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LPC Lattice Parton Collaboration

The Baryon LCDA from Lattice QCD

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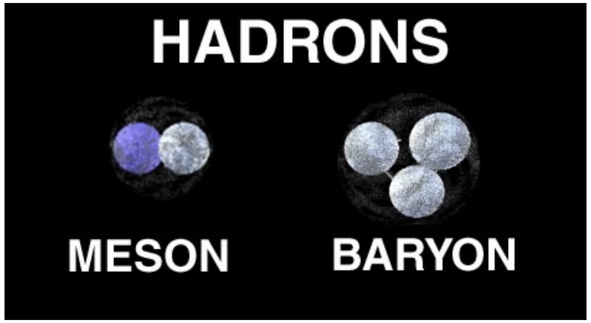
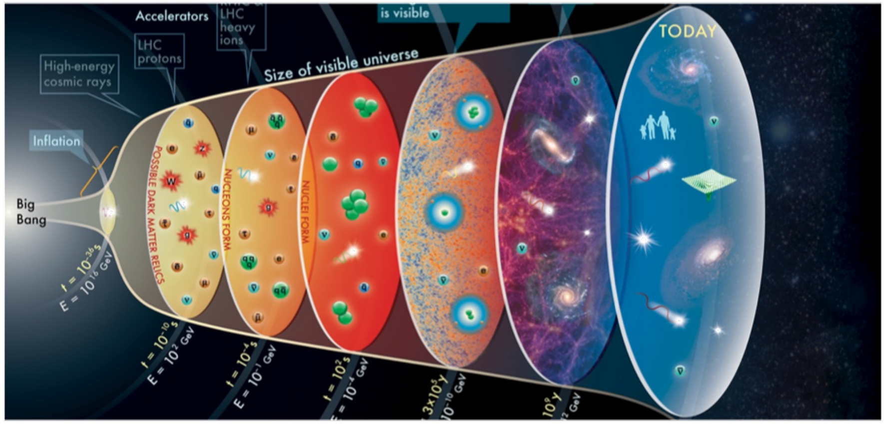
CONTENTS

Motivation

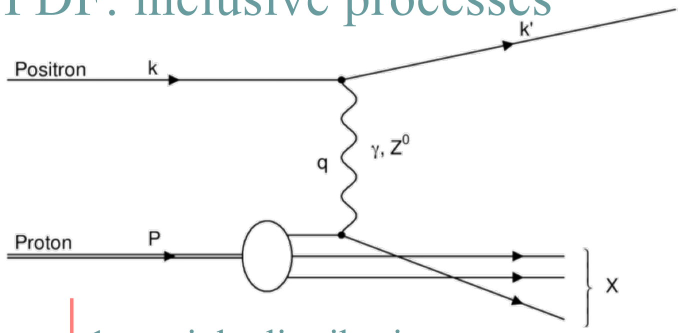
- The visible matter of the Universe is mainly made of baryons.
- Baryons play an important role in the evolution of the Universe, such as baryogenesis and big-bang nucleosythesis.



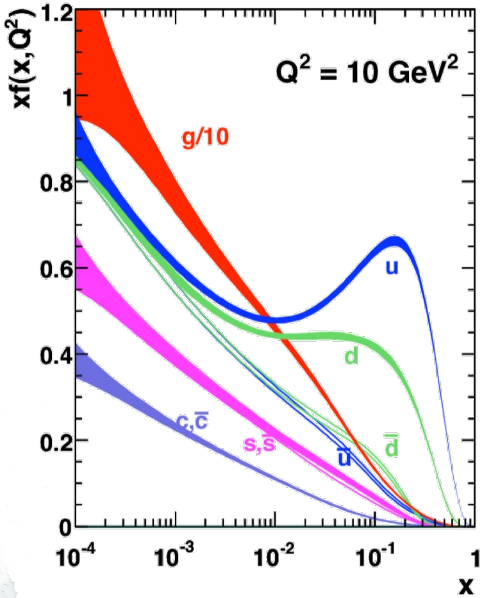


Motivation

PDF: inclusive processes

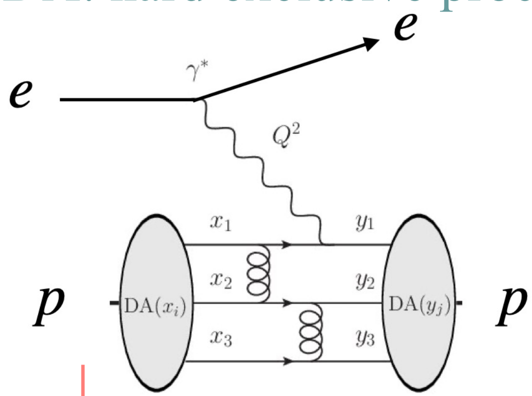


1-particle distributions

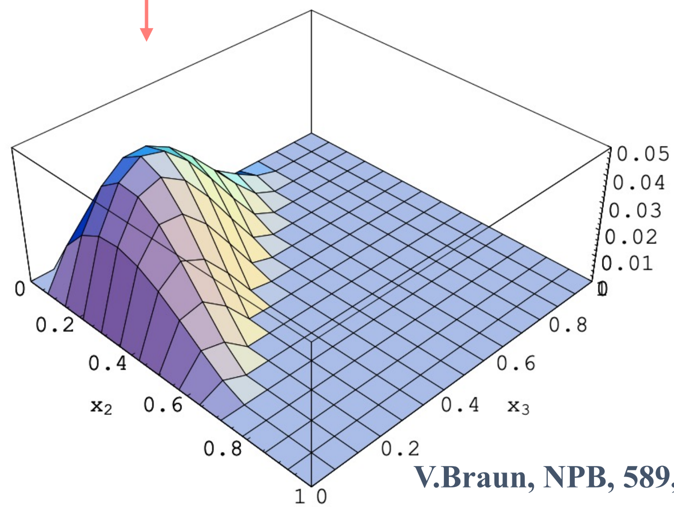


Complementary

LCDA: hard exclusive processes



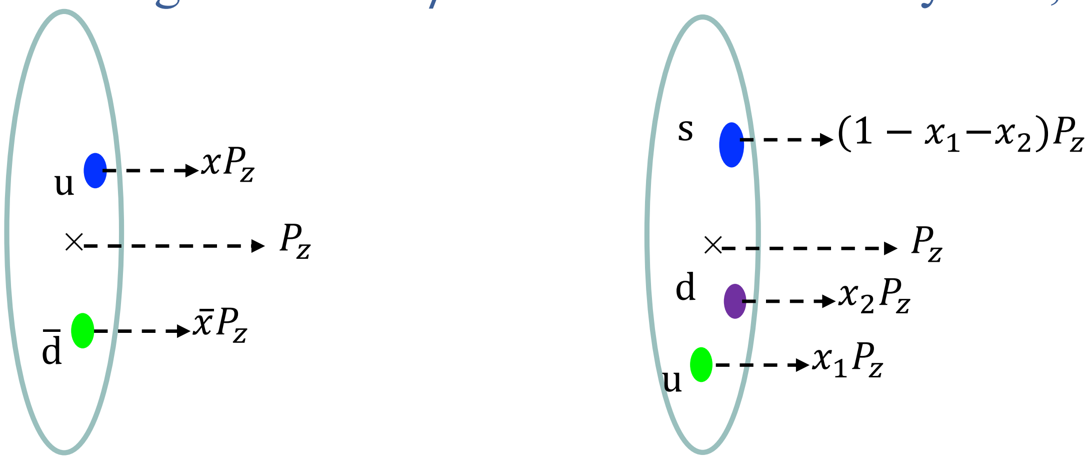
3-particle distributions



V.Braun, NPB, 589,381(2000)

Motivation

Light meson $\pi/K\ldots$ Baryon Λ, proton…



- CKM matrix
- CP violation
- New physics ···

- Sakharov conditions for Baryogenesis:

  1) Baryon number violation

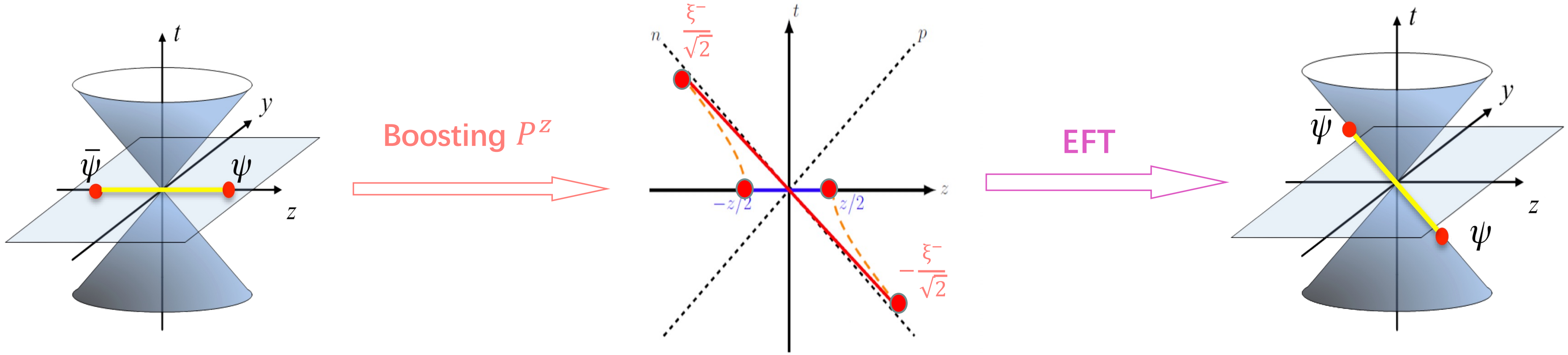
  2) C and CP violation

  3) Out of thermal equilibrium

- CPV well established in K, B and D mesons
- But CPV never established in any baryon
- Indications of CPV $\Lambda_b^0 \to p\pi^-\pi^+\pi^-$, (LHCb) NP13, 391 (2017)

Quasi Distribution

LaMET

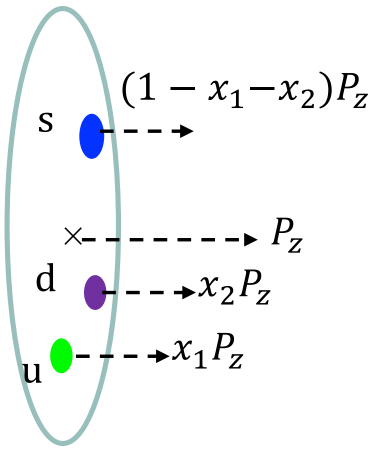


➢ Effective field theory:

- Instead of taking $P^z \to \infty$ calcuation, one can perform an expansion for large but finite P^z:

$$\underbrace{\tilde{q}(x, P^z, \mu)}_{\text{Quasi-DA}} = \int \frac{dy}{|y|} \underbrace{C(x, y, P^z, \mu)}_{\text{Matching kernel}} \underbrace{q(y, \mu)}_{\text{LCDA}} + \underbrace{\mathcal{O}\left(\frac{\Lambda^2, M^2}{(P^z)^2}\right)}_{\text{High power correction}}$$

Quasi Distribution



- Definition of light cone baryon LCDA:

$$\int \frac{d\xi_1^-}{2\pi}\frac{d\xi_2^-}{2\pi} e^{ixp_1^+\xi_1^-} e^{ixp_2^+\xi_2^-} \epsilon^{ijk} \left\langle 0 \left| W^{ii\prime}(\infty,\xi_1^-)\psi_\alpha^{i\prime}(\xi_1^-)\Gamma_{\alpha\beta}W^{jj\prime}(\infty,\xi_2^-)\,\psi_\beta^{j\prime}(\xi_2^-)\,\psi_\gamma^{j}(\infty,0)\right| M(P)\right\rangle$$

$$= if_M(p_1\cdot n)(p_2\cdot n)\phi_M(x_1,x_2).$$

- Corresponding quasi-DA on Euclidean lattice:

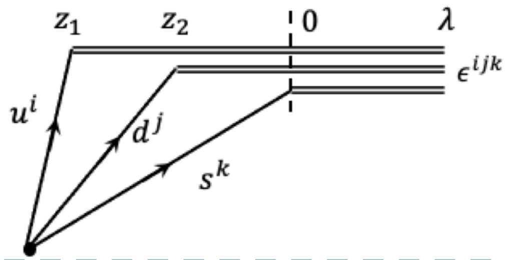
$$f_M(p_1,p_2)\tilde{\Phi}^0(x_1,x_2) = \int \frac{p_{z1}dz_1}{2\pi}\frac{p_{z2}dz_2}{2\pi} e^{-i(x_1p_{z1}z_1+x_2p_{z2}z_2)} \left\langle 0 \left| \tilde{O}\left(z_1,z_2,\tilde{\Gamma}\right)\right| P^z\right\rangle$$

$$\tilde{O}^\Lambda\left(z_1,z_2,\tilde{\Gamma}\right) = \varepsilon^{ijk}U^i(z_1)\,\tilde{\Gamma}D^j(z_2)\,S^k(0)$$

$$= \varepsilon^{ijk}W^{ii'}(\lambda,z_1)\,u_\alpha^{i'}(z_1)\,\Gamma_{\alpha\beta}W^{jj'}(\lambda,z_2)\,d_\beta^{j'}(z_2)\,s_\gamma^k(0)$$

$$\epsilon^{ijk}U^{ii\prime}U^{jj\prime}U^{kk\prime} = det(U)\epsilon^{i\prime j\prime k\prime}$$

$$det(U) = 1$$



Lattice Setup

- Nonlocal 2pt related to baryon quasi DA:

$$C_2^{\Lambda}(z_1, z_2, t; \tilde{\Gamma}, T) \;=\; \mathrm{tr}\{T * \mathrm{tr}_c[\mathrm{tr}_s(\mathrm{qC13}[W(\lambda, z_1)G^u(z_1)C\gamma_5, \tilde{\Gamma}W(\lambda, z_2)G^d(z_2)]) * G^s(0)]\}$$

$$\tilde{\Gamma} = C\gamma_5\gamma^t/\gamma^z, T = (1+\gamma^4)/2$$

- Lattice setup:

CLQCD ensemble, 2 + 1 flavor stout smeared clover fermions and Symanzik gauge action

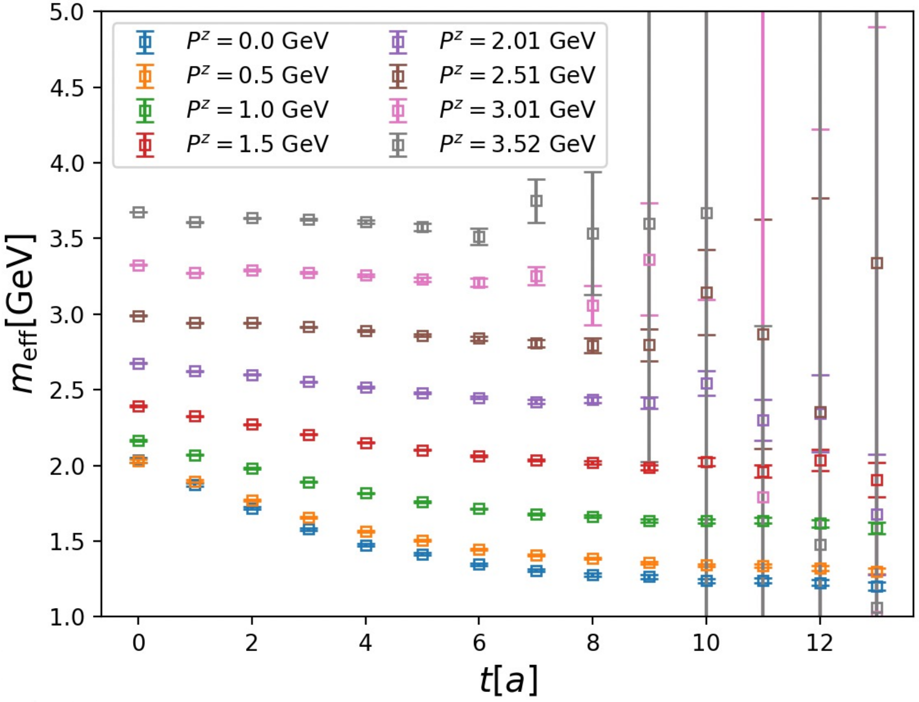
Ensemble	Volume	Lattice spacing	π mass	η_s mass	conf
F32P30	$32^3\times96$	0.077	290MeV	640MeV	777(*32)
H48P32	$48^3\times144$	0.055	300MeV	650MeV	except

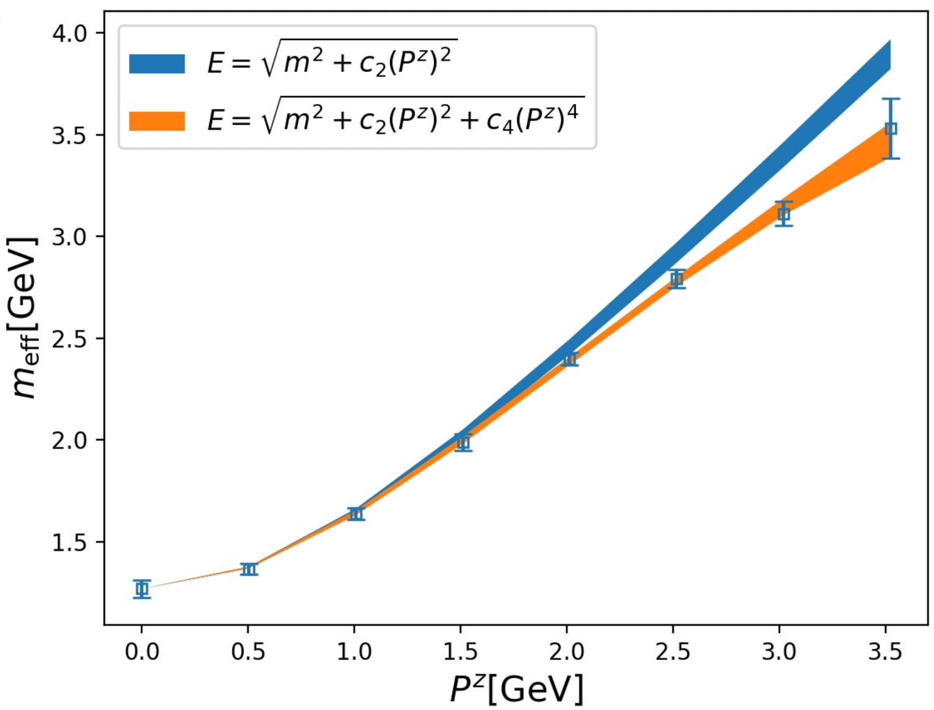
3 momentum: 2.01, 2.51, 3.02 GeV

Lattice Setup

- Dispersion relation:

c2=1.093 (46), c4=-0.0202 (68)





Quasi Distribution

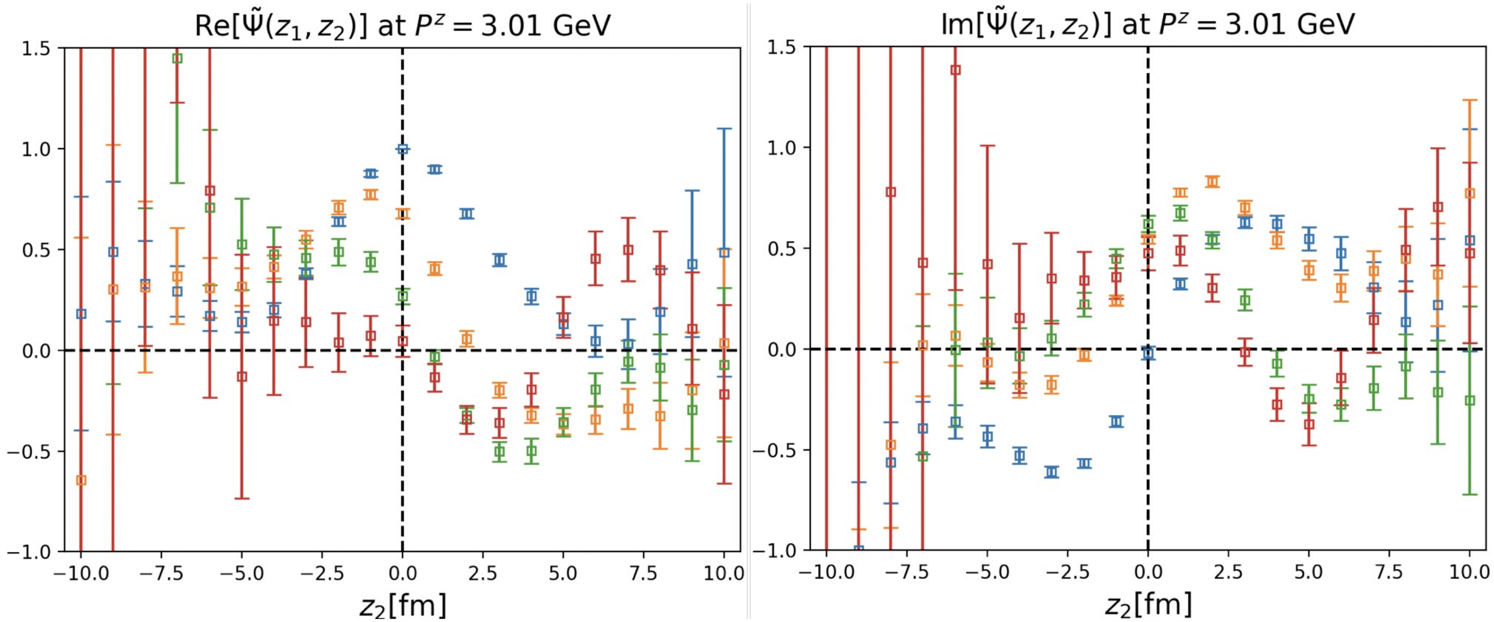
- Baryon (Λ) non-local 2pt correlation function:

$$C_2^{\Lambda}\left(z_1, z_2, t; \tilde{\Gamma}, T\right) = \mathrm{tr}\left\{T * \mathrm{tr}_c\left[\mathrm{tr}_s\left(\mathrm{qC13}\left[G^u\left(z_1\right) C\gamma_5, \tilde{\Gamma} W\left(z_1, z_2\right) G^d\left(z_2\right)\right]\right) * G^s(0)\right]\right\}$$

$$\tilde{\Gamma} = C\gamma_5\gamma^t/\gamma^z$$

$$T = (1+\gamma^4)/2$$

  - Renormalized(Preliminary in ratio scheme) quasi-DA (fixed z_1=0.24fm)



Quasi Distribution

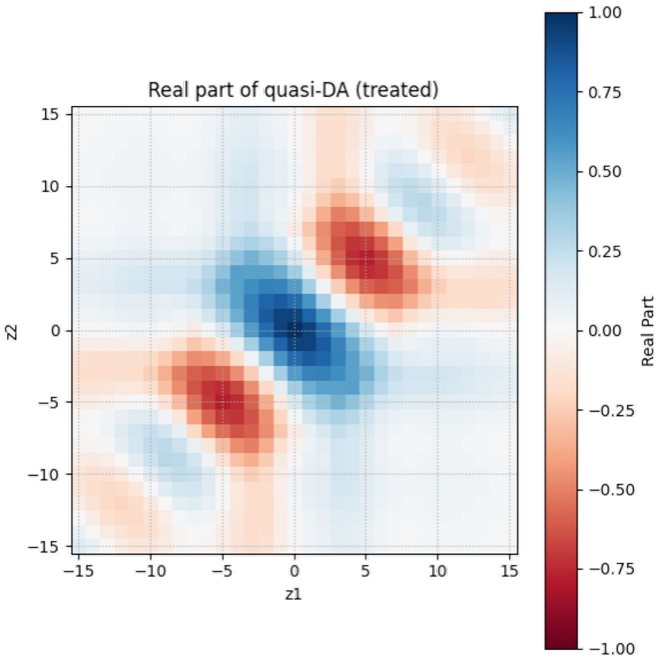
- Baryon (Λ) non-local 2pt correlation function:

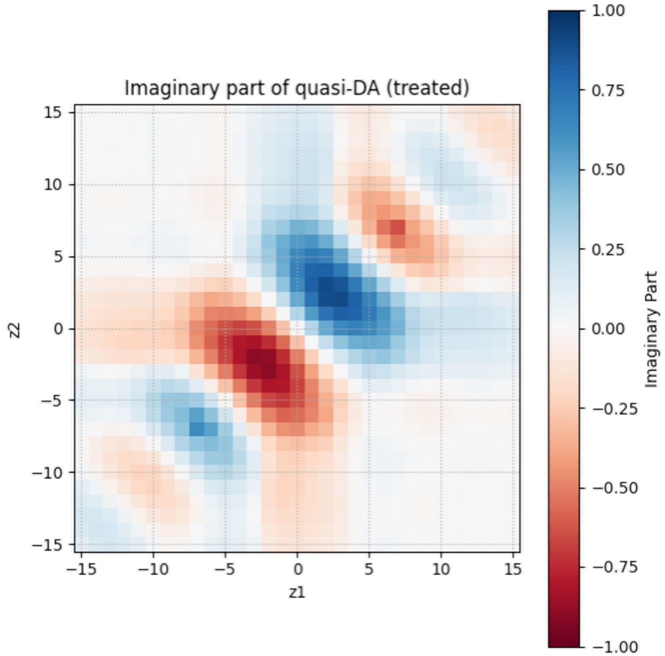
$$C_2^{\Lambda}\left(z_1, z_2, t; \tilde{\Gamma}, T\right) = \mathrm{tr}\left\{T * \mathrm{tr}_c\left[\mathrm{tr}_s\left(\mathrm{qC13}\left[G^u\left(z_1\right) C\gamma_5, \tilde{\Gamma} W\left(z_1, z_2\right) G^d\left(z_2\right)\right]\right) * G^s(0)\right]\right\}$$

$$\tilde{\Gamma} = C\gamma_5\gamma^t/\gamma^z$$

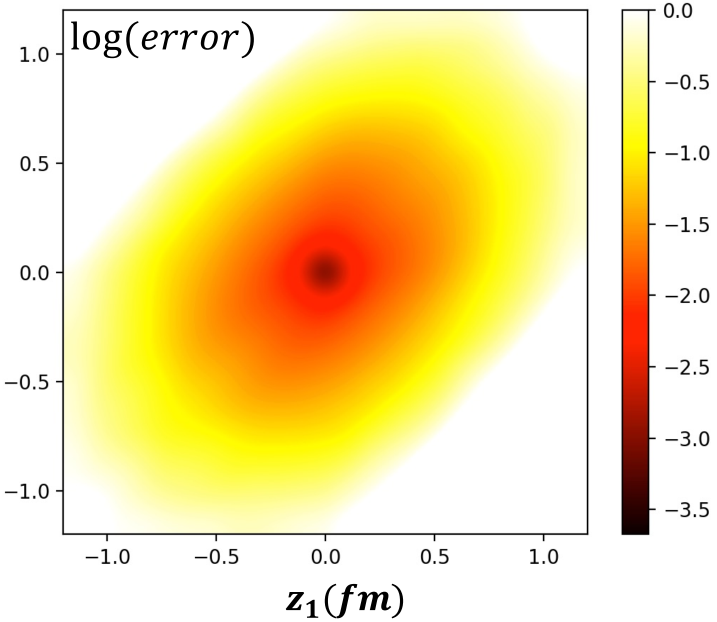
$$T = (1+\gamma^4)/2$$

- Renormalized quasi-DA of Λ :

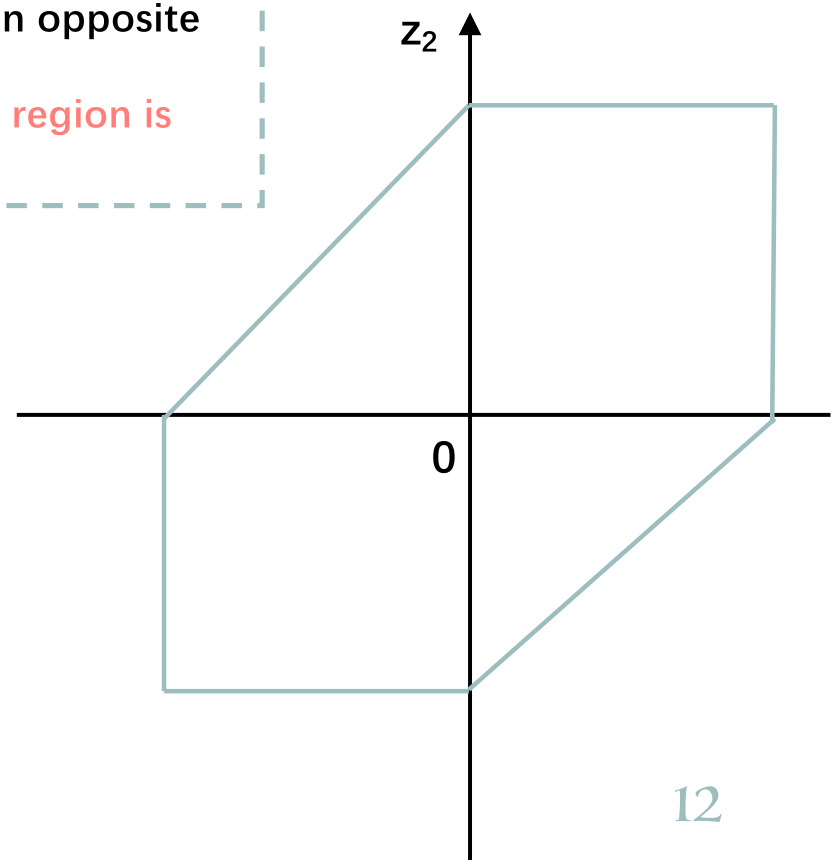


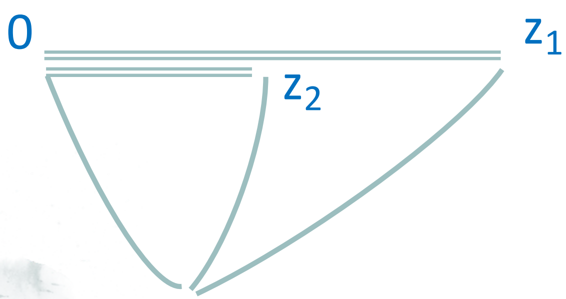


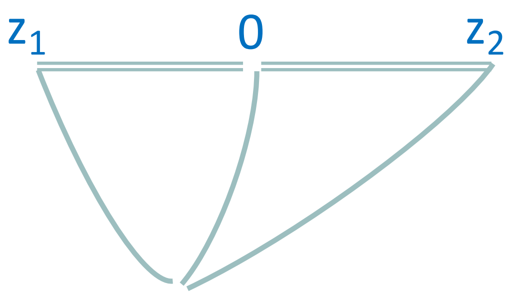
Quasi Distribution



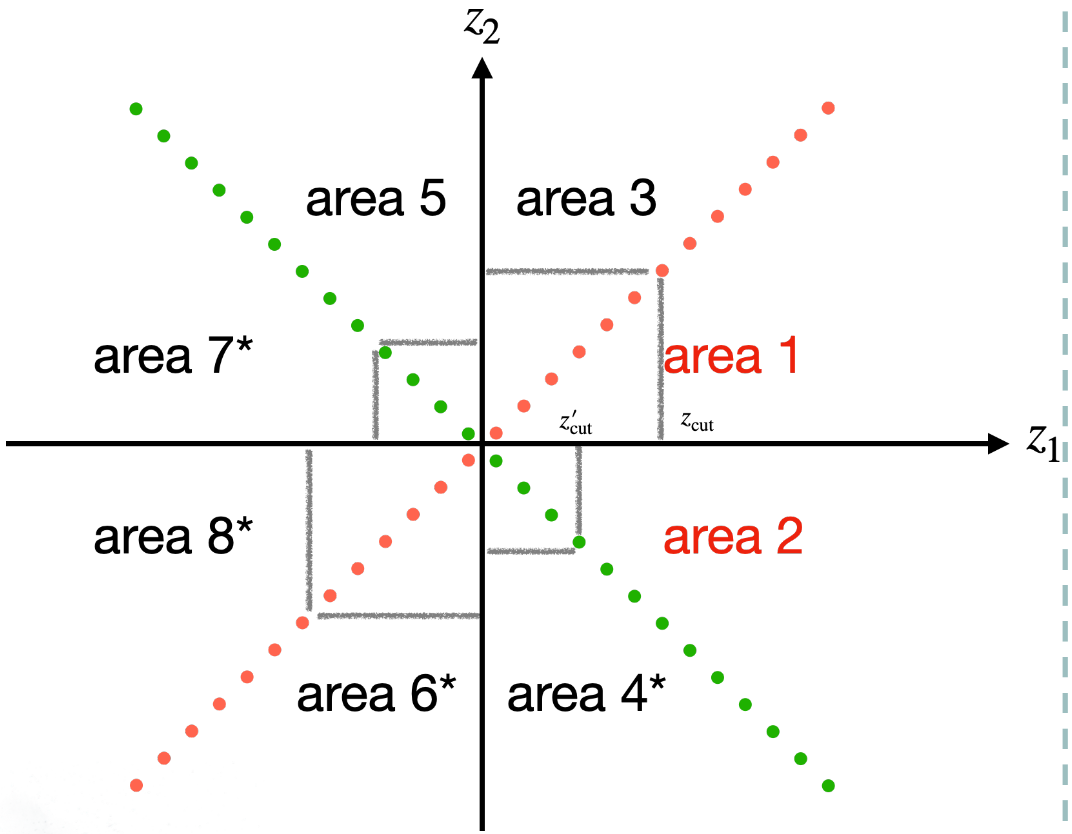
- The Wilson line length(non-local separation) is smaller with z1z2 in same direction than in opposite
- Thus the good signal region is rhombic







Quasi Distribution



➢ Two symmetries for Quasi-DA:

- iso-spin symmetry for "u, d" quarks:

$$\tilde{\phi}(z_1, z_2) = \tilde{\phi}(z_2, z_1)$$

- The constrain by real $\tilde{\phi}(x_1, x_2)$:

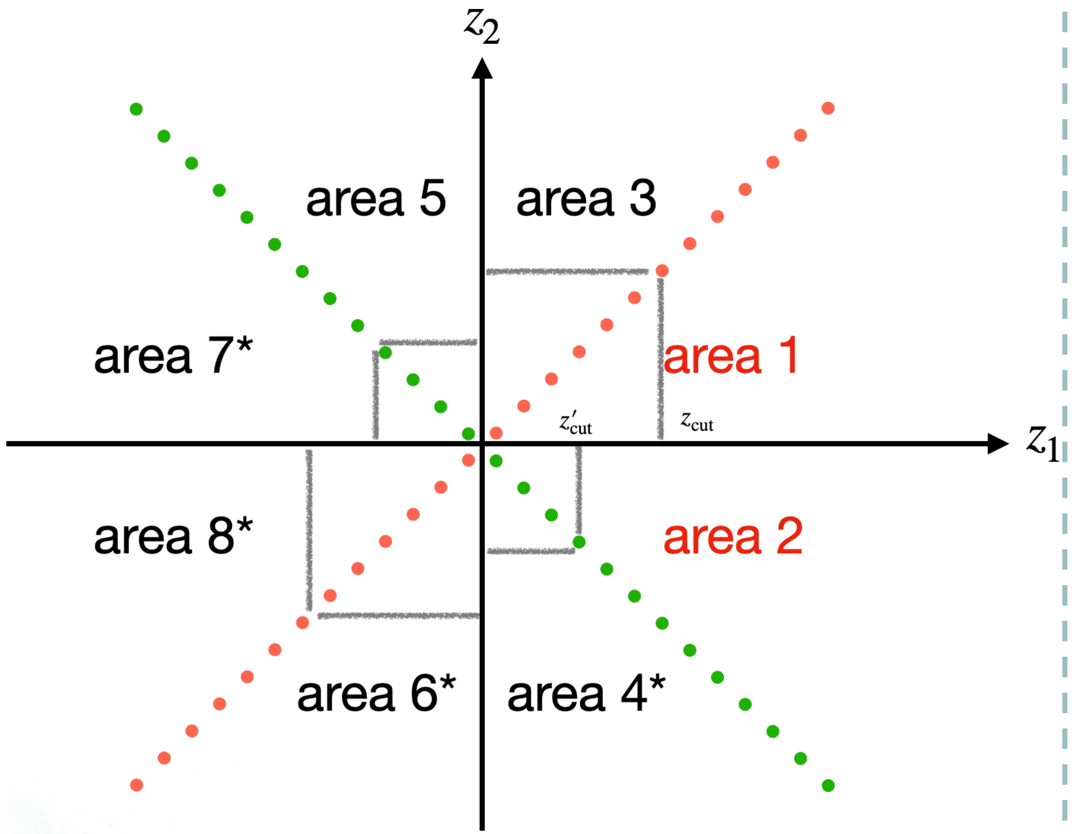
$$\tilde{\phi}(z_1, z_2) = \tilde{\phi}^*(-z_1, -z_2)$$

- ☐ Thus for these areas: $1 = 3 = 6^* = 8^*$, $2 = 4^* = 5 = 7^*$

  only area 1,2 are independent

- ☐ $\tilde{\phi}(z_1, -z_1) = 0$

Quasi Distribution



- Renormalization scheme:
  - Perturbative 0 momentum quasi-DA:

$$\hat{M}_p(z_1, z_2, 0, 0, \mu) = 1 + \frac{\alpha_s C_F}{2\pi}$$

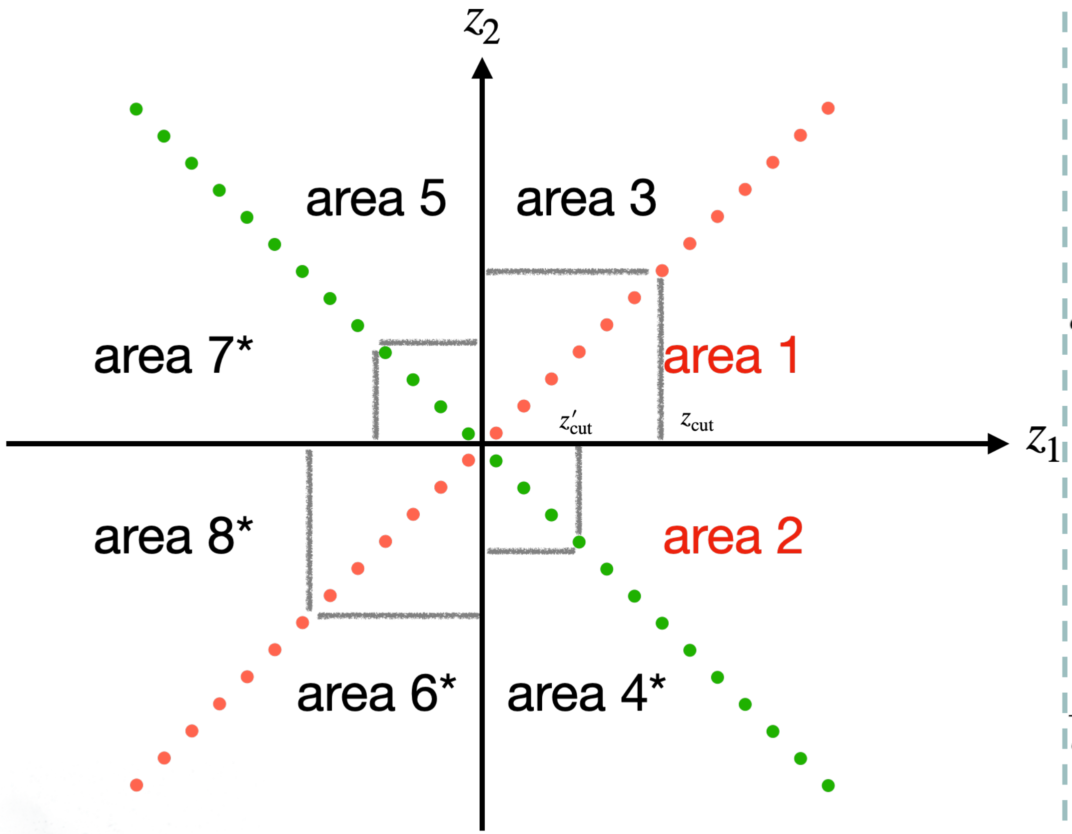
$$\left[\frac{1}{8}\ln\left(\frac{z_1^2\mu^2 e^{2\gamma_E}}{4}\right) + \frac{1}{8}\ln\left(\frac{z_2^2\mu^2 e^{2\gamma_E}}{4}\right) + \frac{1}{4}\ln\left(\frac{(z_1 - z_2)^2\mu^2 e^{2\gamma_E}}{4}\right) + 4\right]$$

1 pole at $z_1 = z_2$

  - Hybrid scheme need a match with the perturbative quasi-DA

At least $a < 0.06fm$ to apply hybrid scheme

Quasi Distribution



➢ Extrapolation & Fourier transformation:

- Asymptotic in momentum space:

$$F(x_1, x_2, d_1, d_2) = C x_1^{d_1} x_2^{d_1} (1 - x_1 - x_2)^{d_2}$$

↓ FT

$$\tilde{\phi}(z_1, z_2; d_1, d_2) = \int_0^1 dx_1 \int_0^1 dx_2\, e^{-ix_1 z_1 P^z} e^{-ix_2 z_2 P^z} C x_1^{d_1} x_2^{d_1} (1 - x_1 - x_2)^{d_2}$$

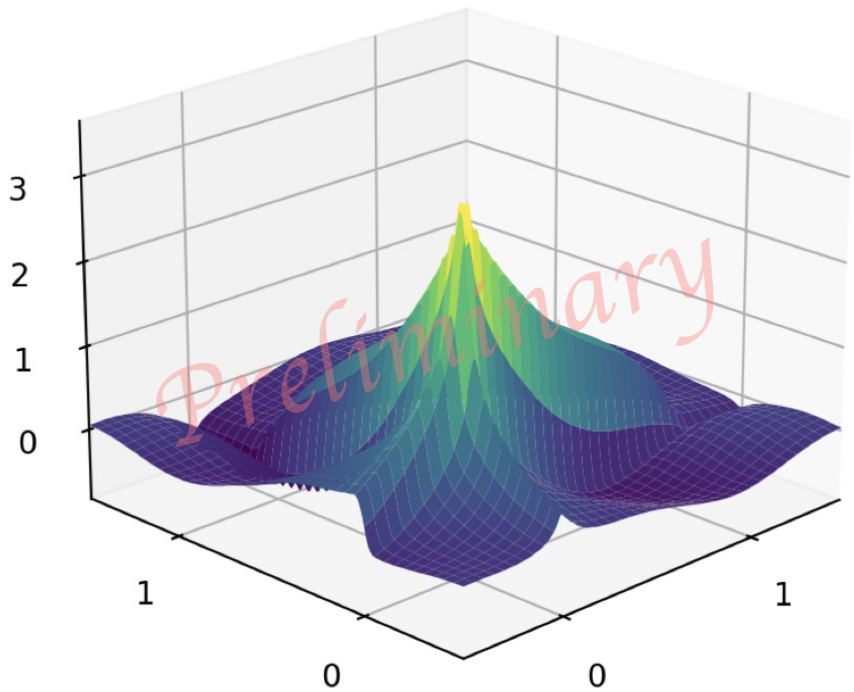
- ▫ Numerically FT as the fit from
  [computing source cost]
- ▫ Analytica FT and simplified as the fit form
  [complicated form for baryon]

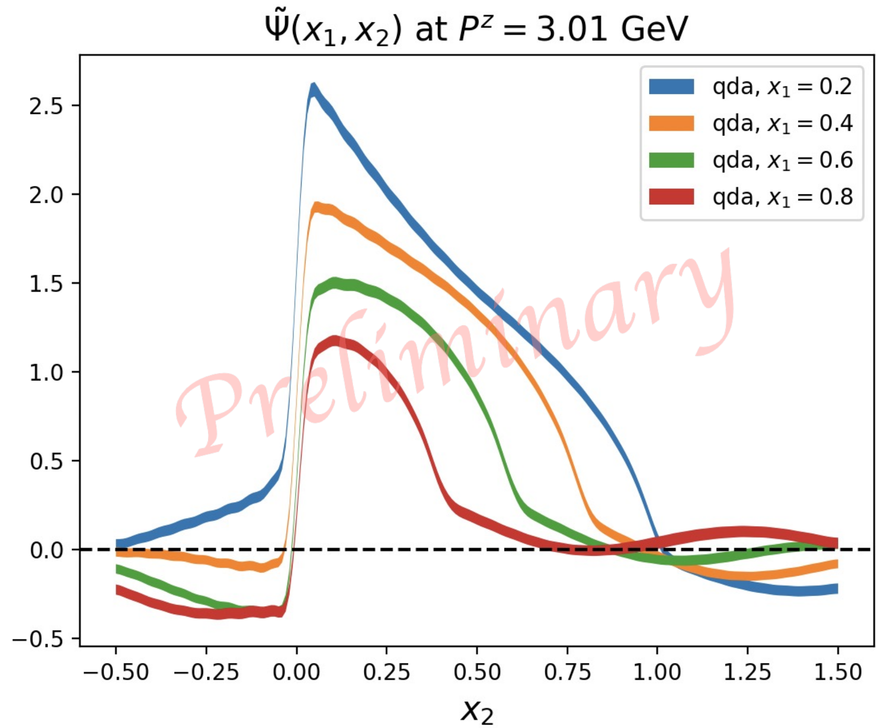
Area2[$z_1 \gg 0, z_2 \gg 0$]:

$$\frac{\Psi(\lambda_1,\lambda_2)}{e^{-\frac{|\lambda_1|}{\lambda_0}-\frac{|\lambda_2|}{\lambda_0}-\frac{|\lambda_1-\lambda_2|}{\lambda_0}}} = c_1\left[\frac{1}{(\lambda_1-\lambda_2)^{d_1}\lambda_1^{d_1}} + \frac{1}{(\lambda_1-\lambda_2)^{d_1}(-\lambda_2)^{d1}}\right] + c_2\frac{\cos\left[\frac{1}{2}(d_1\pi+d_2\pi-2\lambda_1)\right]}{(\lambda_1-\lambda_2)^{d_1}\lambda_1^{d_2}} + c_2\frac{\cos\left[\frac{1}{2}(d_1\pi+d_2\pi+2\lambda_2)\right]}{(\lambda_1-\lambda_2)^{d_1}(-\lambda_2)^{d_2}}$$
$$-i\left(c_2\frac{\sin\left[\frac{1}{2}(d_1\pi+d_2\pi-2\lambda_1)\right]}{(\lambda_1-\lambda_2)^{d_1}\lambda_1^{d_2}} - c_2\frac{\sin\left[\frac{1}{2}(d_1\pi+d_2\pi+2\lambda_2)\right]}{(\lambda_1-\lambda_2)^{d_1}(-\lambda_2)^{d_2}}\right)$$

2-Dimension Matching

- Quasi Distribution in momentum space after FT:





2-Dimension Matching

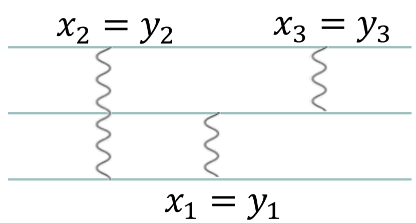
➢ LaMET factorization for Baryon LCDA:

$$\tilde{\phi}(x_1,x_2) = \int_0^1 dy_1 \int_0^{1-y_1} dy_2 C(x_1,x_2,y_1,y_2)\phi(y_1,y_2) + \mathcal{O}\left(\frac{1}{(x_1P^z)^2}, \frac{1}{(x_2P^z)^2}, \frac{1}{[(1-x_1-x_2)P^z]^2}\right)$$

- Matching kernel:

Double plus function

$$\begin{aligned} C(x_1,x_2,y_1,y_2,\mu) &= \delta(x_1-y_1)\,\delta(x_2-y_2) \\ &+\frac{\alpha_s C_F}{2\pi}\left[\left(\frac{1}{4}C_2(x_1,x_2,y_1,y_2) - \frac{7}{8}\frac{-1}{|x1-y1|}\right)\delta(x_2-y_2)\right. \\ &+\left(\frac{1}{4}C_2(x_2,x_1,y_2,y_1) - \frac{7}{8}\frac{-1}{|x2-y2|}\right)\delta(x_1-y_1) \\ &+\left.\left(\frac{1}{4}C_3(x_1,x_2,y_1,y_2) + \frac{1}{4}C_3(x_2,x_1,y_2,y_1) - \frac{3}{4}\frac{-2}{|x_1-y_1-x_2+y_2|}\right)\delta(x_1+x_2-y_1-y_2)\right]_{\oplus}, \end{aligned}$$



C.Han et.al. JHEP 12 044 (2023), JHEP 07 019 (2024)

$$[g(x_1,x_2,y_1,y_2)]_{\oplus} = g(x_1,x_2,y_1,y_2) - \delta(x_1-y_1)\,\delta(x_2-y_2)\int dz_1 dz_2\, g(z_1,z_2,y_1,y_2)$$

2-Dimension Matching

➢ LaMET factorization for Baryon LCDA:

$$\tilde{\phi}(x_1, x_2) = \int_0^1 dy_1 \int_0^{1-y_1} dy_2 C(x_1, x_2, y_1, y_2) \phi(y_1, y_2) + \mathcal{O}\left(\frac{1}{(x_1 P^z)^2}, \frac{1}{(x_2 P^z)^2}, \frac{1}{[(1-x_1-x_2)P^z]^2}\right)$$

- Inverse matching:

  $C(x_1, x_2, y_1, y_2) \rightarrow$ 4 Dimensional tensor $\rightarrow$ Reduce to 2D matrix $\rightarrow$ inverse

- Iterative matching:

$$\tilde{\phi}(x_1, x_2) = \phi(x_1, x_2) + \frac{\alpha_s C_F}{2\pi} \int_0^1 dy_1 \int_0^{1-y_1} dy_2 c^{(1)}(x_1, x_2, y_1, y_2) \phi(y_1, y_2) + \mathcal{O}(\alpha_s^2)$$

↓ The difference between $\tilde{\phi}(x_1, x_2)$ and $\phi(x_1, x_2)$ introduces error at higher order

$$\phi(x_1, x_2) = \tilde{\phi}(x_1, x_2) - \frac{\alpha_s C_F}{2\pi} \int_0^1 dy_1 \int_0^{1-y_1} dy_2 c^{(1)}(x_1, x_2, y_1, y_2) \tilde{\phi}(y_1, y_2) + \mathcal{O}(\alpha_s^2)$$

2-Dimension Matching

- Iterative matching:

$$\phi(x_1,x_2)=\tilde{\phi}(x_1,x_2)-\frac{\alpha_s C_F}{2\pi}\int_0^1 dy_1\int_0^{1-y_1} dy_2 c^{(1)}(x_1,x_2,y_1,y_2)\tilde{\phi}(y_1,y_2)+\mathcal{O}(\alpha_s^2)$$

Kernel:

$$c^{(1)}(x_1,x_2,y_1,y_2)=\left[f(x_1,x_2,y_1,y_2)\right]_{\oplus}$$

$$=f(x_1,x_2,y_1,y_2)-\delta(x_1-y_1)\delta(x_2-y_2)\int_{-\infty}^{\infty}dt_1\int_{-\infty}^{\infty}dt_2 f(t_1,t_2,y_1,y_2)\,.$$

$$\phi(x_1,x_2)=\tilde{\phi}(x_1,x_2)-\frac{\alpha_s C_F}{2\pi}\int dy_1 dy_2\left[f(x_1,x_2,y_1,y_2)\tilde{\phi}(y_1,y_2)-\delta(x_1-y_1)\delta(x_2-y_2)\int dt_1 dt_2 f(t_1,t_2,y_1,y_2)\tilde{\phi}(y_1,y_2)\right]$$

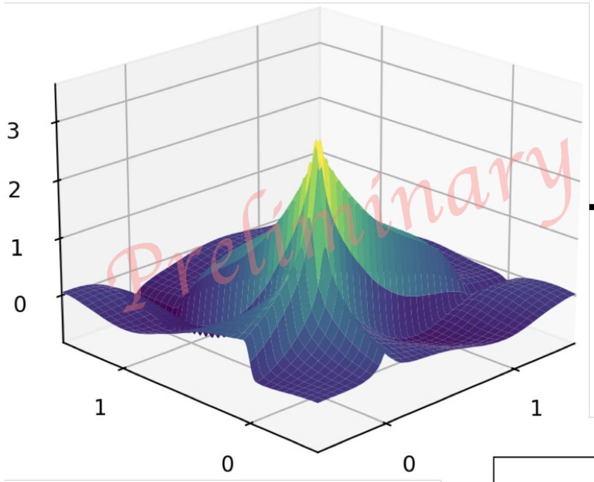
$$=\tilde{\phi}(x_1,x_2)-\frac{\alpha_s C_F}{2\pi}\left[\int dy_1 dy_2 f(x_1,x_2,y_1,y_2)\tilde{\phi}(y_1,y_2)-\int dt_1 dt_2 f(t_1,t_2,x_1,x_2)\tilde{\phi}(x_1,x_2)\right]$$

$$=\tilde{\phi}(x_1,x_2)-\frac{\alpha_s C_F}{2\pi}\left[\int dy_1 dy_2\left[f(x_1,x_2,y_1,y_2)\tilde{\phi}(y_1,y_2)-f(y_1,y_2,x_1,x_2)\tilde{\phi}(x_1,x_2)\right]\right]$$

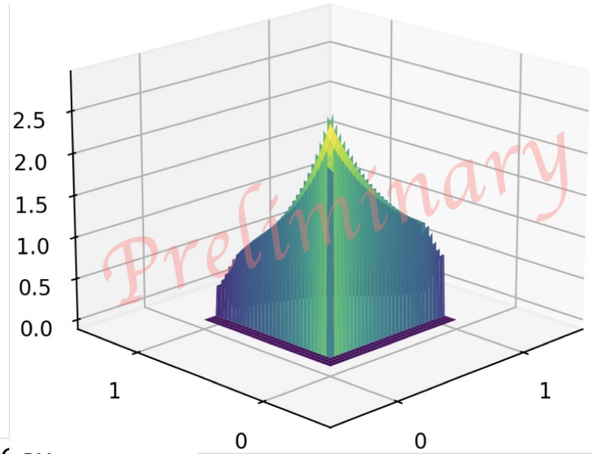
=0 when $x_1,x_2=y_1,y_2$

$x_{1,2}=y_{1,2}$
Poles care canceled

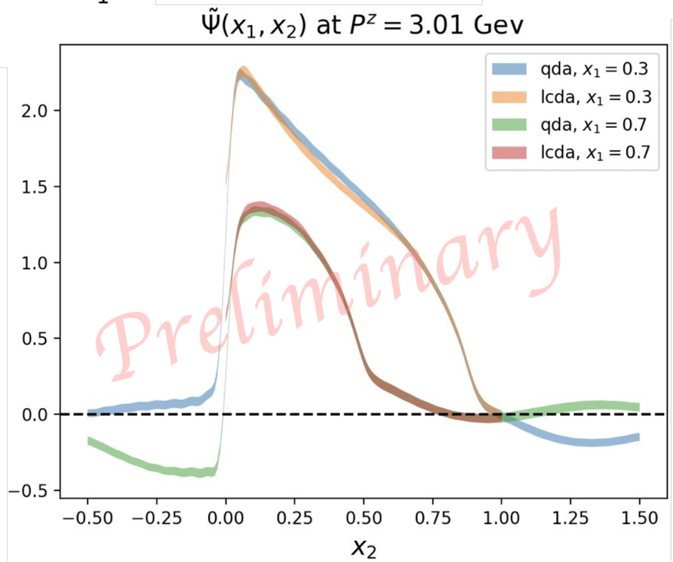
Numerical results



matching



quasi-DA



LCDA

Summary and outlook

- We made the first attempt to implement the numerical computation of baryon LCDA in the LaMET framework.
- The 3-particle distribution cased 3D structure complexity in several parts:
  - Hybrid renormalization (Match with Perturbative Quasi)
  - Extrapolation and Fourier transformation
  - Matching implementation

- [ ] Calculation with smaller lattice spacing (at least < 0.6 fm)

Thanks For Your Attention！