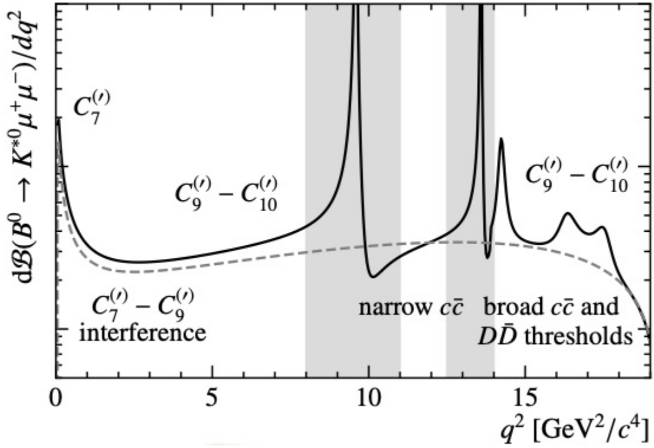
$b \to s\ell^+\ell^-$

Marzia Bordone

CERN

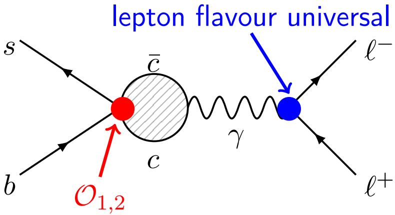
LHC Heavy Flavour WG topical meeting
14.04.2024

Spectrum


$d\mathcal{B}(B^0 \to K^{*0}\mu^+\mu^-)/dq^2$
$C_7^{(\prime)}$
$C_9^{(\prime)} - C_{10}^{(\prime)}$
$C_9^{(\prime)} - C_{10}^{(\prime)}$
$C_7^{(\prime)} - C_9^{(\prime)}$
interference
narrow $c\bar{c}$
broad $c\bar{c}$ and
$D\bar{D}$ thresholds
0
5
10
15
q^2 [GeV$^2/c^4$]

Non-local form factors

$$\mathcal{H}_\lambda(q^2) = iP^\lambda_\mu \int d^4x\, e^{iqx} \langle K^{(*)}(k)| T\left\{\mathcal{J}^\mu_{\textsf{em}}, C_i \mathcal{O}_i(0)\right\} |\bar{B}(k+q)\rangle$$



$$C_9 \to C_9^{\rm eff}(q^2) = C_9 + C_9^{\rm LD}(q^2)$$

How do we parametrise these long-distance effects?

How to estimate C_9^{LD}

- A first-principle calculation of C_9^{LD} is hard and possible only with Lattice QCD

  $\Rightarrow$ No short-term prospects, would require evaluating $B \to K^{(*)} J/\psi$

- We can calculate points at negative q^2 using LCSRs

  $\Rightarrow$ Additional information is needed in the physical region, e.g. from $B \to K^{(*)} J/\psi$ data

- We need to infer a kinematic dependence

  $\Rightarrow$ Can we avoid being model-dependent?

- Experimental data are **essential** to succeed in this task

Binned vs Unbinned

Binned

Unbinned

Binned vs Unbinned

Binned

- No functional form for C_9^{LD} is specified

  $\Rightarrow$ Model-independent

Unbinned

- A specific parametrisation for C_9^{LD} is assumed

  $\Rightarrow$ A model-dependence is introduced

Binned vs Unbinned

Binned

- No functional form for C_9^{LD} is specified

  $\Rightarrow$ Model-independent

- Results are given in terms of binned measurements of angular coefficients, branching ratios, etc.

  $\Rightarrow$ What binning scheme is optimal?

Unbinned

- A specific parametrisation for C_9^{LD} is assumed

  $\Rightarrow$ A model-dependence is introduced

- Results are given in terms of the parametrisation parameters

  $\Rightarrow$ What we can learn?

Binned vs Unbinned

Binned

- No functional form for C_9^{LD} is specified

  $\Rightarrow$ Model-independent

- Results are given in terms of binned measurements of angular coefficients, branching ratios, etc.

  $\Rightarrow$ What binning scheme is optimal?

- Binned analyses are immediately reusable to reinterpret the results in terms of long-distance contributions or NP

Unbinned

- A specific parametrisation for C_9^{LD} is assumed

  $\Rightarrow$ A model-dependence is introduced

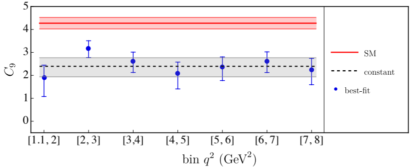
- Results are given in terms of the parametrisation parameters

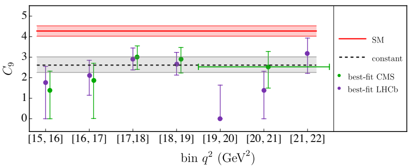
  $\Rightarrow$ What we can learn?

- Is there a way to recast the unbinned results?

An example of unbinned: $B^+ \to K^+\mu^+\mu^-$

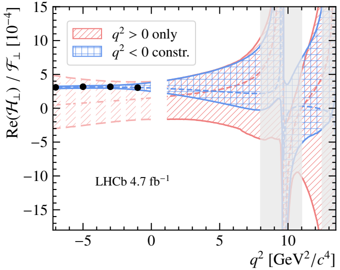
- With binned analysis we can test different parametrisation for C_9^{LD}
- We can study for example the q^2 dependence of C_9 to try to get hints on C_9^{LD}

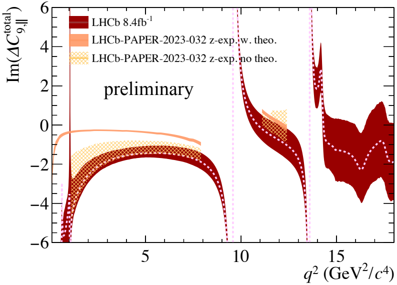




- At low q^2, LHCb and CMS have the same binning scheme
  - $\Rightarrow$ The combination helps in extracting more precise results
- At high q^2, different binning schemes don't allow to combine
  - $\Rightarrow$ The high q^2 region is essential to confirm/reject the patterns that we see at low q^2

Unbinned analysis





- Different ansatz on $C_9^{\rm LD}$ are tested directly on data
- Theory predictions for the long distance can be tested directly against data
- More information in the binned case is available
  - $\Rightarrow$ Can we use this information?

Discussion points

- Binned or unbinned?
  - ⇒ Binned analysis are **necessary** and should be given priority
  - ⇒ We would like to have as many bins as possible, depending on the experimental limitations
- Unbinned analyses are model-dependent but also contain more information
  - ⇒ Is there a way to reinterpret them?
- The high q^2 region is important, and allows us to check the consistency over the full kinematic range
- Would it make sense to have a strategy paper from the HFWG?

Appendix

$B \to K^{(*)}\ell^+\ell^-$

$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (\mathcal{C}_9 \mp \mathcal{C}_{10})\mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2} \left[\mathcal{C}_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

$B \to K^{(*)}\ell^+\ell^-$

$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (\mathcal{C}_9 \mp \mathcal{C}_{10}) \mathcal{F}_\lambda(q^2) + \frac{2m_b M_B}{q^2} \left[\mathcal{C}_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

local: $\mathcal{O}_9, \mathcal{O}_{10}, \mathcal{O}_7$

$B \to K^{(*)}\ell^+\ell^-$

non-local: $\mathcal{O}_1, \mathcal{O}_2$

$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (\mathcal{C}_9 \mp \mathcal{C}_{10}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[\mathcal{C}_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

local: $\mathcal{O}_9, \mathcal{O}_{10}, \mathcal{O}_7$

Exclusive matrix elements

$$\langle H_s|\bar{s}\,\Gamma_\mu b|H_b\rangle = \sum_i S^i_\mu \mathcal{F}_i$$

Exclusive matrix elements

$$\langle H_s|\bar{s}\,\Gamma_\mu b|H_b\rangle = \sum_i S^i_\mu \mathcal{F}_i$$

form factor

scale Λ_{QCD}

independent

Lorentz structures

Exclusive matrix elements

$$\langle H_s | \bar{s}\, \Gamma_\mu b | H_b \rangle = \sum_i S^i_\mu \mathcal{F}_i$$

form factor

scale Λ_{QCD}

independent

Lorentz structures

Form factors determinations

- Lattice QCD
- QCD SR, LCSR ($q^2 - mb^2 \leqslant 0$)

only points at specific kinematic points

Form factors parametrisations

- Analytic properties $\rightarrow$ BGL

data points needed to fix the coefficients of the expansion

The z expansion

- q^2 dependence must be inferred
- q^2 is large compared to $m_B^2 \Rightarrow$ not a good expansion variable
- Conformal variable z

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

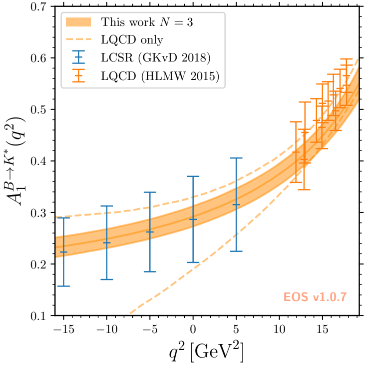
- $t_+ = (m_B + m_{K^{(*)}})^2$ pair production threshold
- $t_0 < t_+$ free parameter that can be used to minimise $|z_{\text{max}}|$
- $|z| \ll 1$, and we can write

$$F_i = \frac{1}{P_i(z)\phi_i(z)} \sum_{k=0}^{n_i} a_k^i z^k$$

$$\sum_{k=0}^{n_i} |a_k^i|^2 < 1$$

Local form factors

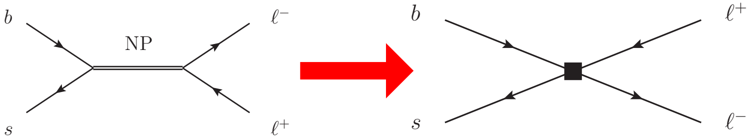
$$\mathcal{F}_\lambda^{(T)} = \langle K^{(*)}(k)|\bar{s}\Gamma_\lambda^{(T)} b|\bar{B}(k+q)\rangle$$



Systematically improvable with new LQCD calculations

See 2305.06301 for details
Other references: 1503.05534, 1811.00983

How to distinguish New Physics from C_9^{LD}



$$C_9^{\mathrm{NP}} = constant$$

- Studying the kinematic dependence of C_9 can give hints on the nature of possible deviations
- It is essential to use precise experimental data in a large q^2 window

Which analysis can we use?

1. **Binned analysis**
   - Results are given in terms of bins of kinematic distributions
   - No model has to be assumed, apart from that in the MC
   - To extract as much information as possible, the more bins the better
2. **Unbinned analysis**
   - A specific model for $C_9^{\mathrm{LD}}(q^2)$ is assumed
   - The results are given in terms of the parameters that are present in the parametrisation
   - The model dependency renders the results difficult to use

q^2 dependence in $C_9^{\rm LD}(q^2)$

- LHCb+CMS data for $B \to K^{(*)}\mu^+\mu^-$ observables are used to extract $C_9^{\rm eff}$

  [1403.8045, 2401.07090, 1606.04731, 2003.04831]

- A model for the charm effects is assumed based on dispersion relation

$$C_9^{\rm eff} = C_9 + \sum_V \eta_V^\lambda e^{i\delta_V^\lambda} \frac{q^2}{(m_V^2)} \frac{m_V \Gamma_V}{m_V^2 - q^2 - i m_V \Gamma_V}$$

- The parameters η_V^λ have to be fixed from $B \to K^{(*)}V$ data

- The phases δ_V^λ are fixed from LHCb analysis [1612.06764]

$B^+ \to K^+\mu^+\mu^-$

V	η_V^K	δ_V^K
J/ψ	32.3 ± 0.6	-1.50 ± 0.05
$\psi(2S)$	7.12 ± 0.32	2.08 ± 0.11
$\psi(3770)$	$(1.3 \pm 0.1) \times 10^{-2}$	-2.89 ± 0.19
$\psi(4040)$	$(4.8 \pm 0.8) \times 10^{-3}$	-2.69 ± 0.52
$\psi(4160)$	$(1.5 \pm 0.1) \times 10^{-2}$	-2.13 ± 0.33
$\psi(4415)$	$(1.1 \pm 0.2) \times 10^{-2}$	-2.43 ± 0.43

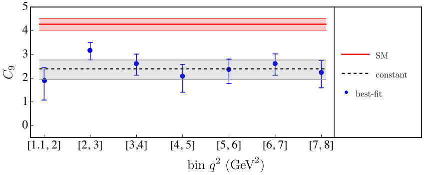
$B \to K^*\mu^+\mu^-$

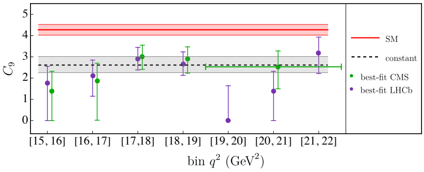
V	Polarization	η_V^λ	δ_V^λ
J/ψ	$\perp$	26.6 ± 1.1	1.46 ± 0.06
	$\parallel$	12.3 ± 0.5	-4.42 ± 0.06
	0	13.9 ± 0.5	-1.48 ± 0.05
$\psi(2S)$	$\perp$	3.0 ± 0.9	3.2 ± 0.4
	$\parallel$	1.11 ± 0.30	-3.32 ± 0.22
	0	1.14 ± 0.06	2.10 ± 0.11

Low- and high-q^2

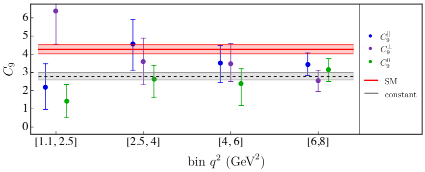
MB, Isidori, Maechler, Tinari, 2401.18007

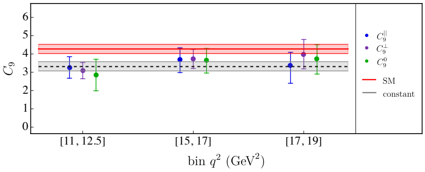
$B^+ \to K^+\mu^+\mu^-$





$B^0 \to K^*\mu^+\mu^-$

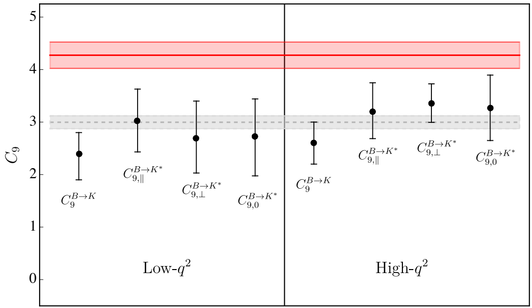




$\Rightarrow$ No evidence of q^2 dependence throughout the bins and the various polarisations

Consistency throughout the whole spectrum

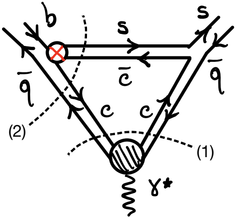
MB, Isidori, Maechler, Tinari, 2401.18007



- Good consistency in the whole q^2 spectrum
- No significant q^2 dependence
- The discrepancy in C_9 is compatible with a short-distance origin
- The method is improvable with finer bins and closer to the resonances

Long-distance effects from analyticity

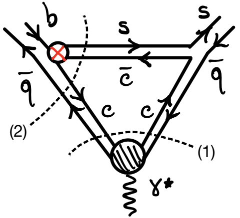
1707.07305, 2011.09813, 2206.03797



When q^2 is large enough to create on-shell states, the amplitude has poles

Long-distance effects from analyticity

1707.07305, 2011.09813, 2206.03797



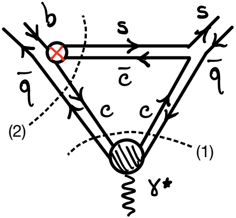
When q^2 is large enough to create on-shell states, the amplitude has poles

The rest should be analytic

$$\mathcal{H}_\lambda \;=\; \frac{1}{\Pi P_i} \sum_k \alpha_k^\lambda z^k$$

Long-distance effects from analyticity

1707.07305, 2011.09813, 2206.03797



When q^2 is large enough to create on-shell states, the amplitude has poles

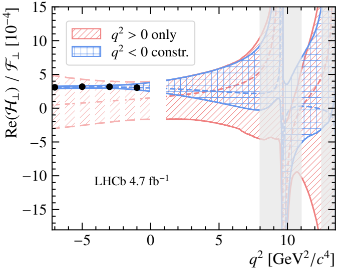
The rest should be analytic

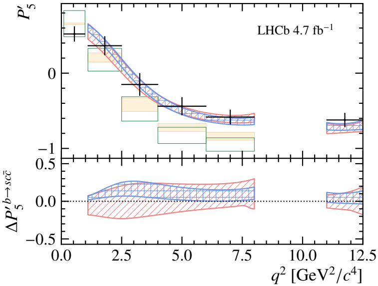
$$\mathcal{H}_\lambda \;=\; \frac{1}{\Pi P_i} \sum_k \alpha_k^\lambda z^k$$

to be determined from data

From theory to unbinned analysis

2312.09102, 2312.09115





- The fit results are given in terms of coefficients of the expansion
- A fit to Wilson coefficients is performed, yielding

$\mathcal{C}_9$	$3.34^{+0.53}_{-0.57}$	$(3.59^{+0.33}_{-0.46})$	$1.9\,(1.8)\,\sigma$
$\mathcal{C}_{10}$	$-3.69^{+0.29}_{-0.31}$	$(-3.93^{+0.27}_{-0.28})$	$1.5\,(0.9)\,\sigma$
$\mathcal{C}_9'$	$0.48^{+0.49}_{-0.55}$	$(0.26^{+0.40}_{-0.48})$	$0.9\,(0.5)\,\sigma$
$\mathcal{C}_{10}'$	$0.38^{+0.28}_{-0.25}$	$(0.27^{+0.25}_{-0.27})$	$1.5\,(1.0)\,\sigma$

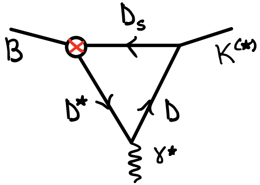
Global significance $1.3(1.4)\sigma$

Is that all?

Is this parametrisation capturing all possible structures?

$\Rightarrow$ Is the analytic structure of the non-local amplitudes different?

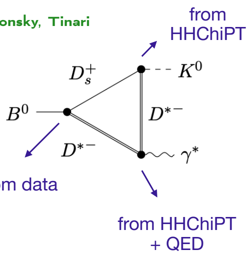
2212.10516



- Rescattering diagrams are known to have a different analytic structure
- How large can these contributions be?

Preliminary estimates:

Isidori, Polonsky, Tinari

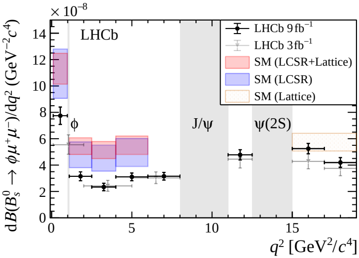


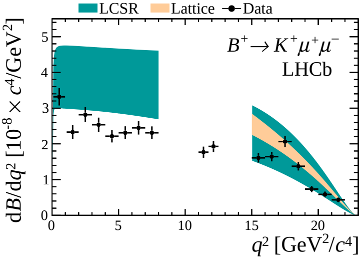
- Reliable at the endpoint
- Preliminary estimate

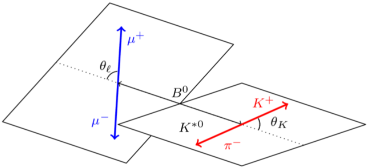
$$\left|\frac{\Delta C_9}{C_9}\right| \leq 3\%$$

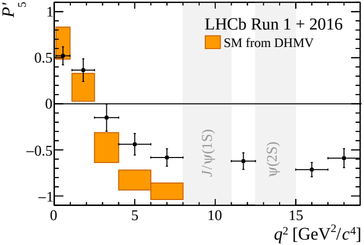
- Extrapolation to the low q^2 region is WIP
- Importance of data ad high q^2

Patterns in $b \to s\mu^+\mu^-$ transitions



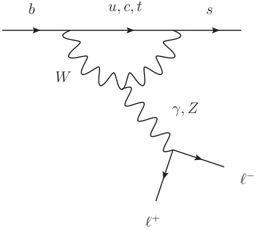






EFT for b decays

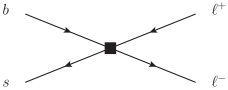
$\Lambda \sim m_t$



$$\mathcal{A}(H_b \to H_s) = \langle H_s | \mathcal{L}_{\rm SM} | H_b \rangle$$

matching
and
running

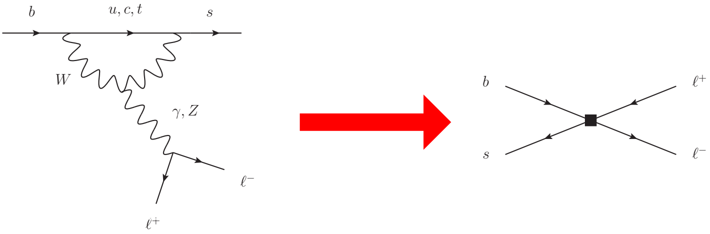
$\Lambda \sim m_b$



$$\mathcal{A}(H_b \to H_s) = \frac{4G_F}{\sqrt{2}} \sum \mathcal{C}_i(\mu) \langle H_s | \mathcal{O}_i | H_b \rangle$$

Energy (Λ)

$b \to s\ell\ell$



$$\mathcal{H}_{\text{eff}} = -4\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[-\mathcal{C}_1 \mathcal{O}_1 - \mathcal{C}_2 \mathcal{O}_2 + \mathcal{C}_7 \mathcal{O}_7 + \mathcal{C}_9 \mathcal{O}_9 + \mathcal{C}_{10} \mathcal{O}_{10} \right]$$

$$\mathcal{O}_1 = (\bar{s}\gamma^\mu P_L b)\,(\bar{c}\gamma_\mu c) \qquad \mathcal{O}_2 = (\bar{s}\gamma^\mu T^a P_L b)\,(\bar{c}\gamma_\mu T^a c)$$

$$\mathcal{O}_9 = (\bar{s}\gamma^\mu P_L b)\,(\bar{\ell}\gamma_\mu \ell) \qquad \mathcal{O}_{10} = (\bar{s}\gamma^\mu P_L b)\,(\bar{\ell}\gamma_\mu \gamma_5 \ell)$$

$$\mathcal{O}_7 = (\bar{s}\sigma^{\mu\nu} P_R b)\,F_{\mu\nu}$$

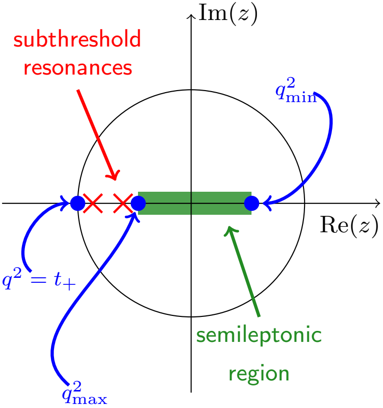
- Wilson coefficients are calculated at NNLO

Gorbahn, Haisch, '04, Bobeth, Gambino, Gorbahn, Haisch, '11

- The running to $\mu = m_b$ is known

The z-expansion and unitarity

[Boyd, Grinstein, Lebed, '95, Caprini, Lellouch, Neubert, '98]



- in the complex plane form factors are real analytic functions
- q^2 is mapped onto the conformal complex variable z

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

- q^2 is mapped onto a disk in the complex z plane, where $|z(q^2, t_0)| < 1$

$$F_i = \frac{1}{P_i(z)\phi_i(z)} \sum_{k=0}^{n_i} a_k^i z^k$$

$$\sum_{k=0}^{n_i} |a_k^i|^2 < 1$$

Lepton Flavour Universality violation

$$R_X = \frac{\mathcal{B}(H_b \to X\mu^+\mu^-)}{\mathcal{B}(H_b \to Xe^+e^-)}$$

- Test of Lepton Flavour Universality, which is one of the building principles of the SM
- With ratios, we reduce hadronic uncertainties at large extent
- For $q^2 \gg m_\ell^2 \to R_X = 1$
- Leading theoretical uncertainty coming from QED effects $\sim 1\%$

MB, Isidori, Pattori, '16
Isidori, Lancerini, Nabeebaccus, Zwicky, '22