Quarks and glue inside hadrons in the instanton vacuum

The hadronic / partonic structure dominated by QCD vacuum

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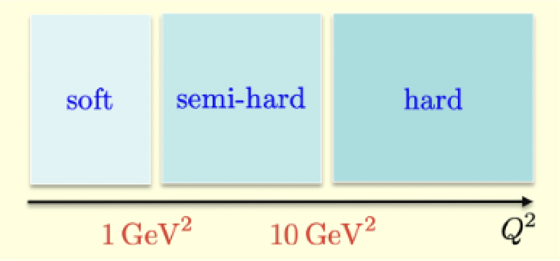
QCD scales



- Hard: perturbative QCD
- Semi-hard: quark-instanton, gluon-instanton coupling
- Soft: quark-meson dynamics (instanton)
- Ultrasoft: χPT, meson dynamics (instanton)

QCD vacuum

How to describe nonperturbative phenomena by QCD?

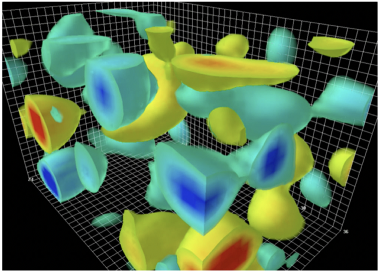
"*Buried treasure in the sand of the QCD vacuum*"

W.-Y. Liu, E. Shuryak, I. Zahed, arXiv:2404.03047 [hep-ph] (2023)

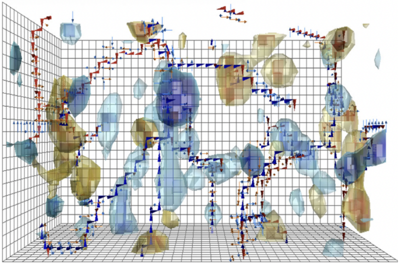
Visualization of QCD Vacuum

- QCD vacuum is topologically active at 0.6 – 0.7 GeV
- Instanton vacuum stabilized at ($n_I = 1$ fm^{-4} and $\rho = 1/3$ fm)
- Zero energy tunnelling event between topological vacua $\Delta n_{CS} = \pm 1$

- Gluon plane waves are almost negligible



P. J. Moran, D. B. Leinweber (2008)

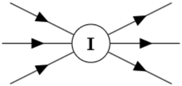


J. C. Biddle, W. Kamleh, D. B. Leinweber (2020)

Instanton Liquid Model

Deep cooling (gradient flow) on lattice: significant reduction on DOFs

$$Z_{N_\pm} = \overbrace{\frac{1}{N_+!N_-!}\prod_{I,A}\int d\rho_I d^4 z_I dU_I n(\rho_I)}^{\text{glue}} \overbrace{\int D\psi D\bar{\psi} e^{-\int d^4x \bar{\psi}\partial\!\!\!/\psi}\prod_{I,A}\Theta_I\Theta_A}^{\text{quarks}-\text{instanton}}$$

$\Theta_I =$ 

- Gluon dynamics encoded in the instanton density $n_\pm$

$$\begin{aligned}\langle\mathcal{O}_{QCD}\rangle_{N_\pm} =& n_+\langle\mathcal{O}_+[q,\bar{q}]\rangle + n_-\langle\mathcal{O}_-[q,\bar{q}]\rangle \\ &+ \frac{n_+^2}{2}\langle\mathcal{O}_{++}[q,\bar{q}]\rangle + n_+n_-\langle\mathcal{O}_{+-}[q,\bar{q}]\rangle + \frac{n_-^2}{2}\langle\mathcal{O}_{--}[q,\bar{q}]\rangle \\ &+ \cdots\end{aligned}$$

Grand Canonical Ensemble: Fluctuations

- $G^2 \to \bar{N} = N_+ + N_-$ and $G\tilde{G} \to \Delta = N_+ - N_-$
- Fluctuations

$$\mathcal{P}(N_+, N_-) \propto \left(\frac{\bar{N}^N}{N!}\right)^{b/4} \frac{1}{\sqrt{2\pi\chi_t}} \exp\left(-\frac{\Delta^2}{2\chi_t}\right) \tag{1}$$

- Topological compressibility: trace anomaly

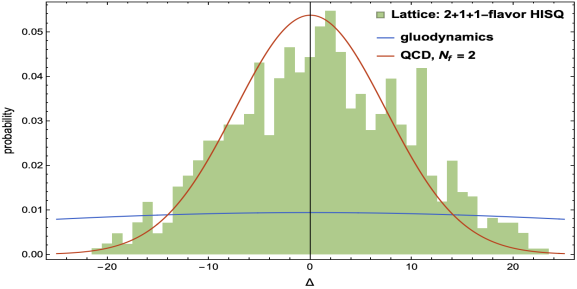
$$\sigma = \frac{4}{b}\bar{N} \tag{2}$$

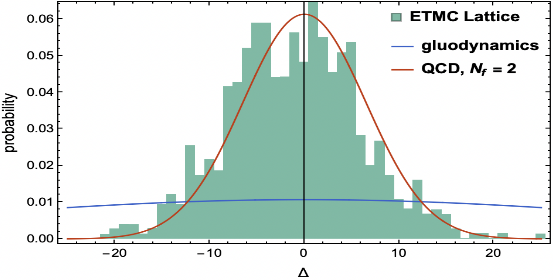
- Topological susceptibility: chiral anomaly

$$\chi_t = \bar{N}\left(1 + N_f \frac{m^*}{m}\right)^{-1} \tag{3}$$

W.-Y. Liu, E. Shuryak, I. Zahed, arXiv:2404.03047 [hep-ph] (2023)

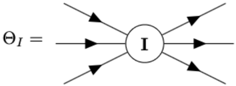
Topological charge on lattice





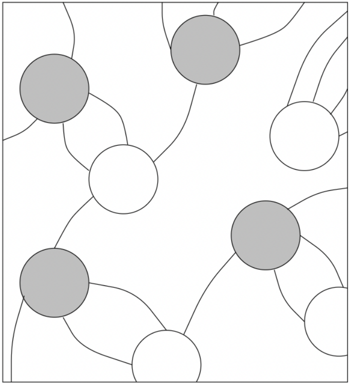
Spontaneous chiral symmetry breaking

N_f light quarks deflected by instantons



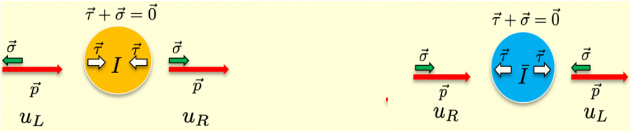
$\langle \bar{q} q \rangle \neq 0$

Delocalization of zero modes
Mott transition (axial charge)



Quarks in Instanton Vacuum

- Constituent massless quarks acquire mass from delocalization



- 't Hooft effective Lagrangian ($N_f = 2$)

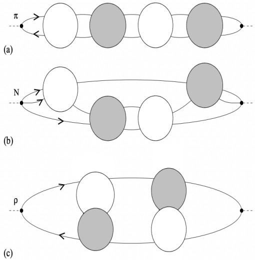
$$\mathcal{L}_I = \frac{G_I}{8N_c^2}\left[(\bar{\psi}\psi)^2 - (\bar{\psi}\tau^a\psi)^2 - (\bar{\psi}i\gamma^5\psi)^2 + (\bar{\psi}i\gamma^5\tau^a\psi)^2\right]$$

- Constituent mass $M \sim 350 - 390$ MeV
- Instanton size $\rho \to$ natural cut-off

G. 't Hooft, Phys. Rev. D 14, 3432 (1976);
W.-Y. Liu, I. Zahed, Phys. Rev. D 107 (2023) 9, 094024

Instanton Molecules

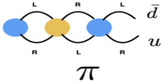
- Instantons are correlated
- Instanton molecules are important
  $\rightarrow$ vector meson (ρ), tensor glueball (T^{++}), odd glueball ($dGGG$)
- Molecule-induceed interations penalized by instanton density



$$\mathcal{L}_{I\bar{I}} = \frac{G_{I\bar{I}}}{2N_c^2}\Big\{4\left[(\bar{\psi}\psi)^2 + (\bar{\psi}\tau^a\psi)^2 + (\bar{\psi}i\gamma^5\psi)^2 + (\bar{\psi}i\gamma^5\tau^a\psi)^2\right] - 4(\bar{\psi}\gamma^\mu\gamma^5\psi)^2$$
$$- \left[(\bar{\psi}\gamma^\mu\psi)^2 + (\bar{\psi}\tau^a\gamma^\mu\psi)^2 + (\bar{\psi}\gamma^\mu\gamma^5\psi)^2 + (\bar{\psi}\tau^a\gamma^\mu\gamma^5\psi)^2\right]\Big\}$$

T. Schäfer, E. Shuryak, and J. Verbaarschot, Phys. Rev. D 51, 1267 (1995); Edward Shuryak, Ismail Zahed, Phys.Rev.D 107 (2023) 3, 034023

Spin-0 meson spectrum

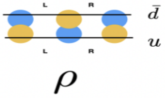


$\rho = 0.3$ fm and $n_{I+A} = 1$ fm^{-4}

Model	m_σ (MeV)	m_{π^0} (MeV)	$m_{\pi^\pm}$ (MeV)
ILM ($N_f = 2$)	683.73	139.4	139.4
ILM ($N_f = 3$)	677.62	142.00	142.00
PDG	$400 - 800$	134.9766(6)	139.57018(35)

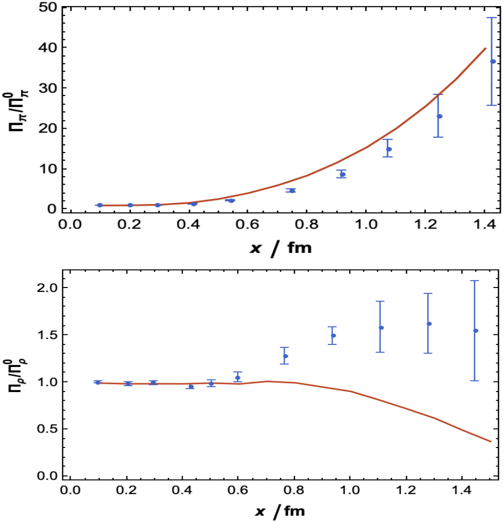
Model	$m_{K^\pm}$ (MeV)	$m_{K^0,\bar{K}^0}$ (MeV)	m_η (MeV)
ILM($N_f = 3$)	458.00	458.00	524.22
PDG	493.677(13)	497.611(13)	547.862(17)

Spin-1 meson spectrum



Model	m_ω (MeV)	m_ρ (MeV)	m_{f_1} (MeV)	m_{a_1} (MeV)
ILM($N_f = 2$)	790.0	790.0	–	–
ILM($N_f = 3$)	787.23	787.23	–	–
PDG	782.65(12)	775.26(25)	1281.9(5)	1230(40)

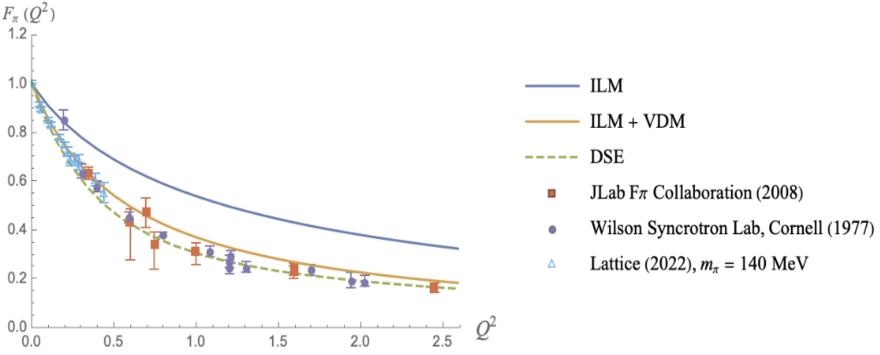
Meson correlator on lattice



Chu et al. 1993b, Leinweber 1995a

Pion Electromagnetic Form Factor

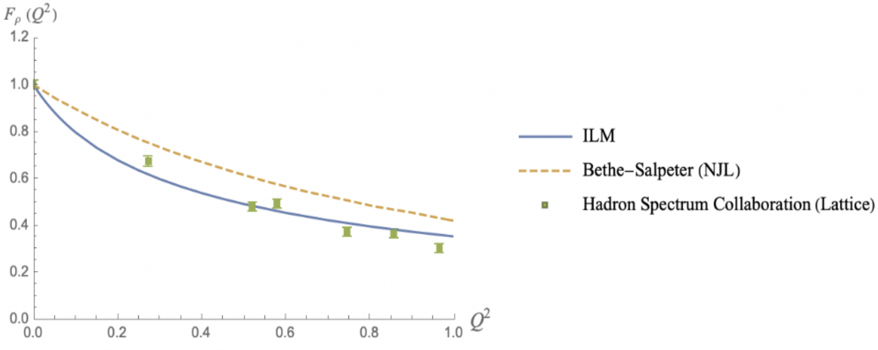
$$\left\langle \pi' \left| J^{\mu}_{EM} \right| \pi \right\rangle = 2F_{\pi}(Q^2)(p+p')^{\mu}$$



Lat: Xiang Gao, Nikhil Karthik, Swagato Mukherjee, Peter Petreczky, Sergey Syritsyn, and Yong Zhao Phys. Rev. D 104, 114515

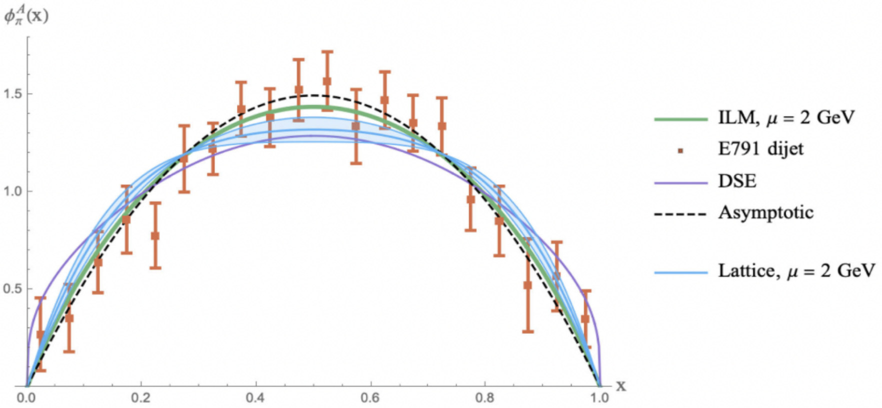
Rho Charge Form Factor

$$F_\rho(Q^2) = \frac{1}{2P^+}\left[\frac{1}{3}\sum_\lambda \langle\rho'|J^+_{EM}|\rho\rangle\right] \quad (4)$$



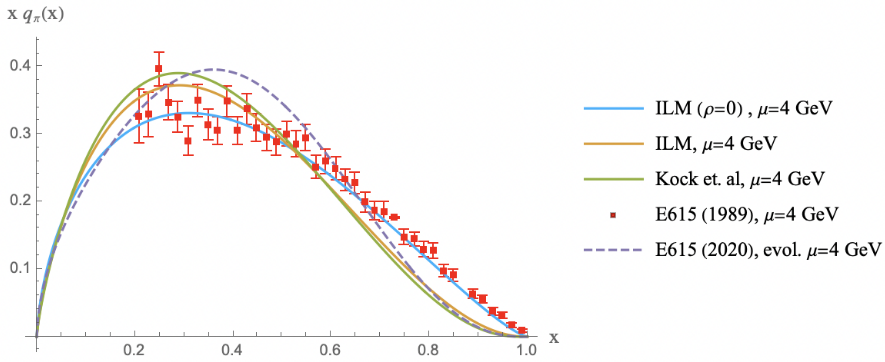
Wei-Yang Liu, Ismail Zahed, Phys. Rev. D 109 (2024), 074029

Pion DA



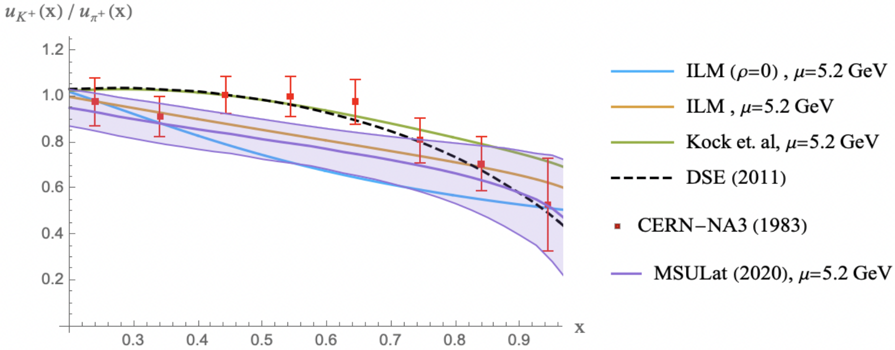
Lat: RQCD (Gunnar S. Bali, Vladimir M. Braun, Simon Bürger, Meinulf Göckeler, Michael Gruber, Fabian Hutzler, Piotr Korcyl, Andreas Schäfer, André Sternbeck Philipp Wein)

Pion PDF



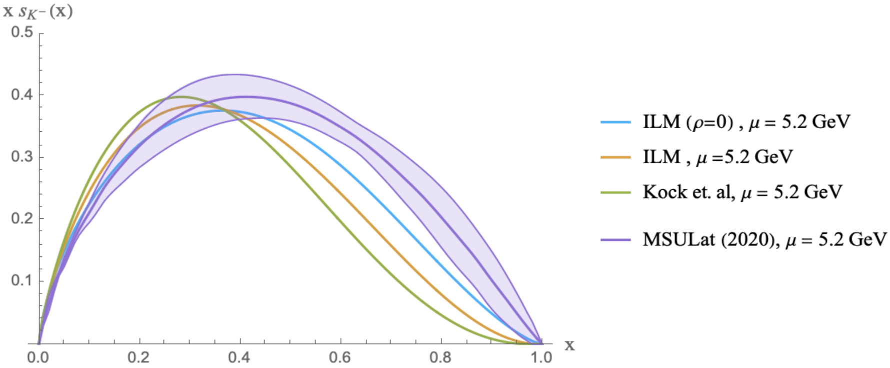
Wei-Yang Liu, Ismail Zahed, Phys. Rev. D 107 (2023) 9, 094024

Kaon PDF



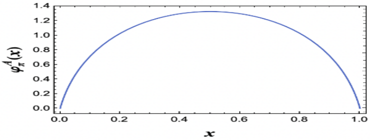
Wei-Yang Liu, Ismail Zahed, Phys. Rev. D 107 (2023) 9, 094024

Kaon PDF

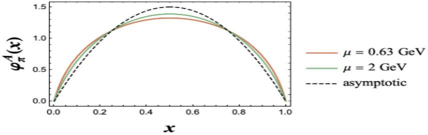


Wei-Yang Liu, Ismail Zahed, Phys. Rev. D 107 (2023) 9, 094024

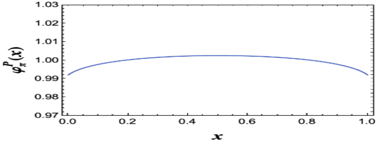
Twist-2 and Twist-3 pion DA



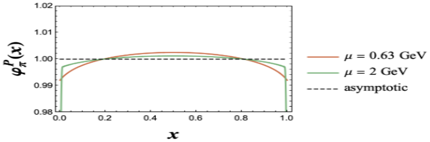
(a)



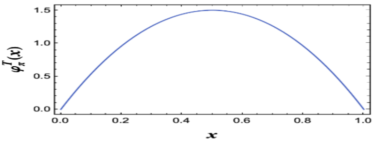
(b)



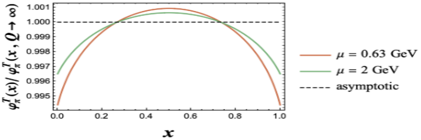
(c)



(d)

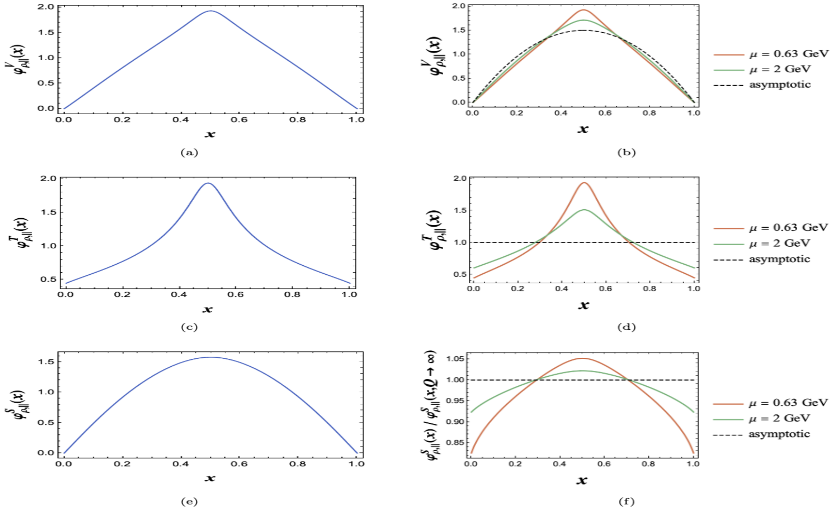


(e)

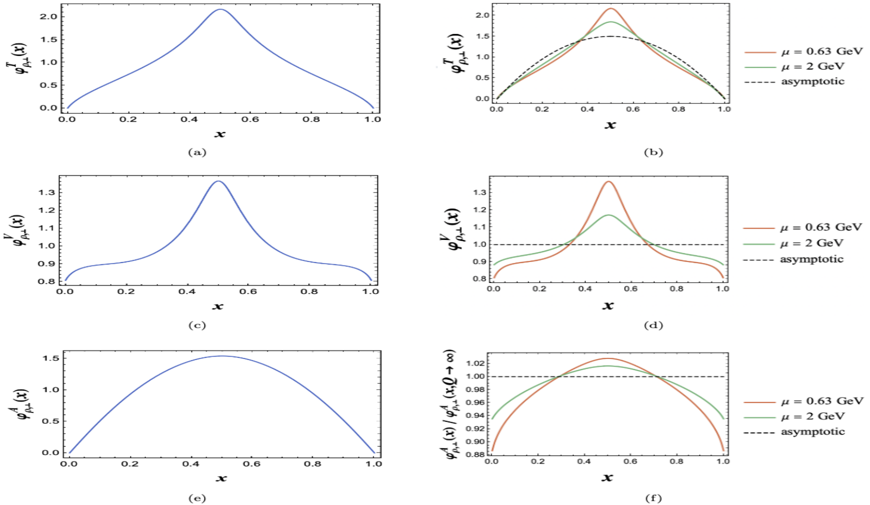


(f)

Twist-2 and Twist-3 $\rho_\parallel$ DA

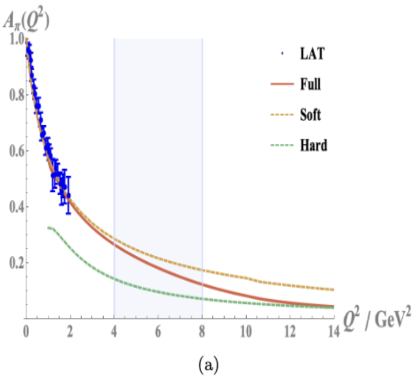


Twist-2 and Twist-3 $\rho_\perp$ DA



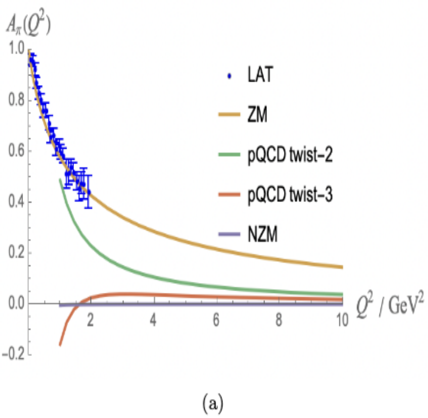
Pion Gravitational Form Factor

$$\langle \pi' \,|\, T^{\mu\nu} \,|\, \pi \rangle = 2\bar{p}^{\mu}\bar{p}^{\nu}\, A_{\pi}(Q^2) + \frac{1}{2}(q^{\mu}q^{\nu} - g^{\mu\nu}q^2)\, D_{\pi}(Q^2)$$



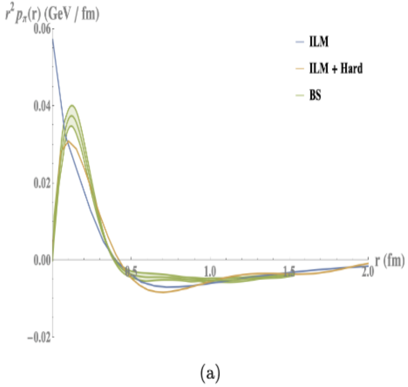
Lattice: Phiala E. Shanahan

Pion Gravitational Form Factor

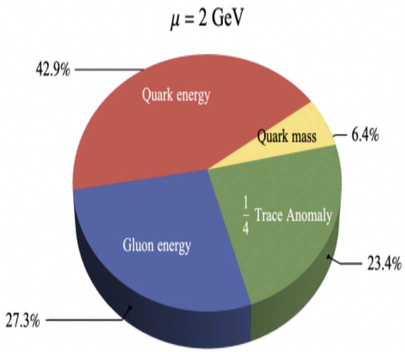


Lattice: Phiala E. Shanahan

related to $D_\pi(Q^2)$ in the coordinate space



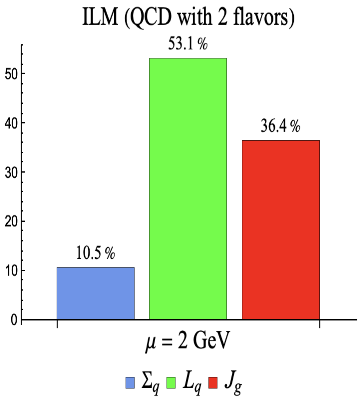
Mass Distribution in Nucleon



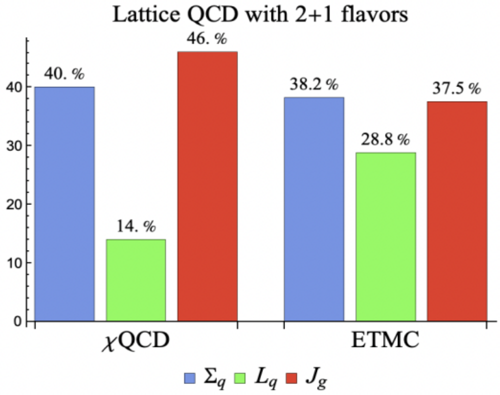
(a) (b)

Spin Sum Rule in Nucleon

$$S_N = \frac{1}{2}\Sigma_q + L_q + J_q$$



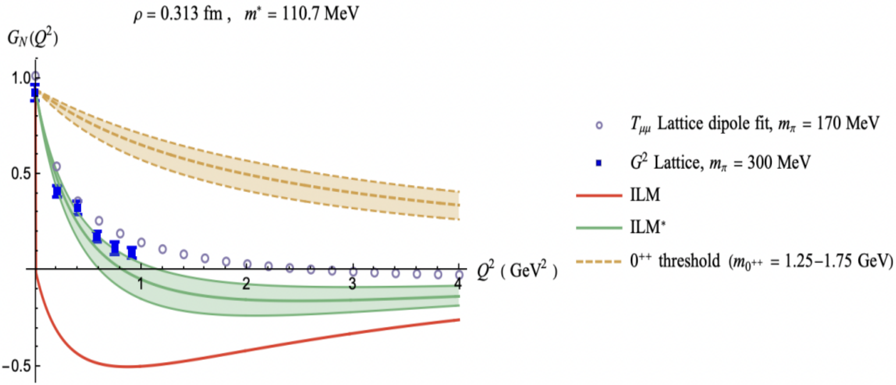
(c)



(d)

Scalar glueball form factor of Nucleon

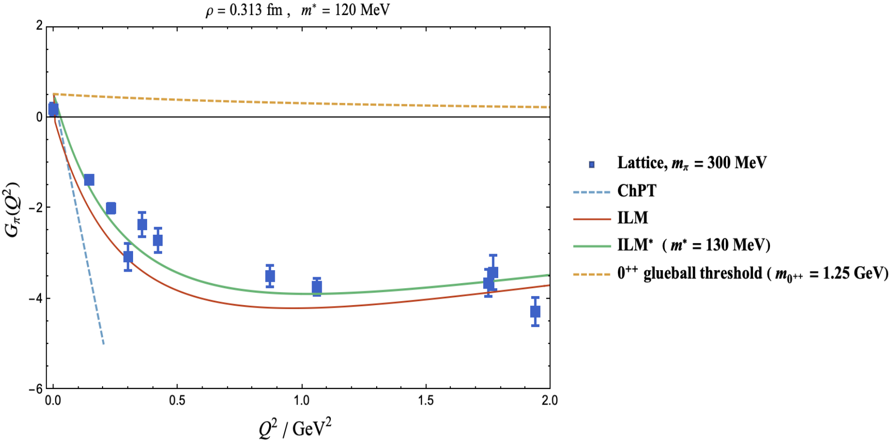
$$-\frac{b}{32\pi^2}\langle N'|g^2 G^2_{\mu\nu}|N\rangle = M_N G_N(Q^2)\bar{u}_s(P')u_s(P)$$



Lattice: χ QCD (Yi-Bo Yang)

Scalar glueball form factor of pion

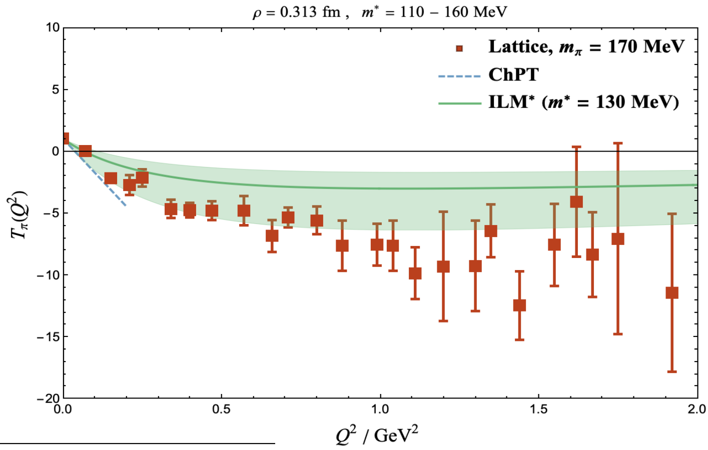
$$-\frac{b}{32\pi^2}\langle\pi'|g^2 G_{\mu\nu}^2|\pi\rangle = 2m_\pi^2 G_\pi(Q^2)$$



Lattice: χ QCD (Yi-Bo Yang)

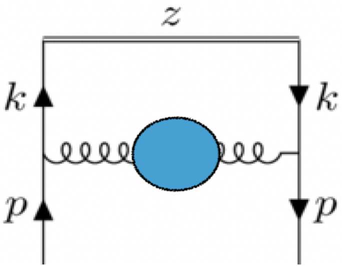
Trace anomaly form factor of pion

$$\frac{\langle \pi' | T^{\mu}{}_{\mu} | \pi \rangle}{2 m_\pi^2} = T_\pi(Q^2) = \left(1 + \frac{Q^2}{4 m_\pi^2}\right) A_\pi(Q^2) + \frac{3Q^2}{4 m_\pi^2} D_\pi(Q^2)$$



Lattice: Phiala E. Shanahan

Instanton on quasi-PDF



Correction to the one-loop:

$$\frac{\alpha_s C_F}{\pi^2} \to \frac{\alpha_s C_F}{\pi^2}\left(1 - 8\pi\sqrt{\kappa}\right)$$

Packing fraction characterizes the nonperturbative power correction

$$\sqrt{\kappa} \sim e^{-4\pi/b\alpha_s} = \rho^2\Lambda_{QCD}^2$$

Yizhuang Liu, Ismail Zahed, arxiv: 2102.07248 [hep-ph]

Instanton Liquid Model Remarks

- QCD vacuum is topologically active at 0.6 – 0.7 GeV (instanton)
- Instanton dominates light meson dynamics: σ, π, K, η, ω, ρ, etc.
- Diquarks $\rightarrow$ light baryons: p, n, Σ, Λ, Ξ
- low energy gluon dynamics is encoded in the instanton density fluctuation
- perturbative gluon dynamics can be recovered by the QCD evolution
- ILM provides solid framework for both quark and gluonic content (glueballs, mesons) inside any light hadrons.
- EM (neutron EDM), axial, gravitational form factors, PDF, DA, TMD, GPD (any light hadrons)
- Not only quark content but also gluon content (notorious for lattice)