北京航空航天大學
BEIHANG UNIVERSITY

Lattice Parton
Collaboration

Exploring the heavy meson distribution amplitudes from lattice QCD

Qi-An Zhang, Beihang University (BUAA)

In collaboration with X.Y. Han, J. Hua, X. Ji, C.-D. Lu, W. Wang, J. Xu and S. Zhao

Aug. 13, 2024 @ LaMET2024

Outline

- **Motivation**
- **Heavy meson LCDAs**
  - **Challenges and recent progresses**
  - **Two-step factorization for heavy meson LCDAs**
  - **LCDAs in QCD and HQET**
- **Summary and outlook**

Motivation: Why LCDAs are important?

- Weak decays of B meson play a crucial role in:
  - **Precise tests of SM**
  - **Searching for NP**
  - **Understanding the origins of CPV**
  - **……**

- $B\to\pi\pi$: **Beneke, Buchalla, Neubert, Sachrajda, 1999; 1422 citations**
- $B\to\pi K$: **Beneke, Buchalla, Neubert, Sachrajda, 2001; 1177 citations**

- $B\to\pi\ell\nu$: **Becher, Hill, 2005; 215 citations**
  Khodjamirian, Mannel, Offen, Wang, 2011; 192 citations
- $B\to K^{(*)}\ell\ell$: **Khodjamirian, Mannel, Pivavorov, Wang, 2010; 486 citations**
- $B\to D\ell\nu$: **HPQCD Collaboration, 2015; 387 citations**

Motivation: Why LCDAs are important?

- Weak decays of B meson play a crucial role in:
  - **Precise tests of SM**
  - **Searching for NP**
  - **Understanding the origins of CPV**
  - **……**

- $B \to \pi\pi$: **Beneke, Buchalla, Neubert, Sachrajda, 1999; 1422 citations**
- $B \to \pi K$: **Beneke, Buchalla, Neubert, Sachrajda, 2001; 1177 citations**

- $B \to \pi \ell v$: **Becher, Hill, 2005; 215 citations**
  Khodjamirian, Mannel, Offen, Wang, 2011; 192 citations
- $B \to K^{(*)}\ell\ell$: **Khodjamirian, Mannel, Pivavorov, Wang, 2010; 486 citations**
- $B \to D\ell v$: **HPQCD Collaboration, 2015; 387 citations**

- Factorization: categories by different characteristic scales

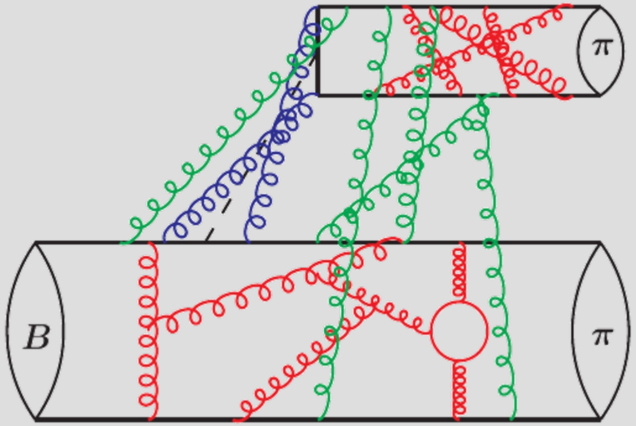
$$\langle \pi(p')\,\pi(q)\,|Q_i|\,\bar{B}(p)\rangle = \underline{f^{B\to\pi}(q^2)} \int_0^1 dx \underline{T_i^{\mathrm{I}}(x)}\,\underline{\phi_\pi(x)} + \int_0^1 d\xi dx dy \underline{T_i^{\mathrm{II}}(\xi,x,y)}\,\underline{\phi_B(\xi)\phi_\pi(x)\phi_\pi(y)}$$

Form factor =

Hard kernel + LCDAs

Hard kernel
(Perturbative)

Meson LCDAs
(Nonperturbative)



Light Meson LCDAs: Research Progresses

➢ Light meson LCDAs have been extensively pursued: (1970s - now)

- **Asymptotic LCDAs**
  Chernyak, Zhitnitsky, 1977; Lepage, Brodsky, 1979;
  Efremov, Radyushkin, 1980
- **Dyson-Schwinger Equation**
  Chang, Cloet, Cobos-Martinez, Roberts, Schmidt, 2013;
  Gao, Chang, Liu, Roberts, Schmidt, 2014;
  Roberts, Richards, Chang, 2021
- **Sum rules**
  Chernyak, Zhitnitsky, 1982; Braun, Filyanov, 1989;
  Ball, Braun, Koike, Tanaka, 1998; Ball, Braun, 1998;
  Khodjamirian, Mannel, Melcher, 2004; Ball,, Lenz, 2007
- **Global Fits**
  Stefanis, 2020; Cheng, Khodjamirian, Rusov, 2020;
  Hua, Li, Lu, Wang, Xing, 2021
- **Models**
  Arriola, Broniowski, 2002, 2006;
  Zhong, Zhu, Fu, Wu, Huang, 2021;
- **Lattice with OPE**
  Martinelli, Sachrajda, 1987; Braun, Bruns, et al., 2016;
  RQCD collaboration, 2019, 2020
- **Lattice with current-current correlation**
  Bali, Braun, Gläßle, Göckeler, Gruber, 2017, 2018;
- **Lattice with LaMET**
  Zhang, Chen, Ji, Jin, Lin, 2017; LP3 Collaboration, 2019;
  Zhang, Honkala, Lin, Chen, 2020; Lin, Chen, Fan, Zhang^2, 2021;
  LPC Collaboration, 2021, 2022; Holligan, Ji, Lin,Su,Zhang, 2023;
  Baker, Bollweg, et·al, 2024; Cloet, Gao, et·al, 2024
- **Quantum Computing**
  QuNu Collaboration, 2023, 2024

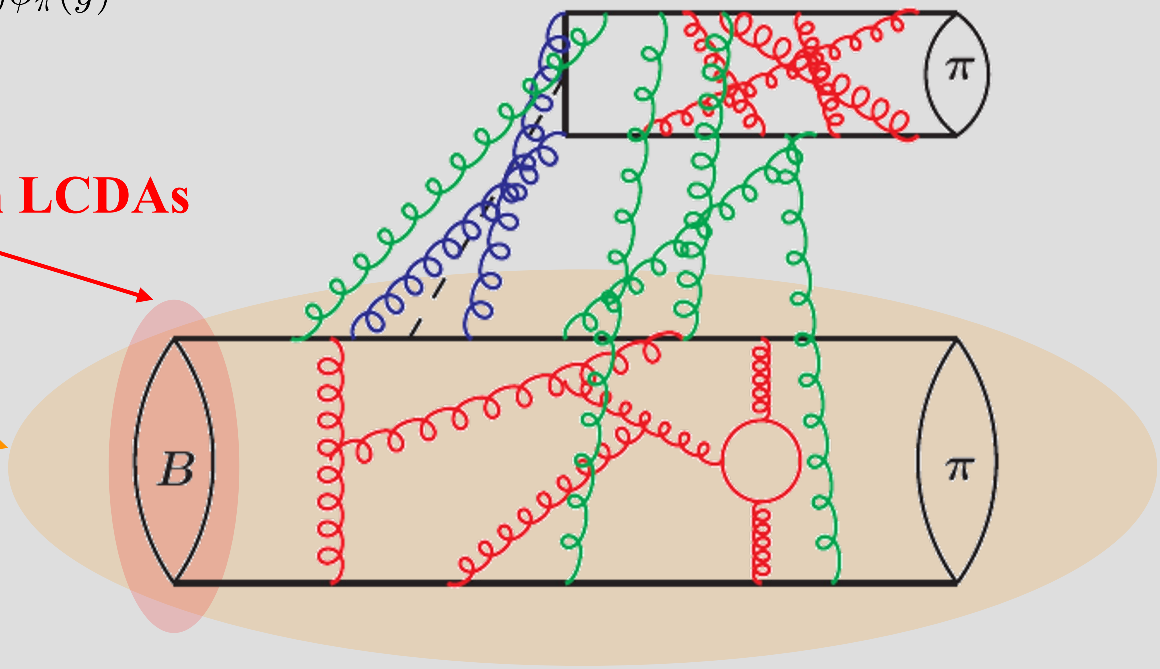
When we focus on the heavy meson LCDAs……

- In current theoretical predictions, the uncertainty of heavy meson LCDA plays a dominant role:

$$\begin{aligned}\langle \pi(p')\,\pi(q)\,|Q_i|\,\bar{B}(p)\rangle =& \underline{f^{B\to\pi}(q^2)}\int_0^1 dx\underline{T_i^{\mathrm{I}}(x)}\phi_\pi(x)\\ &+\int_0^1 d\xi dx dy\underline{T_i^{\mathrm{II}}(\xi,x,y)}\,\underline{\phi_B(\xi)}\phi_\pi(x)\phi_\pi(y)\end{aligned}$$

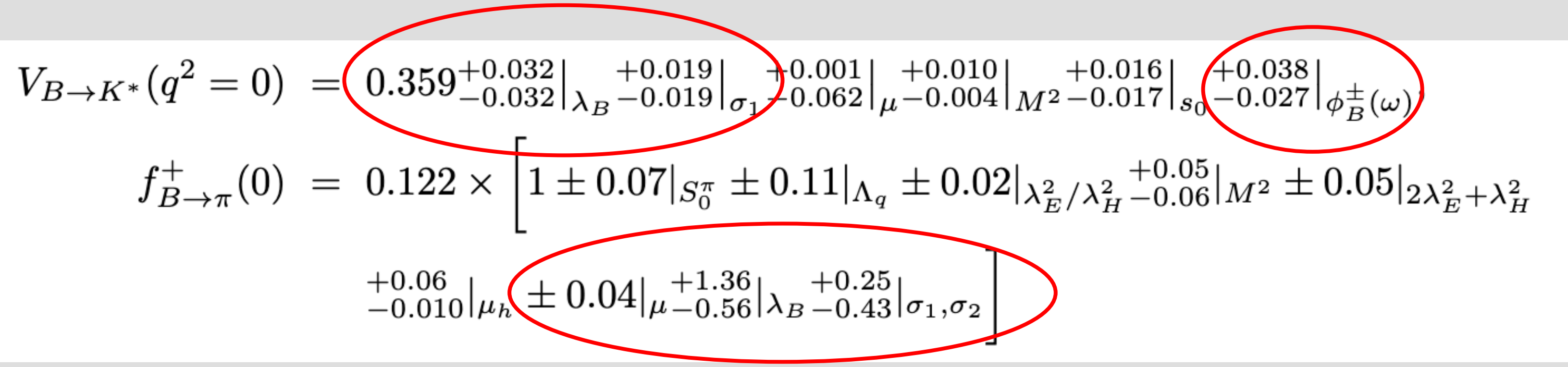
Form factor

Heavy meson LCDAs



When we focus on the heavy meson LCDAs……

- ➢ In current theoretical predictions, the uncertainty of heavy meson LCDA plays a dominant role:



$$\begin{aligned} V_{B\to K^*}(q^2=0) &= 0.359^{+0.032}_{-0.032}\big|_{\lambda_B}{}^{+0.019}_{-0.019}\big|_{\sigma_1}{}^{+0.001}_{-0.062}\big|_{\mu}{}^{+0.010}_{-0.004}\big|_{M^2}{}^{+0.016}_{-0.017}\big|_{s_0}{}^{+0.038}_{-0.027}\big|_{\phi_B^\pm(\omega)} \\ f^+_{B\to\pi}(0) &= 0.122\times\Big[1\pm 0.07\big|_{S_0^\pi}\pm 0.11\big|_{\Lambda_q}\pm 0.02\big|_{\lambda_E^2/\lambda_H^2}{}^{+0.05}_{-0.06}\big|_{M^2}\pm 0.05\big|_{2\lambda_E^2+\lambda_H^2} \\ &\qquad {}^{+0.06}_{-0.010}\big|_{\mu_h}\pm 0.04\big|_{\mu}{}^{+1.36}_{-0.56}\big|_{\lambda_B}{}^{+0.25}_{-0.43}\big|_{\sigma_1,\sigma_2}\Big] \end{aligned}$$

[Gao, Lu, Shen, Wang, Wei, 2020; Cui, Huang, Shen, Wang, 2023]

λ_B and σ_1 denotes the first inverse and log-inverse moment, and $\phi_B^\pm$ denotes other uncertainties from different parameterizations of the B meson LCDA.

Heavy Meson LCDAs: Research Progresses and Challenges

- **The HQET matrix element of heavy meson** [Grozin, Neubert, 1997; Beneke, Feldmann, 2000]

$$\langle 0\,|\bar q_\beta(\xi)[\xi,0]h_{v\alpha}(0)|\,\bar B(v)\rangle = -\frac{i\tilde f_B m_B}{8}\left\{\left[\tilde\phi_B^+(t,\mu)v_+\gamma_- + \tilde\phi_B^-(t,\mu)v_-\gamma_+\right]\gamma_5\right\}_{\alpha\beta}$$

Leading twist Sub-leading twist

- Evolution of φ_B^+ and φ_B^-: [Lange, Neubert, 2003; Bell, Feldmann, 2008]

$$\frac{d}{d\ln\mu}\varphi_B^+(\omega,\mu) = -\frac{\alpha_s C_F}{4\pi}\int_0^\infty d\omega'\gamma_+^{(1)}(\omega,\omega',\mu)\,\varphi_B^+(\omega',\mu)+\mathcal{O}\left(\alpha_s^2\right)$$
$$\frac{d}{d\ln\mu}\varphi_B^-(\omega,\mu) = -\frac{\alpha_s C_F}{4\pi}\int_0^\infty d\omega'\gamma_-^{(1)}(\omega,\omega',\mu)\,\varphi_B^-(\omega',\mu)+\mathcal{O}\left(\alpha_s^2\right)$$

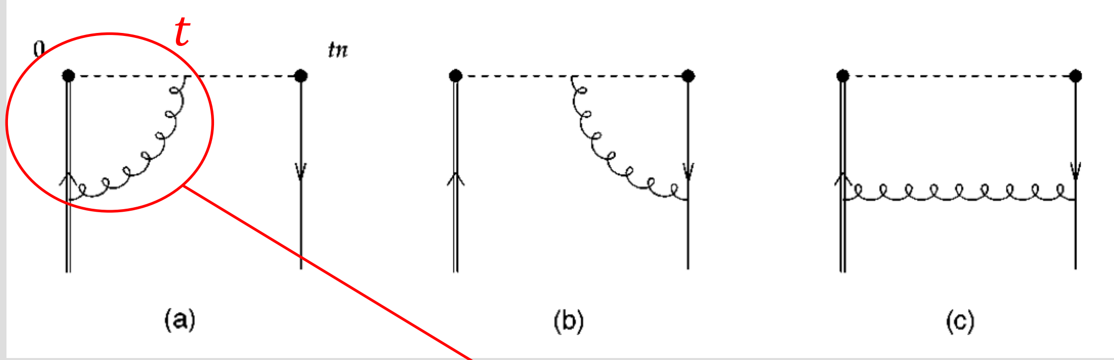
with

$$\gamma_+^{(1)}(\omega,\omega',\mu) = \left(\Gamma_{\text{cusp}}^{(1)}\ln\frac{\mu}{\omega}-2\right)\delta(\omega-\omega') - \Gamma_{\text{cusp}}^{(1)}\,\omega\left[\frac{\theta(\omega'-\omega)}{\omega'(\omega'-\omega)}+\frac{\theta(\omega-\omega')}{\omega(\omega-\omega')}\right]_+,$$
$$\gamma_-^{(1)}(\omega,\omega';\mu) = \gamma_+^{(1)}(\omega,\omega';\mu) - \Gamma_{\text{cusp}}^{(1)}\frac{\theta(\omega'-\omega)}{\omega'}.$$

- Solution of evolution equations: [Bell, Feldmann, Wang and Yip, 2013; Braun, Manashov, 2014]
- RG equations of $\varphi_B^+(\omega,\mu)$ at two-loops: [Braun, Ji, Manashov, 2019; Liu, Neubert, 2020]
- RG equations of the higher-twist B-meson distribution amplitudes: [Braun, Ji, Manashov, 2017]
- NNLO QCD correction to relevant hadronic B-meson decays: [Bell, Beneke, Huber, Li, 2020]

Heavy Meson LCDAs: Research Progresses and Challenges

➢ **Difficulties in calculating the moments of heavy meson LCDA:**



Diverge at $t \to 0$!

$$O_+^{\rm ren}(t,\mu) = O_+^{\rm bare}(t) + \frac{\alpha_s C_F}{4\pi}\left\{\left(\frac{4}{\hat{\epsilon}^2}+\frac{4}{\hat{\epsilon}}\ln(it\mu)\right)O_+^{\rm bare}(t)\right.$$

$$-\frac{4}{\hat{\epsilon}}\int_0^1 du\frac{u}{1-u}\left[O_+^{\rm bare}(ut)-O_+^{\rm bare}(t)\right]$$

[Braun, Ivanov, Korchemsky, 2004]

- Diverge at $t \to 0 \Leftrightarrow$ No local limit
- Non-negative moments $\int dk\, k^n \varphi_+(k)$ for n=0,1,2,… are not related to OPE, and actually they diverge
- Cannot obtain φ_B from lattice QCD through their moments.

Heavy Meson LCDAs: Research Progresses and Challenges

- A theoretical attempt to extract the heavy meson LCDAs in the framework of LaMET:

$$\tilde{\varphi}_B^+(\xi,\mu) \propto \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi} e^{i n_z\cdot v \xi\tau} \langle 0 \left| \boxed{(\bar{q}W_c)(\tau n_z)\,\eta_z\gamma_5\,(W_c^\dagger h_v)(0)} \right| \bar{B}(v)\rangle \quad \Rightarrow \quad \varphi_B^+(\omega,\mu)$$

Connecting the between **equal-time HQET correlator** and **light-cone one**.

- Leading twist matching @ 1-loop: *[Wang², Xu, Zhao, 2020; Xu, Zhang, 2022]*
- Sub-leading twist matching: *[Hu, Wang, Xu, Zhao, 2024]*
- Inverse moment and log moments: *[Xu, Zhang, Zhao, 2022; Hu, Xu, Zhao, 2024]*
- Ioffe-time distributions: *[Zhao, Radyushkin, 2021]*

🤔 Difficult to realize the boosted HQET field (time-like Wilson link) on lattice QCD.

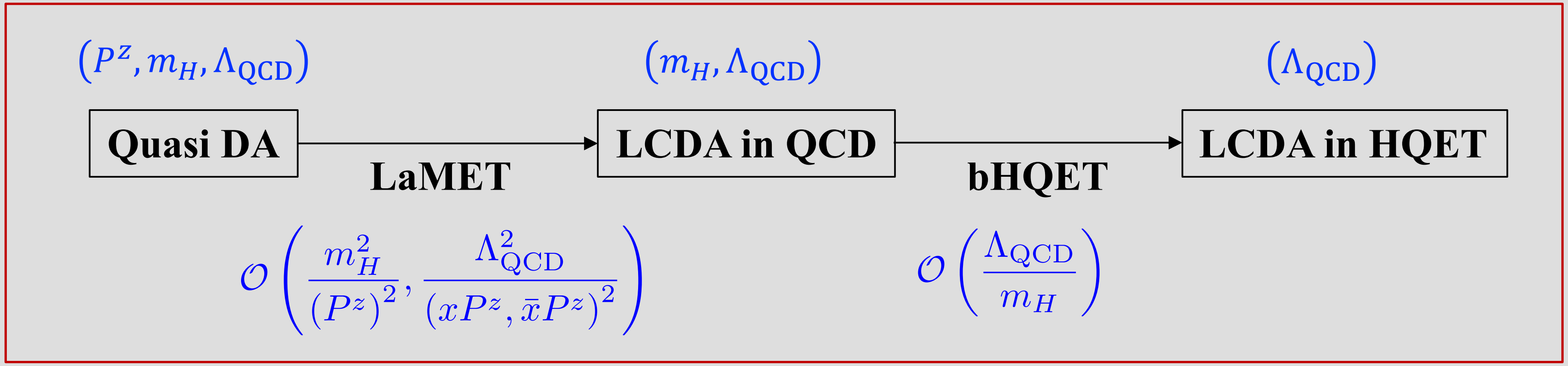
Two-step factorization to access heavy meson LCDA

- Back to the beginning: Quasi DA, which calculated from LQCD
  - Light meson (2 characteristic scales: $P^z, \Lambda_{\text{QCD}}$) → Heavy meson (3 scales: $P^z, m_H, \Lambda_{\text{QCD}}$)

Two-step factorization to access heavy meson LCDA

- ➢ Back to the beginning: Quasi DA, which calculated from LQCD

- Light meson (2 characteristic scales: $P^z, \Lambda_{\mathrm{QCD}}$) → Heavy meson (3 scales: $P^z, m_H, \Lambda_{\mathrm{QCD}}$)



$(P^z, m_H, \Lambda_{\mathrm{QCD}})$

$(m_H, \Lambda_{\mathrm{QCD}})$

(Λ_{QCD})

Quasi DA

LCDA in QCD

LCDA in HQET

LaMET

bHQET

$$\mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(xP^z, \bar{x}P^z)^2}\right)$$

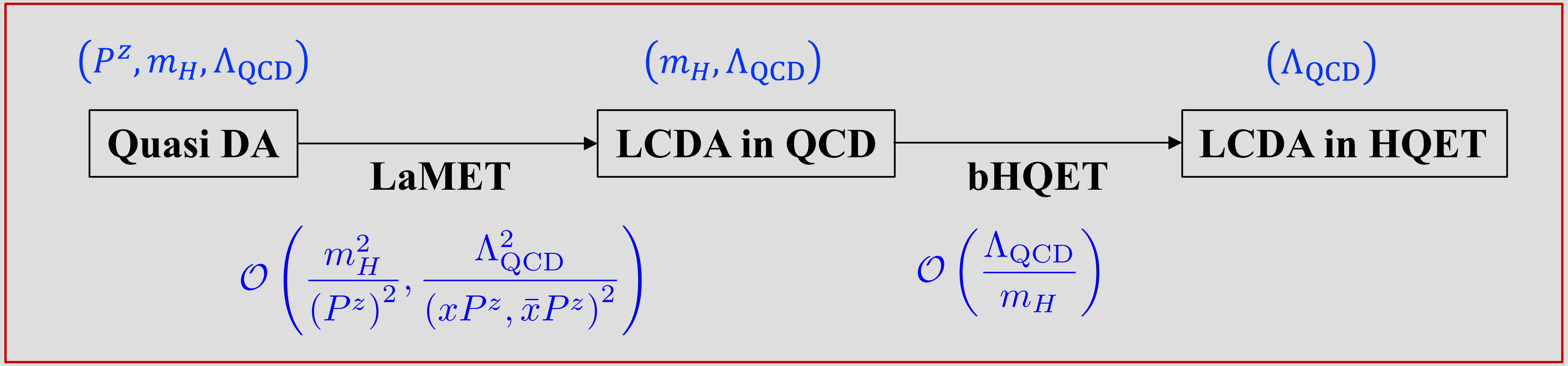
$$\mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{m_H}\right)$$

- A multi-scale processes:

  1. LaMET requires $\Lambda_{\mathrm{QCD}}, m_H \ll P^z$ and finally integrate out P^z;
  2. bHQET requires $\Lambda_{\mathrm{QCD}} \ll m_H$ and integrate out m_H;

⇒ **Hierarchy $\boldsymbol{\Lambda_{\mathrm{QCD}} \ll m_H \ll P^z}$.**

Two-step factorization to access heavy meson LCDA

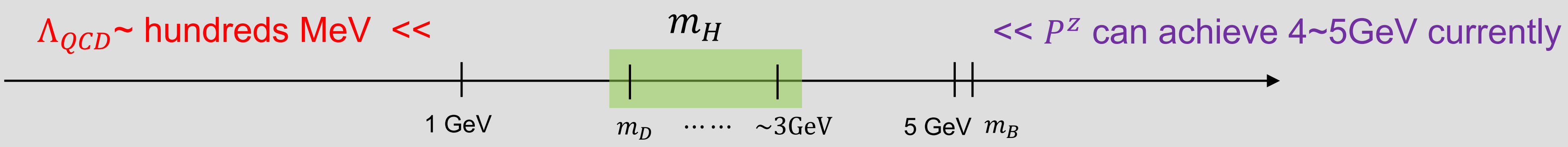


$(P^z, m_H, \Lambda_{\mathrm{QCD}})$ **Quasi DA** $\xrightarrow{\text{LaMET}}$ $(m_H, \Lambda_{\mathrm{QCD}})$ **LCDA in QCD** $\xrightarrow{\text{bHQET}}$ (Λ_{QCD}) **LCDA in HQET**

LaMET: $\mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(xP^z, \bar{x}P^z)^2}\right)$

bHQET: $\mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{m_H}\right)$

⇒ **Hierarchy $\Lambda_{\mathrm{QCD}} \ll m_H \ll P^z$:** A big challenge for lattice simulation



Λ_{QCD}~ hundreds MeV <<

m_H

<< P^z can achieve 4~5GeV currently

Window for lattice simulation

At this stage, the heavy meson could be D, but by no means be the B meson!

Matching I: from quasi DAs to LCDAs in QCD

- Quasi DA $\tilde{\phi}(x, P^z)$, include the scales $\Lambda_{\mathrm{QCD}} \ll m_H \ll P^z$

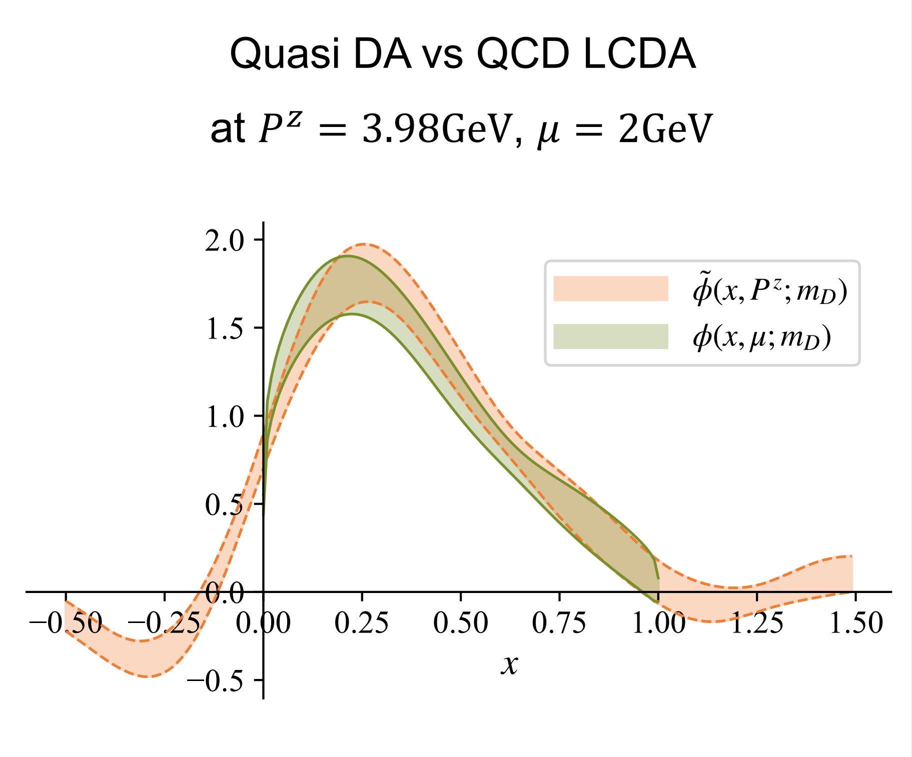
$$\tilde{\phi}(x, P^z) = \int \frac{dz}{2\pi} e^{-ixP^z z} \tilde{M}(z, P^z)$$

- Matching formula in LaMET:

$$\tilde{\phi}(x, P^z) = \int_0^1 C\left(x, y, \frac{\mu}{P^z}\right) \phi(y, \mu) + \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(xP^z, \bar{x}P^z)^2}\right)$$

Liu, Wang, Xu, **QAZ**, Zhao, 2019;
Han, Hua, Ji, Lu, Wang, Xu, **QAZ**, Zhao, 2024

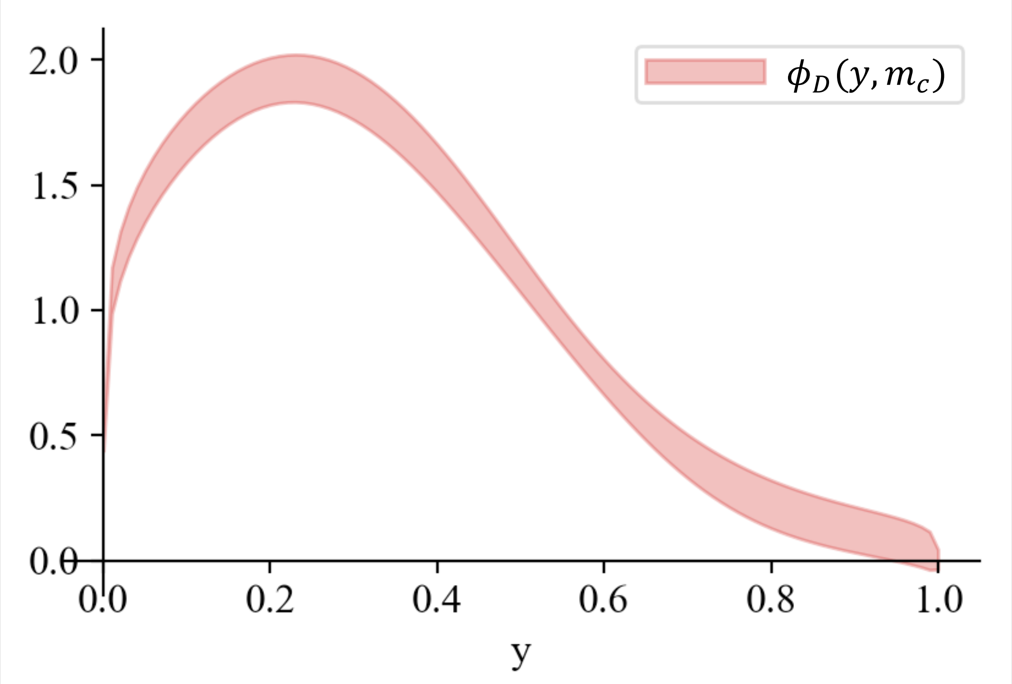
This matching integrate out P^z, obtain the LCDAs in QCD.


Quasi DA vs QCD LCDA
at $P^z = 3.98\mathrm{GeV}$, $\mu = 2\mathrm{GeV}$

LCDAs in QCD

- Heavy meson LCDAs in QCD

$$\phi(y,\mu) = \frac{1}{i f_H}\int_{-\infty}^{+\infty}\frac{d\tau}{2\pi}e^{iyP_H\tau n_+} \times \langle 0\,|\bar{q}(\tau n_+)\not{n}_+\gamma_5 W_c(\tau n_+,0)Q(0)|\,H(P_H)\rangle$$

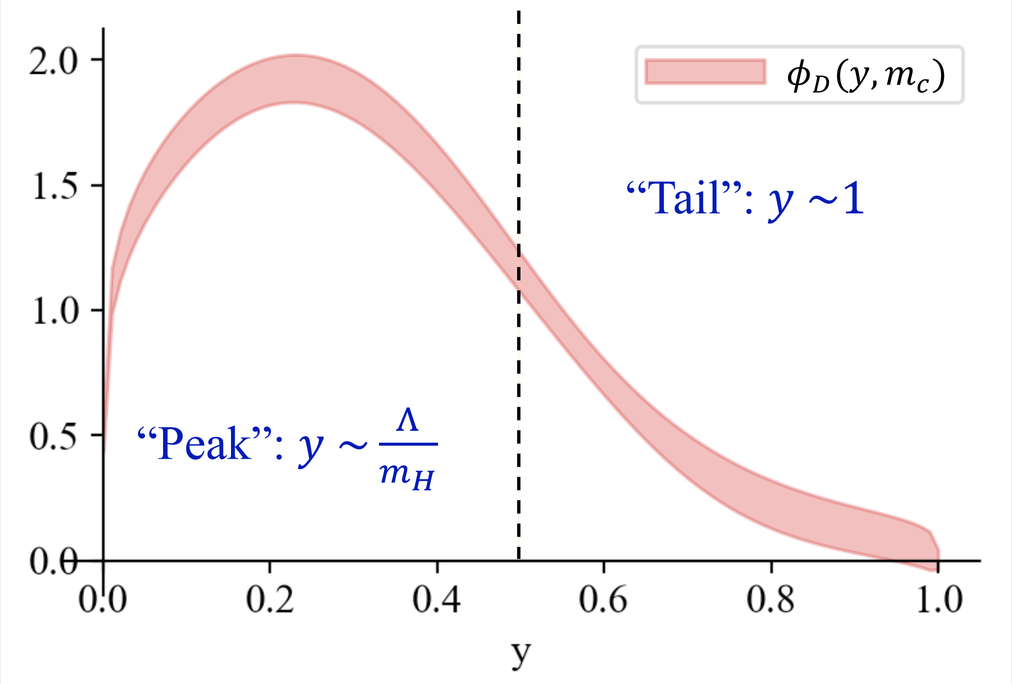


- The peak position dominated by m_H and μ;
- At very large scale $\mu \gg m_H$, asymptotic form;

LCDAs in QCD

➢ Heavy meson LCDAs in QCD

$$\phi(y,\mu) = \frac{1}{if_H}\int_{-\infty}^{+\infty}\frac{d\tau}{2\pi}e^{iyP_H\tau n_+} \times \langle 0\,|\bar{q}(\tau n_+)\not{n}_+\gamma_5 W_c(\tau n_+,0)Q(0)|\,H(P_H)\rangle$$



- The peak position dominated by m_H and μ;
- At very large scale $\mu \gg m_H$, asymptotic form;

- For the scale $\mu \lesssim m_Q$,

  ➩ Light quark carries small momentum fraction $y\sim\Lambda/m_H$

  ⇒ peak region, related to the HQET LCDA;

  [Ishaq, Jia, Xiong, Yang, 2020; Beneke, Finauri, Vos, Wei, 2023]

  ➩ $y\sim O(1)$ region be suppressed in LCDA:

  SCET renormalized matrix element in this region contain only hard-collinear physics, and starts at the one-loop level.

Matching II: connecting LCDAs in QCD and HQET

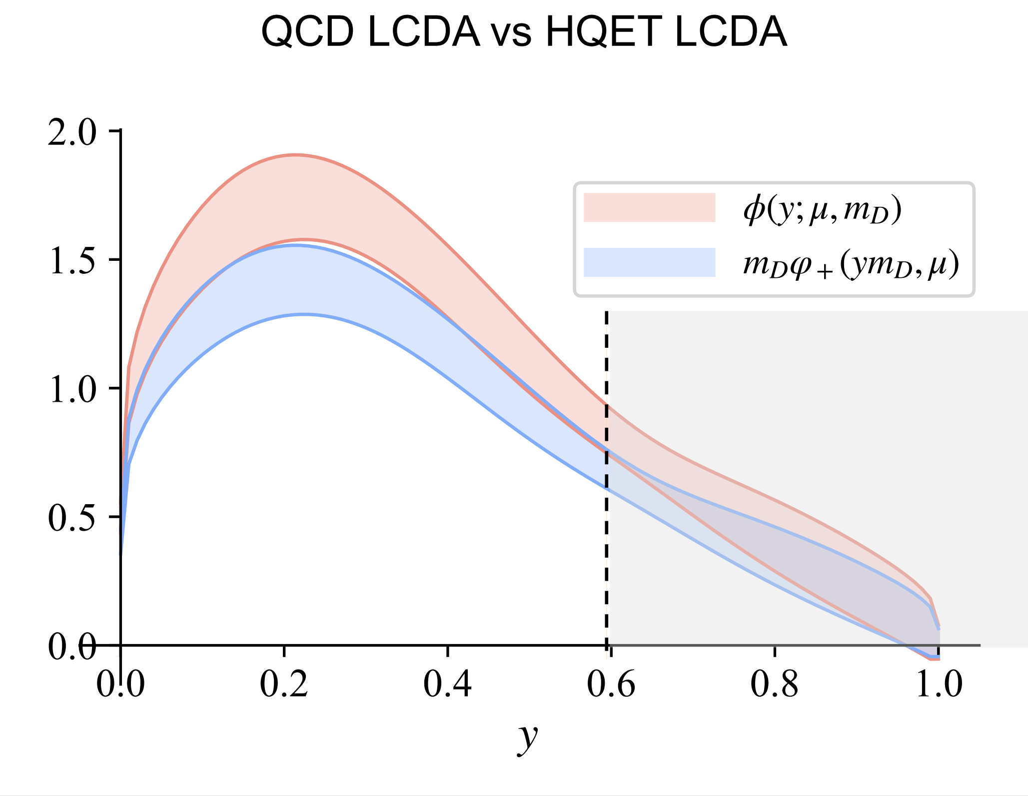
- Leading twist heavy meson LCDA in HQET

$$\varphi^+(\omega,\mu) = \frac{1}{i\tilde{f}_H(\mu)m_H}\int_{-\infty}^{+\infty}\frac{d\eta}{2\pi}\,e^{i\omega n_+\cdot v\eta}$$
$$\times\,\langle 0\,|\bar{q}(\eta n_+)\not{n}_+\gamma_5 W_c(\eta n_+,0)h_v(0)|\,H(v)\rangle$$

connected with the QCD LCDA through a multiplicative factorization in the peak region:

[Beneke, Finauri, Vos, Wei, 2023]

$$\phi\,(y,\mu;m_H) = \frac{\tilde{f}_H}{f_H}J_{\mathrm{peak}}m_H\varphi^+\,(\omega,\mu) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{m_H}\right)$$



Tails of HQET LCDA

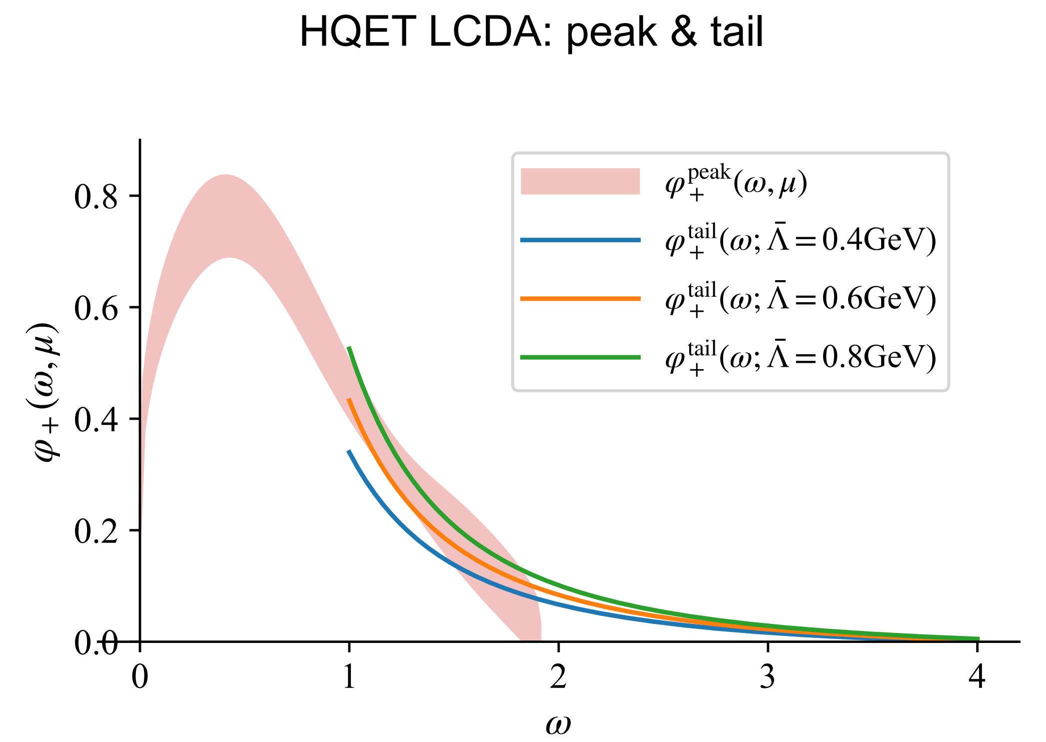
- The tail region of HQET LCDA is perturbative: [Lee, Neubert, 2005]

$$\varphi^{+}_{\text{tail}}(\omega,\mu)=\frac{\alpha_s C_F}{\pi\omega}\left[\left(\frac{1}{2}-\ln\frac{\omega}{\mu}\right)+\frac{4\bar{\Lambda}}{3\omega}\left(2-\ln\frac{\omega}{\mu}\right)\right]$$

where $\overline{\Lambda}\equiv m_H-m_Q^{\text{pole}}$ reflect the power correction, and usually be chosen as 400~600MeV.

- We use the difference between the lines to estimate the power correction.

The final results of HQET LCDA will merge the peak (from LQCD) and tail region (from 1-loop calculation).



Comparison with phenomenological models

- Several commonly used models: [Wang, Shen, 2015; Beneke, Braun, Ji, Wei, 2018; Gao, Huber, Ji, Wang², 2022]

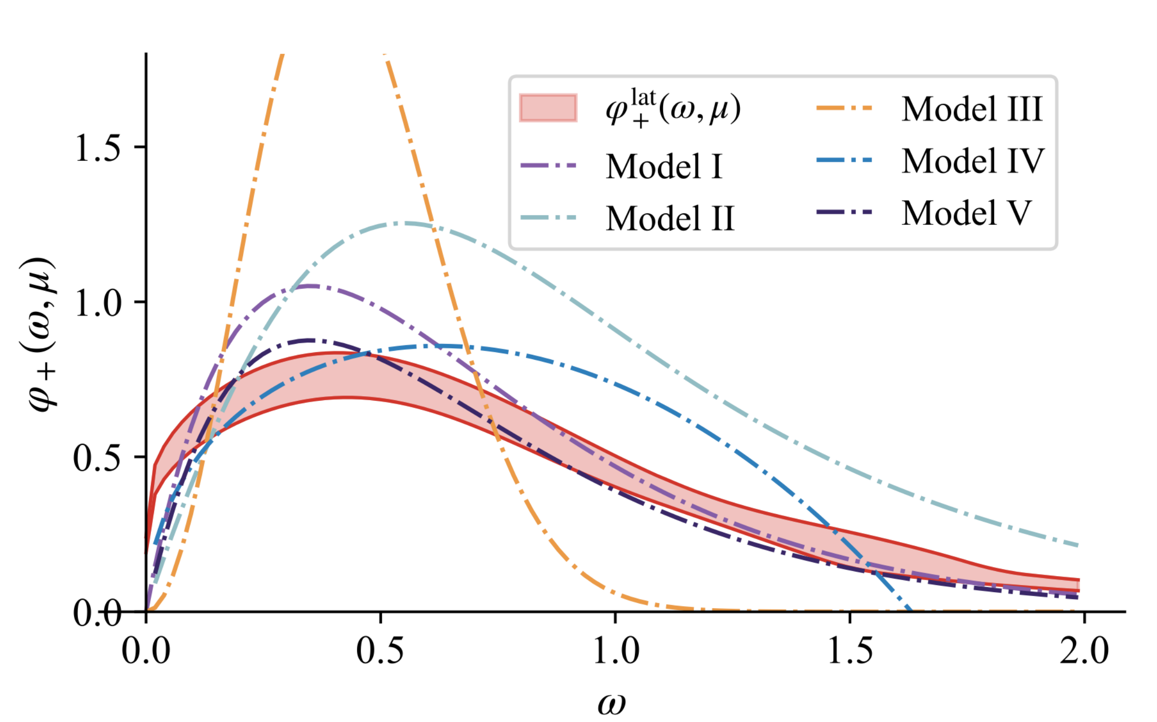
$$\varphi_{\mathrm{I}}^{+}\left(\omega, \mu_0\right)=\frac{\omega}{\omega_0^2} e^{-\omega / \omega_0},$$

$$\varphi_{\mathrm{II}}^{+}\left(\omega, \mu_0\right)=\frac{4}{\pi \omega_0} \frac{k}{k^2+1}\left[\frac{1}{k^2+1}-\frac{2\left(\sigma_B^{(1)}-1\right)}{\pi^2} \ln k\right],$$

$$\varphi_{\mathrm{III}}^{+}\left(\omega, \mu_0\right)=\frac{2 \omega^2}{\omega_0 \omega_1^2} e^{-\left(\omega / \omega_1\right)^2},$$

$$\varphi_{\mathrm{IV}}^{+}\left(\omega, \mu_0\right)=\frac{\omega}{\omega_0 \omega_2} \frac{\omega_2-\omega}{\sqrt{\omega\left(2 \omega_2-\omega\right)}} \theta\left(\omega_2-\omega\right),$$

$$\varphi_{\mathrm{V}}^{+}(\omega, \mu_0)=\frac{\Gamma(\beta)}{\Gamma(\alpha)} \frac{\omega}{\omega_0^2} e^{-\omega / \omega_0} U(\beta-\alpha, 3-\alpha, \omega / \omega_0),$$



Summary and outlook

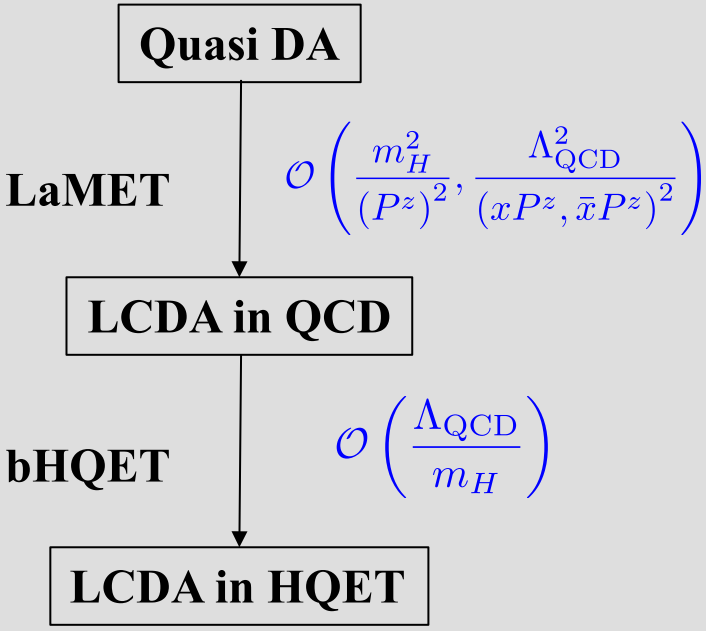
- ✓ Lattice QCD + LaMET provides a powerful tool for exploring the light-like correlations, as LCDAs, PDFs, TMDWFs, TMDPDFs, ……
- ✓ Heavy meson LCDAs: a first lattice-implementable scheme, which can continually be improved.

More importantly, improving the reliability of our results for the next stage……

Summary and outlook

More importantly, improving the reliability of our results for the next stage:

- ➢ The most urgent is to properly <u>control the power corrections</u> within two step factorization:
  - Larger P^z to increase the window for lattice calculations;
  - Consider high power terms within matching.
- ➢ More systematic lattice QCD calculations:
  - Nonperturbative renormalization, continuum and physical mass extrapolation, operator mixing effects, ……
  - More reliable method to merge the peak and tail regions
- ➢ Realize the HQET quasi DA on lattice QCD directly?



Quasi DA

LaMET $\mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(xP^z, \bar{x}P^z)^2}\right)$

LCDA in QCD

bHQET $\mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_H}\right)$

LCDA in HQET

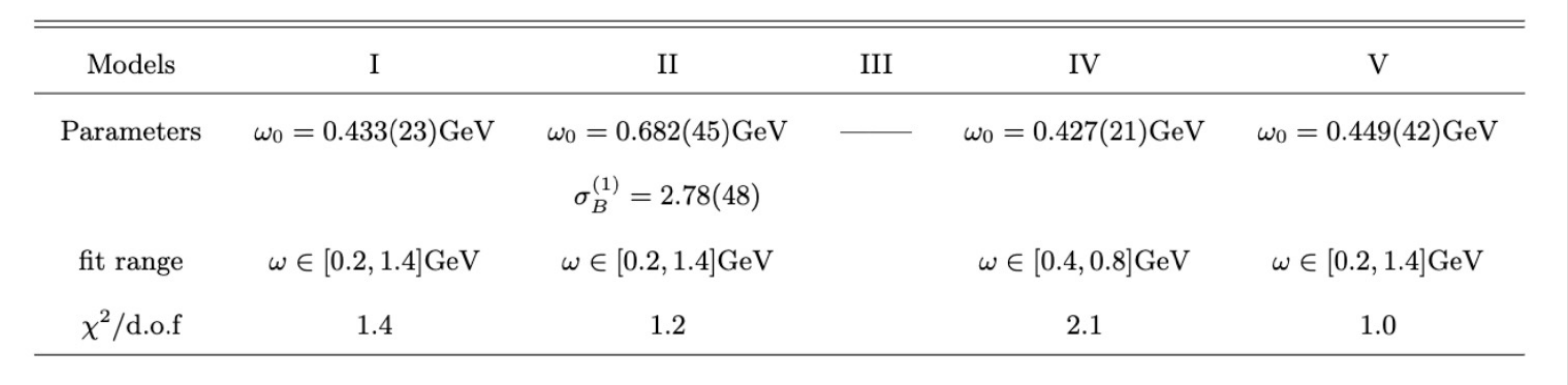
Thanks

Backup slides

First inverse moment

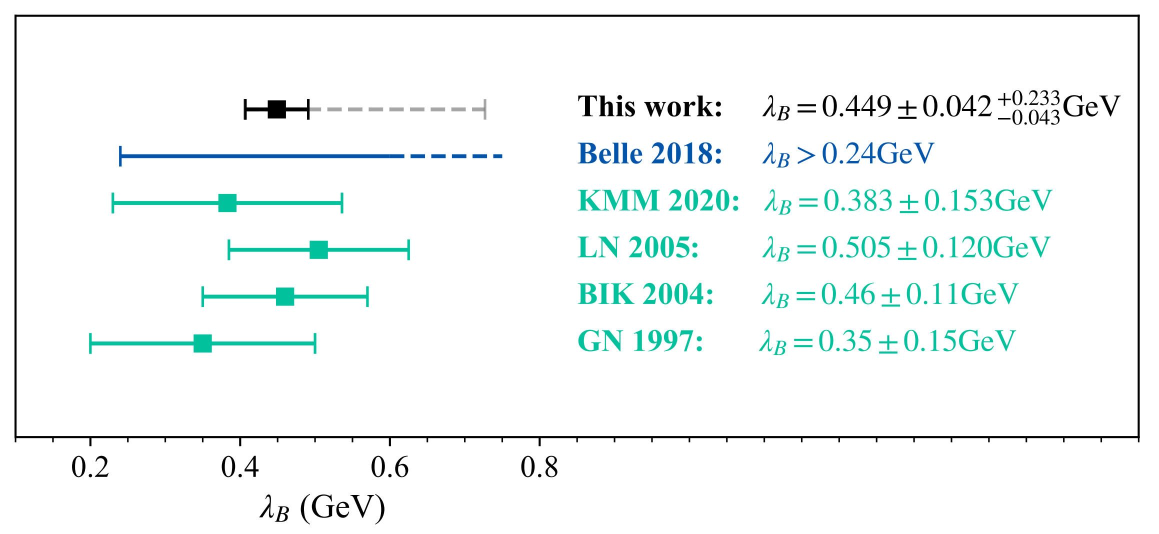
- The first inverse moment

$$\lambda_B^{-1}(\mu) = \int_{-\infty}^{\infty} d\omega \frac{\varphi^+(\omega,\mu)}{\omega}$$



Models	I	II	III	IV	V
Parameters	$\omega_0 = 0.433(23)\mathrm{GeV}$	$\omega_0 = 0.682(45)\mathrm{GeV}$ $\sigma_B^{(1)} = 2.78(48)$	——	$\omega_0 = 0.427(21)\mathrm{GeV}$	$\omega_0 = 0.449(42)\mathrm{GeV}$
fit range	$\omega \in [0.2, 1.4]\mathrm{GeV}$	$\omega \in [0.2, 1.4]\mathrm{GeV}$		$\omega \in [0.4, 0.8]\mathrm{GeV}$	$\omega \in [0.2, 1.4]\mathrm{GeV}$
χ^2/d.o.f	1.4	1.2		2.1	1.0

- The current numerical results are unable to accomplish the integration over full-ω range;
- We determine the λ_B^{-1} by fitting the parameterization forms of different model.



PRD98,112016(2018),
JHEP10,043(2020),
PRD72,094028(2005),
PRD69,034014(2004),
PRD55,272(1997)