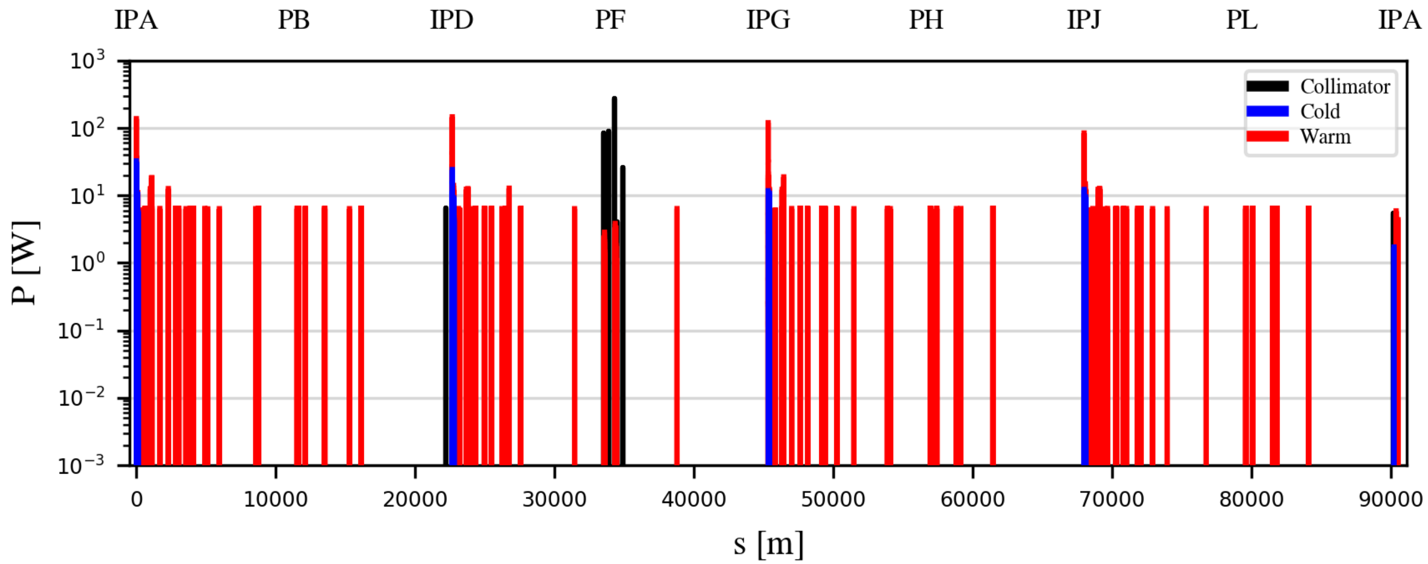
- Loss maps show only Touschek primary losses

- **N.B.** These are the very first results of this type – detailed checks are still ongoing

Beam-beam losses for the Z mode

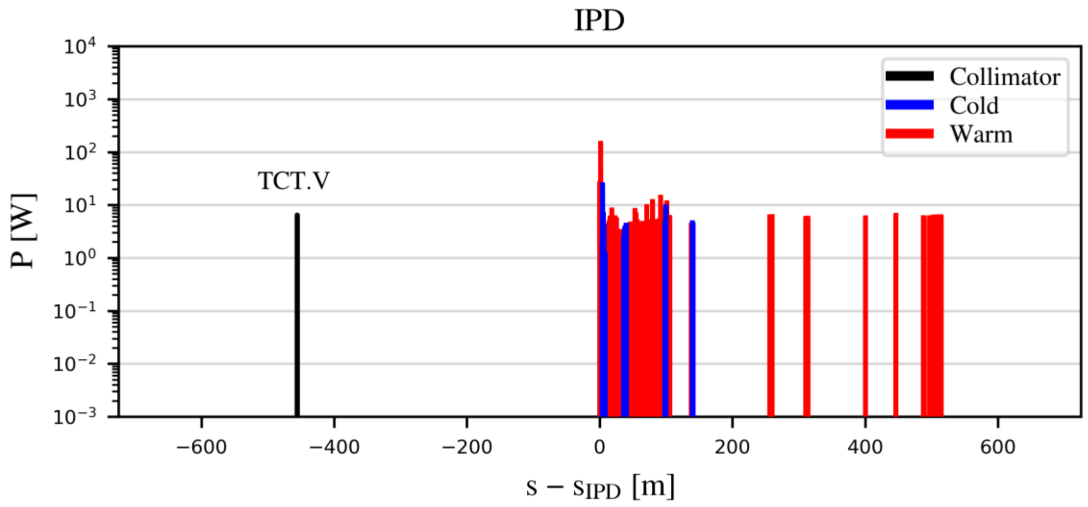
*quantum + lattice + BS + lum.

- Xsuite allows to set-up complex combined-effects simulations: beam-beam + collimation
  - Beam-beam kicks, radiative Bhabha, beamstrahlung in 4 IPs + detailed aperture and collimator model
    - Weak-strong beam-beam model
  - Common technical insertion optics shows significant advantages
    - Absence of strong-vertical emittance blow-up previously observed (BB'24 talk)
    - Beam lifetime* in agreement with expectations: ~14 min vs. ~17 min without aperture and collimators



- Beam-beam losses intercepted by the collimation system in PF
- Beam-beam losses outside PF mostly on elements downstream of the IPs

IR beam-beam losses for the Z mode



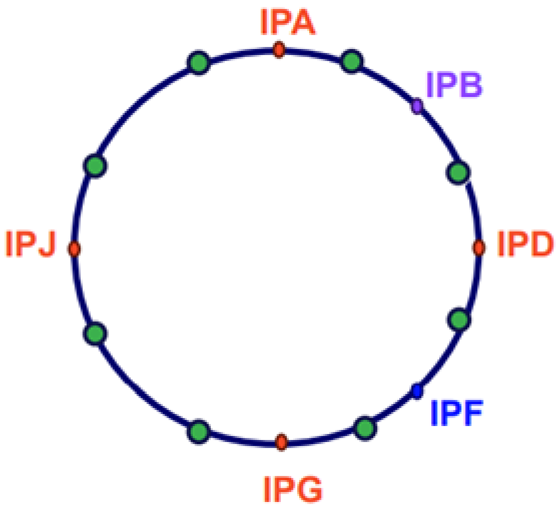
- Beam-beam losses outside PF mostly on elements downstream of the IPs
  - Local losses that cannot be intercepted in PF on a second turn
  - Physics-debris-like collimators downstream of the IPs ?

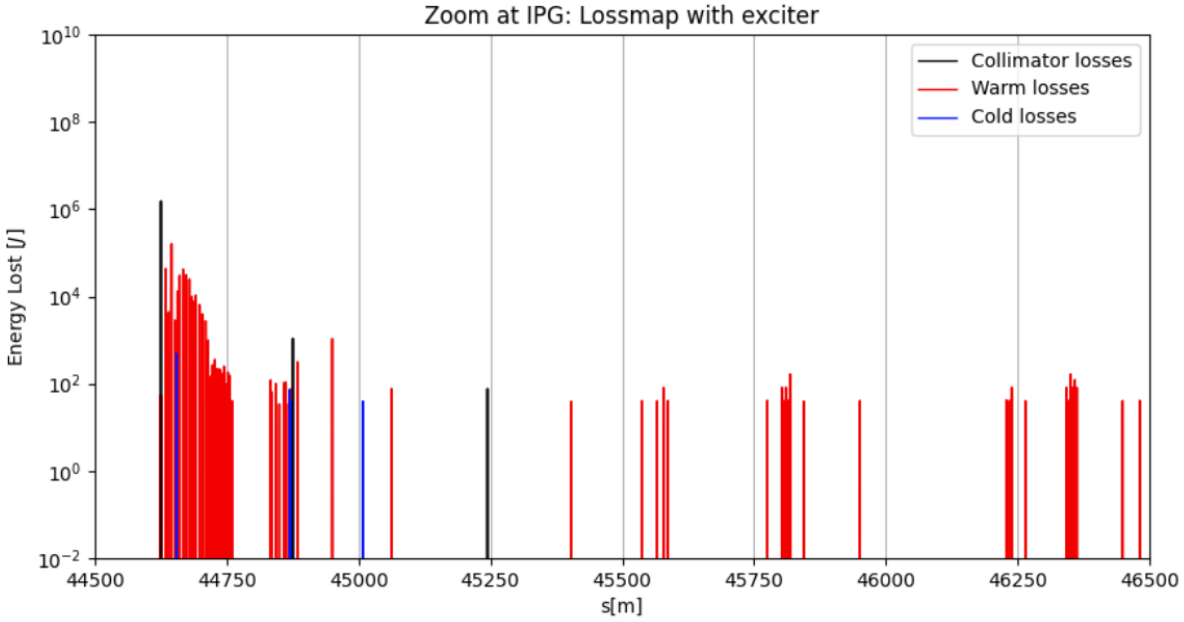
- Physics debris collimators might also suppress losses upstream of the IPs
  - These are losses from the beam halo being populated by beam-beam interactions at previous passages through the Ips

- It is planed to use beam distributions from multi-turn tracking as input to dedicated beam-beam simulations for evaluation of impact on backgorunds

Fast instability losses for the Z mode

- **Studies on fast instability losses have been performed** (G. Nigrelli – FCC physics workshop 2025 talk)
  - Fast instability modeled by synchronized kicks placed along the ring with raising strength
    - Mimic instabilities with different rise times
    - Studied beam loss distributions around the ring and across multiple turns
- **The fast instability develops if the feedback system fails** – feedback system redundancy ?
  - Most likely unacceptable for machine operation
  - Full beam potentially lost within few turns
  - Almost 50% of beam energy lost in one turn, losses of order of MJ in the collimator can be expected





- Potential impact on the detectors is being evaluated

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- **Outlook and next steps**

Outlook and next steps

- **Studies of beam losses and collimation for the FCC-ee**
  - Workflow to evaluate beam-induced backgrounds set up in collaboration with FLUKA and the MDI team
  - First collimation system design available, including beam halo, SR collimators and shower absorbers
  - Simulations of several beam loss scenarios ongoing:
    - Beam halo losses, beam-gas losses, Touschek scattering losses (first preliminary results), beam-beam losses, top-up injection losses, losses from fast instabilities for the **most critical Z mode**
    - **Good collimation performance – residual IR losses safely disposed by MDI collimators**
  - First validation of Xsuite-BDSIM simulation results against measured data from an e+e- collider (SuperKEKB)

- **Next steps**
  - Converge on a complete beam-induced background evaluation campaign
    - Detector tolerances need to be evaluated
  - Study other beam loss scenarios – e.g., failure scenarios
  - Extend the studies to the electron beam (B2)
  - Extend the studies to all beam modes

Thank you!

Backup

Beam-gas losses for the Z mode

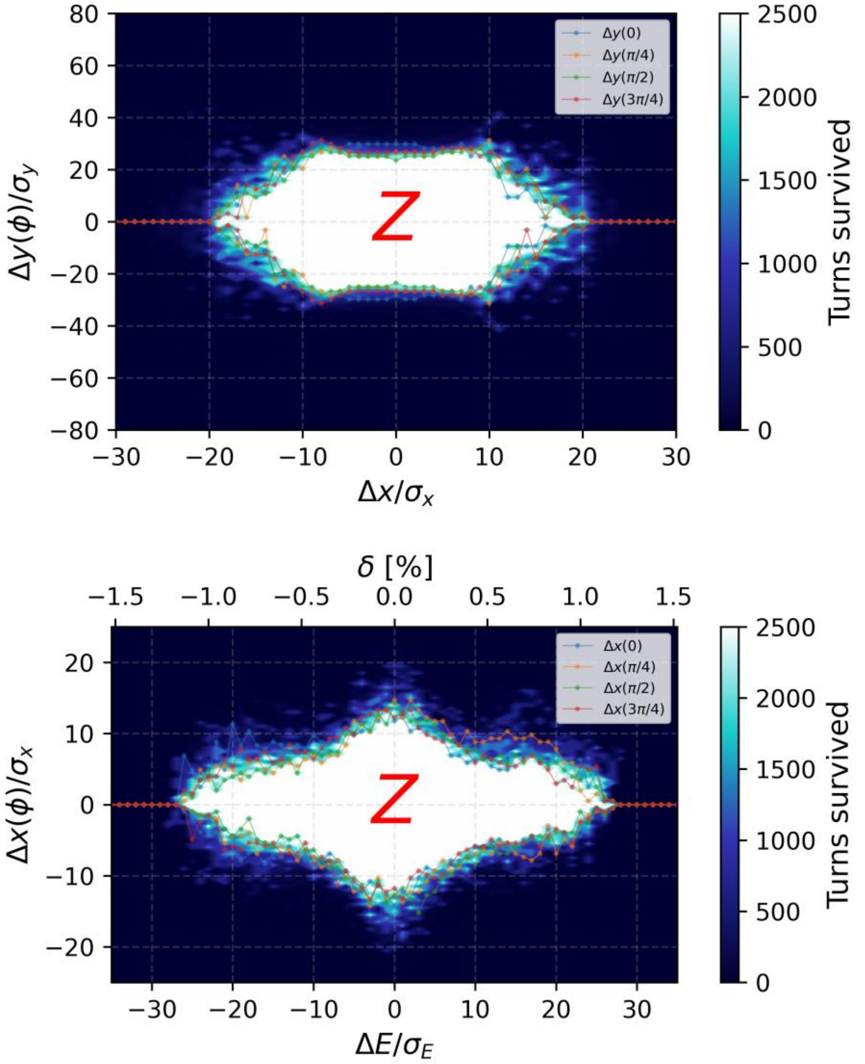
- **Vertical DA in the FCC-ee Z-mode lattice is limited to ~30 σ**
  - Beam is highly sensitive to vertical angular kicks
  - Typical vertical divergence (arcs): $\sigma_{p\gamma} \approx 0.3$ μrad
  - A 10 μrad vertical kick → >30 $\sigma_{p\gamma}$ → particle loss

- **Sensitivity increases near IPs**
  - β_γ reaches several km → $\sigma_{p\gamma}$ further reduced
  - Even smaller kicks can cause losses

- **Horizontal sensitivity is lower**
  - $\sigma_{px} \gg \sigma_{p\gamma}$ (due to much larger ε_x and lower IR-β_x)
  - Same kick → smaller normalized deflection
  - Horizontal DA ≈ vertical DA

- **Bremsstrahlung losses more sensitive to longitudinal DA (~1%)**
  - Caused by photon emission → off-momentum particles

FCC-ee Z V25.2 GHC DA and MA



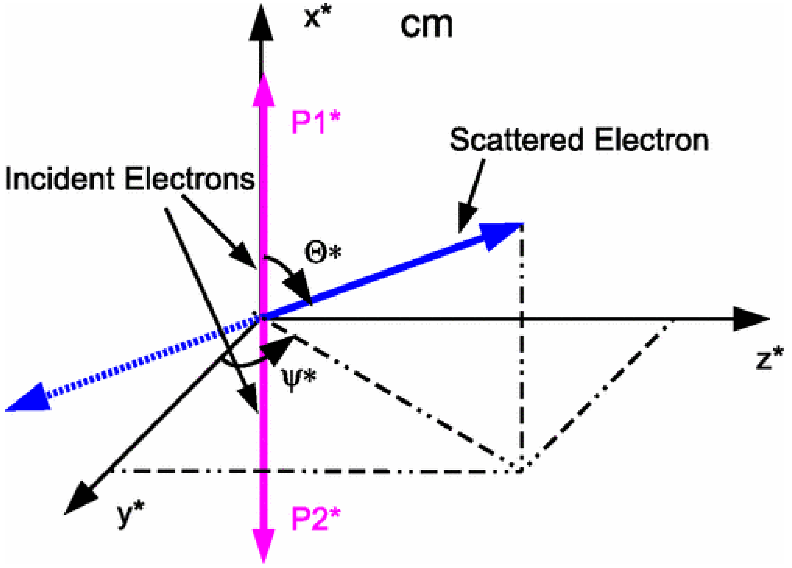
Touschek scattering

- **Touschek scattering**: Coulomb collision between relativistic electrons (or positrons) within a particle beam
  - Causes sudden energy change in scattered particles

- Touschek scattering is a **relativistic effect**
  - Momentum transfer from transverse to longitudinal plane
  - In lab. frame, transfer is amplified by Lorentz factor γ

- Touschek scattering causes **beam losses** if:
  - Energy deviation exceeds RF acceptance
  - Off-energy orbit/oscillation exceeds the dynamic/physical aperture

- Touschek scattering may lead to:
  - Beam lifetime reduction
  - Beam backgrounds



Touschek scattering: theory snapshot

- In center-of-mass (c.m.) system:
  - Scattering probability into solid angle dΩ⁎ given by Møller differential cross section:

$$\boxed{\frac{d\sigma^*}{d\Omega^*}(\Theta^*, \Psi^*) = \frac{r_e^2}{4\gamma^{*2}}\left[\left(1+\frac{1}{\beta^{*2}}\right)^2 \frac{4-3\sin^2\Theta^*}{\sin^4\Theta^*} + \frac{4}{\sin^2\Theta^*} + 1\right]}$$

$$d\Omega^* = \sin\Theta^*\, d\Theta^*\, d\Psi^*$$

re: classical electron radius
Θ*: scattering angle

γ*: Lorentz gamma factor
β*: Lorentz beta factor

NOTE: starred quantities are in the c.m. system

- **Total Touschek scattering rate R** - obtained by integrating:
  - Møller cross section over all possible scattering angles
  - Phase space distribution of all particles in the bunch

$$\boxed{R = 2\int |v|\sigma\rho(\vec{x}_1)\rho(\vec{x}_2)dV,}$$

$$dV = dx_\beta dy_\beta d\Delta z dx'_{\beta 1} dx'_{\beta 2} dy'_{\beta 1} dy'_{\beta 2} d\Delta p_1 d\Delta p_2.$$

v: particle velocity after scattering
σ: total Møller cross section

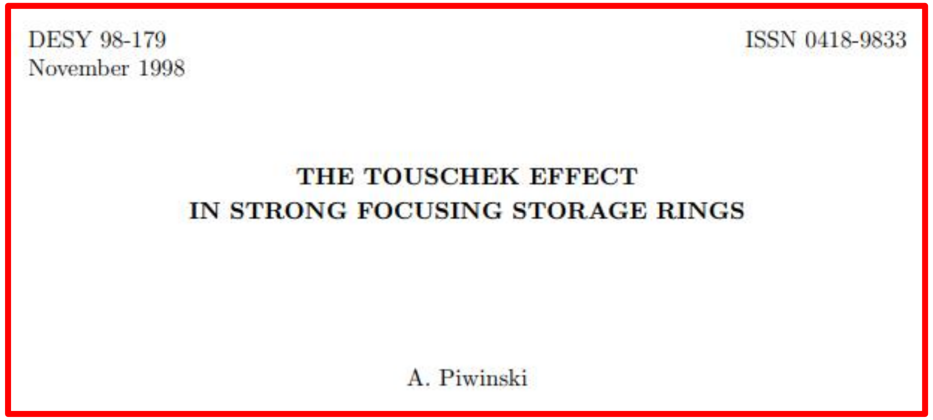
ρ: phase-space density

Touschek scattering: theory snapshot

- **Total Touschek scattering rate R**:

$$R = 2\int |v|\sigma\rho(\vec{x}_1)\rho(\vec{x}_2)dV, \qquad dV = dx_\beta dy_\beta d\Delta z dx'_{\beta 1} dx'_{\beta 2} dy'_{\beta 1} dy'_{\beta 2} d\Delta p_1 d\Delta p_2.$$

- **Piwinski** made a detailed evaluation of the above equation for *R* for a Gaussian-distributed bunch:



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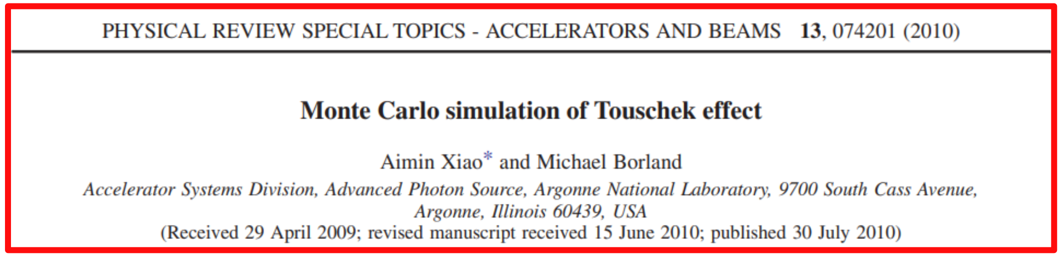
THE TOUSCHEK EFFECT
IN STRONG FOCUSING STORAGE RINGS

A. Piwinski

$$R_P = \frac{r_e^2 c\beta_x\beta_z\sigma_h N_p^2}{8\sqrt{\pi}\beta^2\gamma^4\sigma_x^2\beta_x^2\sigma_z\sigma_s\sigma_p}\int_{\tau_m}^{\infty}\left[\left(2+\frac{1}{\tau}\right)^2\left(\frac{\tau/\tau_m}{1+\tau}-1\right)+1-\frac{\sqrt{1+\tau}}{\sqrt{\tau/\tau_m}}-\frac{1}{2\tau}\left(4+\frac{1}{\tau}\right)\ln\left(\frac{\tau/\tau_m}{1+\tau}\right)e^{-B_1\tau}I_0(B_2\tau)\right]\frac{\sqrt{\tau}\,d\tau}{\sqrt{1+\tau}}$$

Piwinski Touschek scattering rate

Touschek scattering: Monte Carlo simulation



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Monte Carlo simulation of Touschek effect

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- The integral in Piwinski formula can be computed via Monte Carlo integration with N uniformly distributed random points in the n-dimensional volume V (if N is large enough):

$$\int_V f(\vec{x})d\vec{x} \approx \frac{V}{N}\sum_{i=1}^{N} f(\vec{x}_i).$$

- The average scattering rate for particles that results in momentum deviation δ greater than a min. value δm is given by:

$$\boxed{R_{\mathrm{MC}}(|\delta| > \delta_m) = \frac{V}{N}\sum_{i=1}^{M}\left[\frac{v^*}{\gamma^2}\frac{d\sigma^*}{d\Omega^*}\sin\Theta^*\,\rho(\vec{x}_1)\rho(\vec{x}_2)\bigg|_i\right] = \sum_{i=1}^{M} r_i}$$

Monte Carlo
Touschek scattering rate

v*': particle velocity
γ: Lorentz gamma factor

dσ*'/dΩ*': Møller DCS
ρ: phase-space density

N: number of simulated scattering events
M: total number of scattered particles with δ>δm

NOTE: starred quantities are in the c.m. system
NOTE: primed quantities mean "after scattering"
NOTE: when δm = δa, with δa being the local momentum acceptance, the above equation gives the Touschek beam loss rate

Touschek scattering: Monte Carlo simulation

- To use Monte Carlo integration, one needs to generate a series of random scattering events
  - Each event involves a pair of scattering particles

- A gaussian-distributed bunch is considered
  - Nine uniformly distributed random numbers are generated in the normalized phase-space range

$$[-r_x\sqrt{\varepsilon_x},\ r_x\sqrt{\varepsilon_x}] \qquad [-r_y\sqrt{\varepsilon_y},\ r_y\sqrt{\varepsilon_y}] \qquad [-r_z\sqrt{\varepsilon_z},\ r_z\sqrt{\varepsilon_z}]$$

rx, ry, rz: user-defined parameters (default is 3)   ex, ey, ez: geometric emittances

- Let us call these 9 random numbers n0, n1, …, n8. These are assigned as:

particle 1

X1 = n0
X1' = n1
Y1 = n2
Y1' = n3
z1 = n4
δ1 = n5

particle 2

X2' = n6
Y2' = n7
δ2 = n8

Since the two particles are colliding the spatial coordinates x, y, zeta for particle 2 are identical to those of particle 1:

x2 = x1,   y2 = y1,   z2 = z1

X, X', Y, Y': transverse normalized coordinates

z, δ: longitudinal coordinates

Touschek scattering: Monte Carlo simulation

- **Particle 1**: converting from normalized to real phase space coordinates and including dispersive effects:

$$x_1 = \sqrt{\beta_x} X_1 + \delta_1 D_x \qquad x_1' = \frac{X_1' - \alpha_x X_1}{\sqrt{\beta_x}} + \delta_1 D_x' \qquad y_1 = \sqrt{\beta_y} Y_1 + \delta_1 D_y \qquad y_1' = \frac{Y_1' - \alpha_y Y_1}{\sqrt{\beta_y}} + \delta_1 D_y'$$

- **Particle 2**:

$$X_2 = (x_1 - \delta_2 D_x) / \sqrt{\beta_x} \qquad x_2' = \frac{X_2' - \alpha_x X_2}{\sqrt{\beta_x}} + \delta_2 D_x' \qquad Y_2 = (y_1 - \delta_2 D_y) / \sqrt{\beta_y} \qquad y_2' = \frac{Y_2' - \alpha_y Y_2}{\sqrt{\beta_y}} + \delta_2 D_y'$$

- Complete set of coordinates (x, y, z, x1', y1', δ1, x2', y2', δ2) for both particles before scattering defined

- **Phase-space density** (Gaussian-distributed bunch):

$$\rho = \frac{N_p}{8\pi^3 \varepsilon_x \varepsilon_y \varepsilon_z} \exp\left(-\frac{z^2}{2\sigma_z^2} - \frac{\delta^2}{2\sigma_\delta^2}\right) \exp\left(-\frac{x_\beta^2 + (\alpha_x x_\beta + \beta_x x_\beta')^2}{2\sigma_{\beta,x}^2} - \frac{y_\beta^2 + (\alpha_y y_\beta + \beta_y y_\beta')^2}{2\sigma_{\beta,y}^2}\right)$$

  - Computed for each scattering particle

Touschek scattering: Monte Carlo simulation

- Random uniform sampling for Θ* ∈ (0, π/2] and Ψ* ∈ [0, 2π]
  - Møller DCS is computed

- **Phase space volume** computed as:

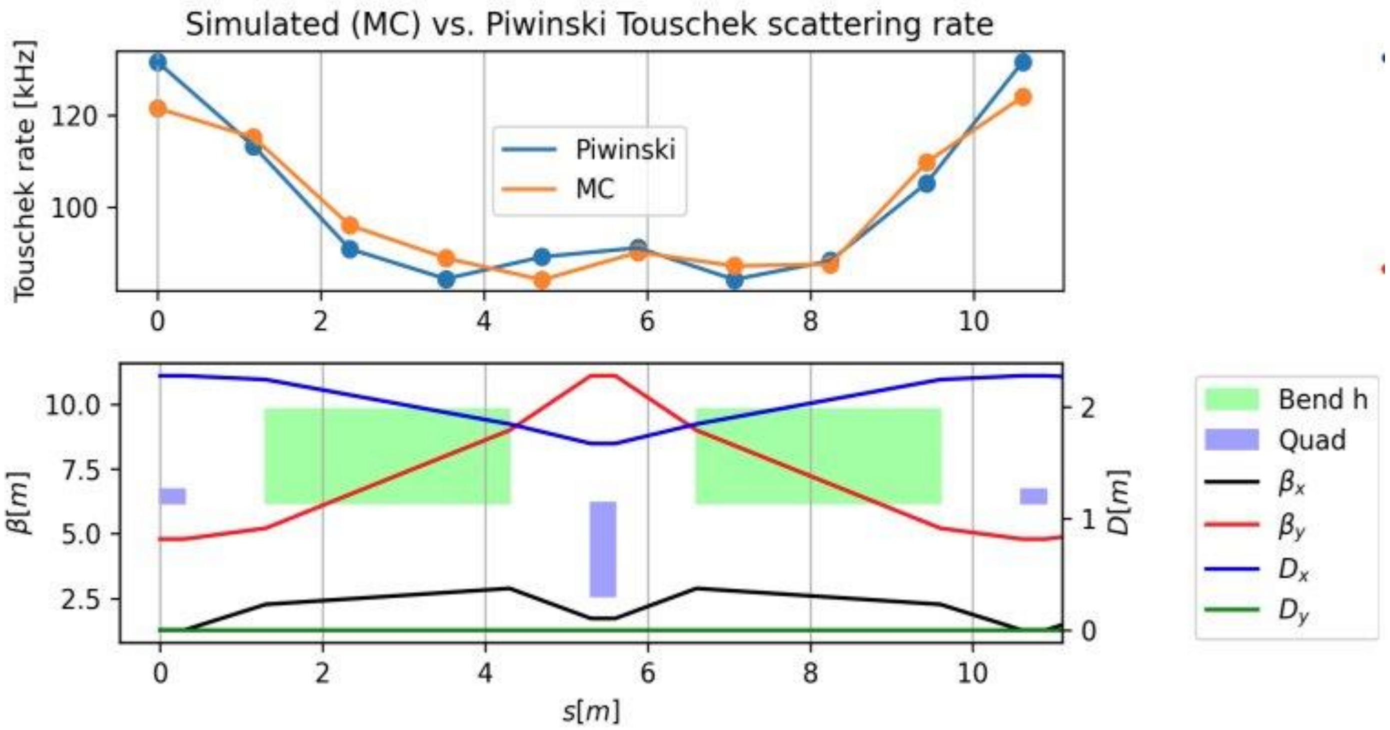
$$V = \frac{\pi^2}{\sqrt{\beta_x \beta_y \beta_z}} \cdot \left(2r_x\sqrt{\varepsilon_x}\right)^3 \left(2r_y\sqrt{\varepsilon_y}\right)^3 \left(2r_z\sqrt{\varepsilon_z}\right)^3$$

- Scattering is applied
  - Transform particle momenta from lab frame to c.m. frame
  - Rotate momenta according to Θ* and Ψ* (in the c.m. frame)
  - Transform back to the lab frame

- All ingredients to compute the Touschek scattering rate (via Monte Carlo integration) available:

$$R_{\mathrm{MC}}(|\delta| > \delta_m) = \frac{V}{N} \sum_{i=1}^{M} \left[\frac{v^*}{\gamma^2} \frac{d\sigma^*}{d\Omega^*} \sin\Theta^* \, \rho(\vec{x}_1)\rho(\vec{x}_2) \Big|_i \right] = \sum_{i=1}^{M} r_i$$

Monte Carlo simulation vs. Piwinski formula



- Excellent agreement between simulated (MC) and Piwinski Touschek scattering rate for a Gaussian-distributed beam

- Level of agreement depends on number of simulated scattering events
  - For the comparison presented here, 1E+7 scattering events are simulated at each scattering center
  - Such value is currently being considered as a standard for these simulations

Xsuite-based Touschek scattering simulations

- Scattering rate simulation validated vs. Piwinski formula

- Using Xsuite particle tracking it is possible to simulate Touschek scattering beam losses
  - To each simulated scattered particle is assigned a total scattering rate *Ri*

$$\boxed{R_i = \frac{r_i}{\sum r_i} \int R_P} \qquad \int R_P = \frac{1}{C} \int R_P(s)\, ds \approx \frac{1}{C} \sum_j R_{P,j}\, \Delta s_j$$

  - The integral of *RP* is taken over each section upstream of each Touschek scattering center, starting from the previous scattering center: this way, the simulated scattered particles accurately represent the scattering rate from the entire upstream section

- The beam-loss rate and location can be calculated by tracking the Touschek-scattered particles and recording all lost particles

- The total loss rate *Rtot* will be the sum of Ri for all the particles lost at any location: $\boxed{R_{\mathrm{tot}} = \sum_i R_i}$
  - The Touschek lifetime can be computed as: $\boxed{\tau = \frac{N_p}{R_{\mathrm{tot}}}}$

Xsuite-based Touschek scattering simulations

- Starting from an Xtrack line, the following procedure is used:

1
An arbitrary number of Touschek scattering centers (Xtrack markers called TMarker) is inserted in the line

2
If not pre-computed, local momentum aperture is estimated by tracking test particles and recording the max. stable δ

3
Piwinski formula to assign to each TMarker the integrated scattering rate from the upstream section

4
Generate pre-scattering particle coordinates at TMarkers, compute phase-space density and phase-space volume

5
SCATTER

6
Compute Monte Carlo scattering rate *RMC* and assign total scattering rate *Ri* to each particle

7
TRACK*

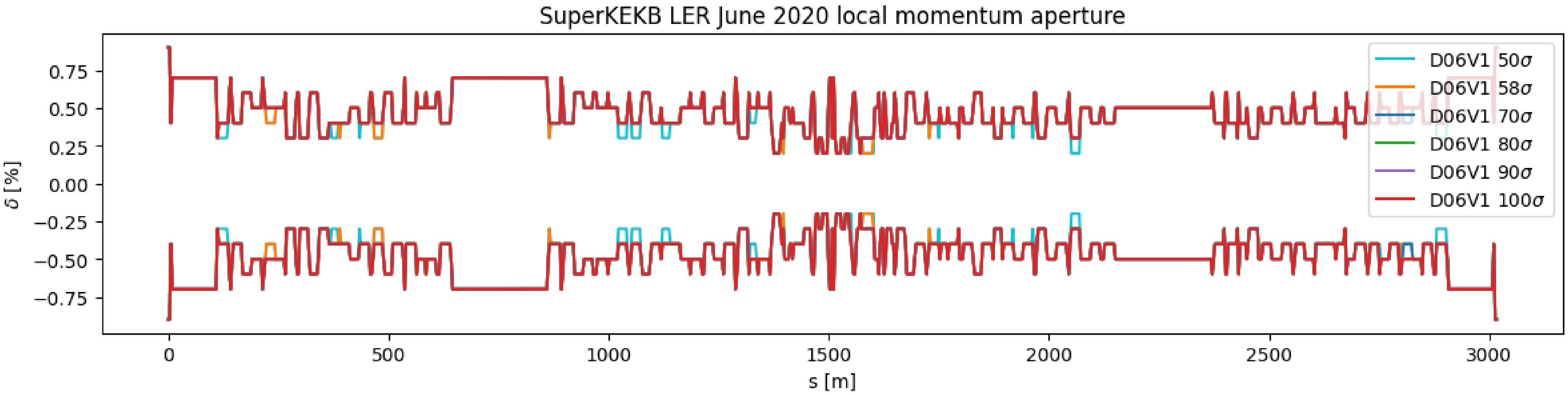
8
Record lost particles

9
Compute Touschek loss rate and Touschek lifetime

***To speed-up simulations, only highest-weight particles that contribute to the 99% of the scattering rate are tracked**

Local momentum aperture

- Tracked for 1000 turns from 1000 distinct s-locations particles starting on the on-momentum closed orbit with different momentum offsets
  - Recorded the maximum stable momentum offset



- Local momentum aperture serves as input for Touschek scattering simulation
  - Required for computing Piwinski and Monte Carlo Touschek scattering rates
- A 0.9 safety factor is applied to the local momentum aperture values to ensure all potentially lost Touschek-scattered particles are tracked—untracked losses would otherwise not contribute to the total Touschek loss rate (lifetime).