Towards Unpolarized Generalized Parton Distributions from Pseudo-Distributions

Hadstruc collaboration (speaker: Hervé Dutrieux)

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Overview

1. **GPDs: physics case and phenomenological challenges**
2. The Hadstruc GPD calculation on the lattice: **arXiv:2405.10304**
3. Perspectives

Generalized parton distributions (GPDs): [Müller et al, 1994], [Ji, 1996], [Radyushkin, 1996]

$$\frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi} e^{ixP^+z^-} \left\langle P_2 \middle| \bar{\psi}^q\left(-\frac{z}{2}\right)\gamma^+\psi^q\left(\frac{z}{2}\right) \middle| P_1 \right\rangle \Big|_{z_\perp=0,\ z^+=0}$$

$$= \frac{1}{2P^+}\bar{u}(P_2)\left(H^q(x,\xi,t)\gamma^+ + E^q(x,\xi,t)\frac{i\sigma^{+\mu}\Delta_\mu}{2M}\right) u(P_1)$$

$$\Delta = P_2 - P_1\,, \quad t = \Delta^2\,, \quad P = \frac{1}{2}(P_1 + P_2)\,, \quad \xi = \frac{P_1^+ - P_2^+}{P_1^+ + P_2^+} = -\frac{\Delta^+}{2P^+}$$

- 3D (x, ξ, t) vs 1D for PDFs
- more GPDs than PDFs
- $\rightarrow$ need to measure more (exclusive vs inclusive)
- $\rightarrow$ experimentally data-driven approach is so far out of reach

- **Hadron tomography** [Burkardt, 2003]:

$$I(x, \mathbf{b}_\perp) = \int \frac{\mathrm{d}^2\Delta_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp \cdot \Delta_\perp} H^q(x, \xi = 0, t = -\Delta_\perp^2)$$

- **Proton's spin decomposition** [Ji, 1996]:

$$\frac{1}{2} = \sum_q \frac{1}{2} \int_{-1}^{1} dx\, x \left[H^q + E^q \right] \Big|_{t=0} + \frac{1}{2} \int_{-1}^{1} dx \left[H^g + E^g \right] \Big|_{t=0}$$

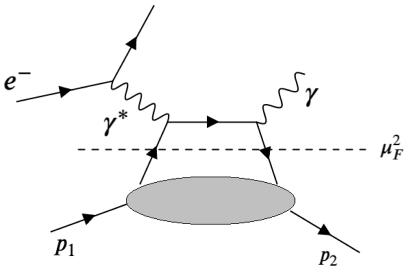
- **Gravitational form factors** [Polyakov, 2003], [Lorcé et al, 2017]: radial energy / pressure

$$\langle P_2 | T_a^{\mu\nu} | P_1 \rangle = \bar{u}(P_2) \Bigg\{ \frac{P^\mu P^\nu}{M} A_a(t) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu}\Delta^2}{M} C_a(t) + M\eta^{\mu\nu} \bar{C}_a(t)$$
$$+ \frac{P^{\{\mu} i\sigma^{\nu\}\rho}\Delta_\rho}{4M} \left[A_a(t) + B_a(t) \right] + \frac{P^{[\mu} i\sigma^{\nu]\rho}\Delta_\rho}{4M} D_a^{GFF}(t) \Bigg\} u(P_1)$$

$$\int_{-1}^{1} \mathrm{d}x\, x\, H^q(x, \xi, t, \mu^2) = A_q(t, \mu^2) + 4\xi^2 C_q(t, \mu^2)$$

DVCS observables parametrized in terms of Compton form factors (CFFs) **[Radyushkin, 1997]**, **[Ji, Osborne, 1998]**, **[Collins, Freund, 1998]**

$$\mathcal{H}(\xi, t, Q^2) = \sum_a \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} T^a \left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2) \right) \frac{H^a(x, \xi, t, \mu^2)}{|x|^{p_a}}$$



- morally "missing" one variable for the x-reconstruction of the GPD from DVCS: **deconvolution problem**
- In a LO analysis,

$$\mathrm{Im}\,\mathcal{H}(\xi, t, Q^2) \longrightarrow H^{q(+)}(x = \xi, t, Q^2)$$

$$\mathrm{Re}\,\mathcal{H}(\xi, t, Q^2) \overset{disp.rel.}{\longrightarrow} \int_{-1}^{1} d\alpha \frac{D^q(\alpha, t, Q^2)}{1 - \alpha}$$

- "fancy" tricks using the entanglement of (x, ξ, μ^2) through evolution equations don't work: **[Bertone, HD, Mezrag, Moutarde, Sznajder, 2021]**

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GPD matrix element **[Bhattacharya et al, 2022]**:

$$\langle P_2|\bar{\psi}\left(-\frac{z}{2}\right)\gamma^\mu\psi\left(\frac{z}{2}\right)|P_1\rangle = \bar{u}(P_2)\Bigg[\gamma^\mu A_1 + z^\mu A_2 + \sigma^{\mu\nu}z_\nu A_3 + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m}A_4 + \frac{\Delta^\mu}{2m}A_5 + \frac{i\sigma^{\alpha\beta}z_\alpha\Delta_\beta}{2m}\left(P^\mu A_6 + \Delta^\mu A_7 + z^\mu A_8\right)\Bigg]u(P_1)$$

Identify the terms that survive in the light-cone limit:

$$H(\nu,\xi,t,z^2) = \lim_{z^2\to 0} A_1 - \xi A_5$$
$$E(\nu,\xi,t,z^2) = \lim_{z^2\to 0} A_4 + \nu A_6 - 2\xi\nu A_7 + \xi A_5$$

Match to the light-cone limit in the short-distance factorization for GPDs **[Radyushkin, 2019]**:

$$\begin{pmatrix} H \\ E \end{pmatrix}(\nu,\xi,t,\mu^2) = \int_{-1}^{1} d\alpha C(\alpha,\xi\nu,\mu^2z^2)\begin{pmatrix} H \\ E \end{pmatrix}(\alpha\nu,\xi,t,z^2) + \text{power corrections}$$

Extracting each amplitude A_k requires to measure matrix elements with various combinations of helicity and gamma structure (kinematic matrix inversion)

Ioffe-time GPDs are great GPDs! [Braun, Gornicki, Mankiewicz, 1995]

x-dependent GPDs

non analytical form

triangular for x > xi (only needs larger momentum fractions)

needs all x range for x < xi

$$\frac{d}{d\ln(\mu^2)} H^q(x,\xi,\mu^2) = \int_0^1 \frac{dy}{y} C\left(\frac{x}{y}, \frac{\xi}{x}, \alpha_s(\mu^2)\right) H^q(y,\xi,\mu^2)$$

$$C\left(\alpha, \frac{\xi}{x}, \alpha_s(\mu^2)\right) = \delta(1-\alpha) + \frac{\alpha_s(\mu^2) C_F}{2\pi}\left\{\theta(1-\alpha)\left[\left(\frac{1+\alpha^2}{1-\alpha}\right)_+ + \mathcal{O}(\xi^2)\right] + \theta(x \leq \xi)\left[-\left(\frac{1}{1-y}\right)_{++} + \ldots\right]\right\}$$

Ioffe-time GPDs

simple analytical form

triangular (only needs smaller Ioffe time range)

$$\frac{d}{d\ln(\mu^2)} H^q(\nu,\xi,\mu^2) = \int_0^1 d\alpha\, K(\alpha, \xi\nu, \alpha_s(\mu^2)) H^q(\alpha\nu,\xi,\mu^2)$$

$$K(\alpha,\xi\nu,\alpha_s(\mu^2)) = \delta(1-\alpha) + \frac{\alpha_s(\mu^2) C_F}{2\pi}\left[\left(\frac{2\alpha}{1-\alpha}\right)_+ \cos(\bar{\alpha}\xi\nu) + \frac{\sin(\bar{\alpha}\xi\nu)}{\xi\nu} - \frac{\delta(1-\alpha)}{2}\right]$$

Conformal moments

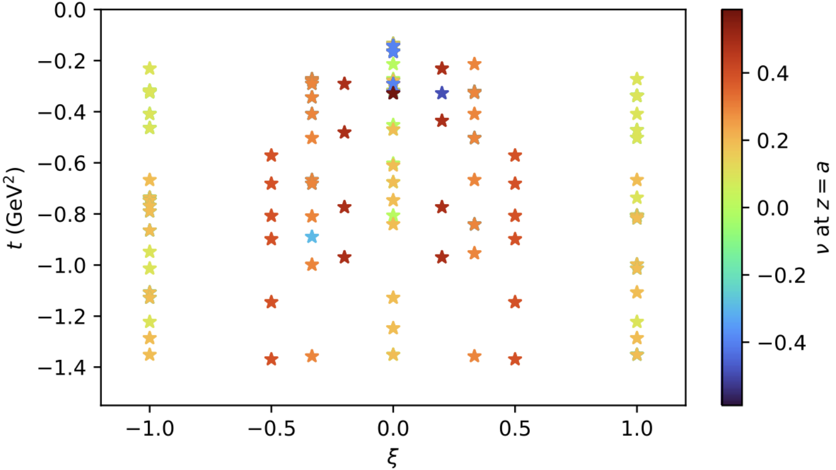
simple analytical form

diagonal (each moment evolves independently)

$$O_n(\xi,\mu^2) \propto \int_{-1}^1 dx\, C_n^{(3/2)}\left(\frac{x}{\xi}\right) H^q(x,\xi,\mu^2)$$

$$O_n(\xi,\mu^2) = O_n(\xi,\mu_0^2)\left(\frac{\alpha_s(\mu^2)}{\alpha_s(\mu_0^2)}\right)^{\gamma_n/(2\pi\beta_0)}$$

ID	a (fm)	m_π (MeV)	β	$m_\pi L$	$L^3 \times N_T$	N_{cfg}	N_{srcs}	rk $(\mathcal{D})$
a094m358	0.094(1)	358(3)	6.3	5.4	$32^3 \times 64$	348	4	64



polynomiality of moments of GPDs:

$$\int_{-1}^{1} dx\, x^{n-1} H(x,\xi,t) = \sum_{k=0 \text{ even}}^{n-1} A_{n,k}(t)\,\xi^k + \mathrm{mod}(n+1,2)\, C_n(t)\xi^n$$

$$A_{1,0}(t) = F_1(t) \quad \text{(elastic form factor)}$$

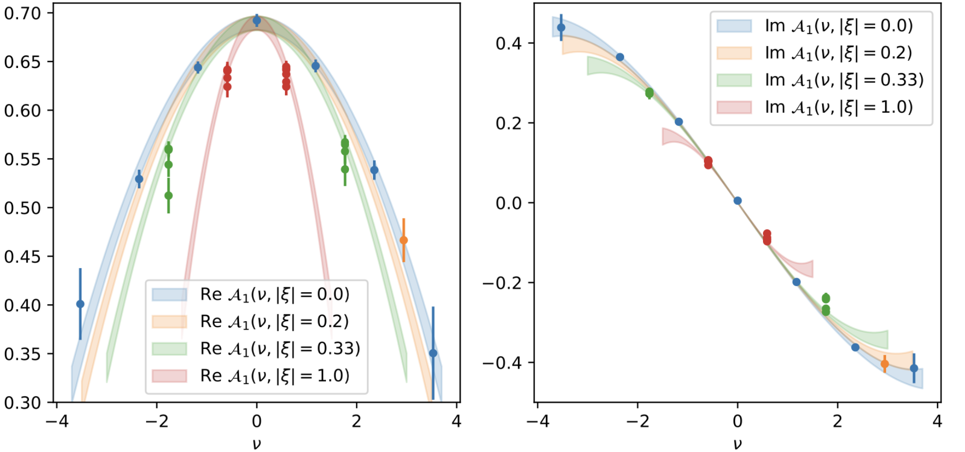
Analysis of D-term can be separated from the rest of the GPD. Hence small Ioffe-time behavior without D-term:

$$\begin{aligned} H(\nu,\xi,t) &= \int dx\, e^{-ix\nu} H(x,\xi,t) \\ &= F_1(t) - i\nu A_{2,0}(t) - \frac{\nu^2}{2}[A_{3,0}(t) + \xi^2 A_{3,2}(t)] + \ldots + \text{power corrections} \end{aligned}$$

With momenta up to 1.4 GeV used in this study, we have signal up to $A_{4,0}$ and $A_{4,2}$.

$$\text{Dipole fit:} \quad A_{n,k}(t) = A_{n,k}(t=0)\left(1 - \frac{t}{\Lambda_{n,k}^2}\right)^{-2}$$

Fixed t and $z = 0.56$ fm, varying $P_{1,2}$ (therefore ν and ξ)



$$H(\nu, \xi, t) = F_1(t) - i\nu A_{2,0}(t) - \frac{\nu^2}{2}[A_{3,0}(t) + \xi^2 A_{3,2}(t)] + ... + \text{power corrections}$$

Pion mass = 0.36 GeV - Proton mass = 1.12 GeV

No continuum limit - signs of discretization errors / light-cone uncertainty

Matching at 2 GeV with leading logarithmic accuracy

Value at t = 0

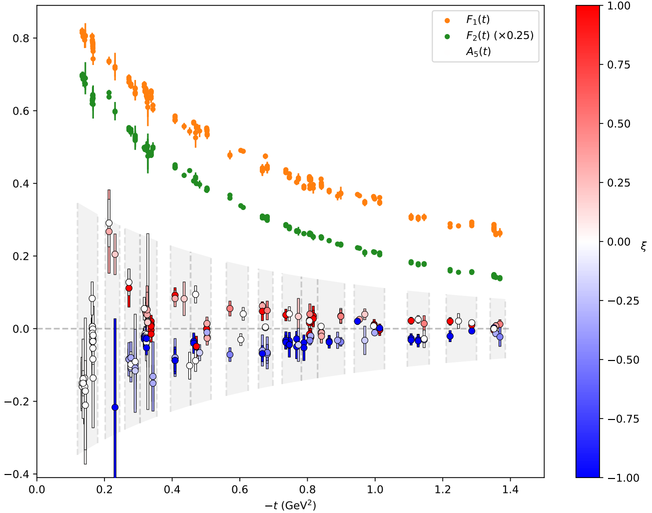
GPD H^{u-d}		GPD E^{u-d}	
$A_{1,0}$: 0.974^{+12}_{-5}		$B_{1,0}$: 3.40^{+7}_{-1}	
$A_{2,0}$: 0.206^{+2}_{-6}		$B_{2,0}$: 0.370^{+9}_{-24}	
$A_{3,0}$: 0.064^{+2}_{-6}	$A_{3,2}$: 0.39^{+11}_{-3}	$B_{3,0}$: 0.063^{+24}_{-8}	$B_{3,2}$: 1.1^{+4}_{-8}
$A_{4,0}$: 0.065^{+5}_{-19}	$A_{4,2}$: 0.5^{+3}_{-3}	$B_{4,0}$: 0.06^{+16}_{-2}	$B_{4,2}$: > 1.1

Dipole mass (GeV)

GPD H^{u-d}		GPD E^{u-d}	
$A_{1,0}$: 1.255^{+3}_{-29}		$B_{1,0}$: 0.987^{+2}_{-6}	
$A_{2,0}$: 1.83^{+9}_{-3}		$B_{2,0}$: 1.39^{+11}_{-5}	
$A_{3,0}$: 2.3^{+2}_{-5}	$A_{3,2}$: 1.10^{+7}_{-11}	$B_{3,0}$: 2.2^{+36}_{-5}	$B_{3,2}$: 0.78^{+77}_{-9}
$A_{4,0}$: > 3.5	$A_{4,2}$: > 0.9	$B_{4,0}$: > 0.6	$B_{4,2}$: 0.5^{+5}_{-2}

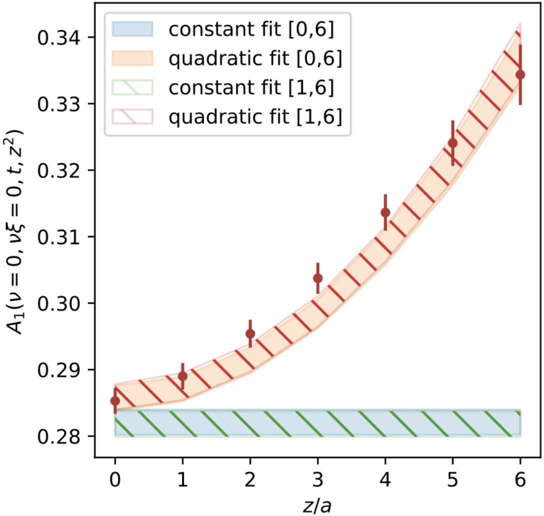
D-term^{u-d}	Value at t = 0	Dipole mass (GeV)
C_2	0.025^{+8}_{-8}	> 2.2

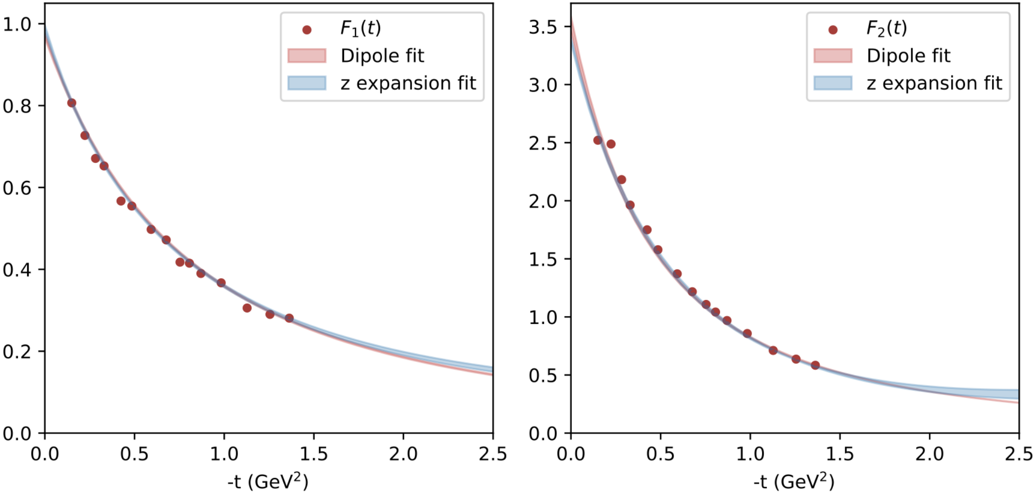
$$\langle p'|\bar{\psi}^q\gamma^\mu\psi^q|p\rangle\Big|_{z=0} = \bar{u}(p')\left[F_1^q(t)\gamma^\mu + F_2^q(t)\frac{i\sigma^{\mu\nu}\Delta_\nu}{2m} + A_5^q(t)\frac{\Delta^\mu}{2m}\right]u(p)$$



probable sign of lattice discretization in A_5 + enhanced sensitivity to excited state contamination

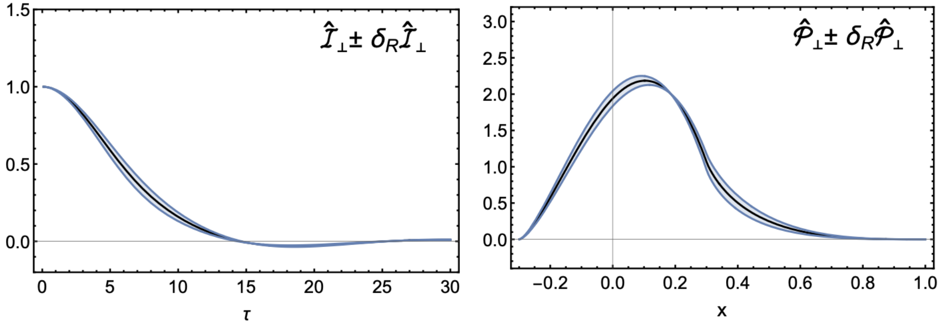
If $p_{f,z} = p_{i,z} = 0$, then $\nu = 0$ and $\nu\xi = 0$, so we have non-local data with signal only of the EFF + whatever parasitic contribution. But are these **power corrections (higher twists)** or **lattice discretization errors**? $A_5(z=0)$ has quite certainly discretization errors.





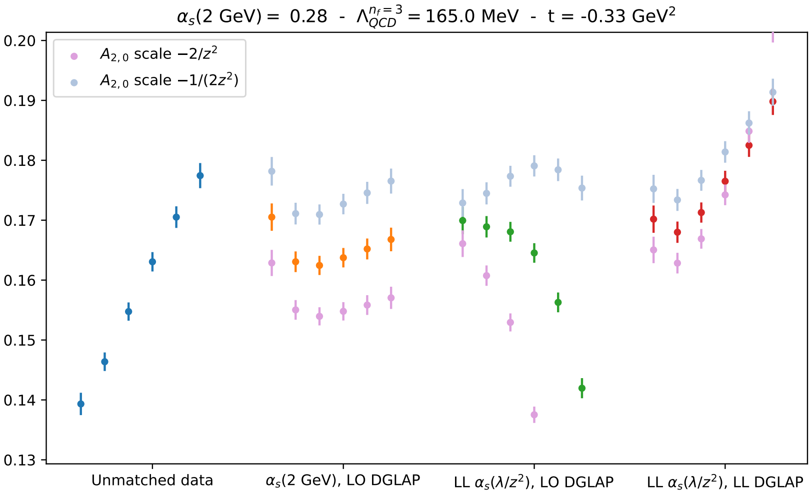
$F_1(t)$
Dipole fit
z expansion fit
-t (GeV^2)
$F_2(t)$
Dipole fit
z expansion fit
-t (GeV^2)

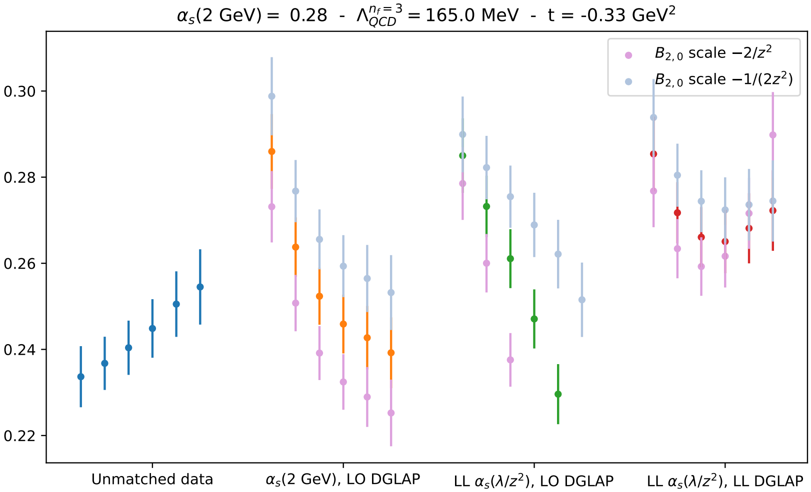
Even with a space-like separation of 1 fm, the power-corrections at $\xi = 0.3$ might be very small (model-dependent estimate of non-perturbative higher-twist contributions through renormalon ambiguity) **[Braun, Koller, Schoenleber, 2024]**:

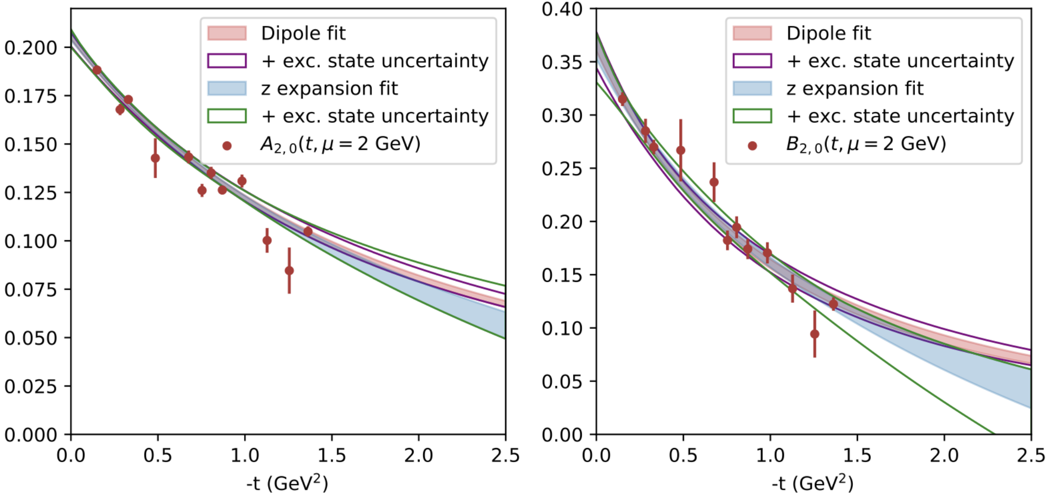


Possible interpretation: higher-twist contribution largely independent on the external momentum, and suppressed by the ratio method

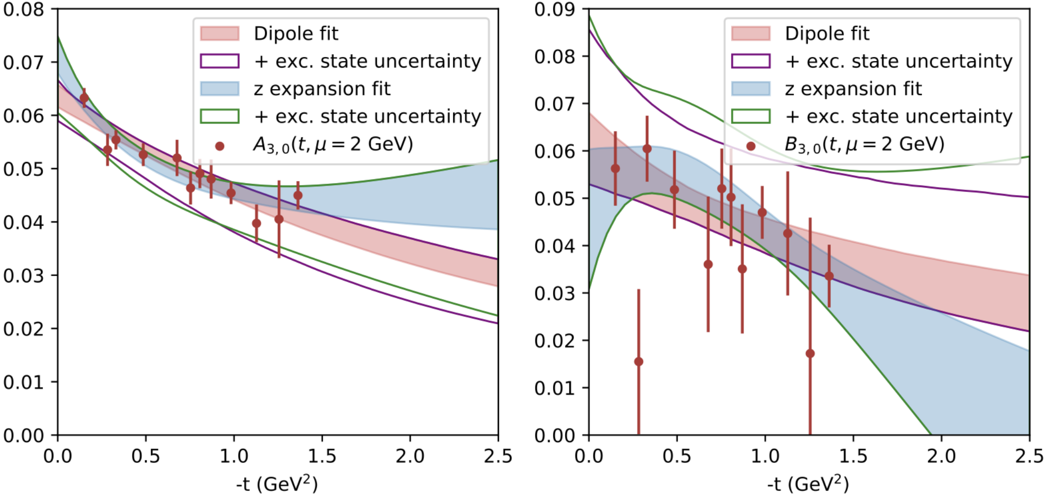
Perturbative matching uncertainty: if there is a strong leading-twist dominance up to separations of 1 fm, what is the perturbative matching kernel worth in this region? But only truly relevant for a few bins of the GFF



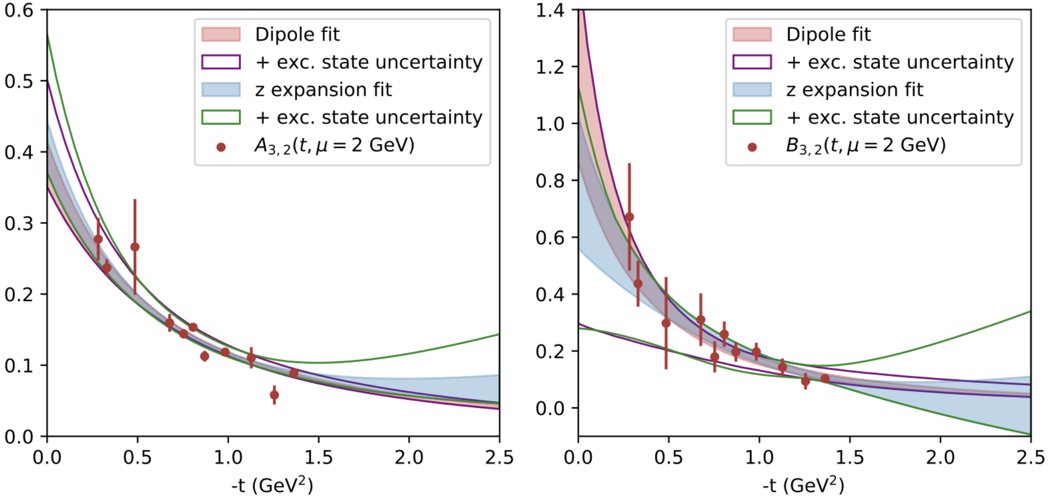

α_s(2 GeV) = 0.28 - $\Lambda_{QCD}^{n_f=3}$ = 165.0 MeV - t = -0.33 GeV^2
$B_{2,0}$ scale $-2/z^2$
$B_{2,0}$ scale $-1/(2z^2)$
0.30
0.28
0.26
0.24
0.22
Unmatched data
α_s(2 GeV), LO DGLAP
LL $\alpha_s(\lambda/z^2)$, LO DGLAP
LL $\alpha_s(\lambda/z^2)$, LL DGLAP



Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$A_{2,0}(t, \mu = 2$ GeV)
-t (GeV^2)
Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$B_{2,0}(t, \mu = 2$ GeV)
-t (GeV^2)



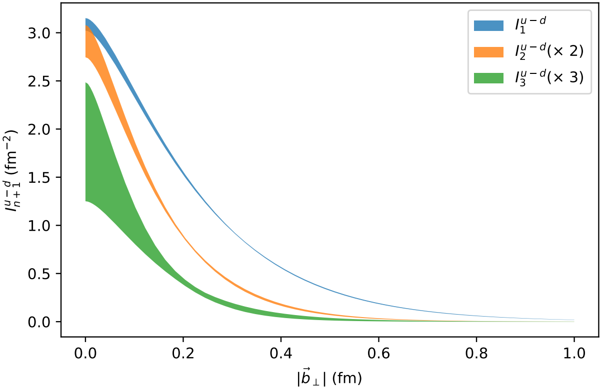
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$A_{3,0}(t, \mu = 2\ \text{GeV})$
0.0
0.5
1.0
1.5
2.0
2.5
-t (GeV^2)
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$B_{3,0}(t, \mu = 2\ \text{GeV})$
0.0
0.5
1.0
1.5
2.0
2.5
-t (GeV^2)



Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$A_{3,2}(t, \mu = 2\text{ GeV})$
$B_{3,2}(t, \mu = 2\text{ GeV})$
-t (GeV^2)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.8
1.0
1.2
1.4
1.5
2.0
2.5

Impact parameter distribution: number density of unpolarized quarks in an unpolarized proton with mom. fraction x and radial distance to the center of longitudinal momentum $\vec{b}_\perp$: **model dependence!**

$$I(x, \vec{b}_\perp) = \int \frac{\mathrm{d}^2\vec{\Delta}_\perp}{(2\pi)^2} e^{-i\vec{b}_\perp \cdot \vec{\Delta}_\perp} H(x, \xi = 0, t = -\vec{\Delta}_\perp^2)$$



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Perspectives on the lattice

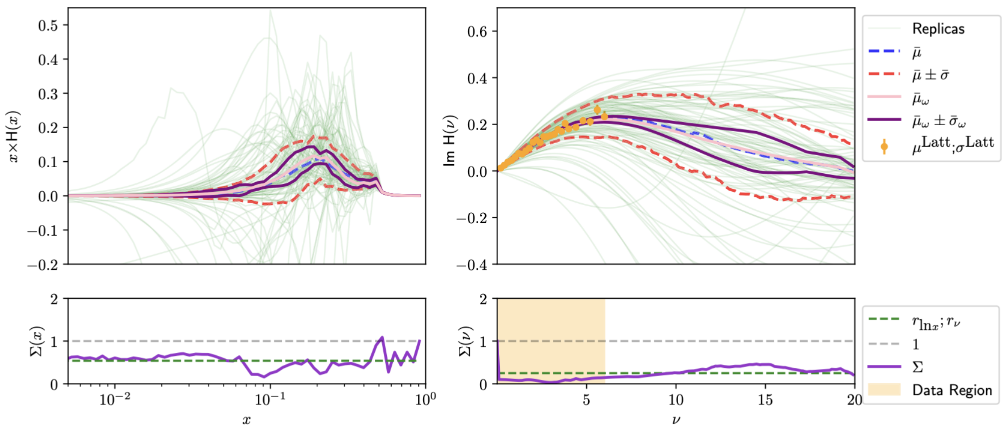
Uncertainties in lattice QCD:

- **Lattice discretization / power corrections**: evidence of issues in the EFFs, needs a continuum limit
- **Excited state / range in Ioffe time**: need a better assurance (GEVP) in order to produce reliable x-reconstruction / high-order moments using large momentum boost
- **Matching uncertainty / power corrections**: proposal to identify a regime of validity of factorization without the use of perturbation theory **[HD, Karpie, Monahan, Orginos, Zafeiropoulos, 2023]** Cf. Joe Karpie's talk earlier today
- **Pion mass / finite volume effects**

A joint lattice - experimental phenomenology

Modest example: using a model whose uncertainty is purely theoretical deconvolution uncertainty (not experimental) and pseudo-lattice data **[Riberdy, HD, Mezrag, Sznajder, 2023]**

$\xi = 0.5$

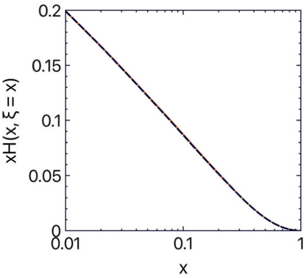


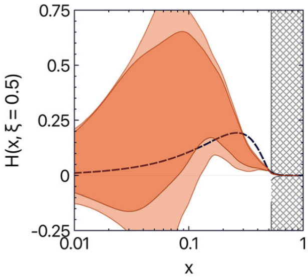
Conclusion

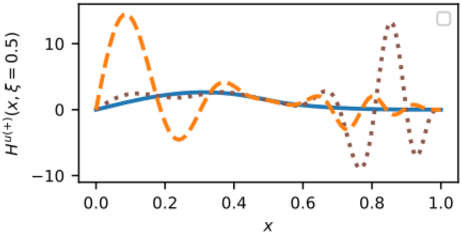
- We have a framework with excellent signal and very large kinematic coverage for GPDs
- There is still a lot of work on purely lattice systematics (but that's ok), power corrections and matching uncertainty is a bigger concern that we start to address
- We have quite a few innovative ideas to explore beyond the dumb powering through the various limits, stay tuned!

Thank you for your attention!

(caricatural) case of uncertainty propagation at NLO: needs to measure DVCS over a range from 1 to 100 GeV^2 with 10^{-5} relative accuracy to discriminate those GPDs.



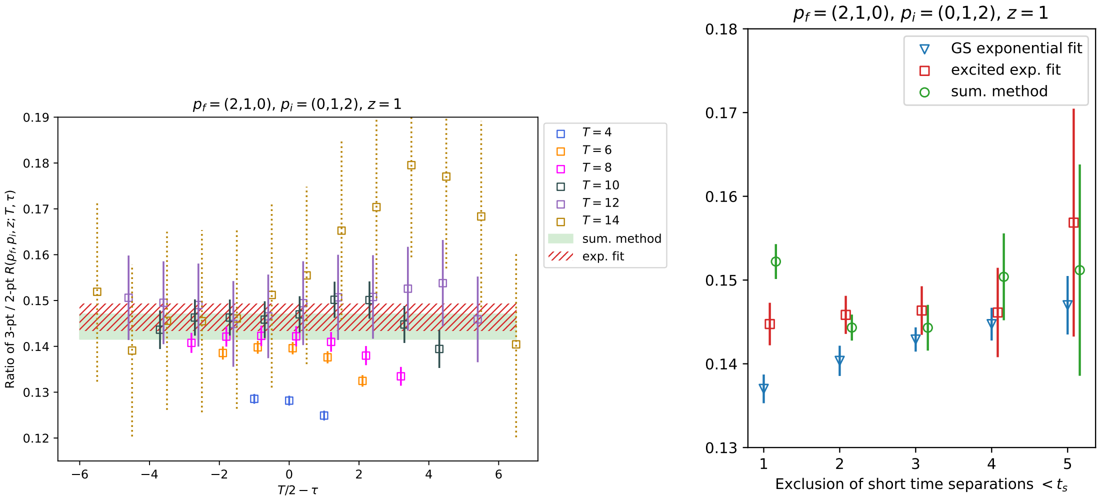


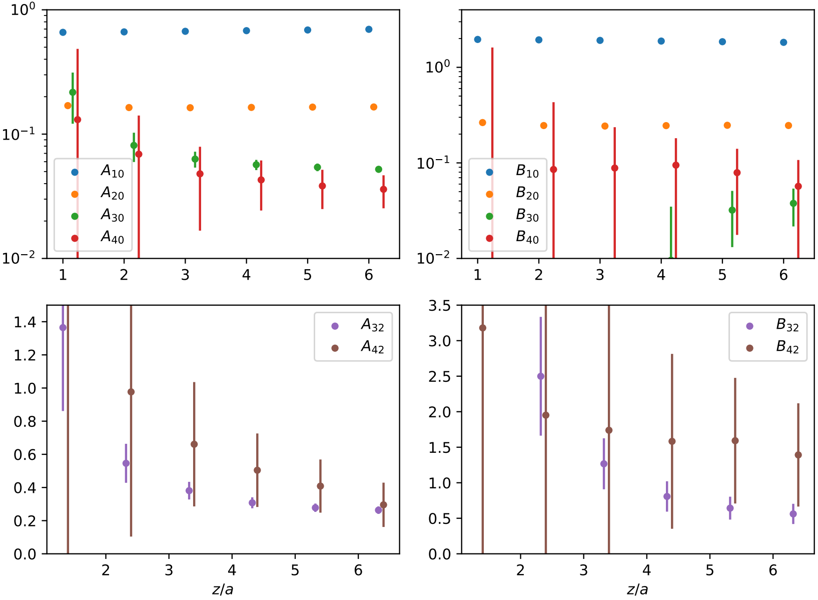


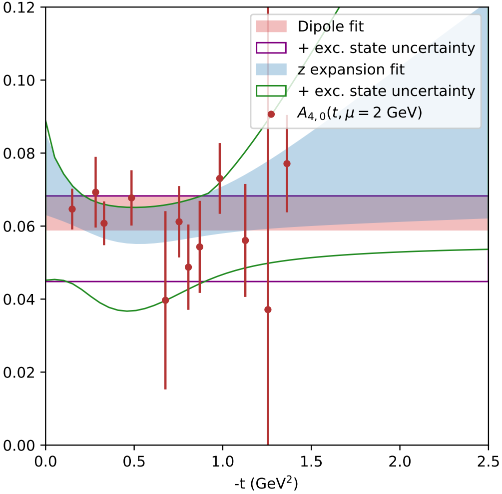
Positivity **[Pire, Soffer, Teryaev, 1998]** for $|x| > |\xi|$ with use of NN **[HD, Grocholski, Moutarde, Sznajder, 2021]**:

- NN model of DD (Lorentz sym)
- Fit DVCS observables produced by a model (no experimental uncertainty)
- Vary the architecture of the NN to probe the functional space

Excited state contamination




A_{10}
A_{20}
A_{30}
A_{40}
B_{10}
B_{20}
B_{30}
B_{40}
A_{32}
A_{42}
B_{32}
B_{42}
z/a
z/a



0.12
0.10
0.08
0.06
0.04
0.02
0.00
0.0
0.5
1.0
1.5
2.0
2.5
-t (GeV^2)
Dipole fit
+ exc. state uncertainty
z expansion fit
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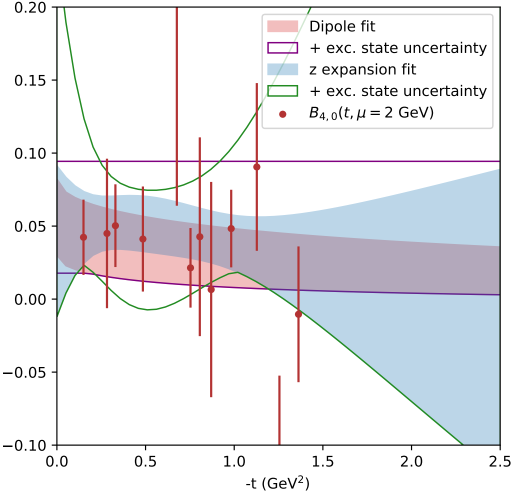
$A_{4,0}(t, \mu = 2$ GeV)



0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
0.0
0.5
1.0
1.5
2.0
2.5
-t (GeV^2)
Dipole fit
+ exc. state uncertainty
z expansion fit
+ exc. state uncertainty
$B_{4,0}(t, \mu = 2$ GeV)