



FUTURE
CIRCULAR
COLLIDER

FCC-ee RF operation scenarios - new baseline

Ivan Karpov and Franck Peauger for FCC SRF WP1 with input from:
Xavier Buffat, Yann Dutheil, Giorgia Favia, Jiquan Guo (JLAB), Mauro Migliorati (INFN),
Katsunobu Oide, Luca Sabato, Roxana Soos, Jorg Wenninger, Frank Zimmermann, Mikhail Zobov (INFN)

Challenging requirements for RF system

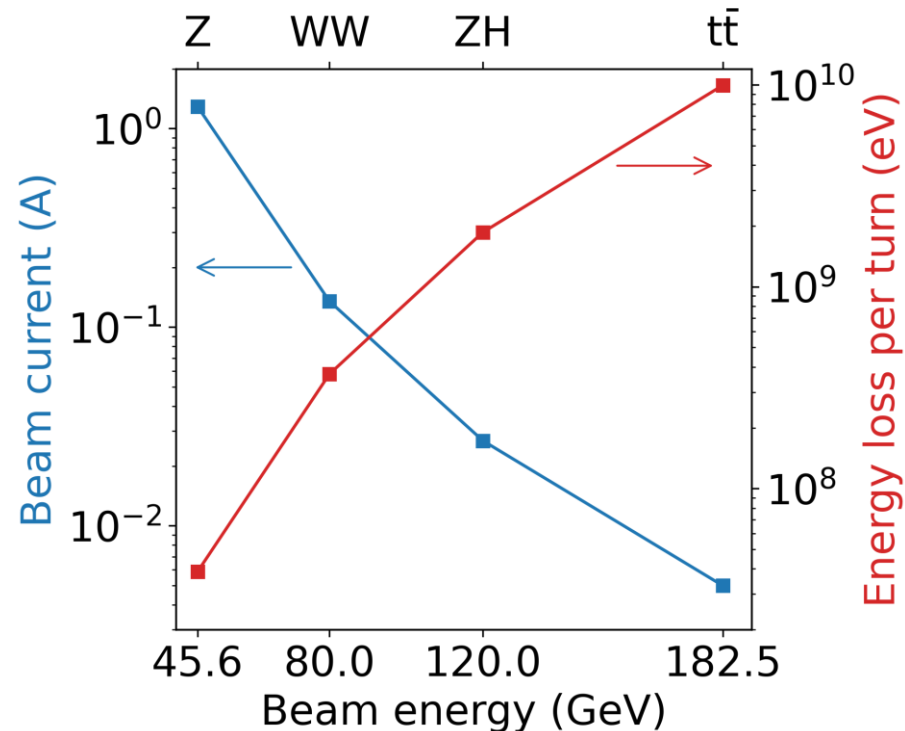
Beam current and energy loss for
50 MW synchrotron radiation (SR) budget

**Low energy operating points:
High beam current and weak SR**

- Low RF voltage
- Beam instabilities
- Significant power losses due to high-order modes (HOM)



A few single-cell cavities at
low RF frequency (400 MHz)



**High energy points:
Low beam current and strong SR**

- High RF voltage to be reached with a small footprint
- Instabilities suppressed by SR
- Small power losses due to low beam current




Many multi-cell cavities at higher
RF frequency (800 MHz)

RF baseline evolution for collider

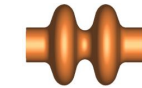
FCC Conceptual design report (2018)

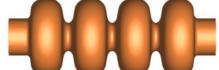
F. Peauger, FCC Week 2022 & FCC FS MTR (2024)

FCC FS Final Report (2025)

Z  x104

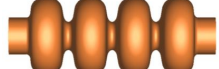
 x112

 x264

WW  x104

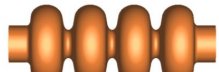
 x264

 x264

ZH  x272

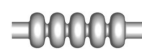
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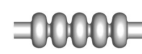
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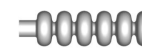
tt̄  x272

 x264

 x264

+  x372

+  x488

+  x408

Common RF for 2 beams

Common RF for 2 beams

Common RF for 2 beams

1. Operational margins
2. Less HOM power for WW with 2-cell cavities
3. Same RF for WW & ZH

1. Same RF for Z, WW & ZH
2. One power source per cavity
3. 6-cell for tt̄ for cost reduction

Reverse Phase Operation (RPO) mode for Z is required to unify Z-WW-ZH RF system

Reverse phase operation

To minimize RF power requirements, the optimal quality factor Q_L needs to be used

Since $Q_L \propto V_{\text{cav}}^2$, cavity voltage, enormous change of Q_L is required switching from Z to WW

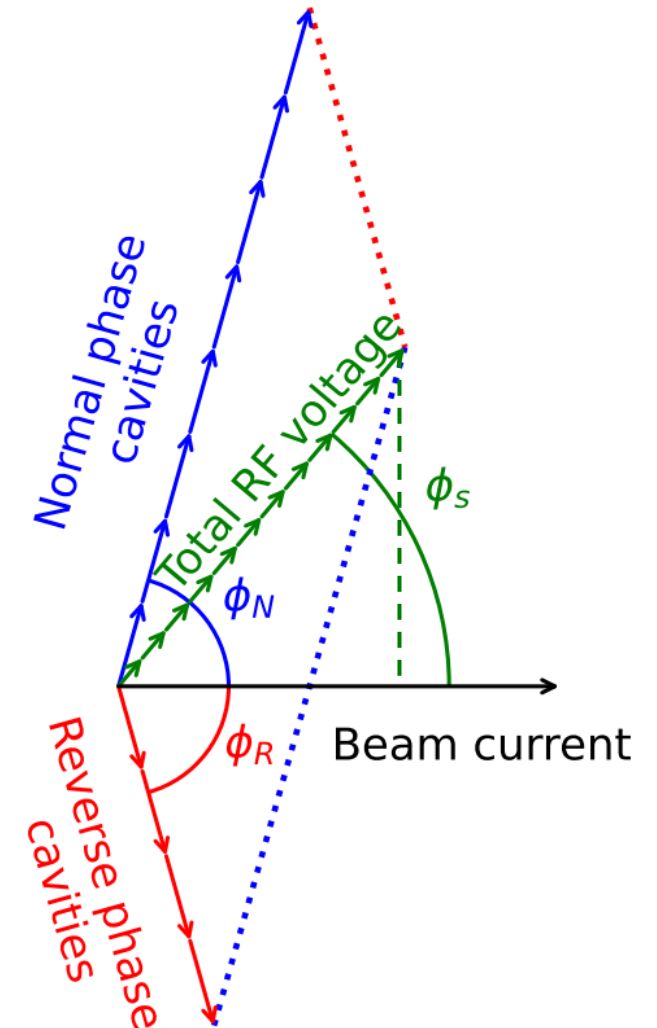
	Total voltage (GV)	Cavity voltage (MV)	Quality factor
Z	0.088	0.67	6.5×10^3
WW	1.05	7.95	9.2×10^5

Reverse phase operation (RPO) mode allows for increasing RF cavity voltage, having optimal static beam loading compensation ([Y. Morita et al., SRF, 2009](#))

- Experimentally verified with high beam loading in KEKB ([Y. Morita et al., IPAC, 2010](#))
- Baseline solution for EIC ESR ([e.g., J. Guo et al., IPAC, 2022](#))

→ Applicability of RPO for FCC-ee needs to be confirmed

A sketch of RPO mode for 12 RF cavities



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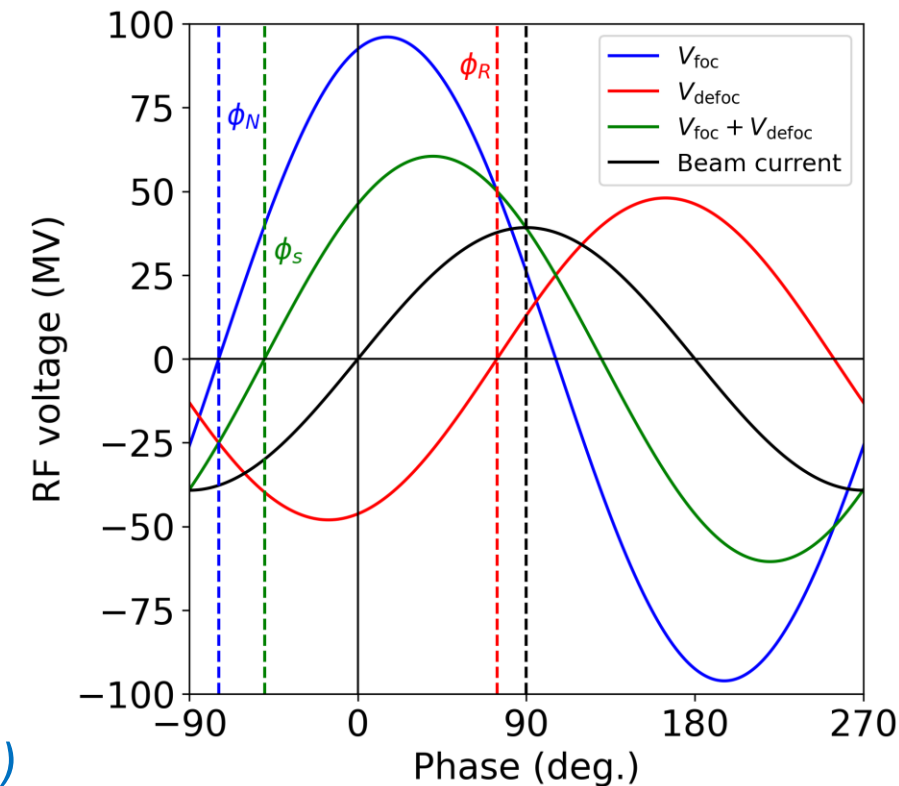
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A sketch of RPO mode for 12 RF cavities



Focusing cavities
Defocusing cavities

RPO validation checklist

- Static beam loading
- Transient beam loading
- Coupled-bunch instabilities
- Higher-order-mode power losses
- Availability aspects

Static beam loading compensation

To minimize RF power requirements optimal quality factor and optimal detuning must be used

$$Q_{L,opt} = \frac{V_{cav}^2 N_{tot}}{2P_{SR}(R/Q)} \quad \Delta\omega_{opt} = -\frac{\omega_{rf}(R/Q)|F_b|I_{b,dc}}{2V_{cav}} \sqrt{1 - \frac{U_0^2}{V_{cav}^2 N_{tot}^2}}$$

To avoid a movable fundamental power coupler design, $Q_{L,opt}$ and V_{cav} must be the same for all cavities for Z, WW, and ZH

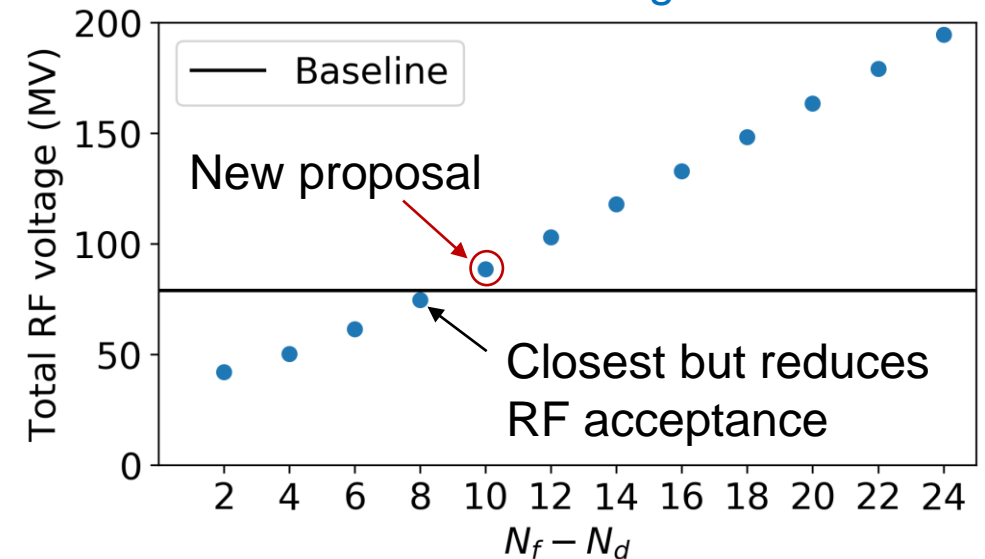
Total RF voltage (e.g., [A. Blednykh et al, 2022](#))

$$V_{tot} = \frac{U_0}{e} \sqrt{1 + \left(1 - \frac{e^2 N_{tot}^2 V_{cav}^2}{U_0^2}\right) \frac{(N_f - N_d)^2}{N_{tot}^2}}$$

can be changed in discrete steps of $N_f - N_d = 2, 4, \dots$

→ RF voltage at Z needs to be slightly increased (89 vs 79 MV)

Possible RF voltages for Z



RPO validation checklist

Static beam loading → New total RF voltage proposed

Transient beam loading

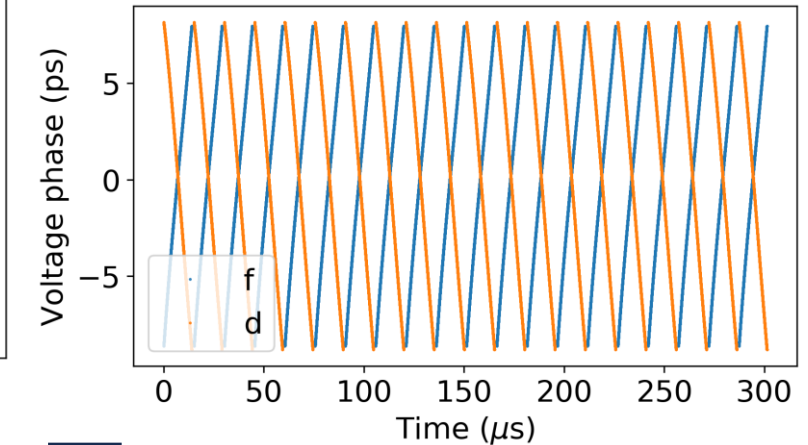
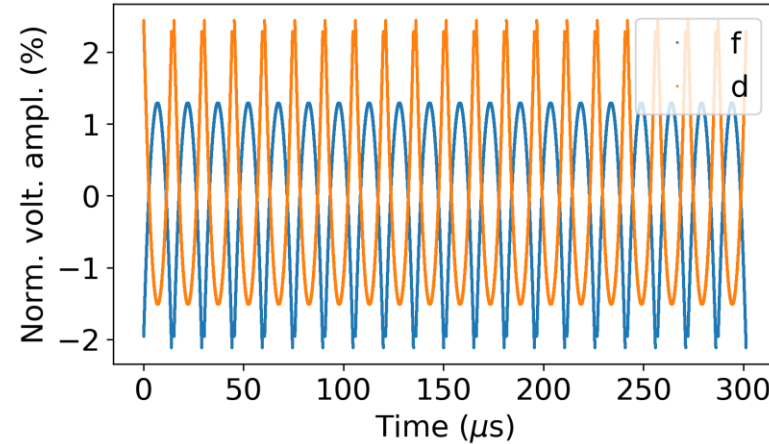
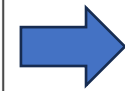
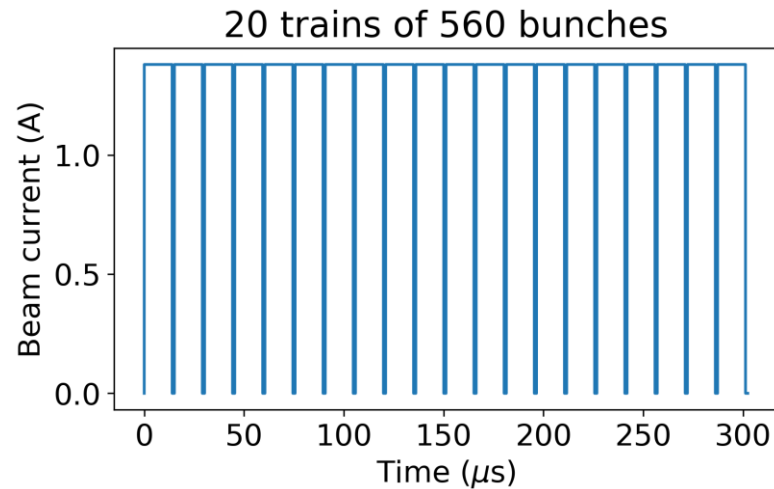
Coupled-bunch instabilities

Higher-order-mode power losses

Availability aspects

Transient beam loading

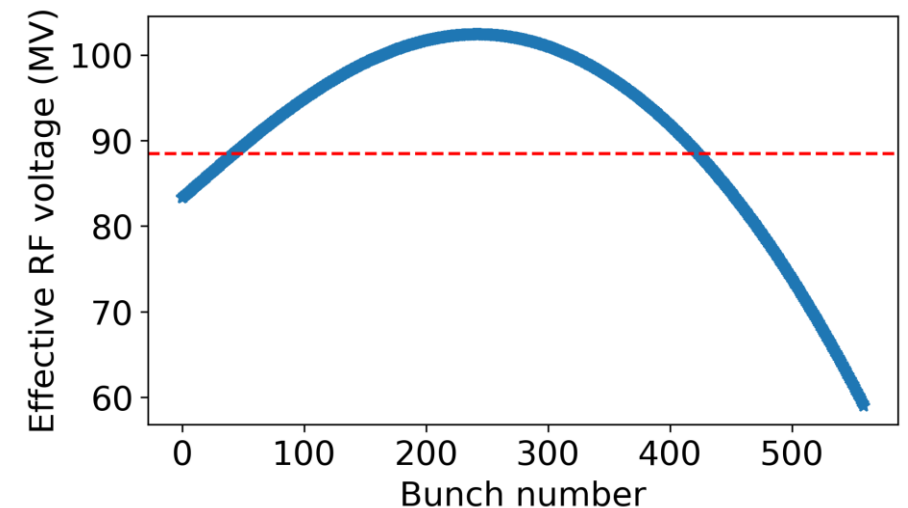
Voltage amplitude and phase transients from small-signal model
 (F. Pedersen, 1992)



N_f	N_d	V_{tot} (MV)	V_{cav} (MV)	$Q_{L,\text{opt}}$
71	61	89	7.95	$9.21\text{e}5$

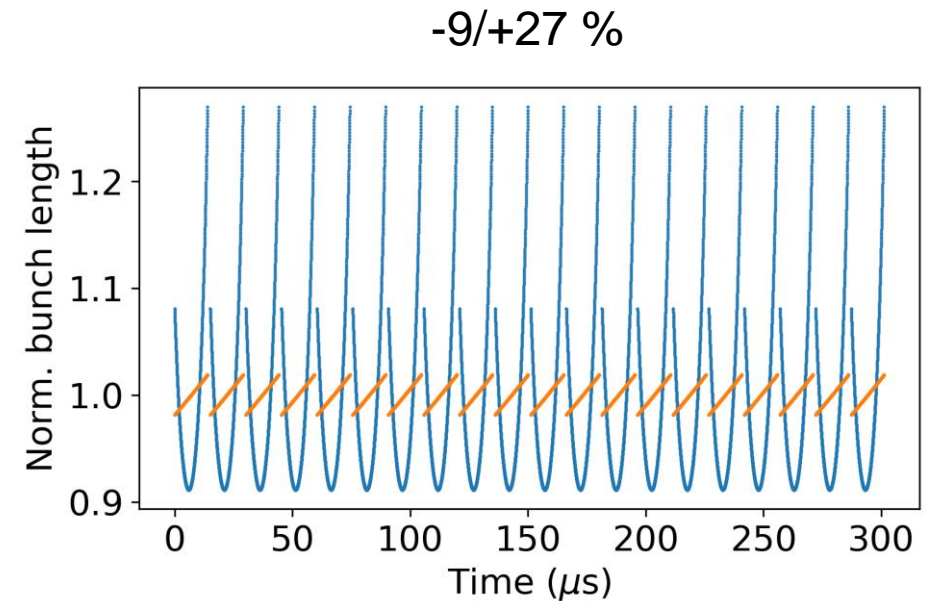
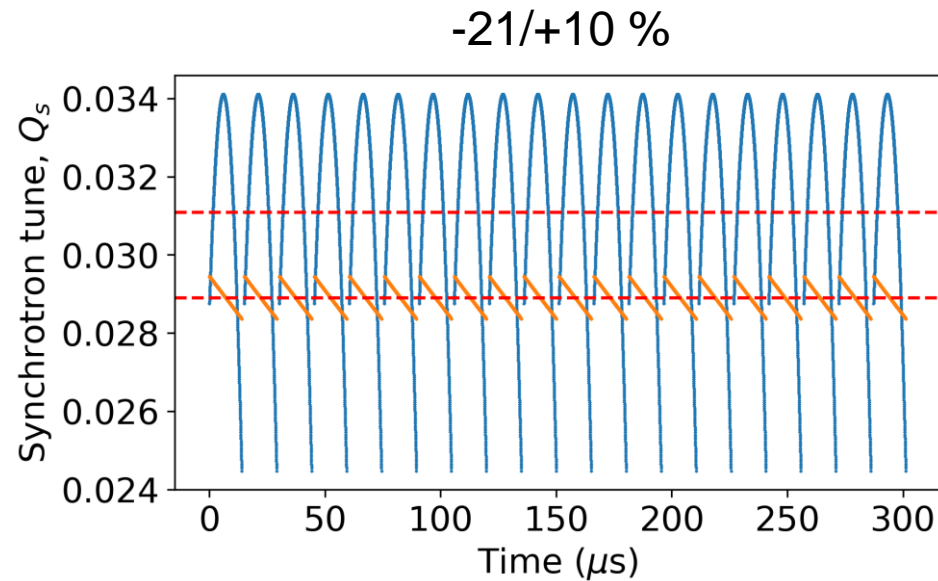
Gaps in beam filling scheme will result in modulation RF parameters

→ Present filling scheme with proposed 88 MV RF voltage results in **~50 %** modulation of the effective RF voltage



Bunch-by-bunch spread of beam parameters

132 2-cell cavities RPO, 56 1-cell cavities normal phase



Synchrotron tune and bunch length spread can significantly degrade beam stability (e.g., due to X-Z instability, [K. Ohmi, 2016](#)):

→ Peak-to-peak spread of **~30%** is a factor of **15** worse compared to 1-cell RF system

Possible mitigations

Synchrotron tune spread according small-signal model

$$\frac{Q_{s,n}}{Q_{s0}} - 1 \approx \frac{N_{\text{tot}} V_{f,n} \sin(\phi_s + \phi_f)}{2 V_{\text{tot}} \sin \phi_s} - \frac{N_{\text{tot}} V_{\text{cav}} \cos(\phi_s + \phi_f)}{2 V_{\text{tot}} \sin \phi_s} \phi_{b,n}$$

where peak-to-peak beam phase spread

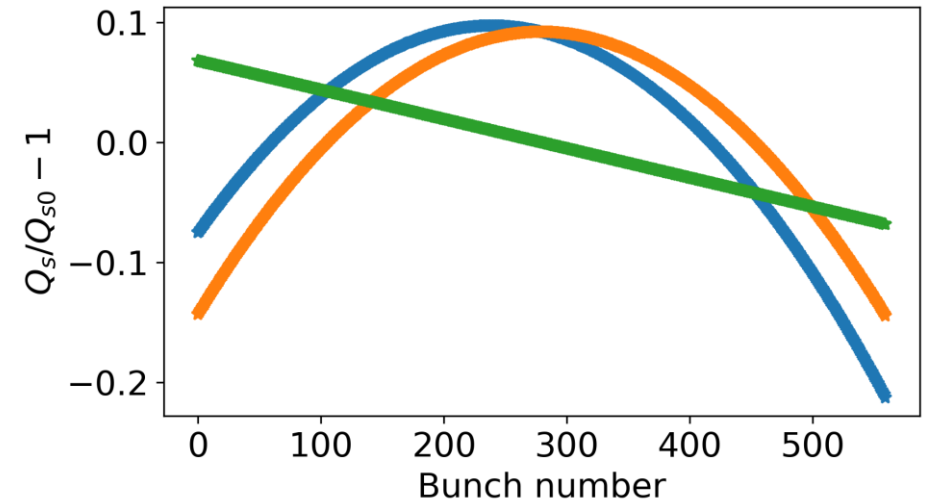
$$\max \phi_{b,n} - \min \phi_{b,n} \propto \frac{\Delta \omega_{\text{opt}} \tau_{\text{gap}} N_{\text{tot}}}{N_f - N_d} \propto \frac{\tau_{\text{gap}}}{V_{\text{tot}}}$$

Higher RF voltage and shorter gaps reduce Q_s spread

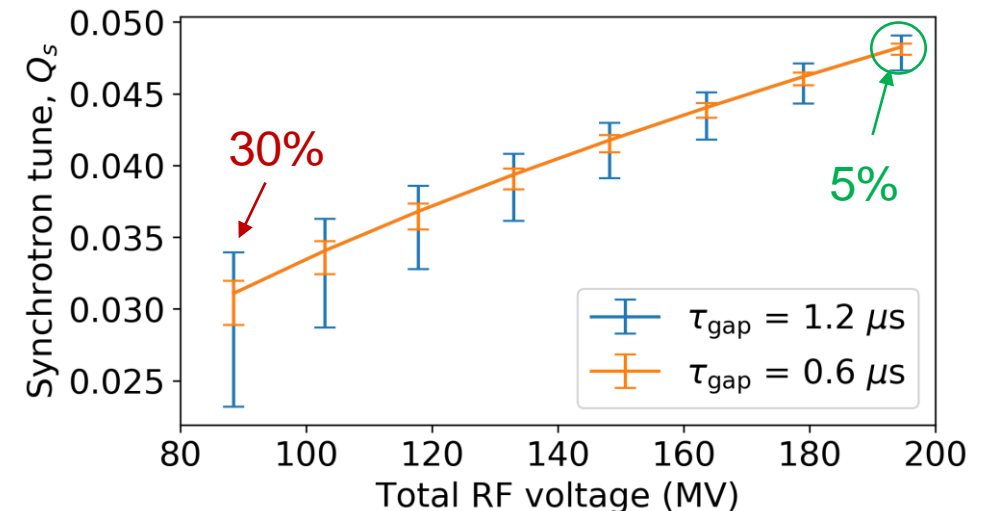
→ Evaluation of option with $V_{\text{tot}} = 200$ MV and original $\tau_{\text{gap}} = 1.2 \mu\text{s}$ showed unacceptable reduction of lifetime to about 20 s compared to 1320 s given by radiative Bhabha scattering ([K. Oide, 09.10.2024](#))

→ A new filling scheme with shorter gaps is necessary for reduction of the spread

Relative contributions to Q_s spread

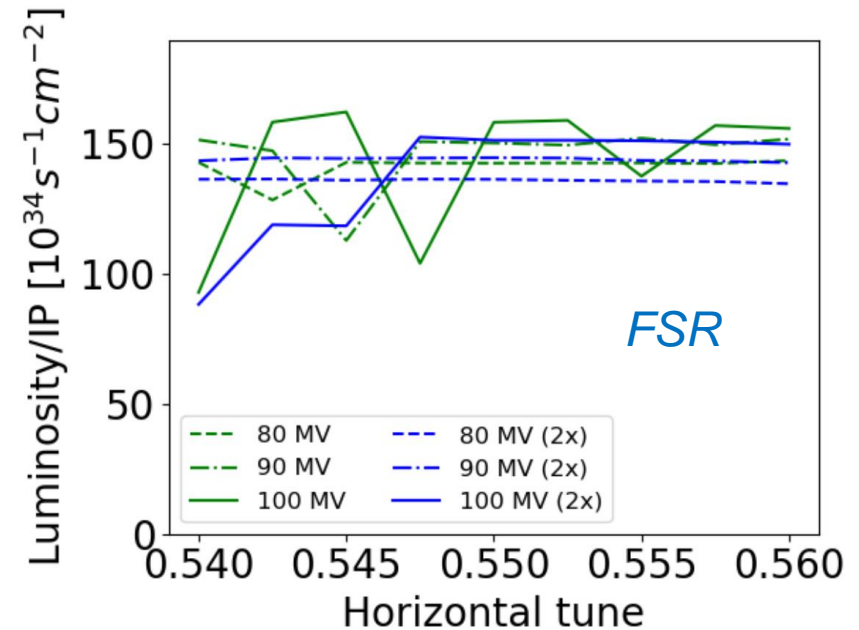
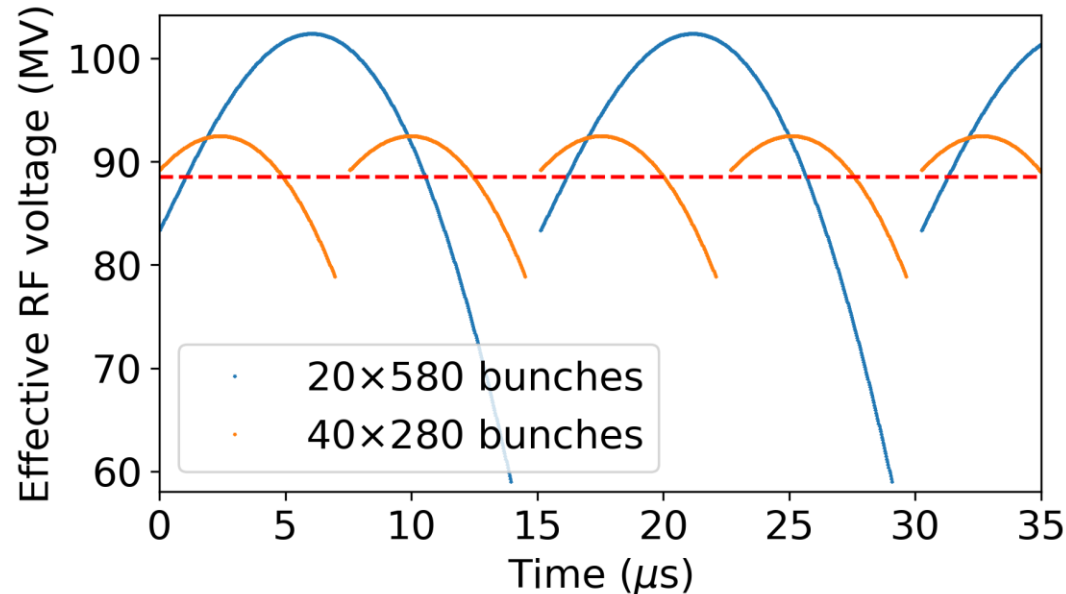


Q_s spread for 1.2 μs gap length



Results for smaller gaps between trains

Injection and extraction systems were adapted to decrease the rise time below 600-ns
([G. Favia, presentation this afternoon](#))

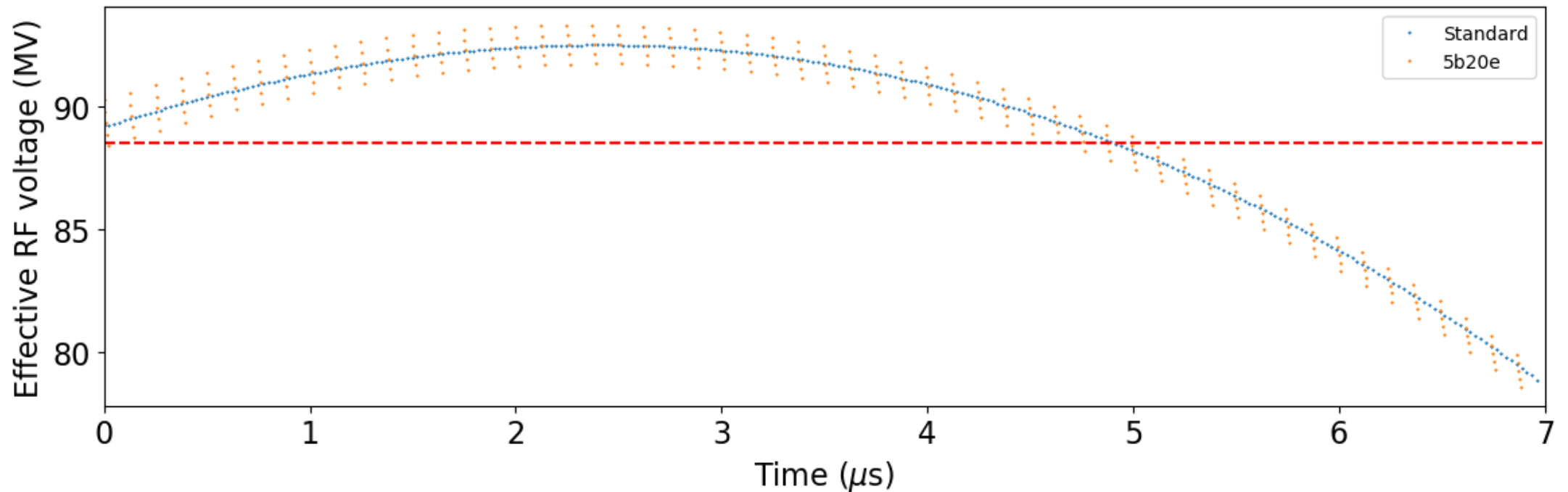
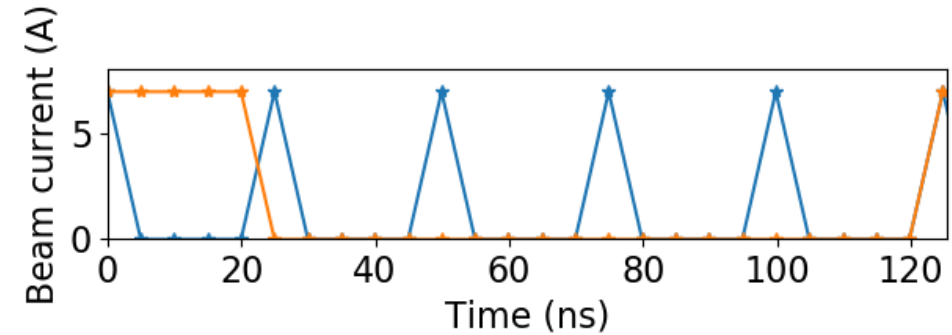


- With a new filling scheme (40 trains of 280 bunches), RF voltage spread was significantly reduced
- Working point can be found for all bunches at nominal intensity, **but not during non-uniform bootstrap injection needed for e-cloud mitigation** ([X. Buffat et al, 24.04.2025](#), also [R. Soos et al, presentation yesterday](#))

Alternative filling schemes

Hybrid filling schemes with 5-ns spacing
micro-trains can suppress e-cloud limitations

[\(L. Sabato, presentation yesterday\)](#)



→ Negligible impact on the RF voltage spread (15.3 % → 16.8 %)

RPO validation checklist

- Static beam loading → New total RF voltage proposed
- Transient beam loading → Shorter beam gaps are introduced
- Coupled-bunch instabilities
- Higher-order-mode power losses
- Availability aspects

Coupled-bunch instabilities due to fundamental mode

Standard analysis: compute growth rates and compare them with synchrotron radiation damping time

For short Gaussian bunches, the growth rate of the mode m is (*J. L. Laclare, 1985*)

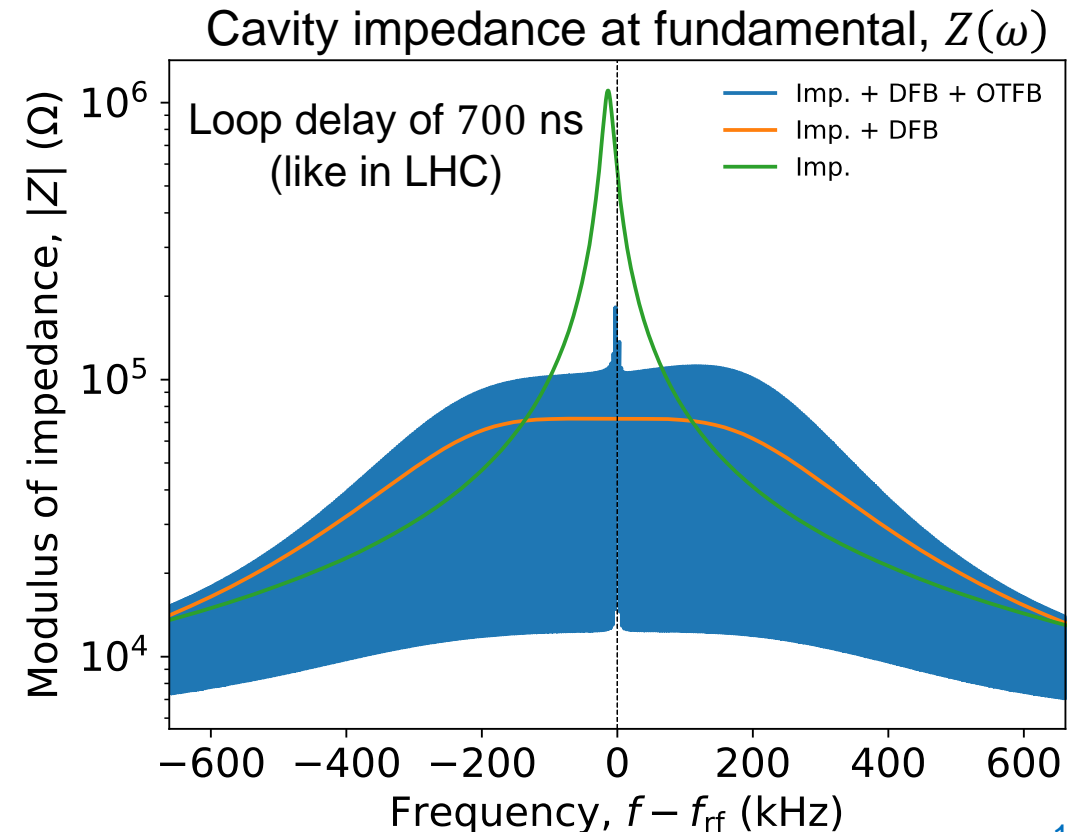
$$\frac{1}{\tau_m} \approx \frac{e\eta I_{b,DC} V_{tot} \omega_{RF}}{4\pi E_b Q_s V_{cav}} \{ \text{Re}[Z_{eff}(\omega_+)] - \text{Re}[Z_{eff}(\omega_-)] \},$$

with $\omega_{\pm} = \omega_{RF} \pm (m + Q_s)\omega_{rev}$

Direct (DFB) and long-delay feedback (OTFB) systems can reduce impedance “seen” by the beam
(*F. Pedersen, 1992*)

$$Z_{eff}(\omega) = \frac{Z(\omega)}{1 + H_{FB}(\omega)Z(\omega)}$$

↑
Feedback transfer function



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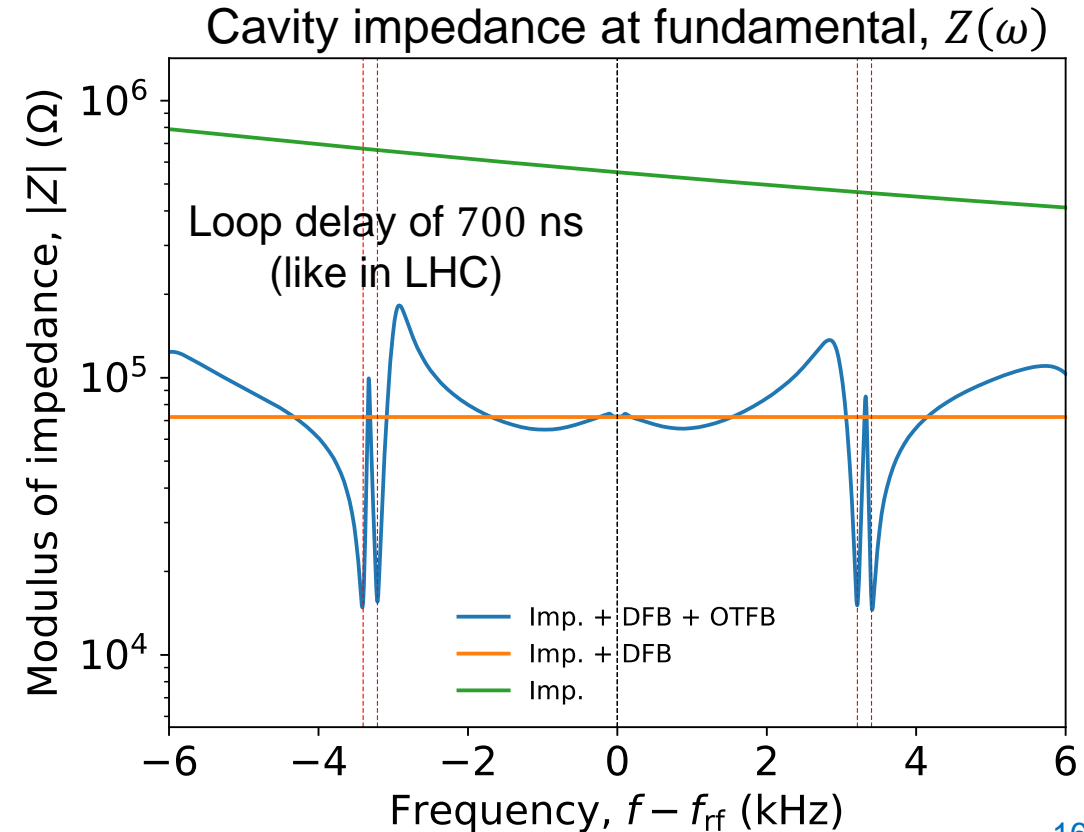
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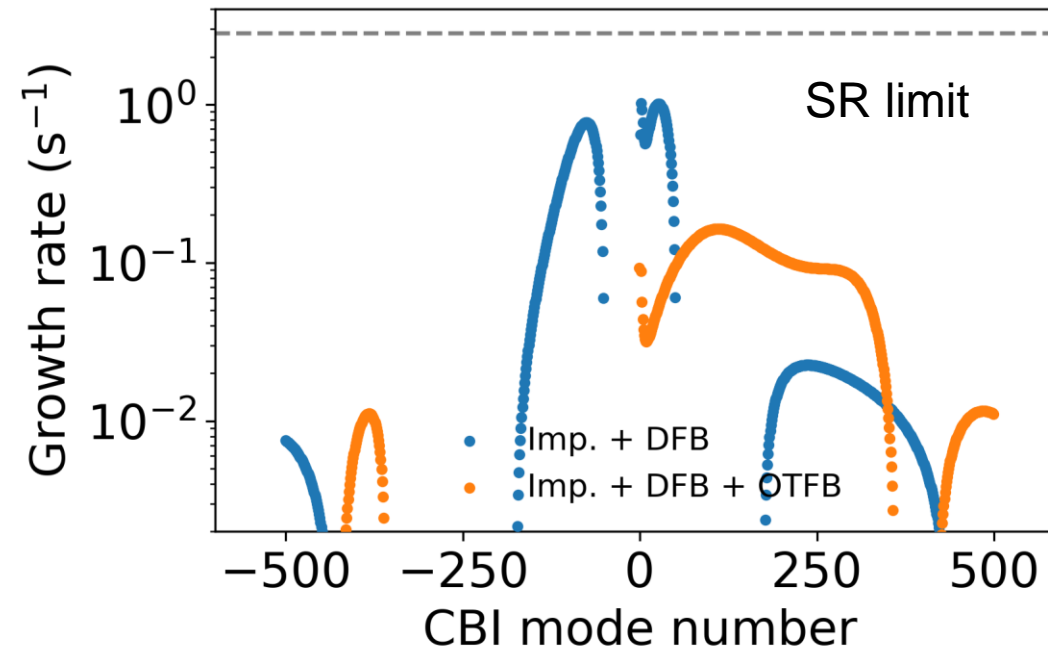
$$Z_{eff}(\omega) = \frac{Z(\omega)}{1 + H_{FB}(\omega)Z(\omega)}$$

↑
Feedback transfer function



Instability growth rates

132 2-cell cavities with RPO



Calculations for loop delay of 700 ns, DFB gain 10, OTFB gain 20:

→ LCBI can be suppressed already with DFB, while OTFB gives additional stability margin

HOM-driven coupled-bunch instabilities

Longitudinal plane:

No trapped HOMs

→ impedance is well below the SR threshold

Updated 2-cell design with suppressed (R/Q) of 0-mode at $f_{r0} = 397.61$ MHz by 2 orders of magnitude

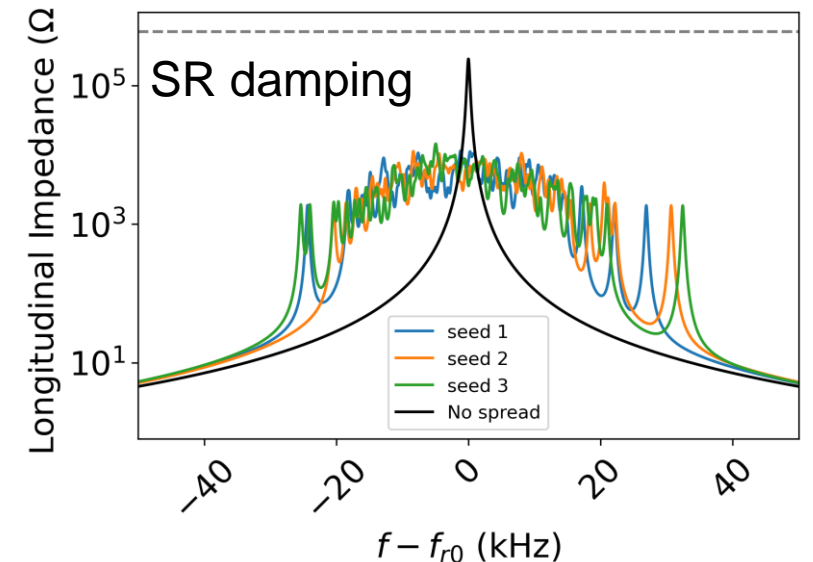
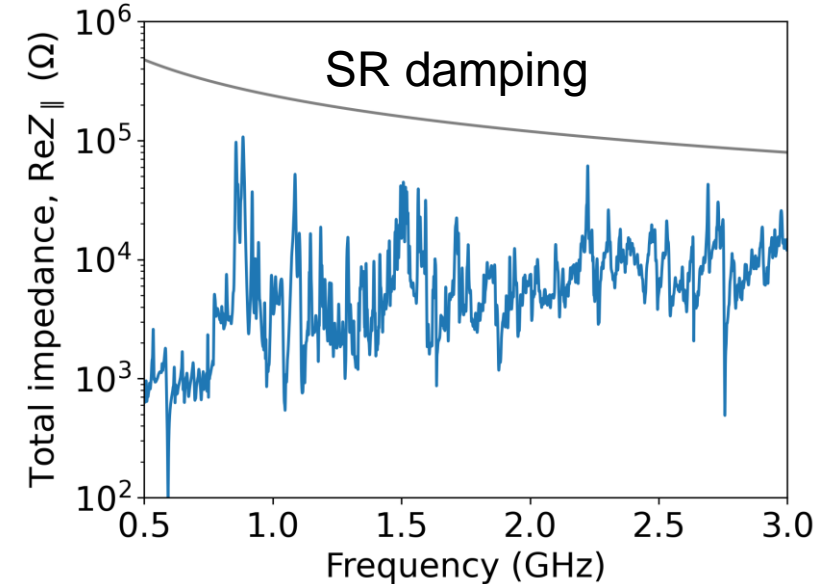
([S. Gorgi Zadeh, 15.08.2024](#))

→ Only twice below SR damping limit for $Q_L = Q_{L,opt}$ (might be larger due to limited bandwidth of circulator)

→ We gain a factor of 20 with 10 kHz frequency spread from cavity to cavity

Transverse plane:

Instabilities growth rates (min. rise time > 100 turns) are within capabilities of transverse feedback system ([W. Hofle, 2024](#))



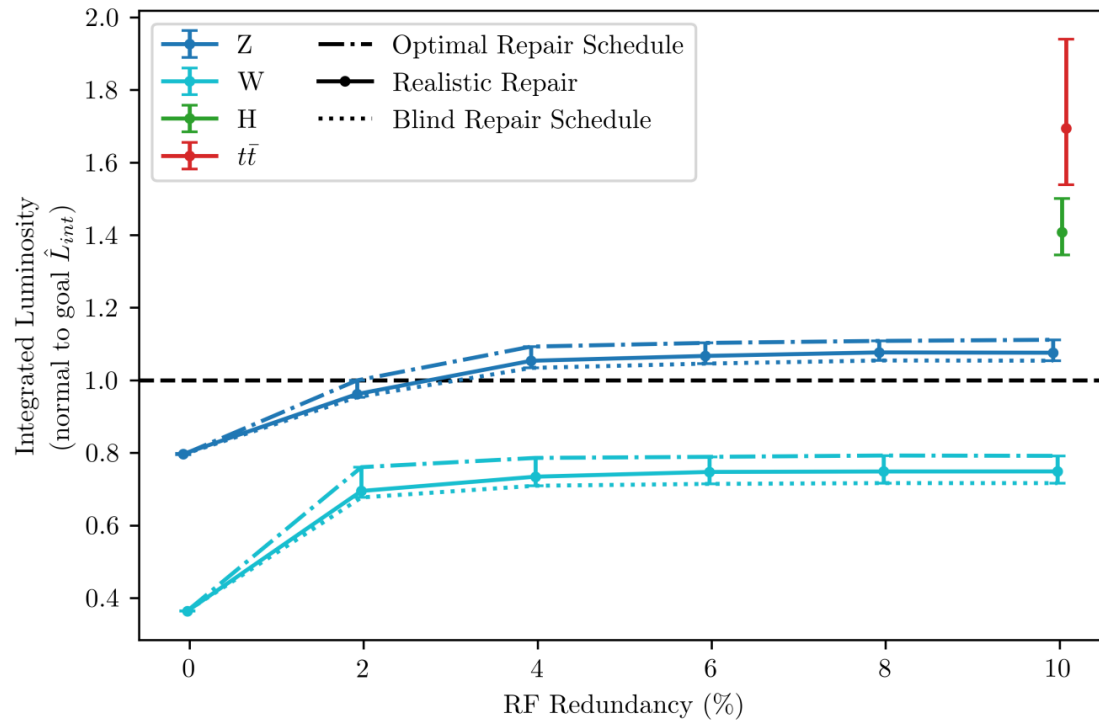
RPO validation checklist

- Static beam loading → New total RF voltage proposed
- Transient beam loading → Shorter beam gaps are introduced
- Coupled-bunch instabilities → Active and passive damping is sufficient
- Higher-order-mode power losses
- Availability aspects

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- Static beam loading → New total RF voltage proposed
- Transient beam loading → Shorter beam gaps are introduced
- Coupled-bunch instabilities → Active and passive damping is sufficient
- Higher-order-mode power losses → (*S. Gorgi Zadeh, presentation this afternoon*)
- Availability aspects

Availability challenges



For H and $t\bar{t}$ a 10% redundancy is assumed based on LEP experience

Availability goals require 10% (minimum 4%) redundancy of the RF system ([J. Heron, FCC Week 2024](#))

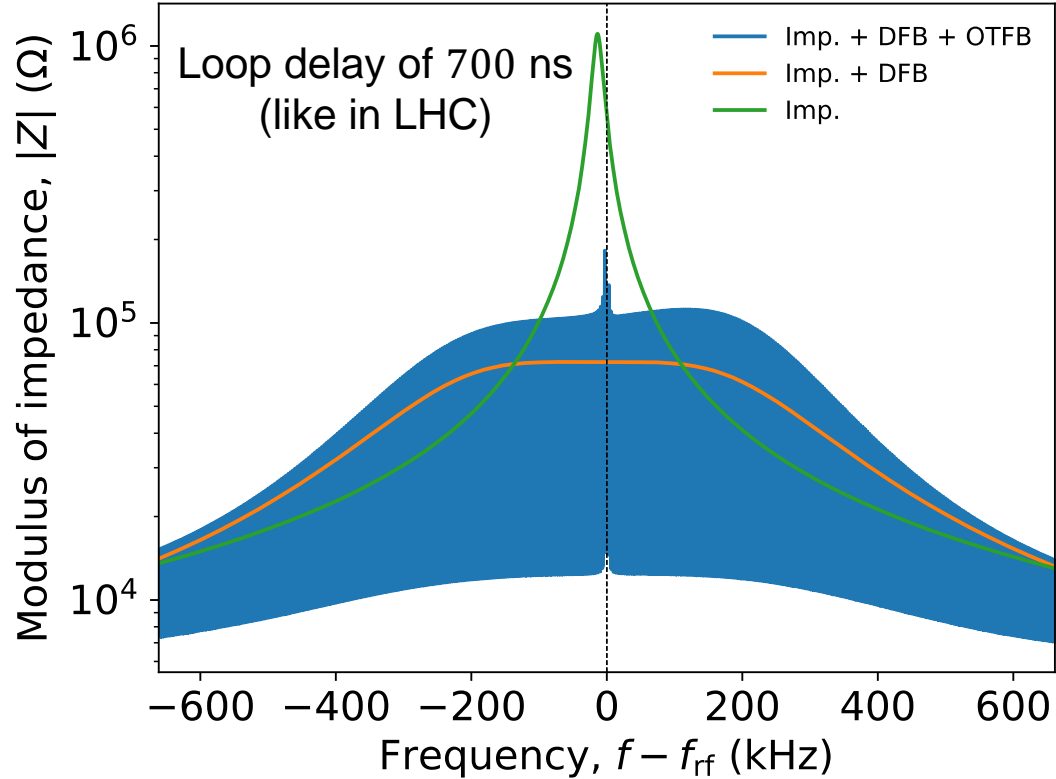
Critical questions for Z mode with RPO:

- Coupled-bunch instability due to fundamental impedance
- Cavity damage due to strong beam-induced fields
- Missing RF voltage (RF power margin)

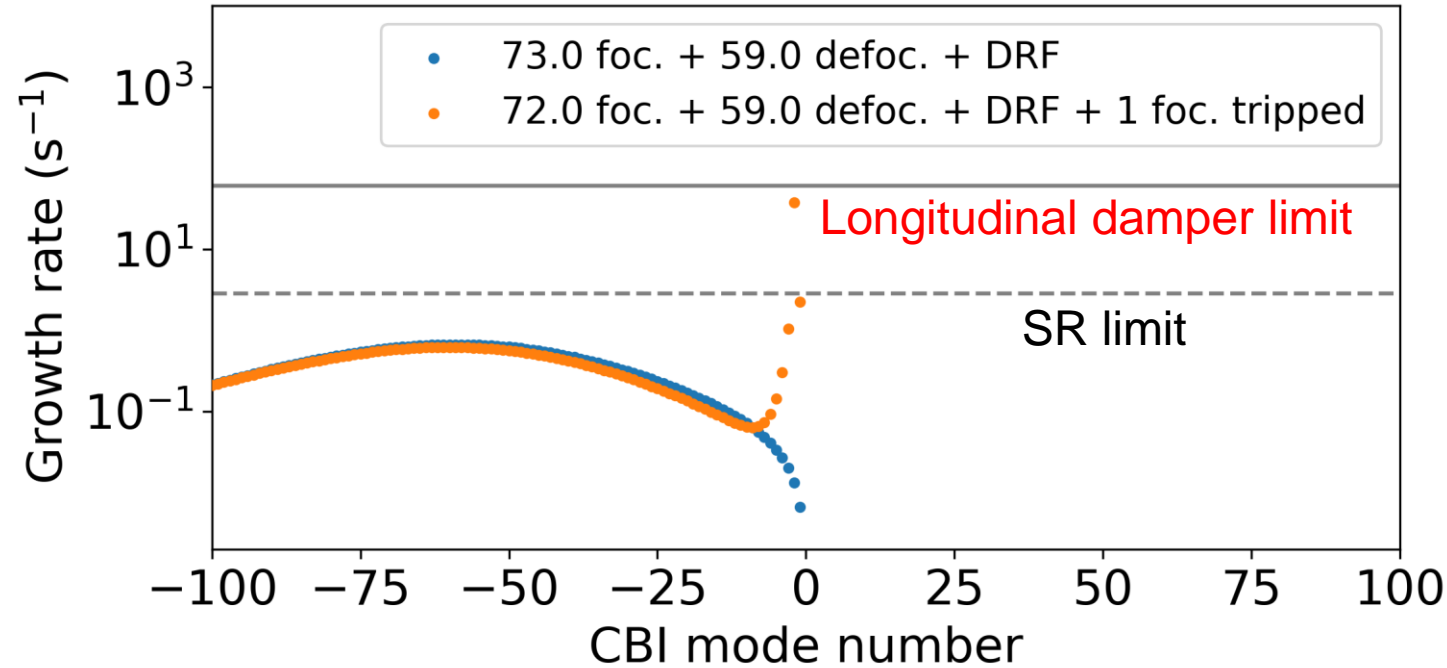
Consider only the case of RF source or LLRF trip (not a quench of the cavity)

Longitudinal coupled-bunch instability

Cavity impedance at fundamental, $Z(\omega)$



Instability growth rates with 1 tripped focusing cavity



Coupled-bunch instability due to fundamental mode could be suppressed by a longitudinal feedback system (main RF system as kicker) with damping time of $2T_s$ (see, [D. Teytelman, FCC week, 2019](#)), but RF power requirements need to be evaluated

→ We are **at the limit** with one missing cavity

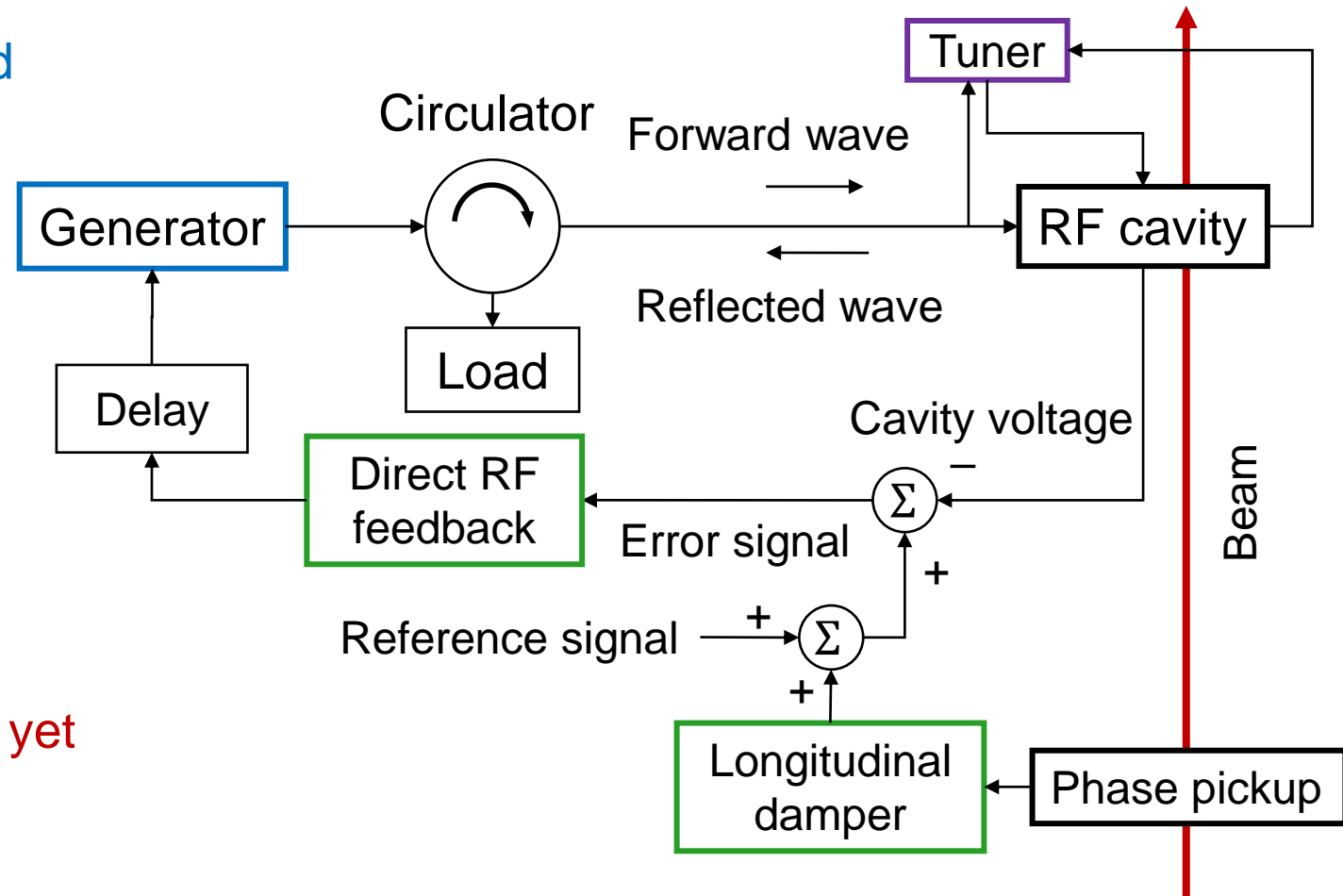
Simplified beam-cavity interaction model

Coupled differential equations are solved for four groups:

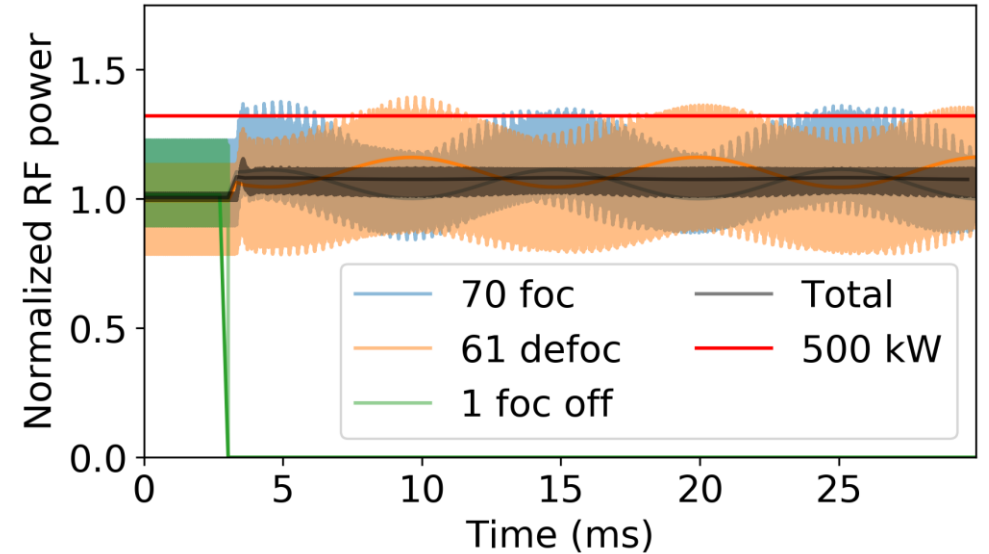
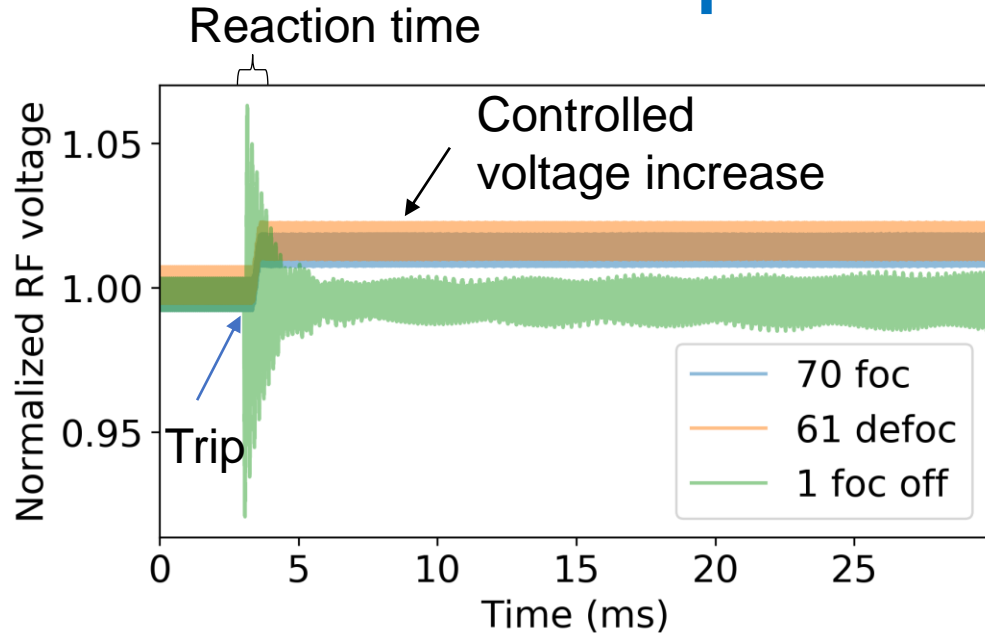
- Focusing (N_f cavities)
- Defocusing (N_d cavities)
- Tripped focusing (N_{off1} cavities)
- Tripped defocusing (N_{off2} cavities)

Combined with longitudinal equations of motion for one particle per bunch

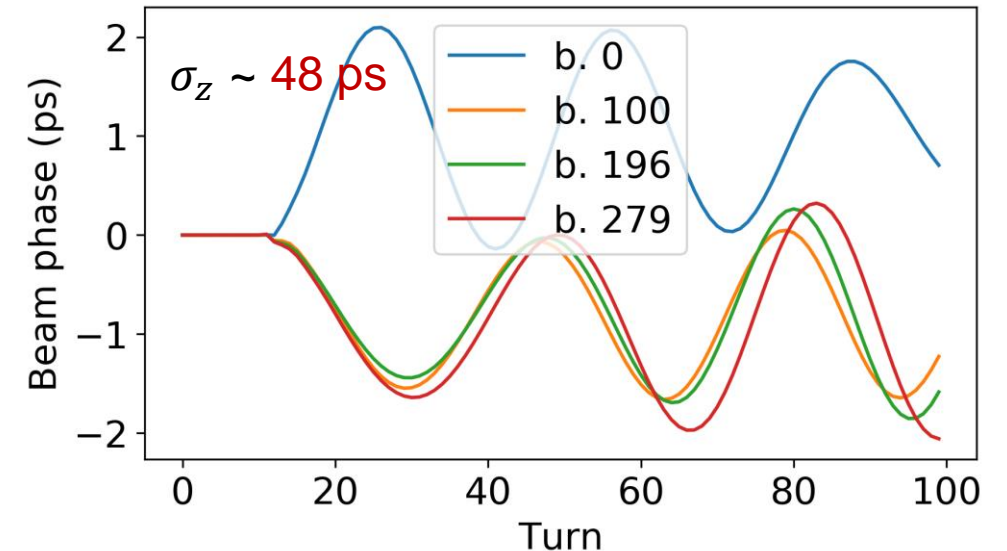
Longitudinal damper is not implemented yet



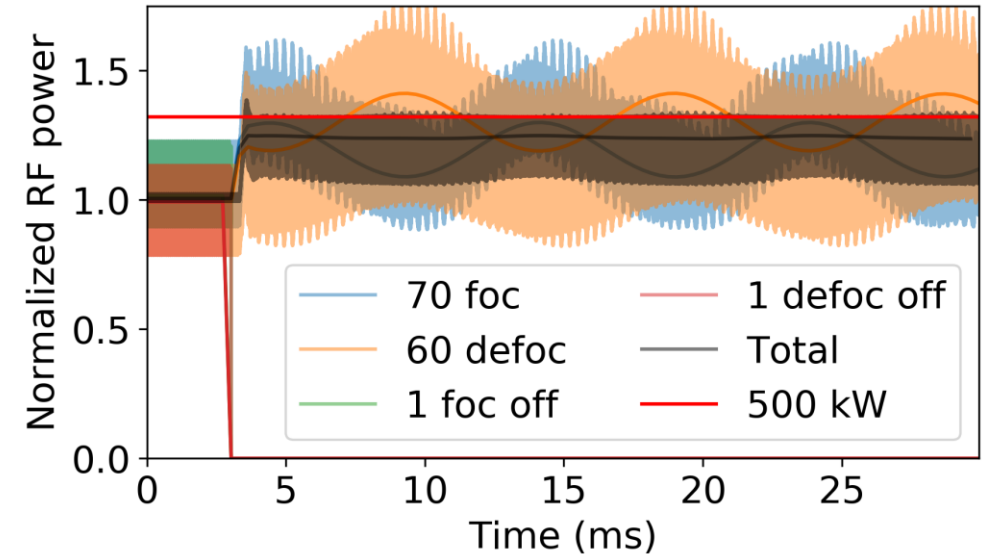
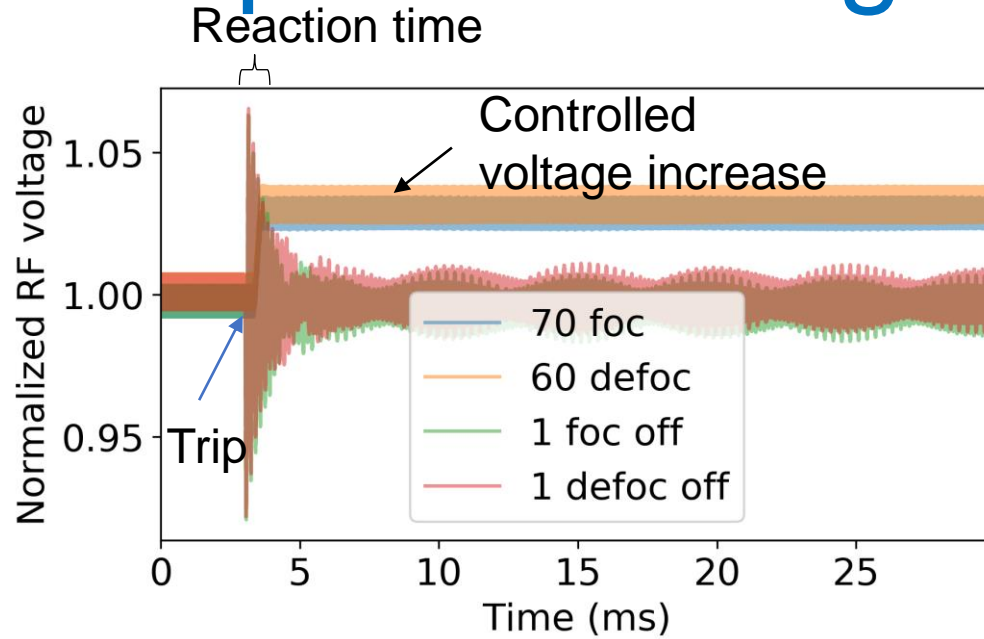
Trip of focusing cavity



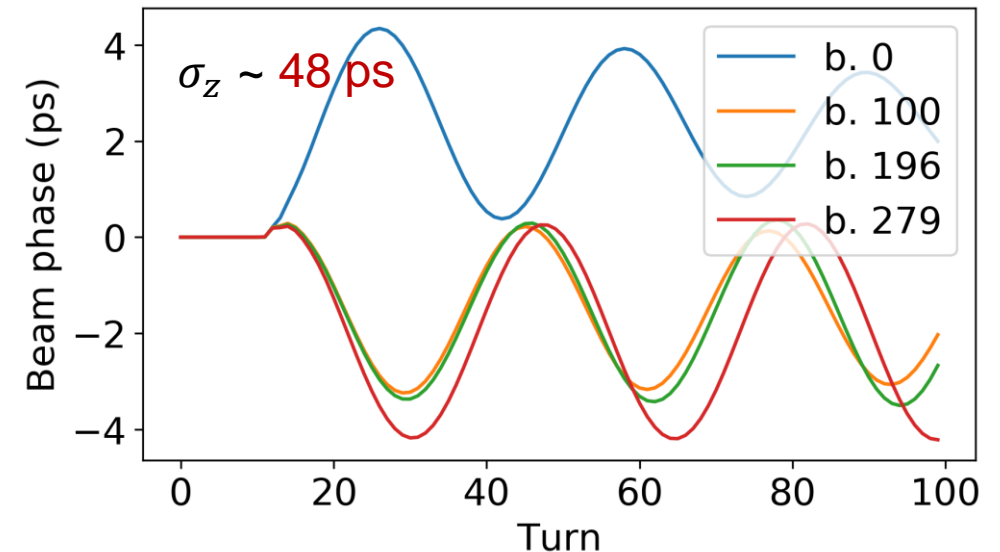
- Short RF voltage transients ~6%
 - Peak power of other cavities is modulated at synchrotron frequency (avg. <15%, peak <32%)
 - Initial bunch oscillation amplitude is <10% of rms bunch length
- New RF power have sufficient margin for a single cavity trip ([I. Syratchev, next presentation](#))



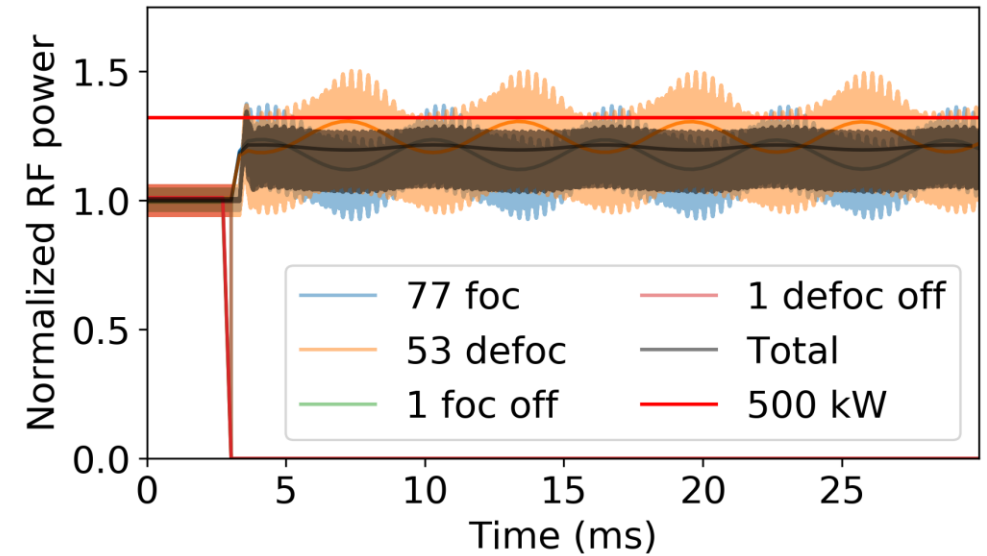
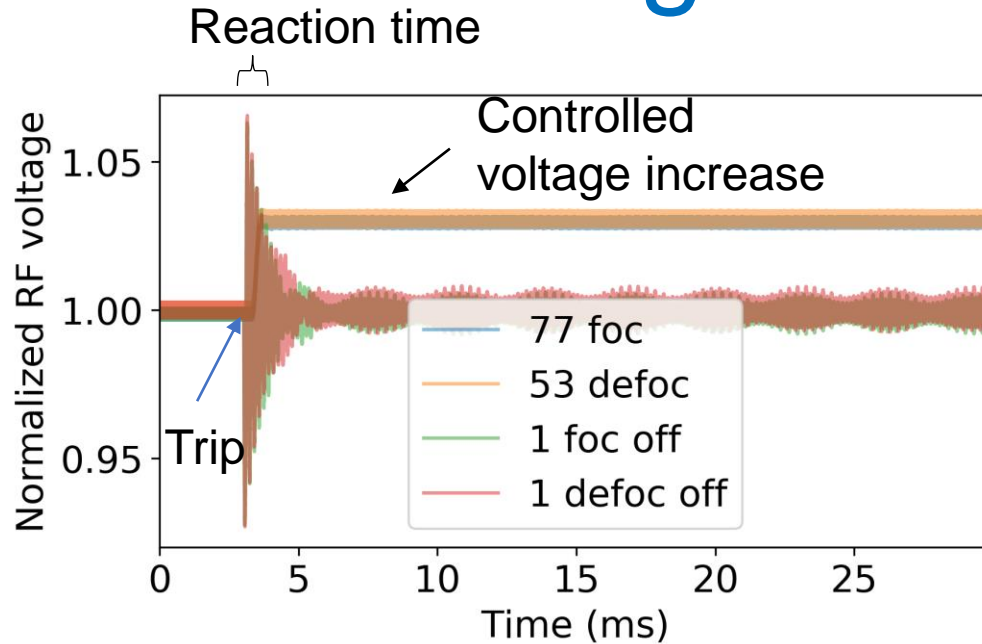
Trip of focusing and defocusing cavities



- Short RF voltage transients ~6%
 - Peak power of other cavities is modulated at synchrotron frequency (avg. <45%, peak <80%)
 - Initial bunch oscillation amplitude is <8% of rms bunch length
- Not enough power margin to handle simultaneous trip of two cavities

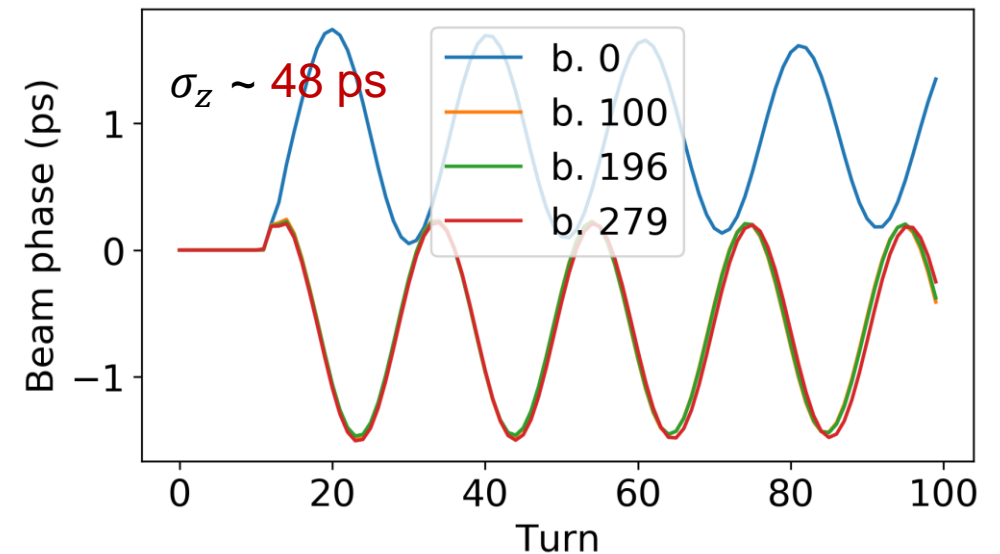


Higher total RF voltage



Moving from 89 to 195 MV reduces RF power transient:

- Is there a room for this in LCC optics?
- Can we profit from relaxed bunch charge for non-uniform filling schemes?



RPO validation checklist

- ☑ Static beam loading → New total RF voltage proposed
- ☑ Transient beam loading → Shorter beam gaps are introduced
- ☑ Coupled-bunch instabilities → Active and passive damping is sufficient
- ☑ Higher-order-mode power losses → ~50 % increase wrt 1-cell design
- ☐ Availability aspects: → Beam can be preserved only in case of a single cavity trip

Summary

Reverse Phase Operation (RPO) mode was the final ingredient to have the same 2-cell 400 MHz RF system for Z, WW, and ZH operating points.

Main aspects were evaluated, and **the most critical item** was identified:

- Required 4% redundancy to achieve integrated luminosity goals cannot be achieved according to dynamic beam-cavity interaction model. RF power margins could be sufficient to preserve the beam only in the case of a single RF source trip. Higher total RF voltage is beneficial for reduction of RF power transients.

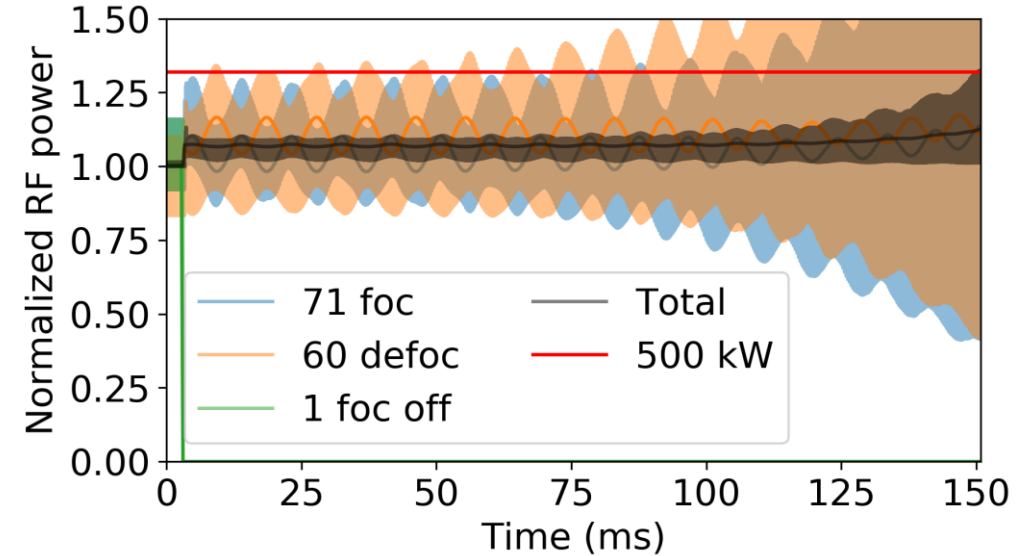
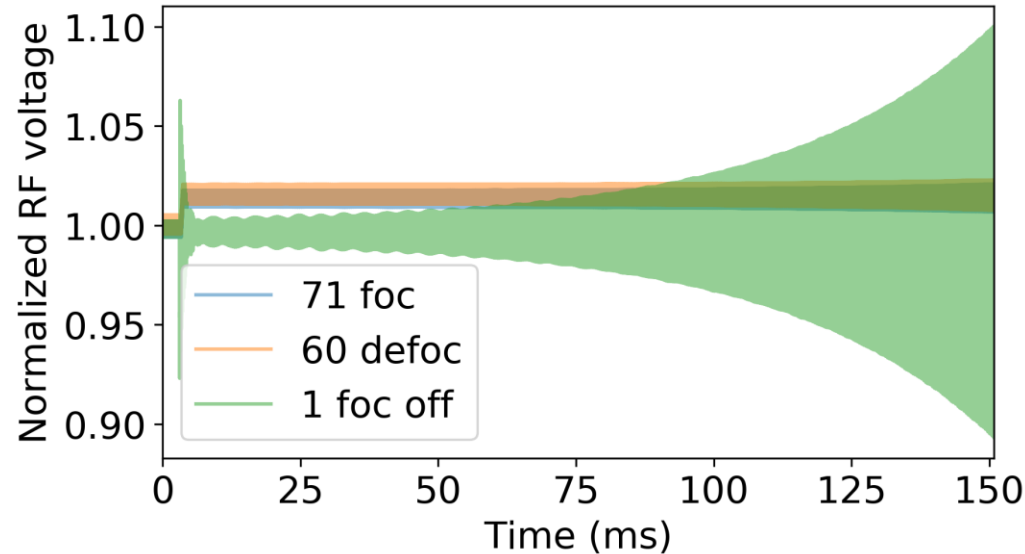
Not covered in this talk:

- RPO is an essential part of FCC-ee high-energy booster (HEB) operation ([L. Valle, presentation tomorrow](#))
- Increased longitudinal impedance requires special attention for Z operating point of HEB ([S. Gorgi Zadeh, presentation this afternoon](#))

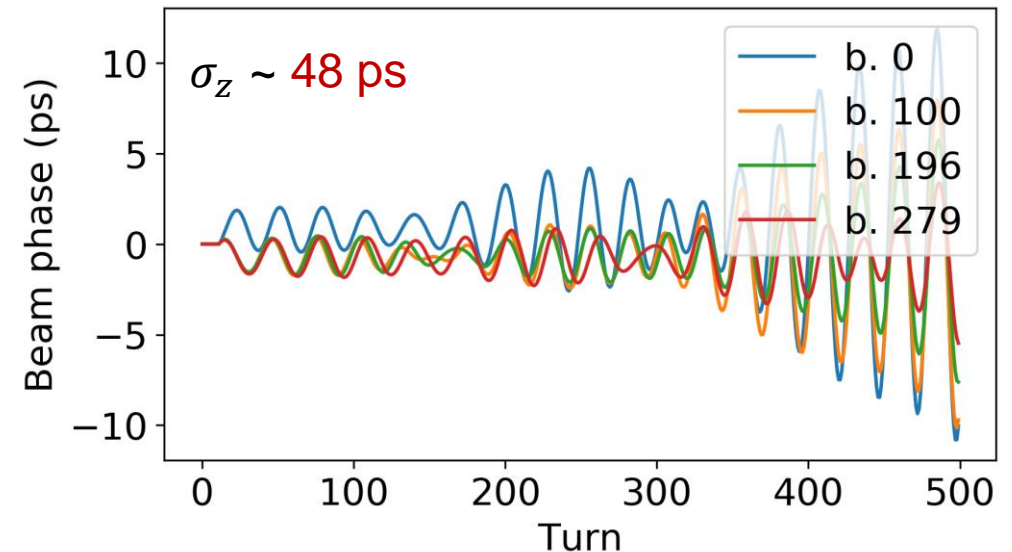
Thank you for your attention!

Backup slides

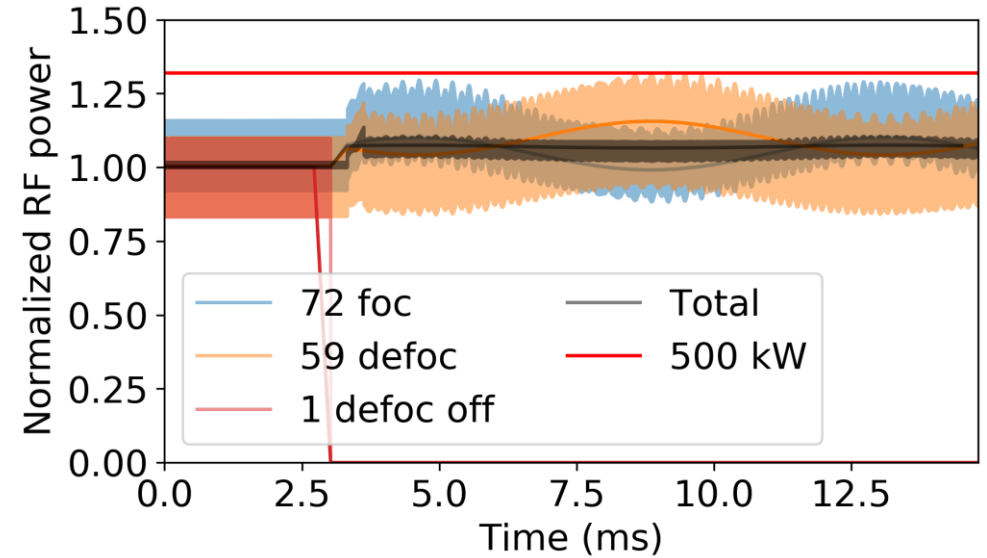
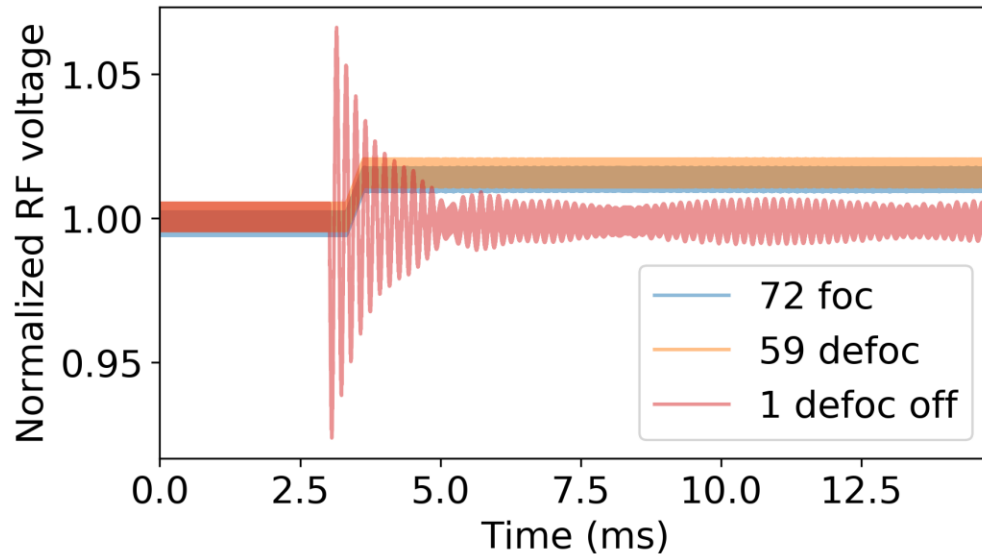
Trip of focusing cavity



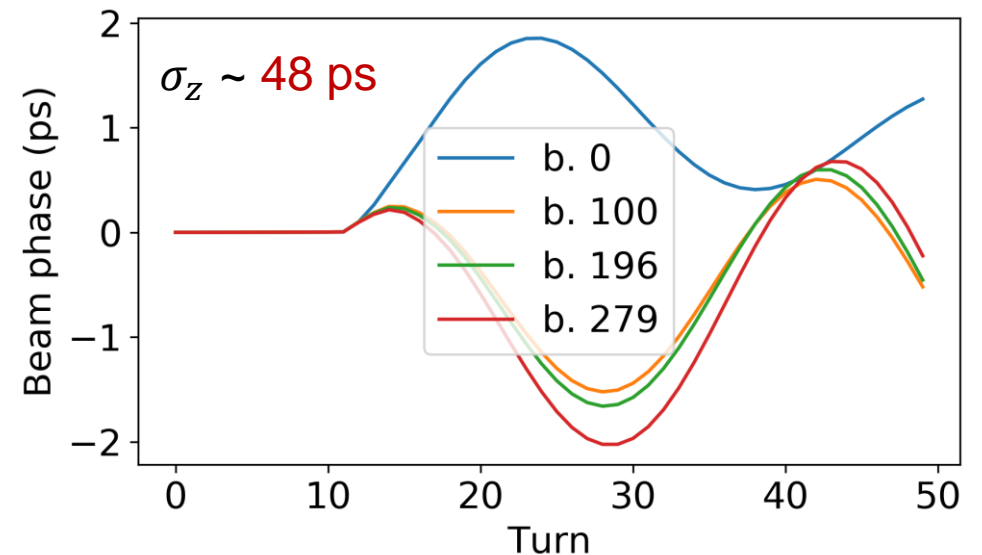
- Short RF voltage transients ~6%
- Peak power of other cavities is modulated at synchrotron frequency (avg. <15%, peak <32%)
- Initial bunch oscillation amplitude is <10% of rms bunch length
- Beam is unstable without longitudinal damper due to uncompensated impedance



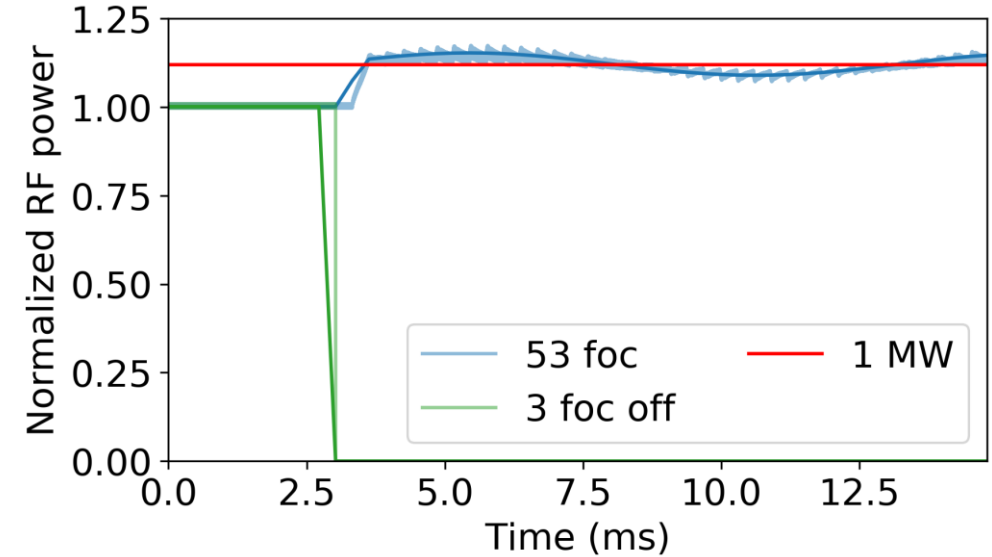
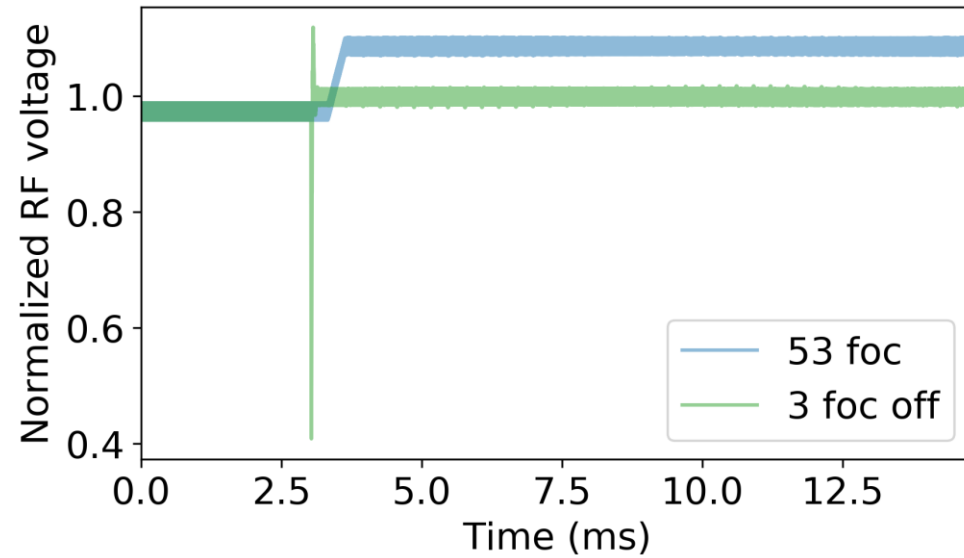
Trip of defocusing cavity



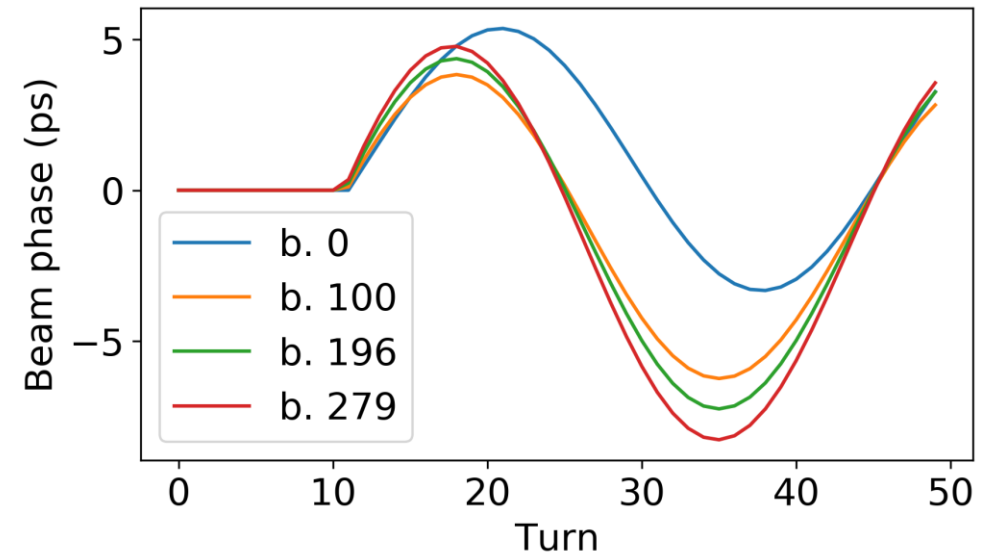
- Similar results to the case of focusing cavity trip
- Simultaneous trip of two cavities of any type leads to significant RF power overshoot



Trip of 3 single-cell cavities (4% redundancy)



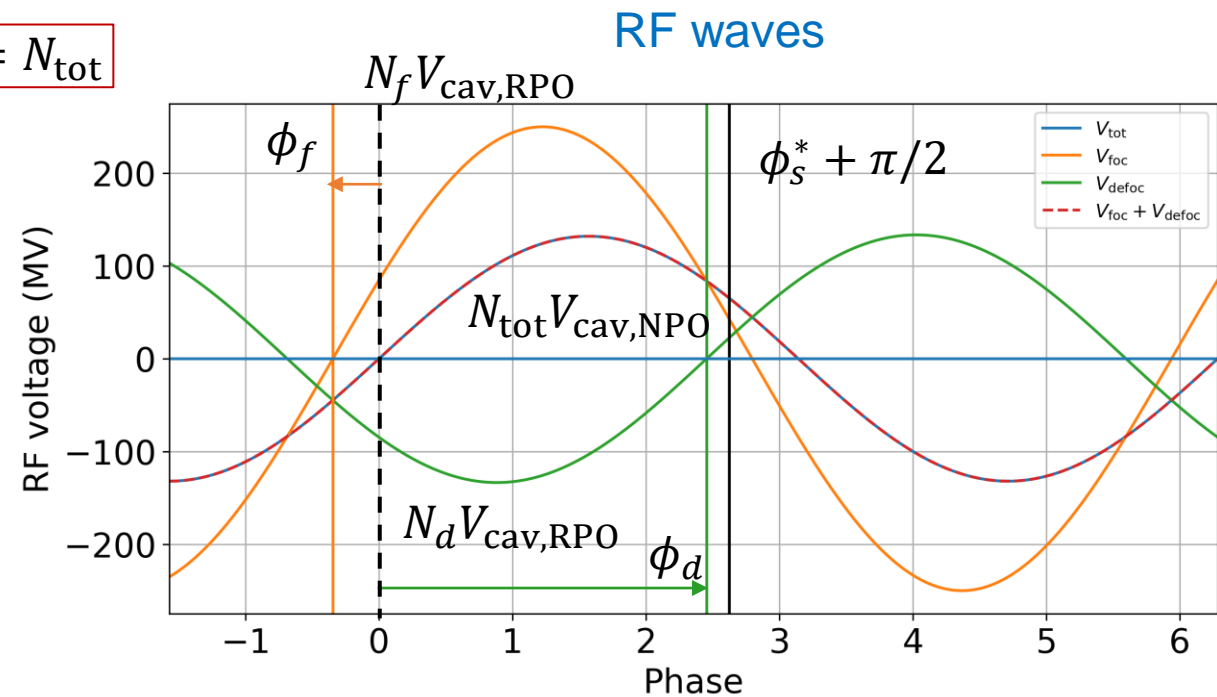
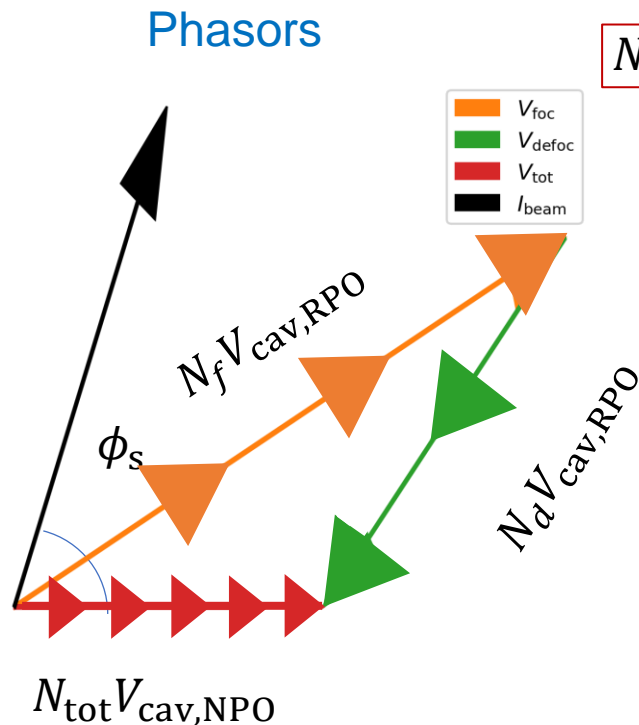
- Smaller relative transient compared RPO scheme
- Absolute peak power modulation exceeds 1 MW level



Reverse phase operation

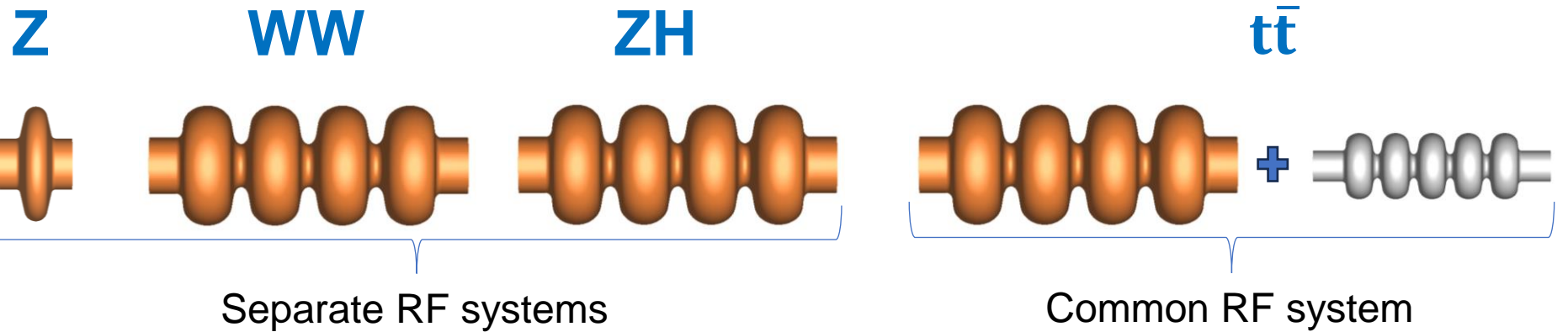
Reverse phase operation (RPO) mode allows for increasing RF cavity voltage having optimal static beam loading compensation ([Y. Morita et al., SRF, 2009](#))

- Experimentally verified with high beam loading in KEKB ([Y. Morita et al., IPAC, 2010](#))
- Baseline solution for EIC ESR ([e.g., J. Guo et al., IPAC, 2022](#))

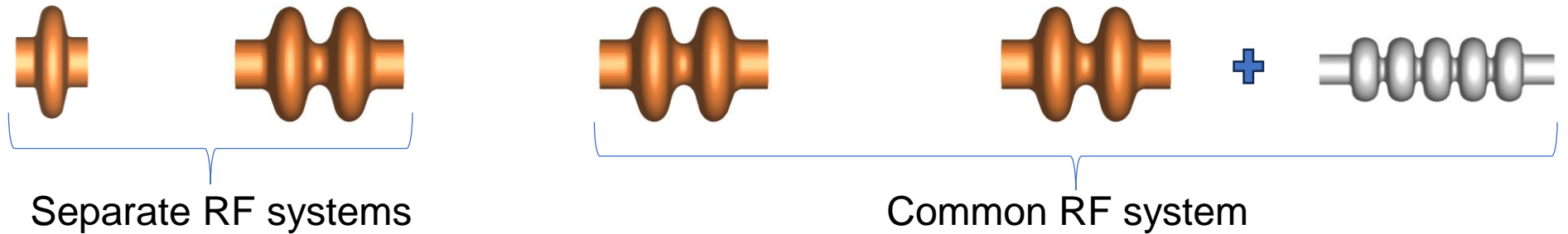


*Electron convention for synchronous phase: $\phi_{s,\text{proton}} = \frac{\pi}{2} + \phi_{s,\text{electron}}$

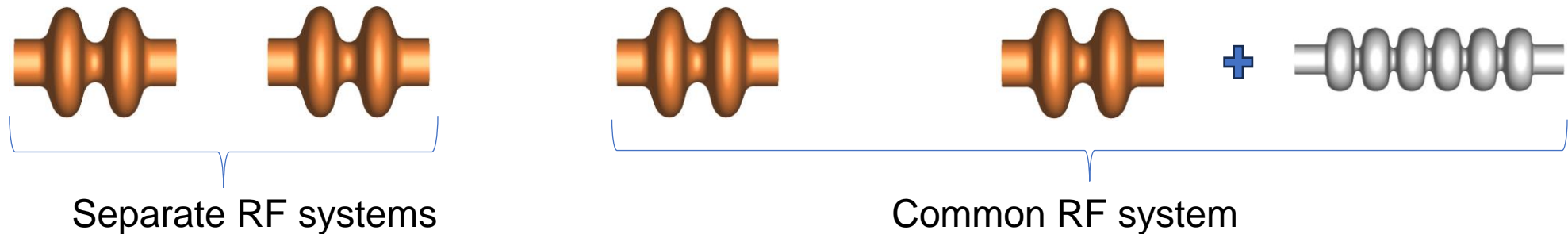
RF baseline evolution



F. Peauger, FCC Week 2022 & FCC FS Mid-Term Report (2024)



FCC FS Final Report (2025)

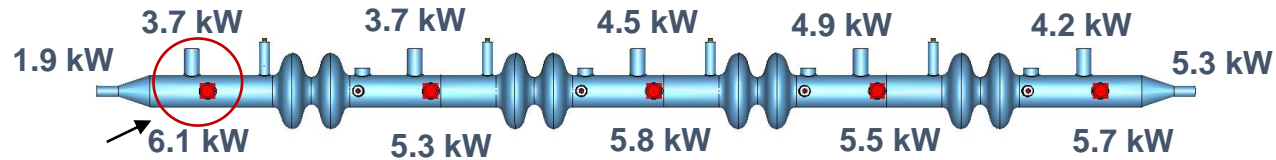


Higher-order mode power

HOM power equation
$$P_{\text{HOM}} = I_{b,DC}^2 \sum_{k=-\infty}^{\infty} \text{Re}[Z(kf_{\text{rev}})] |I_k|^2$$

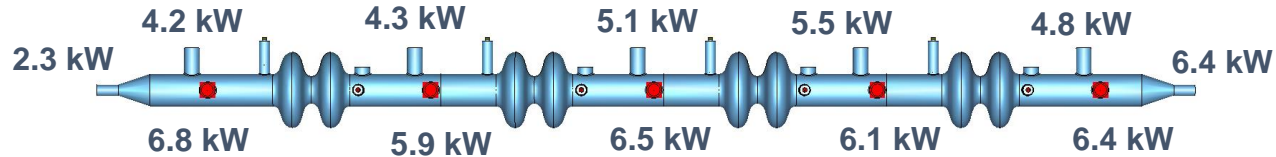
In absence of resonant modes $\rightarrow P_{\text{HOM}} = k_{\parallel, \text{HOM}} I_{b,DC} Q_b$

Option 1: $P_{\text{HOM}} \approx 57.2 \text{ kW}$



2 coaxial
HOM couplers

Option 2: $P_{\text{HOM}} \approx 65.1 \text{ kW}$



Option	V_{tot} (MV)	$\sigma_{z,BS}$ (mm)
1	103.00	14.6
2	117.86	13.7

K. Oide, 06.11.2024

Tapers contribution to the HOM power

400 MHz system	Tapers (100 mm to 300 mm)	Tapers (160 mm to 300 mm)
k_{\parallel} [V/pC] ($\sigma_z = 14.6 \text{ mm}$)	0.174	0.08
P_{HOM} [kW]	7.7	3.6

Unavoidable 15 kW increase in HOM power compared to single-cell design for option 1 and potentially up to 10 kW deposited in resonance mode ([S. Gorgi Zadeh, 23.01.2024](#))

\rightarrow Verification of possible reduction of the tapering angle at the extremities of the cavity string is ongoing ([K. Canderan, 29.10.2024](#))

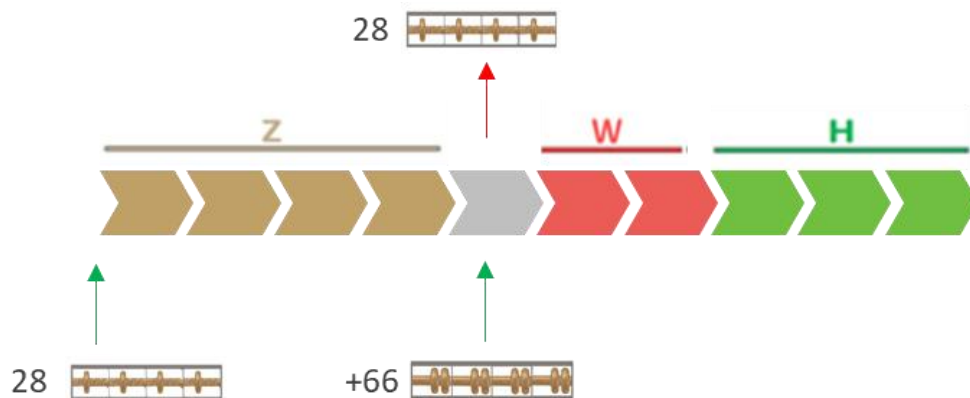
Baseline collider RF system configuration

	Energy (GeV)	Current (mA)	RF voltage (GV)
Z	45.6	1283	0.079
W	80	135	1.05
H	120	26.7	2.1

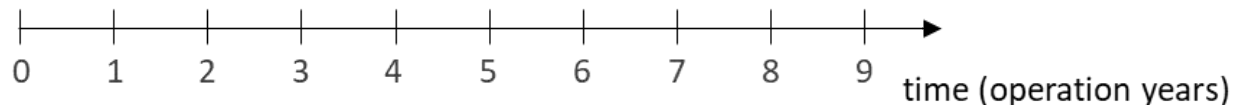
MTR recommendation:

Study options to avoid the 1-cell/2-cell RF cavity reconfiguration between Z and H/WW running to **simplify the SRF system implementation and improve flexibility in the physics programme.**

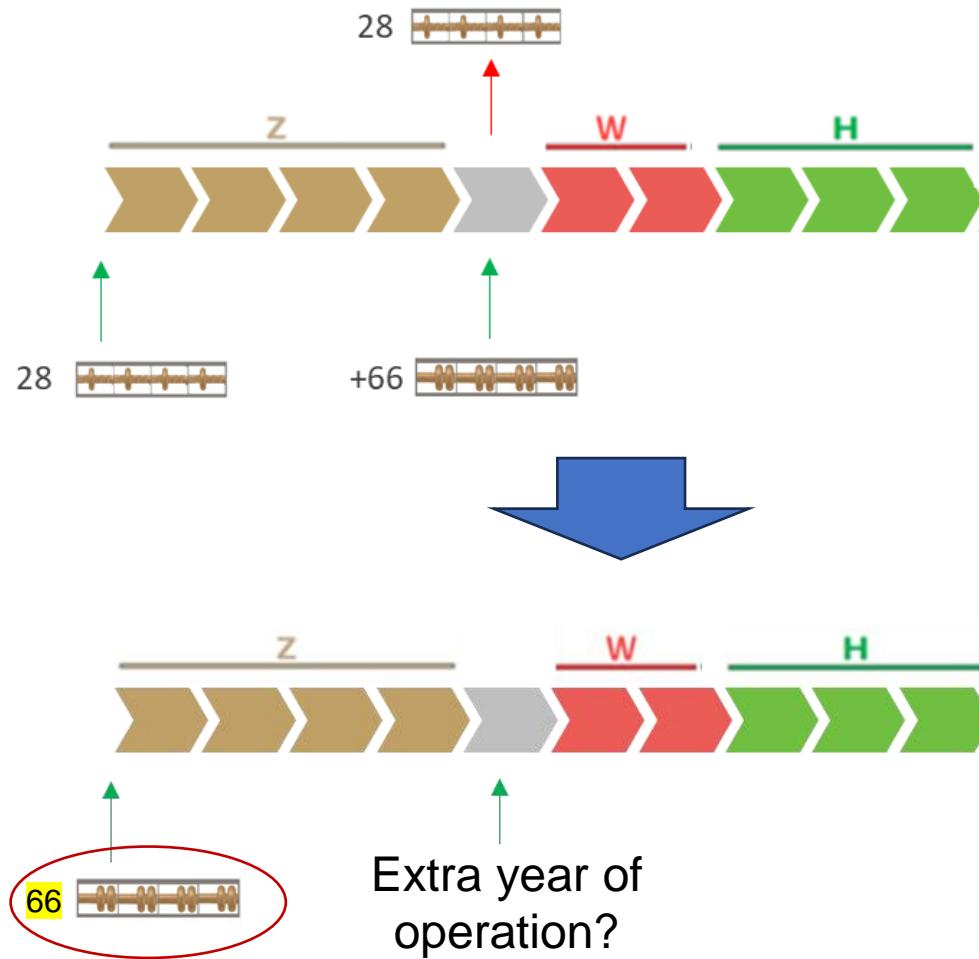
K. Oide, 29.05.2024



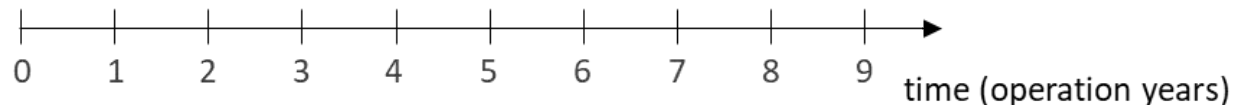
Courtesy of O. Brunner



Baseline collider RF system configuration



Courtesy of O. Brunner



MTR recommendation:

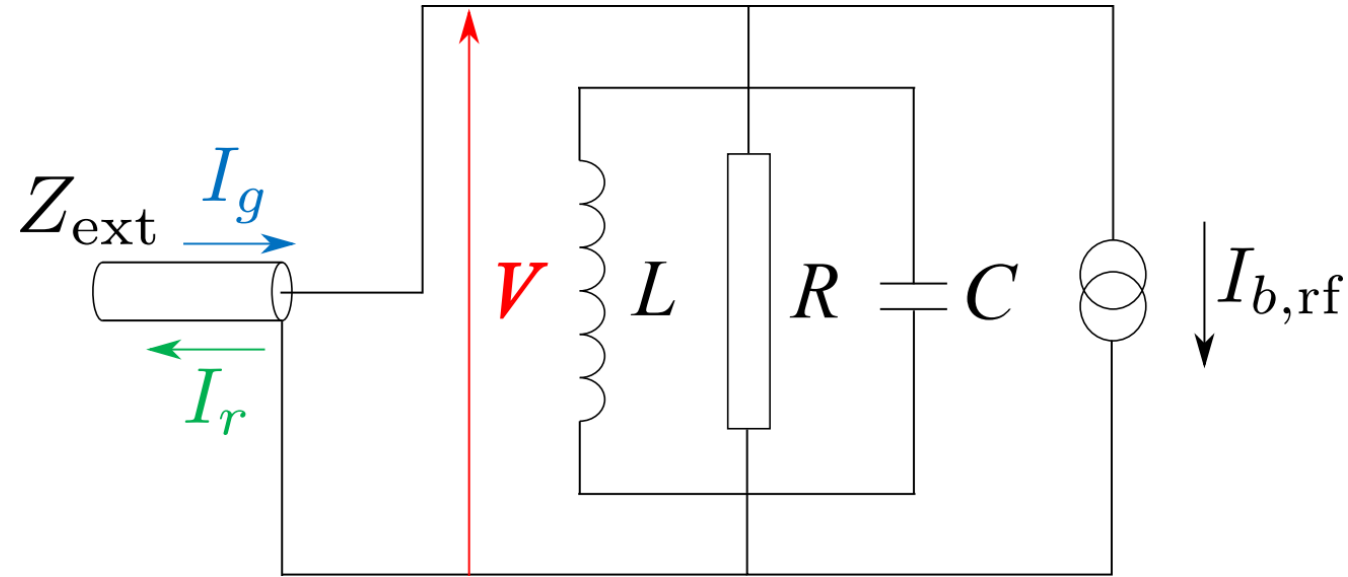
Study options to avoid the 1-cell/2-cell RF cavity reconfiguration between Z and H/WW running to **simplify the SRF system implementation and improve flexibility in the physics programme.**

Proposed solution:

- Installation of 2-cell cavities with individual RF power sources (~0.5 MW level) from the beginning
- Reverse phase operation (RPO) mode for Z, normal mode for W and H (combined RF system for 2 beams)

→ **Applicability of RPO for FCC-ee needs to be confirmed**

Beam loading model: main equation



Generator current

$$I_g = \frac{V}{2(R/Q)} \left(\frac{1}{Q_0} + \frac{1}{Q_{\text{ext}}} - 2i \frac{\Delta\omega}{\omega_{\text{rf}}} \right) + \frac{I_{b,\text{rf}}}{2}$$

Generator power

$$P_g = \frac{1}{2} Z_{\text{ext}} |I_g|^2 = \frac{1}{2} (R/Q) Q_{\text{ext}} |I_g|^2$$

Fixed parameters are V , (R/Q) , Q_0 , ω_{rf} , $I_{b,\text{rf}}$, while V , $\Delta\omega$, and Q_{ext} can be adjusted

Derivations for arbitrary cavity phase (1/2)

Generator current $I_g = \frac{V}{2(R/Q)} \left(\frac{1}{Q_0} + \frac{1}{Q_{\text{ext}}} - 2i \frac{\Delta\omega}{\omega_{\text{rf}}} \right) + \frac{I_{b,\text{rf}}}{2}$

$F_b = 2 \frac{\mathcal{F}[\lambda(t)]_{\omega=\omega_{\text{rf}}}}{\mathcal{F}[\lambda(t)]_{\omega=0}}$

Complex quantities: $I_g, V,$ and $I_{b,\text{rf}} \rightarrow$ $I_g = |I_g|e^{i\phi_L}, V = |V_{\text{cav}}|e^{i\phi_c}, I_{b,\text{rf}} = |F_b|I_{b,\text{dc}}e^{-i\phi_s}$

$$|I_g|e^{i\phi_L} = \frac{|V_{\text{cav}}|e^{i\phi_c}}{2(R/Q)} \left(\frac{1}{Q_{\text{ext}}} - 2i \frac{\Delta\omega}{\omega_{\text{rf}}} \right) + \frac{|F_b|I_{b,\text{dc}}e^{-i\phi_s}}{2} \quad \Bigg| \quad \times e^{-i\phi_c}$$

$$|I_g|e^{i\phi_L - i\phi_c} = \frac{|V_{\text{cav}}|}{2(R/Q)} \left(\frac{1}{Q_{\text{ext}}} - 2i \frac{\Delta\omega}{\omega_{\text{rf}}} \right) + \frac{|F_b|I_{b,\text{dc}}e^{-i\phi_s - i\phi_c}}{2}$$

Then splitting in real and imaginary parts:

Derivations for arbitrary cavity phase (2/2)

$$|I_g|e^{i\phi_L - i\phi_c} = \frac{|V_{\text{cav}}|}{2(R/Q)Q_{\text{ext}}} + \frac{|F_b|I_{b,\text{dc}} \cos(\phi_s + \phi_c)}{2} - i \left[\frac{|V_{\text{cav}}|}{(R/Q)} \frac{\Delta\omega}{\omega_{\text{rf}}} + \frac{|F_b|I_{b,\text{dc}} \sin(\phi_s + \phi_c)}{2} \right]$$

$$P_g = \frac{1}{2} (R/Q) Q_{\text{ext}} |I_g|^2$$

$$= \frac{1}{2} (R/Q) Q_{\text{ext}} \left[\frac{|V_{\text{cav}}|}{2(R/Q)Q_{\text{ext}}} + \frac{|F_b|I_{b,\text{dc}} \cos(\phi_s + \phi_c)}{2} \right]^2 + \frac{1}{2} (R/Q) Q_{\text{ext}} \left[\frac{|V_{\text{cav}}|}{(R/Q)} \frac{\Delta\omega}{\omega_{\text{rf}}} + \frac{|F_b|I_{b,\text{dc}} \sin(\phi_s + \phi_c)}{2} \right]^2$$

Minimized for $Q_{\text{ext,opt}} = \frac{|V_{\text{cav}}|}{|F_b|(R/Q)I_{b,\text{dc}} \cos(\phi_s + \phi_c)}$

= 0 for $\Delta\omega_{\text{opt}} = -\frac{\omega_{\text{rf}}(R/Q)|F_b|I_{b,\text{dc}} \sin(\phi_s + \phi_c)}{2|V_{\text{cav}}|}$

Setting $\phi_c = 0$ recovers classical equations for optimal parameters

Adjusting ϕ_c , $Q_{\text{ext,opt}}$ can be modified to meet certain constraints

The minimum power

$$P_{g,\text{opt}} = \frac{|V_{\text{cav}}||F_b|I_{b,\text{dc}} \cos(\phi_s + \phi_c)}{2}$$

RF power requirements

Constraints:

- The same $Q_{\text{ext,opt}}$ for all cavities to avoid a movable fundamental power coupler design
- The same $P_{g,\text{opt}}$ to have the identical power sources and uniform power distribution (role of variations is under study)

$$Q_{\text{ext,opt}} = \frac{|V_{\text{cav}}|}{|F_b|(R/Q)I_{b,\text{dc}} \cos(\phi_s + \phi_c)}$$

$$P_{g,\text{opt}} = \frac{|V_{\text{cav}}||F_b|I_{b,\text{dc}} \cos(\phi_s + \phi_c)}{2}$$

→ Cavity voltage must be the same for all cavities: $\cos(\phi_s + \phi_{\text{foc}}) = \cos(\phi_s + \phi_{\text{defoc}}) \rightarrow \phi_{\text{foc}} = -2\phi_s - \phi_{\text{defoc}}$

Starting with energy gain per turn

$$N_{\text{foc}}|V_{\text{cav}}| \cos(\phi_s + \phi_{\text{foc}}) + N_{\text{defoc}}|V_{\text{cav}}| \cos(\phi_s + \phi_{\text{defoc}}) = V_{\text{tot}} \cos \phi_s$$

$$N_{\text{foc}} \frac{|V_{\text{cav}}||F_b|I_{b,\text{dc}} \cos(\phi_s + \phi_{\text{foc}})}{2} + N_{\text{defoc}} \frac{|V_{\text{cav}}||F_b|I_{b,\text{dc}} \cos(\phi_s + \phi_{\text{defoc}})}{2} = \frac{|F_b|I_{b,\text{dc}}}{2} V_{\text{tot}} \cos \phi_s$$

$$N_{\text{foc}}P_{g,\text{foc}} + N_{\text{defoc}}P_{g,\text{defoc}} = I_{b,\text{dc}}U_0 = P_{\text{SR}}$$

$$P_{g,\text{opt}} = \frac{P_{\text{SR}}}{N_{\text{tot}}}$$

$$\times \frac{|F_b|I_{b,\text{dc}}}{2}$$

$$\cos \phi_s = \frac{U_0}{V_{\text{tot}}} \quad |F_b| \approx 2$$

$$P_{g,\text{foc}} = P_{g,\text{defoc}} = P_{g,\text{opt}}$$

$$N_{\text{foc}} + N_{\text{defoc}} = N_{\text{tot}}$$

→ No RF power overshoot is needed for RPO if optimal detuning and optimal quality factor are used

Reverse phasing mode equations

Preservation of energy gain

$$N_{\text{foc}}|V_{\text{cav}}| \cos(\phi_s + \phi_{\text{foc}}) + N_{\text{defoc}}|V_{\text{cav}}| \cos(\phi_s + \phi_{\text{defoc}}) = V_{\text{tot}} \cos \phi_s$$

Preservation of synchrotron tune

$$N_{\text{foc}}|V_{\text{cav}}| \sin(\phi_s + \phi_{\text{foc}}) + N_{\text{defoc}}|V_{\text{cav}}| \sin(\phi_s + \phi_{\text{defoc}}) = V_{\text{tot}} \sin \phi_s$$

→ Cavity voltage

$$|V_{\text{cav}}| = \frac{V_{\text{tot}}}{N_{\text{tot}}} \sqrt{\frac{U_0^2}{V_{\text{tot}}^2} + \left(1 - \frac{U_0^2}{V_{\text{tot}}^2}\right) \frac{N_{\text{tot}}^2}{(N_{\text{foc}} - N_{\text{defoc}})^2}}$$

Optimal detuning

$$\Delta\omega_{\text{opt}} = -\frac{\omega_{\text{rf}}(R/Q)|F_b|I_{b,\text{dc}}}{2V_{\text{cav}}} \sqrt{1 - \frac{U_0^2}{V_{\text{cav}}^2 N_{\text{tot}}^2}}$$

See, also [A. Blednykh et al, EIC-ADD-TN-33, 2022](#)

Phases

$$\phi_{\text{foc}} = -\phi_s + \arccos\left(\frac{V_{\text{tot}} \cos \phi_s}{N_{\text{tot}} V_{\text{cav}}}\right) \quad \phi_{\text{defoc}} = -\phi_s - \arccos\left(\frac{V_{\text{tot}} \cos \phi_s}{N_{\text{tot}} V_{\text{cav}}}\right)$$

The aim is to keep V_{cav} , $P_{g,\text{opt}}$, and $Q_{\text{ext,opt}}$ for Z, W, and ZH modes

→ Cavity voltage can be change in discrete steps of $N_{\text{foc}} - N_{\text{defoc}} = 2, 4, \dots$

Critical impact of spread

A coherent beam-beam instability, X-Z instability ([K. Ohmi, 2016](#))

$$\text{Threshold} \propto \frac{Q_s}{\xi_x}$$

Working point chosen between resonances
 $nQ_x + mQ_s = 1$ and $nQ_x + (m + 1)Q_s = 1$

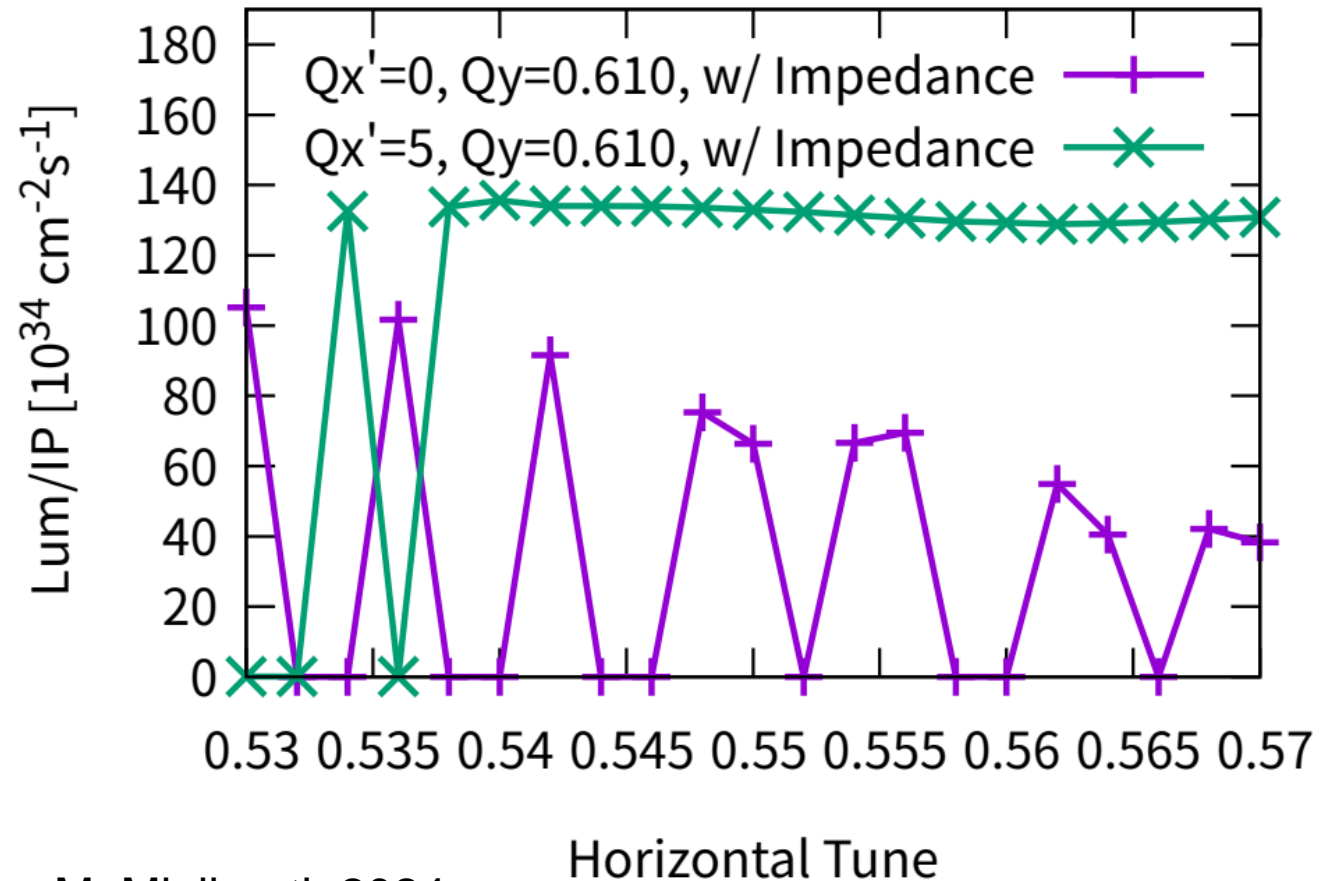
Higher Q_s

→ stronger low order resonance but more space available between them

Bunches are ~50% shorter (assuming the same σ_δ)

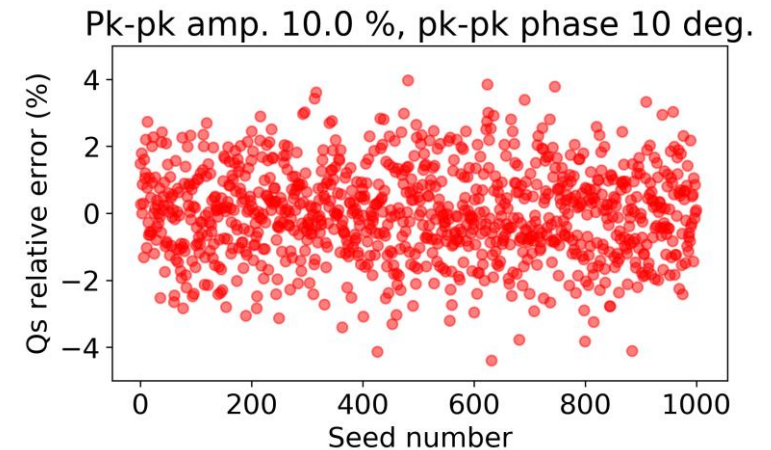
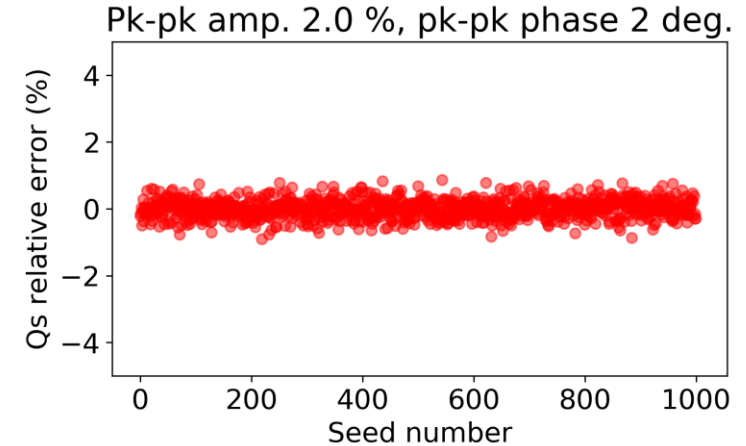
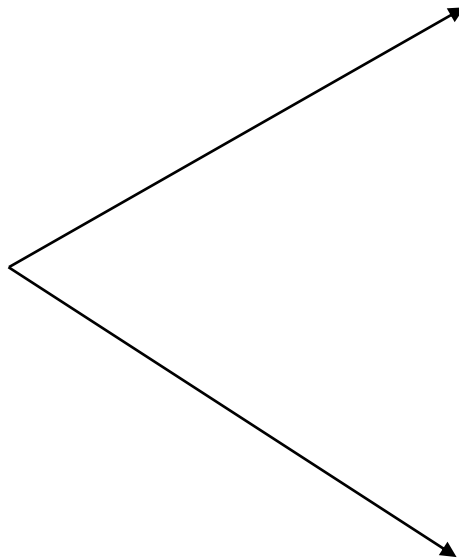
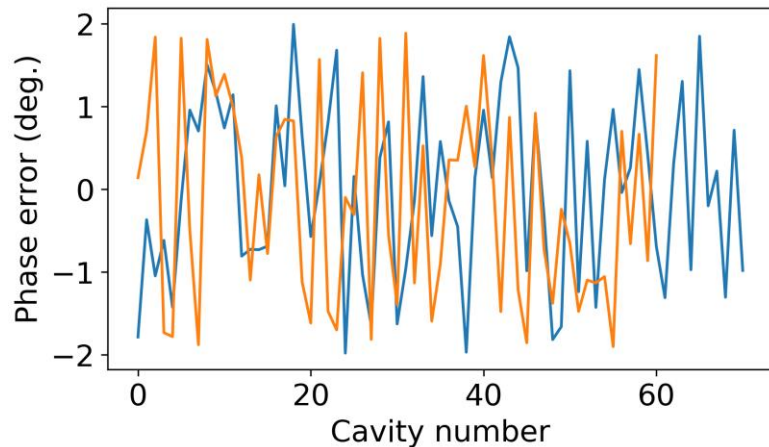
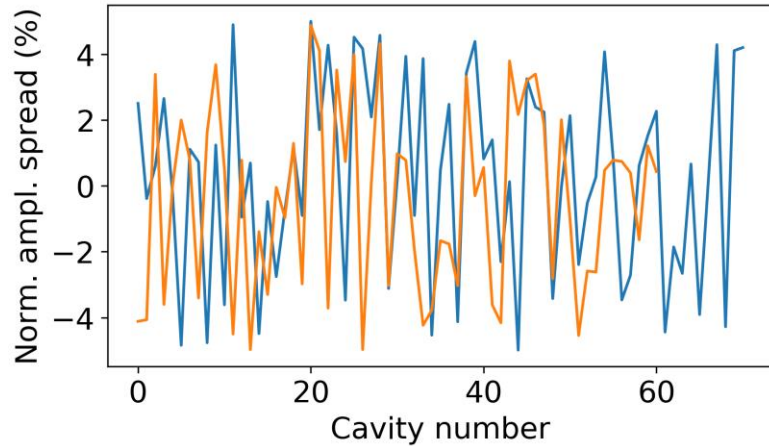
→ stronger beamstrahlung
(ξ_y increase is smaller)

→ stronger impact of longitudinal impedance?



M. Migliorati, 2024

Parameter sensitivity of RPO



Small, but visible impact of parameter spread on global parameters (e.g., Q_s)

Steady-state beam loading

RF power for SRF cavities with circulators is minimized for optimal parameters:

$$\text{Optimal detuning } \Delta\omega_{opt} = -\frac{\omega_{RF}(R/Q) I_{b,DC} \sin \phi_s}{V_{cav}}$$

$$\text{Optimal quality factor } Q_{L,opt} = \frac{V_{cav}}{2(R/Q) I_{b,DC} \cos \phi_s}$$

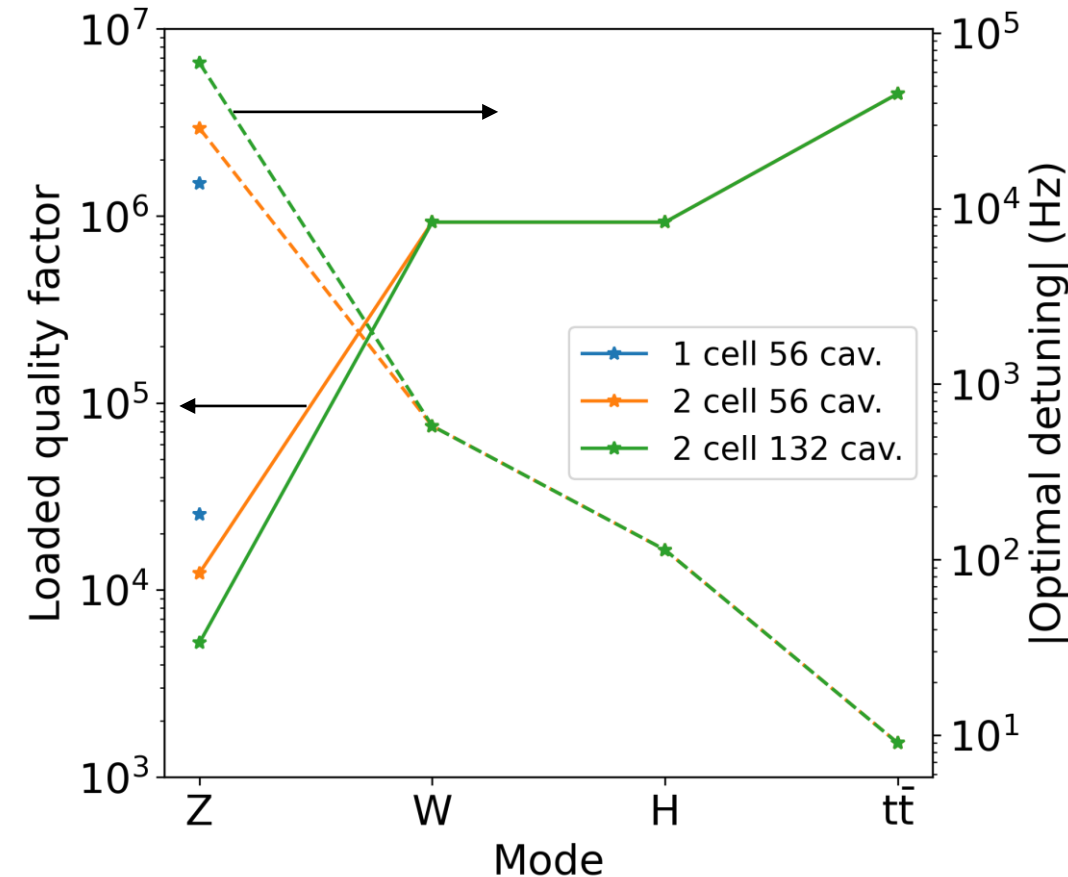
Increasing (R/Q) (43.8→90.6 Ohm) and reducing V_{cav} :

→ Large range for $Q_{L,opt}$ adjustment (a factor of ~75-600) starting from $\sim 5 \times 10^3$: possible FPC solutions under study (*S. Gorgi Zadeh and E. Montesinos, CERN SRF, 2024*;

see also slides of F. Gerigk, FCC week 2024)

→ Increased detuning enhances instability due to fundamental mode

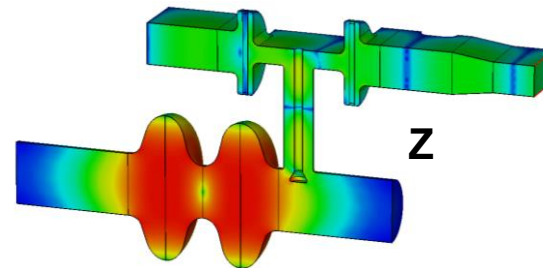
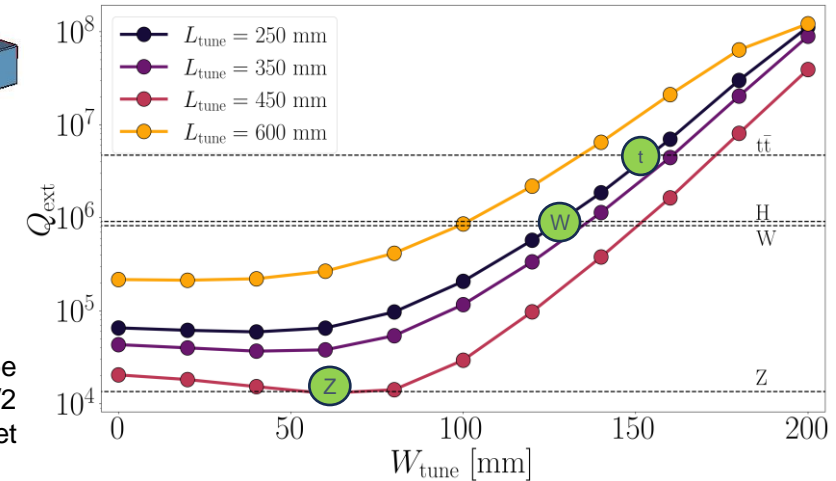
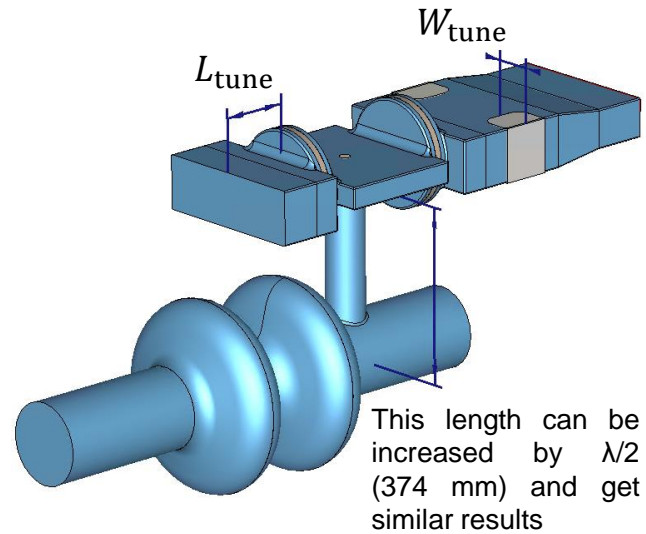
Optimal parameters for different scenarios



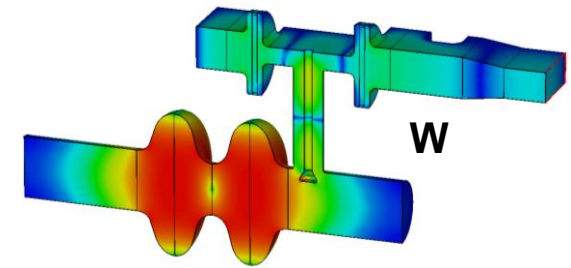
Can the total voltage be increased for Z mode?

New adjustable power coupler concept

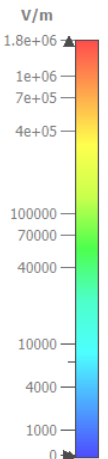
- Two windows (LINAC4-like) are placed outside the cryomodule → integration challenge after cryostating
- A more robust and easier-to-cool down window design → easier to cover all four working points
- Providing ~1 MW in CW operation with variable coupling for Z, W, H, $t\bar{t}$.
- HOM power see: [F. Peauger & I. Karpov](#)
- Prototyping starting.



$L_{\text{tune}} = 450 \text{ mm},$
 $W_{\text{tune}} = 55 \text{ mm},$
 $Q_{\text{ext}} = 1.33e4,$
 $E_{\text{pk-window}} = 0.18 \text{ MV/m},$
 $E_{\text{pk-air}} = 0.22 \text{ MV/m}$



$L_{\text{tune}} = 250 \text{ mm},$
 $W_{\text{tune}} = 127 \text{ mm},$
 $Q_{\text{ext}} = 8.48e5,$
 $E_{\text{pk-window}} = 0.35 \text{ MV/m},$
 $E_{\text{pk-air}} = 0.61 \text{ MV/m}$



E normalized to 1J stored energy