QCD resummation for light quark effects in Higgs boson production and decays

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May 2024

Topics discussed

- Light quark mediated $gg \to H$ amplitude
  - *LL and NLL result*
- Light quark mediated $gg \to Hg$ amplitude
  - *LL result*
  - *jet factorization*
- Effect on production and decays rates

Based on

K. Melnikov, A.A. Penin, *JHEP* ***05****, 172 (2016)*

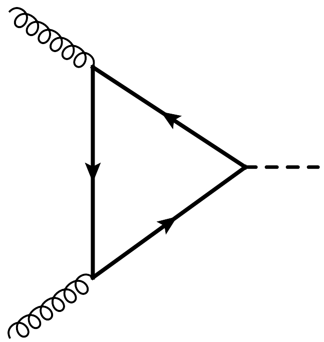
T. Liu, A.A. Penin, *Phys.Rev.Lett.* ***119****, 262001 (2017)*

C. Anastasiou, A.A. Penin, *JHEP* ***07****, 195 (2020)*

T. Liu, A.A. Penin, A. Rechman, *JHEP* ***04****, 031 (2024)*

Light quark mass effect

Leading contribution



$$\propto \alpha_s \ln^2(m_H^2/m_q^2)\frac{m_q^2}{m_H^2}$$

- *large logs at subleading power*

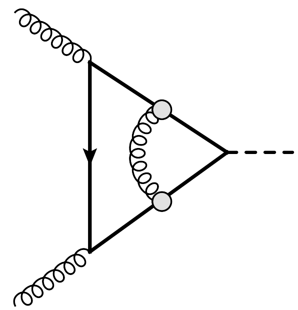
- *effective expansion parameter* $\alpha_s \ln^2(m_H^2/m_q^2) \sim$ $40\,\alpha_s$ *bottom*, $200\,\alpha_s$ *strange*

in real world $100\,\alpha_s$

- *resummation is mandatory*

$gg \to H$ amplitude at LL

- **Non-Sudakov logs** T. Liu, A.A. Penin, Phys.Rev.Lett. **119**, 262001 (2017)



- **Factorization formula** $$\mathcal{M}^b_{gg\to H} = Z_g^{2\,LL}\, g(z)\, \mathcal{M}^{b\,(0)}_{gg\to H}$$

  - *gluon Sudakov factor* $Z_g^{2\,LL} = \exp\left[-\frac{C_A}{\varepsilon^2}\frac{\alpha_s}{2\pi}\frac{\mu^{2\varepsilon}}{Q^{2\varepsilon}}\right]$
  - *non-Sudakov double logarithms*

$$g(z) = 2\int_0^1 \mathrm{d}\xi \int_0^{1-\xi} \mathrm{d}\eta\, e^{2z\eta\xi} = {}_2F_2\,(1,1;3/2,2;z/2)$$

  - *double-log variable* $z = \ (C_A - C_F)\ x,\ \ x = \frac{\alpha_s}{4\pi}L^2,\ L = \ln(m_H^2/m_q^2)$

eikonal color nonconservation

$gg \to H$ amplitude at NLL

C. Anastasiou, A.A. Penin, JHEP **07**, 195 (2020)

$$\mathcal{M}^{b\,NLL}_{gg\to H} = C_b \left(\frac{\alpha_s(m_H)}{\alpha_s(m_b)}\right)^{\gamma_m^{(1)}/\beta_0} Z_g^{2\,NLL} \left[-\frac{3}{2}\frac{m_b^2}{m_H^2} L^2 \mathcal{M}^{t\,(0)}_{gg\to H}\right]$$

Yukawa RG factor   gluon Sudakov form factor   LO amplitude

$$C_b = \left[g(z) + \frac{\alpha_s L}{4\pi}\left(2\gamma_q^{(1)} g_\gamma(z) - \beta_0 g_\beta(z)\right)\right] = 1 + \sum_{n=1}^{\infty} c_n$$

$$c_1 = \frac{z}{6} + C_F\frac{\alpha_s L}{4\pi}\,, \quad c_2 = \frac{z^2}{45} + \frac{z}{5}\frac{\alpha_s L}{4\pi}\left[\frac{3}{2}C_F - \beta_0\left(\frac{5}{6}\frac{L_\mu}{L} - \frac{1}{3}\right)\right],$$

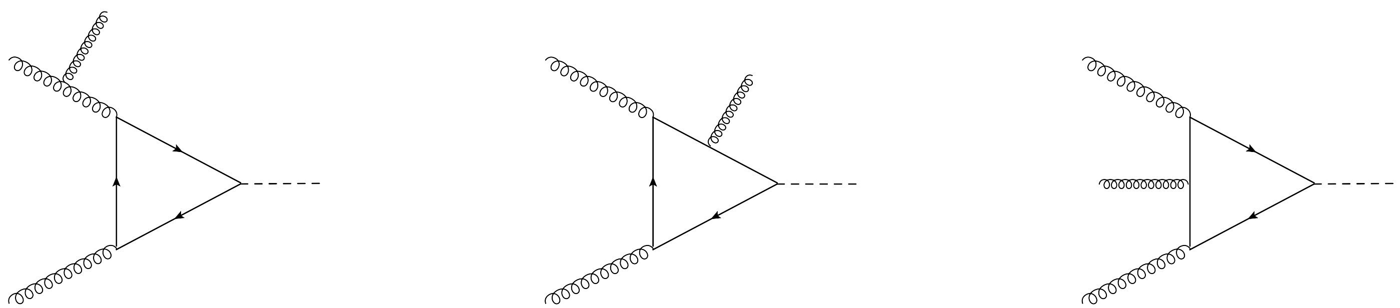
$$c_3 = \frac{z^3}{420} + \frac{z^2}{5}\frac{\alpha_s L}{4\pi}\left[\frac{5}{21}C_F - \beta_0\left(\frac{2}{9}\frac{L_\mu}{L} - \frac{2}{21}\right)\right], \quad \ldots$$

$$L = \ln(s/m_q^2),\ L_\mu = \ln(s/\mu^2)$$

- *main NLL effects:*
  $\alpha_s(\mu) \Rightarrow \alpha_s\left(m_H\left(\frac{m_q}{m_H}\right)^{2/5}\right)$
  $m_q^2(\mu) \Rightarrow m_q(m_q)m_q(m_H)$

$gg \to Hg$ amplitudes

- Kinematics of $g(p_1) + g(p_2) \to g(p_3) + H(p_H)$
  - $m_q^2 \ll p_\perp^2 \ll s, m_H^2$
  - $s \approx m_H^2$
- One-loop amplitudes



  - *single color structure*
  - *two helicity structures*

$gg \to Hg$ **amplitudes**

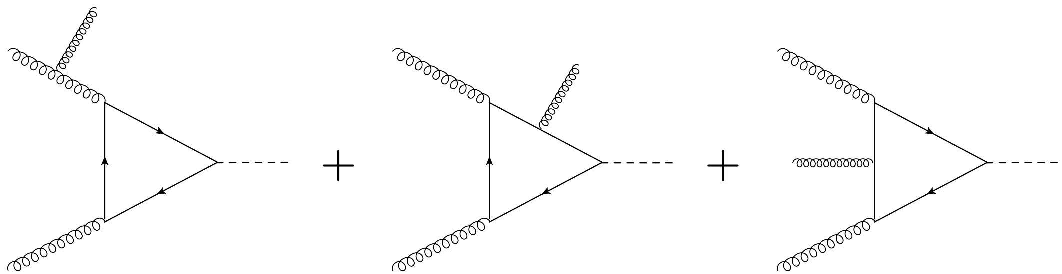
- Amplitudes $M_{++\pm} \propto e^{\frac{\alpha_s}{2\pi} I} \sum_q A^{(q)}_{++\pm}$
- Form factors
  - *large mass expansion* $A^{(t)}_{++\pm} = C_t \sum_{n=0} \left(\frac{\alpha_s}{2\pi}\right)^n \tilde{A}^{(n)}_{++\pm} + \mathcal{O}(m_t^{-2})$
  - *small mass expansion* $A^{(q)}_{++\pm} = \frac{m_q^2}{s} \sum_{n=0} \left(\frac{\alpha_s}{2\pi}\right)^n A^{(n+1)}_{++\pm} + \mathcal{O}(m_q^4)$
- One-loop Sudakov exponent
  - *physical* $I^{(1)}_{\rm ph} = -\frac{C_A}{2\epsilon^2}\left[2\left(\frac{-s}{\mu^2}\right)^{-\epsilon} + \left(\frac{-tu}{s\mu^2}\right)^{-\epsilon}\right]$

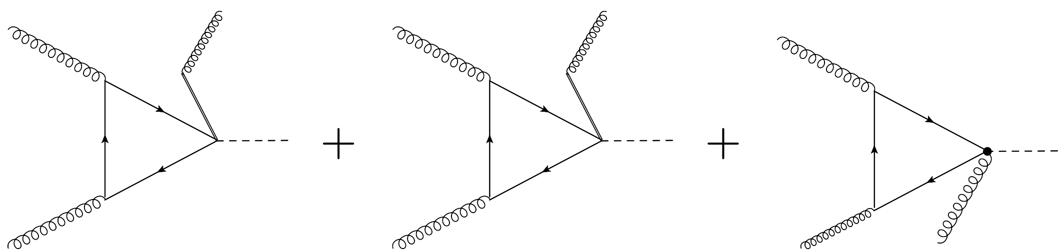
  ➥ *accommodate all the double logs of* $p_\perp$ *and rapidity in* $m_q \to \infty$ *limit*

  - *symmetric* $I^{(1)}_{\rm sym} = -\frac{C_A}{2\epsilon^2}\left[\left(\frac{-s}{\mu^2}\right)^{-\epsilon} + \left(\frac{-t}{\mu^2}\right)^{-\epsilon} + \left(\frac{-u}{\mu^2}\right)^{-\epsilon}\right]$

Factorization

Effective theory decomposition





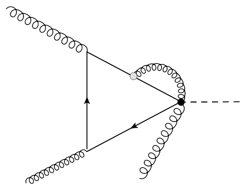
eikonal dipole *soft dipole* *symmetric*

Effective theory decomposition

- Eikonal dipole
  - *standard dipole structure with* $A_{+++} = -A_{++-}$
  - *jet completely factors out from the quark loop*
- Soft dipole
  - *standard dipole structure with* $A_{+++} = -A_{++-}$
  - *quark loop momentum cutoff* $l_\perp^2 < p_\perp^2$
- Symmetric
  - *symmetric tensor structure with* $A_{+++} = 0$
  - *quark loop momentum cutoff* $l_\perp^2 < p_\perp^2$

Double logs

- Soft emission from the factorized gluon line
  - *color is conserved along gluon line*
  - ➥ *only Sudakov double logs*
- Problem reduces to
  - *eikonal dipole:* $g(p_1)g(p_2)H$ *form factor*
  - *soft dipole:* $g(p_1)g(p_2)H$ *form factor with* $p_\perp$ *-dependent UV cutoff*
  - *symmetric:*
    - *unresolved vertex:* $g(p_3)g(p_{1,2})H$ *form factor with* $p_\perp$ *-dependent UV cutoff*
    - *resolved vertex: additional Sudakov-Wilson contribution*
    - ➥ *soft jet factors out*



All-order amplitudes

- ## Leading logarithmic result

$$A^{LL}_{+++} = L^2\left[g(z) - \int_{(1-\tau+\zeta)/2}^{(1+\tau+\zeta)/2} \frac{\mathrm{d}\eta}{2z\eta}\left(e^{2z\eta(1-\eta)} - e^{z\eta(1-\tau-\zeta))}\right)\right]$$

$$A^{LL}_{++-} = -A^{LL}_{+++} - L^2\left[\int_{(1-\tau+\zeta)/2}^{(1+\tau+\zeta)/2} \frac{\mathrm{d}\eta}{2z\eta}\, e^{z((1-\tau)^2-\zeta^2)/2}\left(e^{z\eta(1+\tau+\zeta-2\eta)} - 1\right) + (\zeta \to -\zeta)\right]$$

  - *rapidity variable* $\zeta = \ln(t/u)/L$, *transverse momentum variable* $\tau = \ln(p_\perp^2/m_q^2)/L$

- ## Two loops

  - *after infrared matching agrees with the explicit calculation*

K. Melnikov, L. Tancredi, C. Wever, JHEP 1611, 104 (2016)

All-order amplitudes

Leading logarithmic result

$$A^{LL}_{+++} = L^2\left[g(z) - \int_{(1-\tau+\zeta)/2}^{(1+\tau+\zeta)/2} \frac{\mathrm{d}\eta}{2z\eta}\left(e^{2z\eta(1-\eta)} - e^{z\eta(1-\tau-\zeta))}\right)\right]$$

$$A^{LL}_{++-} = -A^{LL}_{+++} - L^2\left[\int_{(1-\tau+\zeta)/2}^{(1+\tau+\zeta)/2} \frac{\mathrm{d}\eta}{2z\eta}\, e^{z((1-\tau)^2-\zeta^2)/2}\left(e^{z\eta(1+\tau+\zeta-2\eta)} - 1\right) + (\zeta \to -\zeta)\right]$$

- *rapidity variable* $\zeta = \ln(t/u)/L$, *transverse momentum variable* $\tau = \ln(p_\perp^2/m_q^2)/L$

Boundary values

- *low transverse momentum* $p_\perp \sim m_q$: $A^{LL}_{++\pm} = \pm L^2 g(z)$
- *high transverse momentum, central rapidity* $p_\perp \sim m_H\,,\ \ t \sim u$:

$$A^{LL}_{+++} = \frac{1}{2}L^2 g(z), \quad A^{LL}_{++-} = -\frac{3}{2}L^2 g(z)$$

Higgs + Jet production

Partonic cross section near threshold

$$d\sigma^{tb}_{gg\to Hg+X} = -\frac{3m_q^2}{m_H^2} L^2 C_t \, C_b(\tau,\zeta)\, d\tilde{\sigma}^{\rm eff}_{gg\to Hg+X}$$

heavy top effective theory cross section

- *light quark coefficient*

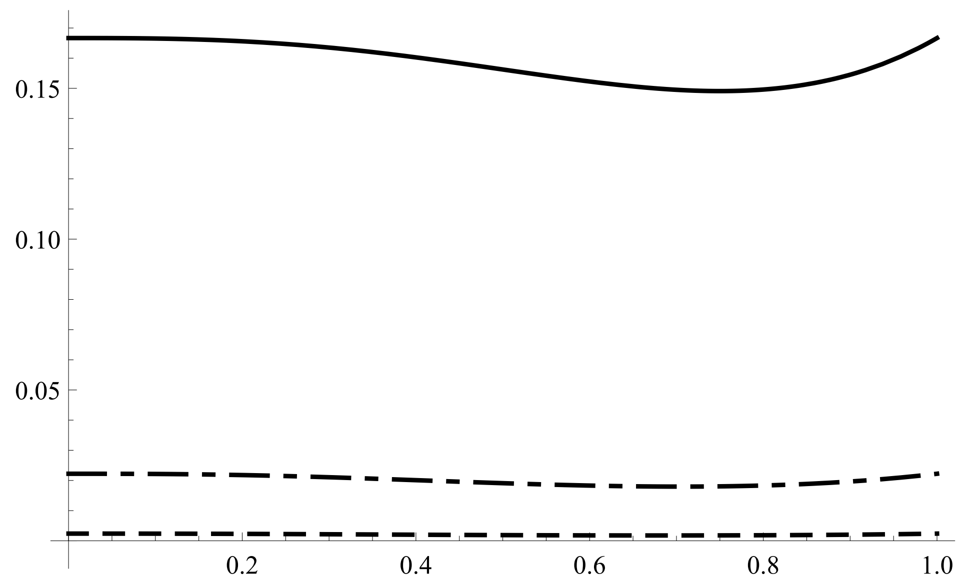
$$C_q(\tau,\zeta) = \frac{A_{+++} - A_{++-}}{2L^2} = 1 + \frac{z}{6}\left(1-\tau^3+\tau^4\right)$$

$$+z^2\left[\frac{1}{45} - \frac{\tau^3}{12} + \frac{\tau^4}{6} - \frac{7\tau^5}{60} + \frac{\tau^6}{30} + \frac{\zeta^2}{12}(\tau^3-\tau^4)\right] + \ldots$$

- *Magic:* $C_q(\tau,\zeta) \approx C_q(0,0) = C_q$

Jet factorization

- *dependence of the* $z^{1,2,3}$ *coefficients on* $m_q < p_\perp < m_H$



➥ *a jet with* $m_q < p_\perp < m_H$ *factors out as a soft jet with* $p_\perp \ll m_q$

Higgs production

- Top-bottom interference through NNLO
  - *NLL threshold*

    $$\delta\sigma^{\rm NNLO}_{pp\to H+X} = -2.18 \pm 0.20 \; pb$$

    C. Anastasiou, A.A. Penin, JHEP ***07****, 195 (2020)*

  - *full result*

    $$\delta\sigma^{\rm NNLO}_{pp\to H+X} = -1.99^{+0.30}_{-0.15} \; pb$$

    M. Czakon, F. Eschment, M. Niggetiedt, R. Poncelet, T. Schellenberger, 2312.09896 [hep-ph]

- Convergence of the logarithmic expansion

	LO	NLO	NNLO	N^{3}LO
$\delta\sigma^{\rm LL}_{pp\to H+X}$	-1.420	-1.640	-1.667	-1.670
$\delta\sigma^{\rm NLL}_{pp\to H+X}$	-1.420	-2.048	-2.183	-2.204
$\delta\sigma_{pp\to H+X}$	-1.06	-1.64	-1.99	

Higgs + Jet production

- Top-bottom interference

$$d\sigma^{tb}_{pp\to Hj+X} = \left[\frac{C_b}{C_t}\left(\frac{\alpha_s(m_H)}{\alpha_s(m_b)}\right)^{\gamma_m^{(1)}/\beta_0}\right]\left(\frac{d\sigma^{tb}_{pp\to Hj+X}}{d\sigma^{tt}_{pp\to Hj+X}}\right)^{\rm LO} d\sigma^{tt}_{pp\to Hj+X}\,.$$

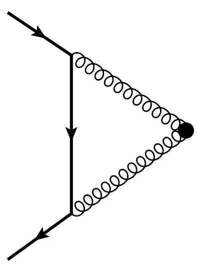
  - *numerically observed in NLO*

    J. M. Lindert, K. Melnikov, L. Tancredi and C. Wever, Phys. Rev. Lett. 118, 252002 (2017)

  ➥ *provides access to higher orders/resummations*

Strange quark contribution to Higgs decays

- Direct decay into strange pairs y_s^2
  - *no logs of* m_s
- Strange-loop mediated decay $y_s m_s$
  - *double-log enhancement* $g(z) \approx 3$ *(in real world* 1.5*)*
- Top-loop mediated decay into strange pairs $y_s m_s$
  - *inverse color flow:*



➥ *double-log suppression* $g(-z) \approx 0.5$ *(in real world* 0.75 *)*