Fundamental Physics

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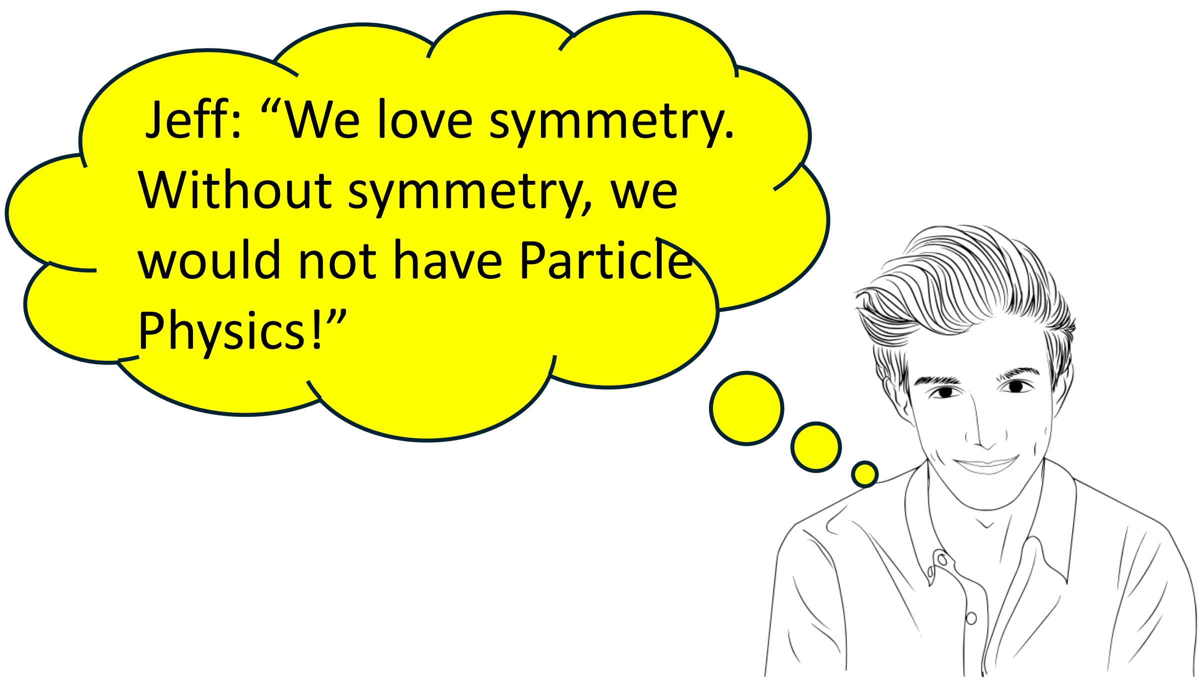
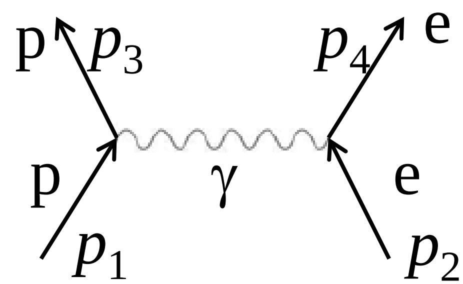
Fundamental Physics

- Gauge theory: from symmetry to interaction
- Some open problems:
- Gravity – how to surpass Einstein?
- Matter-antimatter asymmetry – why have we not been annihilated?

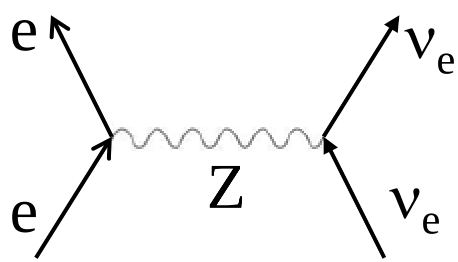


Some interactions in particle physics

- EM: exchange of virtual photons γ
  changes particles' momenta
  γ carries $p = p_3 - p_1 = p_2 - p_4$

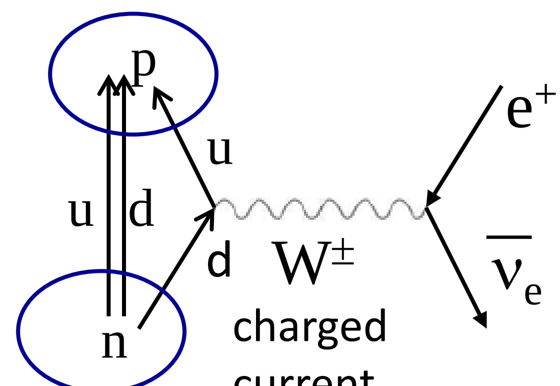


- Weak: exchange of $W^{\pm}$, Z
  May change charge/flavor of a quark



neutral current

Protron: uud

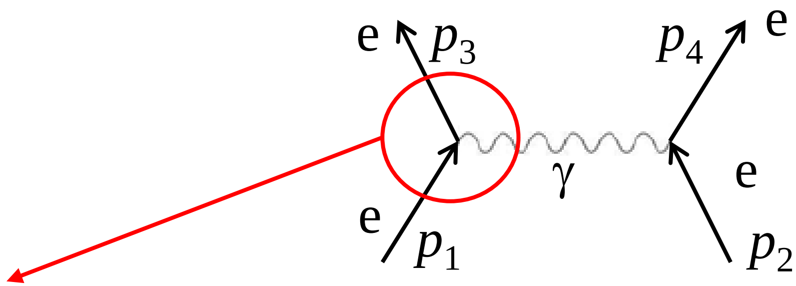


Neutron: udd

$E^2 = m^2 + p^2 \; (c = 1)$

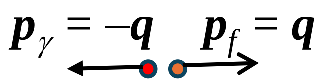
$E_\gamma = p_\gamma \qquad (c = 1)$

Virtual vs. real particles



But: cannot conserve momentum and energy at the same time for the electron to emit a photon!

Rest frame of e before emission: $\boldsymbol{p}_i = 0$, $E_i = m_e$



After emission: if momentum is conserved, $\boldsymbol{p}_\gamma = -\boldsymbol{q} \Rightarrow E_\gamma = q$

$E_f = (m_e^2 + q^2)^{1/2} + E_\gamma > E_i$

Energy is not conserved!

$\therefore\; \gamma \neq$ a real photon. It's a virtual photon allowed by the uncertainty principle – some energy is 'borrowed' from vacuum, returned when the virtual photon is absorbed and disappeared! One never observes a virtual photon.

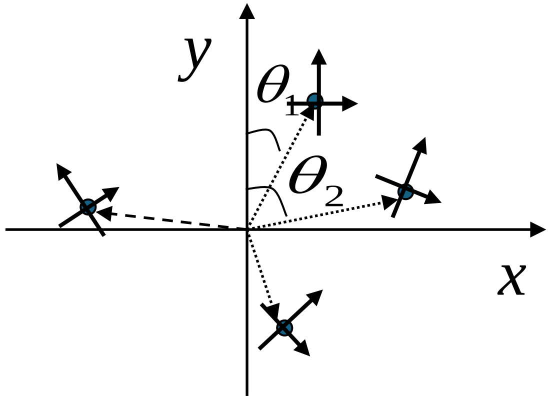
Gauge Symmetry

- **Global gauge symmetry**: a coordinate transformation independent of (x, t) that leaves the system unchanged. E.g. rotating the angle coordinates of all particles

$$\theta_i \rightarrow \theta_i + \theta_o$$

↑ constant, same for all particles anywhere

$\Delta\theta_{ij} \equiv \theta_i - \theta_j$ independent of θ_o



Can we make the symmetry local $\theta_o(x)$?

The choice of coordinate systems should not affect the physics!

- Free particle Schrödinger Equation:

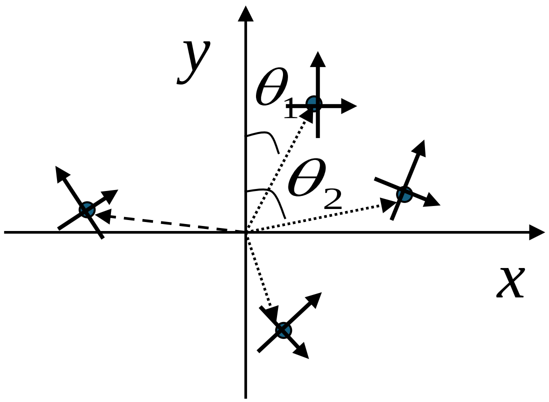
$$i\hbar\partial_t\psi = -\frac{\hbar^2}{2m}\nabla^2\psi \qquad \psi = |\psi|e^{i\theta}$$

Global U(1) symmetry: $\theta \rightarrow \theta + \theta_o$ physics unchanged

Schrödinger Equation is global U(1) invariant.

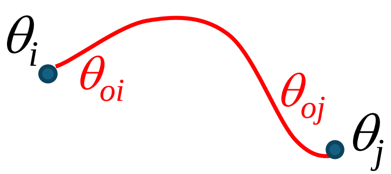
Local Gauge Symmetry

- Can we make the symmetry local?
- $\theta_o(x, t)$ or $\theta_{oi}(t)$?



Yes, the choice of coordinate systems should not affect the physics!

- Yes, as long as each particle also carries $\theta_{oi}(t)$ and let others know: imagine putting in a string between each pair of particles carrying the information $\theta_{oi}(t)$ and $\theta_{oj}(t)$.



$$\Delta\theta_{ij} \equiv \theta_i - \theta_j + \underbrace{\theta_{oi} - \theta_{oj}}_{\nabla\theta}$$

Symmetry: physics independent of coordinate systems

$\rightarrow$ Interaction: keep track of differences

An analogy with global time

Why not?

Each location can freely choose $\theta(x)$ its local time, generally different from the time in other locations

Each location may even change the definition of time (e.g. summer/winter time) $\theta(x, t)$

Need to communicate the time difference for travelers etc. $\nabla\theta$ = a U(1) 'rotation'

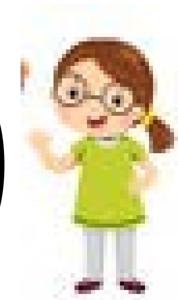
U(1) symmetry

An analogy with colors

$$\begin{pmatrix} r \\ g \\ b \end{pmatrix}$$

Each person can freely choose his/her color scheme, generally different from those for others

$$\begin{pmatrix} r \\ g \\ b \end{pmatrix}$$



Each person may even change the color scheme over time

Need to communicate the color differences

~ rotation matrix λ_{ij} between any two persons

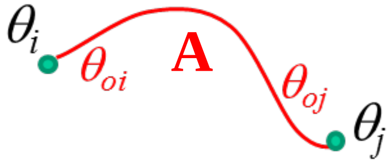
$$\begin{pmatrix} r \\ g \\ b \end{pmatrix}$$

$$\begin{pmatrix} r \\ g \\ b \end{pmatrix}$$

SU(3) symmetry

An Analogy with Exchange Rates (after Prof. Kenneth Young)

- Many different 'dollars': RMB, HK$, US$, …
- Locally, usually only one is used. Numerical values set by convention only (~θ_o)
- Global gauge symmetry: change all denominations by a common factor (e.g. x2) → nothing is changed.
- Local gauge symmetry: change each 'dollar' arbitrarily by **different** factors **locally** e.g. change 1 HK$ to 2 old HK$, 1 US$ to 0.5 old US$, … (~local gauge transformation)
- Must have exchange mechanisms to restore invariance: the **exchange rate (~$\mathbf{A}$)** should change accordingly. E.g. 1US$ = 7.8HK$ →1US$=31.2HK$ (Gauge transformation of $\mathbf{A}$)
- Exchange rate ~ gauge field. Properties of $\mathbf{A}$ determined by local gauge transformations.
- $\mathbf{A}$ in general carries 2 information $(\theta_{oi}, \theta_{oj})$



K. Young, Am. J. Phys. **67**, 862 (1999).

Local Gauge Symmetry

ψ = coordinate wave function = $|\psi|e^{i\theta}$

Local U(1) Gauge Symmetry: $\theta \rightarrow \theta + \phi(x)$

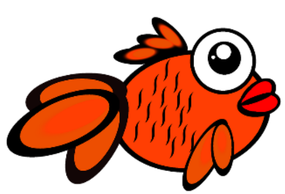
$$\psi \rightarrow \psi' = \psi e^{i\phi(x)}$$

Physics should not depend on the arbitrary phase $\phi(x)$. → local transformation

Free Schrödinger Eq.: $i\hbar\partial_t\psi = -(\hbar^2/2m)\nabla^2\psi$

$$\Rightarrow i\hbar\partial_t\psi' = -(\hbar^2/2m)[\nabla - i\nabla\phi]^2\psi'$$

Free Schrödinger Equation violates local U(1) gauge transformation!



Free Schrödinger Equation

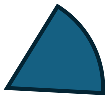
or

local U(1) invariance

Gauge Transformation and Minimal Coupling rule

$\mathbf{B} = \nabla\times\mathbf{A},$

Gauge symmetry of $\mathbf{A}$: $\mathbf{A}' = \mathbf{A} + \nabla\Lambda$



$\Lambda(\mathbf{r}, t)$ = arbitrary function

$\mathbf{B}$ independent of Λ $\because$ $\nabla\times\nabla\Lambda = 0$

$\mathbf{p} = -i\nabla$ (Quantization rule, $\hbar = 1$)

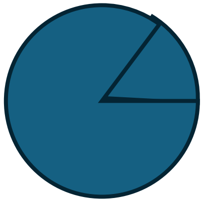
$\psi \rightarrow \psi' = \psi e^{i\phi(x)}$ (local U(1) transformation)

$\mathbf{p}\psi = -i\nabla\psi \rightarrow -i\nabla[\psi e^{i\phi(x)}] = [\mathbf{p} + \nabla\phi(x)]\,\psi e^{i\phi(x)}$

Bad term

Gauge transformations of $\mathbf{A}$ and $\mathbf{p}$ cancel each other if

$\mathbf{p} \rightarrow \mathbf{p} - q\mathbf{A}$ and $q\Lambda = \phi$!



Minimal Coupling rule

$\rightarrow$ Lorentz force (correct EM interaction!)

Rescuring Schrödinger Equation

Coupling the particle to $\mathbf{A}$ ($\mathbf{p} \to \mathbf{p} - q\mathbf{A}$) restores local $\mathrm{U}(1)$ symmetry!

$$i\hbar\partial_t \psi' = -(\hbar^2/2m)[\nabla + (i/\hbar c)q\mathbf{A}]^2 \psi'$$

is local U(1) invariant!

Local $\mathrm{U}(1)$ symmetry $\to$ EM interaction

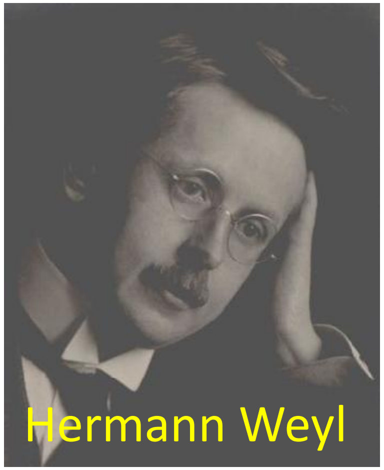
'Phase' = 2x1 matrix (spinor): $\mathrm{SU}(2) \longrightarrow$ Yang-Mills, weak force

'Phase' = 3x1 matrix (color): $\mathrm{SU}(3) \longrightarrow$ strong force

All fundamental interactions are believed to be generated by gauge theories.

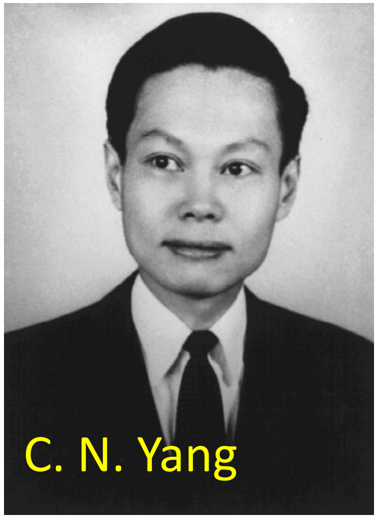
Symmetry $\to$ Dynamics: Gauge theories

Gauge Theories - history



Change of scale (gauge) a local symmetry of General Relativity?

Change of phase for wavefunction $\rightarrow$ EM U(1) gauge theory (Fock, Weyl, London), 1926



Yang & Mills: Non-abelian SU(2) gauge theory 1954

Gauge fields could be massive

But: the mass term of gauge field would violate gauge symmetry. W, Z are massive!

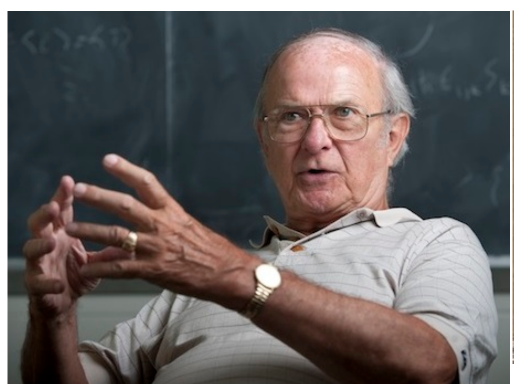
Englert–Brout–Higgs–Guralnik–Hagen–Kibble mechanism

Brout

Englert

Higgs

Guralnik

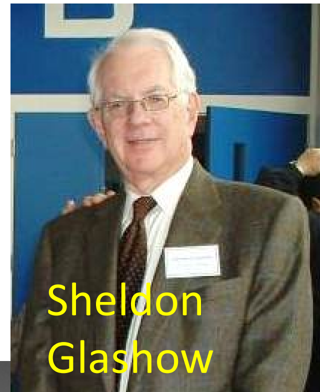


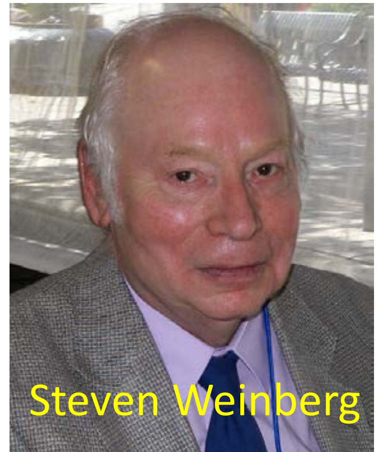
Hagen

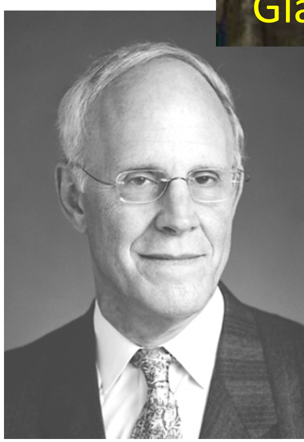
Kibble

Gauge Theories - history

1967: Glashow, Salam, Weinberg: electroweak unification
SU(2)xU(1) gauge theory





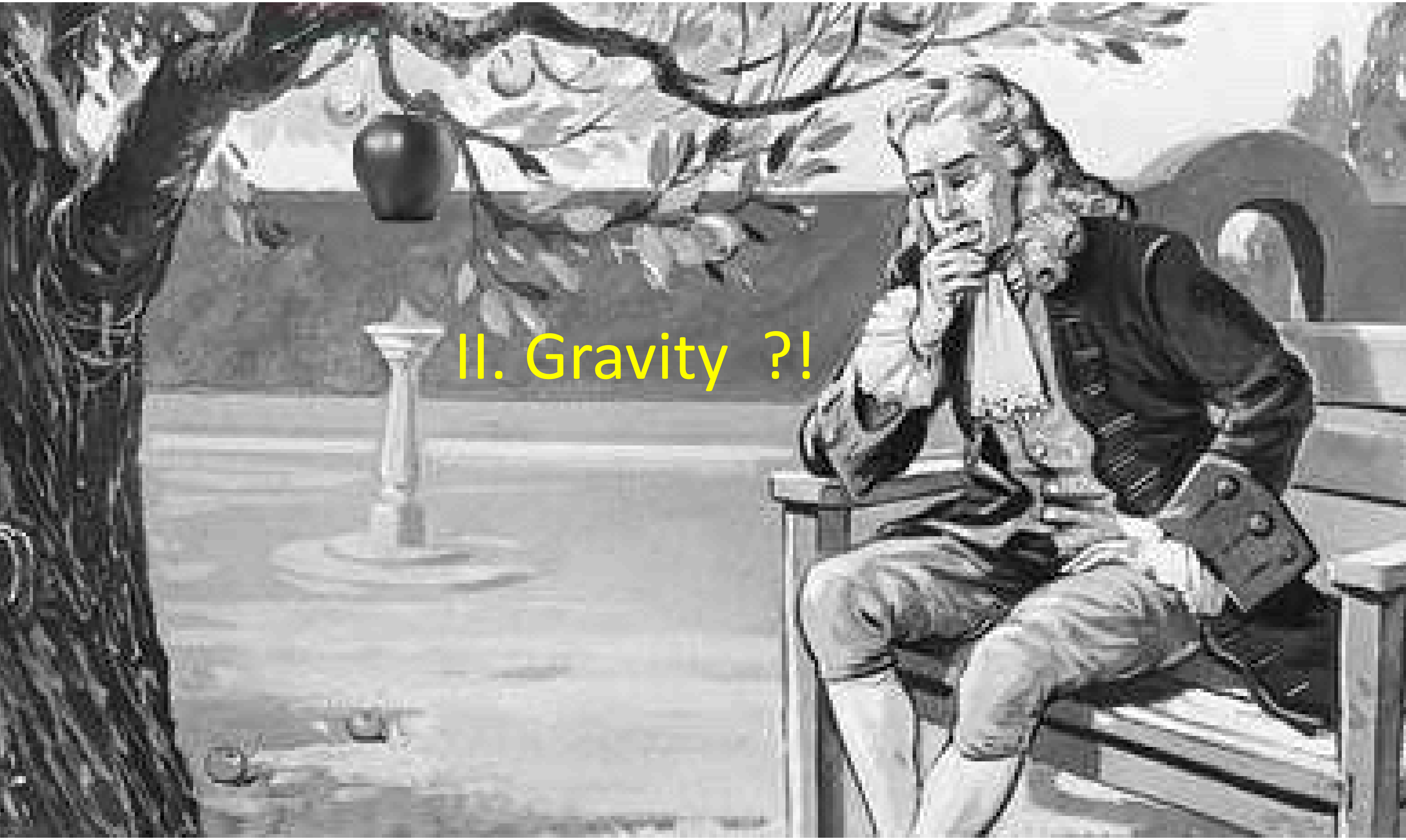

David Gross


David Politzer

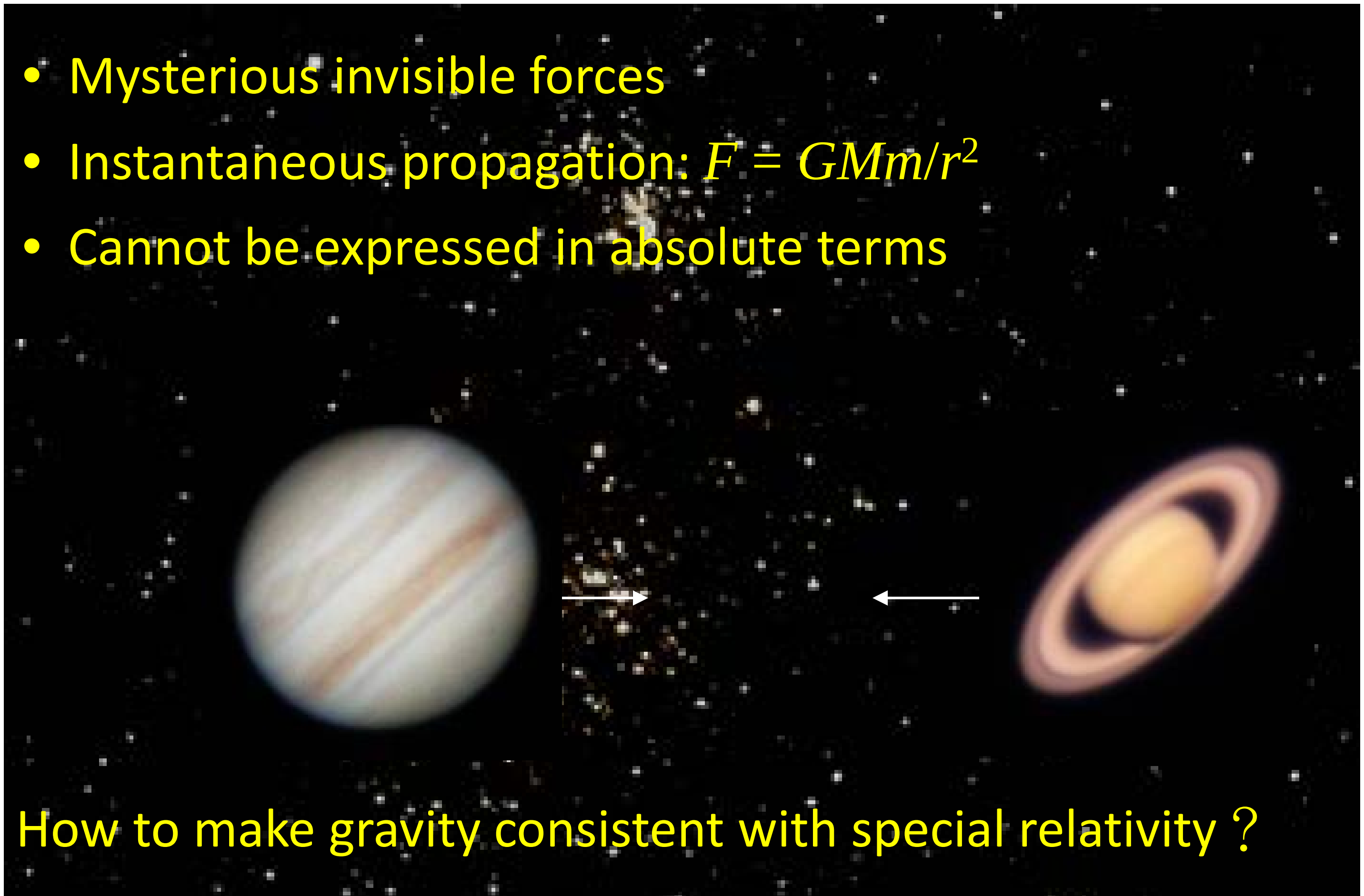

Frank Wilzek

1973: Discovery of asymptotic freedom in strong interaction
SU(3) gauge theory – needed to make field theory consistent,
but only for gauge theories!

II. Gravity ?!



Problems with Newtonian theory



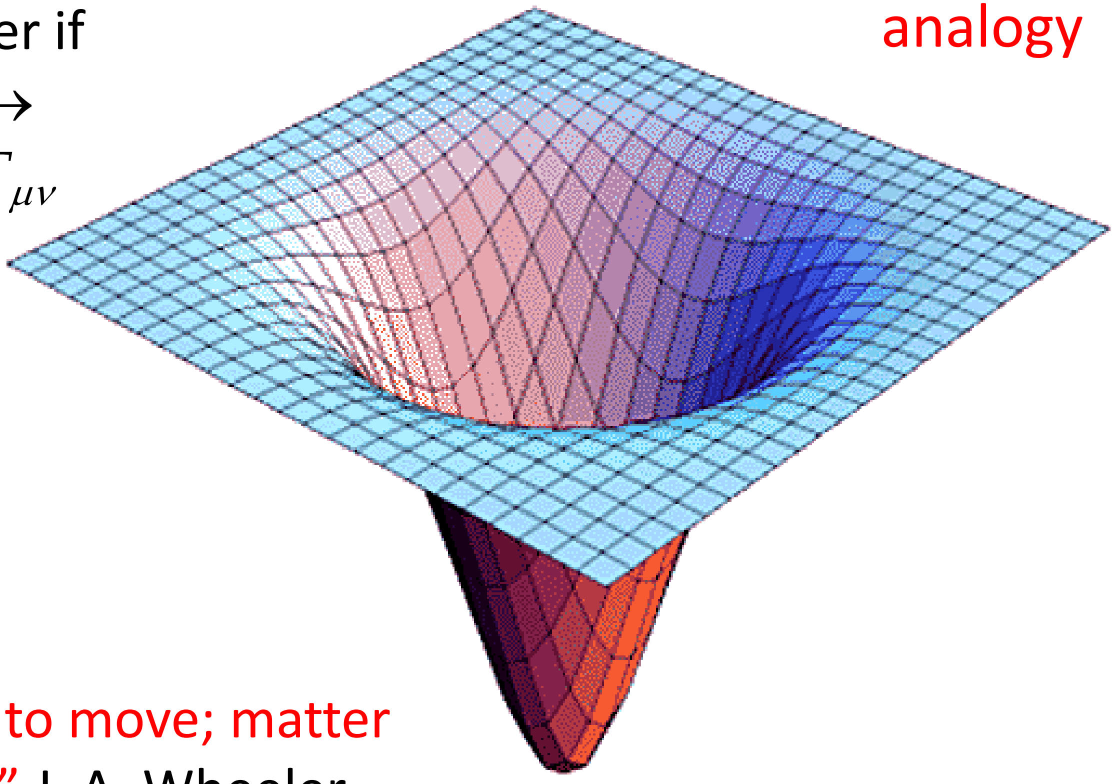
- Mysterious invisible forces
- Instantaneous propagation: $F = GMm/r^2$
- Cannot be expressed in absolute terms

How to make gravity consistent with special relativity？

Mass-energy curves space and time

https://www.youtube.com/watch?v=wrwgIjBUYVc

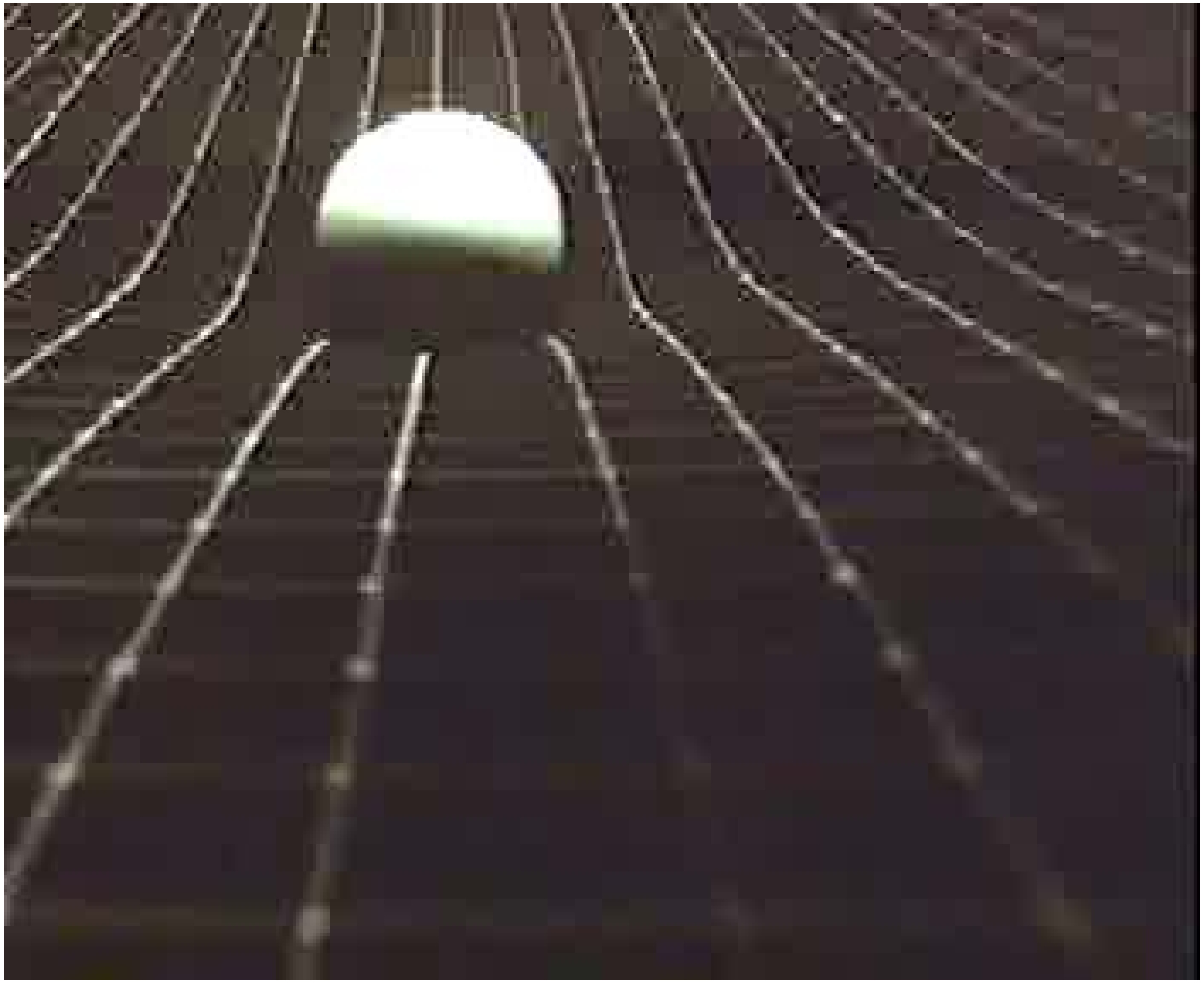
- Matter curves space-time
- Space-time curvature is larger if there's more matter/energy $\rightarrow$ Einstein Equation $G_{\mu\nu} = 8\pi G T_{\mu\nu}$
- Trajectory of an object = **geodesic**



"Spacetime tells matter how to move; matter tells spacetime how to curve" J. A. Wheeler

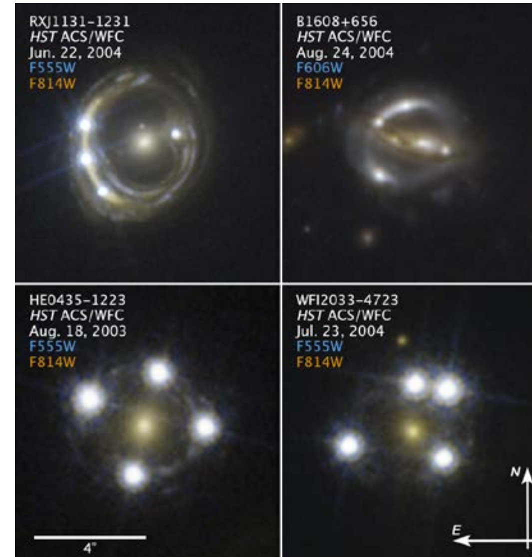
http://www.pitt.edu/~jdnorton/teaching/HPS_0410/chapters/general_relativity_massive/index.html

Space-time curvature

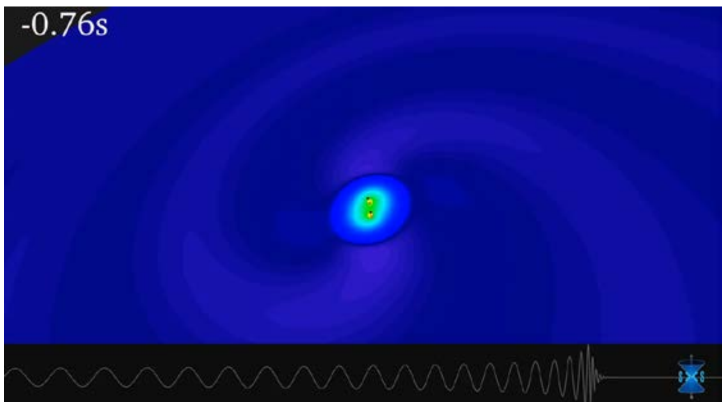


Einstein’s space-time

- Gravitational lens – small warping of space-time
- Gravitational waves – oscillations of space-time
- Expansion of the universe – dynamical space-time
- Black hole – interchanging space and time



Credit: NASA/STScI



Credit: LIGO

Credit: NASA

Credit: NASA's Goddard Space Flight Center/J. Schnittman and B. Powell

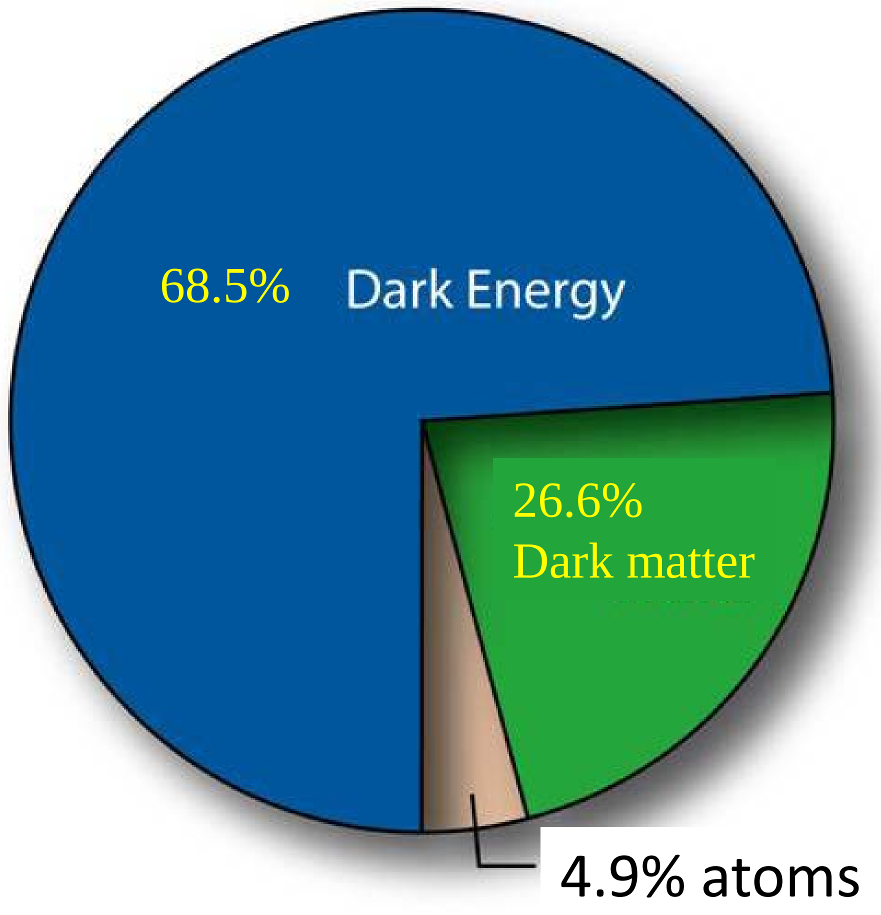
https://svs.gsfc.nasa.gov/14576

BUT …

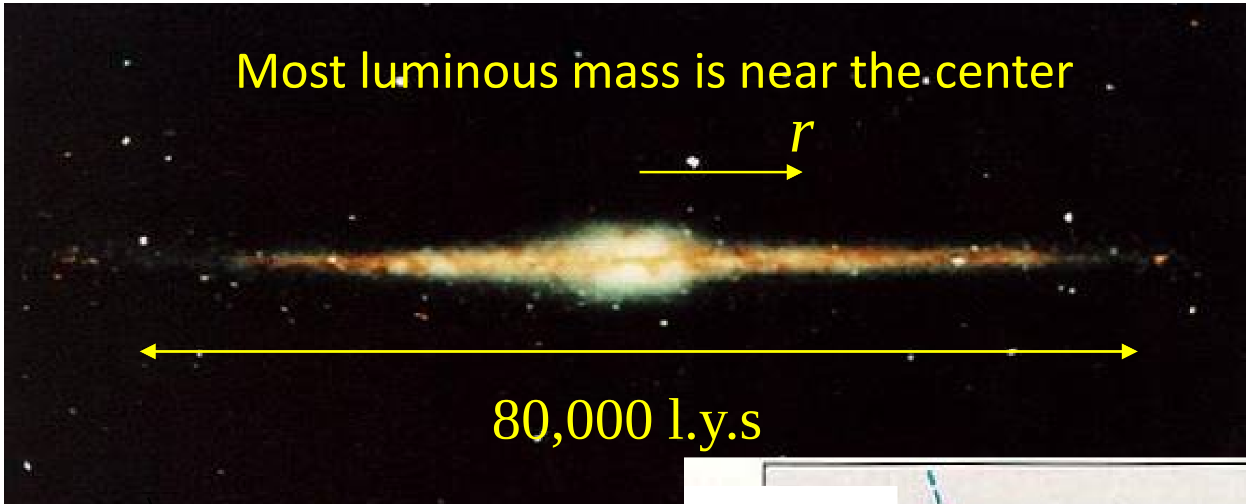
- Nobody knows how to quantize gravity
- GR is not based on gauge field theory
- Gravity is much weaker than other fundamental interactions $\sim 10^{-36}$ EM!
- Dark matter: unknown, invisible sources of gravity
- Dark energy: unknown, invisible source(s) of repulsive gravity

The gravity force of the entire Earth pulling on you is balanced by the EM force between a few atomic layer of Earth's surface and you

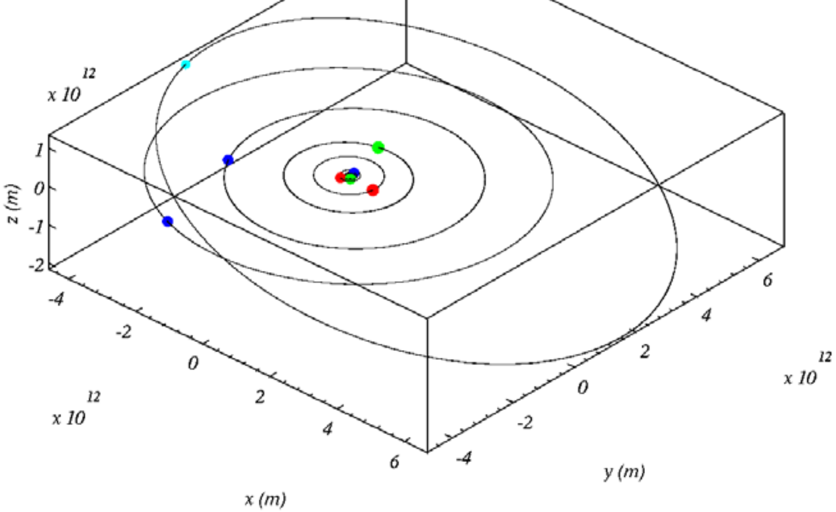
Dark Matter 暗物質

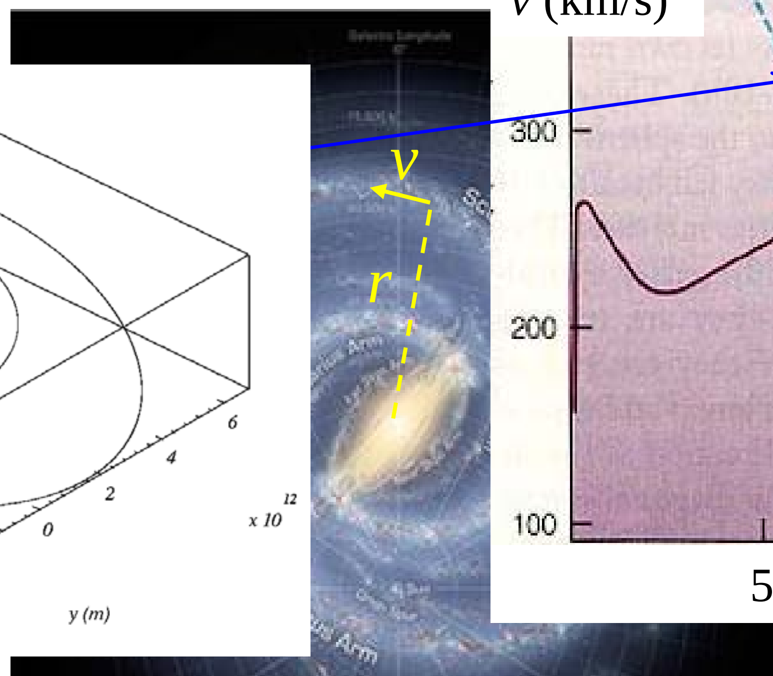


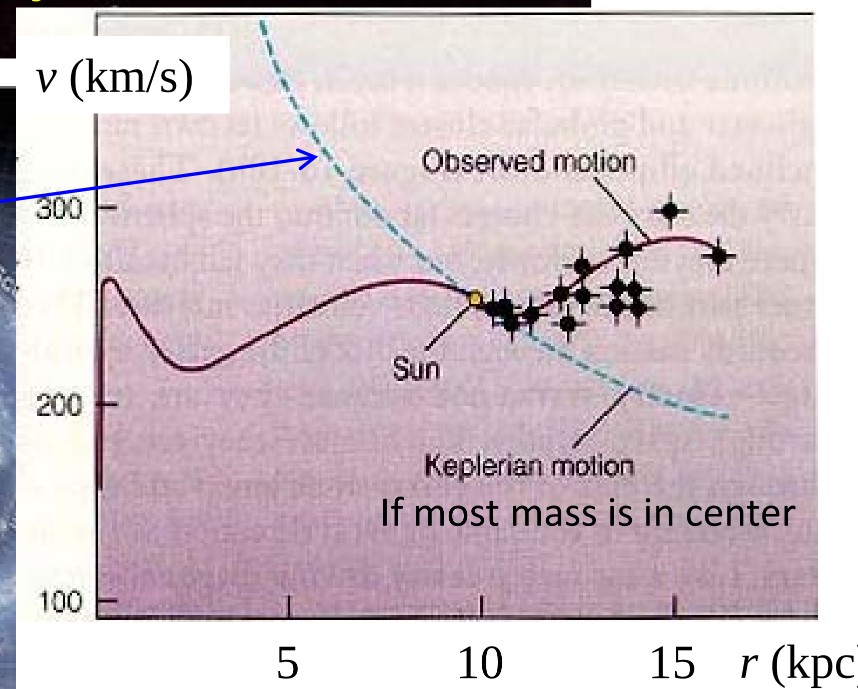
Self-rotation of Milky Way



If most mass is at the center

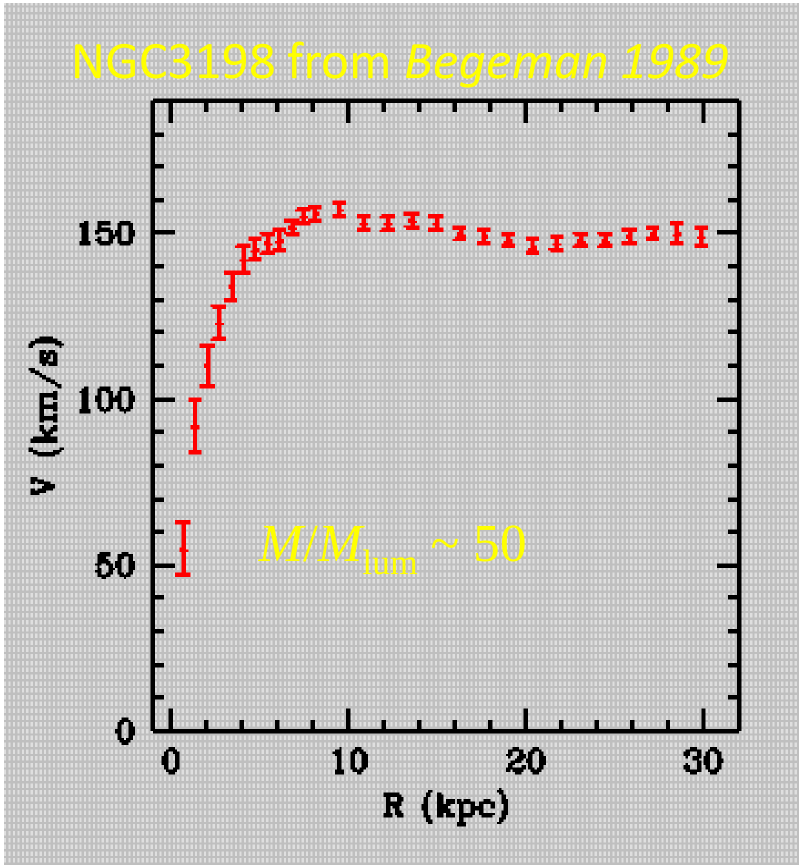




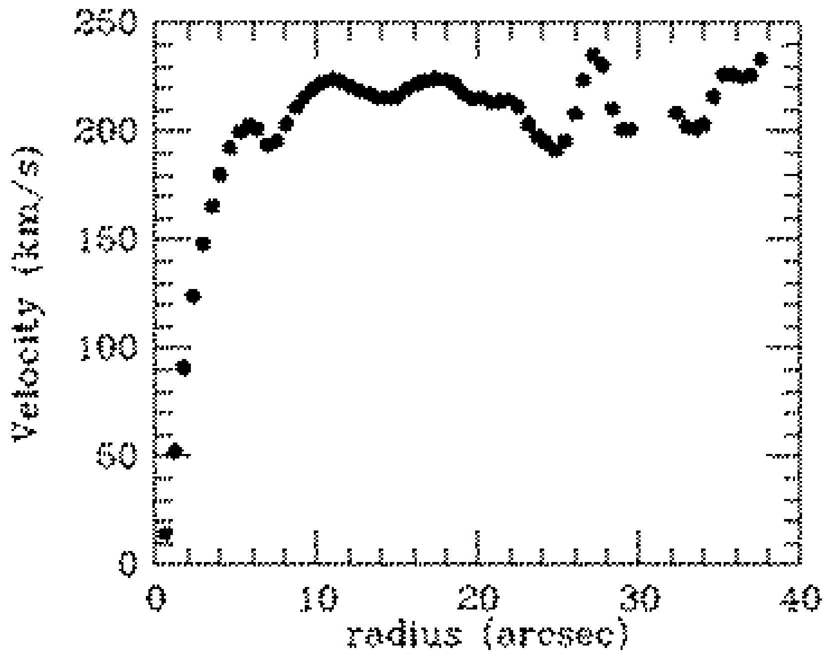


1 kpc ~ 3300 l.y.s

Deduce that Milky Way's mass extends to at least 3-6x10^5 l.y.s, but dark!



UGC9242 from Vogt *et al*.



http://astrosun2.astro.cornell.edu/academics/courses//astro201/rotcurve.htm

circular motion:

For your reference only:

$$\frac{mv^2}{r} = \frac{GM(r)m}{r^2}$$

$M(r)$ = total mass enclosed within r

$$\Rightarrow v(r) = \sqrt{\frac{GM(r)}{r}}$$

$$v \rightarrow \text{constant}$$

$$\Rightarrow M(r) \propto r$$

http://www.astro.queensu.ca/~dursi/dm-tutorial/rot-vel.html

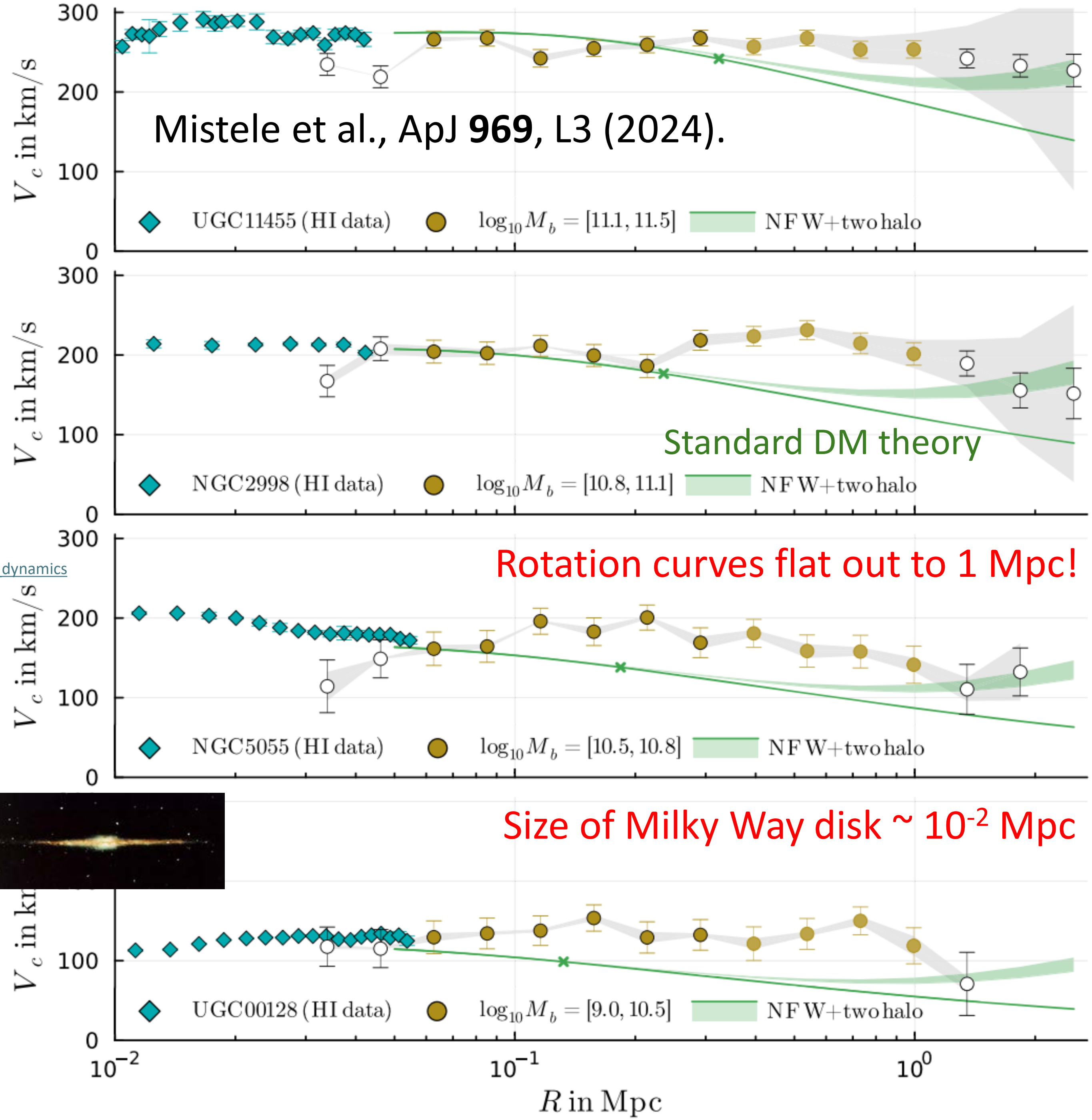
MOND:

$$F = m\mu(a/a_o)a$$

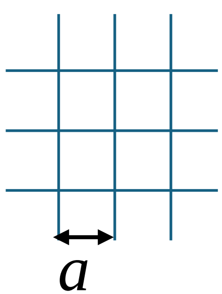
$$\mu(x) \rightarrow 1 \text{ if } a >> a_o$$

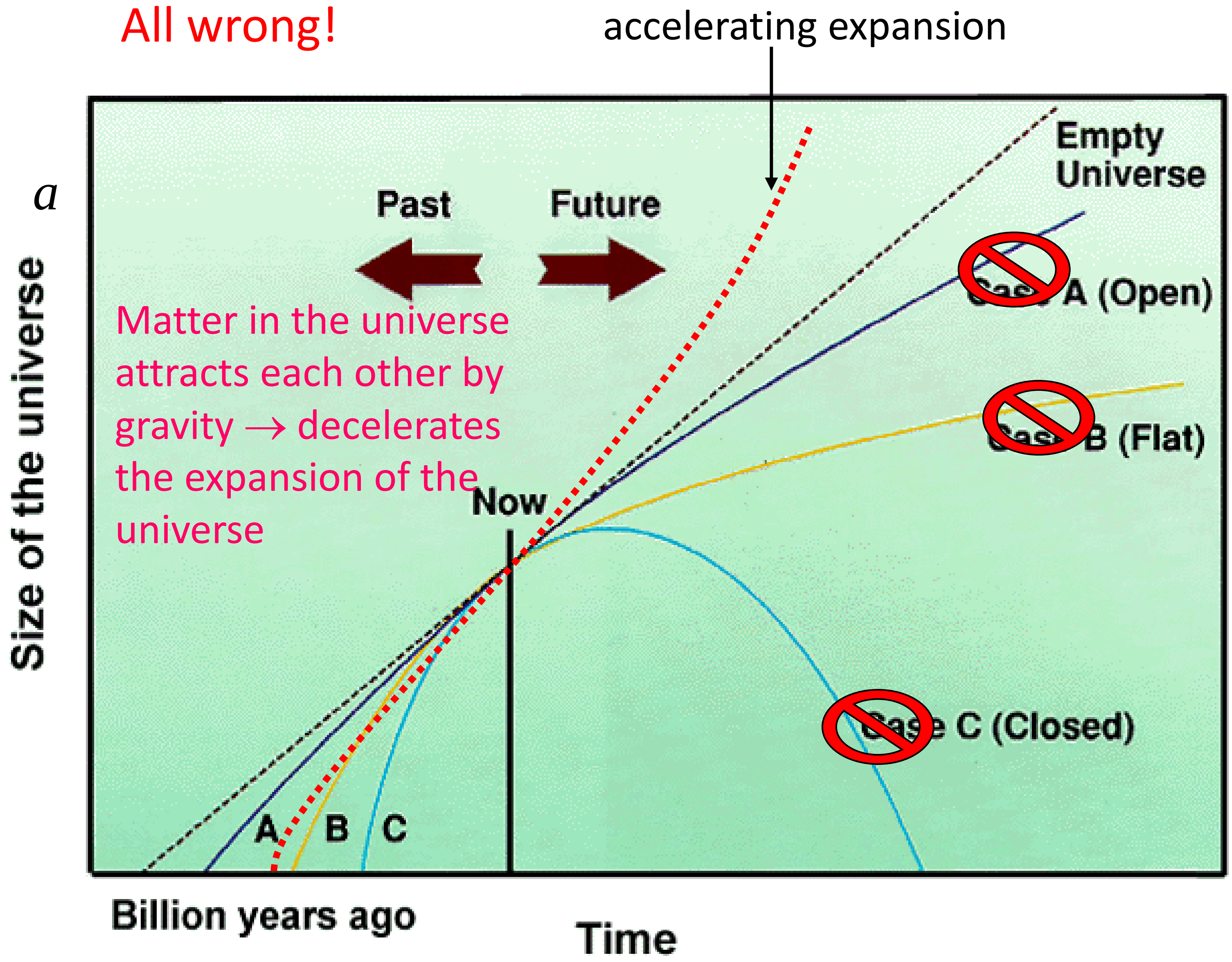
$$\mu(x) \rightarrow x \text{ if } a << a_o$$

https://en.wikipedia.org/wiki/Modified_Newtonian_dynamics



All wrong!


a
a


accelerating expansion
Size of the universe
Past
Future
Empty Universe
Case A (Open)
Case B (Flat)
Case C (Closed)
Now
A
B
C
Billion years ago
Time
Matter in the universe attracts each other by gravity → decelerates the expansion of the universe

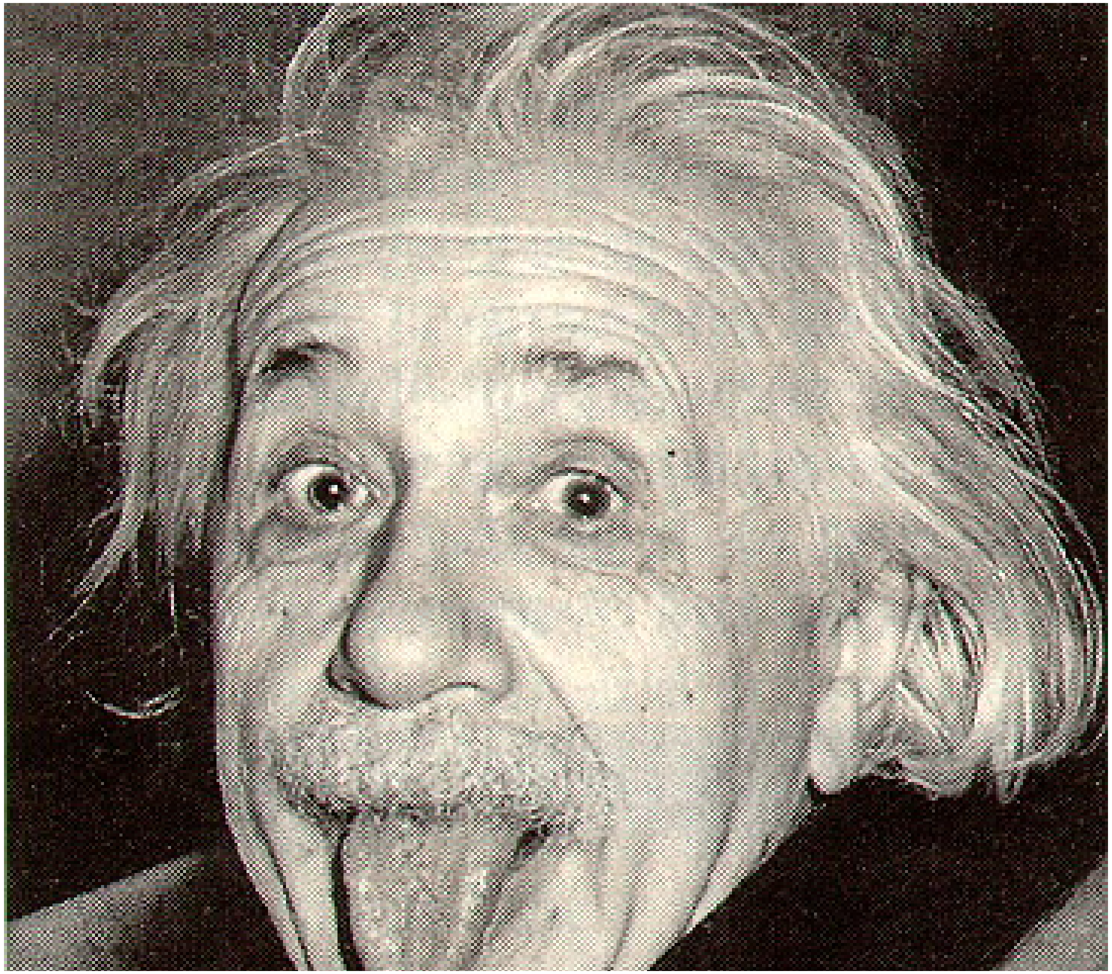
Dark Energy

Need a previously unknown repulsive force (energy) to account for accelerating expansion of the universe!

Zero-point energy? Wrong by 10^{-120}!

Biggest embarrassment in modern physics

Einstein's 'mistake'



Einstein's cosmological constant does represent a vacuum repulsive force

His greatest blunder is probably thinking that he made the greatest blunder by introducing the cosmological constant.

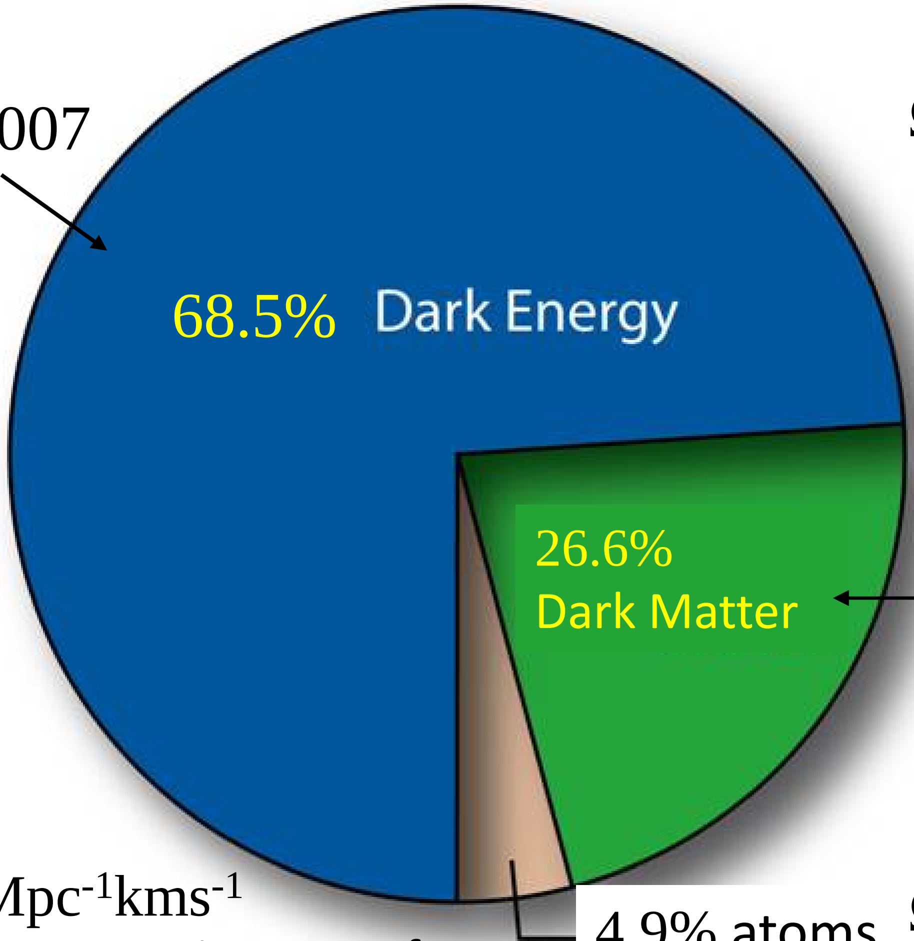
Energy density of the universe

$\Omega_\Lambda = 0.685 \pm 0.007$

Dark energy: repulsive force

$\Omega_K = 0.001 \pm 0.002$

$= 0$ →flat space-time



$\Omega_\Lambda > \Omega_m$ today

→expansion is accelerating; H becomes larger

$\Omega_m = 0.266 \pm 0.007$

attractive force

$H_o = 67.4 \pm 0.5\ \mathrm{Mpc^{-1}kms^{-1}}$

Hubble parameter = expansion rate of the universe today

$\Omega_B = 0.0493 \pm 0.0002$

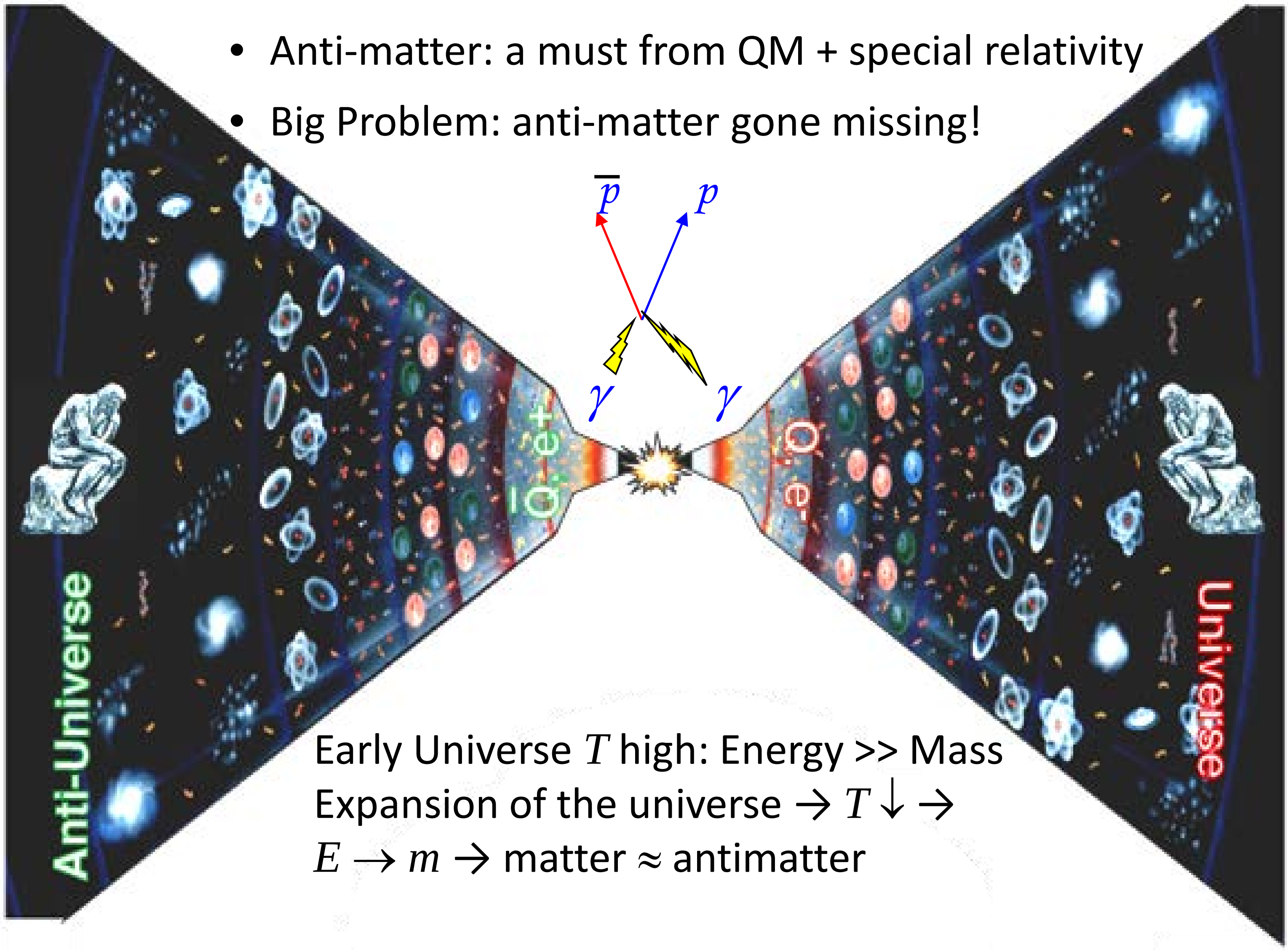
'Baryons': ordinary matter

From Planck 2018 results

III. Matter-antimatter asymmetry – why have we not been annihilated?

Why have we not been annihilated? - a Big Problem with the Big Bang

- Anti-matter: a must from QM + special relativity
- Big Problem: anti-matter gone missing!



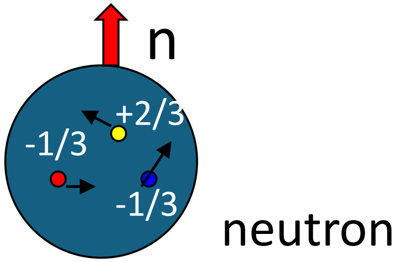
Early Universe T high: Energy >> Mass
Expansion of the universe → $T\downarrow$ →
$E \to m$ → matter ≈ antimatter

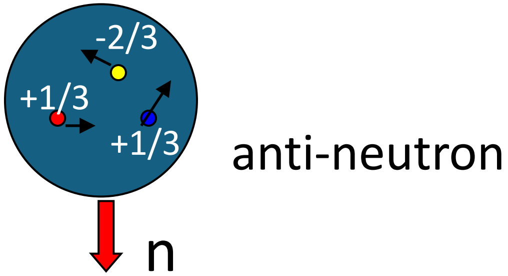
From Prof. Ting's talk in HK

Anti-particles

- Corresponding to each type of particles, there is an anti-particle
- Most properties (e.g. mass) of an anti-particle are the same as the corresponding particle, except that the electric charges are opposite.
- E.g.: electron: -e, positron = anti-particle of electron: +e.
- A neutron is made of quarks, which carry electric charges; an anti-neutron is made of anti-quarks.

Nobel Prize 1933



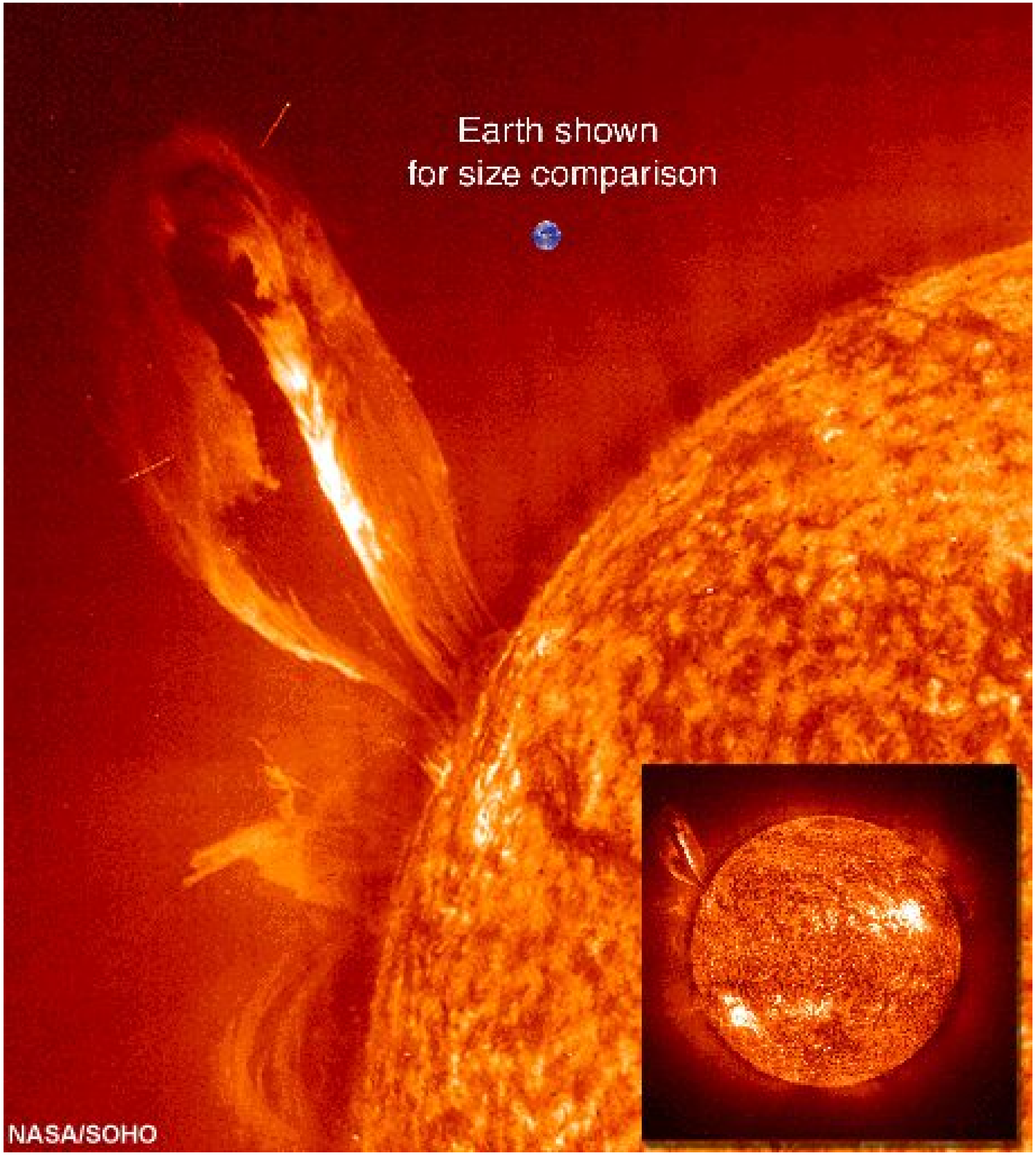


http://cdsweb.cern.ch/record/1177161?ln=sk

Matter dominates anti-matter

How do we know whether Sun is made of matter or anti-matter?

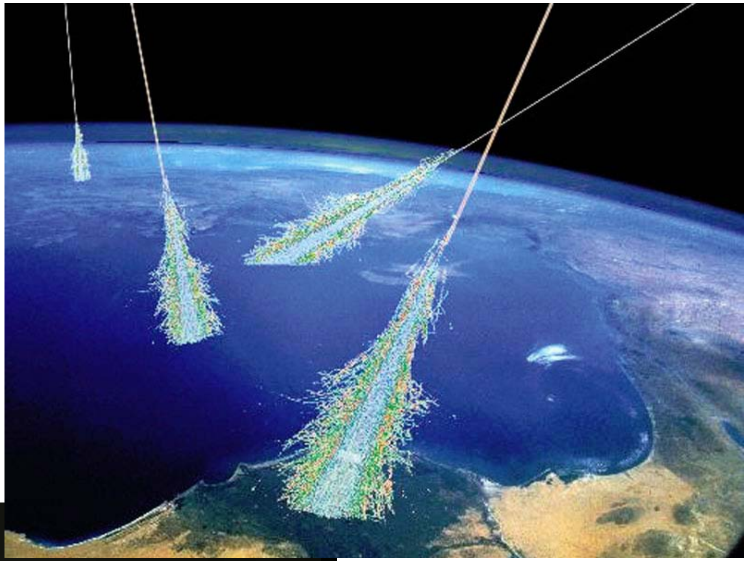
Solar wind is composed of mostly protons and electrons



Where are the anti-matter?

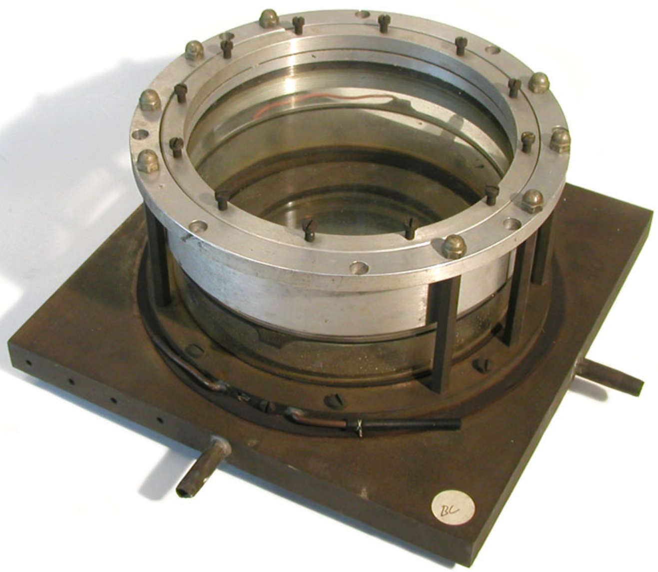
EM interaction is symmetric w.r.t. matter/anti-matter. Electromagnetic signals from distant stars cannot tell us whether they are made of matter or antimatter.

Cosmic Rays



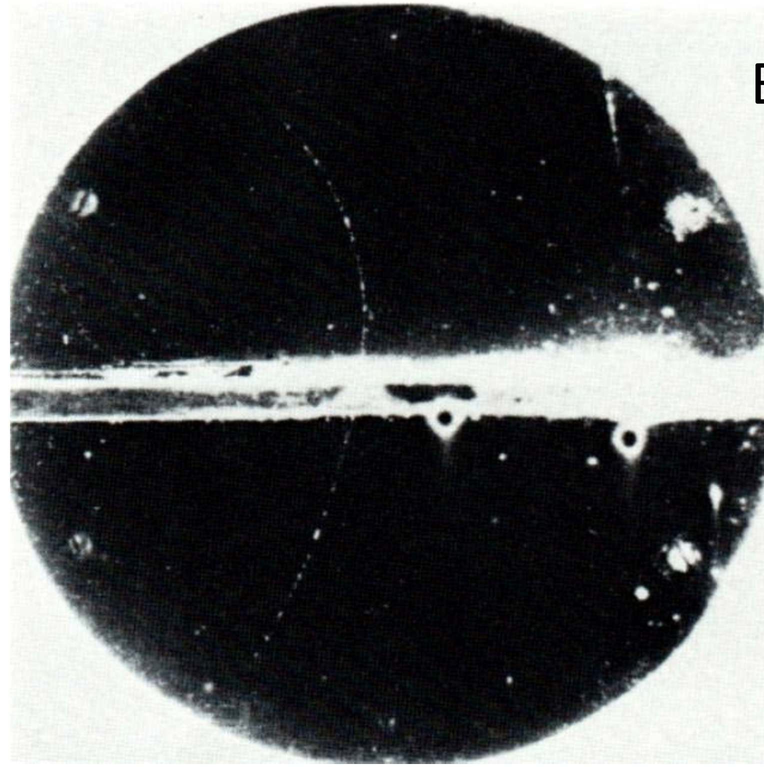
- Cosmic rays = subatomic particles from outside of Solar System
- Most are believed to be generated by dying stars (supernovae)

Discovery of anti-particles

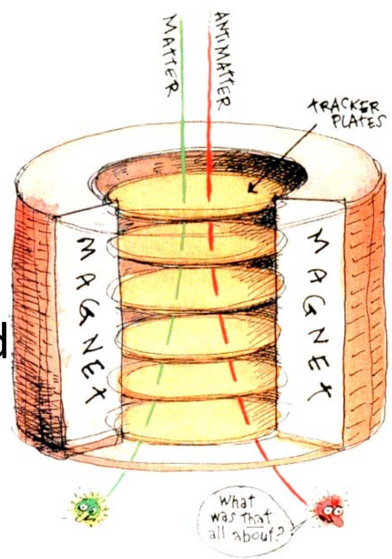


Anderson's cloud chambe[r]

Carl Anderson



A charged particle (charge q, mass m) moving in a magnetic field bends with a radius proportional to q/m, with direction related to the sign of its charge



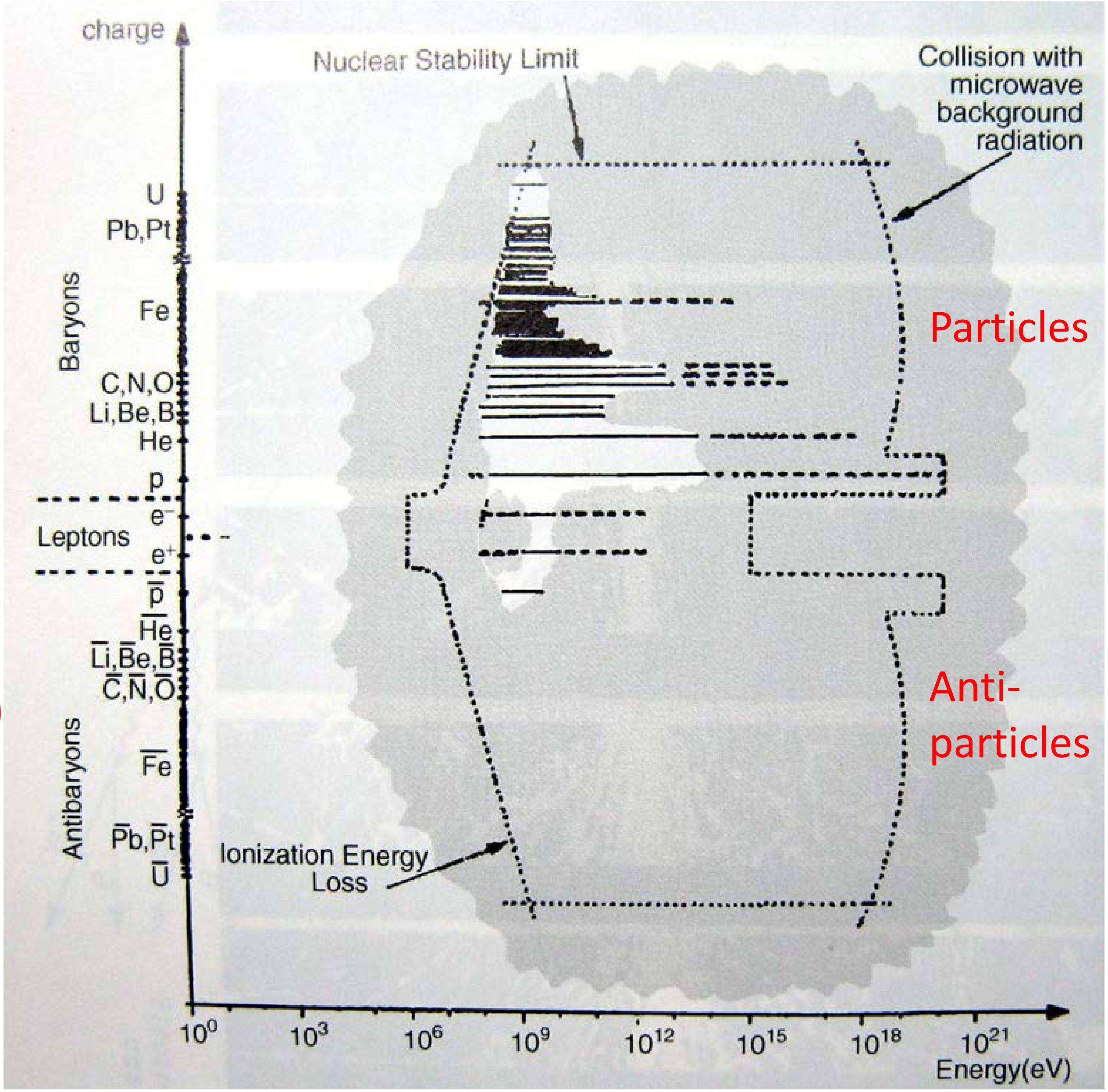
Air and Space
Smithsonian Institution, May 2001

Carl Anderson discovered a particle with the same mass but opposite charge as the electron (1932) = positron 反電子

Composition of cosmic rays

Cosmic rays are made of mostly particles, very few anti-particles

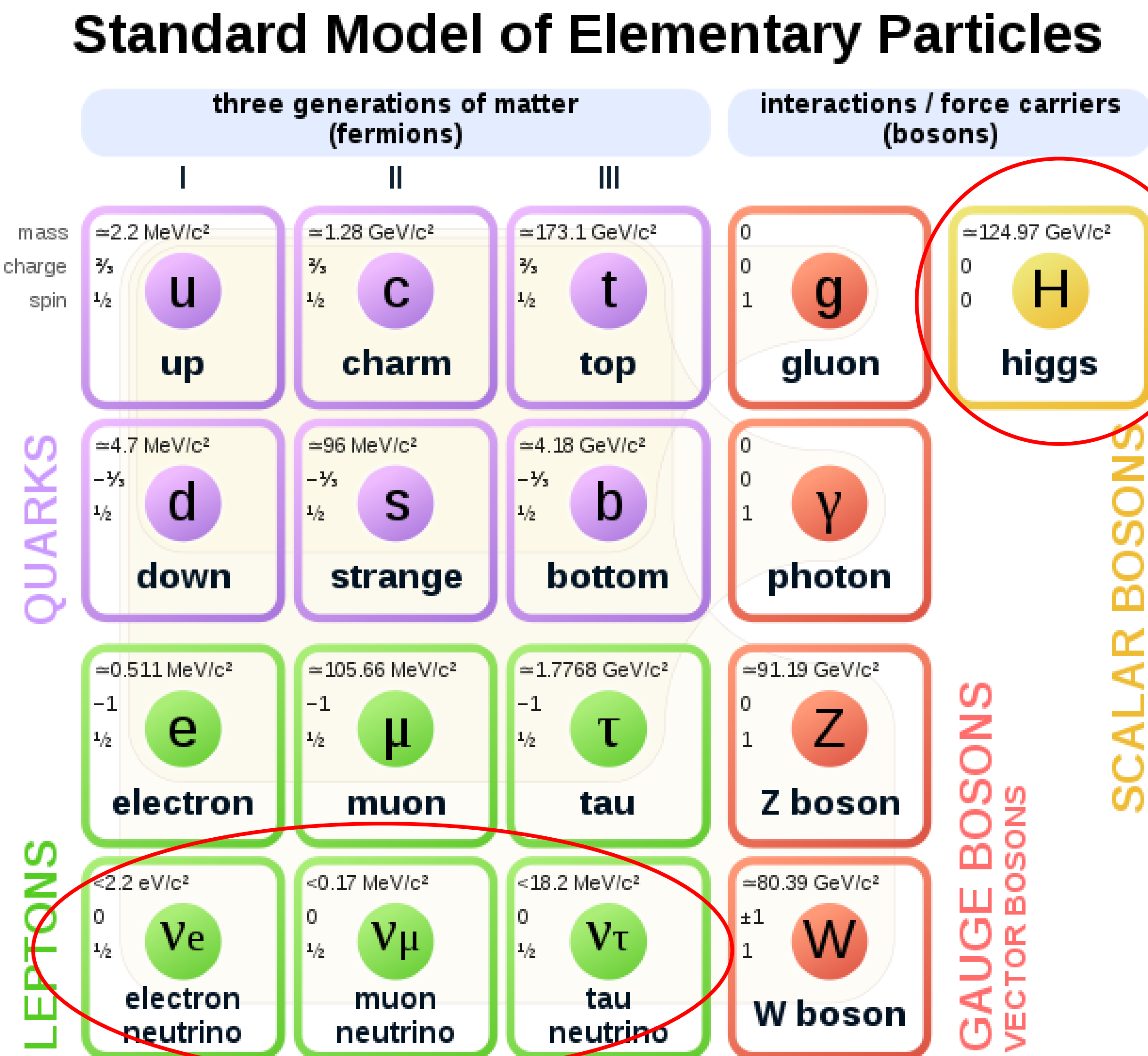
There should be a 10^{-8} asymmetry in matter and anti-matter: CP Violation!
100,000,001 (matter) – 100,000,000 (anti-matter) = our world
Need a new source of CP violation!



R. Schlickeiser, *Cosmic Ray Astrophysics*, 2001

Source of CP Violation?

The least understood particles are:



We are working on this!

We worked on this (Daya Bay)!

From Wikimedia

Fundamental Physics

- Gauge theory: from symmetry to interaction
- Some open problems:
- Gravity – how to surpass Einstein?
- Matter-antimatter asymmetry – why have we not been annihilated?

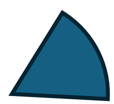
Fundamental Physics

Ming-chung Chu
Department of Physics
The Chinese University of Hong Kong
CERN Teacher Programme (Hong Kong) 2024

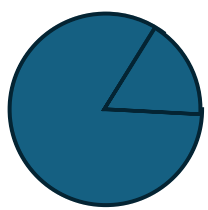
Gauge fields

Free Schrödinger Equation:
violates U(1) symmetry
(~rotation symmetry)

Vector potential $\mathbf{A}$
Gauge field

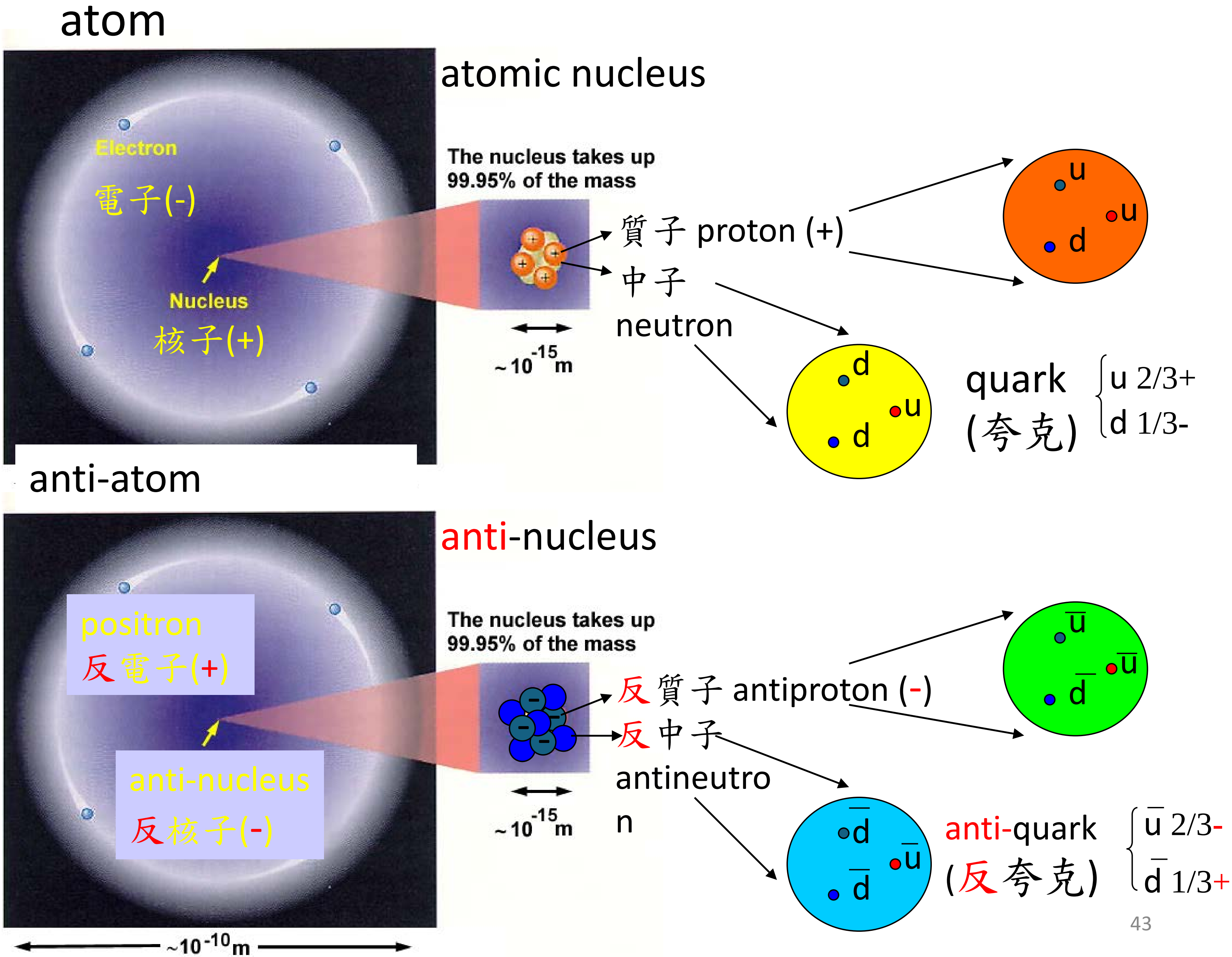


Missing piece, also
violates U(1) symmetry

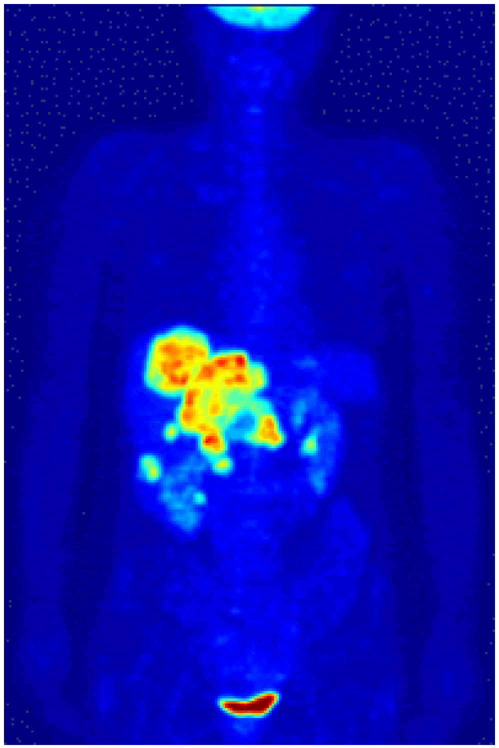


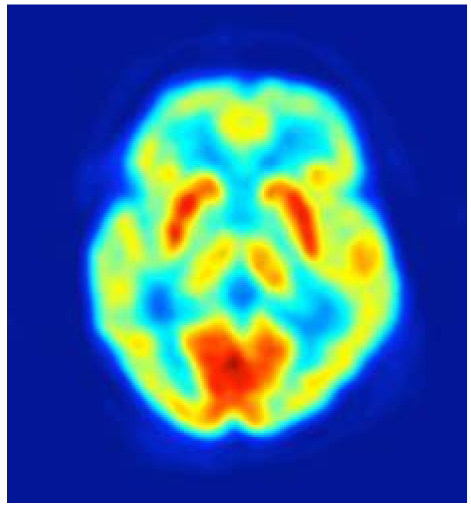
Together restore U(1)
symmetry!

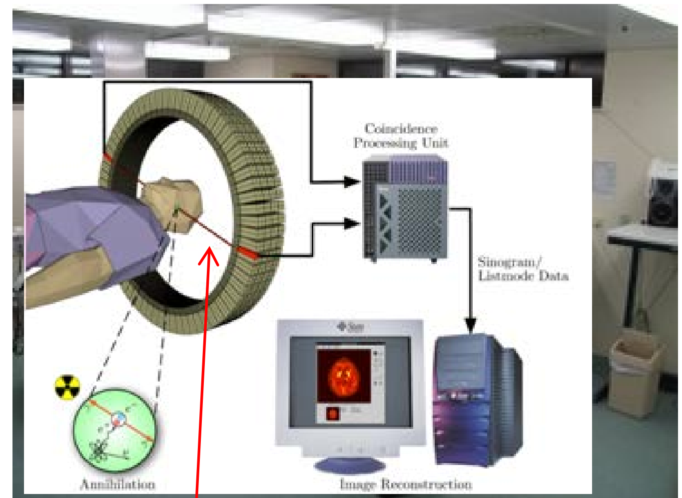
A charged (q) particle in EM field:
$\mathbf{p} \rightarrow \mathbf{p} - q\mathbf{A}/c$ (minimal coupling rule)


atom
atomic nucleus
Electron
電子(-)
Nucleus
核子(+)
The nucleus takes up 99.95% of the mass
~10^-15 m
質子 proton (+)
中子 neutron
u u d
d u d
quark (夸克)
u 2/3+
d 1/3-
anti-atom
anti-nucleus
positron
反電子(+)
anti-nucleus
反核子(-)
The nucleus takes up 99.95% of the mass
~10^-15 m
反質子 antiproton (-)
反中子 antineutron
anti-quark (反夸克)
ū 2/3-
d̄ 1/3+
~10^-10 m

Positron **E**mission **T**omography Scan







Drink liquid with radioactive isotopes that emit positrons $\rightarrow$ annihilate with electrons to give $2\ 511\mathrm{keV}\ \ \gamma$'s.

http://en.wikipedia.org/wiki/Positron_emission_tomography

http://www.crump.ucla.edu/software/lpp/lpphome.html

Dark energy = Casimir energy (zero-point energy)?

spherical conducting shell with radius R: $E_c = 3hc/128\pi R$

E.g. V. Nesterenko, I. G. Pirozhenko, ArXiv:hep-th/9707253v1

Casimir energy density = $\rho_C = 3E_c/4\pi R^3 = dhcR^{-4}$
d = dimensionless number ~ 0.01

R scales with $a(t) \Rightarrow \rho_C \propto a^{-4}$, ~ radiation, but different from cosmological constant ($\propto a^0$)!

Assume R = horizon ~ c/H_o, then $\rho_C \sim dhH_o^4 c^{-3}$

$\rho_C/\rho_\Lambda \sim d' hH_o^2 G/c^5$, d' = dimensionless order-1 number

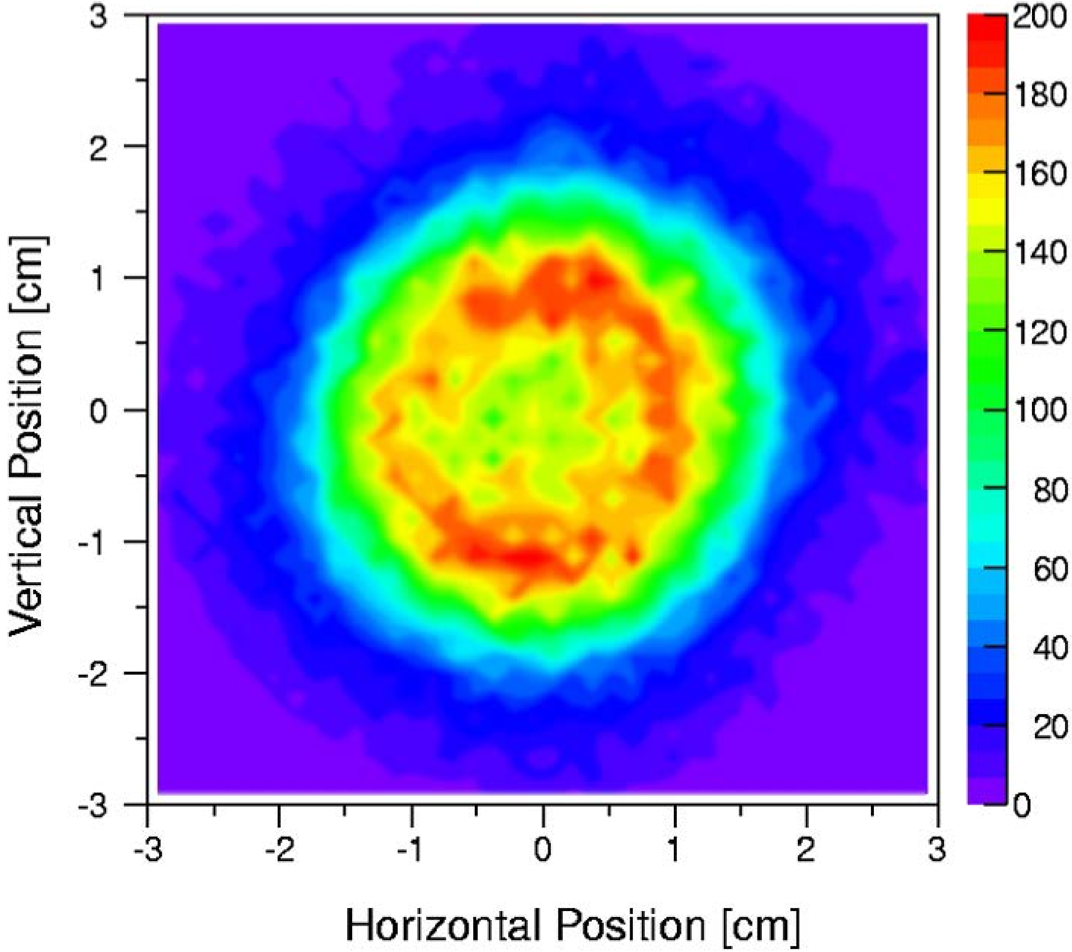
~ (Planck time/age of the universe)^2 ~ 10^{-120}

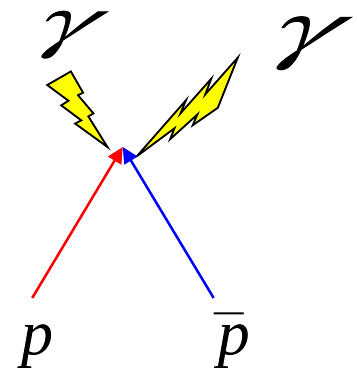
Biggest embarrassment in modern physics!

Particle-anti-particle annihilation (湮滅)

A particle and its anti-particle may annihilate each other , converting all their masses into energy/photons $E = mc^2$.
E.g.: $\overline{p} + p \rightarrow 2$ GeV ~ $3.2\ x10^{-10}$ J.

H atom and anti-H atom annihilate to emit photons (gamma rays)





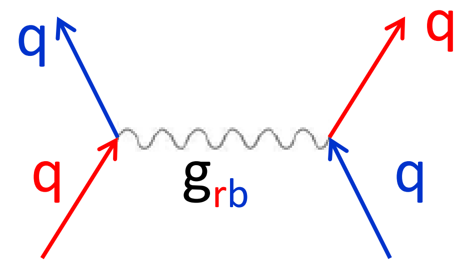
Credit: *Michael Martin Nieto, Michael H. Holzscheiter, Slava G. Turyshev*

Some interactions in particle physics

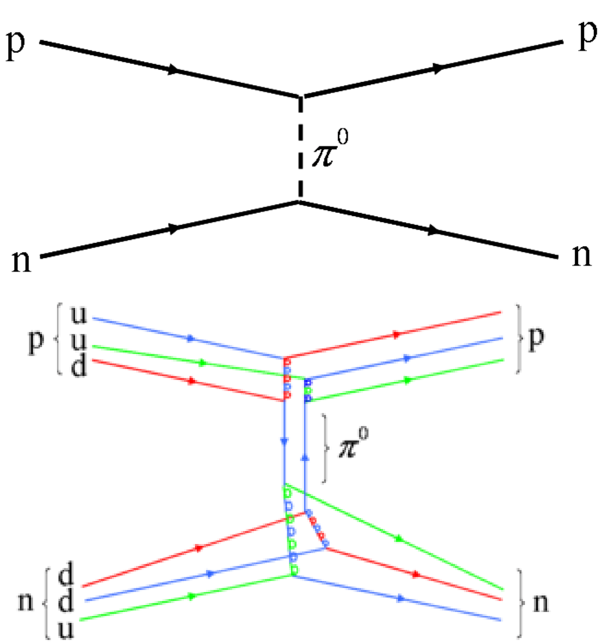
- Strong: exchange of gluons

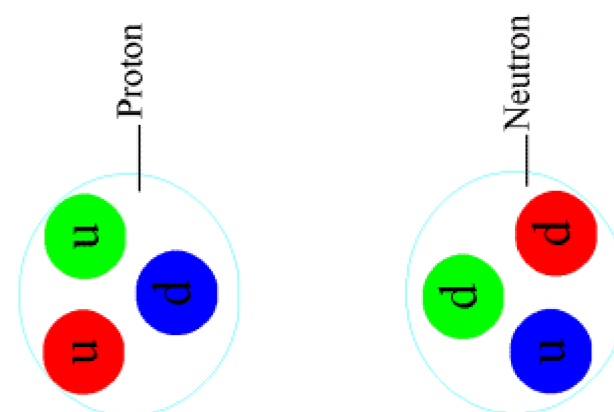
  Strong charge: ‘colors’ (r, g, b)

  A gluon carries 2 colors: changes the color of a quark, but not its ‘flavor’ (u, d, s, …)



- Nuclear: exchange of mesons





from: http://en.wikipedia.org/wiki/Nuclear_force