Lattice QCD Predictions for Meson Electromagnetic Form Factors at High Momenta: Testing Factorization in Exclusive Processes

Peter Petreczky

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- Pion and kaon form-factors

H.-T. Ding, X. Gao, A.D. Hanlon, S. Mukherjee, PP, P. Scior, Qi Shi, S. Syritsyn, R. Zhang, Y. Zhao, arXiv:24.04.04412

- Pion distribution amplitude (DA)

I. Cloët , X. Gao, S. Mukherjee, S. Syritsyn, N. Karthik, P. Petreczky, R. Zhang, Y. Zhao, arXiv:2407.00206

X. Gao, A.D. Hanlon, N. Karthik, S. Mukherjee, PP, P. Scior, S. Syritsyn, Y. Zhao, PRD 106 (2022) 074505

Factorization in exclusive processes

Elastic scattering $eM(P) \to eM(P'),\ M = \pi, K \Rightarrow$ E-M form factors

$$F_M(Q^2) = \int_0^1 \int_0^1 dxdy\ \phi_M(y, \mu_F^2) T_H(x, y, Q^2, \mu_R^2, \mu_F^2) \phi_M(x, \mu_F^2)$$

NNLO, Chen et al, PRL 132 (2024) 201901

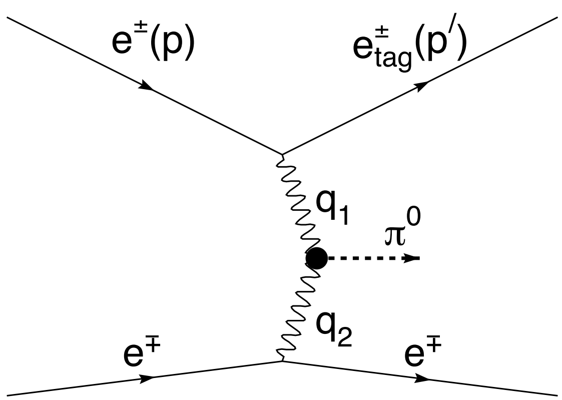
Lepage, Brodsky, PRD 21 (1980) 2157
Farrel, Jackson, PRL 43 (1979) 246

$$i f_M \phi_M(x, \mu) = \int \frac{d\eta^-}{2\pi} e^{ixP^+\eta^-} \langle 0 | \overline{q}'(0) \gamma_5 \gamma_+ W(0, \eta^-) q(\eta^-) | M(P) \rangle$$

$$\mu_F \to \infty : \phi_M(x, \mu_F^2) \to \phi_M^{as} = f_M x(1-x)/\sqrt{2},\ F_M \to 8\pi\alpha_s(Q^2) f_M^2/Q^2$$

Alternative: k_T factorization approach at NLO with higher twist contributions
Cheng et al, PRD 100 ('19) 013007, Chai et al, EPJC 83 ('23) 556

$e^+e^- \to e^+ e^-_{tag} \pi^0 \Rightarrow$ pion transition form factor



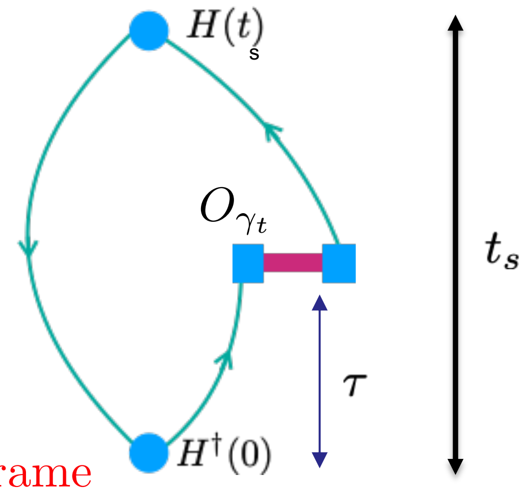
$$F_{tr}(Q^2) = \frac{\sqrt{2} f_M}{6Q^2} \int_0^1 \phi_M(x, \mu) T_{tr}(x, \mu, Q^2)$$

NNLO, Braun et al, PRD 104 (2021) 094007

Lattice QCD setup

2+1 flavor HISQ (HotQCD) lattices, pions boosted along the z-direction
HYP smeared clover action for valence quarks, $m_\pi^{val} \simeq 300$ MeV
64^4, $a = 0.04$ fm, $P_z^{max} = 3$ GeV
64^4, $a = 0.076$ fm, $m_\pi = 140$ MeV, $P_z^{max} = 2.3$GeV

$$P_z = \frac{2\pi}{L} n_z,\ n_z = 0,\ 1,\ 2,\ 3, 4,\ 5,\ 6,\ 7,\ 8, 9$$



Boosted sources (Bali et al, PRD 93 ('16) 094515) with Gaussian size r_G in Coulomb gauge

$$C_{2\mathrm{pt}}^{ss}(t;\mathbf{P}) = \langle M_s(\mathbf{P},t) M_s^\dagger(\mathbf{P},0)\rangle \qquad \mathbf{P}^f = \mathbf{P} = \mathbf{P}^i + \mathbf{q}$$

$$C_{3\mathrm{pt}}(\mathbf{P}^f,\mathbf{P}^i,\tau,t_s) = \langle M_s(\mathbf{P}^f,t_s) O_{\gamma_t}(\tau) M_s^\dagger(\mathbf{P}^i,0)\rangle$$

$$ME = \lim_{t_s\to\infty,\tau\to\infty} R(\tau,t_s), \quad R^{fi} \sim \frac{3\mathrm{pt}}{2\mathrm{pt}}$$

Form factor
Near Breit frame kinematics $\mathbf{P}_f = -\mathbf{P}_i$

Meson DA

$$C_{\mathrm{pt-split}}(\mathbf{P}^i,t_s) = \langle O_{\gamma_5\gamma_3}(t_s) M_s^\dagger(\mathbf{P}^i,0)\rangle$$

$$O_\Gamma(z) = \left[\overline{u}(z)\Gamma W(z,0)u(0) - \overline{d}(z)\Gamma W(z,0)d(0)\right]$$

Iso-vector, no disconnected diagrams for the pion

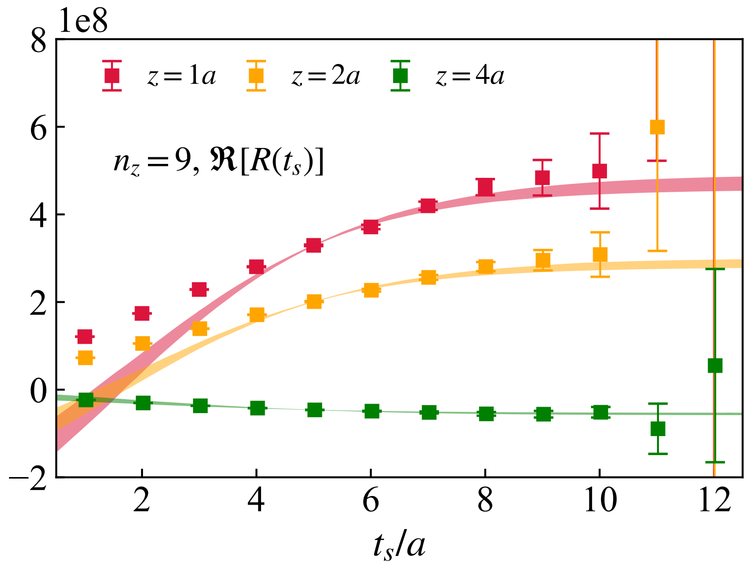
Extrapolation is done using 2 and 3 exp. Fits with energy levels from 2pt functions

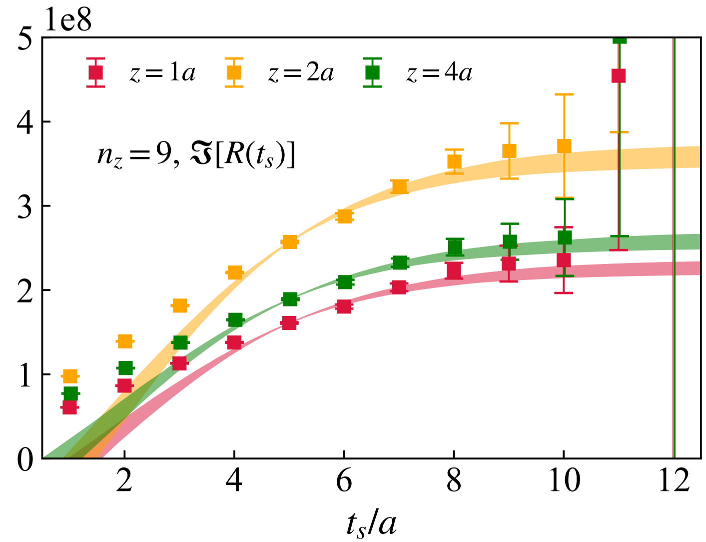
Meson DA matrix elements

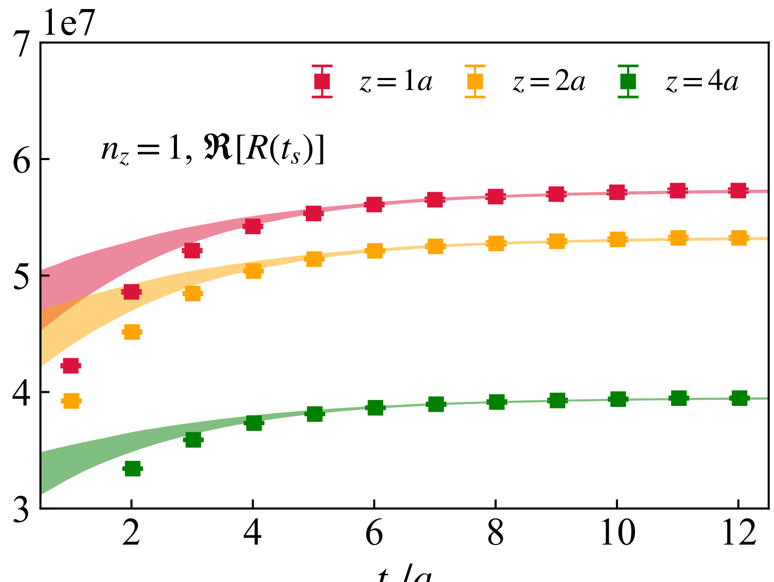
$$R = \frac{C_{\text{pt-split}}(\mathbf{P}, t_s)}{C^{\text{ss}}_{\text{2pt}}(\mathbf{P}, t_s)}$$

$64^4,\ a = 0.076\ \text{fm},\ m_\pi = 140\ \text{MeV},\ P_z^{max} = 2.3\text{GeV}$

$r_G^s = 0.83\ \text{fm},\quad r_G^l = 0.59\ \text{fm}$







$$\langle 0|O_{\gamma_5\gamma_\mu}(z)|M;\mathbf{P}\rangle = P^\mu \tilde{H}(z^2, z\cdot P) + z^\mu m_K^2 \tilde{k}(z^2, z\cdot P)$$

$$h^B(z, P_z) = \frac{1}{P_\mu}\left(\langle\Omega|O_{\gamma_5\gamma_3}(z)|0;\mathbf{P}\rangle - \langle\Omega|O_{\gamma_5\gamma_3}(z)|0;\mathbf{0}\rangle\right)$$

Moments of meson distribution amplitudes

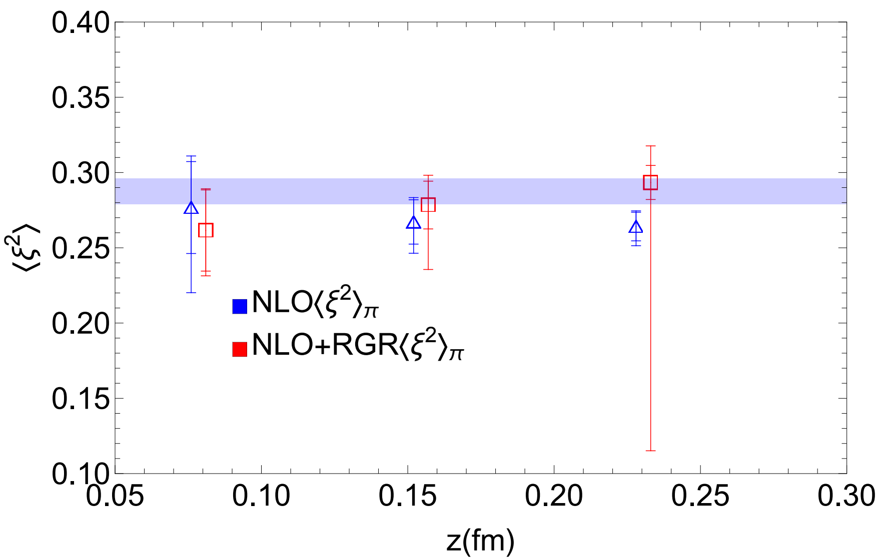
Start with quasi-DA $iP_z H^R\left(z \cdot P_z, z^2, \mu\right) \equiv \langle 0|O_{\gamma_5\gamma_3}(z,\mu)|M;P\rangle$ and use short distance factorization:

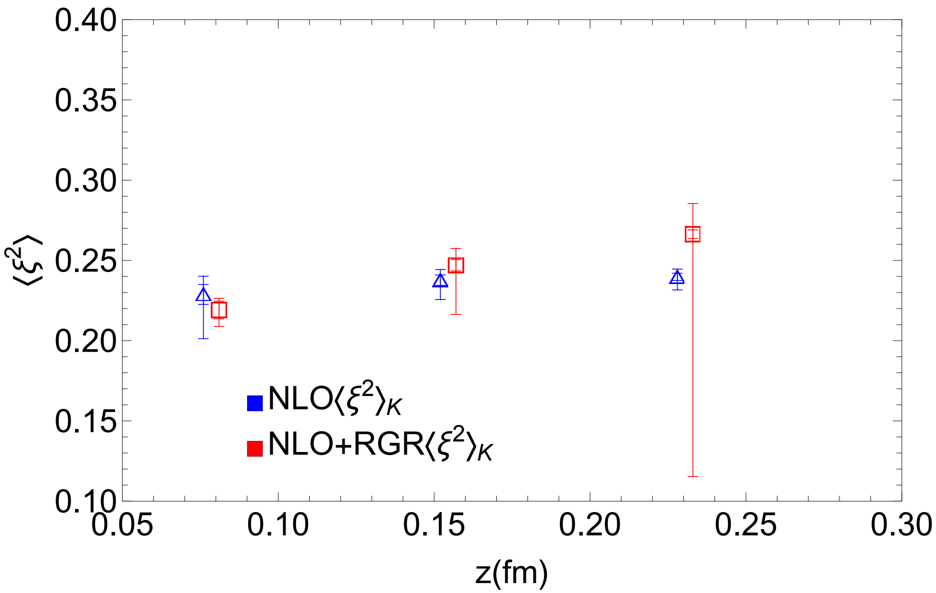
Mellin-OPE

$$h^{\text{tw2}}(\lambda, z^2, \mu) = \sum_{n=0} \frac{(-i\lambda/2)^n}{n!} \sum_{m=0}^{n} C_{n,m}(z^2\mu^2)\langle \xi^m \rangle,\ \lambda = zP_z,\ \langle \xi^n \rangle = \int_0^1 \phi(x,\mu)(2x-1)^n dx$$

Ratio scheme:

$$\mathcal{M}(\lambda, z^2, P^0) \equiv \frac{H^B(\lambda, z^2)}{H^B(\lambda_0, z^2)} = \frac{H^R(\lambda, z^2, \mu)}{H^R(\lambda_0, z^2, \mu)}, \leftrightarrow \frac{h^{\text{tw2}}(\lambda, z^2, \mu)}{h^{\text{tw2}}(\lambda_0, z^2, \mu)},\ \lambda_0 = P_z^0 z \ \Rightarrow \langle \xi^n \rangle_\mu$$





Renormalization and meson quasi-DA in x-space

Hybrid scheme: $$a\delta m = \ln \frac{C_0(z,\mu)/C_0(z-a,\mu)}{h^B(z,0,a)/h^B(z-a,0,a)}$$

renormalon ambiguity when matching lattice scheme to $\overline{MS}$ scheme

$\Rightarrow$ Leading Renormalon Resummation (LRR):

$$C_0(z,\mu) \to C_0^{\mathrm{LRR}}(z,\mu) = 1 + \frac{\alpha_s(\mu)C_F}{2\pi}\left(\frac{3}{2}\ln\frac{z^2\mu^2 e^{2\gamma_E}}{4} + \frac{5}{2} + \delta_{i3}\right)$$

$$-\alpha_s z\mu N_m(1+c_1) + z\mu N_m \frac{4\pi}{\beta_0}\int_{0,\mathrm{PV}}^{\infty} du\, e^{-\frac{4\pi u}{\alpha_s(\mu)\beta_0}} \frac{1}{(1-2u)^{1+b}}(1 + c_1(1-2u) + \ldots)$$

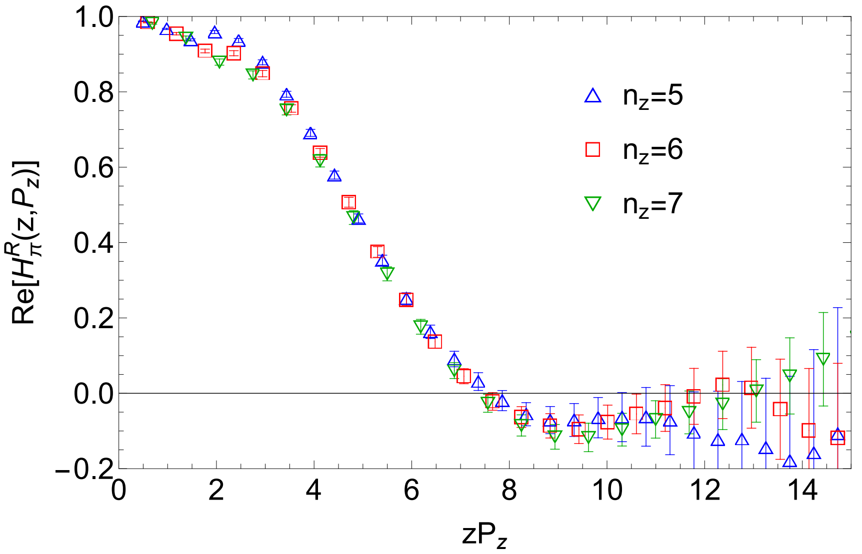
Zhang et al, PLB 844 (‘23) 134081

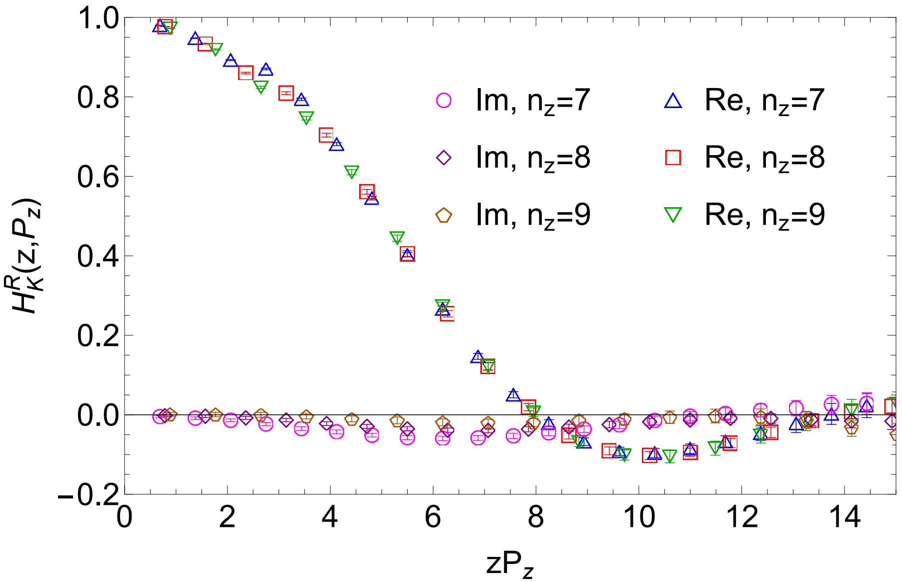
Renormalization Group Resummation (RGR)

$$C_n^{\mathrm{NLO}}(z,\mu) \to C_n^{\mathrm{RGR}} = C_n^{\mathrm{NLO}}(z,\mu = 2z^{-1}e^{-\gamma_E})e^{-I(2z^{-1}e^{-\gamma_E})},$$

$$I(\mu) = \int d\alpha \frac{\gamma_n(\alpha)}{\beta(\alpha)}|_{\alpha=\alpha_s(\mu)}$$

Renormalization and meson quasi-DA in x-space



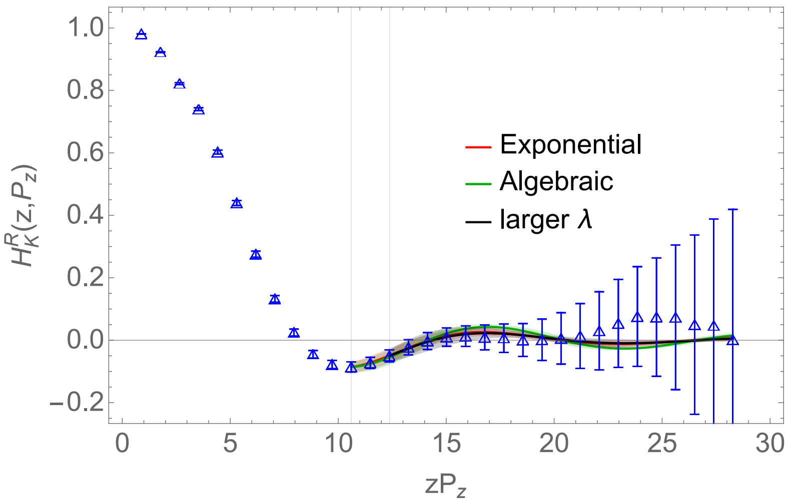


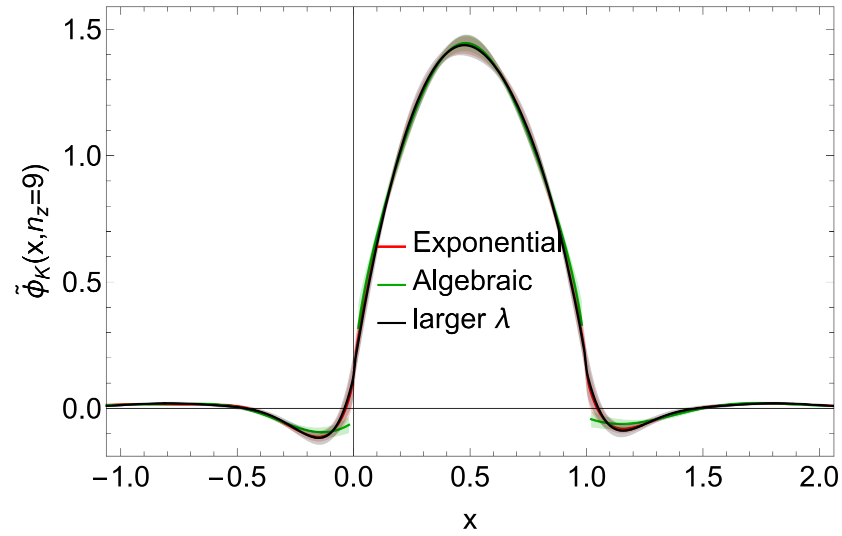
Imaginary part becomes smaller with increasing momenta

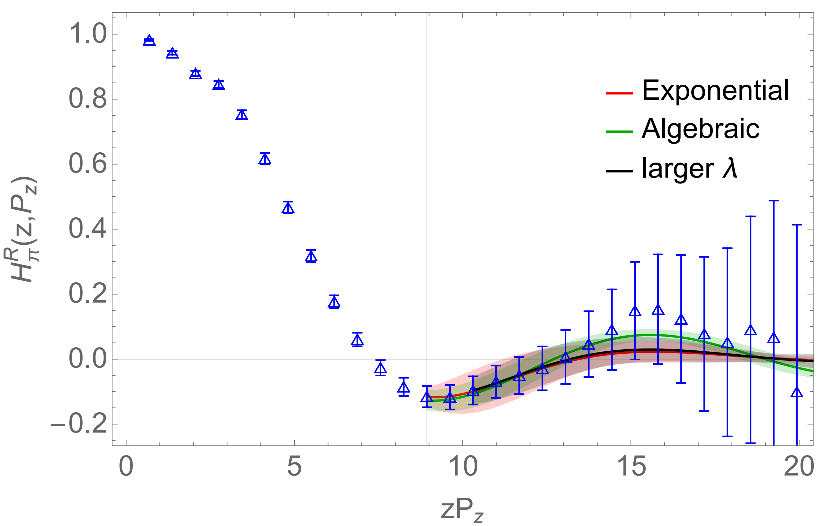
Long tail modeling:

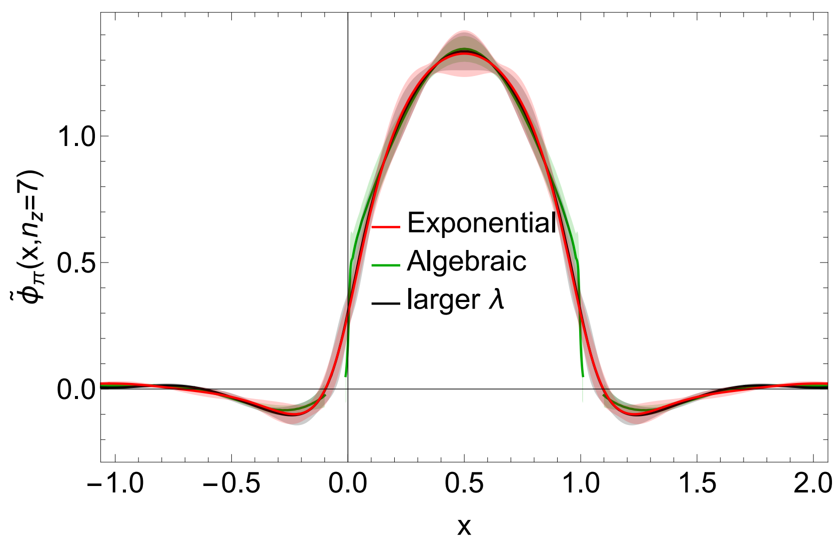
$$H(\lambda) \xrightarrow{\lambda\to\infty} \left(\frac{c_1 e^{-i\lambda/2}}{(-i\lambda)^{d_1}} + \frac{c_2 e^{i\lambda/2}}{(i\lambda)^{d_2}} \right) e^{-\lambda/\lambda_0}$$

Results on quasi-DA in x-space









Meson distribution amplitude from x-space matching

$$\phi_M(x,\mu) = \int_{-\infty}^{\infty} dy\, \mathcal{C}^{-1}(x,y,\mu,P_z)\tilde{\phi}_M(y,P_z) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{x^2P_z^2}, \frac{\Lambda^2_{\rm QCD}}{(1-x)^2P_z^2}\right)$$

Ji et al, NPB 964 (‘21)115311

$$\mathcal{C}(x,y,\mu,P_z) = \delta(x-y) + \frac{\alpha_s(\mu)C_F}{2\pi}\left[\begin{cases} \frac{1+x-y}{y-x}\frac{\bar{x}}{\bar{y}}\ln\frac{(y-x)}{\bar{x}} + \frac{1+y-x}{y-x}\frac{x}{y}\ln\frac{(y-x)}{-x} & x<0 \\ \frac{1+y-x}{y-x}\frac{x}{y}\ln\frac{4x(y-x)P_z^2}{\mu^2} + \frac{1+x-y}{y-x}\left(\frac{\bar{x}}{\bar{y}}\ln\frac{y-x}{\bar{x}} - \frac{x}{y}\right) & 0<x<y<1 \\ \frac{1+x-y}{x-y}\frac{\bar{x}}{\bar{y}}\ln\frac{4\bar{x}(x-y)P_z^2}{\mu^2} + \frac{1+y-x}{x-y}\left(\frac{x}{y}\ln\frac{x-y}{x} - \frac{\bar{x}}{\bar{y}}\right) & 0<y<x<1 \\ \frac{1+y-x}{x-y}\frac{x}{y}\ln\frac{(x-y)}{x} + \frac{1+x-y}{x-y}\frac{\bar{x}}{\bar{y}}\ln\frac{(x-y)}{-\bar{x}} & 1<x \end{cases}\right.$$

$$\left. + \frac{3\mathrm{Si}(z_sP_z(y-x))}{\pi(y-x)}\right]_+^{[-\infty,\infty]},$$

Threshold logs for $x \to y$

$$\mathcal{C}(x,y,\mu,P_z) \xrightarrow{x\to y} H(xP_z,\bar{x}P_z,\mu) \otimes J(|x-y|P_z,\mu) + \mathcal{O}((y-x)^0).$$

Sudakov factor — Jet function

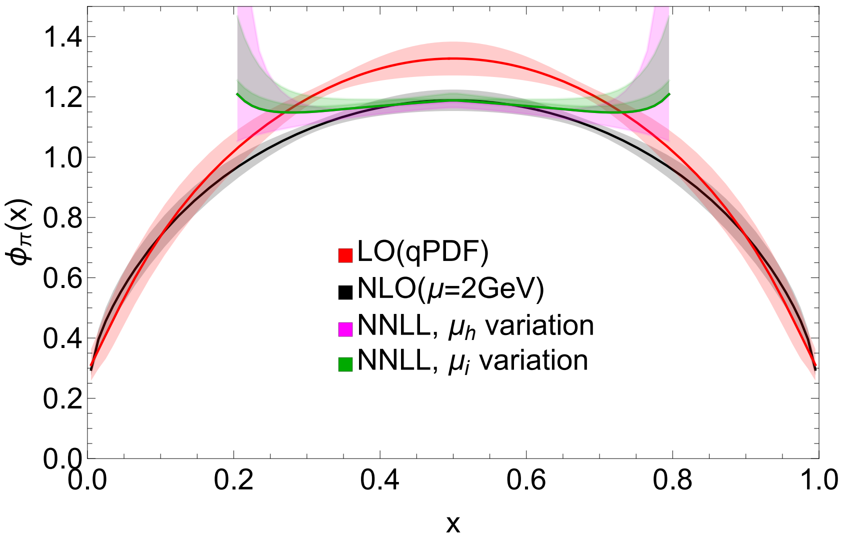
Holligan et al, NPB 993 (‘23)116282

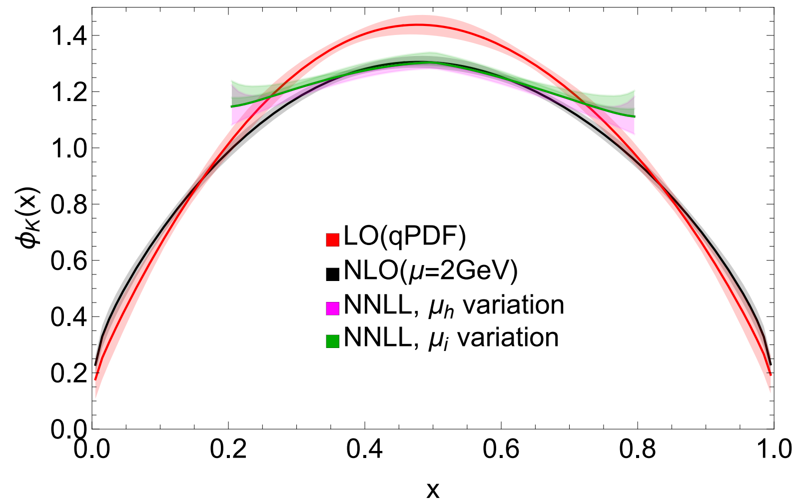
Sudakov factor and Jet function satisfy RG equation with known Γ^{cusp}, γ_S and γ_J $\Rightarrow$ by solving these RG equations and mathcing to the NLO result we resum the threshold logarithms

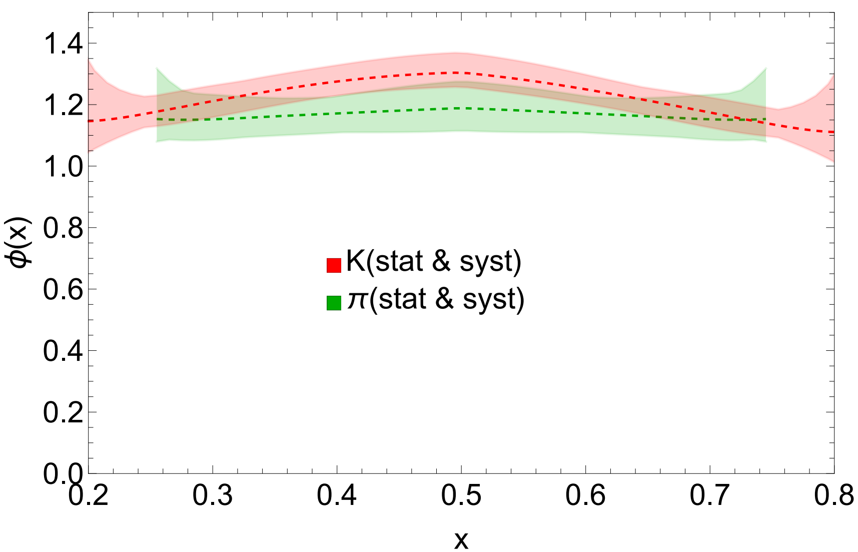
$$C^{-1}_{NLO}(\mu) \to C^{-1}_{\rm TR}(\mu) = C^{-1}_{\rm NLO}(\mu) \otimes J_{\rm NLO}(\mu) \otimes H_{\rm NLO}(\mu) \otimes H^{-1}_{\rm TR}(\mu_{h1},\mu_{h2},\mu) J^{-1}_{\rm TR}(\mu_i,\mu)$$

$$\mu_i = 2\min[x,\bar{x}]P_z, \mu_{h_1} = 2xP_z,\ \ \mu_{h_2} = 2\bar{x}P_z.$$

Meson distribution amplitude from x-space matching







Reliable results for DA in $[x_0, 1-x_0]$

$x_0(K) = 0.2,\ x_0(\pi) = 0.25, (\ P_z^{max} = 1.8 \text{ GeV for } \pi)$

LaMET-SDF complementarity

Ji, arXiv:2209.09332,
Holligan et al, NPB 993 ('23)116282

Assume $\phi(x \to 0,1) \propto A x^m (1-x)^n$

$\Downarrow$

$\phi(x,\mu,m,n)$ in $[0,1]$

Match

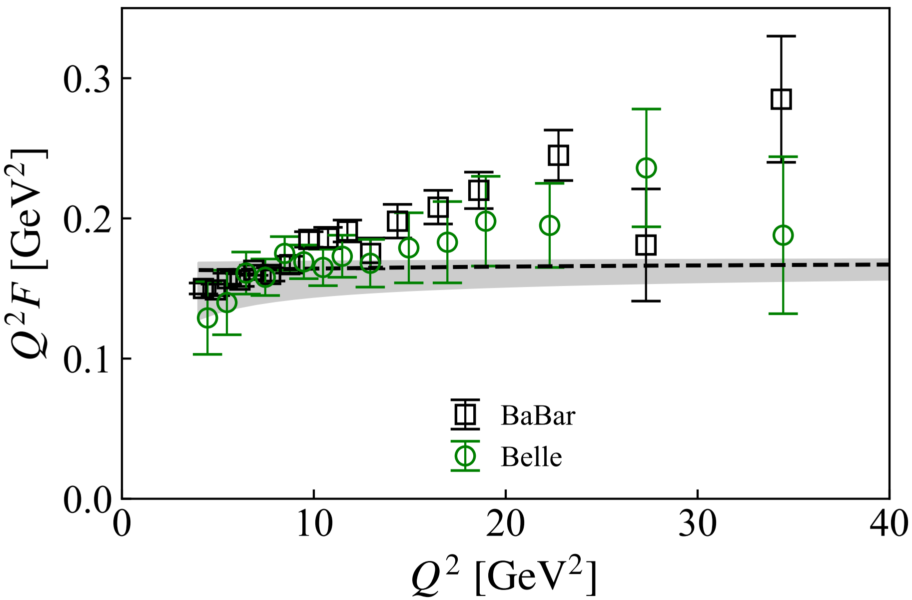
$$H(z,m,n) = \int d\nu \mathcal{Z}(\nu, z^2, \mu, zP_z) \times \int P_z dz\ e^{i(\frac{1}{2}-x)\nu z P_z} \phi(x,\mu,m,n)$$

to $H^R(z,P_z) \Rightarrow n,\ m$

Moments of meson DAs and transition form-factor

$$\langle \xi^i \rangle = \int_0^1 dx \phi_M(x)(2x-1)^i,$$
$$a_i = \int_0^1 dx \phi_M(x) C_i^{3/2}[2x-1] \frac{4(i+3/2)}{3(i+1)(i+2)},$$

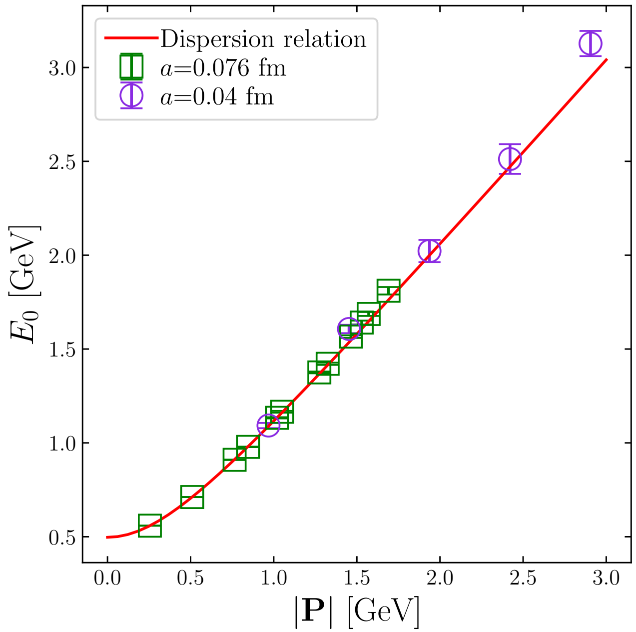
	m	n	a_2	a_4	a_6	$\langle \xi \rangle$	$\langle \xi^2 \rangle$	$\langle \xi^4 \rangle$	$\langle \xi^6 \rangle$
K	0.62(7)	0.58(7)	0.114(20)	0.037(11)	0.019(5)	0.001(10)	0.237(7)	0.115(6)	0.070(5)
π	0.31(6)	0.31(6)	0.196(32)	0.085(26)	0.056(15)	0	0.267(11)	0.139(10)	0.090(8)



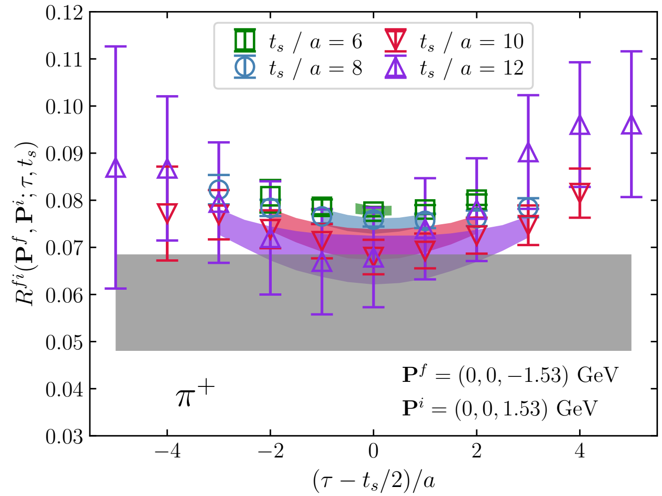
Belle-II, PTEP 2019 (‘19) 123C01
Barbar, PRD 80 (‘09) 052002

Pion and kaon form factor at large momentum transfer

$$R^{fi}(\mathbf{P}^f,\mathbf{P}^i;\tau,t_s) \equiv \frac{2\sqrt{E_0^f E_0^i}}{E_0^f+E_0^i}\,\frac{C_{3\text{pt}}(\mathbf{P}^f,\mathbf{P}^i;\tau,t_s)}{C_{2\text{pt}}(\mathbf{P}^f,t_s)}\left[\frac{C_{2\text{pt}}(\mathbf{P}^i,t_s-\tau)C_{2\text{pt}}(\mathbf{P}^f,\tau)C_{2\text{pt}}(\mathbf{P}^f,t_s)}{C_{2\text{pt}}(\mathbf{P}^f,t_s-\tau)C_{2\text{pt}}(\mathbf{P}^i,\tau)C_{2\text{pt}}(\mathbf{P}^i,t_s)}\right]^{1/2}$$

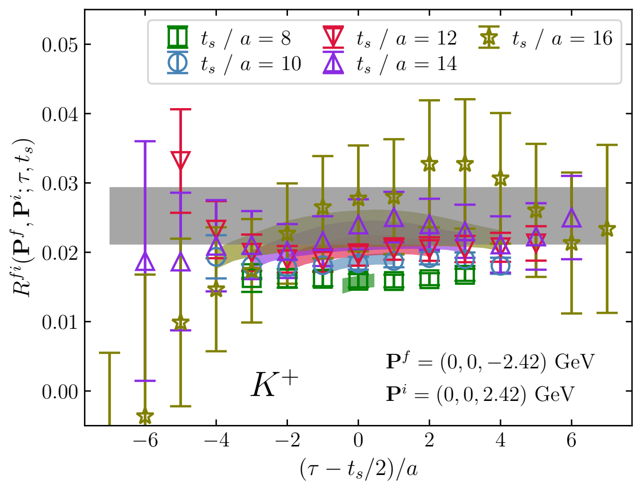


Pion, *a=0.076* fm



350 gauge configurations
32-256 AMA samples

Kaon, *a=0.040* fm

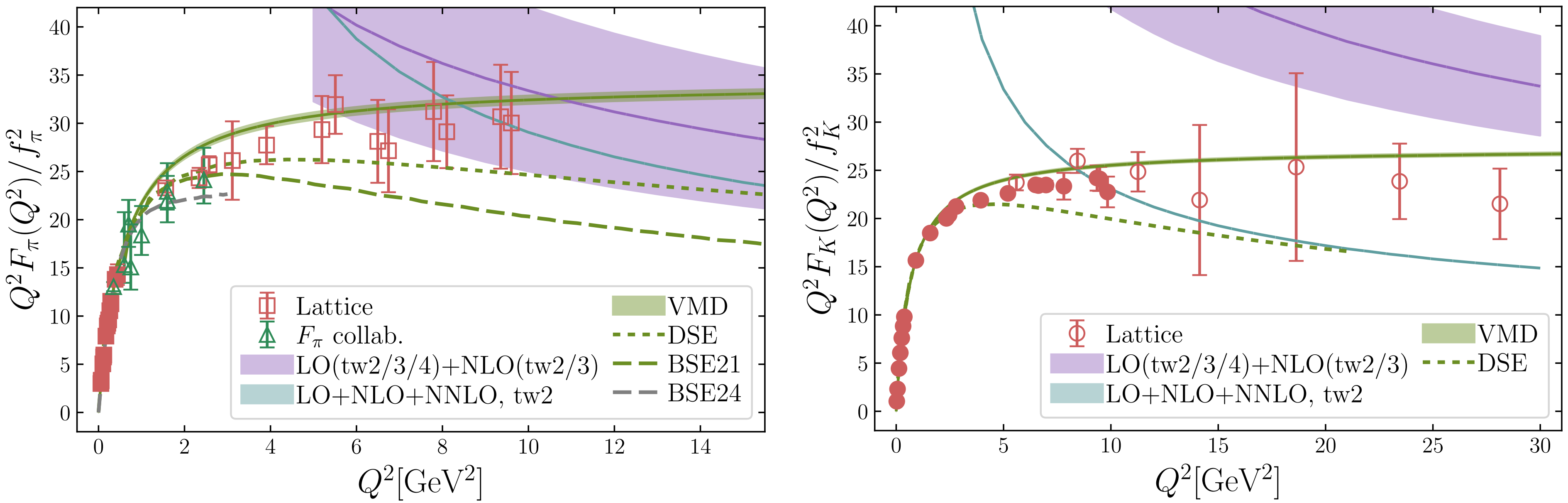


$$a = 0.076\text{ fm}:\ \ r_G^s = 0.83\text{ fm},\ \ r_G^l = 0.59\text{ fm}$$

$$a = 0.040\text{ fm}:\ \ r_G^s = 0.86\text{ fm},\ \ r_G^l = 0.59\text{ fm}$$

Pion and kaon form factor at large momentum transfer

H.-T. Ding, X. Gao, A.D. Hanlon, S. Mukherjee, PP, P. Scior, Qi Shi, S. Syritsyn, R. Zhang, Y. Zhao, arXiv:24.04.04412



- The lattice results are compatible with DSE and VDM model, BSE calculations tend to be on the low side compared to the lattice data for $Q^2 > 3$ GeV^2
- Collinear pQCD factorization approach with lattice DA can explain the high Q2 lattice results on the form factors => verification of QCD factorization in exclusive processes
- pQCD k_T factorization approach with higher twist contributions seems to overpredict the kaon form factor but works for pion form factor

Summary

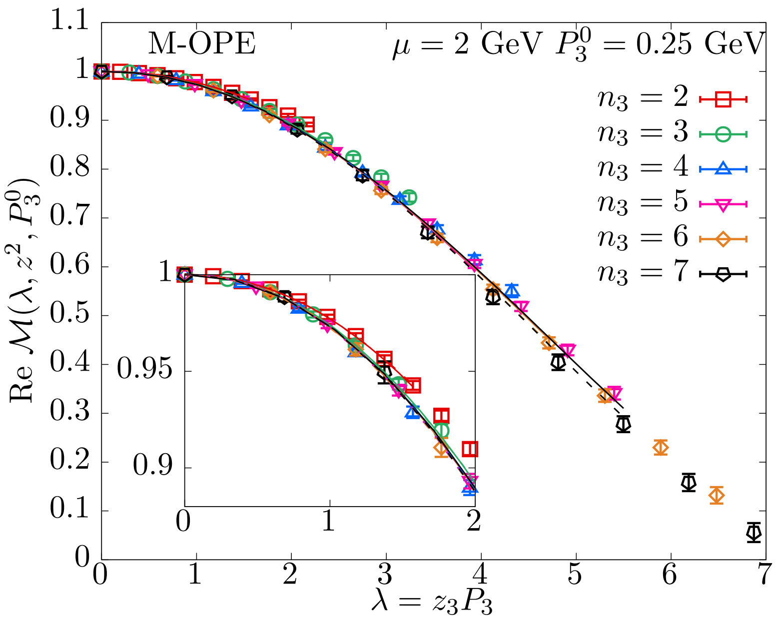
- The *x*-dependence of pion and kaon DA can be from the lattice using LaMET combined with short distance factorization

- The pion DA is very different from the asymptotic regime or flat form and leads to pion transition form factors that agrees with newest results from Belle

- Pion and kaon form factor have been studied at large momentum transfer using lattice QCD providing predictions for Jlab and EIC meson programs; Within errors lattice QCD results agree with NNLO results in collinear factorization scheme with DA obtained from lattice QCD

- pQCD k_T factorization approach with higher twist contributions overpredicts the kaon form factor

BACK-UP Slides



$$\langle x^2 \rangle = 0.2848(52)(71),$$
$$\langle x^4 \rangle = 0.124(11)(20).$$

Very different from asymptotic values:
$\langle x^2 \rangle = 0.2$ and $\langle x^4 \rangle = 0.0857$
or flat DA ($\phi(x) = 1/2$):
$\langle x^2 \rangle = 1/3$ and $\langle x^4 \rangle = 0.2$

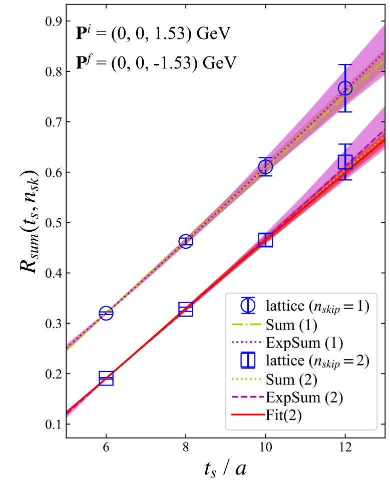
Results are cross-checked with conformal OPE

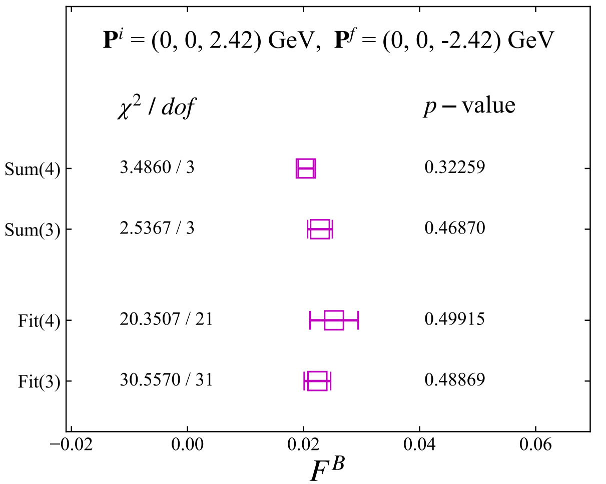
$a_2 = 0.227(18)(23)$, $a_4 = -0.16(13)(30) \Rightarrow$
same Mellin moments

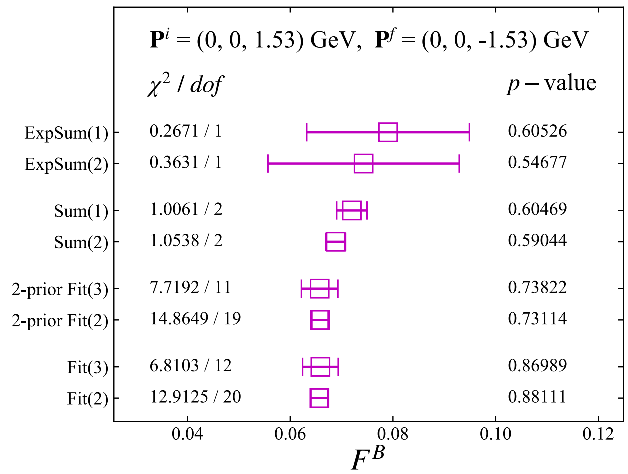
$$R^{fi}_{\mathrm{sum}}(t_s) = \sum_{\tau=n_{\mathrm{sk}}a}^{t_s - n_{\mathrm{sk}}a} R^{fi}(t_s, \tau)$$

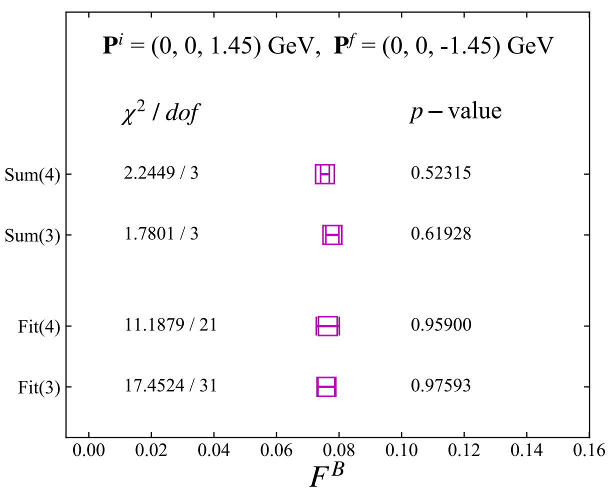
$$R^{fi}_{\mathrm{sum}}(t_s) = nF^B + B_0 + \mathcal{O}(e^{-(E_1-E_0)t_s}), \quad n = t_s - (2n_{\mathrm{sk}} - 1)a$$

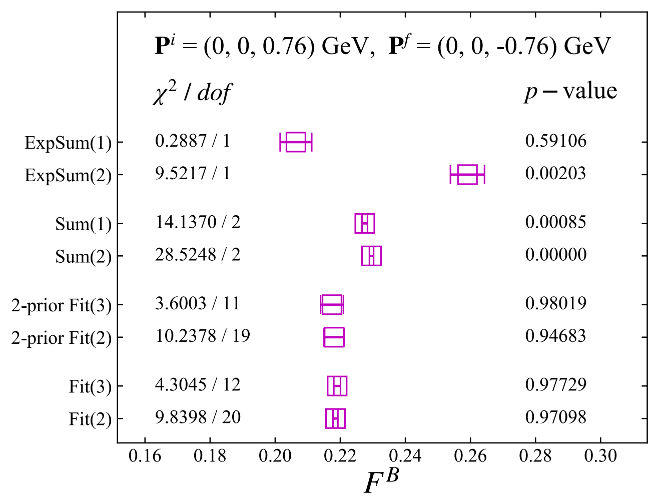
$$R^{fi}_{\mathrm{sum}}(t_s) = nF^B + B_0 + nB_1 e^{-(E_1-E_0)t_s}$$









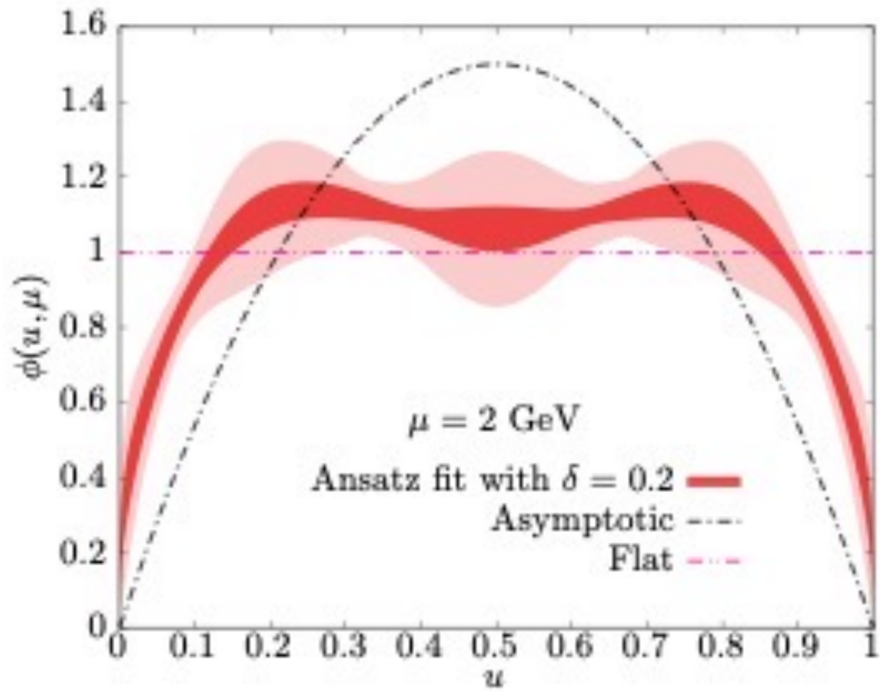


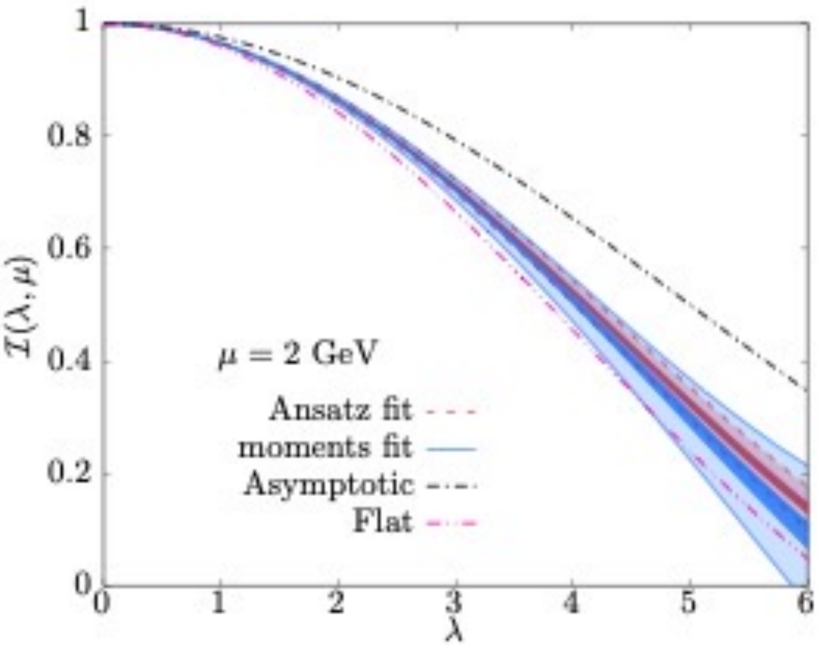
Pion distribution amplitude

Conformal OPE inspired fit form $x = 2u - 1$

X. Gao, A.D. Hanlon, N. Karthik, S. Mukherjee, PP, P. Scior, S. Syritsyn, Y. Zhao, PRD 106 ('22) 074505

$$\phi(u) = \mathcal{N} u^\alpha (1-u)^\alpha \sum_{n=0}^{N_G+1} s_n C_{2n}^{\frac{1}{2}+\alpha}(1-2u), \; s_0 = 1 \quad |s_n| < \delta, \; n > 0$$

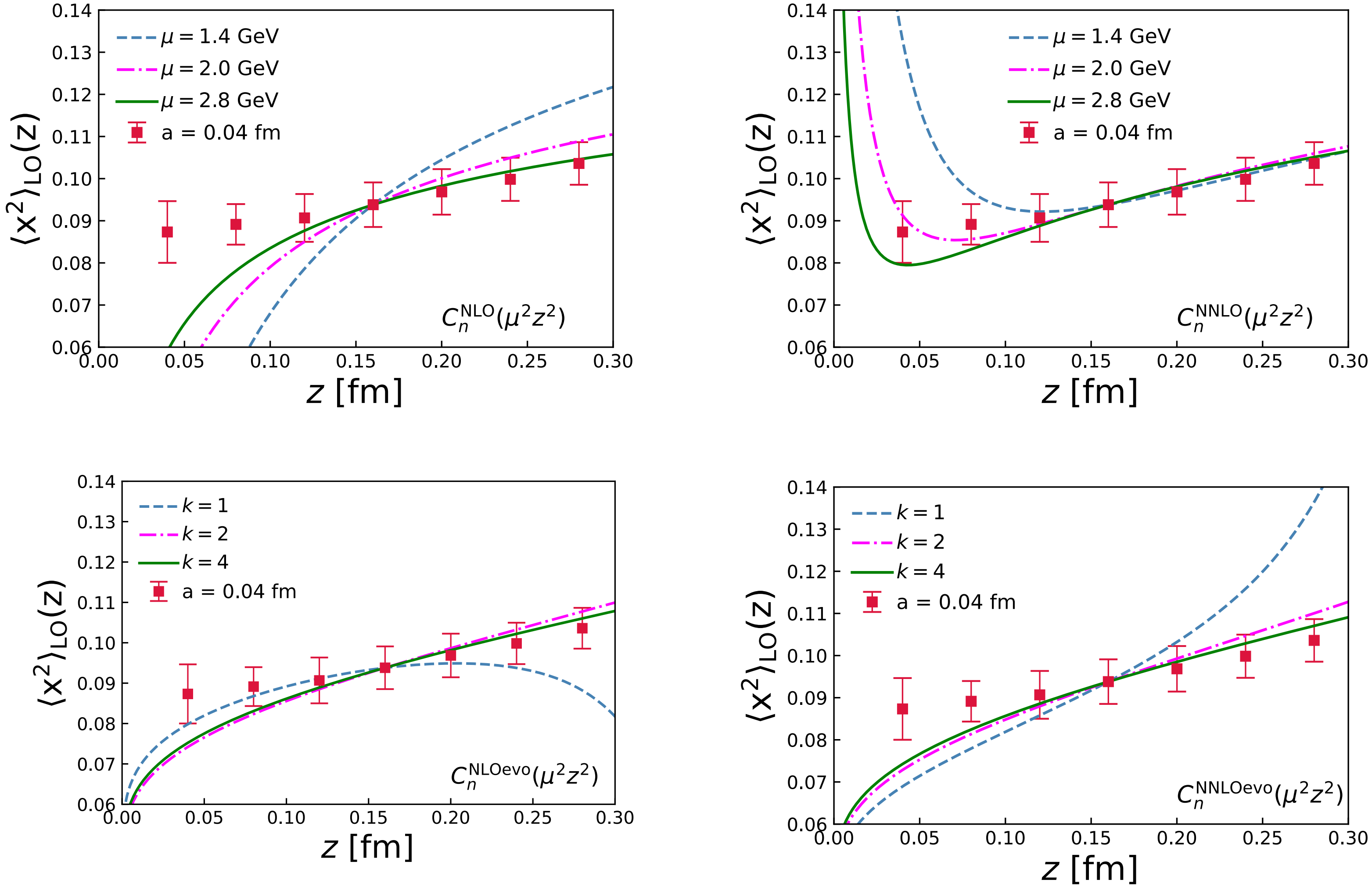




$$\phi(x) \Rightarrow \langle x^2 \rangle = 0.2845(44)(58), \; \langle x^4 \rangle = 0.1497(50)(38)$$

Agreement with direct fits of moments

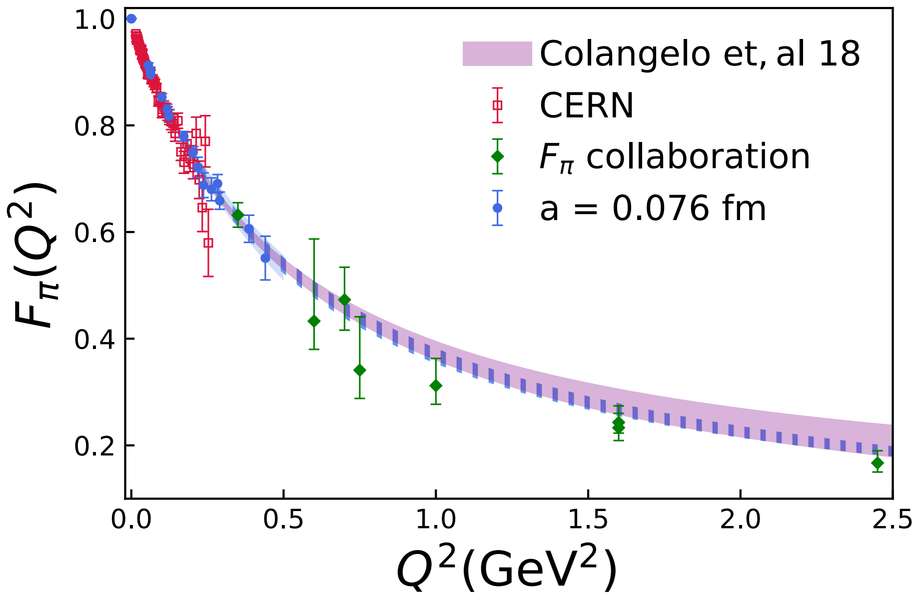
Back-up: perturbative convergence at small z

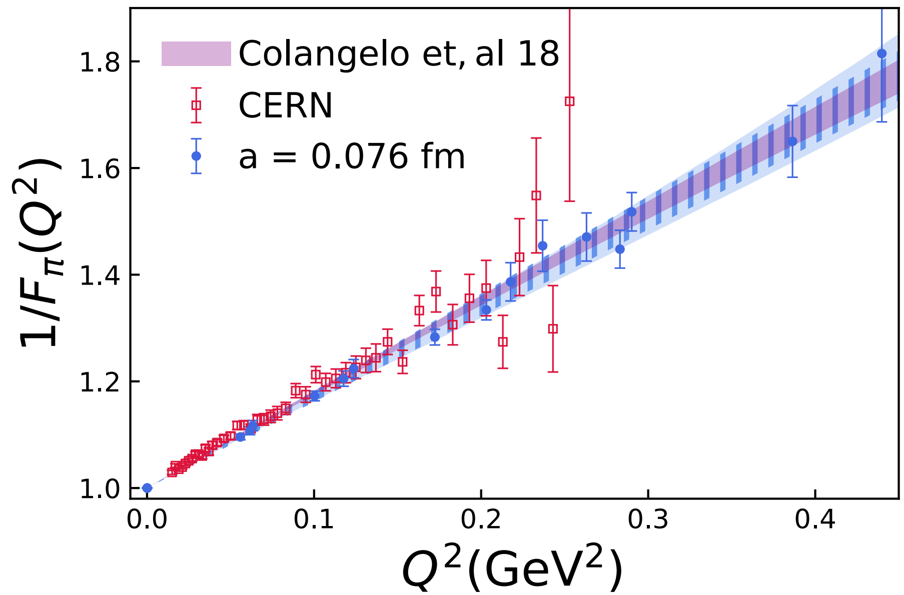


Pion form factor

Form factors are sensitive to the light quark masses

$$64^4,\ a = 0.076\ \text{fm},\ m_\pi = 140\ \text{MeV},\ P_z \sim 2\text{GeV}$$





Lattice and experimental results on the pion form factor agree

Lattice results also agree with the results of the dispersive analysis of the time-like pion form factor

Colangleo, Hofferichter, Stoffer, JHEP 02 (2019) 006

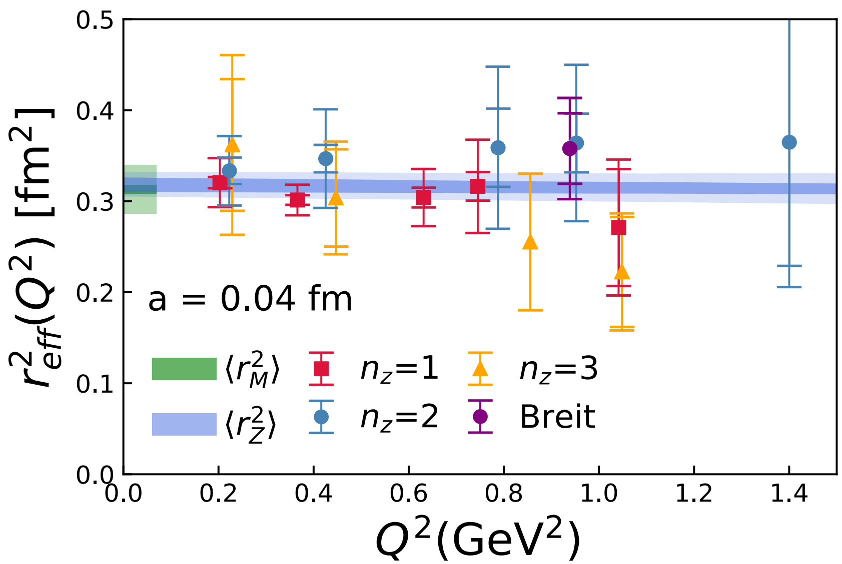
The monopole Ansatz $F_\pi(Q^2) = (1 + Q^2/M^2)^{-1},\ M \simeq 0.8$ GeV works well

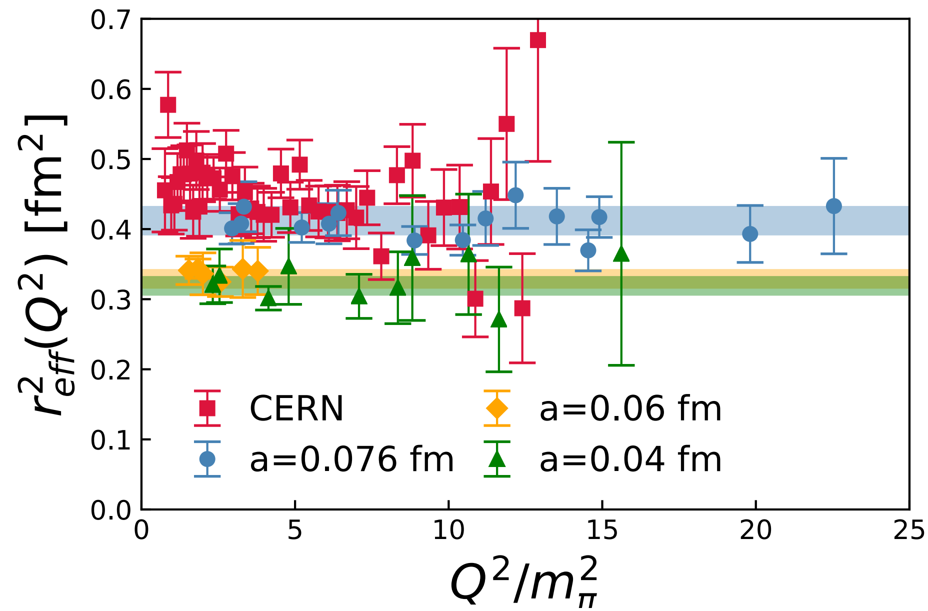
Pion form factor (con't)

Pion charge radius: $\langle r_\pi^2 \rangle = -6 \frac{dF_\pi(Q^2)}{dQ^2}|_{Q^2=0}$ $\langle r_\pi^2 \rangle = 6/M^2$ for monopole fit

The effective radius $r_{eff}^2(Q^2) = \frac{6(1/F_\pi(Q^2) - 1)}{Q^2}$ is constant for all Q^2

→ monopole Ansatz works for extended Q^2 range





Pion form factor is very sensitive to the quark mass

$\langle r_\pi^2 \rangle = 0.42(2)$ fm^2 (monopole fit, z-expansion) $\langle r_\pi^2 \rangle_{PDG} = 0.434(5)$ fm^2