Towards Singlet Quasi-PDF Matching in Hybrid-Ratio Scheme

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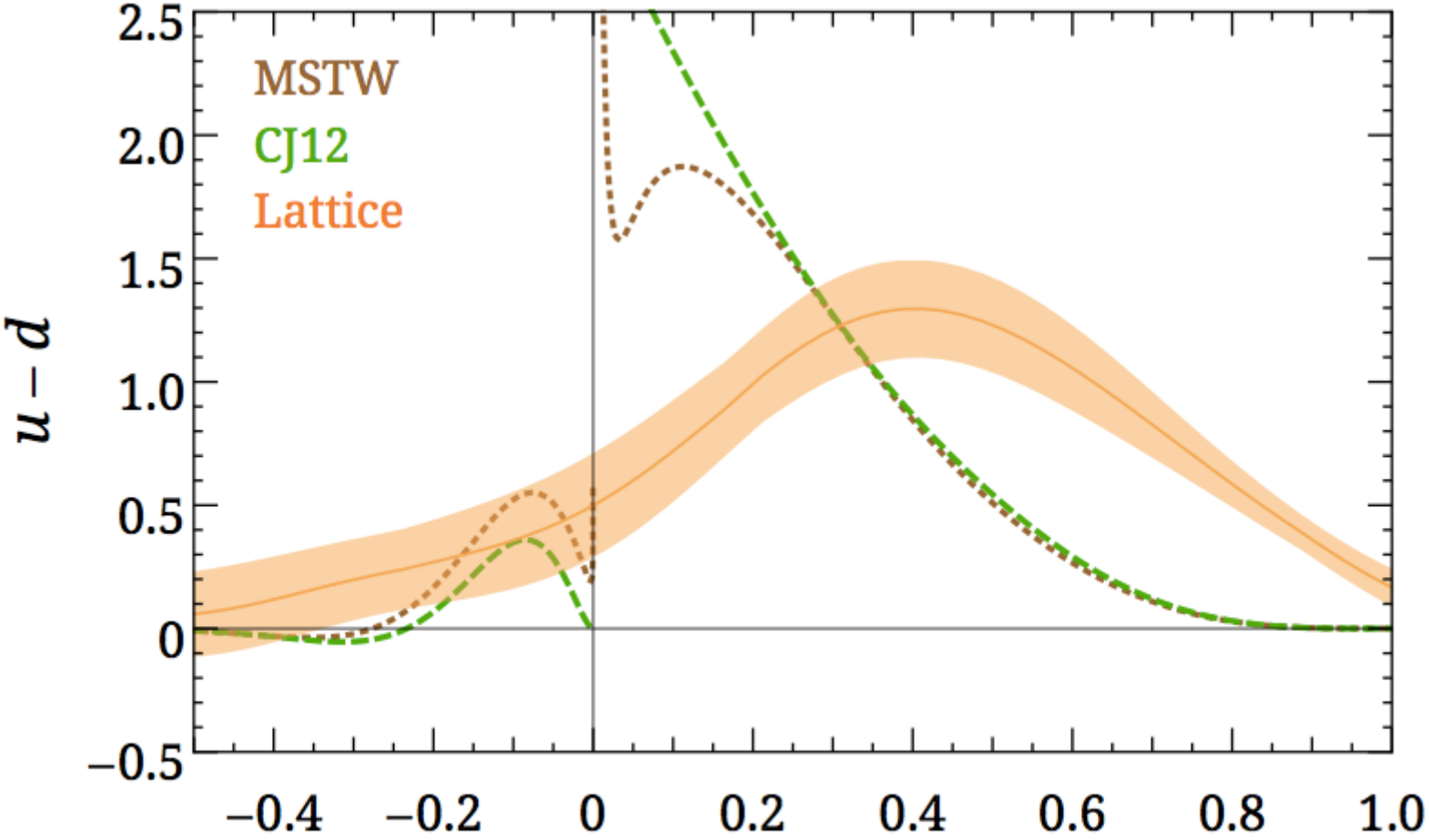
LaMET

Rapid progress since
Xiangdong Ji's seminal paper
in 2013

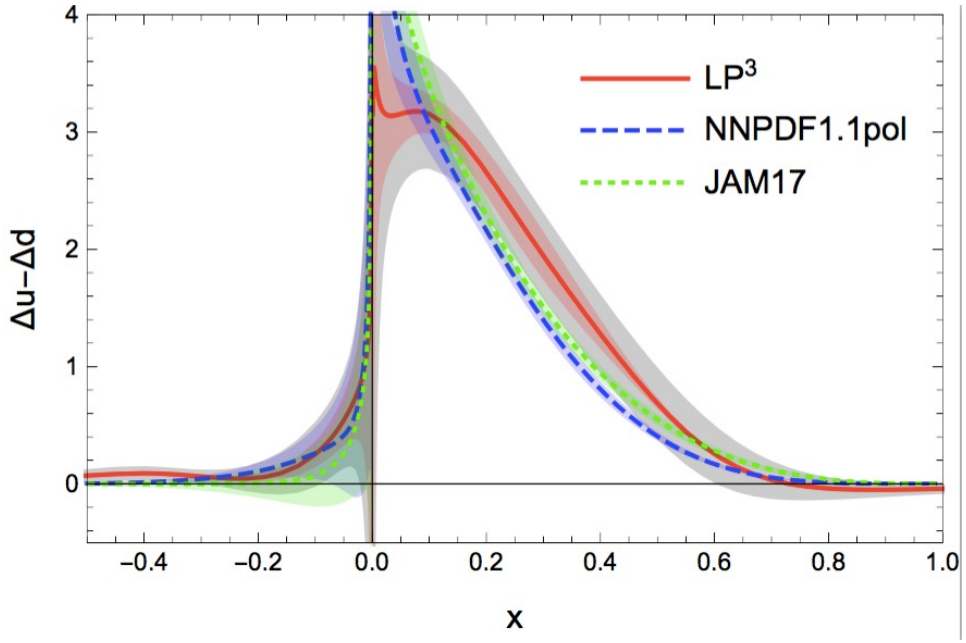
Proton Unpolarized PDF

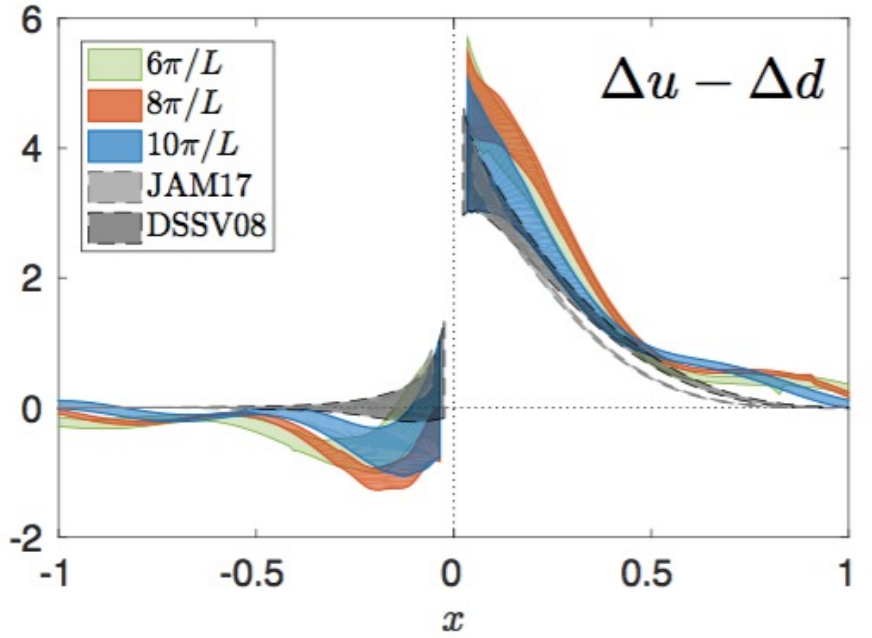
LP3 (1402.1462)



$$q(-x) = -\bar{q}(x)$$

Proton Helicity PDF

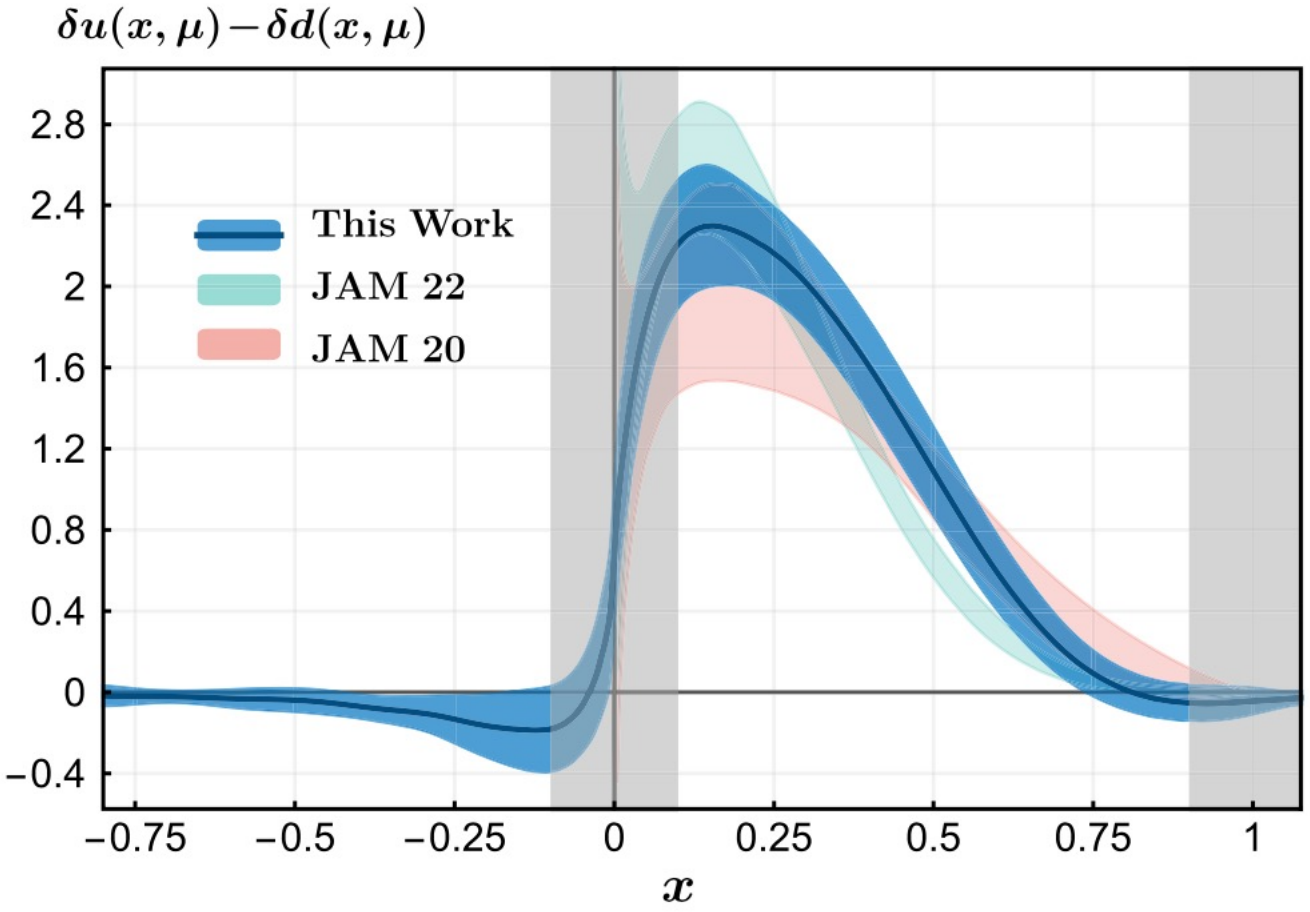




LP3(1807.07431,PRL)

ETMC(1803.02685,PRL)

Proton Transversity PDF in the Continuum and Physical Mass Limit



Yao, Walter et al. (LPC), PRL 2023

This year

- Precision frontier: Resummations, no Wilson line, Lanczos, high precision gauge fixing
- Higher dimensional frontier: TMD, GPD
- Flavor frontier? gluon, u+d, s, c, b(?)

W. Wang, J.-H. Zhang, S. Zhao, and R. Zhu

Balitsky, W. Morris, and A. Radyushkin

J. P. Ma, Z. Y. Pang, and G. P. Zhang

Fei Yao, Yao Ji and Jian-Hui Zhang

LaMET Factorization Theorem (Ji)

(Non-singlet)

$$\tilde{q}(x,\Lambda,P_z)=\int\frac{dy}{|y|}Z\left(\frac{x}{y},\frac{\mu}{P_z},\frac{\Lambda}{P_z}\right)q(y,\mu)+\mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_z^2},\frac{M^2}{P_z^2}\right)+....$$

- Z: UV difference of quasi-PDF and PDF
- PDF in MS-bar, quasi-PDF in lattice spacing---lattice action dependent, slow convergence
- NPR:

  (a) Ratio scheme (Radyushkin)

  (b) RI/MOM (Zhao & Stewart; Constantinou et al) might not have continuum limit (ChQCD)---However, see Yi-Bo's talk!

Hybrid Renormalization

- Ratio scheme: long (> Zs~0.3 fm) Wilson line counterterm has non-perturbative IR effect; replaced by Wilson line mass subtraction scheme (X. Ji, Y. Liu, A. Schäfer, W. Wang, Y.B. Yang, J.H. Zhang, Y. Zhao, 2008.03886)

- Focus on the hybrid-ratio scheme in this talk

Closer look at Ratio Scheme

$$\frac{\langle P|\bar{O}_q^B(z)|P\rangle}{\langle P=0|\bar{O}_q^B(z)|P=0\rangle} = \frac{\langle P|\bar{O}_q^R(z)|P\rangle}{\langle P=0|\bar{O}_q^R(z)|P=0\rangle}$$

- Multiplicative renormlaiztion needed
- smooth limit to z=0, charge renormalized to one.

Singlet PDF

- Mixing: valence quark PDFs do not mix with gluons, but the non-valence ones do

$$
\begin{aligned}
O_q(z) &= -i\partial_z[\bar{\psi}(z)\gamma^t U(z,0)\psi(0) - (z \to -z)] \\
O_g(z) &= F^{ti}(z)U(z,0)F_i^z(0)
\end{aligned}
$$

- Mixing is UV finite for non-zero z:

$$
\frac{\langle P|O_g^B(z)|P\rangle}{\langle P=0|O_g^B(z)|P=0\rangle}\Big|_{a\to 0} = \frac{\langle P|O_g^R(z)|P\rangle}{\langle P=0|O_g^R(z)|P=0\rangle}
$$

Singlet PDF

- Mixing: valence quark PDFs do not mix with gluons, but the non-valence ones do

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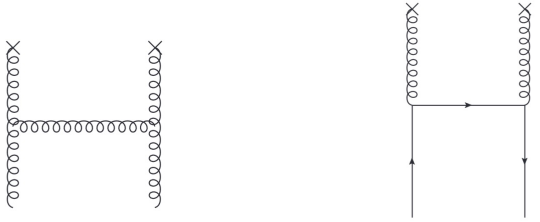
$$\frac{\langle P|O_g^B(z)|P\rangle}{\langle P=0|O_g^B(z)|P=0\rangle}\Big|_{a\to 0} = \frac{\langle P|O_g^R(z)|P\rangle}{\langle P=0|O_g^R(z)|P=0\rangle}$$

- But CT IR divergent!



Removing IR Divergence

- Do it without introducing another scale?

$$\frac{\langle P|O_g^B(z)+O_q^B(z)|P\rangle}{\langle P=0|O_g^B(z)+O_q^B(z)|P=0\rangle}$$

- Counterterm IR divergent free in dim reg. But no multiplicative renormlaiztion.
- Does not work!

Removing IR Divergence

- Introducing a gauge invariant IR regulator:

$$\frac{\langle P|O_g^B(z)|P\rangle_{m_q}}{\langle P=0|O_g^B(z)|P=0\rangle_{m_Q}}$$

$$\frac{1}{a} \gg P^z \gg m_Q \gg \Lambda_{\rm QCD}$$

Hybridization

$$\mathcal{C}^{\text{hybrid-ratio}} = \mathcal{C}^{\text{ratio}} + \delta\mathcal{C}$$

•

comments

$$\langle x \rangle_{\tilde{q}^R} + \langle x \rangle_{\tilde{g}^R} = 1$$

- For quasi-PDFs in MS-bar and for lightcone PDFs
- Enforced as an extra constraint in our hybrid ratio scheme with each term to be finite.