Spin Polarization in the EIC Electron Storage Ring

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Motivation

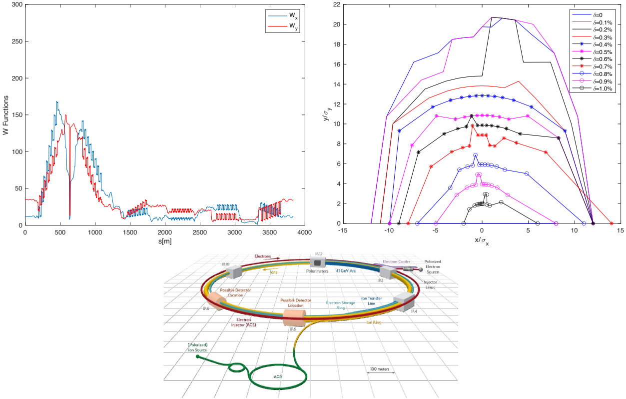


Figure: On the left, the W functions with the semi-local chromatic correction, on the right, the dynamic aperture of the optimized lattice, at the bottom the layout of EIC.

- Can we do similar thing for the spin?
- More information can be found in my talk in FCC Week 2024

Bargmann-Michel-Telegdi Equation in Magnet

The equation that governs the precession of spin vector $\vec{S}$ in presence of the magnetic field is given by $\frac{d\vec{S}}{ds} = \vec{\Omega} \times \vec{S}$ and

$$\vec{\Omega} = -\frac{1 + x/\rho}{1 + \delta}\{[1 + a\gamma_0(1 + \delta)]\vec{b}_\perp + (1 + a)\vec{b}_{||}\} + \frac{1}{\rho}\hat{y}.$$

- $a = 0.00115965218$: the anomalous magnetic moment of electron
- γ_0: Lorentz factor at the designed energy
- $\vec{b}$: instantaneous magnetic field normalized with respect to magnetic rigidity $B_0\rho$
- $\vec{b}_\perp$ and $\vec{b}_{||}$ are respect to the velocity of the moving particle

It is easy to implement this precession in the codes with symplectic integrators. At each integration step, one has to work out a rotation matrix assuming $\vec{\Omega}$ is a constant.

Spin Rotators in the EIC Electron Storage Ring

The spin rotators are shown in

At 18 GeV, only the long modules are used. In each solenoid module, the horizontal spin matching can be achieved by a block-diagonal matrix

$$M_x = \begin{pmatrix} -\cos[2K_sL(1+a)] & -\frac{2}{K_s}\sin[2K_sL(1+a)] \\ \frac{K_s}{2}\sin[2K_sL(1+a)] & -\cos[2K_sL(1+a)] \end{pmatrix},$$

and $M_y = -M_x$ between two equally strength solenoids.

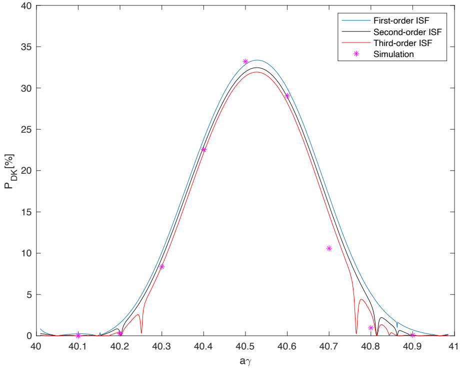
V. Ptitsyn, *Spin Matching derivation*. Brookhaven National Laboratory, 2021

Equilibrium Polarization in the Electron Storage Ring

The polarization is calculated using the Derbenev-Kondratenko formula,

$$P_{DK} = \frac{8}{5\sqrt{3}} \frac{< \oint ds(\hat{n}_y - \gamma\partial\hat{n}_y/\partial\gamma)/|\rho^3| >}{< \oint ds[1 - \frac{2}{9}\hat{n}_z^2 + \frac{11}{18}(\gamma\partial\hat{n}/\partial\gamma)^2]/|\rho^3| >}$$

where $\hat{n}$ is the invariant spin field: $R(z) \cdot \hat{n}(z) = \hat{n}(\mathcal{M} \circ z)$ and $<>$ denotes the average over the orbital distribution in the phase space.



Chromatic Spin in the Lattice

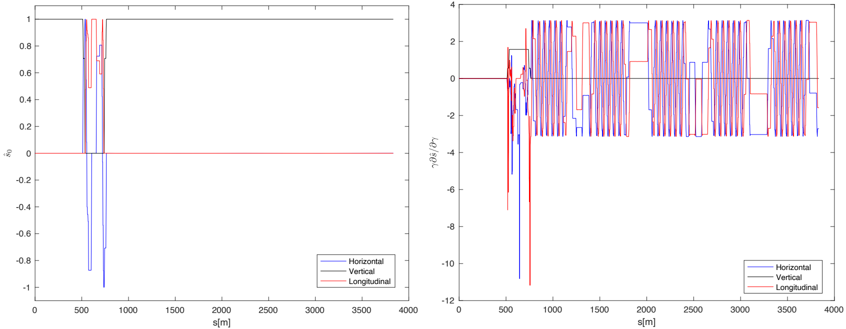


Figure: On the left, the close orbit of spin and on the right, its first derivative respect to δ.

- The spin is not longitudinally matched

Semi-Local Spin Matching in the Lattice

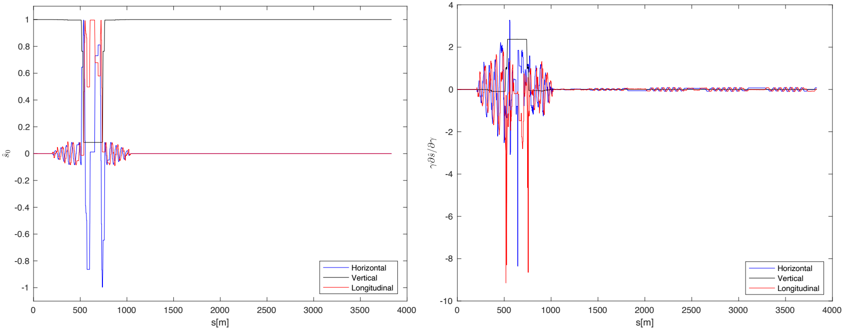
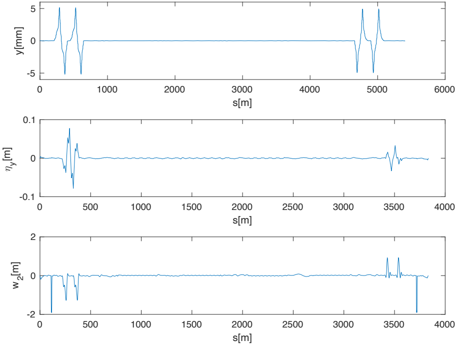


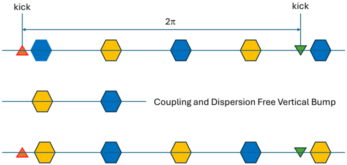
Figure: On the left, the close orbit of spin and on the right, its first derivative respect to δ.

- The spin is longitudinally matched along with the spin closed orbit
- Use four vertical bumps without leaking out coupling and vertical dispersion

Orthogonal Vertical Bumps



where w_2 is the coupling parameter from the horizontal angle to the vertical position.



Equilibrium Polarization after Longitudinal Matching

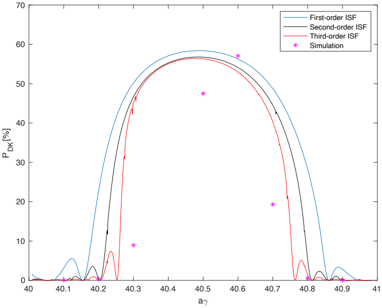


Figure: The Derbenev-Kondratenko polarizations.

- Equilibrium polarization is nearly doubled

Spin Diffusion Vector: $\gamma\partial\hat{n}/\partial\gamma$ in the ESR

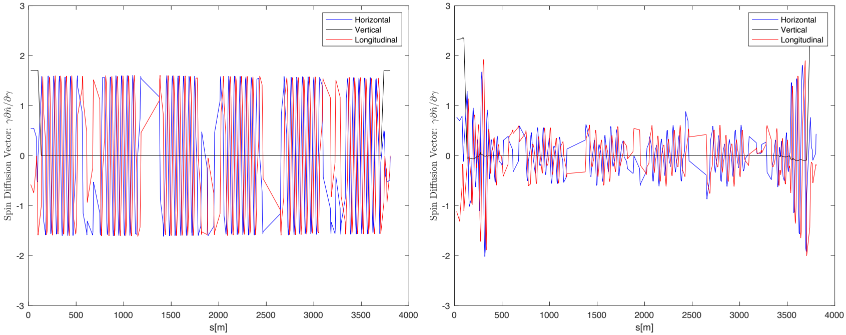


Figure: The Derbenev-Kondratenko diffusion vector without (left) and with (right) the orthogonal vertical bumps.

Dynamic Aperture of the Lattice with the Bumps

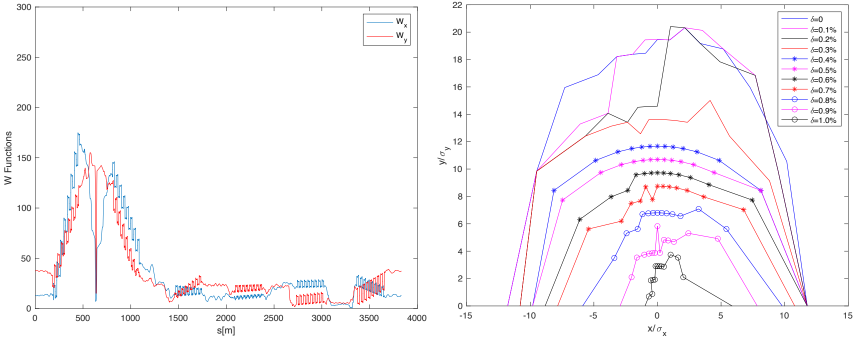


Figure: On the left, the W functions and on the right, the dynamic aperture of the lattice with the orthogonal vertical bumps.

Depolarization Time in the Electron Storage Ring

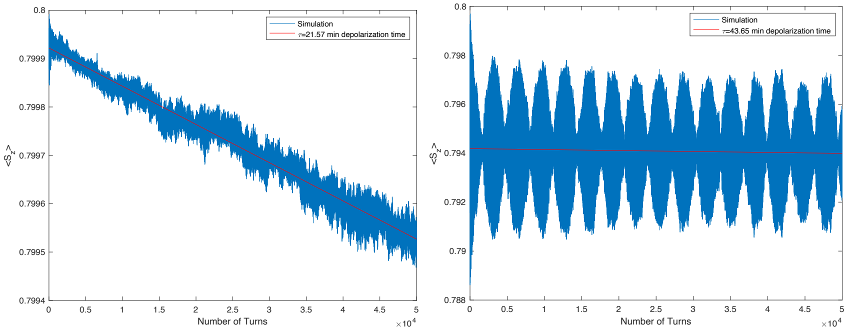


Figure: On the left, an element-by-element tracking without (left) and with (right) the orthogonal vertical bumps.

Yunhai Cai, SLAC-PUB-7793, 1998

Comparison between theory and tracking

The depolarization time can be calculated using

$$\tau_{dep}^{-1} = \frac{55\sqrt{3}cr_e\lambda_C\gamma^5}{144C_{cir}} < \oint ds(\gamma\partial\hat{n}/\partial\gamma)^2/|\rho^3| > .$$

A comparison between the theory and the simulations is summarized in the table

τ_{dep} [minutes]	Without Bumps	With Bumps
First-Order	21.20	76.74
Second-Order	20.27	69.11
Third-Order	19.73	67.43
Tracking	21.57	43.65

Polarization and Time Partially Based on Tracking

Given the inverse of Baier and Katkov polarization time

$$\tau_{BK}^{-1} = \frac{5\sqrt{3} c r_e \lambda_C \gamma^5}{8 C_{cir}} < \oint ds (1 - \frac{2}{9}\hat{n}_z^2)/|\rho^3| >,$$

we have $\tau_{DK}^{-1} = \tau_{BK}^{-1} + \tau_{dep}^{-1}$. If we can use the tracking value for τ_{dep}, to calculate the total polarization time we can use $\tau_{tot}^{-1} = \tau_{BK}^{-1} + \tau_{dep}^{-1}$. Then the equilibrium polarization is given by

$$P_{tot} = P_{BK} \frac{\tau_{tot}}{\tau_{BK}},$$

where

$$P_{BK} = \frac{8}{5\sqrt{3}} \frac{< \oint ds (\hat{n}_y - \gamma \partial \hat{n}_y / \partial \gamma)/|\rho^3| >}{< \oint ds (1 - \frac{2}{9}\hat{n}_z^2)/|\rho^3| >}.$$

At $\gamma a = 40.5$, we have $\tau_{BK} = 33.36$ minutes and $P_{BK} = 84.58\%$. Using the tacking value for the depolarization time $\tau_{dep} = 17.57$ minutes, we have $\tau_{tot} = 13.10$ minutes and $P_{tot} = 33.21\%$.

Filling Time for the Electron Storage Ring

Given the initial polarization P_i and the equilibrium time τ and polarization P_o, the polarization evolves according to

$$P(t) = P_o + (P_i - P_o)e^{-\frac{t}{\tau}},$$

and the average polarization within time T_0 is

$$\bar{P} = P_o + (P_i - P_o)\frac{\tau}{T_0}(1 - e^{-\frac{T_0}{\tau}}).$$

Since we know the required average polarization $\bar{P}$, we solve this equation numerically for the required filling time T_0. Given $\tau = 13.10$ minutes, $P_o = 33.21\%$, $P_i = 80\%$, and $\bar{P} = 70\%$ for the up spin (in the arcs), we have $T_0^+ = 6.57$ minutes. For the down spin, we have $P_i = -80\%$, $\bar{P} = -70\%$, and $T_0^- = 2.46$ minutes.

Larger than 70% Polarization in the ESR

T_0^+ for up spin and T_0^- for down spin in the arcs. We assumed 80% injected polarization.

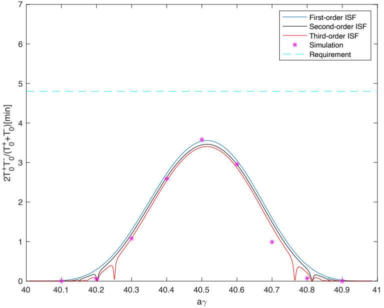


Figure: The stored times without (left) and with (right) the orthogonal vertical bumps.

Requirement: $\frac{1}{T_0^+} + \frac{1}{T_0^-} \leq \frac{2}{4.8}\ \mathrm{min}^{-1}$ (two bunch injection rate)

Conclusion

- Developed normal forms both for orbital motion and spin precession up to arbitrary order
- Applied them to construct invariant spin field: $\hat{n}(z)$
- Used invariant spin field to compute DK equilibrium polarization and polarization time in electron storage rings
- Differential algebra and Lie algebra especially SO(3) Lie algebra are essential
- Developed a semi-local spin matching scheme to improve the polarization in electron storage rings
- We have a nonlinear lattice that is quite close to satisfy the requirements for both dynamic aperture and polarization with one interaction points near 18 GeV

Acknowledgements

- A. W. Chao for continuous encouragements
- D. P. Barber for helpful and stimulating discussions
- D. Sagan and G. H. Hoffstaetter for discussions of quaternion

- The work for the EIC is inspired by the works on the lattice with one interaction point near 18 GeV of Eliana Gianfelice-Wendt with harmonic bumps and Matthew Signorelli using vertical π bumps