# Factorization of quasi GPDs and the mixing channel puzzle

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(In collaboration with J.P.Ma, C.P.Zhang and G.P.Zhang.)
In progress(In collaboration with Y.Ji, F.Yao and J.H.Zhang).

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# GPD

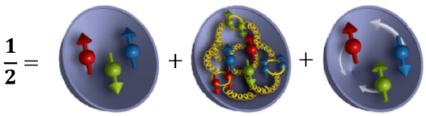
Generalized Parton Distribution:

$$F_q(x,\xi,t) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ixP^+\lambda}\langle p'|\bar{\psi}\left(-\frac{\lambda}{2}n\right)\mathcal{L}_n^\dagger\left(-\frac{\lambda}{2},\infty\right)\gamma^+\mathcal{L}_n\left(\frac{\lambda}{2},\infty\right)\psi\left(\frac{\lambda}{2}n\right)|p\rangle$$
$$= \frac{1}{2P^+}\bar{u}(p')\left[\gamma^+ H_q(x,\xi,t) + i\frac{\sigma^{+\nu}\Delta_\nu}{2m}E_q(x,\xi,t)\right]u(p),$$

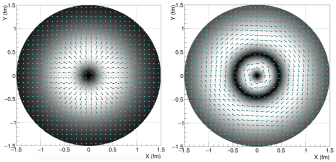
encodes the coupling between longitudinal momentum and transverse position of the parton.

$$J_q = \lim_{t,\xi\to 0}\frac{1}{2}\int \mathrm{d}x x\,[\mathrm{H}^q(x,\xi,t)+\mathrm{E}^q(x,\xi,t)]\,,$$

$$\int \mathrm{d}x x \mathrm{H}(x,\xi,t) = M_2(t) + \frac{4}{5}\xi^2 d_1(t),$$



Ji, 1996




Burkert et al., 2021

# Existing work on the factorization of quasi GPDs:

- Factorization of non-singlet quasi quark GPD in momentum space:

  Ji et al.,2015
  Xiong,Zhang,2015
  Liu et al.,2019

- Factorization of quasi light-front correlators in coordinate and momentum (pseudo) space:

  Radyushkin,2019
  Yao,Ji,Zhang,2023

- Factorization of quasi quark and gluon GPDs in momentum space:

  Ma,Pang,Zhang,2022
  Ma et al.,2023

This talk:

Introduce the factorization of quasi GPDs based on diagram expansion, especially how we derive a gauge-invariant result.

# Outline

- Factorization of quasi quark GPD at one-loop
- Factorization of quasi gluon GPD: derivation of a gauge-invariant form
- The puzzle in the gluon-quark mixing channel
- Summary and outlook

- Factorization of quasi quark GPD at one-loop

Factorization of quasi gluon GPD: derivation of a gauge-invariant form

The puzzle in the gluon-quark mixing channel

Summary and outlook

# Factorization of quasi quark GPD at one-loop

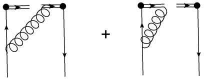
Quasi generalized quark GPD:

$$\mathcal{F}_q(z,\xi,t) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{iz\lambda P^z} \langle p'|\bar{\psi}\left(-\frac{\lambda}{2}n_z\right)\mathcal{L}_z^\dagger\left(-\frac{\lambda}{2}n_z\right)\gamma^z \mathcal{L}_z\left(\frac{\lambda}{2}n_z\right)\psi\left(\frac{\lambda}{2}n_z\right)|p\rangle$$

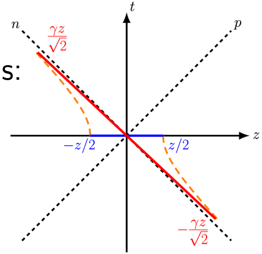
$$= \frac{1}{2P^z}\bar{u}(p')\left[\gamma^z \mathcal{H}_q(z,\xi,t) + i\frac{\sigma^{z\nu}\Delta_\nu}{2m}\mathcal{E}_q(z,\xi,t)\right]u(p),$$

$$\mathcal{F}_q(z,\xi,t) = F_q(z,\xi,t) + \frac{\alpha_s}{2\pi}\int_{-1}^{1} dx\left[C_F h_q(x,\xi,z)F_q(x,\xi,t) + h_g(x,\xi,z)F_g(x,\xi,t)\right],$$

- $h$ is free from collinear divergence,
- soft div cancel after adding real and virtual diagrams:

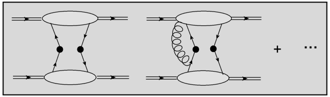


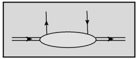
$$\ni \left(\frac{\epsilon(x-z)}{(x-z)}\right)_+, \left(\frac{\epsilon(x-z)\ln(x-z)^2}{x-z}\right)_+$$



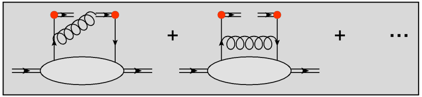
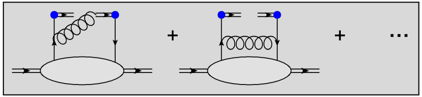
# Factorization of quasi quark GPD at one-loop

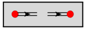
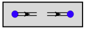
Hadronic tensor $W^{\mu\nu}$ in Drell-Yan process :

$W^{\mu\nu}_{\rm tree} =$ 

 $: \int \frac{d^4\xi}{(2\pi)^4} e^{-i\xi\cdot k_A} \langle p' | [\bar{\psi}(\xi)]_i [\psi(0)]_j | p \rangle,$

(In analogy)Matching coefficient $h_q(x,\xi,z)$ in $\mathcal{F}_q$:

$h_q(x,\xi,z) =$  $-$ 

 $:\bar{\psi}(-\frac{\lambda}{2}n_z)\mathcal{L}^\dagger_z(-\frac{\lambda}{2}n_z,\frac{\lambda}{2}n_z)\gamma^z\psi(\frac{\lambda}{2}n_z)$  $:\bar{\psi}(-\frac{\lambda}{2}n)\mathcal{L}^\dagger_+(-\frac{\lambda}{2}n,,\frac{\lambda}{2}n)\gamma^+\psi(\frac{\lambda}{2}n)$

- We need to subtract the double-counting contribution: the exchanged gluon is collinear to $l$ direction,
- The matching above is between operators.

# Factorization of quasi quark GPD at one-loop

Factorization of unpolarized quasi quark GPD (JHEP 08 (2022) 130):

$$\mathcal{F}_q(z,\xi,t) = F_q(z,\xi,t) + \frac{\alpha_s}{2\pi}\int_{-1}^{1} dx\Big[C_F h_q(x,\xi,z)F_q(x,\xi,t) + h_g(x,\xi,z)F_g(x,\xi,t)\Big],$$

$$\begin{aligned}
h_q(x,\xi,t) = {} & \left(\ln\frac{(2P^z)^2}{\mu^2}+1\right)\left[\frac{|z+\xi|}{4\xi(x+\xi)} - \frac{|z-\xi|}{4\xi(x-\xi)} + \frac{|x-z|}{2(x^2-\xi^2)}\right] + \frac{|z+\xi|}{4\xi(x+\xi)}\ln(z+\xi)^2 - \frac{|z-\xi|}{4\xi(x-\xi)}\ln(z-\xi)^2 \\
& + \frac{|x-z|}{2(x^2-\xi^2)}\ln(x-z)^2 + \frac{1}{2}\Bigg\{\Bigg\{\left(\frac{\epsilon(x-z)}{(x-z)}\ln\frac{(2(x-z)P^z)^2}{e\mu^2}\right)_+ - \frac{\epsilon(x-z)}{x+\xi}\ln\frac{(2(x-z)P^z)^2}{e\mu^2} \\
& + \frac{|z+\xi|}{(x-z)(x+\xi)}\ln\frac{(2(z+\xi)P^z)^2}{e\mu^2} + \delta(x-z)\Big[-\ln\frac{(2P^z)^2}{e\mu^2}\ln\frac{(x+\xi)^2}{1-x^2} - 2\ln^2|x+\xi| \\
& + \ln^2(1+x) + \ln^2(1-x) + \frac{1}{3}\pi^2\Big]\Bigg\} + (\xi\to-\xi)\Bigg\}.
\end{aligned}$$

$\ln\mu$ dependence → **Evolution kernel of light-cone GPD** ✓

$\Delta^\mu \to 0$ → **Matching coefficient of quasi pdf** ✓

Factorization of quasi quark GPD at one-loop

- Factorization of quasi gluon GPD: derivation of a gauge-invariant form

The puzzle in the gluon-quark mixing channel

Summary and outlook

# Factorization of quasi gluon GPD

## Derivation of a gauge invariant form (in Feynman gauge)

Quasi generalized gluon GPD:

$$\mathcal{F}_{gU}(z,\xi,t) = \frac{1}{P_z}\int \frac{d\lambda}{2\pi} e^{izP_z\lambda}\langle p'|g_{\perp\mu\nu}\tilde{G}^{*z\mu}\left(-\frac{1}{2}\lambda n_z\right)\tilde{G}^{z\nu}\left(\frac{1}{2}\lambda n_z\right)|p\rangle \quad \tilde{G}^{z\mu}(x) = \mathcal{L}_z(x) G^{z\mu}(x),$$

$$= -\frac{1}{2P_z}\bar{u}(p')\left[\gamma^z \mathcal{H}_g(z,\xi,t) + \frac{i\sigma^{z\alpha}}{2m}\Delta_\alpha \mathcal{E}_g(z,\xi,t)\right]u(p),$$

$$\boxed{\mathcal{F}_{gU}(z,\xi,t) = \frac{1}{P_z}\int d^4k \frac{1}{2}\mathcal{M}_{\mu\nu}(k_1,k_2)\int \frac{d^4y}{(2\pi)^4} e^{iy\cdot k}\langle p'|G^{c,\nu}\left(-\frac{y}{2}\right)G^{c,\mu}\left(\frac{y}{2}\right)|p\rangle,}$$

Factorization of hard processes: choice of gauge

- Axial gauge: No unphysical gluon and ghost, the unphysical singularities complicate the proof of factorization and perturbative calculation (Collins,Soper and Sterman,1985,1988).
- Covariant gauge: "superleading" contribution appears (superficially)(Collins 2008), ghost attachments are needed to get a gauge-invariant result.

# Factorization of quasi gluon GPD

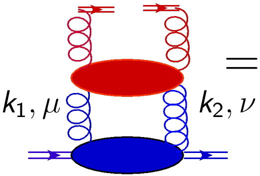
## Derivation of a gauge invariant form (in Feynman gauge)

Quasi generalized gluon GPD:

$$\mathcal{F}_{gU}(z,\xi,t) = \frac{1}{P_z}\int \frac{d\lambda}{2\pi} e^{izP_z\lambda}\langle p'|g_{\perp\mu\nu}\tilde{G}^{*z\mu}\left(-\tfrac{1}{2}\lambda n_z\right)\tilde{G}^{z\nu}\left(\tfrac{1}{2}\lambda n_z\right)|p\rangle \quad \tilde{G}^{z\mu}(x) = \mathcal{L}_z(x)G^{z\mu}(x),$$

$$= -\frac{1}{2P_z}\bar{u}(p')\left[\gamma^z\mathcal{H}_g(z,\xi,t) + \frac{i\sigma^{z\alpha}}{2m}\Delta_\alpha\mathcal{E}_g(z,\xi,t)\right]u(p),$$

$$\boxed{\mathcal{F}_{gU}(z,\xi,t) = \frac{1}{P_z}\int d^4k\,\frac{1}{2}\mathcal{M}_{\mu\nu}(k_1,k_2)\int \frac{d^4y}{(2\pi)^4}e^{iy\cdot k}\langle p'|G^{c,\nu}\left(-\tfrac{y}{2}\right)G^{c,\mu}\left(\tfrac{y}{2}\right)|p\rangle,}$$

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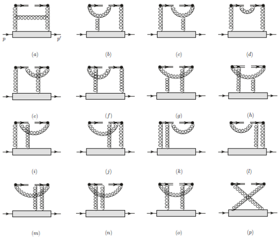

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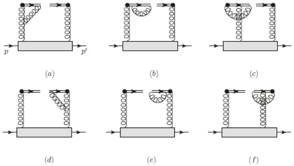
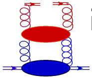
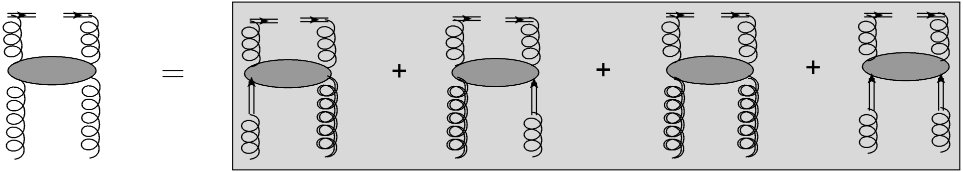
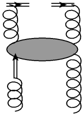


# Factorization of quasi gluon GPD

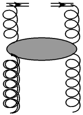
## Derivation of a gauge invariant form (in Feynman gauge)

Decompose the gluon in  into K and G gluon (Grammer and Yennie,1973):





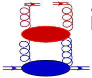
$$: \mathcal{M}^{\mu\nu}(k_1, k_2)\frac{k_{1\mu} n_\alpha}{n\cdot k_1}$$

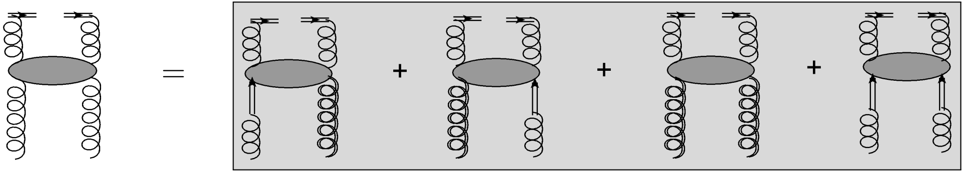


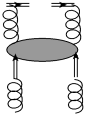
$$: \mathcal{M}^{\mu\nu}(k_1, k_2)\left(g_{\mu\alpha} - \frac{k_{1\mu} n_\alpha}{n\cdot k_1}\right)$$

# Factorization of quasi gluon GPD

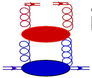
## Derivation of a gauge invariant form (in Feynman gauge)

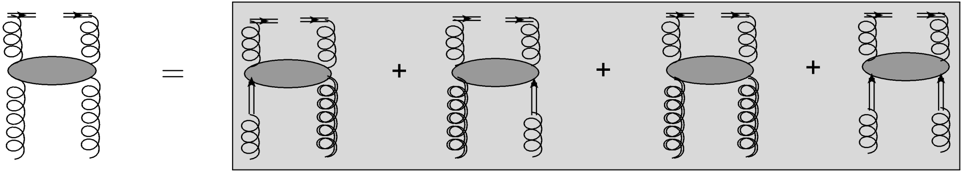
Decompose the gluon in  into K and G gluon (Grammer and Yennie,1973):

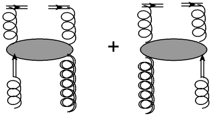


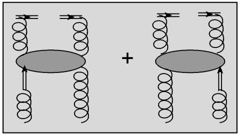
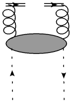
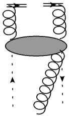
- $\mathcal{M}^{\mu\nu}(k_1, k_2) \sim \frac{1}{N_c^2-1} \int d^4x_1 d^4x_2 e^{-ik_1\cdot x_1 + ik_2\cdot x_2} \langle 0| T\left(G^{a,\nu}(x_2) G^{a,\mu}(x_1) \mathcal{O}\right)|0\rangle\Big|_{\text{amp}}$ amputated Green's function $\rightarrow$ $k_{1\mu} k_{2\nu} \mathcal{M}^{\mu\nu}(k_1, k_2) = 0.$
- The superleading contribution is zero $\left(\mathcal{M}^{--}(\hat{k}_1, \hat{k}_2) = 0\right)$.

Decompose the gluon in  into K and G gluon (Grammer and Yennie,1973):





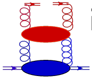
$$\frac{1}{\xi_G}\langle 0|\, T^*\big(G^{a,\nu}(x_2)\partial_\mu G^{a,\mu}(x_1)\mathcal{O}\big)\,|0\rangle \overset{\text{BRST}}{=} -\langle 0|\, T^*\big((D^\nu)^{ab}C^b(x_2)\bar{C}^a(x_1)\mathcal{O}\big)\,|0\rangle,$$

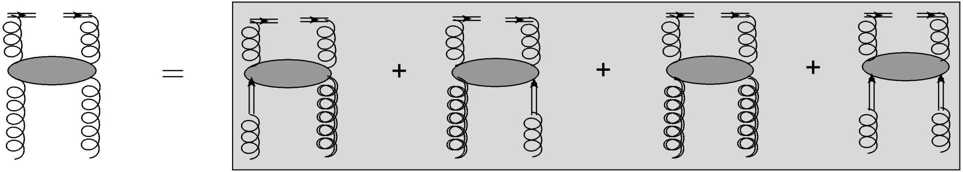
 $= \quad k_2^\nu \times$  $-$  $\times k_2^2$

The BRST symmetry of Green's function is used $\big(0 = \delta_B\langle 0|\, T^*\big(G^{a,\nu}(x_2)\bar{C}^a(x_1)\mathcal{O}\big)\,|0\rangle\big)$.

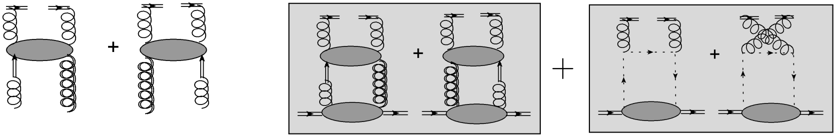
# Factorization of quasi gluon GPD

## Derivation of a gauge invariant form (in Feynman gauge)

Decompose the gluon in  into K and G gluon (Grammer and Yennie,1973):



$$-\langle p|\partial_\beta G^{c,\beta}\left(-\tfrac{y}{2}\right) n\cdot G^{c}\left(\tfrac{y}{2}\right)]|p\rangle + \langle p'|\left[\bar{C}^{a}\left(-\tfrac{y}{2}\right)(n\cdot D)^{ab}C^{b}\left(\tfrac{y}{2}\right)\right]|p\rangle = \langle p'|\delta_B\left[\bar{C}^{a}\left(-\tfrac{y}{2}\right) n\cdot G^{a}\left(\tfrac{y}{2}\right)\right]|p\rangle,$$

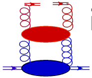


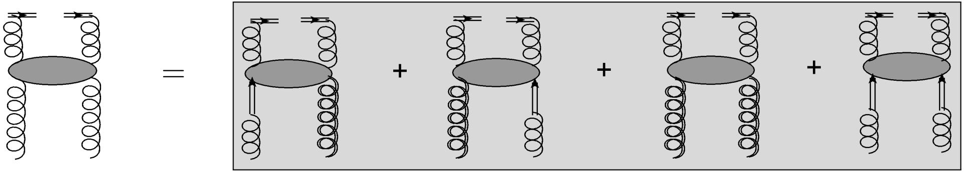
$= \mathcal{O}(g_s).$

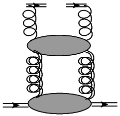
The contribution from ghost matrix element cancels the gauge-variant(KG) term.

# Factorization of quasi gluon GPD

## Derivation of a gauge invariant form (in Feynman gauge)

Decompose the gluon in  into K and G gluon (Grammer and Yennie,1973):





- $$\mathcal{F}_{gU}(z,\xi,t) = \frac{1}{\sqrt{2P^+}} \int \frac{dk^+}{k_1^+ k_2^+} \mathcal{M}_{\mu\nu}(\hat{k}_1, \hat{k}_2) \times \int \frac{dy^-}{2\pi} e^{iy^- k^+} \langle p' | G^{c,+\nu}\left(-\frac{y^-}{2}n\right) G^{c,+\mu}\left(\frac{y^-}{2}n\right) | p \rangle + \cdots ,$$
- $\partial^2 G^{a,\mu}(x) + \mathcal{O}(g_s) = 0$,(in Feynman gauge),$\langle \delta_B(\mathcal{O}) \rangle_{\text{physical}} = 0$ is used.

For details, see JHEP 04 (2023) 001.

# Factorization of quasi gluon GPD

## Derivation of a gauge invariant form (in Feynman gauge)

- For the nonlocal gauge-invariant operator $\tilde{\mathcal{O}}^{\mu\nu} = \tilde{G}^{*z\mu}\left(-\frac{1}{2}\lambda n_z\right)\tilde{G}^{z\nu}\left(\frac{1}{2}\lambda n_z\right)$,

  we verify it will mix with gauge-invariant, BRST-variant and EOM operator:

  $\tilde{\mathcal{O}}^{\mu\nu} \sim [\mathcal{O}_g] + [\delta_B \mathcal{O}_B] + [\mathcal{O}_{EOM}]$,

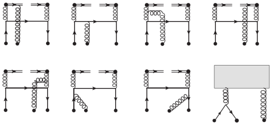
  For the local gauge-invariant $\mathcal{O}$, above mixing pattern has been strictly proven (Joglekar, Lee,1976;Joglekar,1977;Collins,1984).

- For $\mathcal{O}(g_s)$ term in the EOM of gluon field:

$$\partial^2 G^{a,\nu}(x) = g_s\bar{\psi}(x)\gamma^\mu T^a\psi(x) + g_s f^{abc}(\partial^\nu \bar{C}^b)(x)C^c(x) + g_s f^{abc} G^b_\rho(x)G^{c,\rho\nu}(x)$$
$$+g_s f^{abc}((\partial_\mu G^{b,\mu})(x)G^{c,\nu}(x) + G^{b,\mu}(x)\partial_\mu G^{c,\nu}(x))$$

$\surd$ $\surd$

  we need to consider the contribution from matrix elements of three-parton and four-parton.



Factorization of quasi quark GPD at one-loop

Factorization of quasi gluon GPD: derivation of a gauge-invariant form

- The puzzle in the gluon-quark mixing channel

Summary and outlook

# The puzzle in the gluon-quark mixing channel

Ambiguity in transforming the matching(evolution) kernel from coordinate to momentum space(Phys.Lett.B 413 (1997) 114;Phys.Lett.B 406 (1997) 161;JHEP 11 (2023) 021):

$$O_g(z_1,z_2)=\int_0^1 d\alpha\int_0^{\bar\alpha}d\beta\, C_{gq}(\alpha,\beta,\mu^2z_{12}^2)\,O_q^{\text{l.t.}}(z_{12}^{\alpha},z_{21}^{\beta})$$

$$C_{gq}(\alpha,\beta,\mu^2z_{12}^2)=\frac{-2ia_sC_F\Gamma(-\epsilon)\eta^{\epsilon}}{z_{12}}\Big[2D_2+2\epsilon D_1\big(\delta(\alpha)+\delta(\beta)\big)+D_1\delta(\alpha)\delta(\beta)\Big],$$

pseudo ↓ $\qquad C'_{gq}=z_{12}C_{gq}\Big|_{\epsilon=0},\qquad \eta=-(z_{12}^2\mu^2e^{\gamma_E})/4\qquad$ ↓ quasi

$$\mathcal{P}_g(\tau,\xi,\mu^2z_{12}^2)=\int_{-1}^1 dy\,\mathcal{C}_{gq}(\tau_1,\tau_2,y_1,y_2;\mu^2z_{12}^2)H_q(y,\xi,\mu),$$

$$\mathbb{H}_g(x,\xi,\tfrac{\mu}{P_z})=\int_{-1}^1 dy\,\mathbb{C}_{gq}(x_1,x_2,y_1,y_2;\tfrac{\mu}{P_z})H_q(y,\xi,\mu),$$

$$\mathcal{C}_{gq}=iP^z\int_{-\infty(1+i\epsilon)}^{\tau_1}e^{i(t-\tau_1)P^zz_{12}}dt\int_0^1 d\alpha\int_0^{\bar\alpha}d\beta\,C'_{gq}(\alpha,\beta)\delta(\tau_1-\bar\alpha y_1-\beta y_2)$$

$$\mathbb{C}_{gq}=\int_0^1 d\alpha\int_0^{\bar\alpha}d\beta\int\frac{dz_{12}}{2\pi}e^{iz_{12}P^z(x_1-\bar\alpha y_1-\beta y_2)}\frac{1}{z_{12}}C'_{gq}(\alpha,\beta,\mu^2z_{12}^2)$$

- The result of $f(x)=\int\frac{dz_{12}}{2\pi}e^{iz_{12}x}\frac{1}{z_{12}}$ has an ambiguity: $\frac{df(x)}{dx}=i\delta(x)$,$f(x)=i\int dx\,\delta(x)+C$.

# The puzzle in the gluon-quark mixing channel

Ambiguity in transforming the matching(evolution) kernel from coordinate to momentum space(Phys.Lett.B 413 (1997) 114;Phys.Lett.B 406 (1997) 161;JHEP 11 (2023) 021):

$$O_g(z_1,z_2)=\int_0^1 d\alpha\int_0^{\bar\alpha} d\beta\, C_{gq}(\alpha,\beta,\mu^2 z_{12}^2)\, O_q^{\text{l.t.}}(z_{12}^{\alpha}, z_{21}^{\beta})$$

$$C_{gq}(\alpha,\beta,\mu^2 z_{12}^2)=\frac{-2i a_s C_F\Gamma(-\epsilon)\eta^{\epsilon}}{z_{12}}\Big[2D_2+2\epsilon D_1\big(\delta(\alpha)+\delta(\beta)\big)+D_1\delta(\alpha)\delta(\beta)\Big],$$

$$C'_{gq}=z_{12}C_{gq}\big|_{\epsilon=0},\qquad \eta=-(z_{12}^2\mu^2 e^{\gamma_E})/4$$

$$\mathcal{P}_g(\tau,\xi,\mu^2 z_{12}^2)=\int_{-1}^{1} dy\,\mathcal{C}_{gq}(\tau_1,\tau_2,y_1,y_2;\mu^2 z_{12}^2)H_q(y,\xi,\mu),$$

$$\mathcal{C}_{gq}=iP^z\int_{-\infty(1+i\epsilon)}^{\tau_1} e^{i(t-\tau_1)P^z z_{12}}dt\int_0^1 d\alpha\int_0^{\bar\alpha} d\beta\, C'_{gq}(\alpha,\beta)\delta(\tau_1-\bar\alpha y_1-\beta y_2)$$

$$\mathbb{H}_g(x,\xi,\tfrac{\mu}{P_z})=\int_{-1}^{1} dy\,\mathbb{C}_{gq}(x_1,x_2,y_1,y_2;\tfrac{\mu}{P_z})H_q(y,\xi,\mu),$$

$$\mathbb{C}_{gq}=\int_0^1 d\alpha\int_0^{\bar\alpha} d\beta\int\frac{dz_{12}}{2\pi}e^{iz_{12}P^z(x_1-\bar\alpha y_1-\beta y_2)}\frac{1}{z_{12}}C'_{gq}(\alpha,\beta,\mu^2 z_{12}^2)$$

- $\int d\alpha\int_0^{\bar\alpha} d\beta\int\frac{dz_{12}}{2\pi}e^{iz_{12}P^z(x_1-\bar\alpha y_1-\beta y_2)}C_{gq}(\alpha,\beta,\mu^2 z_{12}^2)=\mathbb{C}_{gq}^{\text{momentum}}$ (using dim reg:$f(x)=\int\frac{dz_{12}}{2\pi}e^{iz_{12}x}\frac{(z_{12}^2)^{\epsilon}}{z_{12}}$).

# The puzzle in the gluon-quark mixing channel

Matching the Mellin moments in coordinate and pseudo space(hep-ph/0504030):

$$\lim_{\xi\to 0}\int_0^1 dx x^{j-1}\mathcal{C}_{gq}^{\text{matched}}\left(2\frac{x}{\xi}-1,\frac{2}{\xi}-1\right)=-\frac{\gamma_{gq}(j)}{j},\quad j\geq 1$$

l.h.s:to be determined, r.h.s:generated from results in coordinate space.

$\mathcal{C}_{gq}^{\text{matched}} \stackrel{\text{FT}}{\to} \mathbb{C}_{gq}^{\text{matched}}$ vs $\mathbb{C}_{gq}^{\text{momentum}}(\Delta\mathbb{C}_{gq}=\mathbb{C}_{gq}^{\text{matched}}-\mathbb{C}_{gq}^{\text{momentum}})$ :

- $\Delta\mathbb{C}_{gqL}=-6a_s C_F\epsilon(x), \Delta\mathbb{C}_{gqU}=0$(for quasi pdf).
  $\Delta\tilde{f}_{gL}=-6a_s C_F\epsilon(x)g_A \stackrel{\text{FT}}{\to} \Delta\langle\tilde{O}_{gL}\rangle\sim\frac{-3i}{z_{12}}g_A$(difference in coordinate space).
- Matching relation for forward quasi-LF correlator:
  $\langle\tilde{O}_{gL}(z_{12},0)\rangle=\frac{1}{z}C_{gq}\otimes\langle O_{qL}^s\rangle, \langle O_{qL}^s(z_{12},0)\rangle=\frac{1}{2}\left[O_{qL}(z_{12},0)+O_{qL}(0,z_{12})\right].$
  $\xrightarrow{\text{modified to}}$ $\langle\tilde{O}_{gL}(z_{12},0)\rangle_{\text{reg}}=\frac{1}{z}C_{gq}\otimes\langle O_{qL}^s-g_A\rangle,$ ?

Factorization of quasi quark GPD at one-loop

Factorization of quasi gluon GPD: derivation of a gauge-invariant form

The puzzle in the gluon-quark mixing channel

- Summary and outlook

# Summary and Outlook

- The factorization of quasi GPDs at one-loop is derived, with emphasis on how to obtain a gauge-invariant result.
- (Direct)calculation of the Wilson coefficient $C_n^{gq}$ is needed: $\mathcal{H}_g(\zeta,\mu^2 z^2) = \sum_n C_n^{gq}(\mu^2 z^2)\frac{(-i\zeta)^n}{n!}a_{q,n+1}(\mu)$, as a justification of choosing matching coefficients in coordinate(momentum) space.
- In (forward)quasi-LF correlator, certain combination maximally reduces the higher-twist contamination. (Balitsky,Morris,Radyushkin,2019,2022) Extending the analysis to the nonforward case is worth exploration.

# Thanks!